# The Geometers of God

## Mathematics and Reaction in the Kingdom of Naples

***By Massimo Mazzotti****

**ABSTRACT**

The controversy about whether analytic or synthetic methods should be employed to solve geometrical problems was particularly lively in Naples during the first half of the nineteenth century. I investigate the origin of this controversy, arguing that, far from being based on a mere technical divergence, it was deeply rooted in the cultural and social milieu of the protagonists. I also show that the Neapolitan debate cannot be reduced to an opposition between "traditional" and "modern" mathematicians. The establishment and institutionalization of the "ancient" synthetic school was in fact accomplished in response to recent developments in mathematics, by practitioners who were themselves competent in the algebraic method. The proposed interpretation of the controversy links choices about problem-solving methods to more general orientations regarding the cultural and social changes that shattered the kingdom in the age of the French Revolution.

*Among the sciences, the mathematical ones are those which have taken the more false and disastrous direction. They were the first to be included in the assault of the philosophers against Christianity. . . ; they have become deadly weapons in the hands of impiety and pride; they have broken every restraint; they have unchained all the passions; they have eroded the foundations of society and order.*
*—Giacchino Ventura (1824)*

IN THE LAST TWENTY YEARS, some scholars in the history of mathematics have paid particular attention to the relation between the production and transmission of mathematical knowledge and the wider cultural and social context. The epistemological isolation of mathematics characteristic of much traditional historiography has been brought

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into question by reference to the causal significance of the conditions in which mathematicians make decisions about the foundations, boundaries, and practice of their discipline. The study of how contingent and local factors contribute to mathematical knowledge becomes crucial if we want to argue that mathematics is similar to other kinds of human knowledge. Indeed, specific historical investigations undermine the image of a necessary development of mathematical knowledge along a predetermined path.[1]

This essay is a contribution to the program of "contextualization" of mathematical knowledge. I shall describe the problem-solving methods of two opposed schools of mathematicians. Then I shall maintain that, although they shared a common heritage of mathematical and philosophical traditions, the two groups of practitioners made very different uses of them because of their contending social interests. This led members of the schools to hold two different "images" of mathematics and, consequently, to practice mathematics in two different ways.[2]

## THE NEAPOLITAN CONTROVERSY

The summer of 1839 saw what was probably the last mathematical *disfida* to take place in Italy. It was not uncommon, during the Renaissance, to organize public contests between mathematicians. Originally, these *disfide* had either the form of *cartelli*—that is, exchanges of letters containing problems to solve—or of *duelli*—public "duels" where problems were solved before an audience. This glorious tradition came to an end with an obscure episode in one of the most conservative European states of the period, the Kingdom of Naples.[3] The 1839 contest was also an interesting episode in the debate between the two schools that dominated mathematics in Naples: the "synthetic" and the "analytic."

The synthetic school had grown around the charismatic figure of Nicola Fergola (1752–1824) (see Figure 1); its members held mathematical chairs at the Royal University of Naples and controlled the mathematical activity of the Royal Academy of Sciences. The analytic school, not so well institutionalized, was made up of teachers from the Scuola di Applicazione del Corpo di Ingegneri di Ponti e Strade, an institute of higher education designed to train civil engineers, on the model of the French École des Ponts et Chaussées.

[1] In the social history of mathematics, the pioneering work is Henk J. M. Bos and Herbert Mehrtens, "The Interactions of Mathematics and Society in History: Some Exploratory Remarks," *Historia Mathematica*, 1977, *4*:7–30. See also the essays in Mehrtens, Bos, and Ivo Schneider, eds., *Social History of Nineteenth-Century Mathematics* (Boston: Birkhäuser, 1981); Donald MacKenzie, *Statistics in Britain, 1865–1930: The Social Construction of Scientific Knowledge* (Edinburgh: Edinburgh Univ. Press, 1981); Joan Richards, *Mathematical Visions: The Pursuit of Geometry in Victorian England* (San Diego, Calif.: Academic, 1988); Steven Shapin, "Robert Boyle and Mathematics: Reality, Representation, and Experimental Practice," *Science in Context*, 1988, *2*:23–58; Mario Biagioli, "The Social Status of Italian Mathematicians (1450–1600)," *History of Science*, 1989, *27*:41–69; and Mehrtens, *Moderne—Sprache—Mathematik: Eine Geschichte des Streits um die Grundlagen der Disziplin und des Subjeckts formaler Systeme* (Frankfurt am Main: Suhrkamp, 1990). For an overview of both the social history of mathematics and recent empiricist trends in philosophy of mathematics see Richards, "The History of Mathematics and *L'esprit humain*: A Critical Reappraisal," *Osiris*, N.S., 1995, *10*:122–135. For the epigraph see Gioacchino Ventura, "Elogio di Nicola Fergola" (1824), in *Raccolta di elogi funebri e lettere necrologiche* (Genoa: G. D. Rossi, 1852), p. 84. Here and throughout the essay, translations are mine unless otherwise indicated.

[2] An "image" of mathematical knowledge is the set of fundamental beliefs, metaphysical and philosophical, that inform the practice of a community of mathematicians.

[3] The official denomination of the kingdom, after the Bourbon Restoration (1815), was "Kingdom of the Two Sicilies," because it included both the former Kingdom of Naples and the former Kingdom of Sicily. I will employ the denomination "Kingdom of Naples" even when referring to the age of the Restoration. For an overview of mathematics in the Italian states in the nineteenth century see Umberto Bottazzini, *Va' Pensiero: Immagini della matematica nell'Italia dell'ottocento* (Bologna: Il Mulino, 1994).

***Figure 1.*** *Nicola Fergola. Engraving from a posthumous edition of his* Teorica de' miracoli (*Naples: Flauti, 1843), facing page xix.*

The work and teaching of the synthetic school centered on pure geometry, treated according to the synthetic methods of classical mathematics, and on historical reconstructions of the works of the ancient geometers. The analytics favored the use of analysis in geometry, and their interest was focused on the application of calculus to empirical problems.[4]

[4] Among the students and followers of Fergola were Felice Giannattasio (1759–1849), Giuseppe Scorza (1781–1843), Giuseppe Sangro (ca. 1775–ca. 1835), Vincenzo Flauti (1782–1863), Francesco Bruno (?–1862), and Ferdinando De Luca (1783–1869). Among the more active members of the analytic school were Ottavio Colecchi (1773–1847), Salvatore De Angelis (1789–1850), Francesco Paolo Tucci (1790–1875), and Fortunato Padula (1815–1881).

The hostilities erupted in 1810, following critical remarks on Fergola's work by Ottavio Colecchi (1773–1847), a teacher of calculus at the Scuola di Applicazione, and continued through the second and third decades of the century.[5] Colecchi and his colleagues at the Scuola di Applicazione criticized Fergola for the preeminence he accorded to pure geometry and accused him of being dismissive of the advances made by modern analysis. For their part, the members of Fergola's school argued for the intellectual superiority of their own geometrical research; they saw contemporary analysis as dangerously contaminated by empirical considerations and in need of more rigorous foundations. What is puzzling, from our point of view, is the radical nature of this debate, which even extended to the sphere of morals. The synthetics accused the analytics of "moral depravity" and of holding an "antiscientific attitude." Their moral emphasis pointed to the allegedly "corrupting" effect of analysis on the minds of young students. Indeed, according to the synthetics, the "analytic turn" of mathematics in the eighteenth century was responsible not only for the general decay of mathematics but also for the degeneration of morals, the social order, and religious orthodoxy.

The Neapolitan controversy is an underexplored episode in the history of Italian science. The process of erasure that has taken place with regard to the names and works of the members of the synthetic school is, in itself, a noteworthy phenomenon. The very existence of a Neapolitan synthetic school—let alone its competition with the analytics—was unknown to the historian Gino Loria around 1890. Loria embarked on his research only because he found a reference to the school in an 1835 history of geometry by Michel Chasles; he eventually published an essay that remains the only complete survey of the work of the synthetic school.[6]

Historians have provided resources that could help explain the Neapolitan controversy and similar debates between supporters of geometrical and algebraic methods in other countries. It has been maintained, for example, that some mathematicians are naturally "synthetic minded," whereas others are "analytic minded." But the explanatory power of such an argument is comparable to that of the *virtus soporiphera* of opium. More interestingly, such debates have been presented as struggles between different generations of mathematicians. In the Neapolitan case, then, we find on the one side the members of the synthetic school, blindly devoted to the defense of pure geometry in the age of Lagrange and Cauchy; on the other side are the analytic challengers, the young "outsiders" well informed about recent developments in mathematics in the other Italian states and in

[5] Ottavio Colecchi, "Riflessioni sopra alcuni opuscoli che trattano della funzioni fratte e del loro risolvimento in funzioni parziali," *Memorie della Biblioteca Analitica*, 1810, *2*:249–269, 329–376.

[6] "Le goût de cette Géométrie, qui a donné tout d'éclat aux sciences mathématiques jusques il y a près d'un siècle, surtout dans la patrie de Newton, s'est affaibli depuis, et aurait presque disparu, si les géomètres italiens ne lui fussent restés fidèles. On doit, de nos jours, au célèbre Fergola, et à ses disciples, MM. Bruno, Flauti, Scorza, plusieurs écrits importants sur l'Analyse géométrique des Anciens, qui s'y trouve rétablie dans sa pureté originarire": Michel Chasles, *Aperçu historique sur l'origine et le développement des méthodes en géométrie* (1835; Paris: Gauthier-Villars, 1875), p. 46. For Loria's survey see Gino Loria, *Nicola Fergola e la scuola che lo ebbe a duce* (Genoa: Tipografia del Regio Istituto Sordo-Muti, 1892). Information on the debate between the synthetic and analytic schools in Naples can be found in Federico Amodeo, *Vita matematica napoletana*, 2 vols., Vol. 1 (Naples: Giannini, 1905), Vol. 2 (Naples: Tipografia dell'Accademia Pontaniana, 1924). See also Massimo Galluzzi, "Le scuole del Fergola e del Ruffini," in *Storia d'Italia*, Suppl. 3: *Scienza e tecnica nella cultura e nella società dal Rinascimento a oggi*, ed. Gianni Micheli (Turin: Einaudi, 1980), pp. 1008–1019. Recently, the early work of Fergola has been studied in Giovanni Ferraro and Franco Palladino, "Sui manoscritti di Nicolò Fergola," *Bollettino di Storia delle Scienze Matematiche*, 1993, *13*(2):147–197; and Ferraro and Palladino, *Il calcolo sublime di Eulero e Lagrange eposto col metodo sintetico nel progetto di Nicolò Fergola* (Naples: La Città del Sole, 1995).

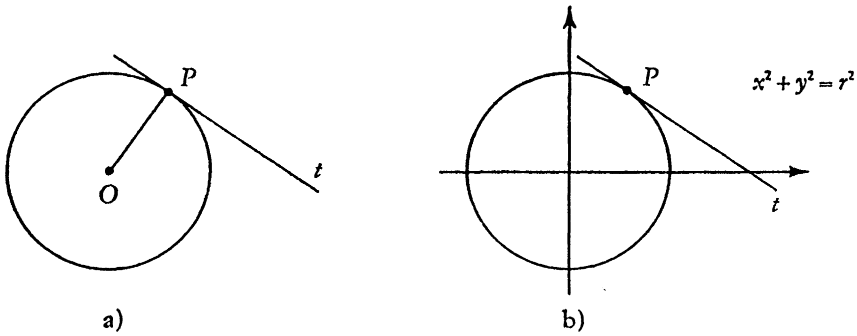


***Figure 2.*** *Two solutions to a problem.*

France. I shall return to the limits of such an interpretation; first, let me try to clarify the meaning of the terms "analysis" and "synthesis" in this context.[7]

The focal question of the Neapolitan controversy was, What is the "proper" method for solving geometrical problems? "Proper," here, refers to didactic and research purposes. Consider the solutions provided by synthetic and analytic textbooks to a simple problem from the theory of conic sections. It calls for the tangent to be traced to a given circle $C$ at a given point $P$. One way to solve this problem is to trace the radius $OP$ and then trace the line $t$ passing through this point and perpendicular to the radius $OP$ (see Figure 2*a*). Alternatively, we can choose an appropriate system of coordinates (see Figure 2*b*); with respect to these coordinates, the equation of the curve will be

$$x^2 + y^2 = r^2,$$

and the point $P$ will be individuated by the pair $(a,b)$. Now we only have simultaneously to solve this equation and the general equation of a straight line passing through the point $P$,

$$y - b = m(x - a),$$

to calculate the gradient $m$ of the tangent line $t$. The solutions of this system are the points of intersection between the straight line $t$ and the circle. In the case of the tangent, the two points of intersection coincide. If we then solve the system, we find that the equation of the tangent line is

$$ax + by = r^2.$$

In the first method of solving the problem, we have utilized a specific feature of the circle—namely, the fact that the tangent to a circle at a point $P$ is always perpendicular to the radius $OP$. If we were to consider the case of a different kind of curve—a parabola, for example, or an ellipse—we should have to employ some (different) characteristic feature of the curve in question to solve the same kind of problem. For example, to find the tangent at a certain point $P$ in the case of the ellipse, we should consider the following

[7] On the notions of "analysis" and "synthesis" in mathematics see Michel Otte and Marco Panza, eds., *Analysis and Synthesis in Mathematics: History and Philosophy* (Dordrecht: Kluwer, 1997). For the explanation based on "mindedness" see Loria, *Nicola Fergola*; for the generational explanation see Galluzzi, "Scuole del Fergola e del Ruffini."

property: the tangent at $P$ is the bisector of the angle between the straight lines connecting the two foci to $P$.

The second problem-solving method is a very different one. The procedure is unchanged whether we are considering the tangent to a parabola or to an ellipse. In this sense it is more general and easier to employ. Nevertheless, it should be noted that when we use this second method information about the specific properties of the curve being considered is not only unnecessary but much more difficult to establish. So the fact that the tangent in $P$ is perpendicular to $OP$ can be stated only if we recognize the validity of the algebraic equation

$$b/a\ (-a/b) = -1,$$

where $b/a$ is the gradient of the radius $OP$ and $-a/b$ is the gradient of the line $t$. While the second method is completely adequate for solving the problem, it fails to provide information that is immediately evident when using the first method.

The first method employed in this simple example was held by the synthetic school to be the proper method for solving a geometrical problem; it included both the classical procedures of geometrical analysis and geometrical synthesis. Following traditional usage, its proponents referred to it as the "synthetic method." The second method, an approach to geometry developed since the seventeenth century and based on the use of algebraic algorithms, was recommended by teachers who were members of the analytic school; they referred to it as "two- or three-coordinate geometry" or as the "analytic method." Following the eighteenth-century usage of the term "analysis," by "analytic methods in geometry" they meant the employment of algebra and calculus in order to solve geometrical problems.[8]

The two methods can be characterized in different ways, depending on what criterion is given the most weight. Neapolitan mathematicians from both schools seemed to take the degree of generality as the main discriminating criterion. On this basis, they characterized the two methods as follows:

The synthetic method is *specific:* every problem to be solved calls for a different construction; thus the geometer requires skill, knowledge, and experience, which can be gained only through long training. This method relies on the intuition of the geometer, who must choose the construction most suitable for a particular problem. Although demanding, the synthetic method was held by its supporters to be the only natural and sound method of reasoning in mathematics; they presented analysis as an artificial tool whose operations are not only epistemologically subordinate to geometry but also much more complex and counterintuitive.

The analytic method is *general:* every problem can be put in the form of an equation,

[8] On methods in geometry see Julian L. Coolidge, *A History of Geometrical Methods* (New York: Dover, 1963); Wilbur Knorr, *The Ancient Tradition of Geometric Problems* (Boston: Birkhäuser, 1986); and Carl Boyer, *History of Analytic Geometry* (New York: Scripta Mathematica, 1956). On the history of analysis in the eighteenth and nineteenth centuries see Ivor Grattan-Guinness, *The Development of the Foundations of Mathematical Analysis from Euler to Riemann* (Cambridge, Mass.: MIT Press, 1970); and Umberto Bottazzini, *The Higher Calculus: A History of Real and Complex Analysis from Euler to Weierstrass* (New York: Springer, 1986). On the eighteenth century see also Henk Bos, "Calculus in the Eighteenth Century: The Role of Applications," *Bulletin of the Institute of Mathematics and Its Applications*, 1977, *13*:221–227; and Craig Fraser, "The Calculus as Algebraic Analysis: Some Observations on Mathematical Analysis in the Eighteenth Century," *Archive for History of Exact Sciences*, 1989, *39*:317–336. A more philosophical investigation of analysis is Jaako Hintikka and Unto Remes, *The Method of Analysis: Its Geometrical Origins and Its General Significance* (Boston: Reidel, 1974).

and then solved, by following the same steps. Through solving the "equation of the problem" we obtain, mechanically, *all* the possible solutions. Nothing is left to the intuition of the geometer. The analytics stressed that this method is "easy," "mechanical," and easily learned. The fact that it often required great ingenuity to formulate and solve the equation was not considered an essential problem. The point was to show that an extension of the "empire" of analysis to the field of geometry is possible, and indeed desirable, because it would render the problem-solving procedure entirely mechanical.

In fact, in the writings of the Neapolitan mathematicians we find *three* different problem-solving methods: the synthetic method ("ancient"), the mixed method ("Cartesian"), and the purely algebraic method ("Lagrangian"). The synthetic school considered use of the first two methods legitimate, whereas the analytic school accepted only the third. The analytics regarded "il signor de la Grange"—Joseph-Louis Lagrange (1736–1813)—as the founder of their method. His work on triangular pyramids (1773) and his well-known *Mécanique analytique* (1788) had a great influence on both the topics and the methods of their research.[9] The synthetics accepted the Cartesian method because, although it employs algebraic algorithms, the connection with the geometrical figures is never forgotten. The analytic tools play a subsidiary, economic role and could, in principle, be eliminated. The synthetics did not reject the use of analytic tools per se. Their concerns were foundational: they wanted their reasoning always to be geometrical, that is, relative to geometrical entities and founded on geometrical truths. Geometrical intuition cannot, according to the synthetics, be entirely replaced by a mechanical procedure.

## MATHEMATICIANS AT WAR

In the contest of 1839, the glove was thrown down by the leader of the synthetic school, Vincenzo Flauti (1782–1862). A former pupil of Fergola, he was at the time of the contest a powerful professor at the Royal University of Naples and the perpetual secretary of the Royal Academy of Sciences. In April 1839 Flauti presented three geometrical problems for solution. His intention was to prove the superiority of the synthetic method over the analytic; indeed, the problems had been carefully chosen with that goal in mind. The first one called for the geometrical construction of an equation to solve what was known as "Cramer's problem." "Succeeding in this work," Flauti remarks, "would complete the research on a famous problem, repeatedly treated and generalized by our school, for which we still lack an adequate analytic solution." An elegant synthetic solution for the problem had been provided by a pupil of Fergola's in 1786; Flauti must have thought that counterposing such a solution to the complexity of the construction of the equation would provide strong evidence in favor of the synthetic problem-solving method. The second problem was taken from the classical problems of "contacts": it called for three circles to be inscribed in a triangle under specified conditions. The problem could be solved according to each of the three methods. But, Flauti states, "we address this problem especially to the sagacious students of this very modern method [the purely analytical one], to assay its force and extension."[10] Again, the choice was not casual: Nicola Trudi (1811–1881), a

[9] J.-L. Lagrange, "Solutions analytiques de quelques problèmes sur les pyramides triangulaires" (1773), in *Oeuvres de Lagrange*, ed. J. A. Serret, 14 vols., Vol. 3 (Paris: Gauthier-Villars, 1869), pp. 661–692; and Lagrange, *Mécanique analytique* (1788), *ibid.*, Vol. 11 (Paris: Gauthier-Villars, 1888).

[10] Vincenzo Flauti, "Programma destinato a promuovere e comparare i metodi per l'invenzione geometrica presentato a' matematici del Regno delle Due Sicilie nell'aprile 1839," in *Produzioni relative al programma di tre quistioni geometriche proposto da un nostro professore* (Naples, 1840), pp. 3–16, on pp. 12, 13. The synthetic

pupil of Flauti's, had recently found a very elegant, purely geometrical solution for the problem, one much shorter than any synthetic solution offered before. Flauti expected that this solution would be superior to anything the analytics could offer. The third problem required three spheres to be inscribed in a pyramid under specified conditions. It was, to Flauti's mind, a natural extension of the second.

The reply of the analytics was quick. Fortunato Padula (1815–1881) published a response in June 1839. He opened with a historical and methodological introduction, in which all the theoretical assumptions of the synthetic school, as well as its reconstruction of the evolution of mathematical knowledge, were criticized and rejected. Then he provided the geometrical construction of the equation of the first problem—but he also remarked that this operation is not essential to the solution of the problem itself, "and in this consists the great superiority of Algebra over Geometry." He went on to solve the second problem analytically, expressing its conditions algebraically in the form of three equations and obtaining from them all the possible solutions. Padula also provided the construction of the equation but not the final geometrical "proof" of his results, which he held was simply not necessary. Finally, he remarked that the third problem is overdetermined; one could certainly try to solve the family of problems deriving from those terms, but, he continued, "we are not interested" in this kind of work. The point is that "we certainly consider the researches in pure Geometry as beautiful pieces of work, but they are too sterile and scarcely interesting for those who have already accomplished their elementary courses and want to practice mathematics in the present century, when this science, applied to Natural Philosophy, Constructions, Industrial Mechanics, etc. grows every day, so that it is almost impossible to be informed on every new development."[11] Padula concluded by announcing that, because of the limited interest of pure geometry, he would never again discuss the advantages of the Lagrangian method relative to a particular geometrical problem.

Padula was not new to this kind of provocative publishing. Indeed, his *Collection of Geometrical Problems Solved by Means of the Algebraic Analysis,* published in 1838, was probably the immediate impetus for Flauti's contest. Following the example of Lagrange's geometrical works, Padula solved algebraically a number of problems in plane geometry that were usually employed in the teachings of the synthetic school; he concluded by arguing for the more consistent use of calculus in descriptive geometry.[12]

Padula's polemical response to the 1839 contest makes explicit some important points of the analytic program. The essence of the analytic method, he asserts, is "to put the

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school allowed the use of algebraic methods to solve certain problems but stipulated that in these cases the solving equation should always be "constructed"—that is, a geometrical construction had to be performed, yielding line segments with lengths equal to the roots of the equation. On this practice see Henk J. M. Bos, "Arguments on Motivation in the Rise and Decline of a Mathematical Theory: The 'Construction of Equations,' 1637–ca. 1750," *Arch. Hist. Exact Sci.,* 1984, *30*:331–380. Bos links its decline with the general shift from a "geometrical" to an "analytic" conception of mathematics and places its disappearance around 1750. The reinstatement of this theory was part of the Fergolian restoration of the supremacy of geometry over analysis. "Cramer's problem" called for inscribing in a given circle a triangle whose sides pass through three given points. It had been proposed by the Swiss mathematician Gabriel Cramer (1704–1752) to the Tuscan Castillon (Giovanni F. M. Salvemini [1704–1791]). Castillon presented a synthetic solution for this problem at the Berlin Academy in 1776. Later, Lagrange presented an analytic solution. According to Flauti, Lagrange's solution was incomplete because it lacked the final "construction of the solving equation."

[11] Fortunato Padula, *Risposta di Fortunato Padula al programma destinato a promuovere e comparare i metodi per l'invenzione geometrica presentato a' matematici del Regno delle Due Sicilie* (Naples: Stamperia del Fibreno, 1839), pp. 7, 45–46.

[12] Fortunato Padula, *Raccolta di problemi di geometria risoluti con l'analisi algebrica* (Naples: Stamperia del Fibreno, 1838). Flauti himself had published an influential textbook of descriptive geometry: Vincenzo Flauti, *Elementi di geometria descrittiva* (Rome: Perego Salvioni, 1807).

problem in the form of an equation"; the steps for obtaining the equation are always the same:

1. Assume as unknowns the coordinates of that point, line, angle, etc., that, if known, would determine all the other quantities, solving the problem.
2. Express these quantities as functions of the unknowns.
3. Express the conditions of the problem in "algebraic language"; this yields an equation.

"With a little practice in the application of this method," Padula remarks, "and with the comprehension of its spirit, it will be possible to put any problem in the form of an equation as soon as we read the initial terms. We cannot imagine how this could happen with geometrical analysis." "Two-coordinate analysis" gives the general rules for constructing the formulas. What remains to "the astuteness of the analyst" are only "those particular considerations that lead to elegant constructions."[13] But elegance, in the analytic image, is not essential to the solution of a problem.

Padula's analytic approach to geometrical problems rests on a general view of mathematics according to which "algebra is the foundation on which all the other parts of mathematics are based." This foundational character depends on its being not a "particular science" but a general method for studying the relations between magnitudes. Thanks to Lagrangian "algebraic analysis," Padula continues, "the mathematical sciences have finally been reduced to that unity of principles so sought by philosophers." Padula stresses the importance of enabling students to read works written in the "new language" as early as possible. For this purpose, he supports the use of French and North Italian textbooks of analytic geometry; these should replace more traditional synthetic textbooks on conic sections—which, he says, in the "civilized" countries have almost disappeared. The study of "geometrical abstractions," he claims, does not introduce the student to "the modern development of mathematics" and "to the enormous material advantages that society can obtain from the application of mathematics to the arts, manufactures, industries."[14]

That the analytic method is characterized mainly by its generality and applicability is also emphasized by Flauti. Indeed, it is this generality that makes the analyst "a mere compiler of algebraic formulas" and essentially different from the "true geometer," who is always an "inventor," a discoverer of eternal truths. In both methods, Flauti remarks, the "principle of the research" is the same: we assume as given what is required to solve the problem and then find all its "determinants." But we can proceed in a "direct way" or in an "indirect way": the first "is synthesis, which is what we call the method of invention used by the ancients; it implies a great quantity of knowledge"; the second is "the Cartesian method and any other derived algebraic method," in which "all the art of the geometer is reduced to the mere use of general rules."[15] The mechanical, practical "manipulation" (*maneggio*) of algebraic formulas is always opposed, in Fergola and Flauti, to the intel-

[13] Padula, *Raccolta*, p. 13.

[14] Padula, *Risposta* (cit. n. 11), pp. xxvi, xxxi.

[15] Vincenzo Flauti, preface to Nicola Fergola, *Della invenzione geometrica: Opera postuma di Nicola Fergola ordinata, compiuta e corredata d'importanti note dal Prof. V. Flauti: Parte 1* (1807) (Naples: Stamperia privata dell'Autore [V. Flauti], 1842), pp. xii, xxii. In this context, the verb *inventare* (to invent), and the related terms *inventore* (inventor), *invenzione* (invention), and *metodo di invenzione* (method of invention), refer to the Latin verb *invenire* (to discover, to find). Thus, to say that the geometer "invents" geometrical truths means that he discovers them. In fact, Fergola's posthumous book *Della invenzione geometrica* [On geometrical invention] was originally titled *L'arte euristica in sistema scientifico ridotta* [Heuristic art in the form of a scientific system]; Flauti's new title was inspired by Cicero's *On Rhetorical Invention.*

lectual "art of inventing," the discovery of intellectual truths by means of intuition, that is proper to the geometer. Algebraic reasoning and geometrical reasoning are, according to the synthetics, essentially different: they depend on different human faculties and deal with different kinds of objects.

According to the examples presented by Fergola, the synthetic method consists of the following steps:

1. Imagine that we have done what the problem asks ("supposition of the fact").
2. Develop the correct consequences of this supposition.
3. Reduce the problem to another well-known problem or solve it directly.

At this point (after the geometrical analysis, which is "an ontological principle of reduction"), we must proceed to the "geometrical composition" of the problem, that is, the construction of the solution, where the order of the analysis is reversed. This last step is crucial: "the construction is the essential condition for the proper solution of a geometrical problem." Both geometrical analysis and composition must be accomplished according to the ancient criteria of elegance, a feature neglected by the "corrupted taste" of the analytics.[16] The geometer's art of inventing ("one of the most beautiful dianoethic virtues") is certainly a difficult one, requiring "much art, great ingenuity, and very long training." In the case of a particularly difficult problem one might employ algebraic analysis as an aid, but it is essential to present the construction of the equation as the final step in the solution. Algebra's role is purely instrumental; proof consists in "a clear succession of geometrical reasonings treating geometrical quantities." This is why "with the mere knowledge of analytic methods, it does not matter how much ingenuity one has, one will never reach the proper solution of the problem."[17]

Padula's reply to the 1839 contest was rejected by the Class of Mathematics of the Royal Academy, which was responsible for awarding a money prize to the winner. They objected that Padula, in solving the problems, had failed to meet some of the designated criteria: I have cited his refusal to provide a final "proof" for the second problem and his dismissal of the third one. Nicola Trudi was the official winner of the dispute. With him, the synthetic method also "won." Of course, this was far from the last word in the controversy between the two schools—quite the opposite. Their ongoing antagonism is proof that their opposition was much deeper than a merely "technical" disagreement. It could not be resolved by putting the two methods to work and comparing their results.

The criteria for evaluating the "reliability," the "usefulness," or the "didactic effectiveness" of a problem-solving method cannot but depend on more general considerations as to what mathematics is about, the degree of certainty enjoyed by mathematical propositions, and the relations between mathematics and empirical reality. In short, they depend on what we believe the nature of mathematical knowledge to consist in and what we hold its relations with other kinds of human knowledge to be. In the case of the Neapolitan debate, we can make sense of the opposing judgments about the reliability and usefulness of the two methods only by contrasting these valuations against the background of two

[16] Fergola, *Invenzione geometrica*, p. 196. Flauti quotes Edmond Halley to explain what elegance is. "Analisi brevissima et simul perspicua, Synthesi concinna et minime operosa": Vincenzo Flauti, "Considerazioni generali su tre difficili problemi e sul modo di risolverli, lette alla R.A. Accademia delle Scienze di Napoli in agosto 1839," in *Produzioni* (cit. n. 10), p. 21. The quotation is from Halley's preface to *Apollonii Pergaei de sectione rationis . . .* (Oxford, 1706), unpaginated (last page).

[17] Flauti, preface to Fergola, *Invenzione geometrica*, pp. xxiii, xxvi; and Flauti, "Considerazioni," p. 20. On the construction of the equation see note 10, above.

very different conceptions of mathematical knowledge. At the beginning of this essay I referred to "two different images of mathematics," where an image is that set of general beliefs mathematical practices are grounded on. What were clashing, in Naples, were two images of mathematics.

To understand this controversy, we must examine the emergence and institutionalization of the synthetic school. We shall find that the idea of the synthetic school as merely "backward," a leftover from the past, must be challenged. Interpretation of the debate in terms of a clash between different generations of mathematicians cannot tell us the whole story. In fact, the synthetic school had been created by a very recent community of mathematicians, and its geometrical approach dated back only to the 1780s. It is also impossible to sustain the claim that these mathematicians were unaware of contemporary developments in mathematics. They were familiar with the work of Joseph-Louis Lagrange, Pierre-Simon Laplace, Gaspard Monge, and other European mathematicians. Their appeal to "tradition" does not explain their mathematical practice but is itself something that requires an explanation.

## REVOLUTION AND REACTION

The philosophy of mathematics of the synthetic school is rooted in the cultural atmosphere of the 1780s and the 1790s. At that time, Naples was an important center of the Italian Enlightenment. Its intellectual environment was stimulating, and reformist thinkers published a number of works in economy, law, politics, and the social sciences. The reformist programs put forth in such works were connected to the empiricism that characterized Neapolitan philosophy of the period. Among the main points of reference were Locke, Condillac, French sensationalism, and materialism. This emphasis arose largely in response to the works and teaching of Antonio Genovesi (1712–1769), who in 1754 was given a chair of political economy at the Royal University, the first chair of its kind in Europe. Among the members of the so-called school of Genovesi, were first-rank politicians such as Domenico Caracciolo (1725–1789), radical thinkers such as Mario Pagano (1748–1799), and well-known students of law and society such as Giuseppe Galanti (1743–1806), Gaetano Filangieri (1752–1788), and Melchiorre Delfico (1744–1834). The intellectuals and administrators linked to Genovesi shared the aim of "demystifying" their disciplines—that is, eliminating their metaphysical implications. This permitted them to reduce the practice of politics, law, and economics to a matter of rational administration. The sciences, in these plans, played a very important role: they were not sources of eternal truths (at least not primarily) but the means for the cultural and economic progress of the country. In particular, Genovesi's *Discorso sull'utilità delle scienze e delle arti* [Discourse on the utility of the sciences and of the arts] (1764) presented an instrumental conception of science in which social problems had absolute preeminence over theoretical ones. His polemical target was the idea that "pure science," devoid of practical utility, is superior to the applied sciences (or "mixed mathematics," as they were called).

Genovesi and his school were influential during the brief season of the Bourbon "enlightened absolutism," between the 1760s and the 1780s. After 1789, the "facts of France" caused an irremediable rupture between the intellectual class, made up mostly of members of the weak bourgeoisie of the kingdom, and the monarchy, until then its main ally. This ended the reformist efforts of the Bourbons, which had consisted in weakening the feudal system and the economic and cultural presence of the Catholic Church in the kingdom. From the early 1790s onward, the Church was to be the main ally of the monarchy in its opposition to Jacobin tendencies and, later, the diffusion of liberal ideas. The year 1791, which saw the meeting between King Ferdinando IV of the Bourbons and Pope Pius VI—a significant event, given the traditional anti-Church politics of the Neapolitan government—is usually taken as the symbolic date of the reactionary turn in Naples. A few years later, in 1799, Naples was shaken by a revolution that brought about the proclamation of a Jacobin republic (La Repubblica Napoletana) and the abolition of feudalism. But the newborn republic had a short life indeed. After six months of continuous struggle, it was militarily defeated by the Bourbon loyalists and their European allies. The repression that followed the entrance of the "Christian army" into Naples destroyed almost an entire generation of intellectuals and professionals who had been responsible for the insurrection.

In the 1790s Naples was also an important center for the elaboration of early conservative thought. The Neapolitan manifestation of this European phenomenon took the form of "Reactionary Catholicism," which aimed at a return to an "organic," theocratic society based on an idealized image of the Middle Ages.[18] According to this conception the Church and the Crown, sustained by a powerful aristocracy, were the two pillars of society: the dogmas of the Church were as ancient and untouchable as the feudal rights of property. Simultaneously with the growth of Jacobin clubs, reactionary literature flourished in Naples. Its source was a group of Catholic thinkers, mainly ecclesiastics, who would be charged by the monarchy with reorganizing public education and cultural institutions after the revolutionary break of 1799. In the same years, the king chose the more intransigent bishops to lead the dioceses of the kingdom, and the members of Genovesi's school, suspected of atheism and materialism, came under strict police surveillance. Book burning made its sinister reappearance in the squares of Naples.

This reactionary movement was a counteroffensive against the reformist thought of the eighteenth century, which had been identified as responsible for the diffusion of Jacobinism in Naples and for social disorder in general. The traditional, "natural" order of the kingdom, as well as the tradition of the Church, was opposed to the abstract and overly general laws of Universal Reason of the philosophes. Conservative thought presented religion as the only basis of a "natural" human society; the dissemination of atheism by the philosophes led inevitably to Jacobinism and, in the end, to the disappearance of all social order. Moral disorder advanced from philosophy into society. According to a conservative Neapolitan author, the "sacrilegious conspiracy" aimed to drive "kings from society" as well as "God from the universe."[19]

Karl Mannheim claimed that the essential trait of early nineteenth-century conservative thought was its *reactive* character.[20] Indeed, he showed how the (heterogeneous) contents of the conservative position were almost a point-by-point response to the program of the

[18] On so-called Reactionary Catholicism in the Italian states see Luigi Salvatorelli, *Il pensiero politico italiano* (Turin: Einaudi, 1935), Chs. 4, 5, 6; Daniele Menozzi, "Tra riforma e restaurazione: Dalla crisi della società cristiana al mito della cristianità medievale (1758–1848)," in *Storia d'Italia*, Suppl. 9: *La chiesa e il potere politico dal medioevo all'età contemporanea* (Turin: Einaudi, 1986), pp. 767–806; and Menozzi, "Intorno alle origini del mito della cristianità," *Cristianesimo nella Storia*, 1984, *5*:523–562. References about authors linked to Reactionary Catholicism are in *Letteratura Italiana Laterza*, 65 vols., Vol. 46: Vito Lo Curto and Mario Themelly, *Gli scrittori cattolici dalla Restaurazione all'Unità* (Bari: Laterza, 1976).

[19] Gioacchino Ventura, "Elogio di Pio VII," in *Raccolta* (cit. n. 1), p. 15. In fact, the reaction of the Roman Catholic Church against the values of the Enlightenment dates back to the antimodern turn of Pope Clement XIII (1758). But until the 1790s the Church was isolated in its reactionary position, as is clear from the expulsion of the Jesuits from many European countries (in Naples it took place in 1767) and from the official suppression of the Company in 1773.

[20] Karl Mannheim, "Conservative Thought" (1927), in *Essays on Sociology and Social Psychology* (London: Routledge, 1953); and Mannheim, *Ideology and Utopia* (1929; London: Routledge, 1936).

Enlightenment. The very form in which conservative writers expressed their views was the opposite of the rational, "scientific" prose of the philosophes (thus Mannheim can identify a conservative "style of thought"). Another distinctive trait of these conservative authors was their constant attempt to reconnect their critique of the modern world to an older tradition. They saw themselves as the last defenders of a homogeneous, venerable, and natural conception of the world that came under attack during the eighteenth century. The alleged roots of this *prisca sapientia* were found mainly in the Middle Ages. According to the philosophy of history of the conservative authors, the Renaissance was the beginning of a moral and political corruption that led to the Protestant Reformation and, later, the political revolutions; all were part of a single enormous plot against the "natural" society and its religious grounds. In formulating their argument, conservative authors performed an impressive simplification of modern history and, in particular, invented much of the "venerable tradition" that provided the credibility and authority for their arguments. It is only in the context of this specific trend of conservative thought that we can make full sense of the scientific program elaborated by the geometers of the synthetic school in Naples. Their criticisms of the purely analytic approach to mathematics cannot be understood apart from their own philosophical and religious agenda. The defense of the dignity of geometry and the insistence on the preeminence of geometrical over analytical methods that characterized Fergola and his school from the late 1780s on must be investigated together with the contemporary emergence of analysis as the scientific basis of the political ideology of the Neapolitan Jacobins.

## JACOBIN SCIENCE AND MATHEMATICS

The idea of a science that was not "pure" or contemplative, but "modern" and linked to the real needs of the populace, had been spread by the writings of the Neapolitan reformers and by the translations of the French philosophes. By the 1790s, this idea had become a fundamental point in the program of the Neapolitan Jacobins. Most of them were young, well-educated men and women with some training in scientific disciplines. The principal Jacobin club was founded in 1792 by Carlo Lauberg (1762–1834) and Annibale Giordano (1771–1835). Interestingly, both were teachers of mathematics.

Lauberg had been trained at the Reale Accademia Militare (Royal Military Academy) of Naples under the guidance of Colonel Giuseppe Parisi. The influence of Condillac's sensationalism is evident in his first published work, a philosophical essay "on the operations of the human understanding." Interested in chemistry and mathematics, Lauberg become known at the end of the 1780s for his experiments on the extraction of colorants from plants and on the industrial production of hydrogen sulfate. But his researches were opposed by the natural philosophers of the Royal University because he was a follower of the "new chemistry" of Lavoisier. Similarly, his mathematical work, inspired by Lagrange, did not win the favor of the rising star of Neapolitan mathematics, Nicola Fergola. In 1789 Lauberg tried and failed to win a post as a teacher of mathematics at the Royal Military Academy; the position went instead to Annibale Giordano, a young student from Fergola's private studio. That same year Fergola was appointed to his first mathematical chair, at the Liceo del Salvatore, a prestigious Neapolitan college. The appointment was made by the king himself, who had chosen Fergola because he was "persuaded that the virtue and the good behavior of the citizens are made natural through teaching." As we know from his unpublished papers, Fergola had pressured for Lauberg's exclusion from serious consideration in the competition for the job at the Royal Military Academy because of his allegedly reprehensible moral behavior (Lauberg was a defrocked priest).[21]

Lauberg had submitted an essay on mechanics, in which he aimed to prove the fundamental principles of the science of motion "in an easy and direct way [*con vie facili e dirette*]." His approach was to state clearly "the conventions that are at the basis of the science" and derive the laws of mechanics from them "through simple induction." Once this was accomplished, "the solution of every problem is reduced to a matter of pure calculation." These considerations, Lauberg continued, made him decide to study mechanics "by means of new analytic views [*con nuove analitiche vedute*]."[22] Following Lagrange, his aim was to show that all the theorems of mechanics are merely different expressions of a unitary principle, the axiom of the virtual velocities.

Lauberg's analytic approach to mechanics relies on certain fundamental beliefs about the nature of the sciences: "every science is nothing but the combination of all the simple ideas that constitute the complex idea of a phenomenon, and of the conventions that have been established." In the case of mechanics, we know that "all the bodies are active beings, and are in movement: thus there must be a general expression that comprehends all the combinations of simple ideas constituting this phenomenon of the bodies' activity [force, distance, time], which will give us the general equation to obtain the solutions for all the particular problems of mechanics." By means of this general equation, Lauberg argued, we can mechanically obtain the solutions for every particular problem of mechanics, leaving aside any metaphysical considerations about the "essences of the bodies" or the "nature of the forces" and "useless" questions such as "whether the force is intrinsic or extrinsic to the body." Importantly, Lauberg stated that this analytic approach was not limited to mechanics but could be employed in "every science."[23]

Lauberg's later attempts to obtain chairs at the Royal University, in 1791 and 1792, in experimental physics and natural history also failed. In the meantime, he had opened a successful private studio for mathematics and chemistry, where he taught with the young mathematician Annibale Giordano.[24] Giordano was, surprisingly, one of the brightest students from Fergola's studio. He was known for having presented a very elegant and original synthetic solution for Cramer's problem to the Royal Academy in 1786, when he was only sixteen. This solution was later published in the acts of the Società dei XL by Antonio Maria Lorgna and was cited by Chasles and Carnot. (It was the solution that Flauti had in mind when he proposed the first problem of the 1839 contest.) But Fergola would soon be disappointed by his favorite pupil: only a year after winning the 1789 competition for the post at the Royal Military Academy, Giordano joined Lauberg's analytic program.

The new studio attracted a number of students—who also discussed the news from France (see Figure 3). In 1792 it became the secret base of the main Jacobin club in Naples,

[21] Lauberg's first published work is [Carlo Lauberg], *Riflessioni sulle operazioni dell'umano intendimento* (Naples, n.d. [between 1786 and 1789]). The king's recognition of Fergola is noted in [Luigi Telesio], *Elogio di Niccolò Fergola scritto da un suo discepolo* (Naples: Trani, 1830), p. 199. On the pressure to exclude Lauberg see Amodeo, *Vita matematica napoletana* (cit. n. 6), Vol. 2, p. 60.

[22] Carlo Lauberg, *Memoria sull'unità dei principi della meccanica* (Naples, n.d. [1789]); quotations are from the unpaginated dedication.

[23] *Ibid.,* p. 4.

[24] The phenomenon of the *studi privati* (private studios) was typical of the educational system in the Kingdom of Naples during the eighteenth and nineteenth centuries. Given the extreme conservativism of the university curricula, students who wanted to study contemporary authors, both in the natural sciences and in the humanities, had to attend such private schools. Often university professors also ran private studios. These schools were generally tolerated by the authorities, except in particularly critical situations such as the years of the Jacobin plots (1794–1799).

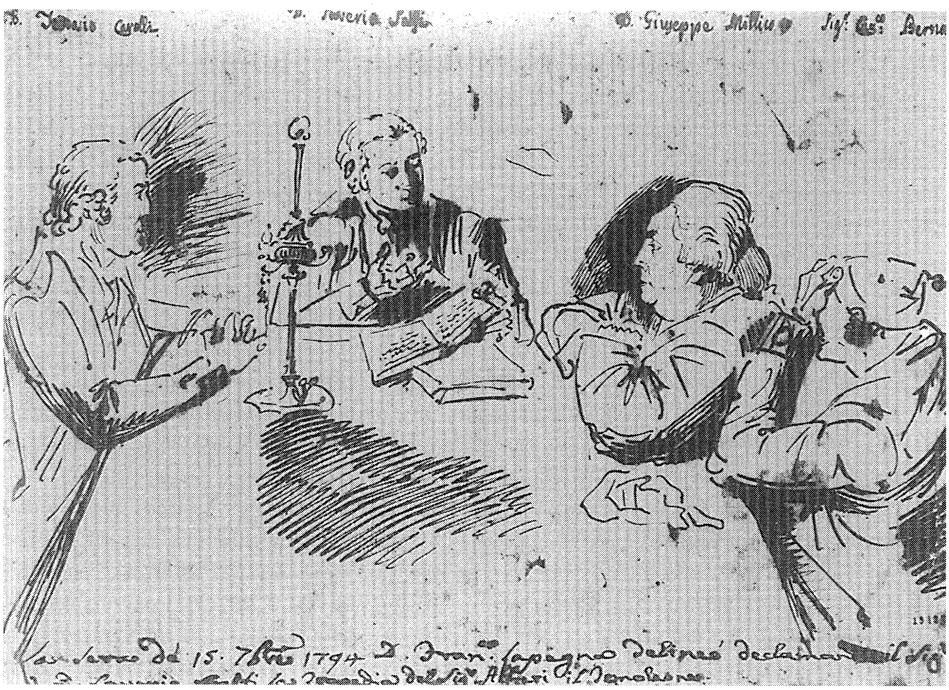

***Figure 3.*** *Secret reunion of Neapolitan Jacobins in 1794. Drawing by Francesco Lapegna. (Naples, Museo di San Martino.)*

Lauberg its leader. He was, according to Benedetto Croce, "the soul of the whole Neapolitan revolutionary movement." Most of the Neapolitan Jacobins received some mathematical training at the studio.[25] In 1792 Lauberg and Giordano published a textbook of mathematics, entitled *Principi analitici delle matematiche* [Analytical principles of mathematics]. It consisted of two parts: "Principles of Arithmetic" and "Principles of Geometry." The sciences of arithmetic and geometry are "analyzed"—that is, reduced to their simple, fundamental notions and laws—and then rebuilt according to the "natural order" in which reality and our thoughts are structured. That "natural order" turns out to be the order of algebra, and of the Lagrangian algebraized calculus. In the preface, the authors clarify their image of mathematics and of science in general. As in Lauberg's 1789 essay on mechanics, sensationalism and analysis are the main resources. The whole of human knowledge, they claim, derives from "very simple sensations," which are the basis for the construction of our concepts. Every science derives from a class of sensations and from the conventions we stipulate about them. Between sensations and general laws there is a "necessary link." This link is given by algebraic reasoning, so that phenomena are "con-

[25] Benedetto Croce, *La Rivoluzione Napoletana del 1799* (Bari: Laterza, 1912), p. 210. Among the Neapolitan Jacobins at Lauberg's studio was the radical thinker Francesco Mario Pagano (1748–1799). Liberal and physiocratic in economics, he became a well-known lawyer in the 1770s. In 1785 he was given the chair of criminal rights at the Royal University of Naples. His most famous book is *Considerazioni sul processo criminale* (1787), in which he suggested reforms of Neapolitan judicial procedures and proposed the establishment of a universal legal code for all the countries of the world. In 1799 he was a member of the Provisional Government of the Republic, responsible for the preparation of the Neapolitan Constitution. At the return of the Bourbons he was sentenced to death and hanged.

nected to each other with analytic order." The science of mathematics, for example, derives from our basic sensations and conventions about magnitudes. Physics is "the generalization of phenomena resulting from the activity of matter"; it is not "that crowd of substances and qualities whose only utility was to teach us words without meaning [*vocaboli privi affatto di idee*]." Similarly, metaphysics has been freed from "the darkness with which the [philosophical] schools had surrounded it." This science consists, in fact, of two main branches: the study of human sensibility, and the investigation of those general physical laws called "cosmological laws." Morals are merely the analysis of the sensations caused by human needs and of the means to manage them; politics is limited to the problem of satisfying individual needs within the context of public needs. The authors conclude that "if Physics, Metaphysics, Morals, Politics are merely the analysis of the effects of the activity of matter, of human sensibility, of the control of this sensibility relative to human needs, as Mathematics is the analysis of quantity; then, Mathematics being an exact science, so also must we consider the other ones, when we regard them without mystery, and from the right point of view." In every science, we must investigate "the natural development of the simple ideas constituting the primitive phenomenon that is the object of the science itself."[26] This natural development of primitive ideas is given, in any area of human knowledge, by the rules of algebra and calculus.

Given these premises, the choice of the analytic method as the preferred method in mathematical textbooks becomes crucial. If we aim, the authors wrote, "to promote public education and to eradicate old prejudices"—and this was a central point in the Jacobin program—"the only way [. . .] is to accomplish this simplification of the sciences, so that, reduced to the analysis of our sensations, they no longer constitute that congeries of isolated truths which is presented by the method of Composition [i.e., the synthetic method]." "These considerations," they continued, "made us regard the textbooks of mathematics and philosophy compiled according to the synthetic methods as unworthy of the education of man." Indeed, "they present a history of single truths, instead of presenting the methods of invention that contributed to the development of the human spirit." The synthetic method presented pupils with a set of disconnected, particular truths (in geometry, the more skilled we are, and the more naturally gifted, the more we can find out new geometrical truths), while the analytic method was a "complete" and universal one. It was, so the argument went, the method that permitted the development of scientific knowledge during the modern era. It was the method that would permit the development of social and political sciences in the near future. "We decided to prepare this textbook according to the view that we have presented above," they concluded, "with the sole aim of being useful for our Country."[27]

This project of radical social, political, and cultural reform was grounded by what the authors called the "generality" of the analytic method. According to the Neapolitan supporters of this method, mathematics is not a question of individual intuitions, of long and difficult training, and its aim is not limited to the discovery of eternal truths in the heavenly kingdom of geometrical entities. On the contrary, mathematics is a universal language, comprehensible to every rational being, that can be applied to every field of human experience. Even the human sciences can be mathematized and reach the degree of certainty proper to mathematics. This implied that the existing social and political order could be

[26] Annibale Giordano and Carlo Lauberg, *Principii analitici delle matematiche* (Naples: Giaccio, 1792), pp. 1–2.
[27] *Ibid.*, p. 2.

criticized as "unscientific" and that it could legitimately be rebuilt on the basis of the new theorems of the sciences of politics and economics. Once their "very simple elements" had been well defined, it would be possible to deduce the laws of the social sciences just as others had deduced those of hydrodynamics.[28]

The rise of the synthetic school coincided with the persecution of Jacobin intellectuals, its apogee with the repression of the Neapolitan republic. Not surprisingly, Lauberg and Giordano were among the first to be sought by the police when the existence of a Jacobin plot was discovered in 1794. About Lauberg, Croce reports: "his home was searched, and his papers, rich in algebraic signs, were confiscated as mysterious documents; but he was hidden, and later he managed to escape." In fact Lauberg went to France; he returned only in 1799, at the time of the revolution. Giordano was arrested. A student of theirs, Emanuele De Deo, found in possession of copies of a clandestine edition of the French Constitution, was hanged in October 1794. The trials of 1794 were only the beginning of a period of violence that ended with the revolution. Lauberg and Giordano played central roles: in January 1799 Lauberg was elected president of the Provisional Republican Government, while Giordano became a member of the Military Committee of the Republic. Lauberg distinguished himself for his radical ideas about antifeudal laws; Giordano, as Amodeo remarks, was "the inspirer of all the projects and operations devoted to promoting the revolution and to destroying the memory of the Bourbon monarchy."[29] But June 1799 saw the defeat of the Republican army, mass executions, and the exile of the survivors. The whole of Neapolitan culture suffered from these momentous events: seven professors at the Royal University lost their lives in the aftermath of the revolution, and eleven others were arrested. Similarly, the Military Academy and many of the private studios paid a high price, in terms of lives lost, for the Republican cause.

## NICOLA FERGOLA: THE ASCETIC MATHEMATICIAN AND HIS SCHOOL

We have already met Nicola Fergola, the founder of the Neapolitan synthetic school. A teacher of philosophy at the Liceo del Salvatore, he was better known for his private studio of mathematics, where pupils received a mathematical education superior to that offered at the Royal University.[30] Fergola had studied mainly on his own, in a private library that

[28] On this point, the link between Neapolitan Jacobinism and the Neapolitan Enlightenment is evident: Genovesi's aim was, indeed, to identify the true "laws of politics and economics." "Politics, like economics, has its own certain and eternal principles: thus, it has its own theorems and its own problems": Antonio Genovesi, *Logica per gli giovinetti* (1766; Turin: Einaudi, 1977), p. 232. On the basis of these alleged laws, Genovesi criticized the feudal social setting of the kingdom as well as its juridical system, which openly favored the aristocracy in its quarrels with local communities. The extreme complexity of the old laws, he showed, in fact functioned to maintain a status quo that originated from an illegal action—namely, the acquisition of feudal rights. While Genovesi approached the creation of the new sciences of economics, politics, and law from a reformist perspective, Lauberg and Giordano held that the principles of these sciences justified revolutionary action. In the metaphor of Francesco Lomonaco, a Jacobin who defended Naples in 1799: "The great tree of the sciences . . . will form leafy branches, which will provide a restful shadow for a humiliated mankind." Francesco Lomonaco, *Rapporto al cittadino Carnot* (1800), in Vincenzo Cuoco, *Saggio storico sulla rivoluzione napoletana del 1799* (1801; Bari: Laterza, 1913), p. 288.

[29] Croce, *Rivoluzione Napoletana* (cit. n. 25), p. 213; and Amodeo, *Vita matematica napoletana* (cit. n. 6), Vol. 2, p. 71. On the conspiracy of 1794 see Tommaso Pedio, *Massoni e giacobini nel Regno di Napoli: Emanuele De Deo e la congiura del 1794* (Bari: Levante, 1986).

[30] In fact, at the university, only a few courses in geometry and arithmetic were given. Traditionally, the faculties of medicine and law attracted the majority of the students and resources. The social and academic prestige of disciplines such as medicine and law was decidedly higher than that enjoyed by natural philosophy and mathematics. On teaching at the University of Naples in the eighteenth and early nineteenth centuries see Michelangelo Schipa, "Il secolo decimottavo," in *Storia dell'università di Napoli* (Naples: Ricciardi, 1924), pp. 433–466; and Alfredo Zazo, "L'ultimo periodo borbonico," *ibid.*, pp. 467–588.

regularly received the published proceedings of the major European academies. By learning about mathematical developments elsewhere and sharing his knowledge with his students, Fergola "brought Naples back into Europe," as his biographer said. The first works on the application of integral calculus to physical problems published in Naples were by Fergola; the fact that they did not appear until the 1780s is symptomatic of the state of mathematics in the kingdom. Fergola's early interests, in the 1770s and 1780s, were very wide. He taught and did research in both synthetic and analytic geometry, paying special attention to the physical applications of calculus. At this stage, he praised works by authors such as Lagrange and D'Alembert. Around 1786, however, Fergola began to show a particular interest in the "forgotten methods of the ancients."[31] In the following years, his interest was to be directed almost exclusively toward pure geometry and synthetic methods. His main concern became to solve geometrical problems according to "the way of the ancients," which he characterized as "elegant," "intuitive," and "certain," as opposed to the unreliable procedures of an analysis that still lacked secure logical foundations. In his publications, which became increasingly didactic, he continued to provide analytical solutions for problems, but epistemological priority was given to the synthetic method that he now considered the proper method for work in geometry. Analytic procedures were allowed only if they could, at least in principle, be replaced by "synthetic reasoning." The preeminence of synthetic methods became even more pronounced in the teaching and research of Fergola's pupils—for whom, by about 1800, mathematics had become synonymous with synthetic and descriptive geometry. Naples was, once again, out of Europe.

In the summer of 1799, after the republican defeat, Fergola was back from his quiet sojourn in the countryside, where he had retired at the beginning of the political upheaval. At this point he found himself, along with other loyalist professors, charged by the government with restructuring the system of public education in the kingdom. The restored Bourbons bestowed on Fergola and his pupils mathematical chairs at the university and posts at the military and naval academies. Their control of the mathematics section of the Royal Academy of Sciences was now unchallenged. As is clear from the letters of appointment, the king was looking for "reliable" professors, and Fergola had always been a loyalist. "Not agreeing with the new political aspirations," Amodeo says, "he remained forgotten and isolated during the whole revolutionary epic." None of his other pupils followed the example of Giordano. According to a friendly biographer, himself a member of Fergola's school, the behavior of the other members—particularly Vincenzo Flauti and Giuseppe Scorza, who were "always highly esteemed by the government"—was "more than sufficient to compensate for the loss of Giordano."[32]

Fergola was a fervent Catholic whose faith often approached fanaticism. His religious practices were perfectly representative of the new Catholicism that had been introduced in the eighteenth century by the Jesuit popular missions in response to the attacks brought by the philosophes against the superstitious practices of the populace in Catholic countries. All those aspects of Catholicism that had been dismissed as superstitious by the critics—

[31] [Telesio], *Elogio di Fergola* (cit. n. 21), p. 32. Among Fergola's early works on applications of calculus see Nicola Fergola, "Risoluzione di alcuni problemi ottici" [read in 1780], in *Atti della Reale Accademia delle Scienze e delle Belle-Lettere di Napoli dalla fondazione sino all'anno 1787* (Naples: Campo, 1788), pp. 1–14; and Fergola, "La vera misura delle volte a spira" [read in 1783], *ibid.*, pp. 65–84. On the turn toward the "ancients" see the introductory note to Fergola, "Nuovo metodo da risolvere alcuni problemi di sito e di posizione" [read in 1786], *ibid.*, pp. 119–138.

[32] Amodeo, *Vita matematica napoletana,* Vol. 2, p. 141; and [Luigi Telesio], *Appendicetta all'Elogio di Niccolò Fergola* (Naples, 1836), pp. 26–27.

the cult of the saints, the cult of the Holy Virgin, and the excessive manifestations of popular religiosity — were supported and intensified.[33] The devotion to the miracle of Saint January, the liquefaction of the dried blood of the saint that takes place periodically in the cathedral of Naples, was a typical example of this baroque religiosity. Fergola, a devotee of the Virgin Mary, regularly took part in popular religious processions in Naples. He was also well known by his fellow citizens for his ascetic style of living. Reluctant to receive honors or accept other academic positions, he spent his life alone, according to his vow of chastity; his only family was the pupils of the school. Those who visited his house reported the frugality of his strictly vegetarian meals and the lack of the sorts of comforts one would expect in the home of a renowned professor. The religious dimension characterized not only Fergola's private life but also his activity as leader of a school of mathematics. In his mind, the function of the school went well beyond the transmission of mathematics and natural philosophy. The chief goal was to help the pupils in their spiritual growth, in order to make them good Christians and good citizens. This is why Fergola was concerned with the whole of his pupils' education, including their moral behavior and their religious and political ideas. The transmission of mathematical knowledge was important as part of a wider formative process. Mathematics, as it was practiced in Fergola's school, was a "spiritual science," a science that brings its practitioners very close to the mind of the Almighty. It was also a powerful resource in the fight against atheism and materialism.

In 1804, in reply to rationalistic criticisms, Fergola wrote an essay defending Saint January's miracle and the possibility of miracles in general.[34] This work, which has not been investigated by historians interested in Fergola's mathematical activity, contains a series of aphorisms on philosophy and religion that are of some interest for understanding the background of his research. Fergola argues for the spirituality of the soul and for the nonmaterial origin of human thoughts. He rejects materialism, in its many variants, because empirical reality is too complex to be reduced to material causality. These themes reemerge in Fergola's lectures on Newton's *Principia*, published in 1792–1793 at the explicit request of the government. They contain a full, clear presentation of Newton's work, integrated with more recent materials from Euler, D'Alembert, Lagrange, and others (the first volume is divided into two parts, on mechanics and statics; the second is devoted to the "science of fluids"). The general perspective is deeply religious. It is interesting to compare the part on mechanics with the almost contemporary essay on mechanics by Lauberg. As we have seen, Lauberg rejected as "useless" metaphysical questions related to notions such as "force": Are forces intrinsically or extrinsically related to bodies? What is the real nature of forces? These very questions are central to Fergola's presentation of mechanics, however. We are interested in studying forces, he explains, because they are "the soul that informs the great [material] body of the universe and gives it life"; this study permits us to discover "the laws of the universe" and, at the same time, "the deep knowledge of who rules and sustains it." Indeed, according to Fergola, the source of all the forces acting in

[33] See Louis Châtellier, *La religion des pauvres* (Paris: Aubier, 1993). On the social role of Catholicism and of the Church in the Kingdom of Naples see Giuseppe Galasso and Carla Russo, eds., *Per la storia sociale e religiosa del mezzogiorno d'Italia,* 2 vols. (Naples: Guida, 1980, 1982).

[34] Nicola Fergola, *Teorica de' miracoli* (Naples: Flauti, 1839). A second edition was printed in Milan in 1842. It presents a selection from Fergola's manuscripts. It contains an introduction by the editor (Vincenzo Flauti), a biography of Fergola, the essay on miracles, the "Discorso-apologetico sul miracolo di San Gennaro" [Apologetical speech on Saint January's miracle], and an anthology of Fergola's aphorisms on philosophy and religion. The third edition, printed in Naples in 1843, also contains a proof of the spirituality of the soul by Flauti: "Il sentimento ed il pensiero essere incompatibili alla materia" [Feelings and thoughts are incompatible with matter].

the universe is the "Hand of the Living God." And whereas Lauberg criticized Maupertuis for his apologetic use of the principle of minimal action ("What is the link," Lauberg asks, "between distance times velocity [$D \times V$] and the existence of God?"), Fergola sees in that very principle "the most powerful argument" against the atheists.[35]

Fergola considered his study of the immaterial forces that give life to the physical universe as intimately linked to his efforts to prove the spirituality of the soul or the impossibility that matter can produce thoughts. As the great machine of the universe is inexplicable materialistically, so the spiritual dimension of human life is irreducible to its material dimension. Fergola's various writings can be seen as parts of a complex antimaterialistic project. Interestingly, this project includes themes from Genovesi's philosophy of nature as well. Fergola had been a pupil of Genovesi, but his use of Genovesi's empiricism was very different from that of the reformers or the Jacobins. In Fergola's hands, this empiricism is developed in an antimaterialistic direction: the emphasis is on the gap between phenomenal reality, the subject of empirical investigations, and metaphysical reality, the subject of theological consideration.[36]

Fergola's lectures on Newton would be employed by Reactionary Catholic authors to argue for the fallible character of empirical knowledge and the limits of scientific investigation. In fact, a look at Fergola's few correspondents and intimates shows that he was in close contact with the group of Catholic thinkers that prepared and implemented the new educational policies of the government after the revolution. That he was in close personal contact with members of the Accademia Arcivescovile, where Neapolitan Reactionary Catholicism was originally elaborated, and a member of the Accademia di Religione Cattolica, founded in 1801 in Rome by Pius VI with explicit antimodern aims, enables us to position him in the network of those intellectuals who, in the Italian states, opposed the secularization of science and society.

In understanding Fergola and his school as organically connected to a wider reactionary intellectual movement, we should not lose sight of the important point that during the first years of his work on the "synthetic program," between 1786 and 1789, he was intellectually isolated. Forms of antimodern thought were certainly present in Naples at that time; I have noted the reactionary position of the Roman Catholic Church. But because of the "Roman" connection, they were viewed with suspicion by most of the members of the government, which was still largely anticlerical in outlook. An episode symbolic of this outlook was the visit of the marquis Domenico Caracciolo to Fergola's school in 1786. Former ambassador in Turin, London, and Paris, then viceroy of Sicily, and, at the time of his visit, prime minister (1786–1789), Caracciolo was a man of the Neapolitan Enlightenment. He had studied with Genovesi and had been in close contact with the men of the *Encyclopédie*, in particular with D'Alembert. He was also a close friend and an admirer of Lagrange. In fact, Caracciolo had brought Lagrange, then a young mathematician, from Turin to Paris and had introduced him in the drawing rooms of the local aristocracy. Caracciolo's diplomatic and political action was mainly devoted to reducing the economic and cultural

[35] [Nicola Fergola], *Prelezioni sui Principi matematici della filosofia naturale del cavalier Isacco Newton per uso dell'Università interna del Real Convitto del Salvatore,* 2 vols., Vol. 1 (Naples: Porcelli, 1792), Vol. 2 (Naples: Di Bisogno, 1793), Vol. 1, pp. 24, 25; and Lauberg, *Principi della meccanica* (cit. n. 22), p. 25.

[36] A similar elaboration of empiricist themes can be found in the writings of Johan Georg Hamann (1730–1788). Hamann provided an "irrational" and antimaterialist reading of Hume to oppose the intrusion of quantitative science into the sphere of social and individual life. He claimed that the artificial conceptual systems of the philosophes, their material and causal explanations, cannot capture the real life of the universe. This is accessible only through a series of intuitive insights. See Isaiah Berlin, *The Magus of the North: J. G. Hamann and the Origins of Modern Irrationalism* (London: Fontana, 1994).

power of the Catholic Church and to counterbalancing the great economic and juridical power of the barons in the countryside. Among other things, he was responsible for the abolition of the tribunal of the Holy Inquisition in Sicily and for the abolition of the formal vassalage of the king of Naples to the pope. During his visit to Fergola's school, this anticlerical freethinker argued that some of the mathematical problems treated by the synthetics would be better addressed by the analytic method and praised the works of his French friends. Only a few years before, in 1781, Caracciolo had tried to convince Lagrange to join the Royal Academy of Sciences and to lead mathematical research in the kingdom.[37]

In the reformist atmosphere of the 1780s, Fergola's synthetic approach faced similar criticisms because of its refusal to accept the algebraic turn that had taken place in France.[38] At that time Fergola was already popular—but as a private teacher of mathematics and as a teacher of philosophy at a *liceo.* It is significant that, as late as 1786, he was refused a pension by the Royal Academy of Sciences; it went instead to an obscure professor at the Royal University. The key period for his rise to importance seems to be around 1789, corresponding to the radicalization of the political debate. At that time, the various forces within the conservative movement—some members of the upper hierarchy of the Church, officials of some cultural institutions, isolated thinkers, and some representatives of the feudal aristocracy in the countryside—rallied around the monarchy. Meanwhile, reformist thinkers, unorthodox Catholics such as the Jansenists, the "enlightened" aristocracy of the city of Naples, Freemasons, and revolutionaries were all grouped under the label of "Jacobins" and put under strict police surveillance. In the course of this simultaneous process of radicalization and reaction, the synthetic school became what we might describe as the "scientific component" of the Reactionary Catholic movement, which supported the new alliance between throne and altar.

## APOLOGETICAL EMPIRICISM

How did mathematicians become central to Reactionary Catholicism—which, like other forms of conservative thought, has been described as denying the value of all modern science, particularly mathematics?[39] The fact is that the new cultural elite, or at least the majority of it, did not oppose "modern science" *tout court.* The successes of the applied sciences had been too conspicuous to be denied. Further, mathematics was held to be too important a weapon to be ceded to the Jacobins. Thus powerful elements in both Church and government supported the research and teaching of Fergola, not only because they approved of his personal behavior, but also because they valued his work. As we have seen, Fergola's work in natural philosophy and his theological views of the soul were

[37] On the visit see [Telesio], *Elogio di Fergola* (cit. n. 21), pp. 59–60; on Caracciolo see Benedetto Croce, *Uomini e cose della vecchia Italia* (Bari: Laterza, 1956), pp. 82–111. For Lagrange's reply to the invitation see Lagrange, *Oeuvres,* ed. Serret (cit. n. 9), Vol. 14 (Paris: Gauthier-Villars, 1892), pp. 279–282.

[38] Luigi Telesio, one of Fergola's pupils and later his biographer, describes an episode that occurred in the late 1780s. Some officers of the French army visited Fergola's school and questioned the students in order to assess their mathematical knowledge. According to Telesio, the officers were surprised by the students' competence in classical methods and by their ability to use those methods, rather than the more common analytic procedures, to solve geometrical problems. See [Telesio], *Elogio di Fergola,* p. 78.

[39] Mannheim himself, in describing the "conservative style of thought" in political writers, philosophers, historians, and novelists, excludes the sciences from his analysis. Empirical sciences and mathematics are presented as part of the "liberal style of thought." There is no space, in his view, for a "conservative style of thought" in science.

linked by their common antimaterialistic focus. Mathematics fit into this apologetic project through proponents' delineation of a "pure mathematics" that was rigidly demarcated, on ontological and epistemological grounds, from the empirical sciences.

Let us approach the point through the work of two ecclesiastics whose apologetical writings made use of Fergola's mathematical works: Francesco Colangelo (1764–1836) and Gioacchino Ventura (1792–1861). Colangelo was a member of the Congregazione dell'Oratorio in Naples, an institution dedicated to the education of the young. The Oratorio had been a center for the elaboration of Reactionary Catholic thought in Naples. Since 1780 it had hosted the meetings of the Accademia Arcivescovile, with its explicit antiencyclopedistic aims. Colangelo, who was in contact with a number of conservative Italian thinkers, was interested in science, philosophy, and history, as his rich output testifies. His contribution to Reactionary Catholicism is well exemplified by his reflections on the revolution of 1799 and by his *Apology for the Christian Religion* (1818). But he also wrote on Vico and published a history of Neapolitan mathematicians and philosophers.[40] During the Restoration Colangelo would become bishop of Castellamare e Lettere (1821), minister of education (1824–1831), and director of the Royal Printing Office (1833). During the Jacobin period and the early Restoration he maintained close relations with Fergola. In 1804 he asked Fergola to prepare a "scientific" defense of the possibility of miracles. In response, Fergola produced the essay on miracles later published by Flauti. But 1804 also saw the publication of an interesting book by Colangelo himself on the progress of the sciences.[41] Here he maintained that the so-called freedom of philosophizing or "freedom of thought" that had characterized the eighteenth century was, in fact, an obstacle to such progress. The book is an attack on the autonomy of scientific practice from religion: such autonomy is presented as a loss, not a gain, for science. Colangelo claims that only a scientist who is also a good Christian can achieve relevant scientific results. His point depends on a fundamental division of reality into two sorts: empirical and metaphysical. These ontological regions are fields for two essentially different kinds of investigation: the empirical sciences, whose objects of study are "secondary causes"; and the theological sciences, which deal with the "primary cause," corresponding to the will of God. Mathematics lies in between, somewhat closer to the theological than to the empirical side.

Colangelo also used Fergola's work in his treatment of the modalities according to which God rules the universe. God is the "monarch of the universe"—that is, "a free master" and not "a mechanical agent." This characterization of God as an absolute monarch, whose will is the source of all natural law, assumes particular significance given contemporaneous debates about turning the Bourbon monarchy into a constitutional monarchy. It can indeed be argued that the whole debate on the possibility of miracles, which was particularly intense around the turn of the nineteenth century, was related to the political issue of whether the king was subject to his own law or was himself the source of sovereignty. As Fergola, and Colangelo after him, argued, if we admit that a miracle is an event that breaks some natural law, and if we admit that God can break his own laws according to his will,

[40] See Francesco Colangelo, *Riflessioni storico-politiche su la Rivoluzione accaduta a Napoli nel 1799* (Naples: Orsino, 1799); Colangelo, *Apologia della religione cristiana, compilata dalle risposte degli antichi Padri della Chiesa alle accuse fatte al cristianesimo* (Naples: Orsino, 1818); Colangelo, *Saggio di alcune considerazioni sull'opera di G. B. Vico intitolata Scienza nuova* (Naples: Trani, 1822); and Colangelo, *Storia dei filosofi e matematici napoletani e delle loro dottrine da' Pitagorici al secolo XVIII dell'era volgare,* 3 vols. (Naples: Trani, 1833–1834).

[41] Francesco Colangelo, *L'irreligiosa libertà di pensare, nemica del progresso delle scienze* (Naples: Orsino, 1804).

then miracles are possible. Such questions were much less abstract than they might now seem. King Ferdinando IV "legitimately" abolished the Sicilian constitution (1817) and, after the political turmoil of 1820–1821, the constitution of the kingdom.[42]

What is most relevant to our present purposes is Fergola's influence on Colangelo's treatment of the question of the certainty of the sciences. Not surprisingly, given his view of the different levels of reality and their corresponding forms of knowledge, Colangelo argued that it is impossible to reach certainty in empirical investigations. Certainty, as we have it in mathematics, is not a feature of the other sciences, where we can talk only of different degrees of "moral certainty" and "we can proceed only by means of *probability, conjectures,* and *approximation* to the truth." This empiricist position, which I call "apologetical empiricism," is characterized by its advocacy of the limited use of mathematical tools in the investigation of nature, by the practice of inductive generalization, and, consequently, by the recognition that scientific knowledge is essentially fallible and revisable. It is discussed at length in another book by Colangelo, published immediately after the Bourbon Restoration established by the Vienna Congress. The title may seem surprising: *Galileo come guida per la gioventù studiosa* [Galileo as a guide for the young student]. Colangelo portrays the Tuscan philosopher as a champion of the (apologetical) empirical investigation of nature and extracts some rules for "contemplators of nature" from his works. These correspond closely with the "canons" for investigating natural phenomena given by Fergola in his lectures on Newton. Again we are told that the empirical sciences can make use of mathematics, but with extreme caution. They must "not cover the natural data with a great quantity of analytical formulas, which are not part of nature itself [*non le appartengono*]. Colangelo cautions against looking at nature through the distorting artificial (mathematical) systems created by the minds of metaphysicians. In contrast to this analytic and mathematizing approach to empirical investigation, he proposes a procedure based on contemplation, data collection, and inductive generalization. The book is dedicated to Fergola, "a *maestro,* and a friend."[43]

Gioacchino Ventura's apologetical writings provide another example of the use of Fergola's mathematical works. A former Jesuit, Ventura was a member of the congregation of the Theatines—an order dedicated to the reform of clerical life through asceticism and apostolic work—when he met Fergola. He was a well-known speaker and controversialist who would later be given a chair of canon law at the University of La Sapienza, in Rome,

[42] *Ibid.,* p. 289. Isaac Newton is one of Fergola's resources in the theological discussion on miracles. Fergola quotes largely from the "Scholium generale" of the *Principia,* where God is said to rule "non ut anima mundi, sed ut universorum dominus. Et propter dominioum suum, dominus deus Παντοκρατωρ, id est Imperator universalis, dici solet": Isaac Newton, *Philosophiae naturalis Principia mathematica* (1687), 2 vols., Vol. 2 (Cambridge: Cambridge Univ. Press, 1972), p. 760. Fergola remarks: "The Lord rules heavens and nature as a sovereign. He didn't write the destinies of things like constitutional laws of the universe, to which He is not subject. He is not the god of the Stoics, *quid scripsit fata, sed sequitur qui semel iussit et semper paret.*" Fergola, *Teorica de' miracoli* (cit. n. 34), p. 54. The political implications of the theological debates over God's will and God's wisdom have been pointed out in Steven Shapin, "Of Gods and Kings: Natural Philosophy and Politics in the Leibniz-Clarke Disputes," *Isis,* 1981, *72*:187–215.

[43] Colangelo, *Irreligiosa libertà di pensare,* p. 357; and Francesco Colangelo, *Galileo come guida per la gioventù studiosa* (Naples: Orsino, 1815), pp. 6–7. See also [Fergola], *Prelezioni sui Principia matematica* (cit. n. 35), Vol. 2, p. 319. What I call "apologetical empiricism" has some similarities with Popkin's "mitigated scepticism." See Richard H. Popkin, *The History of Scepticism from Erasmus to Spinoza* (Berkeley: Univ. California Press, 1979), pp. 129–150. It should be noted that the two positions were elaborated with opposite aims (i.e., against the "scientism" of Condillac, Condorcet, and the Neapolitan Jacobins, and in response to the *crise pyrrhonienne*). Moreover, the stress on the metaphysical relevance of mathematics and on the necessity of integrating scientific investigations with religious values (e.g., "humility") seems to differentiate "apologetical empiricism" from earlier phenomenalistic conceptions of scientific knowledge. See also note 35, above.

by Pope Leo XII. His publications include apologetical, philosophical, and political works. A fervent admirer of the Middle Ages as the most complete form of Christian society, he was responsible for the introduction of French Traditionalism in Italy, mainly through his journal, *Enciclopedia Ecclesiastica e Morale Cattolica* (1821–1823), which was published in Naples. Ventura made the official speech at Fergola's public funeral, held in a baroque Theatine church, in 1824. His presentation of Fergola's life and works is of some interest to us. He stresses the ascetic life of this *maestro* and portrays him as proof that it is possible to be both a good mathematician and a good Christian. Indeed, Ventura argues that only a Christian can penetrate mathematical truth and investigate empirical reality properly. In the hands of atheists such as Laplace and D'Alembert, mathematics and science are terrible weapons for the destruction of certainty and, in the end, of society (see the epigraph to this essay, which is taken from Ventura's speech). Ventura does not enter into the details of the mathematical work of the synthetic school. Nevertheless, he repeats the Fergolian condemnation of the preeminence of analysis over geometry in eighteenth-century mathematics and warns against the use of analysis in the natural and social sciences. The "cold algebraists," he says, reduced the once-noble science of mathematics to mere mechanical calculation, without any deeper meaning. While Fergola "sees God behind the circle and the triangle," these atheists "see only the nothingness behind their formulas." Mathematics as practiced by Fergola and his pupils is a "spiritual science," whereas "algebraized" mathematics is a "material science."[44] Attributions of "spirituality" and "materiality" to the different practices of mathematics seemed unproblematic to men such as Colangelo and Ventura. In their usage the adjectives "spiritual" and "material" have at least a threefold reference. They refer to what we would call "pure" and "applied" mathematics, to different human faculties (intellect and sensibility), and to different orders of reality (metaphysical and empirical). Consequently, they imply moral judgments. To clarify this link between mathematics, the theory of knowledge, and morals, we must return, for the last time, to Fergola and his school.

## PURE MATHEMATICS AS "ANTIMODERN" MATHEMATICS

Fergola stressed the purity of the science of mathematics and contrasted it to the impure, applied character of mixed mathematics—that is, those sciences where mathematics is put to work (mainly through applications of integral and differential calculus). No one before him, at least in Naples, had presented the boundary between the two spheres as sharp and insuperable. More important, no one had stated that "real" mathematicians should deal exclusively with pure mathematics. In fact, the best Neapolitan mathematicians of the eighteenth century had worked mostly in applied mathematics—as had Fergola himself in the early part of his career.

What characterizes the realm of pure mathematics—which, for Fergola, meant pure geometry—is the presence of "certainty." Mathematical propositions are absolutely true, and the theorems are proved with absolute certainty. By contrast, when mathematics is applied—that is to say, when mathematical reasoning is employed with objects that are not abstract, simple, and pure geometrical entities—this certainty is lost. Material reality is too complex to be captured by mathematical reasoning. It is, of course, possible to use

[44] Ventura, "Elogio di Fergola" (cit. n. 1). On Ventura see Paolo Cultrera, *Della vita e delle opere del Rev. P. D. G. Ventura* (Palermo, 1877); and Mario Tesini, *Gioacchino Ventura: La chiesa nell'età delle rivoluzioni* (Rome: Studium, 1988).

mathematical reasoning in fields like physics, but the results are only hypothetical and always revisable. In the more complicated human sciences—politics, morals, religion, and the sciences of society—such application is impossible. The politician, for example, cannot simply calculate the solution to his problems. Political reality cannot be managed by means of abstract, universal laws because of the complexity of its "objects." The politician, then, has to rely on his own practical experience, on his intuition, on his knowledge of the history of the elements that are part of the problem.[45] To sum up, the "apologetical empiricism" of the natural and social sciences is the inevitable result of the limitation of intuitive certainty to pure mathematics and metaphysics.

Fergola traces boundaries between geometry and algebra (the first being ontologically superior, the language of God, and intuitively apprehensible; the second being an artificial, human creation and merely instrumental) and between pure and applied mathematics (where, respectively, certainty is and is not attainable). These boundaries in mathematics are then employed as a "scientific" basis for tracing boundaries between the different areas of human knowledge and putting them in a hierarchical order. Metaphysics, pure mathematics, and the empirical sciences do not overlap. This is not a question of degree of certainty: the truths of faith, of geometry, and of the empirical sciences are essentially different because they refer to different "orders" of reality.[46] One result of Fergola's boundary-drawing strategy, and of the stress on pure mathematics, is to make impossible the application of mathematical reasoning to fields such as metaphysics or the sciences of society. It is important to remember that, in the second half of the eighteenth century, some philosophes had proclaimed the birth of a new discipline, the "mathematics of society," which they claimed would provide rules for designing a "scientific" society.[47] This was exactly the plan of the Neapolitan Jacobins.

[45] These arguments—the opposition between "abstract" laws and "concrete" practice, for example—are common among conservative European authors. The Neapolitan reactionary intelligentsia made reference, in particular, to works by Edmund Burke (1729–1797); Nicola Spedalieri (1740–1795), theorist of the theocratic society; Pietro Tamburini (1737–1827), who stressed the empirical nature of political knowledge; Karl Ludwig von Haller (1768–1854); and Augustine Barruel (1741–1820), one of the founders of the theory of the "great philosophical plot" against religion and society. They also appealed to the "Traditionalists" Joseph de Maistre (1753–1821), Luis de Bonald (1754–1840), and Félicité de Lamennais (1782–1854).

[46] Fergola presents a division between different levels of reality, and a corresponding hierarchy among the human faculties, which reflects the Thomistic theory of knowledge. According to Fergola, faith, intellect, and sensibility deal, respectively, with theology (metaphysics), pure mathematics, and the empirical sciences. This threefold structure is analogous to that presented in the fifth question of Aquinas's comments on the *De Trinitate* of Boethius, which deals with the division of the theoretical sciences. See Thomas von Aquin, *In librum Boethii De Trinitate quaestiones quinta et sexta,* ed. Paul Wyser (Fribourg: Société Philosophique, 1948), pp. 23–51.

[47] Étienne Bonnot de Condillac (1714–1780) aimed to investigate morals and metaphysics with the exactness proper to geometry. He considered the analytic method the most appropriate mode of reasoning because it is simpler and easier than the synthetic one. Algebra became, for Condillac, the universal language that can be employed in any field of human experience. Analytic reasoning creates the proper link between simple sensations and complete knowledge, avoiding the pernicious metaphysical systems of past philosophies. See Étienne Bonnot de Condillac, *Traité des sensations* (1754), in *Oeuvres philosophiques de Condillac,* ed. George Le Roy, 4 vols., Vol. 1 (Paris: Presses Univ. France, 1947), pp. 218–314; and Condillac, *La langue des calculs* (1798), *ibid.,* Vol. 2 (Paris: Presses Univ. France, 1948), pp. 417–529. Marie Jean Antoine, marquis de Condorcet (1743–1794), introduced the notion of "social mathematics." This was a new science that aimed to apply mathematical tools to the human sciences. Its immediate objectives were to describe society by means of statistics and to establish scientific economic theory. Condorcet was explicitly attacked by Fergola and other Reactionary Catholic authors; see, e.g., Ventura, "Elogio di Fergola" (cit. n. 1), p. 97. Particularly important from our point of view are Marie Jean Antoine, marquis de Condorcet, "Tableau général de la science qui a pour object l'application du calcul aux sciences physiques et morales" (1795), in *Oeuvres de Condorcet,* ed. M. F. Arago, 12 vols., Vol. 1 (Paris: Firmin Didot Frères, 1847), pp. 539–573; and Condorcet, *Essai sur l'application de l'analyse à la probabilité des decisions rendues à la pluralité des voix* (Paris, 1785), where Condorcet applied analysis to the voting process. See also Gilles Granger, *La mathématique sociale du Marquis de Condorcet* (Paris: Presses Univ.

It can be argued that the image of mathematics shared by the members of the synthetic school enjoyed the essential features of what Mannheim described as the "conservative style of thought." Conservativism, for Fergola and his pupils, was not simply a political option or a generic reason for studying ancient geometers while ignoring contemporary ones. Conservativism shaped the very content of the mathematical knowledge produced and transmitted in the school: what they shared was a conservative or antimodern image of mathematics.

I have already made reference to the reactive character of conservative thought in its political and social forms. The same reactive character, the same form of argumentation, point by point, can be seen in the work of Fergola and his pupils. Their enemy is the pragmatic, instrumental, universally applicable mathematics employed by the reformists and Jacobins to support their arguments. They understood this "modern" mathematics as centered on the application of integral and differential calculus to empirical reality—natural and social. According to the synthetics, the question of "certainty" is not a central issue for the modern analytics, who give priority to the practical success of applications. Modern mathematics is then said to be *general,* because applicable to every kind of object, and *abstract,* because its formulas lack intuitive content. This, in turn, makes it unreliable, because it lacks certain foundations. What, in contrast, characterizes the synthetic image of mathematics? Fergola describes his shift toward geometry as motivated by a foundationalist aim. In fact, his work is a very early attempt to "rigorize" the calculus and to ground the whole of modern mathematics on the secure base of geometrical intuition. As Flauti writes, we can "try to make [the modern algebraic method] rigorous, giving it a geometrical connection [*nesso geometrico*]," so that, if geometry "can see its principles extended by algebraic Analysis, nevertheless it rewards [analysis] largely through making its results clear and supporting them."[48] The science of geometry, according to the synthetic image, is founded on *intellectual intuition,* and that is the final guarantee of its absolute certainty. As Fergola maintains, geometrical intuition is an intellectual (dianoethic) virtue, the highest achievement of the human mind. It is the result of an act of contemplation. The true mathematician is essentially a contemplator. First, he contemplates nature, in order to glimpse some fundamental geometrical truths (nature is, after all, the second book of God, and it is written in geometrical characters). Second, he contemplates pure geometrical entities, in order to discover the eternal truths of geometry. Geometry is a "pure," "intellectual," "spiritual" science because its axioms, reasonings, and conclusions are completely independent of matter. According to this image, in which geometrical intuition and geometrical reasoning have absolute preeminence, algebra is a merely instrumental branch of mathematics, a useful language that allows us to reach certain geometrical results in an "easier" way. But the use of algebra is legitimate and reliable only when its formulas refer directly to geometrical entities, their final source of meaning and the ground of their results. This implies that any use of algebraic algorithms outside the geometrical context is illegitimate. Their conclusions, in such cases, cannot be considered certain. What corrupts the purity of mathematics in these applications is precisely the presence of matter. This is why modern mathematics is said to be a *material* science: it deals with material reality and thus employs the mathematical apparatus merely as a rhetorical weapon. No certain result can be achieved through such a science.

France, 1956); and Keith M. Baker, *Condorcet: From Natural Philosophy to Social Mathematics* (Chicago: Univ. Chicago Press, 1975).

[48] Flauti, preface to Fergola, *Invenzione geometrica* (cit. n. 15), p. xi.

Considerations about the intuitiveness, the perspicuity, and the concreteness of geometry, as opposed to the abstract, artificial character of algebra, should sound familiar to Mannheim's readers. The first members of the pairs (natural/artificial, concrete/abstract, particular/general) are in fact essential to his definition of the conservative style of thought. Moreover, the preeminence of teaching over original research and the importance of didactic considerations are typical of Fergola as well as of conservative political authors. If we move to the historical perspective of the synthetic school the analogy is, again, striking. The history of mathematics, according to the synthetic school, is the history of the corruption of a venerable, homogeneous tradition attacked by the perverse philosopher-mathematicians of the eighteenth century. They are the "cold algebraists," the "calculators" who reduced mathematics to a practical instrument deprived of any deeper metaphysical meaning. The loss of meaning is seen as a consequence of the "diversion from geometry [*aberrazione dalla geometria*]" that took place in the eighteenth century.

I began this essay by noting what seemed obscure accusations of moral depravity addressed to the analytics by the synthetics in the 1830s. We can now make sense of such accusations.[49] Analysis had been the privileged tool employed by modern mathematicians in their attempt to apply mathematics to any and every aspect of reality. The synthetics, along with other conservative authors, opposed this "algebraization" of natural and social reality. They considered this illegitimate extension of analysis an act of hubris, of arrogant violence, deriving from moral corruption.[50] Moderns do not respect the boundaries between different disciplines—that is, between different human faculties and different orders of reality. Not everything, according to the synthetics, can be explained by means of mathematical reasoning.

As for politically conservative authors, so too for Fergola and his school: the present (specifically, for Fergola, the present condition of mathematics) was simply the result of the corruption of the past. Luigi Telesio's apologetical biography of Fergola contains an engraving of the mathematician, accompanied by the following distich: "Scrutari Veteres Felix, Felicior Idem/Ante Novos Omnes Ire Mathematicos."[51] In mathematics, as in political and religious matters, the modern age had to be surpassed—and this could be achieved only through a return to the original sources of human knowledge. The future could only be in the past.

## CONCLUSIONS

This essay has had a dual purpose. First, I wanted to present a relatively unfamiliar episode in the history of mathematics. This episode represents an interesting variation in the wider

[49] When Padula was writing, in the 1830s, there was yet another interesting component: the debate about the social status of engineers (like Padula) and the limits of their authority. Behind this debate lay the fundamental question of control of the territory of the kingdom, which was contested between the central government and local elites. The controversy over the new professional figure of the engineer and his mathematical education is treated in Massimo Mazzotti, "Engineers and Romantic Painters: Science, Art, and Administration in the Kingdom of Naples (1830–1840)," unpublished MS, 1998.

[50] The term *ibrido* (hybrid) appeared in the Italian language during the eighteenth century to indicate plants or animals obtained from the reproduction of exemplars of different species. But the classic moral connotation (Latin, *hybrida;* Greek, *hy'bris*) echoes in Flauti's words when he refers to the analytics' work as "hybrid advancements [*ibridi progressi*]": Flauti, "Programma" (cit. n. 10), p. 10. See *Dizionario etimologico italiano* (Florence: Barbera, 1952). On "scientistic hubris" see Friedrich A. Hayek, *The Counter-Revolution of Science: Studies on the Abuse of Reason* (Glencoe, Ill.: Free Press, 1952), pp. 105–116.

[51] "Happy to study the ancients, more happy/to surpass all the modern mathematicians": [Telesio], *Elogio di Fergola* (cit. n. 21), p. 4.

debate between supporters of synthetic methods and supporters of analytic methods in geometry and, more generally, in the early nineteenth-century controversy as to whether mathematics is essentially a "pure" or an "applied" science (indeed, "pure mathematics" as an academic discipline was born in these years).

The second purpose was to show that my protagonists' choices to privilege one or the other problem-solving method depended *essentially* on contemporary cultural and political struggles. In particular, I have tried to show how the synthetic school of Fergola, devoted to the study of the ancient authors, to the historical reconstruction of their lost works, and to the imitation of their elegant problem-solving methods, was a *new* phenomenon in the Neapolitan mathematical milieu of the late eighteenth century. The school, and its purely geometrical approach to mathematics, was in fact a *reactive* phenomenon. Concealing this reactive character was part of Fergola's strategy. The school presented itself as continuing a venerable, unitary (but in fact wholly imaginary) tradition that allegedly flowed from the Greek geometers to the Renaissance Italian geometers, to Galileo, to Viète, to Descartes, and to Newton. At the beginning (1786–1789), the purely geometrical interests of the school had been contested by the Francophile and reformist intellectuals who dominated the cultural life of the Kingdom of Naples. The historical circumstances of the 1790s opened the way for the sudden success of Fergola's teaching and allowed him to create a popular, well-institutionalized school.[52] What made Fergola's teaching and research so appealing to the government and the Church was his exemplary loyalty and his fanatical faith. Moreover, the content of his work suited the reactionary cultural politics of the Bourbons, who pointed to him as an example of how to do good science without contradicting religious dogmas or entering into dangerous questions about social and political life. Fergola and his school also provided the reactionary movement with new intellectual resources, mainly by legitimating the discipline of "pure mathematics" (i.e., pure geometry), characterized by the absolute certainty of its reasoning and by its metaphysical relevance. The restriction of "certainty" to pure mathematics, the denial of the possibility that mathematics might be used as a universal key for solving every kind of problem, became—in Naples as elsewhere—the basis for the wider conservative response to the revolutionary effects of the principles of Universal Reason.

[52] For an overview on scientific thought in Naples in the late seventeenth and early eighteenth centuries see Fabrizio Lomonaco and Maurizio Torrini, eds., *Galileo e Napoli* (Naples: Guida, 1987).