Dynamic Asset Allocation in a Conditional Value-at-risk Framework

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature: ……………………………………………………………..

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Abstract

The thesis first extends the original Black-Litterman model to dynamic asset allocation area by using the expected conditional equilibrium return and conditional covariances based on three volatility models (the DCC model, the EWMA model and the RW model) into the reverse optimisation of the utility function (the implied BL portfolio) and the maximised Sharpe ratio optimisation model (the SR-BL portfolio). The momentum portfolios are inputted as the view portfolios in the Black-Litterman model. The thesis compares performance of the dynamic implied BL portfolio and the dynamic SR-BL portfolio in the single period and multiple periods with in-sample analysis and out-of-sample analysis. The research finds that dynamic BL portfolios can beat benchmark in in-sample and out-of-sample analysis, the dynamic implied BL portfolio always show better performance than the dynamic SR-BL portfolio. The empirical VaR and CVaR of the dynamic SR-BL portfolios are much higher than that of the dynamic implied BL portfolio. The dynamic BL portfolios based on the DCC volatility model perform best in contrast to other two volatility models.

In the aim of improving performance of SR-BL portfolios, the thesis further constructs dynamic BL portfolios based on two new optimisation models including maximised reward to VaR ratio optimisation model (MVaR-BL portfolios) and maximised reward to CVaR ratio optimisation model (MCVaR-BL portfolios) with assumption of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. The thesis compares performance of the dynamic MVaR-BL portfolio and the dynamic MCVaR-BL portfolio in the single period and multiple periods with in-sample analysis and out-of-sample analysis. There are three main findings. Firstly, both the MVaR-BL portfolio and the MCVaR-BL portfolio could improve the dynamic SR-BL portfolio performance at moderate confidence levels. Secondly, the MVaR-BL portfolio and the MCVaR-BL portfolio show similar performance with normal distribution assumption, the MCVaR-BL portfolio performs better than the MVaR-BL with t-distribution assumption at certain confidence levels in single period and multiple periods. Thirdly, the performance of the DCC-BL portfolio with t-distribution assumption is superior to the performance of the DCC-BL portfolio with normal distribution assumption.

As the result of higher empirical VaR and CVaR of dynamic SR-BL portfolios, the thesis develops to constrain VaR and CVaR in construction of dynamic BL portfolios with assumption of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. The research studies the effect of assumptions of two distributions, three confidence levels and levels of the VaR constraint and the CVaR constraint on dynamic BL portfolios. Both in-sample performance and out-of-sample performance could be improved by imposing constraints, and they suggest adding moderate CVaR constraints to maximal Sharpe ratio optimisation model with t-distribution at certain confidence level could obtain the best dynamic DCC-BL portfolio performance in the single period and multiple periods. The performance evaluation criterion (higher Sharpe ratio, reward to VaR ratio, and reward to CVaR ratio) would affect the choice of optimisation models in dynamic asset allocation.

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List of Abbreviations

BL	Black-Litterman Portfolio
CSR	Conditional Sharpe Ratio
CVaR	Conditional Value-at-Risk
CVaR-adjusted BL	Black-Litterman Portfolio with CVaR Adjusted Equilibrium Return
CVaR-BL	Black-Litterman Portfolio with CVaR Constraint
DCC	Dynamic Constant Correlation Model
ECSR	Expected Conditional Sharpe Ratio
EWMA	Exponentially Weighted Moving Average Model
Implied BL	Black-Litterman Portfolio with Implied Reverse Optimisation
MCVaR-BL	Black-Litterman Portfolio with Maximal Reward to CVaR Ratio
MVaR-BL	Black-Litterman Portfolio with Maximal Reward to VaR Ratio
N	Normal Distribution
PT	Portfolio Turnover
RW110	Rolling Window Estimator with Window Length of 110
RW50	Rolling Window Estimator with Window Length of 50
SR-BL	Black-Litterman Portfolio with Maximal Sharpe Ratio
t	t-Distribution
VaR	Value-at-Risk
VaR-adjusted BL	Black-Litterman Portfolio with VaR Adjusted Equilibrium Return
VaR-BL	Black-Litterman Portfolio with the VaR Constraint
Variance-adjusted BL	Black-Litterman Portfolio with Variance-Adjusted Equilibrium Return
$\mu / CVaR$	Expected Excess Return to Conditional Value-at-Risk Ratio
μ / VaR	Expected Excess Return to Value-at-Risk Ratio

CHAPTER 1
INTRODUCTION

1.1 Background and Rationale

It is well known that the concepts of portfolio optimisation and diversification play an important role in the development and understanding of financial markets and financial decision-making. In 1952, Markowitz made a breakthrough with the publication of the theory of portfolio selection. He suggested that investors should consider the trade-off between risk and return to determine the allocation of assets. Risk is measured as the standard deviation of returns around their expected values. The idea is based on the theory that a portfolio's riskiness depends on the covariances of its constituents instead of only on the average riskiness of its separate holdings. Building on Markowitz's work, Sharpe (1964) and Lintner (1965) designed the Capital Asset Pricing Model (CAPM) to describe asset returns. Since then, the modern portfolio theory has been gradually developed and applied to the financial markets. However, the portfolio suffers from problems of unrealistic weights such as extreme weights (Green and Hollifield, 1992), corner solutions of highly concentrated portfolios (Frost and Savarino, 1988; Grupa and Eichhorn, 1998; Grauer and Shen, 2000), and the sensitivity of the solution to inputs (Best and Grauer, 1991; Best and Grauer, 1992; Black and Litterman, 1992; Broadie, 1993) in the practice in the use of Markowitz's mean-variance optimisation. The main reason for these problems is estimation errors in the expected returns as a key input of the mean-variance model (Merton, 1980; Michaud, 1989; Chopra and Ziemba 1993). It is necessary to use some robust estimates of input parameters or else resort to new models for optimisation problems to achieve reliability, stability, and robustness with regard to estimation errors or modelling errors. Several researchers have proposed that the robust estimates should include the Bayesian approach (Zellner and Chetty, 1965; Brown, 1976; Frost and Savarino, 1986; Black and Litterman, 1990; Polson and Tew, 2000; Pástor, 2000), a shrinkage estimator (Jorion, 1985; Jorion, 1986; Chopra, 1993; Ledoit and Wolf, 2003; Ledoit et al., 2004) and factor models (Fama and French, 1992; Fama and French, 1996). Other researchers have focused on optimisation modelling areas such as portfolio resampling (Michaud, 1998; Scherer, 2002; Scherer, 2004; Michaud, 2008; Harvey et al., 2008) and robust optimisation techniques (Fabozzi et al., 2007; Fabozzi et al., 2010).

Since mean-variance analysis only uses a single set of estimates from prior information, Zellner and Chetty (1965) develop the Bayesian approach which combines prior distribution and posterior distribution into a single estimate to solve the parameter uncertainty problem. The prior distribution reflects an investor's knowledge about the probability, before external information sources are observed. After new information is provided, the investor adjusts their beliefs about the probability to obtain the posterior distribution. The Bayesian approach assumes that the expected returns are unknown and random. Three main methods have been proposed to calculate the prior means, such as shrinking the tangency portfolio towards the global minimum-variance portfolio (Jorion, 1986), shrinking the tangency portfolio towards the market portfolio (Pástor, 2000; Pástor and Stambaugh, 2000), and shrinking the tangency portfolio towards the market portfolio, but with the tangency portfolio based on subjective investor forecasts instead of sample means (Black and Litterman, 1990). Herold and Maurer (2003) confirm the superior out-of-sample performance of the Bayesian approach in contrast to the mean-variance portfolio approach, and also suggest using the promising Black-Litterman approach (Black and Litterman, 1990; Bevan and Winkelmann, 1998; He and Litterman, 1999; Satchell and Scowcroft, 2000; Drobetz, 2001; Idzorek, 2004) in tactical asset allocation.

A mean-variance analysis which uses standard deviation as a measure of risk has conceptual difficulties, given the undesirable properties of satiation and increasing absolute risk aversion (Huang and Litzenberger, 1988). Besides, asymmetric return distributions make standard deviation an intuitively inadequate risk measure because standard deviation equally penalises desirable upside and undesirable downside returns (King and Wadhwani, 1990). It is well known that the distribution of asset returns is not normal (Mandelbrot, 1963; Fama, 1965; Müller et al., 1998; Rachev and Mittnik, 2000; Rachev et al., 2008). Both academics and practitioners have paid attention to measuring alternative risk such as the safety first strategy (Roy, 1952), semivariance (Markowitz, 1959), lower partial moment (Bawa, 1975), mean absolute deviation (Konno and Yamazaki, 1991), Value-at-Risk (VaR) (Jorion, 1997; Ahn et al., 1999; Basak and Shapiro, 2001), and Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev, 2000; Rockafellar and Uryasev, 2002). Note that VaR fails to meet the coherence of risk measures (Artzner et al., 1999) and convexity properties (Tasche, 2002; Föllmer and Schied, 2002). However, CVaR has tractable properties, including coherent risk measures, easy implementation, and it takes into consideration the entire tail that exceeds VaR on average. In addition, it is not appropriate to consider only the first and second moment of the distribution in portfolio optimisation, which might increase extreme risks (Sornette et al., 2000; Amin and Kat, 2003) and might lead to loss of utility for investors (Cremers et al., 2005). Because of the shortcomings of mean-variance optimisation, several researchers have introduced VaR (Huisman et al., 1999; Campbell, 2001; Favre and Galeano, 2002) and CVaR (Souza and Gokcan, 2004; Agarwal and Naik, 2004) to extend portfolio optimisation techniques under fat-tail distributions. Alexander and Baptista (2001, 2002, 2004) thoroughly study the implications of VaR and CVaR constraints on the mean-variance model, based on theoretical work.

It is widely agreed that financial asset return volatilities and correlations are time-varying, with persistent dynamics. Asset return volatilities enter as an important ingredient in many applications, such as portfolio optimisation and market risk measurement. Perhaps the most popular approaches used to model the conditional covariance matrix of returns are the multivariate GARCH class of models. These models include the Vech and Diagonal Vech models (Bollerslev et al., 1988), the BEKK model (Engle and Kroner, 1995), the Constant Correlation model (Bollerslev, 1990), the Factor ARCH model (Engle et al., 1990), and the Dynamic Conditional Correlation model (Engle and Sheppard, 2001). However, the Vech model and the BEKK model suffer from the curse of large dimensionality, and the Diagonal Vech models, the Constant Correlation model and the Factor ARCH model have cross-equation restrictions on the elements of the covariance matrix (Harris et al., 2007). Other approaches such as rolling estimators of the sample covariance matrix, the exponentially weighted estimator and multivariate stochastic volatility models (Harvey et al., 1994) can also be used to estimate the conditional covariance matrix. In the portfolio optimisation world, portfolio managers usually work on a large number of assets to diversify the unsystematic risk; the relationship and the co-movement among those assets will directly affect the performance of the whole portfolio. The choice of volatility models is an art.

Nowadays, the Black-Litterman (BL) model, an intuitive model based on the desire to combine neutral market equilibrium returns and individual active views, has become the most popular method to estimate the expected return in practice. This model creates stable and intuitively appealing mean-variance efficient portfolios based on investors' subjective views, and eliminates the input sensitivity of the mean-variance optimisation. More and more portfolio managers and financial advisors are choosing this model to support their investment decisions. There are a number of recent research papers which apply the BL model to comply with asset return regularities, such as non-normal distributions and volatility clustering (Giacometti et al, 2007; Meucci, 2006, 2007, 2008; Martellini and Ziemann, 2007; Beach and Orlov, 2007; Palomba, 2008), which can be evaluated by alternative risk measures (Martellini and Ziemann, 2007; Lejeune, 2011; Veress et al., 2012), and incorporated with trading strategies (Fabozzi et al., 2006; Babameto and Harris, 2009), along with other, wider applications (Becker and Gürtler, 2009; Da Silva et al., 2009; Cheung, 2009; Giacometti and Mignacca, 2010; Munda and Strasek, 2011; Mishra et al., 2011; Fernandes et al., 2011; Braga and Natale, 2012).

However, the literature regarding taking VaR and CVaR into account in the BL model is rather limited. In the application of the BL model, Bevan and Winkelmann (1998) analyse portfolio risk by tracking error and the market exposure, and mention that VaR can also be used to measure BL portfolios. Giacometti et al. (2007) focus on generating VaR and CVaR adjusted equilibrium returns with different assumptions of asset return distributions (the normal distribution, the t-distribution, and the stable distributions) in the BL model. Lejeune (2011) regards the VaR and trading requirements as constraints on optimising a BL portfolio in constructing a fund-of-funds following an absolute return strategy. None of the studies impose CVaR constraints on a BL portfolio and no empirical work has been done in inputting the BL expected return into an optimiser which maximises the alternative performance measures, such as reward to VaR ratio and reward to CVaR ratio. One of the research aims is to fill these gaps, supported by empirical work to compare the performance of BL portfolios when optimised by different methods.

Moreover, few documents focus on extending the application of the BL model into the use of volatility models. Beach and Orlov (2007) utilise GARCH models to derive views as an input into the BL model, while Palomba (2008) introduces multivariate GARCH estimates for large-scale tactical asset allocation, expecting to view returns inputted into the BL approach with tracking error constraints. Strictly speaking, tactical asset allocation from their work is driven by time-varying expected view returns; they do not actually involve any estimation of a conditional BL expected return. Bollerslev et al. (1988) argue that investors may have common expectations on returns, which are variable conditional expectations instead of constants. They introduce the multivariate GARCH process into the CAPM to estimate conditional returns. Since nothing in the literature generalises the BL model in a dynamic framework with an estimation of conditional BL expected returns based on volatility models, another research aim is to extend the original BL model into the dynamic asset allocation area.

1.2 Research Aims and Questions

Overall, there are two main research aims. The first research aim is to extend the original BL model into a dynamic framework to make conditional expectations on returns, and then construct a dynamic BL portfolio that can beat a benchmark portfolio. The second research aim is to construct dynamic BL portfolios with VaR and CVaR taken into account, with the objective of improving portfolio performance. On the one hand, VaR and CVaR could be used in performance measures which could then become the optimisation target; for example, the dynamic, unconstrained BL portfolio allocates assets with maximal performance measures such as reward to VaR ratio and reward to CVaR ratio. On the other hand, VaR and CVaR could be used as a constraint on the portfolio optimisation model with a maximal Sharpe Ratio (SR), and a risk-constrained BL portfolio could be formed.

The research addresses the following questions:

1. If I introduce the volatility models into the BL model, which one should I choose to construct a BL portfolio with better performance? What is the impact of the volatility models on the construction of the BL portfolio?
2. If I construct dynamic, unconstrained BL portfolios, which performance measures should be maximised in the optimisation process to achieve

better performance? What is the impact of the choice of volatility models, distribution assumptions and confidence levels on the performance of unconstrained BL portfolios?

3. If I impose risk constraints, such as VaR and CVaR, on a dynamic BL portfolio, which constrained BL portfolio has the better performance? What is the impact of the choice of volatility models, distribution assumptions and confidence levels on the performance of VaR-constrained BL portfolios and CVaR-constrained BL portfolios? Will the constrained BL portfolios overcome the unconstrained BL portfolios?

In order to carry out a thorough evaluation of the unconstrained BL portfolio performance and the risk-constrained BL portfolio performance, the research evaluates both the single period performance and the multiple-period performance, based on an in-sample analysis and an out-of-sample analysis. In addition, there are plenty of volatility models to be selected in constructing a dynamic BL portfolio; the research narrows the choice of volatility models to include only rolling window estimators, exponential weighted estimators, and DCC models to simplify the covariance matrix forecasting process in asset allocation. Besides, this research studies the dynamic asset allocation on a monthly basis instead of other high frequency cases. The relaxation of these limitations can be accepted, but are beyond the scope of this thesis.

1.3 The Contributions of this Thesis

This research makes several contributions. At first, the thesis originally constructs a dynamic BL portfolio starting from a conditional estimation of equilibrium returns, combined with the view portfolios generated from dynamic momentum strategies based on three volatility models including rolling window estimators, exponential weighted estimators and DCC models. The thesis then uses the reverse optimisation implied in the BL model and uses a maximal SR optimisation to get the weight solutions of the implied BL portfolio and the SR-BL portfolio. The thesis evaluates the performance of these two portfolios in a single period and then over multiple periods, based on an in-sample analysis and an out-of-sample analysis, and makes a comparison with the benchmark portfolio. In single period, when the realised return is negative, the thesis adopts the adjusted conditional SR to make the performance evaluation valid, together with the portfolio turnover and the conditional reward to the CVaR ratio to evaluate the single period performance. Over multiple periods, the performance measures take the empirical VaR and CVaR into account and utilise reward to VaR ratios and reward to CVaR ratios to rank performance when compared with the rank from the SR. The research finds that, firstly, by using momentum strategy to generate views, the dynamic BL portfolio could generate a superior in-sample performance and out-of-sample performance to the benchmark portfolio. In addition, the dynamic BL portfolio has more balanced and realistic weight solutions than the traditional mean-variance portfolio. In addition, it could be suggested that the use of the DCC model is the best choice when constructing a BL portfolio with the best in-sample and out-of-sample performances over multiple periods. However, this suggestion cannot be robust in different single periods. Another interesting finding is that the in-sample and out-of-sample performances in the implied BL portfolio always outperform the SR-BL portfolio over multiple periods, although this finding is not robust in different single periods. Higher fat-tail risks are reflected in a highly negative skewness and a higher kurtosis appearing in the SR-BL portfolio over multiple periods, and the empirical VaR and CVaR are greater.

Secondly, the thesis takes action to further construct the dynamic, unconstrained BL portfolio with maximal reward to VaR ratio (MVaR-BL portfolio) and maximal reward to CVaR ratio (MCVaR-BL portfolio) and, for the first time to my knowledge, with an interpretation of the mean-VaR efficient frontier and the mean-CVaR efficient frontier. The thesis utilises the volatility models and the parametric method to estimate the VaR and CVaR in the asset allocation process with normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. In addition, the thesis studies the impact of different volatility models, distribution assumptions, and confidence levels on weights solutions, single period performance and multiple-period performance in the in-sample and out-of-sample analyses. Furthermore, the thesis also makes three pairs of comparison among the MVaR-BL portfolio, the MCVaR-BL portfolio and the SR-BL portfolio. The main findings include, first of all, that both the MVaR-BL portfolio and the MCVaR-BL portfolio could perform better than the SR-BL portfolio over the single period performance and the multiple period performance in the in-sample and out-of-sample analyses. Secondly, in normal distribution, there is a slight difference between the MVaR-BL portfolio and the MCVaR portfolio in an out-of-sample analysis; in t-distribution, the MCVaR-BL portfolio could overcome the MVaR-BL portfolio in certain circumstances. In addition, it could be suggested that the use of the DCC model in the MVaR-BL portfolio and the MCVaR-BL portfolio is the best choice in constructing a BL portfolio with the best in-sample and out-of-sample performance over multiple periods. However, this suggestion cannot be robust in different single periods. Finally, the performances of the MVaR-BL portfolio and the MCVaR-BL portfolio with a t-distribution assumption are superior to performances with a normal distribution assumption.

Thirdly, the thesis develops the study of adding VaR constraints and CVaR constraints to the dynamic, unconstrained BL portfolio, from understanding the VaR bounds and CVaR bounds on the mean-variance efficient frontier, to practical empirical work with originality. After building a VaR-constrained BL (VaR-BL) portfolio and a CVaR-constrained BL (CVaR-BL) portfolio based on three volatility models, the thesis compares the in-sample performance and the out-of-sample performance among volatility models, and then investigates in-depth to examine the effect of constraints, distribution assumption, and confidence levels on the risk-constrained BL portfolio weight solutions and portfolio performance. It can be found that both the in-sample performance and the out-of-sample performance of an SR-BL portfolio could be improved by imposing VaR and CVaR constraints, and it suggests that adding moderate CVaR constraints to the maximal SR optimisation model with t-distribution at certain confidence level could obtain the best dynamic BL portfolio performance in a single period and over multiple periods, based on the DCC model. In addition, the thesis also follows the method of Giacometti et al. (2007) in constructing a risk-adjusted BL portfolio with an estimation of VaR-adjusted equilibrium return and CVaR-adjusted equilibrium return on the out-of-sample basis; the thesis further makes a comparison between unconstrained BL portfolios and risk-constrained BL portfolios. The results from the thesis reflect that the risk-adjusted performance in both the VaR-adjusted BL portfolio and the CVaR-adjusted BL portfolio are better than most of the unconstrained BL portfolios, but the active performance fails to beat the MVaR-BL portfolio and the MCVaR-BL portfolio. Moreover, the VaR-adjusted BL portfolio and the

CVaR-adjusted BL portfolio have quite a limited ability to outperform the VaR-constrained BL portfolio and the CVaR-constrained BL portfolio in the t-distribution at a moderate level of constraints.

1.4 Structure of the Thesis

The thesis contains seven chapters, including this introduction as Chapter 1. Chapter 2 gives an overview of the asset allocation theory and its application, including risk measures.

Chapter 3 provides a thorough review of the BL model, with well-explained mathematical formulae and the underlying intuition, extensions and applications of the BL model summarised from the literature. This chapter also discusses the closely related papers written by Giacometti et al. (2007) and Lejeune (2011).

Chapter 4 analyses the dataset and examines the time series property of the excess return calculated from the data. Then, Chapter 4 demonstrates the methodology of constructing a dynamic BL portfolio, involving an estimation of conditional equilibrium return in the first step, inputting the view portfolios in the second step, and generating the BL expected return in the third step, based on an estimation of the covariance matrix via the RW model, the EWMA model and the DCC model. Chapter 4 also illustrates the construction of an unconstrained BL portfolio with maximal reward to VaR ratio and maximal reward to CVaR ratio. The method of adding VaR constraints and CVaR constraints in the maximal SR optimiser are also interpreted in this chapter.

Chapter 5 concentrates on showing the empirical results of the dynamic BL portfolios in the in-sample framework, following the methodology illustrated in Chapter 4. There are four sections in this chapter. Section 5.1 shows details of the construction of a dynamic, unconstrained BL portfolio, including the MVaR-BL portfolio and the MCVaR-BL portfolio; explains the optimisation process; investigates and analyses the effect of distributions and confidence levels on the weight solutions, the single period performance and multiple-period performance, and makes a comparison between the unconstrained BL portfolios. Following the same structure as Section 5.1, Section 5.2 works on the VaR-constrained BL portfolio, while Section 5.3 works on the CVaR-constrained BL portfolio. Conclusions are made in each section.

Chapter 6 follows the same structure as Chapter 5 but focuses on working the dynamic, unconstrained BL portfolio, the VaR-constrained BL portfolio and the CVaR-constrained BL portfolio in the out-of-sample framework. In addition, Section 6.4 follows the method of Giacometti et al. (2007) in constructing variance-adjusted, VaR-adjusted and CVaR-adjusted BL portfolios, and evaluating the single period and multiple-period performances in contrast to the unconstrained BL portfolio and the risk-constrained BL portfolio.

Chapter 7 summarises the research. It also addresses the limitations of the research and gives some suggestions for future research.

CHAPTER 2
ASSET ALLOCATION FRAMEWORK AND RISK MEASURES

The research in this chapter has been well documented. The main contribution of this chapter is to summarise the basic theory of asset allocation and review its application (Section 2.1), as well as the risk measures (Section 2.2).

2.1 Mean-Variance Analysis and Modern Portfolio Theory

In 1952, Markowitz founded a quantitative framework for portfolio selection, which had a profound impact on the financial industry; he measured portfolio return and portfolio risk by the use of mean returns, variance and covariance under a set of assumptions. The derived portfolio variance formula indicates the importance of diversification in reducing total risk of the portfolio in investment. He also defined an efficient frontier where every portfolio on the frontier maximises the expected return for a given level of risk, or minimises the variance for a given expected return. His model is now widely recognised as the cornerstone of the modern portfolio theory.

2.1.1 Classical Mean-Variance Framework

2.1.1.1 Assumptions

The Markowitz model is developed on the basis of several assumptions about investor behaviour. Firstly, investors wish to maximise the returns from a total set of investments for a given level of risk. In other words, investors aim to maximise one-period expected utility, which is the function of expected return and the expected variance, demonstrating diminishing marginal utility of wealth. The utility curve represents the investor’s sensitivity to changing wealth and risk. Secondly, all investors are risk averse. It means that they will choose the asset with the lower level of risk given same level of expected return; similarly, they prefer higher returns to lower returns for a given risk level. Thirdly, investors think that each investment can be represented by a probability distribution of expected returns over holding periods. Fourthly, investors estimate the risk of the portfolio based on the average squared deviation around the expected return.

2.1.1.2 Mathematical Model

During the investment process, Markowitz considers investors' short-sighted behaviour. He thinks the investor will construct portfolio at time t and hold it for a time horizon of Δt. Only at time $t+\Delta t$, will the investor adjust his investment, according to the performance in the period of Δt. In the classical mean-variance model based on one period, the unknown parameters are estimated from the sample of available data, and the sample estimates are regarded as the unbiased true parameters. Suppose the returns r_t on the N assets at time t, where $\mathbf{r}_t = (r_{1,t}, r_{2,t}, \cdots r_{N,t})'$, follows multivariate normal distribution with a $N \times 1$ vector of assets' expected returns $\boldsymbol{\mu}$, and a $N \times N$ covariance matrix Cov_{ij} with element of σ_{ii}^2 as the variance of each asset i, and σ_{ij}^2 as the covariance of each asset between asset i and asset j. Note that the vector of expected returns $\boldsymbol{\mu}$ is expressed as $\boldsymbol{\mu} = E(\mathbf{r})$, the variance-covariance matrix Cov_{ij} can be defined a $\boldsymbol{\Sigma} = E[(\mathbf{r}-\boldsymbol{\mu})(\mathbf{r}-\boldsymbol{\mu})']$, and the correlation coefficient can be denoted by $\rho_{ij} = \frac{Cov_{ij}}{\sigma_i \sigma_j}$, which can vary only in the range -1 to 1. The $N \times 1$ vector of the weight of each asset i is denoted by $\mathbf{w}$, where $\mathbf{w} = (w_1, w_2, \cdots w_N)'$. The portfolio's return at time t is given by $r_{p,t} = \mathbf{w}'\mathbf{r}_t$. The expected return of the portfolio is computed as:

$$\mu_p = E(r_p) = \mathbf{w}'E(\mathbf{r}) = \mathbf{w}'\boldsymbol{\mu} \quad (2.1)$$

and the variance of the portfolio is defined as:

$$\sigma_p^2 = \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \quad (2.2)$$

The variance for a portfolio of assets is the sum of the weighted average of the individual variance and the weighted covariance between all the assets in the portfolio. If the correlation is not perfect (positive correlation equal to 1), the variance of the portfolio is less than the sum of variance of each asset in the portfolio. In other words, portfolio risk can be diversified by investing into different assets with lower correlation coefficient.

Following Markowitz's argument, a rational investor would choose the portfolio with minimum variance from the set of all possible portfolios for a given level of expected return, and the optimisation problem is a constrained minimisation problem:

$$\min_{\mathbf{w}} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \tag{2.3}$$

$$\text{subject to } \mathbf{w}'\boldsymbol{\mu} \geq \mu_0 \text{ and } \mathbf{w}'\mathbf{1} = 1$$

where $\mathbf{1}$ is $N \times 1$ vector of ones, and μ_0 is the portfolio's target expected return. Note that the first constraint requires that the expected return should at least achieve at the target expected return μ_0, and the second constraint is the budget constraint, which satisfies investing all of wealth.

Alternatively, an investor would choose the portfolio with a maximum expected return for a given level of risk; therefore, the optimisation problem can also be expressed as a constrained maximisation problem:

$$\max_{\mathbf{w}} \mathbf{w}'\boldsymbol{\mu} \tag{2.4}$$

$$\text{subject to } \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \leq \sigma_0^2 \text{ and } \mathbf{w}'\mathbf{1} = 1$$

where σ_0^2 is the portfolio's maximum acceptable risk. Note that the first constraint requires that the portfolio risk should be less than the maximum acceptable risk.

In addition, the mean-variance analysis can also be formulated in another way with the aim of maximising expected utility. The formulation can be written as:

$$\max_{\mathbf{w}} E(U) \tag{2.5}$$

$$\text{subject to } \mathbf{w}'\mathbf{1} = 1$$

where $E(U) = \mu_p - \frac{\delta}{2}\sigma_p^2 = \mathbf{w}'\boldsymbol{\mu} - \frac{\delta}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$ is the expected quadratic utility function[1]. δ is the risk aversion coefficient, which reflects the investors' tolerance for taking additional risk which is compensated for one unit of increase in expected return. When δ is large, a portfolio with more exposure to risk becomes more highly penalised, leading to less risky portfolios; conversely, a small δ implies a small penalty from the contribution of portfolio risk, the portfolio would be more risky.

2.1.1.3 Efficient Frontier

Setting varying values of μ_0 in (2.3), σ_0^2 in (2.4) and δ in (2.5) to solve the optimisation problem would produce a sequence of portfolios on the curve, which represents that the relation between portfolio risk and portfolio return is the mean-variance frontier on the curve *lmn* in Figure 2.1. The upward-sloping portion of the curve is the efficient frontier (curve *lm* in Figure 2.1), which provides the best possible trade-off between expected return and risk. These three formulations generate the same efficient frontier. On the efficient frontier, there is the global minimum variance portfolio which has the smallest variance. The point *m* in Figure 2.1 denotes the global minimum variance portfolio. It can be obtained by solving the optimisation problem:

$$\min_{\mathbf{w}} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \qquad (2.6)$$

$$\text{subject to } \mathbf{w}'\mathbf{1} = 1$$

which has the solution:

$$\mathbf{w}_m = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \qquad (2.7)$$

[1] Fabozzi et al. (2007) show other commonly used utility functions and conclude that the portfolio optimisation problem is not sensitive to changes of utility function in normal and Student-t distribution.

2.1.2 Mean-Variance Analysis with Risk-Free Asset and Capital Asset Pricing Model

Building on the mean-variance portfolio theory, Sharpe (1964) and Lintner (1965) design an equilibrium model, the Capital Asset Pricing Model (CAPM), with an assumption of the existence of risk-free assets. The risk-free asset has zero variance. The covariance of the risk-free asset with any risky asset will always equal zero. Their primary assumptions also include:

- Investors can borrow or lend at the risk-free rate of return;
- All investors will choose an optimal portfolio on the Markowitz efficient frontier;
- All investors possess homogeneous expectations;
- All investors have the same one period time horizon;
- All investments are infinitely divisible;
- There are no taxes or transaction costs;
- There is no inflation or any change in interest rates; and
- Capital markets are in equilibrium.

Suppose that the risk-free asset exists, and that the expected rate of return earned on the risk-free asset is R_f, the rate of return earned on each risky asset is $\mathbf{r}$, the proportion of the portfolio invested in risky portfolio is $\mathbf{w}_r$, and $1-\mathbf{w}_r'\mathbf{1}$ is a risk-free asset. The portion of risk-free assets can be positive or negative if risk-free borrowing and lending are allowed. The average rate of return μ_p and the variance σ_p^2 of the portfolio, when the risk-free asset is combined with the portfolio of risky assets, can be expressed as:

$$\mu_p = E(r_p) = \mathbf{w}_r' E(\mathbf{r}) + (1-\mathbf{w}_r'\mathbf{1})R_f = R_f + \mathbf{w}_r'(E(\mathbf{r}) - R_f) \qquad (2.8)$$

$$\sigma_p{}^2 = \mathbf{w}_r' \mathbf{\Sigma}_r \mathbf{w}_r \qquad (2.9)$$

where $\mathbf{\Sigma}_r$ is the covariance of the risky asset portfolio. The variance of portfolio that combines the risk-free asset with risky assets is the linear proportion of the variance of the risky asset portfolio, because the risk-free asset has zero variance and is uncorrelated with risky assets. From the view of investors, they prefer selecting a portfolio with the highest expected excess return per unit of risk on the efficient frontier. In other words, the Sharpe Ratio (SR), which is calculated as the ratio between expected excess return and standard deviation, could be used to measure portfolio performance, and the optimal portfolio should have the maximal SR. Therefore, in practice, the portfolio problem can also be expressed as the maximisation of the SR:

$$\max_{\mathbf{w}} \frac{\mathbf{w}'(E(\mathbf{r})-R_f)}{\sqrt{\mathbf{w}'\boldsymbol{\Sigma}_r\mathbf{w}}} \tag{2.10}$$

$$\text{subject to } \mathbf{w}'\mathbf{1}=1$$

The solved weights of the investor's optimal portfolio would be given by:

$$\mathbf{w}_r^{\ *} = \frac{\boldsymbol{\Sigma}_r^{\ -1}(E(\mathbf{r})-R_f)}{\mathbf{1}'\boldsymbol{\Sigma}_r^{\ -1}(E(\mathbf{r})-R_f)} \tag{2.11}$$

This optimal portfolio is the tangency portfolio referred to as the market portfolio. We can draw the line from the risk-free rate to the efficient frontier at the point where the line is tangent to the efficient frontier. This line is called the capital market line. The graph of the capital market line is in Figure 2.2. The point M represents the market portfolio. The expression for the capital market line can be shown as:[2]

$$E(r_p) = R_f + (\frac{E(r_M)-R_f}{\sigma_M})\sigma_p \tag{2.12}$$

where $E(r_M)$ is the expected return of the market portfolio, and σ_M is the standard deviation of the market portfolio.

CAPM is a model that determines the expected rate of return on a risky asset $E(r_i)$. The systematic risk measure for the individual risky asset is the covariance with the market portfolio $Cov_{i,M}$. The formula for the risk-return relationship is denoted by:

---

[2] The expected variance for a two-asset portfolio is $\sigma_p^2 = w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\rho\sigma_1\sigma_2$; when the risk-free asset is combined with a risky asset portfolio (market portfolio), the expected variance is $\sigma_p^2 = w_r^{\ 2}\sigma_M^2$, thus, $w_r = \frac{\sigma_p}{\sigma_M}$, and the formula (2.8) can be rewritten as formula (2.12).

$$E(r_i) = R_f + (\frac{E(r_M) - R_f}{\sigma_M^2})Cov_{i,M} \tag{2.13}$$

$$= R_f + (\frac{Cov_{i,M}}{\sigma_M^2})(E(r_M) - R_f) \tag{2.14}$$

$$= R_f + \beta_i(E(r_M) - R_f) \tag{2.15}$$

β_i, which is equal to $\frac{Cov_{i,M}}{\sigma_M^2}$, is a measure of systematic risk. The market portfolio has a beta of 1. There is a linear relationship between the expected return and the systematic risk; Figure 2.3 plots the security market line in this linear relationship. In equilibrium, all assets and all portfolios of assets should plot on the security market line.

2.1.3 Criticisms of the Mean-Variance Approach

The simplicity and the intuitive appeal of the mean-variance approach has attracted significant attention from academia and industry. However, contrary to its theoretical reputation, Markowitz's classical framework has not been used extensively by practitioners as a tool for optimising a large-scale portfolio, due to its numerous implementation difficulties.

The impracticality is that extreme weights or corner solutions from the mean-variance model may be inconvenient in asset allocation, since the investor can neither assign unrealistic weights to each asset, nor diversify risk by investing different assets. Imposing constraints on portfolio weights could alleviate this problem and enable the portfolio to perform better (Frost and Savarino, 1988; Grupa and Eichhorn, 1998; Grauer and Shen, 2000). Discussions regarding the non-short selling constraints can be found in the literature (Jagannathan and Ma, 2003). Additionally, the sensitivity of portfolio weights (Best and Grauer, 1991; Best and Grauer, 1992; Black and Litterman, 1992; Broadie, 1993) is an annoying problem for practitioners as well, as they have to pay significant amounts of transaction costs to buy and sell stocks with weights dramatically changed. The main reason for these problems is the estimation errors in the inputs of the mean-variance model. The accuracy of the estimation of input data will heavily affect the weights allocated to each asset in the mean-variance optimisation, called 'estimation-error maximisers' (Michaud, 1989). Michaud argues that the optimised portfolios tend to overweight (underweight) assets with large (small) expected returns, negative (positive) correlations and small (large) variances. Merton (1980) demonstrates that historical returns are bad proxies for expected returns. He also demonstrates evidence that the estimated variance and covariance from the historical data will be much more accurate than the corresponding expected return estimates. Similarly, Chopra and Ziemba (1993) verify that the impact of estimation errors on the expected returns on portfolio choice dominates that of estimation errors in variances and covariance. Therefore, they suggest paying attention to estimate, 'less noisy' expected returns, followed by a good estimation of variance. To address these problems, robust estimates of the input parameters for optimisation problems become an important research issue. It is advisable to use the Bayes-Stein shrinkage estimator (Jorion, 1985) or the Bayesian estimator (Frost and Savarino, 1986) as alternative estimators of expected return to reduce estimation risk and improve out-of-sample portfolio performance. However, except for estimation error, Green and Hollifield (1992) explain that the high correlation among assets result from the dominance of a single factor in the covariance of asset returns triggering the extreme weights. Therefore, it cannot ignore the impact of correlations on portfolio weights. Fabozzi et al. (2008) suggest using a factor model to model covariance and correlations and therefore deal with the issue of highly correlated assets.

Another significant problem is the computational difficulty associated with inputs of the expected returns and the expected variance-covariance structure for all assets in the investment universe. For example, if there were 100 assets, it would be burdensome for a practitioner to compute 4,950 parameters in the covariance matrix. In practice, it is impossible for portfolio managers to estimate reliable returns for all assets. Estimation errors exist when they anticipate an expected return by using a simple average of historical sample returns. In addition, it is widely agreed that financial asset return volatilities and correlations are time-varying, with persistent dynamics. Asset return volatilities become an important ingredient in many applications, such as portfolio optimisation and market risk measurement. The most popular approach to modelling the conditional covariance matrix of returns is the multivariate GARCH class of models. These models include the Vech and Diagonal Vech models (Bollerslev et al., 1988), the BEKK model (Engle and Kroner, 1995), the

Constant Correlation model (Bollerslev, 1990), the Factor ARCH model (Engle et al., 1990), and the Dynamic Conditional Correlation or DCC model (Engle and Sheppard, 2001). However, the Vech model and the BEKK model suffer from the curse of large dimensionality, and the Diagonal Vech models, the Constant Correlation model and the Factor ARCH model have cross-equation restrictions on the elements of the covariance matrix (Harris et al., 2007). Other approaches, such as rolling estimators of the sample covariance matrix, exponentially weighted estimators and multivariate stochastic volatility models (Harvey et al., 1994) can also be used to estimate the conditional covariance matrix. In the portfolio optimisation world, a portfolio manager would usually work on a large number of assets to diversify the unsystematic risk; the relationship and the co-movement among those assets will directly affect the performance of the whole portfolio. The choice of volatility models is an art.

In addition, from the perspective of investor perception against risk and distribution of asset returns, investors usually prefer a larger profit to a small or negative profit and, obviously, their perception of risk is not symmetric around the mean. The use of variance as a measure of risk becomes a critical weakness of the mean-variance approach. Besides, it is well known that the distribution of asset returns is not normal (Mandelbrot, 1963; Fama, 1965; Müller et al., 1998; Rachev and Mittnik, 2000; Rachev et al., 2008). It is not appropriate to consider only the first and second moment of distribution in portfolio optimisation. Both academics and practitioners focus their attention on meeting the requirement of alternative risk measures for portfolio optimisation, such as the 'safety first' strategy (Roy, 1952), semivariance (Markowitz, 1959), lower partial moment (Bawa, 1975), mean absolute deviation (Konno and Yamazaki, 1991), VaR (Jorion, 1997; Ahn et al., 1999; Basak and Shapiro, 2001), and CVaR (Rockafellar and Uryasev, 2000; Rockafellar and Uryasev, 2002). Because of the shortcomings of mean-variance optimisation, several researchers have introduced VaR (Huisman et al., 1999; Campbell, 2001; Favre and Galeano, 2002) and CVaR (Souza and Gokcan, 2004; Agarwal and Naik, 2004) to extend portfolio optimisation techniques under fat-tailed distributions. Alexander and Baptista (2001, 2002, 2004) thoroughly study the implications of VaR and CVaR constraints on the mean-variance model based on theoretical work. I will provide a detailed introduction of VaR and CVaR risk measures in Section 2.2.

2.1.4 Extension of the Traditional Mean-Variance Approach

Several extensions have been developed to address the issues discussed in Section 2.1.3. Fabozzi et al. (2008) provide a complete review of these extensions. These extensions mainly work in two directions. One direction to obtain robust estimates is the Bayesian approach (Zellner and Chetty, 1965; Brown, 1976; Frost and Savarino, 1986; Black and Litterman, 1990; Polson and Tew, 2000; Pástor, 2000), including the shrinkage estimator (Jorion, 1985; Jorion, 1986; Chopra, 1993; Ledoit and Wolf, 2003; Ledoit et al., 2004), and factor models (Fama and French, 1992; Fama and French, 1996). The other direction focuses on modelling area such as portfolio resampling (Michaud, 1998; Scherer, 2002; Scherer, 2004; Michaud, 2008; Harvey et al., 2008), and robust optimisation techniques (Fabozzi et al., 2007; Fabozzi et al., 2010). Robust portfolio optimisation is not within the scope of this thesis; Fabozzi et al. (2007, 2010) investigate and illustrate the recent advances in a comprehensive literature review of robust portfolio selection with uncertainty parameters. In this section, I will briefly introduce the Bayesian approach.

Bayesian approach

While the mean-variance analysis uses only a single set of estimates from prior information, the Bayesian approach combines the assessed information from external information with a single estimate. Founded by Savage (1954) on this idea, the Bayesian approach was developed into a general framework to solve the parameter uncertainty problem (Zellner and Chetty, 1965). It is important to understand the ‘prior distribution’ and the ‘posterior distribution’ in the Bayesian framework. The ‘prior distribution’ reflects an investor’s knowledge of the probability before external information sources are observed. After new information is provided, the investor would adjust their beliefs about the probability. This new probability distribution is the ‘posterior distribution’. The Bayesian rule, which allows the forecasting process to combine external information and subjective views with traditional prior information, could be applied to calculate the new probability distribution. Naturally, a posterior distribution of expected return can be obtained by integrating a forecast from empirical data with a prior distribution. Specifically, the Bayesian approach shrinks the mean estimators away from the sample means and towards some prior values, to generate a weighted average value as the estimate of expected return. The Bayesian approach assumes that the expected returns are unknown and random. There are three main methods proposed to calculate the prior means, such as shrinking the tangency portfolio towards the minimum-variance portfolio (Jorion, 1986), shrinking the tangency portfolio towards the market portfolio (Pástor, 2000; Pástor and Stambaugh, 2000), and shrinking the tangency portfolio towards the market portfolio but with the tangency portfolio, based on subjective investor forecasts instead of sample means (Black and Litterman, 1990). Herold and Maurer (2003) confirm the superior out-of-sample performance of the Bayesian approaches in contrast to the mean-variance portfolio. They also suggest using the promising Black-Litterman (BL) approach to tactical asset allocation. The BL model estimates the expected returns (mean of posterior distribution) based on the market equilibrium return (prior information), combined with an investor's views (new information). I will introduce the BL model in detail in Chapter 3.

Suppose that an investor has information-based beliefs about the mean vector and the covariance matrix of excess returns, while the prior for the mean vector of the normal distribution is multivariate normal, and the conjugate prior for the covariance matrix of a multivariate normal distribution is the inverse Wishart distribution in statistics:

$$\boldsymbol{\mu} \,|\, \boldsymbol{\Sigma} \sim N(\boldsymbol{\pi}, \frac{1}{\theta}\boldsymbol{\Sigma}) \tag{2.16}$$

$$\boldsymbol{\Sigma} \sim \mathrm{W}^{-1}(\mathbf{X}, v) \tag{2.17}$$

where θ represents the confidence an investor places on the value of $\boldsymbol{\pi}$, while v reflects the confidence about $\mathbf{X}$. The lower θ and v are, the less confidence and the higher the uncertainty about those values.

In Fabozzi et al. (2008), the mean of the predicted excess returns $\tilde{\boldsymbol{\mu}}$ and their covariance matrix $\tilde{\boldsymbol{\Sigma}}$ can be computed by, respectively:

$$\tilde{\boldsymbol{\mu}} = \frac{\theta}{T+\theta}\boldsymbol{\pi} + \frac{T}{T+\theta}\hat{\boldsymbol{\mu}} \tag{2.18}$$

$$\tilde{\mathbf{\Sigma}} = \frac{T+1}{T(v+N-1)}(\mathbf{X} + (T-1)\hat{\mathbf{\Sigma}} + \frac{T\theta}{T+\theta}(\boldsymbol{\pi} - \hat{\boldsymbol{\mu}})(\boldsymbol{\pi} - \hat{\boldsymbol{\mu}})') \tag{2.19}$$

where N is the number of assets, and T is the number of historical observations. The predictive mean $\tilde{\boldsymbol{\mu}}$ is a weighted average of the prior mean $\boldsymbol{\pi}$ and the sample mean $\hat{\boldsymbol{\mu}}$ which is shrunk towards the prior mean. $\hat{\mathbf{\Sigma}}$ is the sample covariance matrix. The stronger the investor's belief in the prior mean (the higher $\frac{\theta}{T+\theta}$), the larger the degree to which the prior mean influences the predictive mean.

2.2 Risk Measures

Based on Markowitz's idea of portfolio selection, the construction of portfolios should target maximising expected returns at a certain level of risk. After inputting the estimates of expected returns and the covariance matrix into the portfolio optimisation model, the optimal portfolio can be constructed. However, mounting evidence has shown that asset returns are not normal for most financial assets; therefore, it is not appropriate to only use the mean and standard deviation to reflect the property of the joint asset return distribution. The fat-tail risks give rise to negative skewness and high kurtosis which cannot be captured by the standard deviation of the portfolio. Consequently, the classical mean-variance approach should not be regarded as a better asset allocation model.

In this section, I will review and provide a brief overview of the most common downside risk measures such as Value-at-Risk (VaR) and Conditional Value-at-Risk, or CVaR (Rockafellar and Uryasev, 2000; Rockafellar and Uryasev, 2002) used in practice for portfolio selection. Other downside risk measures can be found in the literature including the 'safety first' strategy (Roy, 1952), semivariance (Markowitz, 1959), Low Partial Moment (Bawa, 1976; Fishburn, 1977; Price et al., 1982). Recent literature also provides other risk measures including convex measures (Föllmer and Schied, 2002; Frittelli and Rosazza Gianin, 2002), generalised deviation measures (Rockafellar et al., 2006), proper and ideal risk measures (Stoyanov et al., 2007; Rachev et al., 2008). Discussions regarding other risk measures are beyond the scope of this thesis.

2.2.1 Value-at-Risk

In 1994, JP Morgan developed Value-at-Risk or VaR, which has become a standard risk measure in financial risk management due to its conceptual simplicity, ease of computation, and ready applicability. In recent years, it has been widely applied to risk management in the financial industry and in banking regulatory mechanisms (Jorion, 1997; Dowd, 1998; Saunders, 2000). As we all know, a large number of financial institutions hold net positions in a variety of assets. For prudential reasons, they need to measure the overall market risk of their portfolio, which is usually referred to as Value-at-Risk or VaR. In addition, the Basel Committee on Banking Regulation uses VaR to set minimum capital adequacy requirements to cover market risk. VaR measures the predicted maximum loss at a specified probability level over a certain time horizon. For example, if a portfolio has a ten-day, 99% VaR of £100,000, it means that the largest loss of the portfolio could be expected to be £100,000 with 99% certainty over the next ten days. If the portfolio has a daily VaR of £100,000 at a 1% critical value, this implies that it will lose more than £100,000 in only one day out of 100.

Mathematically, VaR can be expressed as:

$$VaR_{\beta}(R_p) = \min\{\alpha \mid P(R_p \le \alpha) \ge \beta\} \qquad (2.20)$$

It means that the VaR_{β} is at the value α, such that the probability P that the maximum portfolio loss R_p is, at most α, is at least at confidence level of β, such as 99%, 95% and 90%. At the outset, the existing methods to measure VaR include historical simulation-bootstrapping techniques, the variance-covariance model, the Monte Carlo simulation, stress testing, and extreme value theory. Each method can generate different results from the others. It is prudent to choose the appropriate method to estimate VaR based on the underlying assumptions, as well as the mathematical models and quantitative techniques used. For example, if the distribution of security returns is assumed to be normal, but the real distribution is not normal, the use of the variance-covariance model with a normality assumption will increase the estimation error. In a nutshell, a large element of judgement is required in practical implementation. Cuthbertson and Nitzsche (2005) clearly demonstrate how to measure VaR in the implementation of different methods. In this thesis, I will only explain the simple variance-covariance method.

Variance-Covariance Method

The variance-covariance method can be categorised into parametric models, which assume a particular distribution for the portfolio return distribution, and use knowledge of that distribution to compute the appropriate quantile. The $VaR_{\beta,t}$ which measures the maximum loss expected in normal market conditions at time t , is calculated as:

$$VaR_{\beta,t} = -(q_{1-\beta}\hat{\sigma}_t + \hat{\mu}_t) \tag{2.21}$$

where $\hat{\mu}_t$ and $\hat{\sigma}_t$ are the portfolio-forecasted mean and standard deviation using sample data until $t-1$, $q_{1-\beta}$ denotes the $(1-\beta)$-quantile of the distribution of the portfolio return.

There are some defects in the variance-covariance method. Firstly, it relies heavily on distributional assumptions about portfolio returns. If the asset returns are multivariate normal, for example, it can be expected that the actual return would be less than $\hat{\mu}_t - 1.65\hat{\sigma}_t$ only one time in 20 (i.e.. 5% certainty or 5% of the time). If the portfolio return distribution is non-normal, the value of $q_{1-\beta}$ would change, corresponding to the assumed distribution. Note that there is an underlying assumption that a linear relationship exists between the portfolio return and the asset returns. Secondly, this method may yield poor approximation for 'non-linear' portfolios containing options. However, we cannot neglect the strength of this method. It is not only easy to use for simple portfolios, but also removes the strict requirement for large amounts of data. Furthermore, it would be straightforward to incorporate volatility clustering into this approach by replacing the unconditional estimate of volatility $\hat{\sigma}_t$ with the conditional volatility $\hat{\sigma}_{t|t-1}$ from the model. Since the forecast of volatilities and correlations would be inputted into the model, many multivariate volatility models can be applied to estimating VaR. For example, Billio and Pelizzon (2000) introduce a multivariate switching regime model to estimate the VaR of both single assets and portfolios; they demonstrate that a switching regime specification is more accurate than the other two methods used by EWMA and

GARCH (1, 1) models. Lee et al. (2006) use a multivariate DCC-GARCH model to measure VaR and find that the DCC-GARCH (1, 1) model is preferable. Santos et al. (2012) find that multivariate GARCH models outperform their univariate counterparts to forecast portfolio VaR on an out-of-sample basis. Specifically, the DCC model with Student-*t* distribution seems to be the most appropriate specification when implemented to estimate the VaR of the real portfolios.

Artzner et al. (1999) propose that coherent risk measures should satisfy several properties including monotonicity, subadditivity, positive homogeneity, and translational invariance. These properties can be expressed thus: if there are only positive returns, then the risk should be non-positive; the risk of a portfolio of two assets should be less than or equal to the sum of the risks of the individual assets. If the portfolio is increased c times, the risk becomes c times larger, and cash or another risk-free asset does not contribute to portfolio risk. Under these restrictive rules, some popular risk measures, including standard deviation and semideviation-type risk measures, would not be coherent. Because the standard deviation violates the monotonicity property and the semideviation-type risk measures, it cannot satisfy the subadditivity requirement.

In addition, VaR has suffered from some shortcomings. Firstly, it is not subadditive. It does not hold that the VaR of a portfolio of two assets, A and B, should be less than or equal to the sum of the VaR of the individual asset A and asset B. In this case, it is not consistent with the concept of diversification in portfolio theory. Secondly, it is too complex and time-consuming to construct the optimal portfolio when we solve the non-smooth and non-convex function to calculate VaR with scenarios. Thirdly, VaR ignores the worst case scenario that the losses may be beyond the VaR value in the left tails. Obviously, the rational investor would make a wiser investment in the portfolio with a shorter left tail, rather than the one with a longer left tail, if the portfolio has the same expected return. These undesirable features motivate the development of Conditional Value-at-Risk.

2.2.2 Conditional Value-at-Risk

Conditional Value-at-Risk (CVaR) is a coherent risk measure defined by the formula:

$$CVaR_{\beta}(R_p) = E(R_p \mid R_p \geq VaR_{\beta}(R_p)) \tag{2.22}$$

CVaR measures the expected amount of losses in the tail of the distribution of possible portfolio losses beyond the portfolio VaR. CVaR can also be referred to as expected shortfall, expected tail loss, and tail VaR. CVaR is always at least as large as VaR. CVaR has tractable properties: it is a coherent risk measure, it is easy to implement and it takes into consideration the entire tail that exceeds VaR on average.

Following Rockafellar and Uryasev (2000), the portfolio loss can be defined as the minus return $-\mathbf{w'r}$, with the assumption that the distribution of $\mathbf{r}$ is continuous. For a given portfolio, the probability of the loss not exceeding a threshold α is given by $\Psi(\mathbf{w},\alpha) = \int_{-\mathbf{w'r} \leq \alpha} p(\mathbf{r})d\mathbf{r}$. Given a confidence level of β, the VaR associated with the portfolio is defined as $VaR_{\beta} = \min\{\alpha \mid \Psi(\mathbf{w},\alpha) \geq \beta\}$. CVaR is defined as the conditional expectation of the loss of the portfolio exceeding or equal to VaR, that is:

$$CVaR_{\beta} = \frac{1}{1-\beta}\int_{-\mathbf{w'r} \geq VaR_{\beta}} -\mathbf{w'r}p(\mathbf{r})d\mathbf{r} \tag{2.23}$$

It seems difficult to calculate CVaR from formula (2.23), due to its convoluted and implicit expression. Rockafellar and Uryasev (2000) demonstrate that CVaR is subadditive and can be introduced as the following convex optimisation problem: $CVaR_{\beta} = \min F_{\beta}(\mathbf{w},\alpha)$, where $F_{\beta}(\mathbf{w},\alpha)$ is expressed as:

$$F_{\beta}(\mathbf{w},\alpha) = \alpha + \frac{1}{1+\beta}\int [-\mathbf{w'r} - \alpha]^{+} p(\mathbf{r})d\mathbf{r} \tag{2.24}$$

where $[-\mathbf{w'r} - \alpha]^{+} = \max\{-\mathbf{w'r} - \alpha, 0\}$. Moreover, $F_{\beta}(\mathbf{w},\alpha)$ is shown to be convex and continuously different with respect to $\mathbf{w}$ and α. An interior algorithm can efficiently solve the convex programming problem.

When the return distribution is not normal, achieving the minimal variance comes at the price of taking large, extreme risks under a mean-variance framework (Sornette et al., 2000; Amin and Kat, 2003); in order to take fat-tail risks into consideration, some researchers have introduced VaR into optimal portfolio selection. Campbell et al. (2001) allocate assets by maximising the expected return, subject to the constraint that the expected maximum loss should meet the VaR limits set by the risk manager. Their findings highlight the impact of non-normal characteristics of the expected return distribution on the optimal asset allocation. Favre and Galeano (2002) incorporate modified VaR, which utilises the Cornish-Fisher expansion to approximate the quantile, in the computation of the optimal portfolio based on the framework of Huisman et al. (1999). Gaivoronski and Pflug (2005) apply VaR to optimal portfolio selection, with an emphasis on solving VaR optimisation problems and present a smoothing algorithm in the computation of mean-VaR efficient portfolios. Meanwhile, other researchers pay attention to introducing CVaR into an optimal portfolio selection and make comparisons with the mean-variance approach and the mean-VaR framework. Agarwal and Naik (2004) develop a mean-CVaR framework for hedge funds, and find that the mean-variance framework underestimates the tail risk of the hedge fund, compared with the mean-CVaR framework. Bertsimas et al. (2004) examine the properties of CVaR and its relation to other risk measures. They demonstrate that the mean-CVaR optimisation problem can be solved efficiently as a convex optimisation problem and a linear optimisation problem in a sample version. They also show that the portfolios constructed by the mean-CVaR approach can outperform those generated by the mean-variance approach. Souza and Gokcan (2004) constructed a mean-CVaR efficient frontier and plotted it on a mean-variance graph to identify a ‘skew gap’ that captures the effect of negative skew inherent in hedge fund strategies. Alexander and Baptista (2001, 2002, 2004) thoroughly study the implications of imposing VaR and CVaR constraints on the mean-variance model based on theoretical work. They show that, when the CVaR bound is larger than the VaR bound, or when a risk-free security is present, a CVaR constraint could dominate a VaR constraint as a risk management tool. Yamai and Yoshiba (2005) compare the properties of VaR and CVaR, and analyse their estimation errors. They stress that both risk measures have benefits and drawbacks, and suggest complementing VaR with CVaR for effective financial risk management.

2.3 Conclusions

Despite the great influence and theoretical impact of Markowitz’s modern portfolio theory, which captures the two fundamental economic insights of risk-return trade-off and diversification, the mean-variance optimisation approach is confronted by several criticisms in practice. Practitioners are reluctant to use this approach because of its numerous implementation difficulties in the estimation of inputs, and the unreliable solutions of weights in assets. The limitations of the mean-variance analysis have stimulated numerous extensions in robust estimates of moments of returns, including expected returns, the covariance matrix and risk measures. The Bayesian approach has become known as a superior method to estimate the prior means to achieve a better out-of-sample performance than the mean-variance method; specifically, it is suggested that the Black-Litterman model be applied to tactical asset allocation (Herold and Maurer, 2003). The next chapter will examine the interpretation of the Black-Litterman approach in asset allocation.

In addition, exploiting the predictability of the covariance matrix in conditional volatility models also indicates an interesting direction; however, the choice of volatility models is an art. Moreover, the use of alternative risk measures other than variance is necessary to capture asymmetric property of returns and to measure tail risks in an asset allocation approach. Although VaR has been widely applied to risk management in the financial industry and in banking regulatory mechanisms, it fails to satisfy the subaddtivity coherent risk measures criterion, and ignores the worst case scenario that losses may be exceed the VaR value in the left tails, while CVaR remedies these drawbacks and becomes a coherent tool to measure risk. Both the mean-VaR framework and the mean-CVaR framework have been widely developed and implemented in portfolio optimisation when returns distribution has a fat-tail. Yamai and Yoshiba (2005) suggest complementing VaR with CVaR to support more comprehensive risk management, with a consideration of pros and cons in each framework. Chapter 4 will show the methods of constructing Black-Litterman portfolio with both VaR and CVaR.

Figure 2.1 Feasible Set and Markowitz Efficient Set

The figure plots the feasible set and the efficient set of the Markowitz portfolio selection theory. $E(r_p)$ is the expected return and σ_p is the standard deviation of returns. Point *m* denotes the global minimum variance portfolios. The curve *lmn* and the area within are the feasible set but only the curve above *m* is efficient, as lower standard deviations for a given return or higher returns for a given standard deviation. The curve above *m* is referred to as efficient set or efficient frontier.

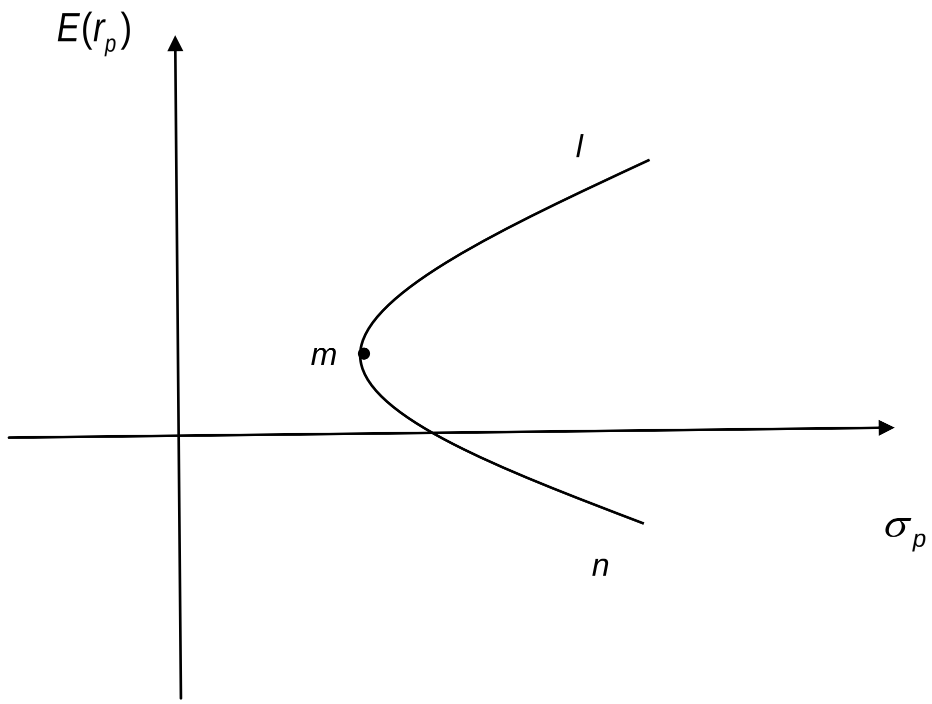


Figure 2.2 Capital Market Line and Efficient Frontier

The figure plots the capital market line (CML) and the efficient frontier. $E(r_p)$ is the expected return and σ_p is the standard deviation of returns. R_f is the risk-free rate. Point *m* denotes the global minimum variance portfolios. The curve *lmn* and the area within are the feasible set but only the curve above *m* is efficient, as lower standard deviations for a given return or higher returns for a given standard deviation. The curve above *m* is referred to as efficient set or efficient frontier. Tangency point M denotes the market portfolio. The line from R_f to M is the new efficient frontier when there is a risk-free asset.

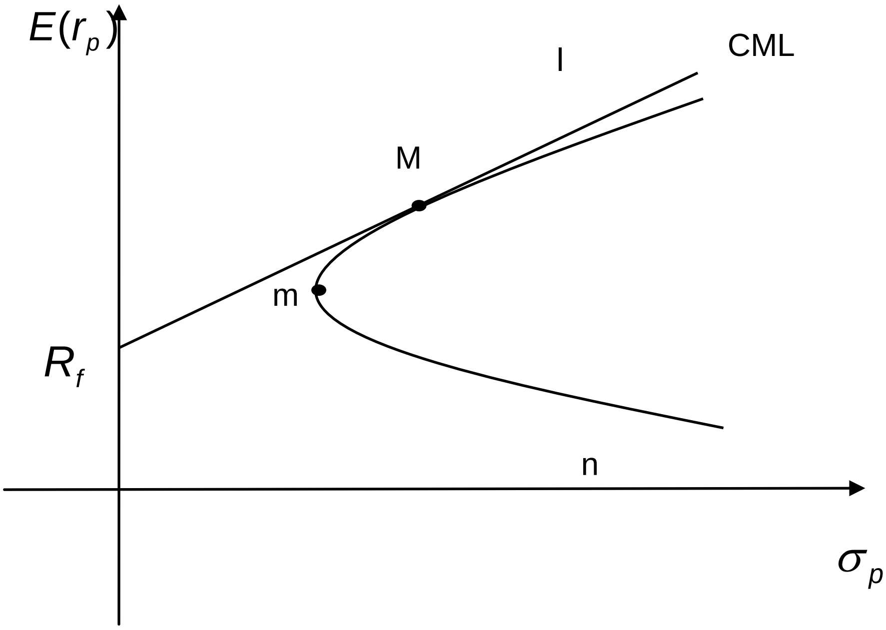
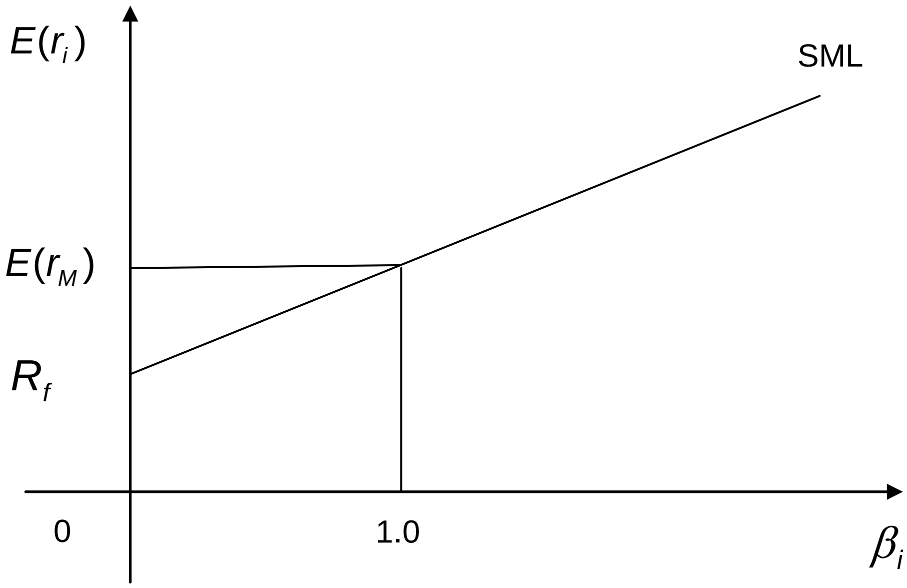


Figure 2.3 Security Market Line

The figure plots the security market line (SML). $E(r_M)$ is the expected return of the market portfolio, and β_i is the standardised measure of systematic risk of the asset *i*, which is the covariance of an asset with the market portfolio, divided by the variance of the market portfolio. R_f is the risk-free rate. $E(r_i)$ is the expected return of the asset *i*.



CHAPTER 3
LITERATURE REVIEW OF THE BLACK-LITTERMAN MODEL

3.1 Introduction

With the application of the Bayesian methodology, the Black-Litterman (BL) model as proposed by Black and Litterman (1990) has an appealing strength in that it can overcome the limitations of the traditional mean-variance model.

Firstly, in the BL model, the absolute or relative views on the expected returns from portfolio managers can be inputted into the model. In the traditional mean-variance model, expected returns and covariances of all assets have to be estimated, however, it is not realistic for investors to estimate every single parameter for all assets in large-scale investment. Comparatively speaking, it is more intuitive and practical for financial industry insiders to seek information from a few investment assets and generate some views in the BL model. This is the main reason why practitioners prefer using the BL model. Secondly, the BL model can mitigate the problem of highly concentrated portfolios, input sensitivity, and estimation error maximisation yielded by the classical mean-variance optimisation model. Several studies contribute to clarifying the intuition behind the BL model and illustrate the practical nature of the BL model (Black and Litterman, 1991, 1992; Bevan and Winkelmann, 1998; He and Litterman, 1999; Satchell and Scowcroft, 2000; Drobetz, 2001; Idzorek, 2004). Nowadays, the BL model is widely used in the industry because of its robustness; more and more portfolio managers and financial advisors employ this model to support their investment decisions. There has been a surge in recent research papers which generalise the BL model to comply with asset return regularities, including non-normal distribution and volatility clustering (Giacometti et al., 2007; Meucci, 2006, 2007, 2008; Martellini and Ziemann, 2007; Beach and Orlov, 2007; Palomba, 2008); to be evaluated by alternative risk measures (Martellini and Ziemann, 2007; Lejeune, 2011; Veress et al., 2012); to incorporate into trading strategies (Fabozzi et al., 2006; Babameto and Harris, 2009), and to a broad range of other applications (Becker and Gürtler, 2009; Da Silva et al., 2009; Cheung, 2009; Giacometti and Mignacca, 2010; Munda and Strasek, 2011; Mishra et al., 2011; Fernandes et al., 2011; Braga and Natale, 2012).

The aim of this chapter is to provide a detailed introduction to the BL model with the mathematical formulae and underlying intuition explained. I also summarise the most recent studies that extend and apply the BL model to different directions. I give a brief introduction to some new approaches and provide some critical comments.

The main contribution of this chapter is to provide a complete picture of the literature and the mathematical models of the BL model. In addition, based on my own critical evaluation, I point out the weakness of some extended BL models. Furthermore, motivated by the lack of literature regarding applying the BL model to a dynamic environment with some alternative risk constraints, I propose a dynamic BL model which starts from a conditional equilibrium return. Besides, the alternative risk constraints such as VaR and CVaR could be introduced in the optimisation model to construct a risk-constrained, dynamic BL portfolio. I will introduce the methodology in Chapter 4.

In the following sections, I will introduce the BL model in detail in Section 3.2. Section 3.2.1 describes the first step of the BL model in obtaining the implied market equilibrium. Section 3.2.2 shows the approach of translating the investors' view to fit the BL model. Section 3.2.3 displays the formula of the Black-Litterman expected return and covariance with the combination of the views in the Bayesian framework. Section 3.2.4 constructs the unconstrained optimal portfolio and explains the economic intuition behind the model followed by He and Litterman (1999). Section 3.3 discusses extensions of the BL model. In this section, I emphasise introducing the method of Fabozzi et al. (2006) to utilise the momentum strategy in the BL model. I also discuss two recent papers written by Giacometti et al. (2007) and Lejeune (2011), which are closely related to my research topics. Section 3.4 concludes this chapter.

3.2 The Black-Litterman Model

In 1990, Black and Litterman published their original work and proposed a superior asset allocation approach, which started from the market equilibrium returns incorporated with additional investor views to form a new mixed estimate of expected returns. Black and Litterman (1991, 1992) expanded the new BL approach to allocate assets at a set of neutral weights and adjust towards views from investors with limited details discussed. However, it was not easy to understand how to empirically realise this asset allocation process and reproduce results. Additional studies have been developed to introduce the BL model and to make it more accessible to practitioners (Bevan and Winkelmann, 1998; He and Litterman, 1999; Satchell and Scowcroft, 2000; Drobetz, 2001; Idzorek, 2004). Bevan and Winkelmann (1998) build on the BL model to allocate assets, and show in clear detail how to construct an optimal Black-Litterman portfolio. After the construction of an unconstrained optimal portfolio, they measure portfolio risks using tracking error and market exposure. They mention that VaR has the same explanations as the tracking error when the asset returns are symmetric. He and Litterman (1999) reveal the mystery of the Black-Litterman approach: it displays the clear economic intuition of the model in that the optimal unconstrained portfolio is the scaled weights of market equilibrium portfolio weights added up to a weighted sum of view portfolios. Satchell and Scowcroft (2000) devote much effort to demystifying the BL model and provide a detailed derivation of the formula in the model. Unlike the paper published by He and Litterman (1999), with its emphasis on the mathematics in the BL model, Drobetz (2001) pays attention to simple samples to lay out the intuition behind the BL model, which avoids the deficiencies of the traditional mean-variance approach to portfolio optimisation. Idzorek (2004) presents step-by-step instructions for practitioners to implement the BL model and obtain returns which could be reasonably expected. All of these researchers contribute to improving the practical implementation of the original BL model. Walters (2009) carries out a thorough survey of studies of the BL approach with a clear explanation of the derivation and implied principles.

The basic idea in the BL model is to combine the equilibrium expected returns with investor views, which means that the Black-Litterman portfolio gravitates to a neutral market capitalisation weighted portfolio that tilts in the direction of assets favoured in the views investors have expressed. The degree of confidence investors have in the views will reflect the extent of the deviation from the equilibrium expected returns.

The first step in the BL model is to find the implied market equilibrium return by utilising the market capitalisation weights based on CAPM theory (Sharpe, 1964) and reverse optimisation. The CAPM assumes that all investors hold the market portfolio combined with cash in equilibrium. Then the investor views are an additional input to the model. When there are no specific views on assets, the expected returns of assets can be regarded as the market equilibrium returns. Starting from holding the market portfolio, investors can add specific views. The expected returns of each asset are estimated by using the Bayesian mixed estimation (Theil, 1971) to combine the implied equilibrium return and investor views. The next procedure is to optimise the assets in the mean-variance optimisation (Markowitz, 1952) with the posterior expected returns of each asset inputted.

3.2.1 The Implied Equilibrium Return

Firstly, the BL model assumes that the $N \times 1$ excess return vector $\mathbf{r}$ follows a multivariate normal distribution with $N \times 1$ expected excess return vector $\boldsymbol{\mu}$ and $N \times N$ covariance matrix of excess returns $\boldsymbol{\Sigma}$:

$$\mathbf{r} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{3.1}$$

The variance-covariance matrix is assumed to be known and is estimated traditionally with the unbiased historical estimator. However, the vector of expected returns is a random vector that follows a normal distribution with known parameters $\boldsymbol{\pi}$, τ and $\boldsymbol{\Sigma}$:

$$\boldsymbol{\mu} \sim N(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma}) \tag{3.2}$$

$\boldsymbol{\pi}$ is the $N \times 1$ expected equilibrium return vector of the market portfolio, and serves as a neutral reference point. The scale parameter τ indicates the uncertainty of the CAPM prior. The smaller value of τ, the higher the confidence in the estimation of the implied equilibrium return. There are several assumptions to set the value of τ. Black and Litterman (1992), He and Litterman (1999), Lee (2000) and Idzorek (2005) all claim that the solution to this practical problem is to impose τ to be close to zero because they believe that the uncertainty in the mean is less than the uncertainty in the return, and they use small values of τ ranging from 0.01 to 0.05. Fabozzi et al. (2006)

choose τ equal to 0.1. Conversely, Satchell and Scowcroft (2000) provide an analytical method which sets $\tau = 1$. Shi and Irwin (2005) demonstrate instead that, theoretically, the parameter has to be equal to T^{-1}, where T is the number of observations of asset returns. Mankert (2006) provides a new approach on the value τ from the point of view of sampling theory. Fabozzi et al. (2008) propose a different approach to selecting τ which satisfies the equation:

$$\|\mathbf{S}\| = \tau \|\mathbf{\Sigma}\| \tag{3.3}$$

where $\|.\|$ means the matrix norms[3], and the matrix $\mathbf{S}$ is the covariance matrix of $\mathbf{r}_t - \boldsymbol{\pi}_t$, where $\mathbf{r}_t$ is the $N \times 1$ vector of observed returns on N assets at time t, and $\boldsymbol{\pi}_t$ is the $N \times 1$ vector of equilibrium returns on N assets at time t, calculated on rolling basis.

Assuming that the capital market is in equilibrium and clear, according to the CAPM theory explained in Chapter 2 equation (2.15), $\boldsymbol{\pi}$ could be given by:

$$\boldsymbol{\pi} = \boldsymbol{\beta}(E(r_M) - E(r_f)) \tag{3.4}$$

where $E(r_m) - E(r_f)$ is the market risk premium, $\boldsymbol{\beta}$ is the vector of asset betas. Betas describes the correlated volatility of assets in relation to the volatility of the market portfolio and it can be written as:

$$\beta_i = \frac{Cov_{i,M}}{\sigma_M^2} \tag{3.5}$$

where $Cov_{i,M}$ is the covariance of risky assets and the market portfolio, and σ_M^2 is the variance of the market portfolio return. Note that $\sigma_M^2 = \mathbf{w}_M^{'}\mathbf{\Sigma}\mathbf{w}_M$, and $\mathbf{w}_M$ is the $N \times 1$ vector of market capitalisation weight, and $\mathbf{\Sigma}$ is the $N \times N$ vector of variance covariance matrix of asset excess returns.

Defining the risk aversion coefficient expressed as:

$$\delta = \frac{E(r_M) - E(r_f)}{\sigma_M^2} \tag{3.6}$$

[3] Fabozzi et al. (2008) show that a simple example of the matrix norm is called Euclidean norm, which can be calculated by the square root of the sum of squared elements in the matrix.

which is a measure for the rate at which the investor is willing to accept additional risk for a one unit increase in expected return. Bevan and Winkelmann (1998) explain the process of adjusting excess return to achieve a target Sharpe Ratio (SR) of 1. Black and Litterman (1992) use an SR closer to 0.5. Satchell and Scowcroft (2000) and He and Litterman (1999) set δ as a positive constant. They assume the world average risk aversion to be 2.5. Drobetz (2001) sets the value of the risk aversion coefficient to be 3. Idzorek (2004) sets a risk premium of 3% divided by the market portfolio variance to calculate the risk aversion coefficient of 3.07. Beach and Orlov (2007) calculate $\delta = 2.01$ for the world portfolio. Babameto and Harris (2009) use a value of 3.5% for the global market risk premium to get δ equal to 1.79. Dimson et al. (2007) forecast a geometric world risk premium of 3%-3.5%. The investment bank uses the risk premium of 4%-5%.

Then, the equilibrium excess return $\boldsymbol{\pi}$ can be denoted by:

$$\boldsymbol{\pi} = \delta \boldsymbol{\Sigma} \mathbf{w}_M \tag{3.7}$$

And the implied market capitalisation weights can be expressed as:

$$\mathbf{w}_M = \frac{1}{\delta} \boldsymbol{\Sigma}^{-1} \boldsymbol{\pi} \tag{3.8}$$

3.2.2 Investor Views

Investors can possess several views on the market returns of some assets in a portfolio, which differ from the implied equilibrium return. One of the most important parts is to translate these views into the Black-Litterman formula. Investors do not have to specify views on all of assets. The uncertainty of the views has the random error terms vector $\boldsymbol{\varepsilon}$ which follows the normal distribution with a mean of zero and the covariance matrix $\boldsymbol{\Omega}$. Note that these error terms are unknown and independent. The investor's views can be expressed as:

$$\mathbf{q} = \mathbf{P}\boldsymbol{\mu} + \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon} \sim N(0, \boldsymbol{\Omega}) \tag{3.9}$$

Let K be the total number of the views, $\mathbf{P}$ be the $K \times N$ matrix of view portfolios, and $\mathbf{q}$ be the $K \times 1$ vector of expected returns on the view portfolios.

Idzorek (2005) states that the error term $\boldsymbol{\varepsilon}$ would be a positive or negative value other than 0 except when the investor possesses 100% confidence about the expressed view. The element entered into the BL model is the variance of each error term υ_{ii}, which constitute $\mathbf{\Omega}$, where $\mathbf{\Omega}$ is a $K \times K$ diagonal covariance matrix with off-diagonal elements usually setting equal to zero. The general case of $\mathbf{\Omega}$ can be given by:

$$\mathbf{\Omega} = \begin{pmatrix} \upsilon_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \upsilon_{ii} \end{pmatrix} \quad (3.10)$$

He and Litterman (1999) explain that the assumption of a diagonal $\mathbf{\Omega}$ matrix is not a restriction because the $\mathbf{\Omega}$ matrix can always be transferred as the $\mathbf{\Omega} = \mathbf{X}\hat{\mathbf{\Omega}}\mathbf{X}^{-1}$ format to set $\hat{\mathbf{\Omega}}$ as a diagonal matrix. The uncertainty of the views could be denoted by the variances of the error terms $\boldsymbol{\varepsilon}$. The larger the variance of the error term υ_{ii}, the greater the uncertainty of the view.

In specifying the relative weighting of each individual asset for each view related to more than two assets, He and Litterman (1999) and Idzorek (2005) use a market capitalisation weighted scheme, which sets the position equal to the value of the asset's market capitalisation divided by the total market capitalisation; Satchell and Scowcroft (2000), however, use an equal weighted scheme. Meucci (2006) proposes that the matrix of asset weights within each view is invertible, and considers extending the BL method to non-normally distributed markets and views. Fabozzi et al. (2006) utilise the momentum trading strategy to set weights for each asset.

The choice of the diagonal elements of $\mathbf{\Omega}$, υ_{ii} is also a practical issue in the use of the BL model. Idzorek (2005) follows the method of He and Litterman (1999) which set the confidence of the view (the ratio of υ_{ii} / τ) as equal to the variance of the view portfolio $\mathbf{P\Sigma P'}$. Fabozzi et al. (2008) provide two methods to obtain υ_{ii}. The first method is to calculate through the backtesting procedure, which I will explain in Section 3.3.1. The second way is to derive the implied standard deviation from a statistical assumption about the distribution of a view. For example, if an investor has a belief that stock B will outperform stock A by 4% within an interval from 2% to 6% at a 90% confidence level then, with the assumption of normal distributed view, it can be derived that the implied standard deviation would be 1% based on basic statistical knowledge. Therefore, υ_{ii} would be equal to squared standard deviation at 0.0001.

In the BL framework, portfolio managers can input absolute or relative views. For example, the belief that the expected return of stock B is 1.5% can be regarded as the absolute view. The belief that the expected returns of stock C are higher than that of stock E by 1% is a relative view. For the above absolute views, it means that we can long stock B, while for the relative view, it means that we can long stock C and short stock E to get a zero-investment view portfolio.

Let me give a simple example to help understanding. The first view is an absolute view whereas the second one is a relative view. I can express the two views together as:

$$\begin{bmatrix} 1.5\% \\ 1\% \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}.$$

The first row of the $\mathbf{P}$ matrix describes the first absolute view and, similarly, the second row represents the second relative view. Since my target is to construct a zero-investment view portfolio, I choose the weights of the second view to add up to zero, but other weighting schemes are also possible. Note that the error terms ε_1, ε_2 do not explicitly enter into the BL model, but their variances do. For example:

$\mathbf{\Omega}_1 = \begin{bmatrix} 0.5\%^2 & 0 \\ 0 & 1\%^2 \end{bmatrix}$ reflects a greater confidence in the views and, conversely,

$\mathbf{\Omega}_2 = \begin{bmatrix} 20\%^2 & 0 \\ 0 & 25\%^2 \end{bmatrix}$ reflects a lower confidence in the views.

3.2.3 Combination of Both Perspectives

In the Bayesian approach, the CAPM prior can be combined with the additional views, the posterior expected returns are distributed as $N(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}})$, where $\hat{\boldsymbol{\mu}}$ is given by:

$$\hat{\boldsymbol{\mu}} = \boldsymbol{\pi} + \tau\boldsymbol{\Sigma}\mathbf{P}'(\tau\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}' + \boldsymbol{\Omega})^{-1}(\mathbf{q} - \mathbf{P}\boldsymbol{\pi}) \qquad (3.11)$$

Implied relations can easily be found from equation (3.11). In the absence of views, the $\mathbf{P}$ matrix is $K \times N$ zeros, then the posterior expected return becomes $\hat{\boldsymbol{\mu}} = \boldsymbol{\pi}$; when the views uncertainty $\boldsymbol{\Omega}$ is small (high confidence in views), the posterior expected returns $\hat{\boldsymbol{\mu}}$ tilt on expected returns in the view portfolios; when the views uncertainty $\boldsymbol{\Omega}$ is large (low confidence in views), the posterior expected return $\hat{\boldsymbol{\mu}}$ is close to implied equilibrium returns.

In the literature of Black and Litterman (1992), an alternative notion is often used:

$$\hat{\boldsymbol{\mu}} = [(\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{P}]^{-1}[(\tau\boldsymbol{\Sigma})^{-1}\boldsymbol{\pi} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{q}] \qquad (3.12)$$

Idzorek (2005) interprets that the equation (3.12) reflects that the new expected return forms as a weighted average of the implied equilibrium return vector $\boldsymbol{\pi}$ and view vector $\mathbf{q}$, while the relative weightings are constituted by a function of the scalar τ and the uncertainty of views $\boldsymbol{\Omega}$. The greater confidence in the views, the lower the weighting in the implied equilibrium return, the higher the weighting in the views, the closer the new expected return towards the views return, and vice versa. As mentioned above, Idzorek (2005) assumes the ratio of υ_{ii} / τ to be equal to the variance of the view portfolio $\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}'$. In this case, only $\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}'$ enters into function of the weighting, the scalar τ will not affect the new vector of expected return anymore. He shows a simple example and explains that this approach could avoid the sensitivity problem resulting from choosing different values of τ. Furthermore, Idzorek (2005) proposes a new method to assign an intuitive level of confidence (0% to 100%) to each view, free from the effect of setting different values of the scalar τ. The magnitude of the tilts away from market capitalisation weights should be controlled by the user-specified confidence level, based on percentage moves of the weights on the interval from 0% confidence to 100% confidence. Then the value of υ_{ii} would be the solution to minimise the sum of the square difference between the target weight vector and the weight calculated from the reverse optimisation, as shown in equation (3.17).

Satchell and Scowcroft (2000) demonstrate that the posterior covariance matrix $\hat{\mathbf{\Sigma}}$, which is the variance of the posterior mean estimate about the actual mean, is given by:

$$\hat{\mathbf{\Sigma}} = ((\tau\mathbf{\Sigma})^{-1}\boldsymbol{\pi} + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{P})^{-1} \quad (3.13)$$

It is the uncertainty in the posterior mean estimate, and is not the covariance of the returns.

3.2.4 Unconstrained Optimal Portfolio

The mean-variance optimisation process starts with an estimation of the expected returns and covariance matrix, since the posterior expected returns have been estimated in the BL model, together with the predictive covariance, the optimal portfolio position could be generated from the optimiser.

The estimated expected returns and covariance can be respectively expressed as:

$$\tilde{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}} \text{ and } \tilde{\mathbf{\Sigma}} = \hat{\mathbf{\Sigma}} + \mathbf{\Sigma} \quad (3.14)$$

For an investor with the risk aversion parameter δ, the maximisation problem can be written as:

$$\max \mathbf{w}'\tilde{\boldsymbol{\mu}} - \frac{\delta}{2}\mathbf{w}'\tilde{\mathbf{\Sigma}}\mathbf{w} \quad (3.15)$$

with the first order condition, it can develop that:

$$\tilde{\boldsymbol{\mu}} = \delta\tilde{\mathbf{\Sigma}}\mathbf{w}^* \quad (3.16)$$

Obviously, the optimal portfolio weights can be given by:

$$\mathbf{w}^* = \frac{1}{\delta}\tilde{\mathbf{\Sigma}}^{-1}\tilde{\boldsymbol{\mu}} \tag{3.17}$$

He and Litterman (1999) derive the optimal portfolio weights as:

$$\mathbf{w}^* = \frac{1}{1+\tau}(\mathbf{w} + \mathbf{P}'\mathbf{\Lambda}) \tag{3.18}$$

The equation (3.18) decomposes the optimal portfolio weights into two parts, scaled by a factor of $\frac{1}{1+\tau}$. One part is the market equilibrium portfolio weights $\mathbf{w}_M = \frac{1}{\delta}\mathbf{\Sigma}^{-1}\boldsymbol{\pi}$. When there are no views, the investor will hold an optimal portfolio with weights of $\frac{\mathbf{w}_M}{1+\tau}$ allocated to each asset. The other part is a weighted sum of the view portfolios. The weight for each portfolio is given by the corresponding element in the vector $\mathbf{\Lambda}$, which is defined as:

$$\mathbf{\Lambda} = \tau\mathbf{\Omega}^{-1}\mathbf{q}/\delta - \mathbf{A}^{-1}\mathbf{P}\frac{\mathbf{\Sigma}}{1+\tau}\mathbf{w} - \mathbf{A}^{-1}\mathbf{P}\frac{\mathbf{\Sigma}}{1+\tau}\mathbf{P}'\tau\mathbf{\Omega}^{-1}\mathbf{q}/\delta \tag{3.19}$$

where:

$$\mathbf{A} = \frac{\mathbf{\Omega}}{\tau} + \mathbf{P\Sigma}/(1+\tau)\mathbf{P}' \tag{3.20}$$

The equation (3.19) reflects the effect of several factors on weights carried in the optimal portfolio. The first factor is the views, which can be observed from the first term. The higher the expected returns on the view portfolio or the lower the confidence of the views, the more weights tilt on the views. The second factor is the covariance between the view portfolio and the market equilibrium portfolio. The third factor shown in the last term is the covariance of the view portfolio with other view portfolios. The negative sign in the front of last two terms indicates an inverse relation between the weights of views and these covariances. In other words, if the covariance between the view portfolio and the market equilibrium portfolio increases, or if the covariance of the view portfolio with other view portfolios increases, then the weight for each view portfolio would decrease. The final optimal portfolio weights would tilt on the market equilibrium portfolio weights. In summary, if we have only partial views on some assets, then, by using a posterior estimate of the variance, we will tilt the posterior weights towards assets with lower variance and away from assets with higher variance. This tilt will not be very large if we are working with a small value of τ.

He and Litterman (1999) also discuss other cases, including changing risk aversion coefficient δ, fixing risk limit σ_0 and adding constraints on the portfolio. When the risk aversion coefficient is not constant, it resorts to the scaled optimal portfolio weights $\mathbf{w}_1^* = \frac{\delta}{\delta_1}\mathbf{w}^*$. When the standard deviation of the portfolio is limited to a specific value σ_0, the solution of the optimal portfolio weights is $\mathbf{w}_2^* = \frac{\sigma_0 \delta}{\sqrt{\tilde{\boldsymbol{\mu}}' \tilde{\boldsymbol{\Sigma}} \tilde{\boldsymbol{\mu}}}}\mathbf{w}^*$. When other constraints are imposed on the portfolio, the optimal portfolio could be yielded by inputting $\tilde{\boldsymbol{\mu}}$ and $\tilde{\boldsymbol{\Sigma}}$ into the portfolio optimiser.

3.3 Extensions of the Black-Litterman Model

Of course, the Black-Litterman model never stops developing. It is well-known that asset returns often show some empirical regularities consisting of thick-tailed distributions, volatility clustering, common movements and persistence in volatilities, thus more and more academic studies focus on improving and extending the BL model in favour of the asset returns properties in recent years.

Several studies are concerned with a clear specification of the required input parameters including equilibrium returns and views. The original BL model holds the assumption that the prior information and the views are jointly normally distributed, however, some asset returns cannot be considered to be normally distributed in reality, with negative skewness or leptokurtosis properties. Giacometti et al. (2007) extend the original BL model to consider the effect of different distributions (normal distribution, t-distribution and stable distribution) and the alternative risk measures on market equilibrium returns. Meucci (2006, 2008) introduces a copula and opinion-pooling methodology to use non-normal views into the BL model, with general application to any market distribution. These approaches enlighten Martellini and Ziemann (2007), who propose incorporating higher instances of hedge fund return distributions into the BL Bayesian approach to construct a portfolio based on the four-moment CAPM. Unlike the work of Giacometti et al. (2007), Martellini and Ziemann utilise the non-parametric approach for general return distributions instead of the parametric method.

With the development of CAPM theory into the Fama and French Factors Model (Fama and French, 1992, 1996), some works apply the Fama and French Models into the BL model. Krishnan and Mains (2005) propose a two-factor BL model which substitutes the original equilibrium return with a multifactor equilibrium return. Gofman and Manela (2010) extend the BL approach to any linear multifactor asset pricing model such as the ICAPM, and further provide a natural Bayesian framework that incorporates an equilibrium model uncertainty into the inference problem. Fabozzi et al. (2006) construct the cross-sectional momentum portfolio as the view portfolio. Jones et al. (2007) consider the use of size, value and momentum factors in constructing a view portfolio inputted into the BL model. Babameto and Harris (2009) incorporate value and momentum trading strategies to track the benchmark at the desired tracking error level under full investment, long-only and beta-neutral constraints.

Furthermore, to take the volatility of asset returns into account, a few researchers have done some theoretical and empirical work to extend the original BL model by incorporating the volatility models. Qian and Gorman (2001) first build a unified theoretical framework to combine both the mean vector and the covariance matrix of investor views into the BL method. They agree that the creative work of the BL method, which uses the conditional distribution implied by the joint equilibrium distribution to adjust the mean vector, can reduce the sensitivity of input resulting from mean-variance optimisation. However, they argue that both the mean vector and the covariance matrix of the view portfolio should be adjusted in the use of conditional distribution, while the BL method fails to adjust the covariance matrix of the views. Beach and Orlov (2007) use EGARCH-M models to generate their views as inputted into the BL model. Palomba (2008) incorporates multivariate FDCC-GARCH forecasts about expected returns and covariance matrices to build a few view portfolios into the BL approach with tracking error constraints in tactical asset allocation. Conservatively speaking, these studies carefully introduce volatility models to obtain reasonable views. However, the market equilibrium excess return covariance matrix is assumed to be constant. They still use the sample historical covariance matrix on a rolling basis to investigate the market equilibrium return.

In the area of controlling BL portfolio risk, most studies refer to standard deviation, market exposure and tracking error (Bevan and Winkelmann, 1998; He and Litterman, 1999; Jones et al., 2007; Braga and Natale, 2007; Palomba, 2008; Babameto and Harris, 2009). There are extremely few studies discussing constraining the alternative downside risks, such as VaR and CVaR in BL portfolio optimisation. Giacometti et al. (2007) consider VaR and CVaR in the BL portfolio, but the research aim is to revise the equilibrium returns to reflect the non-normal character of the asset returns in the use of VaR and CVaR, and to minimise the forecasting error of the equilibrium returns to the realised returns. They do not show the full picture of the risk-constrained optimal portfolio. Martellini and Ziemann (2007) modify the VaR with higher moments to measure the active hedge fund portfolio, and construct the minimum VaR portfolio as the benchmark portfolio to obtain the neutral weights. They report the modified VaR to evaluate the performance of the extended BL model. However, they do not discuss the effects of VaR constraints on the optimal BL portfolio. Lejeune (2011) proposes the new VaR-BL model to construct a fund-of-funds, with the objective of an absolute return within the specific level of VaR. His model also incorporates some specific trading constraints into the optimisation problem, such as diversification, but-in-threshold, liquidity and currency. In his study of VaR constraints, he emphasises the derivation of deterministic equivalent and approximation for the VaR optimisation problem in order to demonstrate that deterministic reformulations are convex. Furthermore, he investigates the computational efficiency of different software solvers to solve the derived optimisation problem. He does not show the empirical results and makes an analysis within the framework of different VaR constraints. Based on the BL model, Veress et al. (2012) obtain forecasts through the Baltic Dry Shipping Index for a number of developed and emerging markets in attempt to enhance optimal portfolios evaluated by the downside risk in form of maximum monthly drawdown and Sortino ratios. They consider the downside risk, however, they focus on using downside risks to evaluate the portfolio performance.

In addition, the number of studies concerned with the application of the BL model in recent years include those by Becker and Gürtler (2009), Da Silva et al. (2009), Cheung (2009), Giacometti and Mignacca (2010), Munda and Strasek (2011), Mishra et al. (2011), Fernandes et al. (2011), and Braga and Natale (2012). Becker and Gürtler (2009) make attempt to integrate the analysts' dividend forecasts into the BL model. Da Silva et al. (2009) propose a remedy to help the portfolio manager reduce unintended trading and take less risk when applying the BL model into active investment management. Cheung (2009) further applies the BL model to several practical issues, and enables the implementation and application of the BL model. Giacometti and Mignacca (2010) investigate stress test analysis of the current managed portfolio in the use of BL framework. Mishra et al. (2011) examine the BL approach in the context of the Indian equity market. Munda and Strasek (2011) use target price to develop the 'Target-to-Real-Price' (TRP) ratio and generate adjusted views returns to input into the BL model. Fernandes et al. (2011) compare the use of a portfolio optimisation methodology from the BL approach and resampling technique. Braga and Natale (2012) propose a new measure for the marginal contribution of each view to the ex-ante tracking error volatility (TEV) in the BL framework.

In the following section, I will introduce the useful approach of using trading strategies in the BL model. I will also briefly present two methods of considering alternative risk measures in the BL framework and make a comparison. In addition, I make some comments about these extensions. Motivated by these methods, I will propose the dynamic BL model with risk constraints and the dynamic BL portfolio optimisation with maximal reward to VaR ratio and reward to CVaR ratio in Chapter 4.

3.3.1 Incorporating Momentum Trading Strategies into the Black-Litterman Model

Fabozzi et al. (2006) incorporate momentum trading strategies to generate investor views as the inputted data of a BL portfolio. Jegadeesh and Titman (1993) find that the momentum that past winners of securities generates would retain their good performance in the near future within a certain period, while the past losers would not change their poor performance. Based on the strategy of buying past winners and simultaneous shorting past losers, they provide the empirical evidence that this strategy could make promising profits on a timescale of three to 12 months. Therefore, Fabozzi et al. (2006) propose constructing a cross-sectional momentum portfolio as views in the BL model. In this section, I will briefly introduce their methods. Firstly, they rank the securities based on their performance over the past nine months; then, a long-short portfolio can be constructed by purchasing good performers and selling bad performers. The quantity used to rank them is their nine-month normalised return:

$$z_{t,i} = \frac{p_{t,i} - p_{t-9,i}}{p_{t-9,i}\sigma_i} \tag{3.21}$$

where, $p_{t,i}$ expresses the price of security i at time t, $p_{t-9,i}$ expresses the price of the security i nine months before t, and σ_i is the volatility of security i.

The top half and the bottom half of securities are allocated weights of $w = \frac{1}{\sigma_i c}$ and $w = -\frac{1}{\sigma_i c}$ respectively. Then, the view matrix $\mathbf{P}$ in the BL model is a single row with elements one of the two quantities above. These weights, calibrated with volatilities, are able to balance the weights among less volatile and more volatile securities. The parameter c is a constant whose role is to constrain the annual long-short portfolio volatility to a certain level (20% in the application). Note that the portfolio weights do not sum to zero with this non-zero-cost long-short portfolio. Since weights are assigned on each security, the expected return of this long-short momentum portfolio as the expected view return $\mathbf{q}$ in the BL model will be calculated.

The next challenge is to decide the confidence level in the views and they use the back-testing procedure. After constructing the momentum portfolios in each period t , they hold it for one month and observe its return $r_{m,t}$ over the holding period. For the same holding period, they observe the realised return $r_{a,t}$ on the portfolio of the actual winners and losers. Then, the residual return is calculated as the difference between $r_{m,t}$ and $r_{a,t}$. The series of residuals could be obtained by moving the evaluation period one month forward and replicating the process. In the end, the level of confidence $\mathbf{\Omega}$ in the view equals to the variance of the series of residual returns.

Indeed, they provide a simple and convenient method for practitioners to introduce momentum strategy into the BL model. However, there are two shortcomings in this method. On one hand, they do not consider the pro-cyclical effect of the momentum strategy and the counter-cyclical effect of the value strategy to combine these two strategies (Bird and Whitaker, 2003). With the impetus from the contrasting properties of the momentum strategy and the value strategy, Babameto and Harris (2009) utilise the combined value-momentum strategy to form a BL portfolio with a promising out-of-sample performance. On the other hand, they assume the volatility to be constant during the holding period when they rank the normalised return. It is not realistic because a significant proportion of the literature shows empirical results that the volatilities of securities are time-varying.

3.3.2 Alternative Risk Measures in the Black-Litterman Approach

Giacometti et al. (2007) relax the assumption of multivariate normal distribution for the returns in the original BL approach to other return distributions, such as t-distribution and stable distribution. They improve the BL model by incorporating non-normal return distributions and alternative risk measures into the CAPM equilibrium returns. They compare the equilibrium returns obtained under different return distributions and different risk measures with the unconditional mean. Their empirical results support evidence of a better forecast by using stable distribution combined with the dispersion risk measure and the CVaR risk measure. I will briefly introduce their methodology.

Giacometti et al. (2007) propose to modify the equation (3.15) to the general case of different return distributions as follows:

$$\max(\mathbf{w}'\boldsymbol{\pi} - \frac{\delta}{2}\vartheta(\mathbf{w}'\mathbf{r})) \tag{3.22}$$

where $\vartheta(\mathbf{w}'\mathbf{r})$ indicates the measure of risk (the variance, the VaR, the CVaR) of the portfolio return $\mathbf{w}'\mathbf{r}$, and the equilibrium returns can be given by:

$$\boldsymbol{\pi} = \delta\boldsymbol{\Sigma}\mathbf{w} \tag{3.23}$$

$$\boldsymbol{\pi} = \frac{\delta}{2}(VaR_{\beta}\frac{\boldsymbol{\Sigma}\mathbf{w}}{\sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}} - E(\mathbf{r})) \tag{3.24}$$

$$\boldsymbol{\pi} = \frac{\delta}{2}(CVaR_{\beta}\frac{\boldsymbol{\Sigma}\mathbf{w}}{\sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}} - E(\mathbf{r})) \tag{3.25}$$

where $\boldsymbol{\Sigma}$ is the covariance matrix under different distributional assumptions, VaR_{β} is the Value-at-Risk for the corresponding distribution at the confidence level of β, $CVaR_{\beta}$ is the Conditional Value-at-Risk for the corresponding distribution at the confidence level of β, $E(\mathbf{r})$ is the expected returns.

Note that they use a different method to set the risk aversion coefficient δ. They set δ equal to the solution of an optimisation problem, which could minimise the sum of the squared error between the neutral equilibrium returns $\boldsymbol{\pi}$ and the day after realisation of return for 20 consecutive months on the basis of a rolling window of 110 months.

Obviously, they pay more attention to revising the equilibrium returns with VaR and CVaR corresponding to different distributions. The impact of choice of distributions and alternative risk measures on the optimal BL portfolio actually result from the estimated equilibrium returns. They evaluate the forecasting performance instead of the unconstrained BL portfolio performance. The risk-adjusted BL portfolio performance is out of their research scope.

3.3.3 A VaR Black-Litterman Model for the Construction of Absolute Return Fund-of-funds

Lejeune (2011) derives the new VaR-BL model, targeting an absolute return to construct a fund-of-funds portfolio. He applies the BL approach to expect asset returns, and then imposes the VaR constraints and specific trading constraints, including diversification, buy-in threshold, liquidity and currency requirements to the probabilistic integer and non-convex optimisation problem. From the perspective of providing a solution method, his work involves two steps. The first step is to derive a deterministic reformulation of the probabilistic problem and the next step is to employ a branch-and-bound algorithm to construct the optimal fund-of-funds. Furthermore, they evaluate the computation contribution of their solution method to confirm that their algorithm technique is efficient, robust and fast. I will summarise their methodology as follows.

The VaR-BL fund-of-funds optimisation problem, which is a non-convex probabilistic integer problem, is given by:

$$\max \mathbf{w}'\tilde{\boldsymbol{\mu}} \tag{3.26}$$

$$\text{subject to } \mathbf{w}'\mathbf{1} = 1$$

$$\mathbf{w}'\tilde{\boldsymbol{\Sigma}}\mathbf{w} \le \sigma_0^2$$

$$P(R_p \le VaR_\beta(R_p)) \ge \beta$$

$$L_i \le w_i \le U_i \quad i = 1,\ldots,N$$

$$|w_i| \ge L_{w_i} \quad i = 1,\ldots,N$$

$$\theta_i = \begin{cases} 1, & if \;\; w_i \ne 0 \\ 0, & if \;\; w_i = 0 \end{cases}$$

Function (3.26) is the object function with target of achieving the maximal absolute return in the optimisation model. The six constraints are the budget constraint, quadratic constraint, VaR constraint, holding constraint, threshold constraint and the integer constraint. The budget constraint means the entirety of the capital is invested. The quadratic constraint ensures the variance of the portfolio does not exceed a prescribed maximal value of σ_0^2. The VaR constraint limits the magnitude of the loss of the capital to be, at most, a specified probability level of $1-\beta$ during a certain period. Taking into the lack of subadditivity property of the VaR risk measure, the author imposes the holding constraints to construct a well-diversified portfolio, which could compensate for the shortcomings of VaR. In the holding constraints, L_i and U_i are vectors specifying the lower and upper bounds of the positions of asset i. The threshold constraint that avoids small investments in a number of assets due to transaction cost, L_{w_i} is the prescribed smallest holding size allowed for asset i. θ_i is a binary variable in the integer constraint.

In order to solve the optimisation problem, the first challenge is to make the probability inequality easily computable. Under the normal distribution assumption of portfolio returns, Lejeune (2011) rewrites the VaR constraint as:

$$\mathbf{w}'\tilde{\boldsymbol{\mu}} + F_{1-\beta}^{-1}\sqrt{\mathbf{w}'\tilde{\boldsymbol{\Sigma}}\mathbf{w}} \geq -\alpha \tag{3.27}$$

where, $F_{1-\beta}^{-1}$ is the $(1-\beta)$-percentile of the normal distribution F. When the probability distribution of the portfolio returns is unknown, the author utilises the well-known probability inequalities to obtain convex approximations.

The second task is to solve the optimisation problem with integer constraints; the author resorts to a non-linear, branch-and-bound algorithm. Non-linear, branch-and-bound algorithm is out of the scope of the thesis.

Lejeune (2011) is the first researcher to impose the VaR constraints on the BL model. He adds diversification constraints to remedy the shortcomings of the VaR constraints, which fails to satisfy the subadditivity property in the coherent measure. However, there is a possible way to impose the CVaR constraints on the BL model, because CVaR, which has the subadditivity property, could overcome the VaR risk measures. Moreover, Rockafellar and Uryasev (2000) propose a new approach for portfolio optimisation to calculate VaR and optimise CVaR simultaneously.

In summary, Giacometti et al. (2007) and Lejeune (2011) enhance the BL model by incorporating the alternative risk measures in two different directions. Giacometti et al. (2007) focus on revising the market equilibrium return, while

Lejeune (2011) devotes his attention to imposing constraints. In addition, both Giacometti et al. (2007) and Lejeune (2011) develop the portfolio optimisation model in a static environment, which means they assume that the covariance matrix would be constant. However, it is well known that constant volatility is not real asset return regularity.

3.4 Conclusions

Having carried out an extensive review of the literature, it is clear that the BL model is an intuitive and practical method in the asset allocation process, and that the BL model has been gradually developed to comply with robust portfolio selection in recent years. The three main directions of enhancement consist of extending the market equilibrium return, various methods of generating views, and constructing constrained BL portfolios. As discussed in Section 3.3, the Fama and French Factor Models could be used to obtain revised market equilibrium returns, to construct a portfolio with trading strategies in order to generate views, and to control the beta measures of the constrained BL portfolio. The multivariate GARCH models are concentrated to produce the view returns and view covariances. Alternative risk measures, such as VaR and CVaR, are suggested to rewrite the market equilibrium return and evaluate portfolio performance. VaR can also be regarded as a limited risk requirement in the construction of the portfolio. Other constraints are generally imposed by tracking error and variety of trading constraints.

However, nowhere in the literature is anything that contributed to generating a conditional CAPM equilibrium return of the BL model in a dynamic environment; or that studied the impact of CVaR constraints on the BL model in comparison with VaR constraints on the BL model. Furthermore, none of the studies constructed a BL portfolio with maximal reward to VaR ratio and reward to CVaR ratio. Therefore, according to the studies about conditional CAPM (Bollerslev et al., 1988) and alternative risk measures such as VaR and CVaR discussed in Chapter 2, it is possible to improve the BL model through these three directions in my thesis. I will propose the new dynamic BL model in Chapter 4.

CHAPTER 4
DATA AND METHODOLOGY

Chapter 3 focusd on the literature of the BL methodology and provided a thorough literature review. According to this review, there is a lack of discussion on the topic of building the dynamic BL model based on conditional expectations. The corresponding dynamic downside risk constrained BL portfolio is also a potential research direction, because no one has made a study of downside risk measures in dynamic BL portfolios. In this chapter, the main purposes are to describe the data used in the empirical study and to introduce the new proposed dynamic BL model with VaR and CVaR taken into account in portfolio optimisation. I propose a dynamic BL asset allocation approach that extends the original BL model to the dynamic case, based on the conditional expectations in CAPM. Taking downside risk measures including VaR and CVaR into account, I also propose other two reward-to-risk ratios, rather than the Sharpe ratio, as the target function in the optimisation problem. In addition, I design a method for investigating the impact of imposing VaR and CVaR constraints on the dynamic unconstrained BL portfolio, with normal distribution and t-distribution at different confidence levels. Furthermore, the portfolio performances are analysed and evaluated in a single period and multi-period, through in-sample analysis and out-of-sample analysis.

In the following sections, Section 4.1 focuses on the task of data description and analysis of the excess return data property of non-normality and time series property. Section 4.2 provides a detailed introduction to the new dynamic BL model framework with VaR and CVaR implemented in portfolio optimisation. Specifically, Section 4.2.1 describes the estimation of time-varying covariance in the use of the Rolling Window method, the EWMA model and the DCC model, which is the indispensible step in asset allocation process. Section 4.2.2 gives the procedures to construct the dynamic BL portfolio. Starting from the estimation of conditional equilibrium returns introduced in Section 4.2.2.1, and translating the investor's views into the BL model explained in Section 4.2.2.2, the BL conditional expected returns and covariance matrix, which can be anticipated from the combination of equilibrium returns and additional views, are described in Section 4.2.2.3. Section 4.2.3 focuses on constructing the unconstrained BL portfolio with the target of maximal reward-to-risk ratios. Section 4.2.4 shows the method of allocating assets in the portfolio with VaR constraints and Section 4.2.5 changes the VaR constraints to CVaR constraints. Section 4.2.6 clarifies in-sample and out-of-sample analysis with BL portfolio performance evaluation, in a single period and over multiple periods.

4.1 Data

The empirical analysis uses monthly price indices and market values for the FTSE 10 industry sectors in the US, UK and Japan, for the period from December 1993 to May 2010. The whole sample has 197 observations. All of these data are collected from DataStream. The selection of FTSE sector indices is to avoid survivorship bias in the FTSE 100, with components of companies that might not always exist in indices. The currency of the price indices and market values is the US Dollar. In addition, I also collect the US one month Treasury Bill rate in the corresponding period from the Kenneth R. French Data Library. I use price indices to compute returns, and subtract the Treasury Bill rate from returns to calculate the excess return: throughout this thesis, I work with the excess returns. The market capitalization of each index is used to generate weights of all the indices in each month for the market benchmark portfolio.

Table 4.1 shows the summary statistics of excess returns for each asset from January 1994 to May 2010. The Jarque-Bera test is used to test the normality property of excess returns at 5% significance level. The null hypothesis of the Jarque-Bera test is that the sample comes from a normal distribution with unknown mean and variance, against the alternative that it does not come from a normal distribution; it is a two-sided goodness-of-fit test, suitable when a fully-specified null distribution is unknown and its parameters must be estimated. For large sample sizes, the test statistic[4] has a chi-square distribution with two degrees of freedom. As can be seen from Table 4.1, the Jarque-Bera test significantly rejects the null hypothesis of normality at 5% significance level for

[4] The test statistic is calculated by $t_{JB} = \frac{n}{6}[s^2 + \frac{(k-3)^2}{4}]$, where n is the sample size, s is the sample skewness, and k is the sample kurtosis.

each asset with p-values that are close to zero, far less than 0.05; it means that the excess return of each asset is not subject to normal distribution. In this case, the assumption of normal distribution in models actually does not apply in practice; it becomes the motivation to relax the normal distribution assumption. In the thesis, I estimate the VaR and CVaR with the normal distribution and t-distribution and compare the difference between the two different distribution assumptions with different confidence levels.

Table 4.2 reports the time series property of excess return for each asset from January 1994 to May 2010, showing the first five autocorrelation coefficients and statistic values of the Ljung-Box test for serial correlation up to 10 lags, the ARCH test of Engle (1982) and the DCC test of Engle and Sheppard (2001). Only a few excess return series display highly significant autocorrelations. In particular, the excess return series in UK Basic Materials, UK Financials, UK Telecom, USA Industrials, Japan Industrials, Japan Technology, and Japan Telecom shows virtually significant autocorrelation. The null hypothesis in the Ljung-Box Q-test is that all autocorrelations up to the tested lags are zero. This null hypothesis is significantly rejected for tests at lags from 1 to 5 and 10 lags. This seems suggest that only these excess returns series might need a conditional mean model. However, the possibilities of non-linear dependence of excess returns and low power of test still exist; there may be non-linear dependence that is picked up by momentum but not by serial correlation. I conduct Engle's ARCH test with one and two lags ARCH models to check for conditional heteroscedasticity. About 20 out of 30 excess return series reject the null hypothesis of no ARCH effects in favour of the alternative ARCH model with one and two lagged squared innovations in Engle's ARCH tests. The ARCH test suggests that there is evidence of significant volatility clustering for most of the assets excess returns. The DCC test is to test the null hypothesis of constant correlation against the alternative of dynamic conditional correlation. According to Table 4.2, the DCC test for 30 excess return series failed to reject the null of a constant correlation in favour of a dynamic structure with p-value bigger than 10%. Interestingly, the DCC test for 18 excess return series selected from 30 excess return series suggests that the data set of 18 assets exhibits significant time varying conditional correlations with p-value less than 1%. It implies that the portfolio constructed by 18 assets actually has the dynamic conditional correlation, and it can be naturally concluded that the portfolio constructed by 30 assets should have the time-varying conditional correlation as well, because these 30 assets contain 18 assets with dynamic conditional correlation. In this case, it makes us doubt the power of the DCC test for 30 excess return series. The non-rejection of the null hypothesis may be due to the lower power of the test. Motivated by the results from Table 4.2, volatility models should be applied into the portfolio construction process.

4.2 Methodology

The results from Table 4.1 and Table 4.2 support the evidence of volatility clustering and non-normality characteristic of asset returns. In order to generalize the BL model in the real world, the volatility models and tail risk measures should be incorporated into the BL model. The following sections develop the new dynamic BL model framework step by step.

4.2.1 Estimation of Time-Varying Covariance

Apparently, the data in Table 4.2 do exhibit volatility clustering, as fluctuations between any two consecutive months are correlated with the adjacent periods. The use of time-varying volatility models is a prerequisite for developing the dynamic asset allocation model. In order to narrow the scope of research, I select two simple and straightforward volatility models, including the Rolling Window method and the EWMA model, to estimate the conditional covariance. In addition, the use of the DCC model could reduce the magnitude of estimated parameters in large-scale assets, and the results of the DCC test as shown in Table 2 confirm the dynamic conditional correlation. Therefore, I also employ the DCC model to make an estimation of the covariance matrix. In this thesis, without consideration of transaction cost, I rebalance the portfolio every month. I use the estimated conditional covariance matrices on each rebalancing date.

4.2.1.1 Covariance Matrix via Historical Rolling Window Estimators

Consider the specification of models for the full N-dimensional conditional distribution of asset (excess) returns $\mathbf{r}_t = (r_{1t}, r_{2t}, \cdots, r_{Nt})'$ with conditional mean zero and conditional covariance matrix $\mathbf{H}_t$:

$$\mathbf{r}_t = \mathbf{H}_t^{\frac{1}{2}} \mathbf{z}_t \tag{4.1}$$

where $\mathbf{z}_t$ is i.i.d with $E(\mathbf{z}_t) = 0$ and $\text{var}(\mathbf{z}_t) = \mathbf{I}_N$. The covariance matrix $\mathbf{H}_t$ is calculated on a window length of M and rolled forward:

$$\mathbf{H}_t = \frac{1}{M-1} \sum_{i=1}^{M} \mathbf{r}_{t-i} \mathbf{r}_{t-i}^{'} \tag{4.2}$$

When selecting the window length M, two considerations have to be balanced. On one hand, I should choose a sample that is as long as possible in order to increase the precision with which we can make the covariance estimation. On the other hand, I should also use a sample that is as short as possible in order to increase the relevance of our recent sample. Decreasing the window length increases the sensitivity of the rolling variance estimator to observations that lie within the window, and consequently increases the volatility of the volatility estimator. The choice of window length is actually a tricky problem in practice, because it would have a big effect on results. The Rolling Window model could capture the time-varying property of volatility and covariance but fail to capture the persistence of volatility and covariance, due to equal weights imposed on both recent and distant observations.

4.2.1.2 Covariance Matrix via Exponential Weighted Estimators

The Exponentially Weighted Moving Average (EWMA) model puts more weight on recent observations and less on the distant past, and remedies the drawbacks of the Rolling Window model to capture the volatility persistence. The covariance matrix can be expressed as:

$$\mathbf{H}_t = \lambda \mathbf{H}_{t-1} + (1-\lambda) \mathbf{r}_{t-1} \mathbf{r}_{t-1}^{'} \tag{4.3}$$

where λ is the decay factor $0 \le \lambda \le 1$ and determines how rapidly the weights on past observations decline; typically it is estimated between 0.92 and 0.96. In *RiskMetrics* (J.P. Morgan, 1994), the decay factor is set to 0.94. The first term of the right hand side of (4.3), $\lambda \mathbf{H}_{t-1}$, determines the persistence in volatility, and the second term, $(1-\lambda) \mathbf{r}_{t-1} \mathbf{r}_{t-1}^{'}$, represents the response of volatility to one-period news. This form of the EWMA estimator is both intuitively appealing and more convenient to implement. The forecasting ability of this model easily overcomes other more sophisticated methods that add more generality (e.g., GARCH-type models).

The most general multivariate GARCH (1, 1) model is

$$vech(\mathbf{H}_t) = vech(\mathbf{C}) + \mathbf{B}vech(\mathbf{H}_{t-1}) + \mathbf{A}vech(\mathbf{r}_{t-1}\mathbf{r}_{t-1}^{'}) \quad (4.4)$$

where the *vech* operator converts the unique upper triangular elements of a symmetric matrix into a $\frac{1}{2}N(N+1)\times 1$ column vector, and **A** and **B** are $\frac{1}{2}N(N+1)\times\frac{1}{2}N(N+1)$ matrices. This model has a total of $\frac{1}{2}N^4+N^3+N^2+\frac{1}{2}N$ parameters. If we use this model in this paper, 432,915 parameters have to be estimated. This is a serious dimensionality problem. In order to reduce the dimensionality, I consider using the Dynamic Conditional Correlation model. In addition, Table 2 shows the results of the DCC test, which implies that the conditional correlation is dynamic, and the DCC model could be used in the estimation of the covariance matrix.

4.2.1.3 Covariance Matrix via Dynamic Conditional Correlation Model

The returns can be either mean zero or the residuals from a filtered time series[5].

$$\mathbf{r}_t \mid \Im_{t-1} \sim N(0,\mathbf{H}_t) \quad (4.5)$$

It is a simple but useful decomposition of the covariance matrix into the correlation matrix pre- and post-multiplied by the diagonal matrix, which can be expressed as:

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t \quad (4.6)$$

Where $\mathbf{H}_t$ is the time-varying covariance matrix, $\mathbf{D}_t$ is the diagonal matrix of time-varying standard deviations from univariate GARCH models, with standard

[5] Engle and Sheppard (2001) explain that the standard errors of the model will not depend on the choice of filtration (ARMA), as the cross-partial derivative of the log-likelihood with respect to the mean and the variance parameters has expectation zero when using the normal likelihood.

deviations σ_i on the i^{th} diagonal, i.e., $\mathbf{D}_t = diag\{\sigma_i\}$. $\mathbf{R}_t$ is the time-varying correlations matrix.

Eagle and Sheppard (2001) propose a decentralized estimation procedure. First, one fits to each asset return an appropriate univariate GARCH model and then standardizes the returns by the estimated GARCH conditional standard deviations. Returns are divided by their conditional volatility to obtain the standardized zero-mean residual $\mathbf{e}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$. Then one exploits the standardized return vector $\mathbf{e}_t$ to model the correlation dynamics with the individual correlations in the $\mathbf{R}_t$ matrix defined by the corresponding normalized elements of $\mathbf{J}_t$.

$$\mathbf{J}_t = (1-\alpha-\beta)\overline{\mathbf{J}} + \alpha(\mathbf{e}_{t-1}\mathbf{e}'_{t-1}) + \beta\mathbf{J}_{t-1} \tag{4.7}$$

$$\mathbf{R}_t = diag(\mathbf{J}_t)^{-\frac{1}{2}}\mathbf{J}_t diag(\mathbf{J}_t)^{-\frac{1}{2}} \tag{4.8}$$

where $\mathbf{J}_t$ is the approximation of the conditional correlation matrix $\mathbf{R}_t$. In the DCC model, $\mathbf{J}_t$ converges to the average correlation $\overline{\mathbf{J}}$. This model is analogous to the multivariate GARCH (1, 1) model (see equation (4.4)), but in terms of volatility-adjusted standardized returns. If α and β are positive with $\alpha + \beta < 1$ and the initial matrix $\mathbf{J}_1$ is positive definite, $\mathbf{J}_t$ is positive semi-definite. As the diagonal elements of $\mathbf{J}_t$ are equal to unity only on average, $\mathbf{J}_t$ is rescaled to calculate the conditional correlation matrix $\mathbf{R}_t = diag(\mathbf{J}_t)^{-\frac{1}{2}}\mathbf{J}_t diag(\mathbf{J}_t)^{-\frac{1}{2}}$. The conditional volatility $\mathbf{D}_t$ and conditional correlations $\mathbf{R}_t$ can be input into equation (4.6) to estimate the conditional covariance matrix $\mathbf{H}_t$.

4.2.2 Dynamic BL Model

In the new dynamic BL model, I define the first and second moments of N asset (excess) returns, conditional on the information set Y, as follows:

$$\mathbf{r}_t = \boldsymbol{\mu}_{BL,t} + \boldsymbol{\varepsilon}_t$$

$$\boldsymbol{\varepsilon}_t \mid Y_{t-1} \sim F(0, \mathbf{V}_t)$$

where (excess) returns vector $\mathbf{r}_t = (r_{1t}, r_{2t}, \cdots, r_{Nt})'$, $\boldsymbol{\mu}_{BL,t}$ is the $N \times 1$ vector of asset expected (excess) returns in BL model in period t, and $\mathbf{V}_t$ is the $N \times N$ covariance matrix, $\boldsymbol{\varepsilon}_t$ is the $N \times 1$ error term vector, $F(.)$ is any location-scale family distribution.

4.2.2.1 Conditional Equilibrium Return

Bollerslev et al. (1988) argue that investors may form common expectations on returns which are variable conditional expectations instead of constants. They utilize the multivariate generalized autoregressive conditional heteroscedastic (GARCH) process into the Capital Asset Pricing Model (CAPM) to estimate returns. The idea behind their method is that the expected returns are proportional to the conditional non-diversifiable risk, which is represented by the conditional covariance of each return with the market portfolio.

Following Bollerslev et al. (1988), let $\mathbf{r}_t$ be the $N \times 1$ vector of excess returns of all assets in the market at time t, and let $\boldsymbol{\pi}_t$ be the $N \times 1$ conditional mean vector and $\mathbf{H}_t$ be the $N \times N$ conditional covariance matrix of these returns given information available at time $t-1$. In addition, define $\mathbf{w}_{t-1}$ to be the $N \times 1$ vector of market capitalization weights at time $t-1$, and the excess return on the market portfolio is denoted by $r_{M,t} = \mathbf{w}_{t-1}'\mathbf{r}_t$. When the CAPM holds, the conditional mean vector $\boldsymbol{\pi}_t$ satisfy the equation as follows:

$$\boldsymbol{\pi}_t = \delta_t \mathbf{H}_t \mathbf{w}_{t-1} \tag{4.9}$$

where δ_t is the risk aversion coefficient. Bollerslev et al. (1988) assumed δ_t to be constant. However, some published works explain that the risk aversion coefficient would be time-varying (Brandt and Wang, 2003; Smith and Whitelaw, 2009; Berardi, 2012). In this thesis, I assume δ_t to be dynamic. In order to make the model simple, I use a simple method (Idzorek, 2004; Babameto and Harris, 2009) to calculate the risk aversion coefficient as the value of the global market risk premium divided by the market variance, as discussed around the Chapter 3 equation (3.6). Note that the market variance $\mathbf{w}_{t-1}'\mathbf{H}_t\mathbf{w}_{t-1}$ is time-varying.

4.2.2.2 Incorporating Momentum Strategies to Generate Views

Following the method of Fabozzi et al. (2006) introduced in Chapter 3 Section 3.3.1, I also utilize the momentum strategies to generate views as the inputted data of the BL model. The difference is that I substitute the constant standard deviation with the time-varying standard deviation to calculate the normalized return in the dynamic framework. In addition, Richard (1997) argues that the momentum effect is strongest at the six-month horizon with annual excess returns exceeding three per cent. Thus, in this thesis, I rank the securities over the past six months and the momentum portfolios are formed on t and hold for 6 months. The normalized six-month return $Z_{t,i}$ is given by:

$$Z_{t,i} = \frac{p_{t-1,i} - p_{t-6,i}}{p_{t-6,i}\sigma_{t,i}} \tag{4.10}$$

where:

$p_{t-1,i}$ is price of country index i at time $t-1$.

$p_{t-6,i}$ is price of country industrial index i six months before t.

$\sigma_{t,i}$ is volatility of country industrial index i at time t.

The top half and the bottom half of the country industrial indexes are allocated weights of $\omega_{i,t} = \frac{1}{\sigma_{t,i}c}$ and $\omega_{i,t} = -\frac{1}{\sigma_{t,i}c}$ respectively. Then, the method of obtaining the $N \times 1$ vector of view weights matrix $\mathbf{P}_t$ at time t, view expected return vector $\mathbf{q}_t$ at time t and the confidence level $\mathbf{\Omega}_t$ in the views at time t are the same as for the Fabozzi et al. (2006) method.

4.2.2.3 Combining Conditional Equilibrium Returns and Views Together

Since all the parameters have been obtained from previous steps, the next essential work in the BL model is to mix the conditional equilibrium return with the views using the Bayesian approach. In the dynamic case, substituting the parameters in formula (3.11) in Chapter 3 with the conditional estimations, the $N \times 1$ vector of conditional expected returns $\boldsymbol{\mu}_{BL,t}$ at time t is given by:

$$\boldsymbol{\mu}_{BL,t} = \boldsymbol{\pi}_t + \tau \mathbf{H}_t \mathbf{P}_t'(\mathbf{P}_t \mathbf{H}_t \mathbf{P}_t' \tau + \mathbf{\Omega}_t)^{-1}(\mathbf{q}_t - \mathbf{P}_t \boldsymbol{\pi}_t) \tag{4.11}$$

where τ is the same scale parameter explained in Chapter 3 section 3.2.1. Correspondingly, rewriting the formula (3.13) and fitting into the formula (3.14), the estimated $N \times N$ vector of covariance matrix $\mathbf{V}_t$ could be denoted by:

$$\mathbf{V}_t = \mathbf{H}_t + ((\tau \mathbf{H}_t)^{-1} \boldsymbol{\pi}_t + \mathbf{P}_t^{'} \boldsymbol{\Omega}_t^{-1} \mathbf{P}_t)^{-1} \quad (4.12)$$

In modelling the covariance matrix of asset returns, $\mathbf{V}_t$, the use of different dynamic models will generate different results of the vector of covariance matrix.

4.2.3 Unconstrained Dynamic BL Portfolio

From formula (4.11) and formula (4.12), the time-varying expected returns and covariance matrix can be estimated. During each single period t, in order to construct the unconstrained BL portfolio, there are two methods to allocate assets. On the one hand, I can simply use the formula (3.17) to find the implied weights of unconstrained BL portfolios. Then, the implied weights $\mathbf{w}_{BL,t}^{*}$ at time t could be given by,

$$\mathbf{w}_{BL,t}^{*} = \frac{1}{\delta_t} \mathbf{V}_t^{-1} \boldsymbol{\mu}_{BL,t} \quad (4.13)$$

On the other hand, the mean-variance asset allocation models can be utilized to form the unconstrained BL portfolios with the estimation of expected returns and covariance matrix as input in each period. The optimisation problem, which is to maximise the Sharpe ratio, is expressed as:

$$\max \frac{\mathbf{w}_{BL,t}^{'} \boldsymbol{\mu}_{BL,t}}{\sqrt{\mathbf{w}_{BL,t}^{'} \mathbf{V}_t \mathbf{w}_{BL,t}}} \quad (4.14)$$

subject to $-1 \leq \mathbf{w}_{BL,t} \leq 2, \mathbf{w}_{BL,t}^{'} \mathbf{1} = 1$

where $\boldsymbol{\mu}_{BL,t}$ is the expected return of Black-Litterman portfolio and $\sqrt{\mathbf{w}'_{BL,t}\mathbf{V}_t\mathbf{w}_{BL,t}}$ is the conditional portfolio standard deviation, $\mathbf{w}_{BL,t}$ is the $N \times 1$ vector of portfolio weights. **1** is the $N \times 1$ vector of ones. The vector of optimal portfolio positions could be solved as:

$$\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}} \tag{4.15}$$

In the previous two methods, standard deviation is used to measure risk of portfolio. I propose to construct portfolios with maximal reward-to-risk ratios where risks are measured by the VaR and CVaR of the portfolio. The portfolio mean-VaR optimisation problem with target ratio between expected excess return and VaR at time t can be written as:

$$\max \frac{\mathbf{w}'_{BL,t}\boldsymbol{\mu}_{BL,t}}{VaR_{\beta,t}} \tag{4.16}$$

$$\text{subject to } -1 \leq \mathbf{w}_{BL,t} \leq 2, \mathbf{w}'_{BL,t}\mathbf{1} = 1$$

where $VaR_{\beta,t}$ is the expected maximum loss on the Black-Litterman portfolio at time t with a certain probability of $1-\beta$. Following Rockafellar and Uryasev (2000), the VaR of BL portfolio at time t can be expressed as:

$$VaR_{\beta,t} = \xi_\beta \sqrt{\mathbf{w}'_{BL,t}\mathbf{V}_t\mathbf{w}_{BL,t}} - \mathbf{w}'_{BL,t}\boldsymbol{\mu}_{BL,t} \tag{4.17}$$

where $\xi_\beta = -F^{-1}(1-\beta)$ and $F(.)$ is the cumulative distribution. β is the confidence level equal to 99%, 95% and 90%.

The portfolio mean-CVaR optimisation problem with target ratio between expected excess return and CVaR at time t can be written as:

$$\max \frac{\mathbf{w}'_{BL,t}\boldsymbol{\mu}_{BL,t}}{CVaR_{\beta,t}} \tag{4.18}$$

$$\text{subject to } -1 \le \mathbf{w}_{BL,t} \le 2, \mathbf{w}_{BL,t}^{'}\mathbf{1} = 1$$

where $CVaR_{\beta,t}$ is the average loss exceeding the expected maximum loss $VaR_{\beta,t}$ at time t on the Black-Litterman portfolio with a certain probability $1-\beta$. With application to Rockafellar and Uryasev's method, CVaR of BL portfolio at time t could be expressed as:

$$CVaR_{\beta,t} = \zeta_{\beta,t}\sqrt{\mathbf{w}_{BL,t}^{'}\mathbf{V}_t\mathbf{w}_{BL,t}} - \mathbf{w}_{BL,t}^{'}\boldsymbol{\mu}_{BL,t} \quad (4.19)$$

where $\zeta_{\beta,t} = \frac{-\int_{-\infty}^{-F^{-1}(1-\beta)} gf(g)dg}{1-\beta}$, g is denoted by $-\mathbf{w}_{BL,t}^{'}\boldsymbol{\mu}_{BL,t} - VaR_{\beta,t}$.

4.2.4 VaR-Constrained Dynamic BL Portfolio

I consider that an investor wishes to maximise the reward-to-risk ratios in each period t subject to downside risk measures constraints. There are two main downside risk measures: VaR and CVaR. The portfolio optimisation problems with the target of obtaining a maximal Sharpe ratio under VaR constraints can therefore be written as follows:

$$\max \frac{\mathbf{w}_{BL,t}^{'}\boldsymbol{\mu}_{BL,t}}{\sqrt{\mathbf{w}_{BL,t}^{'}\mathbf{V}_t\mathbf{w}_{BL,t}}} \quad (4.20)$$

$$\text{subject to } VaR_{\beta,t} \le VaR_0, -1 \le \mathbf{w}_{BL,t} \le 2, \mathbf{w}_{BL,t}^{'}\mathbf{1} = 1$$

where VaR_0 is the target VaR. It is an art to set VaR_0 in the dynamic environment. In single-period analysis, in order to analyse the effect of imposing VaR_0 on the unconstrained BL portfolio, I set the value of VaR_0 equal to decreasing scaling factor k multiplied by VaR of the unconstrained implied BL portfolio at each time t, k could be equal to 0.99, 0.95, 0.90 and reduces sequentially. VaR_0 is not constant during the whole period. The hypothesis is that imposing VaR constraints could improve the unconstrained SR-BL portfolio's performance.

4.2.5 CVaR-Constrained Dynamic BL Portfolio

It is well-known that the VaR fails to satisfy the sub-additivity property of coherent risk measures, which means the portfolio VaR might be bigger than the weighted sum of individual assets VaR. The use of VaR as constraints might be invalid to control the risk; it may happen that although the portfolio VaR seems to meet the target VaR, the target risk level actually is not strict enough to constrain risk at a lower level. In addition, VaR cannot measure the possible loss that is beyond VaR. In risk management, it is better to use CVaR to measure risk and set suitable constraints. Therefore, I propose to add CVaR constraints in the portfolio optimisation process. In the following optimisation problems:

$$\max \frac{\mathbf{w}_{BL,t}^{'}\boldsymbol{\mu}_{BL,t}}{\sqrt{\mathbf{w}_{BL,t}^{'}\mathbf{V}_t\mathbf{w}_{BL,t}}} \tag{4.21}$$

$$\text{subject to } CVaR_{\beta,t} \leq CVaR_0, -1 \leq \mathbf{w}_{BL,t} \leq 2, \mathbf{w}_{BL,t}^{'}\mathbf{1} = 1$$

Similar to set VaR_0, I set the value of $CVaR_0$ equal to decreasing scaling factor k multiplied by the CVaR of the unconstrained implied BL portfolio at each time t, k could be equal to 0.99, 0.95, 0.90 and reduces sequentially. $CVaR_0$ is not constant during the whole period. The hypothesis is that imposing CVaR constraints could improve the unconstrained portfolio performance.

4.2.6 BL Portfolio's Performance Analysis

In the in-sample analysis, I use 197 return observations from January 1994 to May 2010 to estimate 197 time-varying variance covariance matrices of 30 assets by using the Rolling Window method, the EWMA model and the DCC method. I use a natural simulation method to fill in the missing value of rolling window volatilities[6] in the window length. The starting parameter values for the EWMA model and the DCC model are set to be the static covariance matrix estimated in the whole sample, and then the estimation rolls forward by one

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[6]For a window length of M, σ_1^2 is the sample historical variance. $\sigma_t^2 = \frac{1}{M}\left[(M-1)\sigma_{t-1}^2 + \sum_{i=1}^{t-1} r_{t-i}^2\right]$, where $t = 2,\ldots,M$.

month and generates covariance estimates for month 2 (February 1994), and so on until the end of sample (May 2010). For each iteration, the starting parameter values for each model are set to the values estimated in the previous iteration. This procedure results in a total of 197 in-sample estimates. The next step is to construct the momentum portfolio as explained in Section 4.2.2.2: I use price data from December 1993 to May 2010 to get six-month normalized returns, with the initial period of the first six-month normalized return being in May 1994, then I hold the momentum portfolio for six months to calculate the confidence level in the views using a back-testing procedure; the initial value of confidence level in the views is calculated in November 1994. Therefore, the whole period of the dynamic BL portfolio is from November 1994 to May 2010 in in-sample analysis, with 187 estimates. November 1994 is supposed to be the first period to report single period empirical results. However, since I choose the rolling window length of 50 to calculate the covariance matrix, in order to avoid the bias generated from the simulation method, the 51st period, which is in August 1998, should be a better period to make a reasonable comparison between the Rolling Window method, the EWMA and the DCC volatility models. Therefore, I report detailed single period empirical results in August 1998. In order to provide a thorough analysis of the effect of positive and negative view portfolio expected return on the dynamic BL model solution, I also report the other single period results in November 1998. In portfolio performance evaluation, unlike the dynamic conditional results in a different single period, the multiple periods' results emphasise average portfolio performance during the whole period from November 1994 to May 2010. Correspondingly, in order to avoid any bias generated from the simulation method in the window length, I also report multiple periods' performance results during the sub-period from August 1998 to May 2010. Chapter 5 will illustrate the detailed empirical results.

In the out-of-sample analysis, Giacometti et al. (2007) use a window length of 110 in the Rolling Window method; thus, I initially estimate each of the three volatility models using the first 110 observations (from January 1994 to February 2003) to generate a one month ahead out-of-sample forecast of the conditional covariance matrix for month 111 (March 2003). The estimation sample is then rolled forward by one month, the models re-estimate and use this to generate out-of-sample forecasts for month 112 (April 2003), and so on until the end of the sample (May 2010). For each iteration, the starting parameter values for each model are set to the values estimated in the previous iteration. This procedure results in a total of 88 out-of-sample estimates. Then, I construct the momentum portfolio with a holding period of six months to input as the view portfolio into the BL portfolio. Thus, the first period of the construction of the BL portfolio is on August 2003 and the total of out-of-sample estimates is reduced to 82. In order to show the results of portfolio turnover, I report conditional single period results in September 2003 and the multiple periods' average performance results during the period from September 2003 to May 2010. I also compare the proposed dynamic BL portfolio performance with the risk-adjusted BL portfolio proposed by Giacometti et al. (2007). Chapter 6 will show and analyse detailed empirical results.

4.2.6.1 Single Period Optimisation Statistics

In order to make clear comparisons between different optimisation problems from the empirical study, I report the BL portfolio statistics including expected excess return, standard deviation and expected VaR and CVaR in the optimisation problems; I also report the value of maximal expected conditional Sharpe ratio, maximal expected excess return to VaR ratio, and maximal expected excess return to CVaR ratio, which are natural solutions of optimisation problems. Moreover, I would draw the BL portfolio's efficient frontier to illustrate the portfolio selection process.

4.2.6.2 Performance Evaluation

Single period

Following Giamouridis and Vrontos (2007) and Harris and Mazibas (2010), I calculate realized returns, conditional Sharpe ratio and portfolio turnover to assess the BL portfolio performance in the single period.

The realized return r_p^t of the portfolio at time t is calculated as:

$$r_{p,t} = \mathbf{w}_{BL,t}^{'}\mathbf{r}_t \tag{4.22}$$

The conditional Sharpe ratio is computed as:

$$CSR_{p,t} = \frac{r_{p,t}}{\sqrt{\mathbf{w}'_{BL,t}\mathbf{V}_t\mathbf{w}_{BL,t}}} \tag{4.23}$$

The portfolio turnover is defined as:

$$PT_t = \sum_{i=1}^{N} \left| w_{BL_{i,t}} - w_{BL_{i,t-1}} \right| \tag{4.24}$$

This formula means that the sum of the absolute changes in the BL portfolio weights from $t-1$ to t. It implies the measure of the fraction of the portfolio that needs to readjust the weights at the rebalancing period.

In addition, considering the drawbacks of standard deviation and VaR explained in Chapter 2 Section 2.2.1, CVaR has tractable properties: it is a coherent risk measure, it is easy to implement and it takes into consideration the entire tail that exceeds VaR on average. I decide to use the ratio of reward to CVaR ($\frac{r_p^t}{CVaR_{\beta,t}}$) to evaluate portfolio performance in the single period. Note that if the realized returns of the portfolio are negative, the Conditional Sharpe ratio and reward to CVaR ratio will be negative as well. However, using these negative ratios to compare the performance of the portfolio might be incorrect. For example, given the same negative excess return, a larger standard deviation or larger CVaR would lead to a larger Sharpe ratio (less negative Sharpe ratio) or a larger reward to CVaR ratio (less negative reward to CVaR ratio), hence signifying a relatively good performance. Actually, with the same returns, since the portfolio has taken a higher risk, it would be an underperformer. It is necessary to adjust the Sharpe ratio and reward to CVaR ratio when the excess return is negative. Israelsen (2003) proposes a simple method to adjust the Sharpe ratio for performance measurement. He calculates the adjusted Sharpe ratio as the product of negative excess return and percentage of risk. I follow Israelsen (2003)'s method to modify the conditional Sharpe ratio and reward to CVaR ratio in order to compare the performance of the unconstrained BL portfolio. In order to bring the values to a more comparable scale, the adjusted ratios can include a constant multiplier of 100.

The adjusted conditional Sharpe ratio is computed as $\frac{100 * r_{p,t}}{(\sqrt{\mathbf{w}_{BL,t}^{'}\mathbf{V}_t\mathbf{w}_{BL,t}})^{\frac{r_{p,t}}{|r_{p,t}|}}}$,

similarly, the adjusted reward to CVaR ratio is computed as $\frac{100 * r_{p,t}}{(CVaR_{\beta,t})^{\frac{r_{p,t}}{|r_{p,t}|}}}$.

Multiple Periods

Given the optimized weights obtained from the solutions of the previous optimisation problems from the first period to the end, I calculate buy-and-hold returns on the portfolio for a holding period of one month, and repeat the calculation to the end; therefore, I can obtain the time-varying realized returns of the portfolio. I also report the average return of time-varying realized returns, standard deviation, skewness and excess kurtosis. In order to evaluate the performance of the portfolio in multiple periods, I use the Sharpe ratio and the information ratio. In addition, I also employ the ratio between rewards to downside risk measures in order to assess performance. For example, I calculate the return per unit of tail risk, while tail risk is measured by VaR and CVaR based on empirical distribution. With the use of these evaluation criteria, I make comparisons between the benchmark, unconstrained BL portfolios and the constrained BL portfolios. Moreover, I investigate the effect of the choice of different distributions and different confidence levels on the dynamic BL portfolio's performance.

4.3 Conclusions

Having described the dataset and studied the non-normal property and time series property of excess return data, this chapter moves on to focus on the methodology for constructing a dynamic BL portfolio and evaluating the portfolio's performance through in-sample analysis and out-of-sample analysis in single period and multiple periods, based on three volatility models. The proposed dynamic BL portfolios include unconstrained dynamic BL portfolios and constrained dynamic BL portfolios. In the construction of the unconstrained BL portfolios, the weight solutions are generated from: the reverse optimisation implied in the BL model; the optimisation function with target of maximal Sharpe ratio; the optimisation function with target of maximal reward to VaR ratio; and the optimisation function with target of maximal reward to CVaR. In the construction of the constrained BL portfolios, the weight solutions are produced from the optimisation function with the target of maximal Sharpe ratio with VaR constraints and CVaR constraints. Furthermore, this chapter also illustrates the methods of evaluating portfolio performance in both a single period and multiple periods. The in-sample analysis examines the dynamic BL model in samples from November 1994 to May 2010, choosing August 1998 and November 1998 to make in-depth single-period study, while the out-of-sample analysis define the sample in the period from September 2003 to May 2010, simply choosing September 2003 to investigate the dynamic BL model. The following chapters, Chapter 5 and Chapter 6, will explore the empirical work in detail, through in-sample analysis and out-of-sample analysis respectively.

Table 4.1 Summary Statistics for the FTSE Sector Indices Excess Returns

Table 4.1 reports summary statistics for the monthly excess return series on 10 FTSE Sector Indices in UK, US and Japan countries for the period January 1994 to May 2010. The table also reports the statistics of the Jarque-Bera tests. All the statistics confirm the rejection of normality hypothesis at 5% significance level.

	Mean	Median	Standard Deviation	Skewness	Excess Kurtosis	Min	Max	Jarque-Bera	P-value
UK BASIC MATS	0.0009	0.0034	0.0802	-1.0695	5.7373	-0.4448	0.2267	99.0608	0.0000
UK CONSUMER GDS	0.0000	0.0039	0.0728	-0.2152	1.4760	-0.2736	0.2569	20.5856	0.0000
UK CONSUMER SVS	-0.0024	0.0027	0.0523	-0.7789	1.7558	-0.2218	0.1173	32.6288	0.0000
UK FINANCIALS	-0.0020	0.0024	0.0699	-0.7914	5.6979	-0.3888	0.2888	80.3093	0.0000
UK HEALTH CARE	0.0008	0.0050	0.0453	-0.2424	0.5934	-0.1553	0.1499	49.4703	0.0000
UK TECHNOLOGY	-0.0123	-0.0017	0.1210	-0.3899	0.4525	-0.3671	0.2863	58.2601	0.0000
UK INDUSTRIALS	-0.0031	0.0043	0.0705	-1.1488	3.3503	-0.2971	0.1437	44.3357	0.0000
UK OIL & GAS	0.0031	0.0050	0.0607	-0.1751	0.6955	-0.1877	0.1639	44.5982	0.0000
UK TELECOM	-0.0020	0.0027	0.0666	-0.5598	0.7034	-0.2314	0.1561	53.5819	0.0000
UK UTILITIES	0.0010	0.0001	0.0477	-0.1261	0.9412	-0.1840	0.1459	35.3153	0.0000
USA BASIC MATS	0.0018	0.0043	0.0645	-0.5359	2.3774	-0.2853	0.2095	12.6095	0.0018
USA CONSUMER GDS	-0.0015	0.0032	0.0547	-0.7458	1.7533	-0.2290	0.1172	31.0213	0.0000
USA CONSUMER SVS	0.0012	0.0039	0.0504	-0.5470	1.0648	-0.1843	0.1295	40.5632	0.0000
USA FINANCIALS	0.0010	0.0066	0.0646	-1.0445	3.6762	-0.2667	0.1788	39.5728	0.0000
USA HEALTH CARE	0.0028	0.0094	0.0414	-0.7183	0.9942	-0.1351	0.0977	49.9644	0.0000
USA INDUSTRIALS	0.0026	0.0088	0.0546	-0.7378	2.2875	-0.2245	0.1604	22.0377	0.0000
USA OIL & GAS	0.0040	0.0042	0.0544	-0.3279	1.2213	-0.2033	0.1629	29.4997	0.0000
USA TECHNOLOGY	0.0045	0.0140	0.0837	-0.6133	1.0130	-0.3227	0.2017	44.7577	0.0000
USA TELECOM	-0.0026	0.0074	0.0571	-0.2520	1.1609	-0.1530	0.2300	29.8485	0.0000
USA UTILITIES	-0.0018	0.0036	0.0477	-0.5481	0.6412	-0.1457	0.1218	55.5314	0.0000
JAPAN BASIC MATS	-0.0037	-0.0038	0.0704	0.0130	0.7966	-0.2413	0.2045	39.8553	0.0000
JAPAN CONSUMER GDS	-0.0005	0.0012	0.0572	-0.0230	0.8656	-0.1744	0.2067	37.4125	0.0000
JAPAN CONSUMER SVS	-0.0045	-0.0078	0.0515	0.2200	0.1892	-0.1555	0.1447	66.4389	0.0000
JAPAN FINANCIALS	-0.0094	-0.0143	0.0865	0.1602	0.5040	-0.2542	0.2754	51.9803	0.0000
JAPAN HEALTH CARE	-0.0012	-0.0001	0.0505	0.1763	1.3412	-0.1646	0.1974	23.6069	0.0000
JAPAN INDUSTRIALS	-0.0013	0.0049	0.0627	-0.3064	0.2980	-0.2138	0.1642	63.0091	0.0000
JAPAN OIL & GAS	-0.0049	-0.0051	0.0867	-0.2627	0.9109	-0.3160	0.2397	38.0910	0.0000
JAPAN TECHNOLOGY	-0.0028	-0.0029	0.0864	0.0165	0.1153	-0.2190	0.2567	68.3161	0.0000
JAPAN TELECOM	-0.0030	-0.0030	0.0769	0.3943	2.1017	-0.2618	0.3188	11.7277	0.0028
JAPAN UTILITIES	-0.0034	-0.0063	0.0514	0.2053	0.7984	-0.1247	0.2078	41.1697	0.0000

Table 4.2 Time Series Property

Table 4.2 reports the test statistics for autocorrelation, autoregressive conditional heteroskedasticity (ARCH) and dynamic conditional correlation for the full sample from January 1994 to May 2010. The Ljung-Box-Q test statistic for the autocorrelation of up to order 10 is asymptotically distributed as a central Chisquare with ten d.o.f. The ARCH (1) statistic is asymptotically distributed as a central Chi-square with one d.o.f. and the ARCH (4) statistic is asymptotically distributed as a central Chi-square with four d.o.f. The DCC statistic is distributed as a central Chi-square with one d.o.f. *, ** and *** denote significance at 10%, 5% and 1% levels respectively. In DCC test, 30 assets means the sample includes all assets, 18 assets means the sample includes include assets selected with significant autocorrelation in the squared residuals with 1 lag.

	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)	LB-Q(10)	ARCH(1)	ARCH(4)	DCC test	statistic	p-Value
UK BASIC MATS	0.30***	0.19***	0.08***	0.07***	-0.14***	45.27***	39.95***	41.33***	30 Assets	1.4075	0.4947
UK CONSUMER GDS	0.01	0.00	0.10	-0.11	-0.06	14.97	0.16	7.76*	18 Assets	9.0402	0.0109
UK CONSUMER SVS	0.17**	-0.01*	0.02	0.17**	-0.04**	15.11	1.51	4.88			
UK FINANCIALS	0.26***	0.07***	0.09***	0.16***	-0.02***	28.71***	7.85***	26.27***			
UK HEALTH CARE	0.02	0.03	0.00	-0.08	0.02	2.36	1.34	17.29***			
UK TECHNOLOGY	0.12*	0.01	0.08	0.16*	-0.03*	11.64	8.66***	14.71***			
UK INDUSTRIALS	0.12*	-0.01	-0.01	0.06	0.06	10.25	0.01	0.88			
UK OIL & GAS	-0.07	-0.02	-0.01	-0.02	0.01	5.96	3.84*	6.39			
UK TELECOM	0.14**	-0.02	0.2***	0.03**	0.1**	20.09**	4.37**	20.49***			
UK UTILITIES	0.09	0.08	0.05	0.07	-0.03	11.14	3.65*	4.36			
USA BASIC MATS	0.10	0.05	0.02	0.06	-0.07	17.1*	30.15***	30.93***			
USA CONSUMER GDS	0.05	-0.17**	0.01	0.03	-0.01	12.06	0.42	8.74*			
USA CONSUMER SVS	0.11	-0.16**	0.08**	0.04*	-0.05*	9.89	5.57**	11.56**			
USA FINANCIALS	0.13*	-0.04	0.08	0.11	0.08	17.82*	9.88***	32.56***			
USA HEALTH CARE	0.04	-0.05	-0.01	-0.03	0.12	13.94	1.47	12.16**			
USA INDUSTRIALS	0.11	-0.1	0.05	0.2***	0.01**	23.06***	7.78***	23.07***			
USA OIL & GAS	-0.06	0.04	-0.07	0.08	-0.01	9.59	1.44	6.35			
USA TECHNOLOGY	-0.01	0.02	0.13	-0.03	0.03	7.37	22.23***	31.23***			
USA TELECOM	0.02	-0.06	0.12	0.05	0.09	15.75	11.22***	24.83***			
USA UTILITIES	0.11	-0.02	0.09	0.08	0.05	15.38	1.09	15.43***			
JAPAN BASIC MATS	0.10	-0.01	0.13	0.01	0.08	11.12	20.78***	23.33***			
JAPAN CONSUMER GDS	0.07	-0.03	0.17*	0.03	0.01	13.87	2.12	4.19			
JAPAN CONSUMER SVS	0.11	-0.10	0.08	-0.07	0.03	12.27	0.34	2.98			
JAPAN FINANCIALS	0.10	-0.05	0.09	0.00	0.03	8.56	0.07	6.14			
JAPAN HEALTH CARE	0.05	-0.07	-0.03	-0.03	0.02	5.75	0.05	4.82			
JAPAN INDUSTRIALS	0.17**	0.02*	0.14**	0.02**	0.03*	12.12	7.12***	10.91**			
JAPAN OIL & GAS	-0.03	-0.12	0.14*	-0.06*	-0.04	14.55	8.25***	8.79*			
JAPAN TECHNOLOGY	0.13*	0.13**	0.19***	0.06***	0.06***	19.53**	14.62***	19.97***			
JAPAN TELECOM	0.19***	0.03**	0.03*	0.01	0.16*	16.31*	4.11**	12.52**			

CHAPTER 5
IN-SAMPLE DYNAMIC BLACK-LITTERMAN PORTFOLIOS

With the aim of extending the original Black-Litterman (BL) model to comply with the asset return styled facts of volatility clusters, non-normality and asymmetric features, I propose to apply the volatility models to the traditional BL model. In addition, from the perspective of risk management, I also take tail risks into account in the asset allocation process; in other words, I use Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) to measure the tail risk and construct the unconstrained BL portfolio in the pursuit of achieving maximum reward to VaR ratio and reward to CVaR ratio. Furthermore, I investigate the empirical study of the dynamic BL model with risk constraints by constructing a dynamic constrained BL portfolio, firstly with VaR constraints, and then with CVaR constraints. Moreover, the effects of risk constraints, different confidence levels and different assumed distributions on the portfolio performance are analysed.

This chapter concentrates on demonstrating the empirical study of dynamic BL portfolios, following the methodology illustrated in Chapter 4. There are four sections in this chapter.

Section 5.1 outlines the details of a dynamic unconstrained BL portfolio. In this section, Section 5.1.1 describes the benchmark portfolio and analyses the performance of the benchmark portfolio. Section 5.1.2 illustrates how to use three volatility models to estimate the conditional covariance matrix. Section 5.1.3 discusses how to set the dynamic risk aversion coefficient. Section 5.1.4 estimates the implied equilibrium return based on market portfolio. Section 5.1.5 combines views from the momentum strategy with the implied equilibrium return to generate the expected rate of return. Section 5.1.6 forms the dynamic unconstrained BL portfolio with inputs of BL expected return and the BL conditional covariance matrix into portfolio optimisers. Section 5.1.7 makes comparisons between three portfolio optimisation models through efficient frontiers and optimisation statistics. Section 5.1.8 focuses on weight solutions among different portfolio optimisers and analyses the effect of different confidence levels and different assumed distributions on weights solutions.

Section 5.1.9 evaluates all unconstrained BL portfolios in a single period and over multiple periods. Section 5.1.10 makes conlusions.

Section 5.2 focuses on a VaR-constrained dynamic BL portfolio, based on three volatility models in assumed the normal distribution and the t-distribution, at different confidence levels. Section 5.2.1 describes the process of building a VaR-constrained BL portfolio, and creates an explanation through the efficient frontier. Section 5.2.2 evaluates the in-sample performance in a single period and over multiple periods, based on three volatility models. Section 5.2.3 analyses the effect of VaR constraints, distributions and confidence levels on optimisation process, weights, and performances.

Similarly, Section 5.3 develops this to construct a CVaR-constrained dynamic BL portfolio in assumed the normal distribution and the t-distribution, with different confidence levels. Section 5.3.1 illustrates the process of building a CVaR-constrained BL portfolio and makes a comparison with the VaR-constrained BL portfolio through efficient frontier figures. Section 5.3.2 evaluates the in-sample performance in a single period and over multiple periods, based on three volatility models, and contrasts this with the VaR-constrained BL portfolio. Section 5.3.3 also studies the effect of CVaR constraints, distributions and confidence levels on the optimisation process, weights, and performances, and compares with the VaR-constrained BL portfolio. Conclusions are made in each section.

5.1 Construction of the Unconstrained Black-Litterman Portfolio

5.1.1 Benchmark Portfolio

The market portfolio is formed by FTSE 10 sector indices in the US, UK and Japan. The weight of each asset is the market capitalisation of each asset divided by the total market value. The performance of the market portfolio plays an important role in providing a benchmark for comparison with other portfolios.

Table 5.1.1 shows the benchmark portfolio performance and its tail risks. From Panel A, it can be seen that the benchmark portfolio had an average return of 0.08% per month with the standard deviation of 4.3% to get the Sharpe Ratio (SR) of 1.82% from January 1994 to May 2010. With the negative skewness of 0.9079 and a somewhat higher kurtosis of 4.9253, the tail risks of the benchmark portfolio should not be neglected. Panel B shows the results of using the parametric method to estimate the VaR and CVaR of the benchmark portfolio at different confidence levels (90%, 95%, 97.5% and 99%): it can be found that the estimated VaR ranged from 5.43% to 9.93% with the normal distribution assumption, and ranged from 6.52% to 16.04% with the t-distribution assumption. The estimated values of CVaR were a little higher than VaR, within the range 7.47% and 11.39% with the normal distribution assumption and, correspondingly, within the range 10.67% and 22.38% with the t-distribution assumption. The estimated values of VaR (11.87%) and CVaR (17.1%) at a confidence level of 97.5% with the t-distribution assumption were close to the values of empirical VaR (12.85%) and CVaR (16.73%).

Chapter 4 describes the features of the benchmark portfolio, such as non-normality, volatility clustering and dynamic constant correlation (DCC). Therefore, the use of volatility models is necessary. To make it simple to estimate the conditional covariance matrix and to capture some styled facts of the benchmark portfolio, I choose the rolling window (RW) model, the Exponentially Weighted Moving Average (EWMA) model and the Dynamic Constant Correlation (DCC) model to estimate the in-sample conditional covariance matrix. In Section 5.1.2, I would show how to use these three volatility models.

5.1.2 Time-Varying Variance and Covariance Matrix

Firstly, I use the RW model, as explained in Chapter 4, Section 4.2.1.1, to forecast 197 time-varying variance covariance matrices. The choice of the window length is an art, balancing the trade-off between the distant data and recent data. In order to show the effect of the different window lengths on the benchmark volatility, in this section, I choose a window length of 50 and a window length of 100 to make a comparison. The first sample has 50 observations (from January 1994 to February 1998); I calculate the historical sample covariance as the forecasted covariance matrix. On a rolling basis, I would get 147 historical sample covariance matrices (from March 1998 to May 2010). Alternatively, I would only get 97 historical sample covariance matrices (from May 2002 to May 2010) with the 100 window length (from January 1994 to April 2002). In order to make a comparison in the same time horizon, this means 197 time-varying conditional covariances; I use a natural simulation method to fill in the missing values of RW volatilities.[7]

Secondly, I utilise the simple EWMA model, as shown in Chapter 4, Section 4.2.1.2, to make an estimation of 197 conditional covariance matrices, according to the equation (4.3) by setting an initial covariance matrix equal to the whole sample average covariance matrix.

Thirdly, when I use the DCC model[8] (as described in Chapter 4, Section 4.2.1.3) to estimate the conditional covariance matrices, I finish the first step in forecasting the univariate GARCH model based on 197 observations in the whole sample, and then standardise the returns by the estimated GARCH conditional standard deviations.

Although I have estimated conditional covariance matrices for 30 assets during each period, the large dataset of the 30×30 vector of conditional covariance

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[7]For a window length of M, σ_1^2 is the sample historical variance. $\sigma_t^2 = \frac{1}{M}\left[(M-1)\sigma_{t-1}^2 + \sum_{i=1}^{t-1} r_{t-i}^2\right]$, where $t = 2,\ldots,M$.

[8] I go to Prof. Kevin Sheppard's matlab codes of the DCC model in the UCSD GARCH Toolbox to make parameter estimations, which are provided on the website http://www.spatial-econometrics.com/, and have been used by modifying the codes, according to the needs of the analysis.

matrices in 197 periods is not convenient to report upon within this thesis. To study the effect of different volatility models on the portfolio time-varying standard deviation, I firstly calculate the volatility of the market portfolio by $\sigma_{M,t} = \sqrt{\mathbf{w}_{M,t}^{'}\mathbf{H}_t\mathbf{w}_{M,t}}$, where $\mathbf{w}_{M,t}$ is the market capitalisation of each index at time t , and $\mathbf{H}_t$ is the conditional covariance matrix at time t , as can be seen in Figure 5.1.1. The simulation method of filling the missing values in the RW model leads to a less volatile trend in the previous 50 months and 100 months. The trends of monthly volatilities of the benchmark portfolio in the use of different volatility models (DCC, EWMA and RW50) were generally similar, with peaks and troughs falling in the same sub-periods in the whole period from January 1994 to May 2010. The monthly volatilities of the benchmark portfolio stayed in the relatively lower level range of between 2.7% and 4.3% before August 1998. This was followed by a jump to slightly higher volatilities, caused by the adverse effect of the Asian financial crisis; frequent rises and drops of volatilities changed to around 5% (DCC model) and 4.5% (EWMA model and RW50 model) from September 1998 to May 2003. Then, the tendency of the volatilities declined to the lower point of around 2.8% (DCC model) in April 2007 and around 2.2% (EWMA model and RW 50 model) in June 2007. Because of the outbreak of the global financial crisis, it can be observed that the volatilities had sharp increases to a peak of 7.5% in November 2008 (DCC model), 6.9% in May 2009 (EWMA model), and 5.3% in April 2010 (RW50 model). The monthly volatility of the benchmark portfolio calculated by the RW model with a window length of 100 showed a relatively flat trend, close to the level of 4.4% from May 2002 to March 2009, and with a small increase to 4.8% in December 2009. Obviously, compared with other methods, the choice of the window length of 100 in the RW method significantly decreased the volatility of the benchmark portfolio. With this failure to reflect sensitive volatilities in the market, it might not be quite suitable to choose a window length of 100 in dynamic asset allocation. Therefore, I do not use a window length of 100 in the RW method to elaborate the results of the dynamic BL portfolios in the in-sample analysis.

5.1.3 The Risk Aversion Coefficient

As discussed in Chapter 3, Section 3.2.1, the risk aversion coefficient could be calculated by the formula (3.6). Following the method of Idzorek (2004) and

Babameto and Harris (2009), I set the value of the global market risk premium first and then divided it by the market portfolio standard deviation. Dimson et al. (2007) forecast a geometric world risk premium of 3-3.5%. The investment banks normally use a risk premium of 4%-5%. As Babameto and Harris (2009) make a reasonable assumption in setting the value of world risk premium, I choose the same reasonable value of 3.5%; on a monthly basis, the value would be set to be 0.29%. Since the market portfolio standard deviation is time-varying, as shown in Figure 5.1.1, thus, the resulting risk aversion coefficient is time-varying. To avoid the influence of the simulated data of conditional variance in the RW method and to make a comparable analysis, I calculate the risk aversion coefficient from the 51st period (March 1998). Figure 5.1.2 shows a time-varying risk aversion coefficient with the RW, EWMA, and DCC models from March 1998 to May 2010. With the fixed value of the monthly world risk premium, it could be easy to conclude that the trend of the monthly risk aversion coefficients had an inverse relationship with the trend of the monthly volatilities of the benchmark portfolio, as displayed in Figure 5.1.2. The monthly risk aversion coefficients plunged 50% to the lower range of between 0.9 and 2.3 in September 1998, and stayed in this range until the beginning of 2004. The monthly risk aversion coefficients began to climb to their highest points at 3.65 (DCC), 5.59 (EWMA) and 5.91 (RW50) before the credit crisis in 2007, and then descended to the lower level around 1 (RW50) or even below 1 (DCC and EWMA) in 2009. The average monthly risk aversion coefficients were around 2, specifically 1.83 (DCC), 2.10 (EWMA) and 2.11 (RW 50).

5.1.4 The Implied Equilibrium Return

Section 5.1.2 and Section 5.1.3 have provided all the parameters I need to estimate the implied equilibrium return. According to the formula (4.9) in Chapter 4, Section 4.2.2.1, I can compute the implied equilibrium return for each index. The momentum view expected return (discussed in Section 5.1.5) would be initially estimated in August 1998; in order to make the analysis consistent, I would report the implied equilibrium return in August 1998, as shown in Table 5.1.2. The estimated risk aversion coefficients based on three volatility models were 2.2166 (DCC), 1.3004 (EWMA) and 3.5373 (RW50) in August 1998. The implied equilibrium returns of UK industry indices when the DCC model was used showed values more than 0.1% higher than those when the EWMA and RW50 models were used. There were no big differences in the implied equilibrium returns of most of the indices used between the EWMA model and the RW50 model. Black and Litterman (1992) define the implied equilibrium returns as the set of expected returns that would clear the market if all investors have identical views. They suggest investors to use these neutral means as the starting point to input investor views and set optimisation objectives and constraints. The next task is to combine investor views with the market portfolio.

5.1.5 Inputting Views with the Momentum Strategy

In Chapter 4, Section 4.2.2.2, I described how to use a momentum strategy to construct the view portfolio. Fabozzi et al. (2006) set the parameter c as a constant to constrain the annual long-short portfolio volatility to a certain level at 20%. I calculate c equal to 35 to satisfy this requirement. Table 5.1.3 displays the results of the view portfolio weights ($\mathbf{P}$), the expected return of the view portfolio ($\mathbf{q}$), and the confidence variance ($\mathbf{\Omega}$) in August 1998.

As shown in Table 5.1.3, the long-short momentum portfolio based on different volatility models had 15 of the same assets with negative weights (Japanese industrial indices except Japan Utilities, two USA industrial indices including USA Oil & Gas and USA Basic Materials, and four UK industrial indices including UK Basic Materials, UK Consumer Goods, UK Financials and UK Oil & Gas). With another 15 of the same assets in positive positions, the momentum portfolio allocated the smallest positive weight of 6.81% in the DCC model, 7.05% in the EWMA model and 6.88% in the RW model to UK Technology. Simultaneously, it allocated the smallest negative weight of 7.92% in the DCC model, 7.49% in the EWMA model and 9.83% in the RW model to Japan Oil & Gas. For the DCC method, the remaining positive assets had positions ranging between 10.16% (UK Telecom) and 21.24% (USA Health Care), and remaining short positions ranging between 9.86% (Japan Financials) and 15.80% (USA Oil & Gas). For the EWMA method, the remaining positive assets had positions ranging between 10.02% (USA Technology) and 21.90% (USA Utilities), and remaining short positions ranging between 7.85% (Japan Financials) and 17.33% (Japan Health Care). For the RW method, the remaining positive assets had positions ranging between 13.28% (USA

Technology) and 24.18% (USA Consumer Services), and remaining short positions ranging between 10.33% (Japan Financials) and 21.96% (USA Oil & Gas). Comparing positive ranges among the three volatility models, the DCC method and the EWMA method had a similar range, and the RW method increased to a higher level of range, about 3%. Comparing negative ranges among the three volatility models, the EWMA method enhanced the width of position range by about 2% on both sides, more than that of the DCC method, and the RW method increased the level of the largest negative position by 6% in 30 assets more than that of the DCC method. It can be concluded that the use of different volatility models would generate different degrees of effect on the asset positions of the momentum portfolio. The use of the DCC method allocated weights more conservatively than the RW method, and the EWMA method stood in the middle. I can also reach the same conclusion in another period (November 1998) by ranking the portfolio weights from Panel A in Table 5.1.4 in order from the smallest to the largest.

The expected view return was the expected return of the long-short momentum portfolio. As shown in Panel B in Table 5.1.3, the expected return was -6.2% (DCC), -8.62% (EWMA) and -9.90% (RW50) in August 1998. These negative expected returns reflected the negative effect of the Asian financial crisis in 1998. The expected return changed every month. In November 1998, the expected return was 1.91% (DCC), 2.95% (EWMA) and 3.93% (RW 50), as shown in Table 5.1.4, Panel B.

Followed the backtesting method of Fabozzi et al. (2006), as introduced in Chapter 3 Section 3.3.1, I calculated the level of confidence $\boldsymbol{\Omega}$ in the view equal to 0.47% (DCC), 0.50% (EWMA), and 1.10% (RW50). Since the momentum portfolio is the only view inputted into the BL model, the matrix of the level of confidence $\boldsymbol{\Omega}$ has only one element, which is equal to the variance of the series of residual returns. The lower level of confidence means the greater level of certainty of the view, and the expected returns of the BL portfolio would be close to the expected returns of the view portfolio.

It should be remembered that one of the aims in constructing the BL portfolio is to build an active, outperforming portfolio with reasonable weights. A better realised performance of the momentum portfolio than the benchmark portfolio becomes a point of concern. Figure 5.1.3 shows the accumulative returns of the benchmark portfolio and the momentum portfolio from August 1998 to May 2010. The momentum portfolio can beat the benchmark portfolio during two periods from the beginning of 2001 to the end of 2003, and the period of global financial crisis from 2008 to 2010. The momentum portfolio, based on the DCC model, showed a better performance than the EWMA model and the RW50 model before June 2003, and then the performance of the momentum portfolio based on the EWMA model and the RW50 model gradually overtook that of the DCC model. The accumulative returns of the momentum portfolio in the EWMA model and the RW50 model had similar values over the whole period, except that the accumulative returns with the EWMA model had an average of 9.26% higher values than that of the RW50 model in the period after April 2009.

Table 5.1.5 reports the mean, standard deviation and SR to create a performance comparison between the momentum portfolio and the benchmark portfolio from November 1994 to May 2010, and in the sub-period from August 1998 to May 2010. The momentum portfolio always outperformed the benchmark portfolio with a much higher SR. The momentum portfolio based on the EWMA model was superior to the momentum portfolio based on the DCC model and the RW50 model without 50 simulated data in the sub-period.

5.1.6 Black-Litterman Expected Return and Covariance Matrix

According to Chapter 4, Section 4.2.2.2, I could use the momentum portfolio and translate it as the only view to input into the BL model to calculate the BL expected return. Employing the formulae (4.11) and (4.12), I can calculate the expected returns for each index and relevant covariance matrices every month. Table 5.1.6 reports the BL expected returns for each index in August 1998. Table 5.1.7 reports the BL expected returns for each index in November 1998. Most of the BL expected returns in Table 5.1.6 and Table 5.1.7 were positive, except that the use of the EWMA model generated some negative BL expected returns. Black and Litterman (1992) point out that the BL expected returns would tilt to the expected returns in the view portfolios with higher confidence in views from the market neutral equilibrium returns. However, compared with the implied equilibrium returns in relevant periods, the change of BL expected returns in each asset was smaller than 80bp in August 1998, and the change of

BL expected returns in each asset was much smaller, not more than 10bp in November 1998. It seemed that the BL expected returns were less subject to the expected returns of the view portfolios, although I possessed near 100% confidence level of views. The main reason was that the expected return of the view portfolio was not large enough to have a significant influence on expected returns of every asset. For example, the expected return of the view portfolio in the DCC model in August 1998 was 6.2%; if the effect was shared by eight assets, the change would be 0.8% in each asset.

5.1.7 Comparison of Unconstrained Portfolio Optimisation Models

The previous section displayed and compared the weights solutions of different unconstrained BL portfolios. In this section, I will focus on illustrating and analysing the optimisation process in the use of one volatility model (DCC) in a single period (August 1998).

5.1.7.1 Unconstrained Black-Litterman Portfolio Frontier

Figure 5.1.6 plots an unconstrained BL portfolio frontier for three different optimisation models, including maximal SR optimisation, maximal excess return (reward) to VaR ratio optimisation, and maximal excess return (reward) to CVaR ratio optimisation, in August 1998 at a confidence level of 99%. In Figure 5.1.6 (a), the curve above point B is the efficient frontier in the SR-BL model, point A is the tangent portfolio that has the highest SR of 10.32% with an expected excess return of 0.59%, and a standard deviation of 5.68%. Point B is the minimum variance portfolio with minimum standard deviation equal to 2.43%. In Figure 5.1.6 (b), the curve above point D is the efficient frontier in the maximal VaR-BL model. Point C is the tangent portfolio that has the highest reward to VaR ratio of 4.62%, with an expected excess return of 0.59% and a VaR of 12.86%, and point D is the minimum VaR portfolio with a minimum VaR equal to 5.51%. In Figure 5.1.6 (C), the curve above point F is the efficient frontier in the maximal CVaR-BL model. Point E is the tangent portfolio that has the highest reward to CVaR ratio of 4.59%, with an expected excess return of 0.57% and a CVaR of 14.41%; point F is the minimum CVaR portfolio with minimum CVaR equal to 6.33%. Note that all results are based on excess return, so the starting point of the tangent line is zero.

5.1.7.2 Unconstrained Black-Litterman Portfolio Optimisation Statistics

Figure 5.1.6 is a specific example to show that an unconstrained BL portfolio optimisation process is the process to find the tangent portfolio on the efficient frontier. In order to analyse the difference between these optimisation models, I will report both the statistics inputted into the optimisation models and the results produced from the optimisation models in a single period. Table 5.1.8 shows the statistics for unconstrained BL portfolio optimisation in August 1998. According to Panel A, both the implied BL portfolio and the SR-BL portfolio had the same value of expected excess return to VaR ratio (μ / VaR), expected excess return to CVaR ratio ($\mu / CVaR$) and expected conditional Sharpe ratio (ECSR), which were higher than the benchmark portfolio. Investors would bear higher risk to earn higher return with the construction of an SR-BL portfolio compared with the construction of an implied BL portfolio. This can be explained by the reason that the SR-BL portfolio allocated assets more aggressively than the implied BL portfolio in August 1998, as shown in Table 5.1.9.

Panel B and Panel C in Table 5.1.8 report the corresponding statistics of an unconstrained DCC-MVaR-BL portfolio and a DCC-MCVaR-BL portfolio with the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90% in August 1998. In both the normal distribution and the t-distribution, the lower the confidence level, the higher the values of μ / VaR, $\mu / CVaR$ and ECSR in the DCC-MVaR-BL portfolio and the DCC-MCVaR-BL portfolio. For the normal distribution and the t-distribution, the values of μ / VaR, $\mu / CVaR$ and ECSR in the DCC-MVaR-BL portfolio were nearly same as the corresponding values in the DCC-MCVaR-BL portfolio at confidence levels of 99%, 95% and 90%, with a difference no larger than 0.1%. When the normal distribution changed to the t-distribution, the decreasing changes of values of μ / VaR, $\mu / CVaR$ and ECSR in the DCC-MVaR-BL portfolio were much larger at a confidence level of 99% than at the other two confidence levels of 95% and 90%. However, the decreasing changes of values of μ / VaR, $\mu / CVaR$ and ECSR in the DCC-MCVaR-BL portfolio were larger at all three confidence levels of 99%, 95% and 90%. In conclusion, the decreasing changes of values of μ / VaR, $\mu / CVaR$ and ECSR in the DCC-MCVaR-BL portfolio were more subject to the distribution assumption of the t-distribution.

When Panel B and Panel C are compared with Panel A, the DCC-MVaR-BL portfolio and the DCC-MCVaR-BL portfolio cannot generate a larger ECSR than the SR-BL portfolio but can generate larger μ / VaR and $\mu / CVaR$. These results are consistent with the viewpoints of Alexander and Baptista (2003) that the investor who uses the highest reward-to-VaR ratio to optimise the asset cannot maximise the SR, and the standard deviation of the portfolio selected by the maximal reward-to-VaR ratio is higher than the portfolio selected by the maximal SR. In addition, different results shown in different confidence levels also verify the concern of Alexander and Baptista (2003) that the choice of confidence level would have an influence on the reward-to-VaR ratio, and also on the rankings of the portfolio performance based on the different evaluation ratios.

5.1.8 Unconstrained Black-Litterman Portfolio

The main task of this section is to construct the unconstrained BL portfolio by using the asset allocation model. In the asset allocation model, the expected return and the covariance matrix are important inputs. I will impose the BL expected return and the BL covariance matrix onto the portfolio optimiser to obtain the optimal portfolio.

5.1.8.1 Construction of the Implied Black-Litterman Portfolio and the Sharpe Ratio Black-Litterman Portfolio

The simplest method is to use the reverse optimisation, as formula (4.13), to allocate assets without any constraints. This portfolio is called the implied BL portfolio. The other common method is to allocate assets to get the maximal SR as described in optimisation problem (4.14) with the solution (4.15). This portfolio is called an unconstrained SR-BL portfolio. Table 5.1.9 and Table 5.1.10 show weights allocated to each index in the use of both optimisation methods in August 1998 and in November 1998. Unlike the traditional mean-variance method shown in Appendix 5.1.3, which would generate unrealistic extreme weights in assets, BL models can generate balanced and more reasonable results. According to Table 5.1.8 and Table 5.1.9, there was an obvious coincidence in the long or short of each asset between the implied BL portfolio and the SR-BL portfolio, no matter which volatility model was used. However, the percentage of buying or selling assets was slightly different between the implied BL portfolios and the SR-BL portfolios in the use of the same volatility model. The SR-BL portfolio allocated assets with both long positions and short positions a little larger than the implied BL portfolio in August 1998. Conversely, the SR-BL portfolio allocated assets with both long positions and short positions a little smaller than the implied BL portfolio in November 1998. Comparing the equation (4.13) and equation (4.15) in Chapter 4, the numerators of the solution of the BL portfolio are same; the different denominators are risk aversion coefficients δ_t in the implied BL portfolio and vector $\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$ in the SR-BL portfolio, respectively. In the use of the DCC model, the denominator of the SR-BL portfolio was equal to 1.7871, a little lower than the risk aversion coefficients δ of 2.2166 in August 1998. In November 1998, the denominator of the SR-BL portfolio was equal to 1.0202, a little higher than the risk aversion coefficients δ of 0.8949. Different time-varying denominators would lead to different solutions between the implied BL portfolio and the SR-BL portfolio. Appendix 5.1.2 shows time-varying denominators in the implied BL portfolio and the SR-BL portfolio. It can be seen that the values of denominators of the SR-BL portfolio were more volatile when compared with the implied BL portfolio from November 1994 to May 2010. In the implied BL portfolio, the average of the denominators was 1.9266 and the standard deviation was 0.8511. In the SR-BL portfolio, the corresponding values were 1.9553 and 0.9446. Therefore, the maximal SR method would allocate assets more conservatively or more aggressively during different periods, compared with the reverse optimisation method.

By observing weights allocated to assets in the implied BL portfolio among three volatility models in Table 5.1.9, the positions in the implied DCC-BL portfolio ranged from -7.79% at UK Utilities to 12.5% at USA Oil & Gas; the positions in the implied EWMA-BL portfolio ranged from -20.05% at Japan Utilities to 22.07% at USA Oil & Gas. When the RW model was used, the positions ranged from -3.16% at Japan Utilities to 9.03% at USA Oil & Gas. It can be concluded that the use of the EWMA model would generate the most aggressive investing solutions in assets, followed by the DCC model with moderate investing solutions, and then the RW model with conservative investing solutions. This conclusion also applied to the SR-BL portfolio. In addition, the weights in Table 5.1.10 reflected a similar effect of volatility models on weights as well.

Figure 5.1.4 shows the weights of each asset in the benchmark portfolio, momentum portfolio and implied BL portfolio in the use of the DCC model. According to Figure 5.1.4, it can be seen that weights in the implied BL portfolio were allocated in a contrary direction compared with weights in the momentum portfolio in August 1998. The main reason was that the expected return of the momentum portfolio was negative, at -6.20% in August 1998 in the use of DCC model, as shown in Table 5.1.3. In November 1998, the expected return of the momentum portfolio was positive at 1.91% in the use of the DCC model, as shown in Table 5.1.4; weights in the implied BL portfolio tilted to weights in the momentum portfolio following the feature of the BL model in Figure 5.1.5.

Appendix 5.1.4 reports the average value of weights assigned in each index in the unconstrained implied BL portfolio and the SR-BL portfolio in the period from November 1994 to May 2010 and Appendix 5.1.5 reports the standard deviation of time-varying weights in each index. The average positions in the implied DCC-BL portfolio ranged from -0.06% at JAPAN Oil & Gas to 12.5% at USA Health Care; the average positions in the implied EWMA-BL portfolio ranged from -0.02% at UK Utilities to 12.62% at USA Financials. When the RW model was used, the average positions ranged from 0.11% at UK Oil & Gas to 12.68% at USA Healthcare. The average positions in the DCC-SR-BL portfolio ranged from -0.86% at UK Utilities to 11.72% at USA Health Care; the average positions in the EWMA-SR-BL portfolio ranged from -0.56% at Japan Utilities to 12.10% at USA Financials. When the RW model was used, the average positions ranged from 0.20% at UK Oil & Gas to 12.39% at USA Healthcare. Overall, the absolute range was around 12.5%. The average effect of volatility models on weights' range was not significantly different. The use of the EWMA model would generate slightly aggressive investing solutions in assets, followed by the DCC model with moderate investing solutions, and then the RW model with slightly conservative investing solutions. According to Appendix 5.1.5, it can be found that the standard deviation of weights in the SR-BL portfolio was slightly higher than that of weights in the implied BL portfolio when the DCC model and the EWMA model were used. When the RW model was used, the standard deviation of weights was much lower than that of weights with other models used. Weights in the DCC-BL portfolio were most volatile as can be reflected by the highest average standard deviation of 9.07% (the implied BL portfolio) and 11.90% (the SR-BL portfolio).

5.1.8.2 Construction of the MVaR-BL Portfolio

I propose a new method, which is closely related, to maximise the reward-to-risk ratios in the optimisation model. Unlike the SR, in which the risk is measured by standard deviation, I measure risk by using VaR with the optimisation problem displayed in Chapter 4, Section 4.2.3, function (4.16). This unconstrained BL portfolio is named the MVaR-BL portfolio. In this Section, VaR is estimated by the parametric method with the assumption of the normal distribution and the t-distribution at the confidence level of 99% in the use of different volatility models. Then I study the effect of distribution assumptions and three confidence levels on an MVaR-BL portfolio based on the DCC model (DCC-MVaR-BL portfolio).

The results of weights allocated in the MVaR-BL portfolios in August 1998 and in November 1998 can be found in Table 5.1.11 and Table 5.1.12. In August 1998, five assets including UK Technology, UK Telecom, UK Utilities, USA Consumer Goods and USA Telecom had short positions, no matter which volatility models were used. Unlike the RW model which allocated positive weights to four assets such as UK Consumer Services, UK Health Care, USA Utilities and Japan Utilities, both the DCC model and the EWMA model allocated negative weights to these assets. Exceptionally, the EWMA model took short positions on USA Consumer Services, USA Financials and USA Industrials. In November 1998, five assets, including UK Basic Materials, UK Consumer Goods, UK Industrials, UK Oil & Gas and USA Consumer Goods had short positions under three volatility models; the UK Consumer Services index was allocated negative weights in the use of the EWMA model. To sum up, the general direction of long or short of the selected asset was the same in the use of three volatility models under the normal distribution and the t-distribution assumptions. However, the choice of volatility models and distribution assumptions had different effects on the specific position of each asset. According to Table 5.1.11, in August 1998, for the normal distribution, the positions ranged between -6.89% at USA Telecom and 14.63% at Japan Technology in the use of the DCC model; the positions ranged between -28.25% at UK Telecom and 27.07% at Japan Technology in the use of the EWMA model. When the RW model was used, the positions ranged from -3.94% at UK Telecom to 11.24% at USA Health Care. When the distribution assumption changed to the t-distribution, the position range narrowed to between -4.83% at UK Telecom and 10.61% at Japan Financials in the use of the DCC model; the positions range slightly narrowed to between -26.22% at UK Telecom and 22.19% at Japan Technology in the use of EWMA model. When the RW model was used, the positions stayed similar, ranging from -3.99% at UK Telecom to 11.02% at USA Health Care. Therefore, similar to the implied BL portfolio and the SR-BL portfolio, weights allocated in the MVaR-BL portfolio based on the use of the EWMA model were the most aggressive; compared with the other two volatility models, the use of the RW model would generate weights that were more conservative than that of the DCC model under either the normal distribution assumption or the t-distribution assumption. The positions range could be slightly narrower with the the t-distribution assumption compared with the normal distribution in the use of the DCC model and the RW model. The average absolute values of the change of the weights between the normal distribution and the t-distribution were 2.95% (DCC model), 1.97% (EWMA model) and 0.2% (RW model) in August 1998. The average absolute values of the change of weights between the normal distribution and the t-distribution were 1.04% (DCC model), 0.23% (EWMA model) and 0.73% (RW model) in November 1998. Thus, the choice of the the t-distribution assumption would have more impact on weights solutions based on the DCC model and the EWMA model at a confidence level of 99% in the MVaR-BL portfolio.

Compared with the implied BL portfolio in August 1998, the average absolute values of the change of the weights in the MVaR-BL portfolio were 2.01% (DCC model), 6.71% (EWMA model) and 1.91% (RW model) for the normal distribution in August 1998. For the t-distribution, the average absolute values of the change of weights between the implied BL portfolio and the MVaR-BL portfolio were 3.46% (DCC model), 7.75% (EWMA model) and 1.90% (RW model) in August 1998. Compared with the weights solutions in the implied BL portfolio, it was apparent that the use of the EWMA model could generate the most diverse weights solutions in an MVaR-BL portfolio than the use of the DCC model and the RW model. It can also be found that the effect of the use of the RW model on weights solutions was not sensitive to the assumption of distribution in differences between construction of an implied BL portfolio and an MVaR-BL portfolio, but the use of the DCC model and the EWMA model would have a bigger effect on generating different weights solutions with an assumption of the t-distribution. In November 1998, the findings were similar.

Compared with the SR-BL portfolio in August 1998, the average absolute values of the change of the weights in the MVaR-BL portfolio were 2.30% (DCC model), 10.87% (EWMA model) and 2.05% (RW model) for the normal distribution in August 1998. For the t-distribution, the average absolute values of the change of weights between the SR-BL portfolio and the MVaR-BL portfolio were 4.31% (DCC model), 11.47% (EWMA model) and 2.03% (RW model) in August 1998. The finding was similar. The use of the EWMA model could generate the most diverse weights solutions in an MVaR-BL portfolio, contrasting with an SR-BL portfolio, than the use of the DCC model and the EWMA model. Moreover, the change from the normal distribution to the assumption of the t-distribution would not affect the difference of weights solutions between the SR-BL portfolio and the MVaR-BL portfolio in the use of the RW model. However, the effect of the use of the EWMA model on weights solutions was more sensitive to the assumption of distribution in different weights solutions between construction of an SR-BL portfolio and an MVaR-BL portfolio than that when the DCC model was used. In November 1998, the findings were similar.

Appendix 5.1.6 reports average value of weights allocated to each index in the unconstrained MVaR-BL portfolio in the period from November 1994 to May 2010, and Appendix 5.1.7 reports the standard deviation of time-varying weights in each index. According to Appendix 5.1.6, most of average values of weights in each index were positive; the average effect of different volatility models on weights was not significantly different. For the normal distribution, the average absolute position range was 0.1211 in the use of the DCC model, slightly narrower than 0.1285 in the use of the EWMA model and the RW model. When the distribution assumption changed to the t-distribution, the average absolute position range became slightly wider to be 0.1357 (DCC model), 0.1332 (EWMA model), and 0.1282 (RW model). Therefore, Appendix 5.1.6 also confirmed the finding that the choice of the the t-distribution assumption would have more impact on weights solutions based on the DCC model and the EWMA model at a confidence level of 99% in the MVaR-BL portfolio. In addition, Appendix 5.1.7 showed that the DCC-MVaR-BL portfolio had most volatile weight solutions over the full sample with much bigger average standard deviation, and the choice of the t-distribution exaggerated this effect at a confidence level of 99%. However, the average standard deviation in both the EWMA-MVaR-BL portfolio and the RW-MVaR-BL portfolio decreased under the t-distribution, meaning that less volatile weight solutions were allocated.

5.1.8.3 Effect of Distribution Assumption and Confidence Levels on DCC-MVaR-BL Portfolio

Table 5.1.13 shows the positions of each asset in a DCC-MVaR-BL portfolio in August 1998 under the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. For the normal distribution, the positions of assets ranged between -6.89% (USA Telecom) and 14.63% (Japan Technology) at 99% confidence level; between -7.02% (USA Telecom) and 14.61% (Japan Technology) at 95% confidence level, and between -7.01% (USA Telecom) and 14.19% (Japan Technology) at 90% confidence level. For the t-distribution, the positions of assets narrowed the range between -4.83% (UK Telecom) and 10.61% (Japan Financials) at 99% confidence level; between -6.87% (USA Telecom) and 14.55% (Japan Technology) at 95% confidence level, and between -7.03% (USA Telecom) and 14.58% (Japan Technology) at 90% confidence level. When I ranked assets by positions from greater short positions to the bigger long positions, the ranks of each asset for the normal distribution were similar to the ranks for the t-distribution at confidence levels of 95% and 90%. Therefore, it can be seen that the choice of distribution assumption had a slight impact on the weights solutions of the MVaR-BL portfolio at confidence levels of 95% and 90%. Appendix 5.1.8 also confirmed this conclusion and reflected that the choice of the t-distribution would allocate more volatile weights than that of the normal distribution at confidence levels of 95% and 90%. In addition, the weights solutions of the MVaR-BL portfolio for the t-distribution at a confidence level of 99% with the narrowest position range were significantly different with other MVaR-BL portfolios in August 1998, however, the average value of weights in the MVaR-BL portfolio for the t-distribution at a confidence level of 99% had the widest absolute position range over the full sample.

5.1.8.4 Construction of the MCVaR-BL Portfolio

I propose a new optimisation method to maximise the reward-to-CVaR ratio. The optimisation problem is displayed in Chapter 4, Section 4.2.3, function (4.18). This unconstrained BL portfolio is named as the MCVaR-BL portfolio. CVaR is estimated by the parametric method with the assumption of the normal distribution and the t-distribution at a confidence level of 99% in the use of different volatility models.

As can be seen in Table 5.1.14 and Table 5.1.15, the results of weights allocated in an MCVaR-BL portfolio in August 1998 and in November 1998 showed the same direction of long or short selected assets as an MVaR-BL portfolio in the corresponding periods. However, the choice of volatility models and distribution assumptions has different effects on the specific position of each asset. According to Table 5.1.14, for the normal distribution, the positions ranged between -5.99% at USA Telecom and 12.29% at Japan Technology in the use of the DCC model; the positions ranged between -28.28% at UK Telecom and 26.34% at Japan Technology in the use of the EWMA model. When the RW model was used, the positions ranged between -3.62% at UK Telecom and 11.21% at USA Health Care. With the assumption changed to the t-distribution, the range of positions narrowed between -5.15% at UK Telecom and 11.29% at Japan Financials in the use of the DCC model, and the range of positions based on the use of the EWMA model significantly narrowed between -12.11% at UK Technology and 20.10% at Japan Oil & Gas. When the RW model was used, the range of positions slightly narrowed to between -3.55% at UK Telecom and 11.15% at USA Health Care. Therefore, similar to the MVaR-BL portfolio, weights allocated in the MCVaR-BL portfolio based on the use of the EWMA model were the most aggressive when compared with the other two volatility models; the use of the RW model would generate weights that are more conservative than that of the DCC model under either the normal distribution assumption or the t-distribution assumption. In addition, the choice of the t-distribution assumption could narrow the positions range between the maximum short position and the maximum long position in the construction of an MCVaR-BL portfolio, especially in the use of the EWMA model. The calculated average absolute values of the change of the weights between the normal distribution and the t-distribution were 1.01% (DCC model), 6% (EWMA model) and 0.2% (RW model) in August 1998. The average absolute values of the change of the weights between the normal distribution and the t-distribution were 2.16% (DCC model), 1.71% (EWMA model) and 0.08% (RW model) respectively in November 1998. Thus, the choice of the t-distribution assumption would have more impact on weights solutions based on the DCC and EWMA models at a confidence level of 99% in the MCVaR-BL portfolio.

Compared with the MVaR-BL portfolio in August 1998, the average absolute values of the change of the weights in the MCVaR-BL portfolio were 1.67% (DCC model), 0.28% (EWMA model) and 0.17% (RW model) for the normal distribution in August 1998. For the t-distribution, the average absolute values of the change of weights between the MVaR-BL portfolio and the MCVaR-BL portfolio were 0.42% (DCC model), 4.71% (EWMA model) and 0.19% (RW model) in August 1998. It was interesting to find that the use of the DCC model could generate the most diverse weights solutions in the MCVaR-BL portfolio than the use of the EWMA model and the RW model could for the normal distribution. Additionally, the use of the EWMA model could generate the most diverse weights solutions in the MCVaR-BL portfolio than the use of the DCC model and the RW model could for the t-distribution.

Compared with the MVaR-BL portfolio in November 1998, the average absolute values of the change of the weights in the MCVaR-BL portfolio were 0.08% (DCC model), 0.06% (EWMA model) and 0.68% (RW model) for the normal distribution. For the t-distribution, the average absolute values of the change of the weights between the MVaR-BL portfolio and the MCVaR-BL portfolio were 1.68% (DCC model), 1.61% (EWMA model) and 0.03% (RW model). It meant that the weights solutions in the MVaR-BL portfolio and in the MCVaR-BL portfolio had no difference for the normal distribution, no matter which volatility model was selected. However, the weights solutions in the MCVaR-BL portfolio were different to the weights solutions in the MVaR-BL portfolio for the t-distribution in the use of the DCC model and the EWMA model.

Appendix 5.1.9 reports average value of weights allocated to each index in the unconstrained MCVaR-BL portfolio in the period from November 1994 to May 2010, and Appendix 5.1.10 reports the standard deviation of time-varying weights in each index. According to Appendix 5.1.9, most of average values of weights in each index were positive; the average effect of different volatility models on weights was not significantly different. For the normal distribution, the average absolute position range was 0.1227 in the use of the DCC model, slightly narrower than 0.1304 and 0.1284 in the use of the EWMA model and the RW model respectively. When the distribution assumption changed to the t-distribution, the average absolute position range became slightly wider, at 0.1366 (DCC model) and 0.1393 (EWMA model), and slightly narrower, at 0.1282 (RW model). Therefore, Appendix 5.1.9 also confirmed the finding that the choice of the the t-distribution assumption would have more impact on weights solutions based on the DCC model and the EWMA model at a confidence level of 99% in the MCVaR-BL portfolio. In addition, Appendix 5.1.10 showed that the DCC-MCVaR-BL portfolio had most volatile weight solutions over the full sample with much bigger average standard deviation than other MCVaR-BL portfolios in the use of the EWMA model and the RW model. The choice of the t-distribution had the impact of decreasing the average standard deviation on both the DCC-MCVaR-BL portfolio and the RW-MCVaR-BL portfolio, but the impact of increasing the average standard deviation on the EWMA-MCVaR-BL portfolio. Compared to the MVaR-BL portfolio, the MCVaR-BL portfolio in the use of the RW model had similar average absolute position range and less volatile weight solutions, the DCC-MCVaR-BL portfolio had wider average absolute position range and less volatile weight solutions for the t-distribution.

5.1.8.5 Effect of Distribution Assumption and Confidence Levels on DCC-MCVaR-BL Portfolio

Table 5.1.16 shows the positions of each asset in a DCC-MCVaR-BL portfolio in August 1998 under the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. For the normal distribution, the positions of assets ranged between -5.99% (USA Telecom) and 12.29% (Japan Technology) at 99% confidence level; between -6.87% (USA Telecom) and 14.52% (Japan Technology) at 95% confidence level, and between -6.86% (USA Telecom) and 14.36% (Japan Technology) at 90% confidence level. For the t-distribution, the positions of assets ranged between -5.15% (UK Telecom) and 11.29% (Japan Financials) at 99% confidence level; between -5.34% (UK Telecom) and 11.10% (Japan Financials) at 95% confidence level, and between -6.03% (USA

Telecom) and 12.38% (Japan Technology) at 90% confidence level. Therefore, it can be seen that the change of the normal distribution to the t-distribution could narrow the positions range in the DCC-MCVaR-BL portfolio at all three confidence levels in August 1998. In addition, the higher the confidence level, the narrower the position range in the DCC-MCVaR-BL portfolio. Compared with the DCC-MVaR-BL portfolio in Table 5.1.13, most positions in the DCC-MCVaR-BL portfolio had a relatively narrower range in both distributions at each confidence level, except for the t-distribution at a confidence level of 99%.

However, Appendix 5.1.11 shows the average effect that the change of the normal distribution to the t-distribution could widen the absolute position range at the confidence levels of 99% and 95%. In addition, the higher the confidence level, the wider the absolute position range in the DCC-MCVaR-BL portfolio. Besides, the choice of the t-distribution in the DCC-MCVaR-BL could generate relatively less volatile weight solutions over the full sample. Compared with the DCC-MVaR-BL portfolio in Appendix 5.1.8, the DCC-MCVaR-BL portfolio had wider absolute position range and less volatile weight solutions for the t-distribution at higher confidence levels.

5.1.9 Performance Evaluation of the Unconstrained BL Portfolios

As I have constructed the optimal unconstrained BL portfolios, I will evaluate the real performance of the optimal unconstrained BL portfolio in a single period and over multiple periods.

5.1.9.1 Single Period Performance

Single Period Performance in August 1998

Table 5.1.17 reports the results of the unconstrained BL portfolio and the benchmark portfolio for the portfolio evaluation criteria including realised excess return, conditional Sharpe ratio (CSR), portfolio turnover and reward to CVaR ratio in August 1998 and November 1998. Considering the negative realised excess return in August 1998, the CSR and conditional reward to CVaR ratio were adjusted. The ranking rule is that the larger the ratios, the better the portfolio performance. In August 1998, most unconstrained BL portfolios could not beat the benchmark with relative lower adjusted CSR and lower adjusted reward to CVaR ratio, except the implied BL portfolio. Based on an adjusted

CSR portfolio evaluation criterion, the implied BL portfolio outperformed the benchmark in the use of RW50 model. Based on the alternative portfolio evaluation criterion of the adjusted reward to CVaR ratio, the implied BL portfolio in the use of DCC model and EWMA model outperformed the benchmark. The performance of the SR-BL portfolio was worse than the implied BL portfolio because of more aggressive investment allocation, as explained in Section 5.1.8.1. The values of portfolio turnover in the SR-BL portfolio at 3.6583 (DCC model) and 7.0786 (EWMA model) were much higher than those in the implied BL portfolio: 1.6803 (DCC model) and 5.5327 (EWMA model).

At a confidence level of 99%, when the adjusted CSR is used to evaluate the portfolio performance, the MVaR-BL portfolio performed better for the t-distribution assumption than for the normal distribution in the use of DCC model, with the value equal to -0.7311. When the adjusted reward to CVaR ratio is used, the MVaR-BL portfolio performed better for the t-distribution assumption than for the normal distribution in the use of three volatility models. At confidence levels of 95% and 90%, the MVaR-BL portfolio had better risk-adjusted performance for the t-distribution assumption than for the normal distribution assumption in the use of EWMA model and RW50 model. In addition, when the distribution assumption changed from the normal distribution to the t-distribution, the portfolio turnover would be lower.

At a confidence level of 99%, the MCVaR-BL portfolio had a better risk-adjusted performance for the t-distribution assumption than for the normal distribution in the use of all three volatility models. At confidence levels of 95% and 90%, the MCVaR-BL portfolio performed better for the t-distribution assumption than for the normal distribution in the use of the DCC model and the RW50 model. In addition, when the distribution assumption changed from the normal distribution to the t-distribution, the portfolio turnover would be reduced.

Compared with the SR-BL portfolio, the performances of most of the MVaR-BL portfolios and the MCVaR-BL portfolios were better and, surprisingly, the portfolio turnover could be reduced by using alternative optimisation models. Furthermore, let me make a comparison between the MVaR-BL portfolio and the MCVaR-BL portfolio. For the normal distribution assumption, the risk-adjusted performance of the MCVaR-BL portfolio was superior in the use of the

DCC model and the RW50 model at a confidence level of 99%, and the performance of the MCVaR-BL portfolio was better in the use of the EWMA model and the RW50 model at confidence levels of 95% and 90%. For the t-distribution assumption, the MCVaR-BL portfolio outperformed the MVaR-BL portfolio with higher risk-adjusted performance ratios in the use of the EWMA model and the RW50 model at a confidence level of 99%, and the MCVaR-BL portfolio overtook the MVaR-BL portfolio in the use of the DCC model and the RW50 model at a confidence level of 95%. At a confidence level of 90%, the MCVaR-BL portfolio showed a better risk-adjusted performance in the use of all three volatility models. In addition, the MCVaR-BL portfolio always had lower values of portfolio turnover.

Single Period Performance in November 1998

As shown in Table 5.1.17 Panel B, the realised returns of the benchmark and the unconstrained BL portfolios were positive in November 1998; the traditional SR and reward to CVaR ratio can be used to evaluate portfolio performance. Compared with the benchmark portfolio, both the implied BL portfolio and the SR-BL portfolio can beat the benchmark portfolio in the use of the DCC model, and most of the MVaR-BL portfolio and the MCVaR-BL portfolio can beat the benchmark portfolio as well in the use of the DCC model and the RW50 model. Besides, some MVaR-BL portfolios and MCVaR-BL portfolios can outperform the implied BL portfolio and the SR-BL portfolio in the use of the EWMA model and the RW50 model.

In November 1998, in the use of all three volatility models, the MVaR-BL portfolio performed better for the normal distribution assumption than for the t-distribution at a confidence level of 99%; however, the MVaR-BL portfolio performed better for the t-distribution at a confidence level of 95%. However, at a confidence level of 90%, the performance of the MVaR-BL portfolio was better for the t-distribution only in the use of the EWMA model. In addition, when the distribution assumption changed from the normal distribution to the t-distribution, the portfolio turnover is lower.

At confidence levels of 99% and 95%, the MCVaR-BL portfolio performed better for the normal distribution assumption than for the t-distribution in the use of all three volatility models. However, at a confidence level of 90%, the MCVaR-BL portfolio performed better for the t-distribution assumption than for the normal distribution. In addition, the MCVaR-BL portfolios always have lower values of portfolio turnover in the the t-distribution assumption.

For the normal distribution assumption, in contrast to the MVaR-BL portfolio, the MCVaR-BL portfolio underperformed at a confidence level of 99%; conversely, the MCVaR-BL portfolio outperformed at a confidence level of 95%. At a confidence level of 90%, the MCVaR-BL portfolio had a better performance only in the use of the EWMA model. For the t-distribution assumption, the MCVaR-BL portfolio performed better than the MVaR-BL portfolio in the use of the DCC model and the EWMA model only at a confidence level of 90%. Moreover, the MCVaR-BL portfolios had a lower portfolio turnover than the MVaR-BL portfolio.

Overall, although the time-varying performances of the unconstrained BL portfolios cannot give reliable suggestion about which volatility model should be selected to achieve best performance, these single-period performances indeed provided some evidences that the unconstrained implied BL portfolio had a superior performance to the SR-BL portfolio. Moreover, the MVaR-BL portfolio and the MCVaR-BL portfolio could perform better than the implied BL portfolio and the SR-BL portfolio with a choice of a certain volatility model at an acceptable confidence level. Additionally, the MCVaR-BL portfolio could beat the MVaR-BL portfolio in certain circumstances and the MCVaR-BL portfolio could provide a relatively lower portfolio turnover. Therefore, it is better to analyse the average performance over multiple periods to get reliable conclusions.

5.1.9.2 Multiple Periods Performance

In this section, I would like to analyse the performance of the unconstrained BL portfolio in the in-sample basis. I choose the whole sample period of multiple periods from November 1994 to May 2010 (see Table 5.1.18), and also the sub-period from August 1998 to May 2010 (see Table 5.1.19) to make a comparable analysis between the three volatility models.

Table 5.1.18 reports the results of realised unconstrained BL portfolio performance compared with the benchmark portfolio performance in the period from November 1994 to May 2010**.** The benchmark portfolio had the SR of 1.14% with the average return of 0.05%, and standard deviation equal to 4.35%. The negative skewness of -0.9217 indicated that the left tail risk exists in the benchmark portfolio. The reward to VaR ratio and the reward to CVaR ratio were 0.38% and 0.29%, respectively. According to Table 5.1.18, it can be found that all unconstrained BL portfolios outperformed the benchmark portfolio with much higher values of SR, reward to VaR ratio and reward to CVaR ratio.

The unconstrained implied BL portfolio had a relatively higher kurtosis which indicated bigger fat-tail risk than the benchmark portfolio. The unconstrained implied BL portfolio based on the DCC model showed the best performance, with the highest SR (21.7%), information ratio (28.25%), reward to VaR ratio (10.37%) and reward to CVaR ratio (8.15%). By ranking SR and reward to CVaR ratio, the implied EWMA-BL portfolio performed better than the implied RW50-BL portfolio. However, when information ratio and reward to VaR ratio were used to evaluate performance, the implied RW50-BL portfolio outperformed the implied EWMA-BL portfolio.

The performances of the unconstrained SR-BL portfolio were worse than the unconstrained implied BL portfolio. In the unconstrained SR-BL portfolios, the unconstrained DCC-SR-BL portfolio, which had a relatively higher kurtosis of 33.3293, performed best in unconstrained SR-BL portfolios with the highest SR (15.78%), reward to VaR ratio (7.51%) and reward to CVaR ratio (4.16%), followed by the unconstrained RW50-SR-BL portfolio with moderate corresponding ratios. However, when the information ratio was used to evaluate the active portfolio performance, the unconstrained RW50-SR-BL portfolio showed the best performance with an information ratio of 27.31%, followed by the unconstrained DCC-SR-BL portfolio with an information ratio of 17.73%. The unconstrained EWMA-SR-BL portfolio which had the largest negative skewness (-3.8259) and the highest kurtosis (38.7782) performed worst, with the lowest evaluation ratios.

Compared with the SR-BL portfolio, MVaR-BL portfolios could improve performance. In MVaR-BL portfolios, based on evaluation ratios of SR, reward to VaR ratio and reward to CVaR ratio, at three different confidence levels, the risk-adjusted performances of the DCC-MVaR-BL portfolio were better than the EWMA-MVaR-BL portfolio and the RW50-MVaR-BL portfolio in both distribution assumptions. However, when I evaluated the active portfolio performance tracking the benchmark, the RW50-MVaR-BL portfolio was superior to the other portfolios. Furthermore, I will make a comparison between the portfolio performance for the normal distribution and the t-distribution. At a confidence level of 99%, the change from the normal distribution to the t-distribution improved the performance of the DCC-MVaR-BL portfolio and the EWMA-MVaR-BL portfolio. At a confidence level of 95%, the DCC-MVaR-BL portfolio for the t-distribution overtook the performance of the DCC-MVaR-BL portfolio for the normal distribution. At a confidence level of 90%, both the DCC-MVaR-BL portfolio and the RW50-MVaR-BL portfolio for the t-distribution performed better. The MVaR-BL portfolio could also improve active portfolio performance for the t-distribution at all three confidence levels with different volatility models.

Compared with the SR-BL portfolio, MCVaR-BL portfolios could improve performance. In MCVaR-BL portfolios, at all three different confidence levels, the risk-adjusted performances of the DCC-MCVaR-BL portfolio were better than the EWMA-MCVaR-BL portfolio and the RW50-MCVaR-BL portfolio in both distribution assumptions with higher risk-adjusted performance evaluation ratios. However, when I evaluated the active portfolio performance, the RW50-MCVaR-BL portfolio was superior to the other portfolios. Furthermore, I will make a comparison between the portfolio performance for the normal distribution and the t-distribution. At confidence levels of 99% and 95%, the change from the normal distribution to the t-distribution improved the performance of the DCC-MCVaR-BL portfolio and the EWMA-MCVaR-BL portfolio. At a confidence level of 90%, the DCC-MCVaR-BL portfolio for the t-distribution performed better than for the normal distribution. The MCVaR-BL portfolio could also improve active portfolio performance for the t-distribution at three confidence levels with difference volatility models.

Compared with the MVaR-BL portfolio, the MCVaR-BL portfolio could perform better at certain confidence levels in both distribution assumptions. Specifically, for the normal distribution, both the DCC-MCVaR-BL portfolio and the EWMA-MCVaR-BL portfolio showed a better performance than the corresponding MVaR-BL portfolios at a confidence level of 99%, and the DCC-MCVaR-BL portfolio outperformed the DCC-MVaR-BL portfolio at a confidence level of 95%. For the t-distribution, both the risk-adjusted performance and the active portfolio performance in the DCC-MCVaR-BL portfolio were superior to the DCC-MVaR-

BL portfolio at a confidence level of 99%; additionally, both the risk-adjusted performance and the active portfolio performance in the DCC-MCVaR-BL portfolio and EWMA-MCVaR-BL portfolio were better than the MVaR-BL portfolios.

Besides this, it is also worth taking a look at the empirical VaR and empirical CVaR of the unconstrained BL portfolio. In contrast to the benchmark portfolio with values of 13.14% (empirical VaR) and 16.92% (empirical CVaR), the unconstrained BL portfolio in the use of the RW50 model was slightly higher, with empirical VaR around 13.50% and empirical CVaR around 17.50%. However, based on the DCC model and the EWMA model, the empirical VaR and empirical CVaR of the unconstrained BL portfolios were much higher than the benchmark.

Since the unconstrained BL portfolio performances in the sub-period from August 1998 to May 2010 were similar to the performance in the whole period, I reported the results in Table 5.1.19 without further analysis.

5.1.10 Conclusions

There are several primary findings about the unconstrained BL portfolios through the in-sample analysis. Firstly, they benefit from the outperformance of the view portfolio constructed by the momentum strategy; all of the unconstrained BL portfolios based on different optimisation models have shown an attractive performance, no matter if in single period or in multiple periods, when compared with the benchmark portfolio. It is obvious that the unconstrained BL portfolios have the favourable feature of allocating assets with more balanced and realistic weights than the traditional mean-variance method.

Secondly, to decide whether the implied BL portfolio performance performs better than the SR-BL portfolio performance in a single period, a comparison between denominators in both weight solutions function is necessary. In multiple periods, the implied BL portfolio is superior to the SR-BL portfolio. It is worth noticing that the SR-BL portfolio has larger empirical VaR and empirical CVaR over multiple periods. The use of different volatility models would have different degrees of effect on asset positions and performances in the implied BL portfolio and the SR-BL portfolio. The use of the EWMA model would generate the most aggressive investing solutions in assets, followed by the DCC model with moderate investing solutions, and then the RW50 model with conservative investing solutions in some single periods. However, the average effect of different volatility models on average absolute position range is not significantly different. The use of the DCC model could generate most volatile weight solutions. In some single periods, the implied BL portfolio and the SR-BL portfolio based on RW50 might show the best performance. In multiple periods, the unconstrained implied DCC-BL portfolio has a better performance than the other two implied BL portfolios based on the EWMA and RW50 models. However, the unconstrained DCC-SR-BL portfolio could only perform best in risk-adjusted performance while the unconstrained RW50-SR-BL portfolio could have the best active performance.

Thirdly, the use of maximal reward to VaR ratio and maximal reward to CVaR ratio optimisation models could improve the performance of the implied BL portfolio and the SR-BL portfolio in a single period and over multiple periods at acceptable levels of confidence. In the construction of the dynamic MVaR-BL portfolio and dynamic MCVaR-BL portfolio, not only the choice of different volatility models but also the distribution assumptions and confidence levels impose different effects on weights solutions, single period performance and multiple-period performance. Similar to the effect on the SR-BL portfolio, the use of the EWMA model might generate the most aggressive weight solutions, the use of the RW50 model solves most conservative positions, and the use of the DCC model stands in the middle; the MVaR-BL portfolio and the MCVaR-BL portfolio based on the RW50 model might show the best performance in some single periods. However, the average effect of different volatility models on average absolute position range is not significantly different. The use of the DCC model could generate most volatile weight solutions. The change of the normal distribution to the t-distribution could increase the average standard deviation of the DCC-MVaR-BL portfolio but decrease the average standard deviation of the DCC-MCVaR-BL portfolio. In multiple periods, the risk-adjusted performances of the MVaR-BL portfolio and the MCVaR-BL portfolio based on the DCC model are better than the use of the EWMA model in both distribution assumptions. The MVaR-BL portfolio and the MCVaR-BL portfolio based on RW50 have superior active performance than other volatility models. Both the

MVaR-BL portfolio and the MCVaR-BL portfolio perform better for the t-distribution than for the normal distribution based on the DCC model and the EWMA model. Additionally, the MCVaR-BL portfolio could beat the MVaR-BL portfolio in certain circumstances and the MCVaR-BL portfolio could provide a relatively lower portfolio turnover.

5.2 Value-at-Risk-Constrained Black-Litterman Portfolio

As can be concluded from the results of unconstrained BL portfolio performance, the SR-BL portfolios have larger empirical VaR than the benchmark portfolio and the implied BL portfolio. The negative skewness and high kurtosis also reflect larger tail risks.

Motivated by reducing the tail risk, I will impose VaR constraints on the unconstrained SR-BL portfolio. The constraints are set to be the scaling factor k multiplied by VaR_0, which is the VaR of the implied BL portfolio, and k is equal to 0.99, 0.95, 0.90, and 0.80, and reduces sequentially until the SR-BL portfolio unbinds.

5.2.1 Construction of the VaR-Constrained BL Portfolio

In the empirical study of VaR constraints, the distribution assumptions and the confidence levels are important factors to take into account. According to the optimisation problem in formula (4.20), I construct a VaR-constrained BL portfolio with the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%.

5.2.1.1 VaR-Constrained BL Portfolio Frontier

Figure 5.2.1 plots the VaR-constrained BL portfolio frontier with different distribution assumptions (the normal distribution and the t-distribution) and confidence levels (99%, 95% and 90%) as k equal to 0.99. Note that the constraint is equal to $k * VaR_0$, where VaR_0 is the estimated VaR of the implied BL portfolio. VaR_0 is equal to -10.35% in August 1998. The light blue line represents the VaR constraints for the t-distribution; the green line represents the VaR constraints for the normal distribution. The black point M is the minimum variance portfolio, and the red square point T is the tangent portfolio that has the maximum SR. The left direction of the black arrow means the VaR constraints tighten as the VaR constraints line moves to left. Alexander and Baptista (2004) explain that portfolios that meet a VaR constraint should be on or above a line with intercept $-k * VaR_0$ and slope ξ_β (see Chapter 4, equation (4.17)) in the mean-standard deviation frontier. Therefore, the selected portfolio should be on the efficient frontier from point M to the intersection point of the line and the efficient frontier. When the value of $k * VaR_0$ decreases, the intercept of this line increases, and the slope ξ_β increases when the confidence level increases; the constraint would tighten. Setting the same $k * VaR_0$, the slope in the lower confidence level of 90% is apparently smaller than in the higher confidence level of 99%; at same confidence level, the slope for the t-distribution is much higher than for the normal distribution. Thus, constraints in higher confidence level or for the t-distribution are tighter, and become tighter as k reduces.

When k is equal to 0.99, in Figure 5.2.1(a), the tangent portfolio with maximal SR cannot be selected in both the normal distribution and the t-distribution. In Figure 5.2.1(b), the tangent portfolio can be selected for the normal distribution but is omitted for the t-distribution. In Figure 5.2.1(c), the tangent portfolio can be selected in both the normal distribution and the t-distribution.

5.2.1.2 Weights of VaR-Constrained BL Portfolios

Table 5.2.1 and Table 5.2.2 show weights allocated in the VaR-constrained BL portfolio based on the three volatility models in August 1998 and in November 1998.

In August 1998, for the normal distribution, the positions of the DCC-VaR-constrained BL portfolio were narrowest, with a range between -7.06% (USA Consumer Goods) and 16.55% (USA Oil & Gas). The widest range of positions in the EWMA-VaR-constrained BL portfolio was between -26.14% (USA Industrials) and 43.17% (Japan Industrials); the range of positions in the RW50-VaR-constrained BL portfolio was moderate within the interval from -11.11% (USA Industrials) to 22.13% (Japan Industrials). For the t-distribution, the range of positions in the DCC-VaR-constrained BL portfolio was narrowest, between -11.78% (USA Industrials) and 20.32% (USA Oil & Gas). The widest range of positions in the EWMA-VaR-constrained BL portfolio was between -35.83% (USA Industrials) and 59.62% (Japan Industrials); the range of positions in the RW50-VaR-constrained BL portfolio was moderate, within the interval from -11.02% (USA Industrials) to 22.13% (Japan Industrials). In addition, the average absolute value of the change of the weights between the normal distribution and the t-distribution was 5.75% (DCC model), 8.01% (EWMA model) and 0.2% (RW model) in August 1998.

In November 1998, for the normal distribution, the positions in the DCC-VaR-BL portfolio ranged between -4.34% (UK Consumer Services) and 17.65% (USA Health Care), and the relative wider positions range in the EWMA-VaR-BL portfolio was between -6.36% (UK Consumer Services) and 19.23% (USA Health Care). The narrowest positions range was in the RW50-VaR-BL portfolio within the interval from -1.61% (UK Consumer Goods) to 15.14% (USA Health Care). For the t-distribution, the positions in the DCC-VaR-BL portfolio ranged between -7.06% (Japan Technology) and 15.93% (Japan Consumer Services), and the relatively wider positions range in the EWMA-VaR-BL portfolio was between -16.89% (USA Industrials) and 39.61% (Japan Industrials). The narrowest positions range was in the RW50-VaR-BL portfolio within the interval from -1.34% (USA Consumer Goods) to 14.75% (USA Health Care). In addition, the average absolute value of the change of the weights between the normal distribution and the t-distribution were 4.26% (DCC model), 7.12% (EWMA model) and 0.34% (RW model).

Therefore, in some single periods, weights allocated in the EWMA-VaR-BL portfolio were the most aggressive compared with the other two VaR-constrained portfolios in the use of DCC model and RW50 model, under either the normal distribution assumption or the t-distribution assumption. The change from the normal distribution to the t-distribution could widen the positions range in the DCC-VaR-BL portfolio and the EWMA-VaR-BL portfolio. The positions range in the RW50-VaR-BL portfolio was insensitive to the distribution assumption. The choice of the t-distribution assumption would have had more impact on weights solutions based on the DCC model and the EWMA model at a confidence level of 99% in the VaR-constrained portfolio.

Compared with the SR-BL portfolio in Table 5.1.9, the range of positions in the VaR-constrained BL portfolio was wider in August 1998. Conversely, the range of the positions in the VaR-constrained BL portfolio was narrower, in contrast to the implied BL portfolio and the SR-BL portfolio in November 1998. Therefore, the effect of adding VaR constraints on position range was not consistent in different single periods.

Appendix 5.2.1 reports average value of weights allocated to each index in the unconstrained MCVaR-BL portfolio in the period from November 1994 to May 2010, and Appendix 5.2.2 reports the standard deviation of time-varying weights in each index. According to Appendix 5.2.1, it can be found that the EWMA-VaR-BL portfolio had the widest average absolute position range for the t-distribution and the change from the normal distribution to the t-distribution could widen the positions range in the VaR-BL portfolio. It can also be concluded that the DCC-VaR-BL portfolio had most volatile weight solutions for the normal distribution and the EWMA-VaR-BL portfolio had most volatile weight solutions for the t-distribution from Appendix 5.2.2. In addition, the change from the normal distribution to the t-distribution could make weight solutions less volatile in the use of the DCC model and more volatile in the use of the EWMA model and the RW model.

5.2.2 Performance Evaluation

5.2.2.1 Single Period Performance

Table 5.2.3 reports the results of the VaR-constrained BL portfolio performance evaluated by realised return, CSR, portfolio turnover, and reward to CVaR ratio in August 1998 and November 1998. In August 1998, according to Table 5.2.3 Panel A, it can be found that the RW50-VaR-BL portfolio performed best, followed by the DCC-VaR-BL portfolio and then the EWMA-VaR-BL portfolio by ranking adjusted evaluation ratios. The DCC-VaR-BL portfolio had a much lower portfolio turnover than the EWMA-VaR-BL portfolio. When the normal distribution assumption was changed to the t-distribution, the performance of the VaR-constrained BL portfolio became better with an improved adjusted CSR and adjusted reward to CVaR ratio, and reduced portfolio turnover.

Compared with the implied BL portfolio and the SR-BL portfolio in August 1998 (Table 5.1.17), the VaR-constrained BL portfolio could beat these portfolios, with higher adjusted evaluation ratios and a relatively lower portfolio turnover, especially with the t-distribution assumption. Furthermore, the VaR-constrained BL portfolio could outperform the benchmark portfolio with the t-distribution.

In November 1998, ranking the CSR and reward to CVaR ratio from the smallest to largest, I can easily point out that the performance of the RW50-VaR-BL portfolio was superior to the DCC-VaR-BL portfolio and the EWMA-VaR-BL portfolio, and that the DCC-VaR-BL portfolio performed slightly better than the EWMA-VaR-BL portfolio. The change from the normal distribution to the t-distribution could increase the CSR and reward to CVaR ratio and decrease portfolio turnover in the DCC-VaR-BL portfolio and RW50-VaR-BL portfolio. Briefly, the t-distribution assumption had a positive effect on performance of the DCC-VaR-BL portfolio and RW50-VaR-BL portfolio.

In contrast to the SR-BL portfolio in November 1998, all of the VaR-constrained BL portfolios showed a better performance except the EWMA-VaR-BL portfolio with the t-distribution. While the DCC-VaR-BL portfolio overtook the implied BL portfolio with higher evaluation ratios and lower portfolio turnover with the t-distribution, the RW50-VaR-BL portfolio could beat the implied BL portfolio at the price of a higher portfolio turnover. Moreover, the VaR-constrained BL portfolios in the use of the DCC model and the RW50 model performed better than the benchmark portfolio with the normal distribution assumption and the t-distribution assumption.

Overall, these single-period performances indeed provided some evidences that adding the VaR constraint could improve the performance of the unconstrained implied BL portfolio and the SR-BL portfolio. In addition, these single-period performances suggested the use of the RW model and the t-distribution assumption. However, it is still necessary to evaluate average performances over multiple periods to get more reliable conclusion.

5.2.2.2 Multiple Periods Performance

In this section, I would like to analyse the performance of the VaR-constrained BL portfolio in the in-sample basis. I choose the whole sample period as firstly, the multiple periods from November 1994 to May 2010 (see Table 5.2.4 Panel A and Panel B), and the sub-period from August 1998 to May 2010 (see Table 5.2.4 Panel C and Panel D) to make a comparable analysis between the three volatility models.

From Table 5.2.4 Panel A, for the normal distribution, the DCC-VaR-BL portfolio bore the largest fat-tail risks reflected in the highest kurtosis (13.2598), highest VaR (18.95%) and highest CVaR (22.78%). Simultaneously, I can find that the risk-adjusted performance of the DCC-VaR-BL portfolio had a better performance than the other VaR-constrained BL portfolios, with the highest SR (19.22%), reward to VaR ratio (6.99%) and reward to CVaR ratio (5.81%). However, the EWMA-VaR-BL actually had the best active portfolio performance with the highest information ratio (26.65%). From Table 5.2.4 Panel B, for the t-distribution, the DCC-VaR-BL portfolio had a relatively higher kurtosis (10.7419%), the highest VaR (13.27%) and highest CVaR (16.09%) in contrast to the other VaR-constrained BL portfolios. The best risk-adjusted performance and active performance of the DCC-VaR-BL portfolio, with the highest SR (16.55%), reward to VaR ratio (5.95%) and reward to CVaR ratio (4.91%), compensated for taking greater risks. Making the comparison between the performance for the normal distribution and the performance for the t-distribution, I can observe that the change from the normal distribution to the t-distribution actually had a negative effect on multiple period performances, leading to worse performances with lower evaluation ratios in whole periods at a confidence level of 99%.

Compared with the multiple-period performance of the benchmark portfolio and the unconstrained SR-BL portfolio in the whole period in Table 5.1.18, the VaR-constrained BL portfolios perform better with both the normal distribution and the t-distribution. I can conclude that adding the VaR constraint could improve the performance of the SR-BL portfolio and beat the benchmark portfolio.

Since the VaR-constrained BL portfolio performances in the sub-period from August 1998 to May 2010 were similar to performance in the whole period, I just report the results in Table 5.2.4 without further detailed analysis. When I made a comparison with the performance of the unconstrained SR-BL portfolio and the benchmark portfolio in the sub-period, I can also reach a similar conclusion, that adding the VaR constraint could improve the performance of the SR-BL portfolio and beat the benchmark portfolio with certain conditions such as using the DCC model and the EWMA model with the normal distribution assumption.

5.2.3 Effects of VaR Constraints, Distributions and Confidence Levels

As can be seen from Table 5.2.4, the DCC-VaR-BL portfolio performed best among the VaR-constrained BL portfolios. Thus, in order to investigate the effects of VaR constraints, distributions and confidence levels specifically on the VaR-constrained BL portfolio, I would focus on studying the effects of increasingly tight levels of constraints on DCC-VaR-BL portfolios in this section.

5.2.3.1 Effects on Optimisation Model

Table 5.2.5 reports the statistics inputted in the VaR-constrained SR-BL model, such as estimated expected BL return (μ) and standard deviation (based on DCC model), and the results of ECSR, expected excess return to VaR ratio (μ / VaR) and expected excess return to CVaR ratio ($\mu / CVaR$). The decreasing scaling factor multiplied by the estimated VaR of the implied BL portfolio means the increasingly tight levels of constraints.

According to Table 5.2.5, at a confidence level of 99%, for the normal distribution and the t-distribution, the smaller the VaR factor, the tighter the VaR constraint, the lower the ECSR, μ / VaR, and $\mu / CVaR$. All ECSR were lower than the maximal SR of 10.32%. Figure 5.2.1 could interpret the decreasing tendency of the ECSR. It was because the tangent portfolio with maximal SR cannot be selected in both the normal distribution and the t-distribution that the VaR constraint continued to tighten, as can be seen in Figure 5.2.1(a), as well as the situation for the t-distribution at a confidence level of 95% in Figure 5.2.1 (b). At confidence levels of 95% and 90%, evaluation ratios began to decrease at a certain level of VaR factor. The reason was that the tangent portfolio with maximal SR can be selected as shown in Figure 5.2.1 (b) for the normal distribution and, in Figure 5.2.1 (b), in both distributions.

5.2.3.2 Effects on Weights Solutions

Table 5.2.6 shows the positions of each asset in a VaR-constrained BL portfolio in August 1998 for the normal distribution and the t-distribution, at confidence levels of 99% (Panel A), 95% (Panel B), and 90% (Panel C). In Panel A, at the

99% confidence level, the positions range for the normal distribution gradually widened from the interval of -7.06% (USA Consumer Goods) and 16.55% (USA Oil & Gas) at 0.99 VaR factor, to the interval of -9.99% (USA Industrials) and 19.19% (USA Oil & Gas) at 0.7 VaR factor. For the t-distribution, the positions range further widened to the interval of -13.31% (USA Industrials) and 21.30% (USA Oil & Gas) at 0.90 VaR factor.

At the 95% confidence level, for the normal distribution, the positions range stayed between -9.51% (UK Utilities) and 14.70% (USA Oil & Gas) until the VaR factor reduced to 0.80 to move the range upward between -8% (UK Utilities) and 15.23% (USA Oil & Gas), and then widened to the interval of -7.10% (USA Consumer Goods) and 16.39% (USA Oil & Gas). For the t-distribution, the positions range gradually widened from the interval of -7.24% (USA Consumer Goods) and 15.78% (USA Oil & Gas) to the interval of -8.93% (USA Industrials) and 18.51% (USA Oil & Gas) as the constraints tightened.

At the 90% confidence level, for the normal distribution and the t-distribution, the positions range stayed between -9.51% (UK Utilities) and 14.70% (USA Oil & Gas) until the VaR constraints tightened by a product of 0.7 and shifted upward to the range between -7.25% (USA Consumer Goods) and 15.75% (USA Oil & Gas).

Overall, the positions range for the t-distribution was wider than for the normal distribution. The higher confidence level would have the most effect on the positions range. However, the direction of long or short of the selected asset and the rank of positions were less subject to the change of distribution at the same level of confidence.

5.2.3.3 Effects on Portfolio Performance in the Single Period

Table 5.2.7 reports VaR-constrained BL portfolio performance results, including realised excess return, adjusted CSR, portfolio turnover and adjusted reward to CVaR ratio in August 1998. As can be seen, in negative realised excess return, SR would be negative; however, negative SR might be invalid to evaluate the portfolio performance. Thus, I followed Israelsen's (2003) method to adjust the SR. Adjusted CSR is equal to the product of negative realised excess return and the standard deviation multiplied by 100. Similarly, I adjusted the reward to

CVaR ratio equal to the negative realised excess return multiplied by CVaR and a constant of 100.

In August 1998, at the 99% confidence level, I observed that the lower the value of the VaR factor (the tighter the VaR constraints), the better the single period performance (the higher the adjusted CSR and reward to CVaR ratio, the lower the portfolio turnover). With the same VaR constraints, the VaR-constrained BL portfolio for the t-distribution showed a better performance than for the normal distribution. At the 95% confidence level, the VaR-constrained BL portfolio began to show a better performance after the VaR constraints tightened by the VaR factor of 0.9 for the normal distribution. The VaR-constrained BL portfolio performed better as the VaR constraints became tighter for the t-distribution. At the same level of VaR factor, the performance of the VaR-constrained BL portfolio for the t-distribution outperformed expectations with higher evaluation ratios and lower portfolio turnover. At the 90% confidence level, the performance of the VaR-constrained BL portfolio could not improve until quite restrictive VaR constraints were imposed. In Table 5.2.8, which reports VaR-constrained BL portfolio performance results in November 1998, I can reach the same conclusions as in Table 5.2.7. However, it is still necessary to investigate the effect of portfolio performance over multiple periods to get more reliable conclusion.

Compared with the implied BL portfolio and the SR-BL portfolio in Table 5.1.17, I can apparently find that adding relatively restrictive VaR constraints could significantly improve the BL portfolio performance, even beat the performance of the implied BL portfolio and the benchmark portfolio in August 1998 and November 1998. In addition, the SR-BL portfolio within a moderate level of VaR constraints can outperform the MVaR-BL portfolio and MCVaR-BL portfolio in both single periods.

5.2.3.4 Effects on Portfolio Performance in Multiple Periods

Let us move to investigate the effect of confidence levels, distribution assumptions and VaR constraints on the performance of the VaR-constrained BL portfolio over multiple periods between November 1994 and May 2010 (see Table 5.2.9) and in sub-periods between August 1998 and May 2010 (see Table 5.2.10).

As can be seen in Table 5.2.9 Panel A, with the normal distribution assumption at the 99% confidence level, as the VaR limits became tighter, the performance of the VaR-constrained BL portfolio firstly improved and then deteriorated. When the VaR factor was equal to 0.90, the constrained BL portfolio performed best, with the highest SR (19.44%), information ratio (25.90%), reward to VaR ratio (7.22%), and reward to CVaR ratio (5.98%). At the 95% confidence level, as the VaR constraints became more restrictive, the performance of the VaR-constrained BL portfolio could be enhanced by showing increasing the SR, information ratio and reward to CVaR ratio. However, the VaR-constrained BL portfolio showed a worse performance than the VaR-constrained BL portfolio at the 99% confidence level with the same VaR limits until the VaR factor reduced to 0.70; the performance of the VaR-constrained BL portfolio at the 95% confidence level overtook this to reach higher evaluation ratios. When the VaR factor was equal to 0.60, the VaR-constrained BL portfolio performed best, with SR, information ratio, reward to VaR ratio and reward to CVaR equal to 19.45%, 25.81%, 7.27% and 6.02%, respectively. At the 90% confidence level, as the VaR constraints increased, the performance of the VaR-constrained BL portfolio became better, as evaluated by SR, information ratio and reward to CVaR ratio. It showed the best performance at the value of 0.5 in the VaR factor with the SR, information ratio, reward to VaR ratio and reward to CVaR equal to 19.42%, 25.79%, 7.14% and 5.95%, respectively. However, at the same level of VaR limits (except the factor of 0.5), the VaR-constrained BL portfolio at the 90% confidence level performed worse than at the 95% confidence level. Therefore, it can be concluded that adding a moderate level of VaR constraints on the BL portfolio can improve the portfolio performance at each confidence level with the normal distribution assumption. In addition, it also showed that the higher the confidence level, the greater the impact of VaR constraints on BL portfolios to improve performance with the normal distribution assumption. Moreover, it was worth mentioning that the evaluation ratio of reward to VaR ratio might give different rankings of the BL portfolio performance when compared with other evaluation ratios with the normal distribution and the t-distribution assumption at a confidence level of 90%. This finding was consistent with the view of Alexander and Baptista (2003) that the confidence level could have an influence on performance ranking by reward to VaR ratio. It would be prudent to use the reward to VaR ratio to evaluate the portfolio performance because the VaR failed to consider the risk beyond the VaR under non-normality.

As can be seen in Table 5.2.9 Panel B, with the the t-distribution assumption, at the 99% confidence level, the performance of the VaR-constrained BL portfolio performed worse with tighter VaR limits. At the VaR factor of 0.9, the worst performance had the lowest evaluation ratios including SR (15.16%), information ratio (18.04%), reward to VaR ratio (5.32%) and reward to CVaR ratio (4.49%). At the 95% and 90% confidence levels, as the VaR limits became tighter, the active portfolio performance and risk-adjusted performance of the VaR-constrained BL portfolio firstly improved and then deteriorated with turning points at VaR factors of 0.8 and 0.6 respectively. Sometimes, SR and reward to VaR ratio can generate different rankings of portfolio performance. Specifically, for the normal distribution, at the 90% confidence level, the SR increased as the VaR factor decreased to 0.7; however, the reward to VaR ratio decreased, leading to totally different rankings. Alexander and Baptista (2003) suggested that non-normality measures should be used when the portfolio performance ranking from the reward to VaR ratios is significantly different from the ranking by SR. Nevertheless, the rankings of the portfolio performance from the reward to VaR ratio for the t-distribution still contradicted the rankings from the SR. I noticed that ranking from the reward to CVaR ratio could be consistent with SR, so the reward to CVaR ratio could also be used to evaluate the portfolio performance. Portfolio managers should be careful to use different evaluation ratios to evaluate the portfolio performance when tail risks exist in the portfolio.

Making a comparison between the normal distribution and the t-distribution, it can be found that the VaR-constrained BL portfolio actually performed worse for the t-distribution than for the normal distribution because of the negative effects of too much restrictive VaR bound for the t-distribution at a confidence level of 99%. However, at the confidence levels of 95% and 90%, the improved performance of the VaR-constrained BL portfolio for the t-distribution benefited from the positive effect of a more restrictive VaR constraint.

Compared with the implied DCC-BL portfolio in Table 5.1.18, the VaR-constrained BL portfolio underperformed. However, in contrast to the DCC-SR-BL portfolio in Table 5.1.18, the VaR-constrained BL portfolio outperformed with a much higher SR, information ratio and reward to CVaR ratio. Note that the reward to VaR ratio might distort the portfolio performance evaluation because of the limitations of VaR risk measures. In addition, when we made a comparison between the DCC-VaR-BL portfolio, the DCC-MVaR-BL portfolio and the DCC-MCVaR-BL portfolio, we can find that the DCC-VaR-BL portfolio was superior.

Table 5.2.10 shows realised VaR-constrained BL portfolio performance in the sub-period from August 1994 to May 2010. Similar to Table 5.2.9, the tendency of the improving performance of the VaR-constrained BL portfolio as the VaR constraints increased to the moderate level can also be seen in Table 5.2.10. The conclusions from Table 5.2.9 also apply to Table 5.2.10 except the conclusions related to the comparison with the SR-BL portfolio. Thus, I will not analyse or explain the VaR-constrained BL portfolio in detail as shown in Table 5.2.10, but instead focus on comparing Table 5.2.10 with Table 5.1.19. In the sub-period, the VaR-constrained BL portfolio cannot beat the implied BL portfolio, but the VaR-constrained BL portfolio can outperform the SR-BL portfolio with a higher SR and reward to CVaR ratio. In addition, most of the VaR-constrained BL portfolios could perform better than the MVaR-BL portfolios and MCVaR portfolios with the normal distribution assumption.

5.2.4 Conclusions

In the in-sample analysis, the main finding of this section is that adding acceptable levels of VaR constraints on the SR-BL portfolio could improve the realised performance of the SR-BL portfolio in the single period and over multiple periods. I have shown some evidences that the VaR-constrained BL portfolio, especially for the t-distribution and based on the three different volatility models could even overtake the implied BL portfolio, the MVaR-BL portfolio and the MCVaR-BL portfolio in the single period and over multiple periods.

The choice of volatility models, distributions and confidence levels has different influences on weights solutions and performances in the VaR-constrained BL portfolio. In some single periods, the use of the EWMA model tends to allocate assets most aggressively with the widest position range. The change from the normal distribution to the t-distribution could widen the positions range in the

DCC-VaR-BL portfolio and the EWMA-VaR-BL portfolio. The positions range in the RW50-VaR-BL portfolio is insensitive to the distribution assumption. The choice of the t-distribution assumption would have more impact on weights solutions based on the DCC model and the EWMA model at a confidence level of 99% in the VaR-constrained portfolio. The DCC-VaR-BL portfolio allocates most volatile weights for the normal distribution and then allocates less volatile weights for the t-distribution. The single period performance is not consistent with the multiple-period performance. In the single period, the higher confidence level would have a greater impact on VaR constraints on the DCC-BL portfolio to improve performance with the normal distribution assumption. The RW50-VaR-BL portfolio could show a better performance in both distributions than other VaR-constrained portfolios. In addition, the change from the normal distribution to the t-distribution could improve performance in the DCC-VaR-BL portfolio and the RW50-VaR-BL portfolio. However, these conclusions from the single-period performance might not be reliable without supplement of the multiple-period performance. Over multiple periods, the risk-adjusted performance of the DCC-VaR-BL portfolio actually has better performance than other VaR-constrained BL portfolios; the EWMA-VaR-BL portfolio has the best active portfolio for the normal distribution. For the t-distribution, the DCC-VaR-BL portfolio shows the best risk-adjusted performance and active performance but takes greater risks.

5.3 Conditional Value-at-Risk-Constrained Black-Litterman Portfolio

As can be concluded from the results of the unconstrained BL portfolio performance, the SR-BL portfolios have larger CVaR than the benchmark portfolio and the implied BL portfolio. The tail risk reflected by negative skewness and high kurtosis should be considered in the construction of a portfolio. In addition, using the reward to VaR ratio to rank performance might mislead the portfolio manager into choosing the portfolio that actually performs worse, as interpreted in Section 5.2.3.4. Motivated by the better properties of CVaR, which considers risks beyond VaR, I will impose the CVaR constraint on the unconstrained SR-BL portfolio. The constraint is set to be the scaling factor k multiplied by CVaR0, which is the CVaR of the implied BL portfolio. In the dynamic environment, CVaR0 is time-varying. k is equal to 0.99, 0.95, 0.90, and 0.80 and reduces sequentially until the SR-BL portfolio unbinds.

5.3.1 Construction of the CVaR-Constrained BL Portfolio

In the empirical study of CVaR constraints, the distribution assumptions and the confidence levels are important factors to take into account. Before we start to construct the CVaR-constrained BL portfolio, the first important thing is to understand the efficient frontier. In Section 5.3.1.1, I will display the CVaR-constrained BL portfolio frontier and explain the optimisation process.

5.3.1.1 CVaR-Constrained BL Portfolio Frontier

Figure 5.3.1 and Figure 5.3.2 plot the VaR constraints and CVaR constraints on the BL portfolio frontier for the normal distribution and the t-distribution, respectively, at confidence levels of 99%, 95% and 90% as k equal to 0.99. Note that the constraint is equal to $k * CVaR_0$, where $CVaR_0$ is the estimated CVaR of the implied BL portfolio. $CVaR_0$ is equal to -11.93% in August 1998. The green line represents the VaR constraint, and the purple line represents the CVaR constraint. The black point M is the minimum variance portfolio, and the red square point T is the tangent portfolio that has the maximum SR. The left direction of the black arrow means the VaR bound and CVaR bound decreased as k reduces.

Similar to VaR constraints, Alexander and Baptista (2004) illustrate that the portfolios that meet a CVaR constraint should be on or above a line with intercept $-k * CVaR_0$ and slope ζ_β (see Chapter 4, equation (4.19)) in the mean-standard deviation frontier. Therefore, the selected portfolio should be on the efficient frontier from the point M to the intersection point of the line and the efficient frontier. When the value k decreases, the intercept of this line increases, and the slope ζ_β increases when the confidence level increases; the CVaR constraint would tighten. Setting the same $k * CVaR_0$, the slope in the lower confidence level of 90% (Figure 5.3.1 (c)) is apparently smaller than in the higher confidence level of 99% (Figure 5.3.1 (a)); at the same confidence level, the slope for the t-distribution (Figure 5.3.2) is much higher than for the normal distribution (Figure 5.3.1). Therefore, CVaR constraints at a higher confidence level or for the t-distribution are tighter, and become tighter as k reduces. Since $\zeta_\beta > \xi_\beta$, the CVaR constraint, has a larger slope than the VaR constraint, giving rise to a more restrictive constraint, as can be shown in Figure 5.3.1 and Figure

5.3.2, that the line of the CVaR constraint with a larger slope is at the left of the VaR constraint.

When k is equal to 0.99, in Figure 5.3.1(a) and Figure 5.3.2 (a), the tangent portfolio with maximal SR cannot be selected in both the normal distribution and the t-distribution at the 99% confidence level with both VaR and CVaR constraint. At the 95% confidence level, the tangent portfolio can be selected for the normal distribution (Figure 5.3.1 (b)) but omitted for the t-distribution (Figure 5.3.2 (b)). At the 90% confidence level, the tangent portfolio can be selected within both the VaR and CVaR constraint in Figure 5.3.1(c) for the normal distribution, however, the tighter CVaR constraint would exclude the choice of the tangent portfolio but the VaR constraint would still include the tangent portfolio for the t-distribution.

5.3.1.2 Weights of CVaR-Constrained BL Portfolios

In this part, I will show the weights allocated to the CVaR-constrained BL portfolio in August 1998 (Table 5.3.1). With the normal distribution assumption, the position in the DCC-CVaR-BL portfolio ranged between -7.09% (USA Consumer Goods) and 16.56% (USA Oil & Gas); the position in the RW50-CVaR-BL portfolio had a much wider range of between -11.53% (USA Industrials) and 21.92% (Japan Industrials), and the position in the EWMA-CVaR-BL portfolio had the widest range within the interval -27.07% (USA Industrials) and 42.58% (USA Oil & Gas). When the normal distribution assumption changed to the t-distribution assumption, the position range in the DCC-CVaR-BL portfolio widened to the interval between -15.80% (USA Industrials) and 27.03% (USA Consumer Services). Followed by the EWMA-CVaR-BL portfolio which also had a wider position range of between -38.45% (USA Industrials) and 64.33% (Japan Industrials), the positions range in the RW50-CVaR-BL portfolio increased to the widest range of between -58.65% (USA Industrials) and 81.30% (Japan Industrials). In addition, the average value of the absolute difference average absolute value of the change of the weights between the normal distribution and the t-distribution were 9.20% (DCC model), 10.23% (EWMA model) and 17.72% (RW50 model) in August 1998. Thus, the impact of the change of distribution assumption on the positions range in the CVaR-BL portfolios was greater in the use of the EWMA model and the RW50 model, which also generated much a wider range than the use of the DCC model.

Compared with the VaR-BL portfolio in August 1998, the position range in the DCC-CVaR-BL portfolio stayed nearly in the same range, while the position range in the EWMA-CVaR-BL shifted downward by about 76 basis points and the position range in the RW50-CVaR-BL shifted downward by about 32 basis points for the normal distribution. However, for the t-distribution, the position range in the CVaR-BL portfolio was about 10.73% wider than the VaR-BL portfolio based on the DCC model, and about 7.33% wider and 106.80% wider in the use of the EWMA model and the RW50 model, respectively. Undoubtedly, compared with the implied BL portfolio and the SR-BL portfolio in August 1998, the CVaR-BL portfolio would allocate assets with a much wider position range.

In November 1998, for the normal distribution, the positions in the DCC-VaR-BL portfolio ranged between -4.42% (UK Consumer Services) and 17.65% (USA Health Care), and the relative wider positions range in the EWMA-VaR-BL portfolio were between -6.44% (UK Consumer Services) and 19.41% (USA Health Care), and the narrowest positions range was in the RW50-VaR-BL portfolio within the interval from -2.15% (UK Consumer Goods) and 16.99% (USA Health Care). For the t-distribution, the positions in the DCC-VaR-BL portfolio ranged between -10.02% (Japan Technology) and 21.00% (Japan Consumer Services), and the relative wider positions range in the EWMA-VaR-BL portfolio was between -23.02% (USA Industrials) and 52.08% (Japan Industrials); the widest positions range was in the RW50-VaR-BL portfolio, within the interval from -57.18% (USA Industrials) and 61.12% (Japan Industrials). In addition, the average absolute value of the change of the weights between the normal distribution and the t-distribution were 6.59% (DCC model), 9.79% (EWMA model) and 17.33% (RW model).

Compared with the VaR-BL portfolio in November 1998, for the normal distribution, the position range in the DCC-CVaR-BL portfolio stayed nearly in the same range, while the position range in the EWMA-CVaR-BL portfolio was slightly wider by about 28 basis points, and the position range in the RW50-CVaR-BL portfolio was wider by about 2.39%. However, for the t-distribution, the DCC-CVaR-BL portfolio increased the width of the position range by 8.03%, the EWMA-CVaR-BL portfolio increased the width of the position range by a higher value of 18.60%, and the RW50-CVaR-BL portfolio substantially increased the width of the position range by 102.20%.

Therefore, I can find that both the EWMA-CVaR-BL portfolio and the RW50-CVaR-BL portfolio might allocate extreme weights to some assets with a much wider position range for the t-distribution than for the normal distribution. The position range in the DCC-CVaR-BL portfolio would also be slightly wider for the t-distribution than for the normal distribution. In addition, I can conclude that the normal distribution assumption in the CVaR-BL portfolio would not result in a large difference in the position range from the VaR-BL portfolio; however, the t-distribution assumption has the impact of widening the position range in the CVaR-BL portfolio, especially in the use of the EWMA model and the RW model. According to Appendix 5.3.1, the conclusions are similar. The portfolio manager needs to be cautious about imposing CVaR constraints on a BL portfolio based on the EWMA model and the RW model to allocate assets. In addition, from Appendix 5.3.2, it can be found that the DCC-CVaR-BL portfolio would generate most volatile weight solutions for the normal distribution and least volatile weight solutions for the t-distribution compared with other CVaR-BL portfolios. Moreover, the change from the normal distribution to the t-distribution could make weight solutions less volatile in the use of the DCC model and more volatile in the use of the EWMA model and the RW model.

5.3.2 Performance Evaluation

After I analysed the impact of different volatility models and distribution assumptions on weights allocation in the CVaR-BL portfolio in Section 5.3.1, I would investigate the CVaR-BL portfolio performance in single period and over multiple periods in this section.

5.3.2.1 Single Period Performance

Table 5.3.3 shows the CVaR-constrained BL portfolio performance in August 1998 and November 1998 with assumption of the normal distribution and the t-distribution at a confidence level of 99%.

In August 1998, as can be seen in Table 5.3.3 Panel A, the realised returns were negative; adjusted CSR and adjusted reward to CVaR ratio were used to rank the portfolio performance. The RW50-CVaR-BL portfolio showed the best performance with the highest adjusted evaluation ratios, followed by the DCC-CVaR-BL portfolio and then the EWMA-CVaR-BL, which performed the worst. The portfolio turnover in the EWMA-CVaR portfolio with the value of 5.1408 for the normal distribution and 2.4236 for the t-distribution was higher than the DCC-CVaR portfolio with the value of 3.2105 for the normal distribution and 1.6783 for the t-distribution. The change from the normal distribution to the t-distribution improved the portfolio performance with higher adjusted evaluation ratios and lower portfolio turnover.

In November 1998, the realised returns of the CVaR-BL portfolio became positive. Similar to the rank of portfolio performance in August 1998, the RW50-CVaR-BL portfolio performed best with the highest CSR and reward to CVaR ratio, followed by the DCC-CVaR-BL portfolio and then the EWMA-CVaR-BL, which performed worst. The values of portfolio turnover for the normal distribution also gave the same rank, but gave an inverse rank for the t-distribution. The change from the normal distribution to the t-distribution improved the DCC-CVaR-BL portfolio performance with higher evaluation ratios and lower portfolio turnover. The change from the normal distribution to the t-distribution improved the RW50-CVaR-BL portfolio performance to reach higher evaluation ratios at the cost of higher portfolio turnover. The EWMA-CVaR-BL portfolio failed to get any benefit from the the t-distribution assumption, with worse portfolio performance.

Overall, these single-period performances indeed provided some evidences that adding the CVaR constraint could improve the performance of the unconstrained implied BL portfolio and the SR-BL portfolio. For the normal distribution, the performance of the CVaR-constrained BL portfolio was similar to that of the VaR-constrained BL portfolio except that the use of the RW50 model would mean the deterioration of the performance of the CVaR-constrained BL portfolio. For the t-distribution, the performance of the CVaR-constrained BL portfolio was better than that of the VaR-constrained BL portfolio in August 1998; however, the CVaR-constrained BL portfolio cannot beat the VaR-constrained BL portfolio in November 1998. Thus, we cannot be sure if the CVaR-constrained BL portfolio always performs better by evaluating the portfolio performance in a single period. It is necessary to evaluate the portfolio performance over multiple periods.

5.3.2.2 Multiple Periods Performance

In this section, I will evaluate the CVaR-constrained BL portfolio performance and compare the difference among the use of the three different volatility models over multiple periods from November 1994 to May 2010 (see Table 5.3.4, Panel A and Panel B) and the sub-period from August 1998 to May 2010 (see Table 5.3.4, Panel C and Panel D).

As can be seen in Table 5.3.4, Panel A, for the normal distribution, the DCC-CVaR-BL portfolio performed best with the highest SR of 19.21%, reward to VaR ratio of 7.00%, and reward to CVaR ratio of 5.82%. The EWMA-CVaR-BL portfolio showed the second best performance with SR, reward to VaR ratio and reward to CVaR ratio equal to 17.85%, 6.62%, and 4.98%, respectively. The RW50-CVaR-BL portfolio performed worst with the lowest evaluation ratios. However, CVaR-BL portfolios displayed an inverse rank in active portfolio performance evaluated by the information ratios. For the t-distribution (see Panel B), the risk-adjusted performance and the active performance of the CVaR-BL portfolio can be ranked in the order of DCC-CVaR-BL portfolio, EWMA-CVaR-BL portfolio and RW50-CVaR-BL portfolio. The CVaR-BL portfolio for the t-distribution performed worse than the CVaR-BL portfolio for the normal distribution, especially in the use of the RW50 model. Compared with the VaR-BL portfolios, most of the CVaR-BL portfolios underperformed except the RW50-CVaR-BL portfolio for the normal distribution which could beat the corresponding VaR-BL portfolio. It was because the bigger CVaR bound at the high confidence level of 99% was too restrictive to impose a positive effect on improving CVaR-BL portfolio performance. Compared with the multiple-period performance of the benchmark portfolio and the unconstrained SR-BL portfolio in the whole period in Table 5.1.18, the CVaR-constrained BL portfolios performed better for the normal distribution, and only the EWMA-CVaR-BL portfolio could beat the implied EWMA-BL portfolio.

In the sub-period from August 1998 to May 2010, similar to the performance in the whole period, the DCC-CVaR-BL portfolio behaved best, followed by the EWMA-CVaR-BL portfolio and then the RW50-CVaR-BL portfolio. The CVaR-

BL portfolio performed better for the normal distribution than for the t-distribution. Compared with the multiple period performance of the benchmark portfolio and the unconstrained SR-BL portfolio in the whole period in Table 5.1.19, both the DCC-CVaR-BL portfolios and the EWMA-CVaR-BL portfolio for the normal distribution performed better.

5.3.3 Effects of CVaR Constraints, Distributions and Confidence Levels

According to Table 5.3.4, the DCC-VaR-BL portfolio showed the best performance among the VaR-constrained BL portfolios, therefore, in this section, I would concentrate on studying effects of distributions and confidence levels on the DCC-VaR-BL portfolio at increasing tightness levels of constraints.

5.3.3.1 Effects on Optimisation Model

Table 5.3.5 shows the statistics of the inputs and outputs of the optimisation model based on increasing CVaR constraints, two distribution assumptions and three different confidence levels. According to Table 5.3.5, Panel A with the normal distribution, at the 99% confidence level, the ECSR and reward to alternative risk ratios decreased as the CVaR constraints increased, because the choice of portfolio was all below the optimal point with the tighter CVaR constraints. Figure 5.3.1 also reflected this relationship. At the 95% confidence level, the ECSR stayed at the highest ratio of 10.32% from 0.99 CVaR factor to 0.90 CVaR factor, and expected reward to VaR ratio and reward to CVaR ratio stayed at 6.69% and 5.27%, respectively. When the CVaR factor kept decreasing, the CVaR limits became tighter, and the expected evaluation ratios inevitably reduced because the CVaR constraints moved away from the optimal point with maximal SR, as can be seen in Figure 5.3.1 (b). At a confidence level of 90%, the selected optimal portfolio with maximal SR did not change because the CVaR constraints were above the optimal point T in Figure 5.3.1 (c) until the CVaR factor reduced close to 0.6. As the CVaR factor decreased from 0.6, the evaluation ratios decreased.

According to Table 5.3.5, Panel B, with the t-distribution, at a confidence level of 99%, the CVaR constraints were too tight to choose the portfolio with maximal expected SR. When the CVaR factor was larger than 0.90, the selected portfolio was close to the minimum variance portfolio; when the CVaR factor was smaller than 0.90, the CVaR constraint was unbounded to find appropriate weight solutions (see Figure 5.3.2 (a)). At confidence levels of 95% and 90%, the CVaR constraints were not loose enough to contain the optimal portfolio with maximal SR (see Figure 5.3.2 (b) and Figure 5.3.2 (c)), therefore, the expected evaluation ratios would decrease as the CVaR constraints tightened.

Compared with the VaR-constrained BL portfolio optimisation statistics in Table 5.2.5, it can be found that the CVaR-constrained BL portfolio optimisation produced the same results as the VaR-constrained BL portfolio for the normal distribution, but generated portfolios with a smaller standard deviation and lower expected evaluation ratios.

5.3.3.2 Effects on Weight Solutions

Section 5.3.3.1 displayed the statistics for the CVaR-constrained BL portfolios optimisation. In Section 5.3.3.2, I showed the weight solutions of the CVaR-constrained portfolios, and analysed the effect of using different distribution assumptions, different confidence levels and decreasing CVaR constraints on weight solutions.

Table 5.3.6 shows the positions of each asset in the CVaR-constrained BL portfolio in August 1998 under the normal distribution and the t-distribution at a confidence level of 99% (Panel A), 95% (Panel B), and 90% (Panel C). In Panel A, at 99% confidence level, the positions range for the normal distribution gradually widened from the interval of -7.09% (USA Consumer Goods) and 16.56% (USA Oil & Gas) at 0.99 CVaR factor to the interval of -9.97% (USA Industrials) and 19.20% (USA Oil & Gas) at 0.7 CVaR factor. For the t-distribution, the positions range further widened to the interval of -15.80% (USA Industrials) and 22.62% (USA Consumer Services) at 0.99 CVaR factor and shift upward to the interval of -10.25% (USA Consumer Goods) and 29.85% (USA Consumer Services).

At 95% confidence level, for the normal distribution, the positions stayed in the range between -9.54% (UK Utilities) and 14.71% (USA Oil & Gas) until the CVaR factor reduced to 0.80 to move the range upward between -7.20% (USA Consumer Goods) and 16.09% (USA Oil & Gas), and then towards the interval of -6.95% (USA Consumer Goods) and 17.19% (USA Oil & Gas). For the t-distribution, the positions range gradually widened from the interval of -8.13% (USA Industrials) and 18.04% (USA Oil & Gas) to the interval of -12.41% (USA Industrials) and 20.75% (USA Oil & Gas) as the constraints tightened.

At 90% confidence level, for the normal distribution, the positions stayed in the range between -9.54% (UK Utilities) and 14.71% (USA Oil & Gas) until the VaR constraint tightened by a product of 0.7, shifted upward to the range between -7.22% (USA Consumer Goods) and 15.76% (USA Oil & Gas). For the t-distribution, the positions range gradually widened from the interval of -7.14% (USA Consumer Goods) and 15.98% (USA Oil & Gas) at 0.90 CVaR factor to the interval of -9.16% (USA Industrials) and 18.71% (USA Oil & Gas) as the constraints tightened.

Compared with Table 5.2.6, related to the VaR-BL portfolio weights solutions, most of the weight solutions in the CVaR-BL portfolio were slightly different for the normal distribution, leading to the same statistical results in optimisation in Table 5.3.5, Panel A. However, for the t-distribution, the CVaR-BL portfolio allocated each asset with very different positions but the same long and short direction at the 99% confidence level and the 90% confidence level. At the same confidence level, the position range in the CVaR-BL portfolio was wider than the VaR-BL portfolio position range for the t-distribution because of more restrictive CVaR constraints than VaR constraints.

Overall, in the CVaR-constrained BL portfolio, the positions range for the t-distribution was wider than for the normal distribution, a higher confidence level would have the effect of widening the positions range. Since the the t-distribution and the higher confidence level would lead to more restrictive CVaR constraints, we can conclude that the use of a high level of CVaR constraints tightness would have the tendency of widening the position range. However, the direction of long or short of the selected asset and the rank of positions are less subject to the change of distribution at same level of confidence.

5.3.3.3 Effects on Portfolio Performance in the Single Period

In Section 5.3.3.2, I have figured out the weight solutions of the CVaR-BL portfolios. The next important task, in Section 5.3.3.3, is to evaluate the realised

CVaR-BL portfolio performance and investigate the effect of CVaR constraints, distributions, and confidence levels on portfolio performance in the single period (see Table 5.3.7 and Table 5.3.8) and over multiple periods (see Table 5.3.9 and Table 5.3.10).

In August 1998, the realised expected excess return was negative; the performance evaluation may not have much validity in the use of the traditional SR. Therefore, I followed Israelsen's (2003) method to calculate the adjusted SR and adjusted reward to CVaR ratio in the same way. I also computed portfolio turnover to measure the possible transaction cost.

Table 5.3.7 reports the results of portfolio performance evaluation in August 1998 with the normal distribution assumption and the t-distribution assumption at confidence levels of 99%, 95% and 90% as the CVaR constraints increased. At the 99% confidence level, for the normal distribution, the realised CVaR-BL portfolio performance became better with increasing adjusted CSR and adjusted reward to CVaR ratios and decreasing portfolio turnover as the increasing CVaR constraints were imposed. However, the extremely tight CVaR constraints had a negative effect on improving CVaR-BL portfolio performance for the t-distribution. At confidence levels of 95% and 90%, the realised CVaR-BL portfolio did not improve performance until the CVaR constraint increased to a certain level for the normal distribution. Nevertheless, for the t-distribution, the tighter CVaR constraints on the CVaR-BL portfolio could generate a better performance with higher evaluation ratios and lower portfolio turnover. In addition, the CVaR-BL portfolio for the t-distribution performed better than for the normal distribution with the same CVaR bound at the same confidence level. In November 1998 (see Table 5.3.8), the realised excess return was positive; I can also find that imposing more restrictive CVaR constraint could improve single period performance, as reflected in August 1998.

Compared with the VaR-BL portfolio in Table 5.2.7 and Table 5.2.8, the CVaR-BL portfolio performed better with constraints that were more restrictive. In contrast to the implied BL portfolio, some CVaR-BL portfolios with relatively tighter constraints could beat the implied BL portfolio. In addition, the SR-BL portfolio within a moderate level of CVaR constraints can outperform the MVaR-BL portfolio and MCVaR-BL portfolio in both single periods.

5.3.3.4 Effects on Portfolio Performance in Multiple Periods

I have found that a higher confidence level and the t-distribution would increase CVaR limits, and imposing a moderate level of CVaR limits on an SR-BL portfolio could improve realised portfolio single period performance. I would investigate whether imposing CVaR limits on an SR-BL portfolio could improve realised portfolio performance over multiple periods, and the effect of CVaR limits, distribution assumptions and confidence levels on performance in the whole multiple period between November 1994 and May 2010 (see Table 5.3.9) and the sub-period between August 1998 and May 2010 (see Table 5.3.10).

According to Table 5.3.9, Panel A, for the normal distribution, as the CVaR became tighter at 99% confidence level, the performance of the CVaR-BL portfolio first slightly improved to achieve the highest evaluation ratios, including SR (19.44%), information ratio (25.90%), reward to VaR ratio (7.23%) and reward to CVaR ratio (5.98%) at a CVaR factor of 0.90. Then it deteriorated to the worst performance with SR, information ratio, reward to VaR ratio and reward to CVaR ratio equal to 14.40%, 15.83%, 4.93% and 4.28%, respectively. Similarly, at confidence levels of 95% and 90%, the evaluation ratios of the CVaR-BL portfolio gradually climbed to the highest level when the CVaR factor reduced to 0.7 and 0.6 and continued to perform badly as the CVaR factor kept decreasing. For the t-distribution, as can be seen in Table 5.3.9, Panel B, the increasing CVaR constraints cannot improve the CVaR-constrained BL portfolio performance at confidence levels of 99% and 95% because the CVaR constraints were too tight to have a positive effect on performance with a higher confidence level. When the confidence level lowered to 90%, the performance of the CVaR-constrained BL portfolio for the t-distribution was similar to the performance for the normal distribution at 95% confidence level; the increasing CVaR limits first showed a positive effect on performance with increasing evaluation ratios before the CVaR factor decreased to 0.8 and then showed a negative effect on performance. At the same confidence level of 99%, the CVaR-BL portfolio performed worse for the t-distribution than for the normal distribution, resulting from overly restrictive constraints for the t-distribution; however, at confidence levels of 95% and 90%, the CVaR-BL portfolio could perform better for the t-distribution than for the normal distribution at relatively higher CVaR factors. Moreover, the evaluation ratio of reward to VaR ratio might give different rankings to the CVaR-BL portfolio performance compared with other evaluation ratios with the normal distribution and the t-distribution assumption at a confidence level of 90%. Caution should be exercised in using the reward to VaR ratio to evaluate the portfolio performance because the VaR failed to consider the risk beyond the VaR.

In contrast to the VaR-BL portfolio in Table 5.2.9, the CVaR-BL portfolio showed nearly the same performance for the normal distribution at a 99% confidence level. Most of the CVaR-BL portfolios could slightly outperform the VaR-BL portfolio for the normal distribution at 95% and 90% confidence levels at constraints factor higher than 0.7, but the CVaR-BL portfolio could underperform the VaR-BL portfolio when the constraints factor decreased lower than 0.7. For the t-distribution, the CVaR-BL portfolio failed to overtake the VaR-BL portfolio at the 99% confidence level; however, the CVaR-BL could be superior to the VaR-BL portfolio at the 95% confidence level when the constraints factor decreased from 0.99 to 0.90; after the constraint factor kept decreasing from 0.8 to 0.6, the VaR-BL portfolio showed a better performance than the CVaR-BL portfolio. Similarly, at the 90% confidence level, there was a turning constraint factor of 0.6 to show that the CVaR-BL portfolio could be overcome by the VaR-BL portfolio.

Let us compare with the SR-BL portfolio, MVaR-BL portfolio and MCVaR-BL portfolio in Table 5.1.18. When I made a comparison between the CVaR-BL portfolio and the SR-BL portfolio, I found that adding a moderate level of CVaR constraints on the SR-BL portfolio could significantly improve the SR-BL portfolio performance. Moreover, in contrast to the MVaR-BL portfolio and the MCVaR-BL portfolio, the CVaR-constrained BL portfolio performed best with both distributions at all three confidence levels except the MCVaR-BL portfolio with the t-distribution at the 99% confidence level.

Table 5.3.10 reports the realised CVaR-constrained BL portfolio performance in the sub-period from August 1994 to May 2010. Similarly to Table 5.3.9, the tendency of improving performance of CVaR-constrained BL portfolio as the CVaR constraint increased to a moderate level can also be seen in Table 5.3.10. The conclusions from Table 5.3.9 also applied to Table 5.3.10. Thus, I would not analyse and explain CVaR-constrained BL portfolio in detail in Table 5.3.10, but focus on comparing Table 5.3.10 with Table 5.1.19. In the sub-period, the CVaR-constrained BL portfolio cannot beat the implied BL portfolio, but the CVaR-constrained BL portfolio can outperform the SR-BL portfolio with a higher SR, information ratio and reward to CVaR ratio. In addition, most of the CVaR-constrained BL portfolios could perform better than the MVaR-BL portfolios and the MCVaR portfolios for the normal distribution.

5.3.4 Conclusions

In the in-sample analysis, the findings from the CVaR-constrained BL portfolio are similar to the findings from the VaR-constrained BL portfolio as an outperformer, in contrast to unconstrained BL portfolios and the benchmark portfolio. Furthermore, several CVaR-constrained BL portfolios could show an even better performance than the VaR-constrained BL portfolio in single period and multiple periods at a moderate level of CVaR constraint, which is more restrictive than the VaR constraint.

Similarly, the choice of volatility models, distributions, and confidence levels also has different effects on weights solutions and performances in the CVaR-constrained BL portfolio. CVaR constraints at a higher confidence level or for the t-distribution are tighter, and are tighter as k reduces. Based on the EWMA model and the RW model, there is an obvious tendency that the position range in the CVaR-constrained BL portfolio would widen as the CVaR constraints tighten, while the position range in the DCC-CVaR-BL portfolio is slightly sensitive. However, the direction of long or short of the selected asset and the rank of positions are less subject to the change of distribution at the same level of confidence. In some single periods, the RW50-CVaR-BL portfolio might show the best performance, and the change from the normal distribution to the t-distribution could improve the DCC-CVaR-BL portfolio and the RW50-CVaR-BL portfolio performance at the price of carrying inscrutable portfolio turnover. We cannot be sure that the CVaR-constrained BL always performs better for the t-distribution in different single periods. Over multiple periods, risk-adjusted performance and active performance of the CVaR-constrained BL portfolio are not always consistent. The DCC-CVaR-BL portfolio only performs best in risk-adjusted performance for the normal distribution but also could perform best in active performance for the t-distribution at a moderate level of CVaR constraints.

Imposing tightening CVaR constraints on an SR-BL portfolio would have a ‘diminishing effect’ on improving the multiple period performance, which first improves with tighter limits and then deteriorates as the limits become too tight.

Table 5.1.1 Benchmark Portfolio Performance and Tail Risk

This table reports the summary statistics of the benchmark portfolio performance from January 1994 to May 2010. This table also shows the estimated VaR and CVaR of the portfolio at different confidence levels (10%, 5%, 2.5%, 1%), with assumptions of normal distribution and t-distribution.

Panel A:	*Performance Evaluation*					
Expected Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Empirical VaR	Empirical CVaR
0.0008	0.0430	-0.9079	4.9253	0.0182	0.1285	0.1673

Panel B:	*Estimated Tail Risk*		
Normal Distribution:			
VaR(0.99)	0.0993	CVaR(0.99)	0.1139
VaR(0.975)	0.0835	CVaR(0.975)	0.0998
VaR(0.95)	0.0700	CVaR(0.95)	0.0880
VaR(0.90)	0.0543	CVaR(0.90)	0.0747
t-Distribution:			
VaR(0.99)	0.1604	CVaR(0.99)	0.2238
VaR(0.975)	0.1187	CVaR(0.975)	0.1710
VaR(0.95)	0.0909	CVaR(0.95)	0.1370
VaR(0.90)	0.0652	CVaR(0.90)	0.1067

Table 5.1.2 Risk Aversion Coefficient and Implied Equilibrium Return in August 1998

This table reports the risk aversion coefficient δ (Panel A) and implied equilibrium return of each index $\boldsymbol{\pi}$ (Panel B) in August 1998. $\delta = \frac{E(r_M) - E(r_f)}{\sigma_M^2}$, the numerator is market risk premium and the denominator is market variance. $\boldsymbol{\pi} = \delta \mathbf{H}\mathbf{w}$, where δ is the risk aversion coefficient, $\mathbf{H}$ is the conditional covariance matrix in the use of the RW model with a window length of 50, the EWMA model and the DCC model, $\mathbf{w}$ is the market capitalisation weight of each index.

Panel A: Risk Aversion Coefficient

	DCC	EWMA	RW50
Risk Aversion Coefficient	2.2166	1.3004	3.5373

Panel B: Implied Equilibrium Return

	DCC	EWMA	RW50
UK BASIC MATS	0.0041	0.0020	0.0018
UK CONSUMER GDS	0.0036	0.0013	0.0018
UK CONSUMER SVS	0.0027	0.0017	0.0018
UK FINANCIALS	0.0032	0.0027	0.0031
UK HEALTH CARE	0.0018	0.0017	0.0022
UK TECHNOLOGY	0.0051	0.0026	0.0026
UK INDUSTRIALS	0.0040	0.0022	0.0010
UK OIL & GAS	0.0030	0.0024	0.0022
UK TELECOM	0.0036	0.0019	0.0014
UK UTILITIES	0.0018	0.0011	0.0022
USA BASIC MATS	0.0036	0.0029	0.0031
USA CONSUMER GDS	0.0030	0.0037	0.0025
USA CONSUMER SVS	0.0029	0.0031	0.0028
USA FINANCIALS	0.0033	0.0047	0.0036
USA HEALTH CARE	0.0022	0.0029	0.0029
USA INDUSTRIALS	0.0034	0.0032	0.0035
USA OIL & GAS	0.0026	0.0022	0.0022
USA TECHNOLOGY	0.0043	0.0042	0.0044
USA TELECOM	0.0025	0.0021	0.0021
USA UTILITIES	0.0019	0.0007	0.0016
JAPAN BASIC MATS	0.0023	0.0017	0.0018
JAPAN CONSUMER GDS	0.0026	0.0024	0.0030
JAPAN CONSUMER SVS	0.0018	0.0015	0.0021
JAPAN FINANCIALS	0.0032	0.0036	0.0035
JAPAN HEALTH CARE	0.0017	0.0011	0.0023
JAPAN INDUSTRIALS	0.0026	0.0018	0.0023
JAPAN OIL & GAS	0.0028	0.0021	0.0025
JAPAN TECHNOLOGY	0.0031	0.0027	0.0034
JAPAN TELECOM	0.0017	0.0019	0.0024
JAPAN UTILITIES	0.0007	0.0003	0.0012

Table 5.1.3 The Views Portfolio Weights, Expected Return and Confidence Variance in August 1998

This table reports the view portfolio weights ($\mathbf{P}$), the view portfolio expected return ($\mathbf{q}$), and the confidence variance ($\mathbf{\Omega}$) in August 1998, based on three volatility models including the DCC model, the EWMA model and RW model with a window length of 50. The view portfolio is constructed by the momentum strategy and translated into the BL model, following the method of Fabozzi et al. (2006).

Panel A: The View Portfolio Weights ($\mathbf{P}$)

	DCC	EWMA	RW50
UK BASIC MATS	-0.0911	-0.1284	-0.1832
UK CONSUMER GDS	-0.0926	-0.1132	-0.1360
UK CONSUMER SVS	0.1764	0.1900	0.2314
UK FINANCIALS	-0.1553	-0.1401	-0.1807
UK HEALTH CARE	0.1905	0.2019	0.2369
UK TECHNOLOGY	0.0681	0.0705	0.0688
UK INDUSTRIALS	0.1017	0.1129	0.1718
UK OIL & GAS	-0.1331	-0.1471	-0.1936
UK TELECOM	0.1016	0.1300	0.1762
UK UTILITIES	0.1798	0.1824	0.1755
USA BASIC MATS	-0.1386	-0.1447	-0.2007
USA CONSUMER GDS	0.1649	0.1170	0.2083
USA CONSUMER SVS	0.1950	0.1517	0.2418
USA FINANCIALS	0.1652	0.1008	0.1841
USA HEALTH CARE	0.2124	0.1579	0.2291
USA INDUSTRIALS	0.1686	0.1505	0.2179
USA OIL & GAS	-0.1580	-0.1656	-0.2196
USA TECHNOLOGY	0.1162	0.1002	0.1328
USA TELECOM	0.1673	0.1623	0.2173
USA UTILITIES	0.1767	0.2190	0.2357
JAPAN BASIC MATS	-0.1404	-0.0885	-0.1125
JAPAN CONSUMER GDS	-0.1498	-0.1456	-0.1523
JAPAN CONSUMER SVS	-0.1656	-0.1407	-0.1552
JAPAN FINANCIALS	-0.0986	-0.0785	-0.1033
JAPAN HEALTH CARE	-0.1638	-0.1733	-0.1835
JAPAN INDUSTRIALS	-0.1535	-0.1406	-0.1568
JAPAN OIL & GAS	-0.0792	-0.0749	-0.0983
JAPAN TECHNOLOGY	-0.1256	-0.1057	-0.1214
JAPAN TELECOM	-0.1517	-0.1421	-0.1515
JAPAN UTILITIES	0.1653	0.1974	0.1879

Panel B: Expected Return of the View Portfolio ($\mathbf{q}$)

	DCC	EWMA	RW50
Expected Return	-0.0620	-0.0862	-0.0990

Panel C: Confidence Variance of the View Portfolio ($\mathbf{\Omega}$)

	DCC	EWMA	RW50
Confidence Variance	0.0047	0.0050	0.0110

Table 5.1.4 The Views Portfolio Weights, Expected Return and Confidence Variance in November 1998

This table reports the view portfolio weights ($\mathbf{P}$), the view portfolio expected return ($\mathbf{q}$), and the confidence variance ($\mathbf{\Omega}$) in November 1998, based on three volatility models including the DCC model, the EWMA model and the RW model with a window length of 50. The view portfolio is constructed by the momentum strategy and translated into the BL model following the method of Fabozzi et al. (2006).

Panel A: The View Portfolio Weights ($\mathbf{P}$)

	DCC	EWMA	RW50
UK BASIC MATS	-0.0851	-0.1257	-0.1656
UK CONSUMER GDS	-0.0838	-0.1022	-0.1253
UK CONSUMER SVS	-0.1686	-0.2062	-0.2208
UK FINANCIALS	-0.0988	-0.1295	-0.1594
UK HEALTH CARE	0.1414	0.1845	0.2204
UK TECHNOLOGY	-0.0527	-0.0690	-0.0653
UK INDUSTRIALS	-0.0859	-0.1157	-0.1500
UK OIL & GAS	-0.1060	-0.1310	-0.1611
UK TELECOM	0.1135	0.1334	0.1666
UK UTILITIES	0.1671	0.1640	0.1784
USA BASIC MATS	-0.1183	-0.1528	-0.1808
USA CONSUMER GDS	-0.0840	-0.1174	-0.1537
USA CONSUMER SVS	0.1180	0.1463	0.1922
USA FINANCIALS	-0.0748	-0.1027	-0.1345
USA HEALTH CARE	0.1365	0.1524	0.1972
USA INDUSTRIALS	-0.1136	-0.1469	-0.1849
USA OIL & GAS	-0.1324	-0.1553	-0.1889
USA TECHNOLOGY	0.0795	0.0949	0.1184
USA TELECOM	0.1195	0.1492	0.1884
USA UTILITIES	0.1768	0.2132	0.2336
JAPAN BASIC MATS	0.0729	0.0864	0.1048
JAPAN CONSUMER GDS	-0.1465	-0.1452	-0.1486
JAPAN CONSUMER SVS	0.1314	0.1293	0.1420
JAPAN FINANCIALS	-0.0553	-0.0692	-0.0865
JAPAN HEALTH CARE	0.1638	0.1283	0.1535
JAPAN INDUSTRIALS	0.1292	0.1417	0.1535
JAPAN OIL & GAS	0.0636	0.0708	0.0887
JAPAN TECHNOLOGY	-0.0790	-0.0977	-0.1141
JAPAN TELECOM	0.1043	0.1338	0.1410
JAPAN UTILITIES	0.0820	0.1240	0.1527

Panel B: Expected Return of the View Portfolio ($\mathbf{q}$)

	DCC	EWMA	RW50
Expected Return	0.0191	0.0295	0.0393

Panel C: Confidence Variance of the View Portfolio ($\mathbf{\Omega}$)

	DCC	EWMA	RW50
Confidence Variance	0.0047	0.0050	0.0110

Table 5.1.5 Portfolio Performance of the Momentum Portfolio and Benchmark Portfolio

This table shows the average return, standard deviation and Sharpe Ratio (SR) of the constructed momentum portfolio and the benchmark portfolio from November 1994 to May 2010, and in the sub-period from August 1998 to May 2010. Note that the initial period for constructing the momentum portfolio is in November 1994, because I use the six-month interval price data from December 1993 to May 1994 to calculate the normalised return to create the ranking and then use the subsequent six months as the holding period from June 1994 to November 1994. To avoid the noise from the simulated data of conditional variance in the RW method with a window length of 50 and to make a comparable analysis, I evaluate the portfolio performance from the 56th period (August 1998).

Panel A: Nov-94 - May-10

	DCC	EWMA	RW50	Benchmark
Average Return	0.0058	0.0069	0.0161	0.0005
Standard Deviation	0.0536	0.0579	0.0967	0.0435
Sharpe Ratio	0.1078	0.1200	0.1661	0.0114

Panel B: Aug-98 - May-10

	DCC	EWMA	RW50	Benchmark
Average Return	-0.0003	0.0006	0.0000	-0.0014
Standard Deviation	0.0503	0.0541	0.0597	0.0467
Sharpe Ratio	-0.0066	0.0111	0.0000	-0.0305

Table 5.1.6 The BL Expected Returns for Each Index in August 1998

This table reports the BL expected return $\boldsymbol{\mu}_{BL}$ for each index in August 1998 in the use of three volatility models. $\boldsymbol{\mu}_{BL,t} = \boldsymbol{\pi}_t + \tau \mathbf{H}_t \mathbf{P}_t (\mathbf{P}_t' \mathbf{H}_t \mathbf{P}_t \tau + \boldsymbol{\Omega}_t)^{-1} (\mathbf{q}_t - \mathbf{P}_t' \boldsymbol{\pi}_t)$, where τ is set to be 0.1.

	DCC	EWMA	RW50
UK BASIC MATS	0.0059	0.0039	0.0025
UK CONSUMER GDS	0.0042	0.0022	0.0023
UK CONSUMER SVS	0.0020	0.0006	0.0016
UK FINANCIALS	0.0026	0.0020	0.0029
UK HEALTH CARE	0.0013	0.0005	0.0020
UK TECHNOLOGY	0.0025	-0.0015	0.0003
UK INDUSTRIALS	0.0039	0.0018	0.0007
UK OIL & GAS	0.0041	0.0032	0.0029
UK TELECOM	0.0014	-0.0021	0.0005
UK UTILITIES	0.0011	-0.0008	0.0017
USA BASIC MATS	0.0037	0.0032	0.0031
USA CONSUMER GDS	0.0021	0.0019	0.0020
USA CONSUMER SVS	0.0018	0.0007	0.0020
USA FINANCIALS	0.0018	0.0017	0.0023
USA HEALTH CARE	0.0011	0.0009	0.0021
USA INDUSTRIALS	0.0027	0.0020	0.0031
USA OIL & GAS	0.0032	0.0023	0.0023
USA TECHNOLOGY	0.0030	0.0030	0.0039
USA TELECOM	0.0012	-0.0009	0.0010
USA UTILITIES	0.0009	-0.0018	0.0002
JAPAN BASIC MATS	0.0051	0.0088	0.0051
JAPAN CONSUMER GDS	0.0045	0.0049	0.0049
JAPAN CONSUMER SVS	0.0041	0.0059	0.0046
JAPAN FINANCIALS	0.0069	0.0105	0.0068
JAPAN HEALTH CARE	0.0035	0.0050	0.0043
JAPAN INDUSTRIALS	0.0049	0.0063	0.0048
JAPAN OIL & GAS	0.0079	0.0103	0.0060
JAPAN TECHNOLOGY	0.0055	0.0078	0.0059
JAPAN TELECOM	0.0037	0.0056	0.0046
JAPAN UTILITIES	0.0018	0.0025	0.0025

Table 5.1.7 The BL Expected Returns for Each Index in November 1998

This table reports the BL expected return $\boldsymbol{\mu}_{BL}$ for each index in November 1998 in the use of three volatility models. $\boldsymbol{\mu}_{BL,t} = \boldsymbol{\pi}_t + \tau \mathbf{H}_t \mathbf{P}_t (\mathbf{P}_t{}' \mathbf{H}_t \mathbf{P}_t \tau + \boldsymbol{\Omega}_t)^{-1} (\mathbf{q}_t - \mathbf{P}_t' \boldsymbol{\pi}_t)$, where τ is set to be 0.1.

	DCC	EWMA	RW50
UK BASIC MATS	0.0017	0.0005	0.0015
UK CONSUMER GDS	0.0014	-0.0004	0.0008
UK CONSUMER SVS	0.0012	0.0006	0.0012
UK FINANCIALS	0.0024	0.0021	0.0024
UK HEALTH CARE	0.0017	0.0018	0.0018
UK TECHNOLOGY	0.0026	-0.0006	0.0013
UK INDUSTRIALS	0.0019	0.0005	0.0010
UK OIL & GAS	0.0016	0.0012	0.0017
UK TELECOM	0.0018	0.0021	0.0015
UK UTILITIES	0.0011	0.0013	0.0013
USA BASIC MATS	0.0019	0.0018	0.0024
USA CONSUMER GDS	0.0031	0.0027	0.0029
USA CONSUMER SVS	0.0026	0.0026	0.0027
USA FINANCIALS	0.0041	0.0036	0.0038
USA HEALTH CARE	0.0023	0.0029	0.0028
USA INDUSTRIALS	0.0027	0.0028	0.0031
USA OIL & GAS	0.0014	0.0012	0.0017
USA TECHNOLOGY	0.0035	0.0043	0.0041
USA TELECOM	0.0023	0.0026	0.0024
USA UTILITIES	0.0010	0.0005	0.0008
JAPAN BASIC MATS	0.0032	0.0033	0.0029
JAPAN CONSUMER GDS	0.0015	0.0024	0.0028
JAPAN CONSUMER SVS	0.0018	0.0026	0.0027
JAPAN FINANCIALS	0.0040	0.0046	0.0046
JAPAN HEALTH CARE	0.0015	0.0027	0.0026
JAPAN INDUSTRIALS	0.0019	0.0023	0.0025
JAPAN OIL & GAS	0.0028	0.0042	0.0039
JAPAN TECHNOLOGY	0.0030	0.0032	0.0035
JAPAN TELECOM	0.0020	0.0023	0.0027
JAPAN UTILITIES	0.0019	0.0019	0.0018

Table 5.1.8 Statistics for Unconstrained BL Portfolio Optimisation in August 1998

This table reports both the statistics inputted into the optimisation models, such as estimated expected BL return and standard deviation (based on the DCC model), and the results produced by the optimisation models, including Expected Conditional Sharpe Ratio (ECSR), expected excess return to VaR ratio (μ / VaR) and expected excess return to CVaR ratio ($\mu / CVaR$). An implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem. MVaR-BL portfolio is constructed by achieving maximal excess return to VaR ratio in the optimisation problem. MCVaR-BL portfolio is constructed by achieving maximal excess return to conditional VaR ratio in the optimisation problem. Both VaR and CVaR are estimated by the parametric method with assumption of normal distribution ('N') and t-distribution ('t') at confidence levels 99%, 95%, and 90%.

Panel A: Unconstrained Implied BL and SR-BL Statistics in Aug-98

	Benchmark	Implied BL	SR-BL
Expected Return	0.0029	0.0048	0.0059
Standard Deviation	0.0370	0.0465	0.0568
VaR	0.0812	0.1035	0.1264
CVaR	0.0935	0.1193	0.1456
μ / VaR	0.0357	0.0464	0.0464
$\mu / CVaR$	0.0310	0.0403	0.0403
ECSR	0.0784	0.1032	0.1032

Unconstrained MVaR-BL and MCVaR-BL Statistics in Aug-98

Panel B: Normal Distribution

	0.99		0.95		0.90	
	MVaR-BL	MCVaR-BL	MVaR-BL	MCVaR-BL	MVaR-BL	MCVaR-BL
Expected Return	0.0059	0.0057	0.0060	0.0059	0.0059	0.0059
Standard Deviation	0.0578	0.0562	0.0581	0.0576	0.0574	0.0573
VaR	0.1286	0.1251	0.0896	0.0889	0.0677	0.0676
CVaR	0.1482	0.1441	0.1139	0.1129	0.0949	0.0948
μ / VaR	0.0462	0.0459	0.0666	0.0666	0.0873	0.0872
$\mu / CVaR$	0.0401	0.0398	0.0524	0.0524	0.0623	0.0622
ECSR	0.1027	0.1021	0.1028	0.1028	0.1029	0.1028

Panel C: t-Distribution

	0.99		0.95		0.90	
	MVaR-BL	MCVaR-BL	MVaR-BL	MCVaR-BL	MVaR-BL	MCVaR-BL
Expected Return	0.0052	0.0054	0.0059	0.0055	0.0060	0.0058
Standard Deviation	0.0512	0.0532	0.0577	0.0541	0.0580	0.0563
VaR	0.1867	0.1940	0.1170	0.1098	0.0829	0.0806
CVaR	0.2621	0.2724	0.1788	0.1677	0.1389	0.1351
μ / VaR	0.0278	0.0279	0.0506	0.0502	0.0719	0.0714
$\mu / CVaR$	0.0198	0.0199	0.0331	0.0329	0.0429	0.0426
ECSR	0.1015	0.1017	0.1028	0.1019	0.1028	0.1022

Table 5.1.9 Weights in the Unconstrained Implied BL Portfolio and the SR-BL Portfolio in August 1998

This table reports the weights assigned in each index in August 1998. Weights in the unconstrained implied BL portfolio are calculated by $\mathbf{w}^*_{BL,t} = \frac{1}{\delta_t}\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$ .The SR-BL portfolios allocate asset to achieve the maximal SR in the optimisation problem, weights can be calculated by $\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}$.

	DCC		EWMA		RW50	
	Implied BL	SR-BL	Implied BL	SR-BL	Implied BL	SR-BL
UK BASIC MATS	0.0532	0.0648	0.1469	0.1989	0.0484	0.0546
UK CONSUMER GDS	0.0503	0.0614	0.1268	0.1825	0.0342	0.0402
UK CONSUMER SVS	-0.0647	-0.0789	-0.1798	-0.2621	-0.0285	-0.0334
UK FINANCIALS	0.1180	0.1436	0.1922	0.2776	0.0806	0.0931
UK HEALTH CARE	-0.0594	-0.0723	-0.1802	-0.2407	-0.0173	-0.0182
UK TECHNOLOGY	-0.0257	-0.0313	-0.0664	-0.0967	-0.0073	-0.0085
UK INDUSTRIALS	-0.0365	-0.0447	-0.1060	-0.1399	-0.0243	-0.0275
UK OIL & GAS	0.0702	0.0857	0.1630	0.2306	0.0467	0.0543
UK TELECOM	-0.0286	-0.0350	-0.1165	-0.1791	-0.0174	-0.0212
UK UTILITIES	-0.0779	-0.0954	-0.1831	-0.2619	-0.0275	-0.0315
USA BASIC MATS	0.0956	0.1171	0.1831	0.2487	0.0710	0.0805
USA CONSUMER GDS	-0.0615	-0.0747	-0.1038	-0.1440	-0.0258	-0.0287
USA CONSUMER SVS	-0.0157	-0.0202	-0.0803	-0.1025	0.0275	0.0329
USA FINANCIALS	0.0310	0.0381	0.0062	0.0079	0.0722	0.0830
USA HEALTH CARE	0.0340	0.0414	-0.0286	-0.0441	0.0888	0.1020
USA INDUSTRIALS	-0.0243	-0.0294	-0.1008	-0.1325	0.0111	0.0157
USA OIL & GAS	0.1205	0.1471	0.2207	0.3242	0.0903	0.1042
USA TECHNOLOGY	0.0421	0.0515	-0.0064	-0.0153	0.0703	0.0811
USA TELECOM	-0.0554	-0.0674	-0.1452	-0.2077	-0.0206	-0.0241
USA UTILITIES	-0.0300	-0.0368	-0.1761	-0.2610	0.0054	0.0055
JAPAN BASIC MATS	0.0846	0.1022	0.1108	0.1474	0.0394	0.0454
JAPAN CONSUMER GDS	0.1030	0.1255	0.1859	0.2464	0.0620	0.0698
JAPAN CONSUMER SVS	0.1036	0.1257	0.1733	0.2426	0.0553	0.0637
JAPAN FINANCIALS	0.0747	0.0916	0.1110	0.1548	0.0483	0.0552
JAPAN HEALTH CARE	0.0961	0.1178	0.2018	0.2885	0.0551	0.0650
JAPAN INDUSTRIALS	0.0908	0.1125	0.1665	0.2752	0.0489	0.0598
JAPAN OIL & GAS	0.0434	0.0531	0.0855	0.1303	0.0256	0.0292
JAPAN TECHNOLOGY	0.0763	0.0928	0.1283	0.1838	0.0404	0.0468
JAPAN TELECOM	0.0836	0.1019	0.1617	0.2278	0.0414	0.0472
JAPAN UTILITIES	-0.0720	-0.0876	-0.2005	-0.2797	-0.0316	-0.0363

Table 5.1.10 Weights in the Unconstrained Implied BL Portfolio and the SR-BL Portfolio in November 1998

This table reports the weights assigned in each index in November 1998. Weights in the unconstrained implied BL portfolio is calculated by $\mathbf{w}^*_{BL,t} = \frac{1}{\delta_t}\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$. The SR-BL portfolios allocate asset to achieve the maximal SR in the optimisation problem, weights can be calculated by $\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}$.

	DCC		EWMA		RW50	
	Implied BL	SR-BL	Implied BL	SR-BL	Implied BL	SR-BL
UK BASIC MATS	-0.0275	-0.0247	-0.0465	-0.0429	-0.0187	-0.0185
UK CONSUMER GDS	-0.0305	-0.0276	-0.0399	-0.0369	-0.0160	-0.0157
UK CONSUMER SVS	-0.0483	-0.0442	-0.0679	-0.0644	-0.0143	-0.0142
UK FINANCIALS	-0.0022	-0.0021	-0.0172	-0.0167	0.0131	0.0127
UK HEALTH CARE	0.0872	0.0785	0.1091	0.1026	0.0640	0.0633
UK TECHNOLOGY	-0.0140	-0.0125	-0.0218	-0.0205	-0.0028	-0.0028
UK INDUSTRIALS	-0.0213	-0.0189	-0.0357	-0.0321	-0.0098	-0.0094
UK OIL & GAS	-0.0398	-0.0354	-0.0525	-0.0488	-0.0218	-0.0215
UK TELECOM	0.0651	0.0584	0.0765	0.0705	0.0450	0.0444
UK UTILITIES	0.0783	0.0705	0.0811	0.0775	0.0383	0.0380
USA BASIC MATS	-0.0234	-0.0211	-0.0405	-0.0386	-0.0036	-0.0039
USA CONSUMER GDS	-0.0164	-0.0147	-0.0322	-0.0299	-0.0062	-0.0065
USA CONSUMER SVS	0.1292	0.1162	0.1442	0.1376	0.1111	0.1101
USA FINANCIALS	0.0846	0.0760	0.0713	0.0668	0.0940	0.0929
USA HEALTH CARE	0.1962	0.1765	0.2063	0.1941	0.1714	0.1699
USA INDUSTRIALS	0.0125	0.0115	-0.0040	-0.0067	0.0298	0.0290
USA OIL & GAS	-0.0140	-0.0129	-0.0264	-0.0264	0.0104	0.0102
USA TECHNOLOGY	0.1473	0.1325	0.1560	0.1457	0.1334	0.1320
USA TELECOM	0.0733	0.0660	0.0889	0.0808	0.0540	0.0529
USA UTILITIES	0.1332	0.1203	0.1529	0.1430	0.0977	0.0965
JAPAN BASIC MATS	0.0434	0.0387	0.0511	0.0466	0.0301	0.0296
JAPAN CONSUMER GDS	-0.0329	-0.0297	-0.0354	-0.0334	0.0033	0.0032
JAPAN CONSUMER SVS	0.0741	0.0673	0.0765	0.0713	0.0429	0.0425
JAPAN FINANCIALS	0.0056	0.0051	-0.0013	-0.0020	0.0146	0.0144
JAPAN HEALTH CARE	0.0807	0.0728	0.0695	0.0647	0.0381	0.0376
JAPAN INDUSTRIALS	0.0652	0.0589	0.0737	0.0697	0.0366	0.0362
JAPAN OIL & GAS	0.0293	0.0264	0.0341	0.0333	0.0172	0.0169
JAPAN TECHNOLOGY	-0.0185	-0.0168	-0.0279	-0.0269	-0.0042	-0.0042
JAPAN TELECOM	0.0518	0.0462	0.0670	0.0642	0.0314	0.0312
JAPAN UTILITIES	0.0432	0.0388	0.0633	0.0577	0.0337	0.0331

Table 5.1.11 Weights in the Unconstrained MVaR-BL Portfolio in August 1998

This table reports weights allocated to each index in the unconstrained MVaR-BL portfolio in August 1998. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

Aug 98	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.1000	0.1417	0.0352	0.0816	0.1406	0.0359
UK CONSUMER GDS	0.0466	0.1266	0.0348	0.0368	0.1201	0.0348
UK CONSUMER SVS	-0.0398	-0.0351	0.0150	-0.0104	-0.0265	0.0146
UK FINANCIALS	0.0505	0.0636	0.0447	0.0180	0.0233	0.0423
UK HEALTH CARE	-0.0619	-0.0765	0.0100	-0.0211	-0.0681	0.0093
UK TECHNOLOGY	-0.0343	-0.0832	-0.0131	-0.0241	-0.1041	-0.0154
UK INDUSTRIALS	-0.0297	0.0479	0.0170	0.0145	0.0569	0.0178
UK OIL & GAS	0.1016	0.1037	0.0302	0.0439	0.0773	0.0300
UK TELECOM	-0.0381	-0.2825	-0.0394	-0.0483	-0.2622	-0.0399
UK UTILITIES	-0.0687	-0.1266	-0.0254	-0.0426	-0.1294	-0.0262
USA BASIC MATS	0.0713	0.0628	0.0272	0.0333	0.0485	0.0253
USA CONSUMER GDS	-0.0573	-0.0723	-0.0013	-0.0343	-0.0737	-0.0028
USA CONSUMER SVS	0.0053	-0.0499	0.0543	0.0315	-0.0470	0.0521
USA FINANCIALS	0.0261	-0.0029	0.0836	0.0453	-0.0219	0.0791
USA HEALTH CARE	0.0269	0.0161	0.1124	0.0723	0.0159	0.1102
USA INDUSTRIALS	0.0053	-0.0275	0.0432	0.0186	-0.0231	0.0400
USA OIL & GAS	0.1152	0.1108	0.0559	0.0623	0.0803	0.0553
USA TECHNOLOGY	0.0567	0.0221	0.0868	0.0620	-0.0037	0.0829
USA TELECOM	-0.0689	-0.1680	-0.0288	-0.0429	-0.1625	-0.0301
USA UTILITIES	-0.0381	-0.0818	0.0105	-0.0032	-0.0684	0.0095
JAPAN BASIC MATS	0.0872	0.1153	0.0393	0.0755	0.1613	0.0442
JAPAN CONSUMER GDS	0.1210	0.0976	0.0527	0.0827	0.0900	0.0538
JAPAN CONSUMER SVS	0.0902	0.1294	0.0492	0.0715	0.1368	0.0525
JAPAN FINANCIALS	0.1118	0.1462	0.0677	0.1061	0.2034	0.0691
JAPAN HEALTH CARE	0.0527	0.1212	0.0360	0.0390	0.1202	0.0383
JAPAN INDUSTRIALS	0.1281	0.1677	0.0462	0.0835	0.1495	0.0496
JAPAN OIL & GAS	0.0498	0.0869	0.0313	0.0973	0.1592	0.0351
JAPAN TECHNOLOGY	0.1463	0.2707	0.0674	0.0956	0.2219	0.0690
JAPAN TELECOM	0.1008	0.1862	0.0497	0.0615	0.1559	0.0525
JAPAN UTILITIES	-0.0567	-0.0102	0.0077	-0.0059	0.0295	0.0113

Table 5.1.12 Weights in the Unconstrained MVaR-BL Portfolio in November 1998

This table reports weights allocated to each index in the unconstrained MVaR-BL portfolio in November 1998. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

Nov 98	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	-0.0434	-0.0757	-0.0237	-0.0247	-0.0741	-0.0092
UK CONSUMER GDS	-0.0429	-0.0900	-0.0366	-0.0300	-0.0904	-0.0184
UK CONSUMER SVS	0.0040	-0.0128	0.0045	0.0052	-0.0139	0.0097
UK FINANCIALS	0.0155	0.0178	0.0269	0.0205	0.0180	0.0315
UK HEALTH CARE	0.0608	0.0566	0.0390	0.0458	0.0540	0.0327
UK TECHNOLOGY	-0.0076	-0.0313	0.0049	-0.0257	-0.0378	-0.0056
UK INDUSTRIALS	-0.0330	-0.0830	-0.0243	-0.0208	-0.0821	-0.0083
UK OIL & GAS	-0.0262	-0.0384	-0.0188	-0.0157	-0.0375	-0.0099
UK TELECOM	0.0419	0.0793	0.0399	0.0222	0.0746	0.0247
UK UTILITIES	0.0311	0.0648	0.0282	0.0165	0.0609	0.0150
USA BASIC MATS	-0.0072	-0.0182	0.0044	0.0036	-0.0166	0.0139
USA CONSUMER GDS	-0.0089	-0.0174	-0.0014	-0.0017	-0.0168	0.0064
USA CONSUMER SVS	0.0835	0.0831	0.0835	0.0799	0.0820	0.0822
USA FINANCIALS	0.1119	0.0997	0.1123	0.1128	0.1002	0.1146
USA HEALTH CARE	0.1780	0.1750	0.1600	0.1638	0.1730	0.1533
USA INDUSTRIALS	0.0517	0.0590	0.0570	0.0540	0.0588	0.0581
USA OIL & GAS	0.0230	0.0083	0.0230	0.0292	0.0086	0.0303
USA TECHNOLOGY	0.1343	0.1500	0.1304	0.1209	0.1510	0.1253
USA TELECOM	0.0552	0.0807	0.0483	0.0354	0.0761	0.0318
USA UTILITIES	0.0788	0.0876	0.0751	0.0676	0.0828	0.0641
JAPAN BASIC MATS	0.0319	0.0521	0.0334	0.0402	0.0566	0.0316
JAPAN CONSUMER GDS	0.0203	0.0288	0.0285	0.0262	0.0310	0.0306
JAPAN CONSUMER SVS	0.0401	0.0555	0.0368	0.0404	0.0565	0.0334
JAPAN FINANCIALS	0.0179	0.0234	0.0322	0.0480	0.0297	0.0389
JAPAN HEALTH CARE	0.0450	0.0579	0.0290	0.0363	0.0580	0.0250
JAPAN INDUSTRIALS	0.0147	0.0277	0.0211	0.0179	0.0303	0.0212
JAPAN OIL & GAS	0.0235	0.0658	0.0315	0.0337	0.0705	0.0258
JAPAN TECHNOLOGY	0.0079	0.0063	0.0116	0.0173	0.0111	0.0169
JAPAN TELECOM	0.0357	0.0217	0.0145	0.0306	0.0226	0.0147
JAPAN UTILITIES	0.0624	0.0659	0.0289	0.0504	0.0630	0.0198

Table 5.1.13 Effect of Distribution Assumptions and Confidence Levels on MVaR-BL Portfolio Weights

This table shows positions of each asset in the MVaR-BL portfolio in August 1998 under normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. Note that the covariance matrix applied to the MVaR-BL model is the DCC covariance matrix in this table.

	Normal Distribution			t-Distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
UK BASIC MATS	0.1000	0.0993	0.0904	0.0816	0.0985	0.0968
UK CONSUMER GDS	0.0466	0.0478	0.0459	0.0368	0.0463	0.0469
UK CONSUMER SVS	-0.0398	-0.0413	-0.0494	-0.0104	-0.0408	-0.0439
UK FINANCIALS	0.0505	0.0525	0.0658	0.0180	0.0526	0.0566
UK HEALTH CARE	-0.0619	-0.0623	-0.0625	-0.0211	-0.0621	-0.0629
UK TECHNOLOGY	-0.0343	-0.0360	-0.0438	-0.0241	-0.0354	-0.0390
UK INDUSTRIALS	-0.0297	-0.0323	-0.0468	0.0145	-0.0320	-0.0375
UK OIL & GAS	0.1016	0.1037	0.1106	0.0439	0.1028	0.1066
UK TELECOM	-0.0381	-0.0394	-0.0344	-0.0483	-0.0371	-0.0376
UK UTILITIES	-0.0687	-0.0685	-0.0682	-0.0426	-0.0685	-0.0687
USA BASIC MATS	0.0713	0.0718	0.0779	0.0333	0.0725	0.0742
USA CONSUMER GDS	-0.0573	-0.0588	-0.0590	-0.0343	-0.0572	-0.0591
USA CONSUMER SVS	0.0053	0.0029	-0.0018	0.0315	0.0048	0.0015
USA FINANCIALS	0.0261	0.0249	0.0315	0.0453	0.0270	0.0265
USA HEALTH CARE	0.0269	0.0259	0.0272	0.0723	0.0271	0.0257
USA INDUSTRIALS	0.0053	0.0036	0.0053	0.0186	0.0054	0.0040
USA OIL & GAS	0.1152	0.1180	0.1284	0.0623	0.1171	0.1219
USA TECHNOLOGY	0.0567	0.0530	0.0499	0.0620	0.0562	0.0522
USA TELECOM	-0.0689	-0.0702	-0.0701	-0.0429	-0.0687	-0.0703
USA UTILITIES	-0.0381	-0.0378	-0.0361	-0.0032	-0.0378	-0.0379
JAPAN BASIC MATS	0.0872	0.0887	0.0891	0.0755	0.0872	0.0890
JAPAN CONSUMER GDS	0.1210	0.1214	0.1197	0.0827	0.1209	0.1217
JAPAN CONSUMER SVS	0.0902	0.0924	0.0944	0.0715	0.0905	0.0933
JAPAN FINANCIALS	0.1118	0.1117	0.1107	0.1061	0.1112	0.1114
JAPAN HEALTH CARE	0.0527	0.0558	0.0650	0.0390	0.0538	0.0584
JAPAN INDUSTRIALS	0.1281	0.1292	0.1285	0.0835	0.1282	0.1299
JAPAN OIL & GAS	0.0498	0.0508	0.0465	0.0973	0.0486	0.0484
JAPAN TECHNOLOGY	0.1463	0.1461	0.1419	0.0956	0.1455	0.1458
JAPAN TELECOM	0.1008	0.1031	0.1052	0.0615	0.1011	0.1044
JAPAN UTILITIES	-0.0567	-0.0559	-0.0618	-0.0059	-0.0578	-0.0584

Table 5.1.14 Weights in the Unconstrained MCVaR-BL Portfolio in August 1998

This table reports weights allocated to each index in the unconstrained MCVaR-BL portfolio in August 1998. The weight in the MCVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to CVaR ratio. Correspondingly, CVaR is also estimated by the parametric method with the assumption of normal distribution and t distribution at the confidence level of 99%.

Aug 98	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.1005	0.1447	0.0341	0.0875	0.0450	0.0327
UK CONSUMER GDS	0.0481	0.1288	0.0339	0.0387	0.0146	0.0312
UK CONSUMER SVS	-0.0197	-0.0327	0.0155	-0.0144	-0.0272	0.0142
UK FINANCIALS	0.0220	0.0581	0.0411	0.0180	0.0035	0.0393
UK HEALTH CARE	-0.0407	-0.0763	0.0111	-0.0268	-0.0191	0.0109
UK TECHNOLOGY	-0.0220	-0.0833	-0.0109	-0.0268	-0.1211	-0.0173
UK INDUSTRIALS	0.0073	0.0533	0.0176	0.0128	-0.0119	0.0156
UK OIL & GAS	0.0657	0.1019	0.0267	0.0490	0.0092	0.0258
UK TELECOM	-0.0549	-0.2828	-0.0362	-0.0515	-0.1174	-0.0355
UK UTILITIES	-0.0564	-0.1297	-0.0248	-0.0478	-0.0636	-0.0244
USA BASIC MATS	0.0437	0.0626	0.0248	0.0359	0.0158	0.0234
USA CONSUMER GDS	-0.0488	-0.0707	-0.0014	-0.0382	-0.0517	-0.0026
USA CONSUMER SVS	0.0174	-0.0484	0.0546	0.0270	-0.0011	0.0531
USA FINANCIALS	0.0263	-0.0031	0.0813	0.0407	0.0157	0.0788
USA HEALTH CARE	0.0473	0.0162	0.1121	0.0655	0.0703	0.1115
USA INDUSTRIALS	0.0070	-0.0255	0.0416	0.0163	0.0083	0.0400
USA OIL & GAS	0.0789	0.1073	0.0527	0.0652	0.0291	0.0517
USA TECHNOLOGY	0.0541	0.0184	0.0842	0.0607	0.0466	0.0824
USA TELECOM	-0.0599	-0.1684	-0.0269	-0.0485	-0.0805	-0.0268
USA UTILITIES	-0.0216	-0.0805	0.0118	-0.0095	-0.0282	0.0111
JAPAN BASIC MATS	0.0849	0.1174	0.0422	0.0796	0.1847	0.0476
JAPAN CONSUMER GDS	0.1016	0.0960	0.0526	0.0878	0.0767	0.0540
JAPAN CONSUMER SVS	0.0829	0.1281	0.0503	0.0748	0.1213	0.0535
JAPAN FINANCIALS	0.1133	0.1557	0.0691	0.1129	0.1911	0.0712
JAPAN HEALTH CARE	0.0436	0.1187	0.0366	0.0409	0.0990	0.0389
JAPAN INDUSTRIALS	0.1061	0.1623	0.0461	0.0896	0.1162	0.0495
JAPAN OIL & GAS	0.0895	0.0935	0.0339	0.1018	0.2010	0.0388
JAPAN TECHNOLOGY	0.1229	0.2634	0.0649	0.1035	0.1346	0.0663
JAPAN TELECOM	0.0814	0.1812	0.0489	0.0658	0.0911	0.0509
JAPAN UTILITIES	-0.0204	-0.0062	0.0122	-0.0106	0.0481	0.0143

Table 5.1.15 Weights in the Unconstrained MCVaR-BL Portfolio in November 1998

This table reports weights allocated to each index in the unconstrained MCVaR-BL portfolio in November 1998. The weight in the MCVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to CVaR ratio. Correspondingly, CVaR is also estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

Nov 98	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	-0.0419	-0.0755	-0.0102	0.0049	-0.0340	-0.0085
UK CONSUMER GDS	-0.0418	-0.0905	-0.0197	0.0010	-0.0467	-0.0176
UK CONSUMER SVS	0.0043	-0.0132	0.0094	0.0192	-0.0026	0.0099
UK FINANCIALS	0.0163	0.0178	0.0312	0.0388	0.0265	0.0317
UK HEALTH CARE	0.0596	0.0559	0.0331	0.0302	0.0394	0.0324
UK TECHNOLOGY	-0.0079	-0.0330	-0.0045	0.0054	-0.0397	-0.0065
UK INDUSTRIALS	-0.0317	-0.0830	-0.0093	0.0120	-0.0389	-0.0076
UK OIL & GAS	-0.0252	-0.0381	-0.0105	0.0007	-0.0230	-0.0096
UK TELECOM	0.0407	0.0781	0.0257	0.0184	0.0388	0.0240
UK UTILITIES	0.0301	0.0639	0.0159	0.0090	0.0271	0.0143
USA BASIC MATS	-0.0061	-0.0178	0.0133	0.0242	0.0018	0.0144
USA CONSUMER GDS	-0.0079	-0.0173	0.0059	0.0166	-0.0034	0.0068
USA CONSUMER SVS	0.0836	0.0828	0.0823	0.0868	0.0791	0.0822
USA FINANCIALS	0.1126	0.0998	0.1144	0.1236	0.1065	0.1147
USA HEALTH CARE	0.1771	0.1745	0.1537	0.1523	0.1575	0.1530
USA INDUSTRIALS	0.0522	0.0590	0.0580	0.0615	0.0576	0.0582
USA OIL & GAS	0.0237	0.0084	0.0299	0.0409	0.0200	0.0306
USA TECHNOLOGY	0.1338	0.1505	0.1257	0.1239	0.1347	0.1250
USA TELECOM	0.0539	0.0796	0.0328	0.0246	0.0418	0.0310
USA UTILITIES	0.0781	0.0863	0.0648	0.0650	0.0636	0.0635
JAPAN BASIC MATS	0.0315	0.0533	0.0317	0.0126	0.0529	0.0316
JAPAN CONSUMER GDS	0.0207	0.0295	0.0305	0.0263	0.0329	0.0307
JAPAN CONSUMER SVS	0.0397	0.0558	0.0335	0.0203	0.0471	0.0332
JAPAN FINANCIALS	0.0191	0.0250	0.0383	0.0280	0.0474	0.0394
JAPAN HEALTH CARE	0.0439	0.0579	0.0252	0.0131	0.0437	0.0249
JAPAN INDUSTRIALS	0.0147	0.0285	0.0212	0.0114	0.0292	0.0211
JAPAN OIL & GAS	0.0231	0.0671	0.0261	0.0012	0.0573	0.0256
JAPAN TECHNOLOGY	0.0083	0.0077	0.0166	0.0121	0.0223	0.0172
JAPAN TELECOM	0.0349	0.0219	0.0146	0.0078	0.0209	0.0147
JAPAN UTILITIES	0.0608	0.0650	0.0203	0.0083	0.0400	0.0195

Table 5.1.16 Effect of Distribution Assumptions and Confidence Levels on MCVaR-BL Portfolio Weights

This table shows positions of each asset in the MVaR-BL portfolio in August 1998 under normal distribution and t-distribution at confidence levels of 99%, 95%, and 90%. Note that the covariance matrix applied to the MCVaR-BL model is the DCC covariance matrix in this table.

	Normal Distribution			t-Distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
UK BASIC MATS	0.1005	0.0978	0.0938	0.0875	0.0926	0.1004
UK CONSUMER GDS	0.0481	0.0462	0.0453	0.0387	0.0435	0.0480
UK CONSUMER SVS	-0.0197	-0.0413	-0.0445	-0.0144	-0.0147	-0.0202
UK FINANCIALS	0.0220	0.0536	0.0597	0.0180	0.0186	0.0225
UK HEALTH CARE	-0.0407	-0.0622	-0.0627	-0.0268	-0.0312	-0.0412
UK TECHNOLOGY	-0.0220	-0.0359	-0.0396	-0.0268	-0.0223	-0.0231
UK INDUSTRIALS	0.0073	-0.0332	-0.0400	0.0128	0.0128	0.0061
UK OIL & GAS	0.0657	0.1034	0.1067	0.0490	0.0541	0.0667
UK TELECOM	-0.0549	-0.0366	-0.0343	-0.0515	-0.0534	-0.0550
UK UTILITIES	-0.0564	-0.0685	-0.0683	-0.0478	-0.0500	-0.0565
USA BASIC MATS	0.0437	0.0730	0.0762	0.0359	0.0376	0.0442
USA CONSUMER GDS	-0.0488	-0.0573	-0.0575	-0.0382	-0.0422	-0.0493
USA CONSUMER SVS	0.0174	0.0045	0.0027	0.0270	0.0239	0.0169
USA FINANCIALS	0.0263	0.0274	0.0299	0.0407	0.0344	0.0257
USA HEALTH CARE	0.0473	0.0271	0.0273	0.0655	0.0591	0.0466
USA INDUSTRIALS	0.0070	0.0055	0.0059	0.0163	0.0120	0.0065
USA OIL & GAS	0.0789	0.1180	0.1233	0.0652	0.0695	0.0800
USA TECHNOLOGY	0.0541	0.0560	0.0544	0.0607	0.0572	0.0536
USA TELECOM	-0.0599	-0.0687	-0.0686	-0.0485	-0.0524	-0.0603
USA UTILITIES	-0.0216	-0.0377	-0.0372	-0.0095	-0.0130	-0.0220
JAPAN BASIC MATS	0.0849	0.0873	0.0878	0.0796	0.0810	0.0854
JAPAN CONSUMER GDS	0.1016	0.1208	0.1206	0.0878	0.0923	0.1024
JAPAN CONSUMER SVS	0.0829	0.0907	0.0918	0.0748	0.0777	0.0836
JAPAN FINANCIALS	0.1133	0.1110	0.1103	0.1129	0.1110	0.1134
JAPAN HEALTH CARE	0.0436	0.0543	0.0579	0.0409	0.0412	0.0442
JAPAN INDUSTRIALS	0.1061	0.1283	0.1287	0.0896	0.0951	0.1071
JAPAN OIL & GAS	0.0895	0.0481	0.0457	0.1018	0.0964	0.0888
JAPAN TECHNOLOGY	0.1229	0.1452	0.1436	0.1035	0.1098	0.1238
JAPAN TELECOM	0.0814	0.1013	0.1025	0.0658	0.0716	0.0825
JAPAN UTILITIES	-0.0204	-0.0583	-0.0613	-0.0106	-0.0121	-0.0207

Table 5.1.17 Single Period Unconstrained BL Portfolio Performance Evaluation

This table reports the results of unconstrained BL portfolios and the benchmark portfolio for the portfolio evaluation criteria, including realised excess return, Conditional Sharpe Ratio (CSR), Portfolio Turnover (PT) and reward to CVaR ratio. The standard deviation is estimated by conditional covariance matrix of three volatility models. An implied BL portfolio is constructed by reverse optimisation of the utility function. SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem. The MVaR-BL portfolio is constructed by achieving maximal return to VaR ratio in the optimisation problem. The MCVaR-BL portfolio is constructed by achieving maximal return to CVaR ratio in the optimisation problem. Both VaR and CVaR are estimated by the parametric method in the optimisation model with assumptions of normal distribution ('N') and t-distribution ('t') at confidence levels of 99%, 95% and 90%. Panel A reports results in August 1998. Panel B reports results in November 1998. August 1998 is the first period to construct the portfolio in the use of the RW method with a 50 window length, therefore, there are no results of PT shown for RW50. Following Israelsen's (2003) method, the CSR and the reward to CVaR ratio in August 1998 were adjusted to make a comparison.

	Realised Excess Return			Adjusted CSR			Portfolio Turnover			Adjusted Reward to CVaR		
Panel A: Aug 98	DCC	EWMA	RW50	DCC	EWMA	RW50	DCC	EWMA	RW50	DCC	EWMA	RW50
Benchmark	-0.1420	-0.1420	-0.1420	-0.5135	-0.6705	-0.4065	N/A	N/A	N/A	-3.3845	-3.8028	-3.0993
Implied BL	-0.1156	-0.1255	-0.1223	-0.5382	-1.1718	-0.3874	1.6803	5.5327	N/A	-2.7711	-4.6971	-2.5276
SR-BL	-0.1412	-0.1803	-0.1415	-0.8025	-2.4199	-0.5193	3.6583	7.0786	N/A	-4.1322	-9.7018	-3.3848
99% Confidence Level:												
MVaR-BL N	-0.1466	-0.1434	-0.1439	-0.8476	-1.4563	-0.5060	3.2693	4.0572	N/A	-4.4069	-5.9384	-3.4207
MVaR-BL t	-0.1428	-0.1383	-0.1422	-0.7311	-1.5057	-0.5091	2.3449	4.0130	N/A	-3.9866	-5.9249	-3.3794
MCVaR-BL N	-0.1431	-0.1447	-0.1427	-0.8049	-1.4771	-0.5007	2.9954	3.9932	N/A	-4.1935	-6.0319	-3.3712
MCVaR-BL t	-0.1440	-0.1328	-0.1414	-0.7665	-1.2651	-0.5076	2.4305	3.0912	N/A	-4.1168	-5.1364	-3.3508
95% Confidence Level:												
MVaR-BL N	-0.1451	-0.1434	-0.1466	-0.8426	-1.4731	-0.5322	3.3499	4.2666	N/A	-4.3500	-5.9822	-3.5677
MVaR-BL t	-0.1463	-0.1430	-0.1444	-0.8440	-1.4511	-0.5076	3.2838	4.0765	N/A	-4.3908	-5.9130	-3.4389
MCVaR-BL N	-0.1462	-0.1430	-0.1446	-0.8427	-1.4509	-0.5089	3.2905	4.0782	N/A	-4.3848	-5.9106	-3.4482
MCVaR-BL t	-0.1429	-0.1479	-0.1418	-0.7729	-1.5214	-0.5044	2.8456	3.9529	N/A	-4.1016	-6.2411	-3.3555
90% Confidence Level:												
MVaR-BL N	-0.1439	-0.1416	-0.1516	-0.8269	-1.4890	-0.5678	3.3894	4.6409	N/A	-4.2759	-5.9746	-3.8108
MVaR-BL t	-0.1448	-0.1437	-0.1476	-0.8393	-1.4862	-0.5363	3.3524	4.3470	N/A	-4.3329	-6.0248	-3.6069
MCVaR-BL N	-0.1457	-0.1432	-0.1459	-0.8356	-1.4630	-0.5242	3.3269	4.1741	N/A	-4.3498	-5.9488	-3.5262
MCVaR-BL t	-0.1428	-0.1440	-0.1435	-0.8048	-1.4655	-0.5005	3.0196	4.0323	N/A	-4.1849	-5.9799	-3.3921

Table 5.1.17 (continued)

	Realised Excess Return			Conditional Sharpe Ratio			Portfolio Turnover			Conditional Reward to CVaR Ratio		
Panel B: Nov 98	DCC	EWMA	RW50	DCC	EWMA	RW50	DCC	EWMA	RW50	DCC	EWMA	RW50
Benchmark	0.0510	0.0510	0.0510	0.8960	1.0277	1.3658	N/A	N/A	N/A	0.5064	0.6276	1.0510
Implied BL	0.0485	0.0481	0.0485	0.9344	0.9129	1.3247	0.9230	2.6271	0.9230	0.5123	0.5210	0.9882
SR-BL	0.0444	0.0448	0.0479	0.9132	0.9124	1.3257	1.5992	2.5124	0.9144	0.5213	0.5206	0.9898
99% Confidence Level:												
MVaR-BL N	0.0492	0.0571	0.0533	0.9342	0.9870	1.3715	0.9752	1.7999	0.4565	0.5397	0.5882	1.0601
MVaR-BL t	0.0482	0.0578	0.0517	0.8752	0.9778	1.3265	0.5066	1.7276	0.2917	0.4890	0.5795	0.9909
MCVaR-BL N	0.0493	0.0574	0.0519	0.9338	0.9858	1.3313	0.9545	1.7825	0.3009	0.5393	0.5870	0.9981
MCVaR-BL t	0.0510	0.0534	0.0516	0.8874	0.9488	1.3233	0.0341	1.2606	0.2865	0.4991	0.5528	0.9862
95% Confidence Level:												
MVaR-BL N	0.0488	0.0561	0.0532	0.9322	0.9843	1.3665	1.0434	1.9550	0.5735	0.5379	0.5856	1.0523
MVaR-BL t	0.0492	0.0569	0.0533	0.9342	0.9871	1.3726	0.9906	1.8109	0.6010	0.5397	0.5883	1.0619
MCVaR-BL N	0.0491	0.0568	0.0533	0.9342	0.9871	1.3731	0.9967	1.8151	0.6023	0.5397	0.5882	1.0627
MCVaR-BL t	0.0482	0.0577	0.0518	0.8766	0.9824	1.3285	0.7449	1.7557	0.2954	0.4901	0.5837	0.9939
90% Confidence Level:												
MVaR-BL N	0.0488	0.0524	0.0535	0.9342	0.9529	1.3838	1.1535	2.0095	0.6099	0.5396	0.5565	1.0799
MVaR-BL t	0.0486	0.0558	0.0533	0.9307	0.9826	1.3725	1.0571	1.9724	0.5873	0.5366	0.5840	1.0618
MCVaR-BL N	0.0489	0.0563	0.0530	0.9331	0.9855	1.3615	1.0300	1.8964	0.5605	0.5387	0.5867	1.0443
MCVaR-BL t	0.0493	0.0573	0.0533	0.9340	0.9866	1.3706	0.9638	1.7908	0.4547	0.5395	0.5877	1.0587

Table 5.1.18 Unconstrained BL Portfolio Performance in Multiple Periods (Nov 94 – May 10)

This table shows realised unconstrained BL portfolio performance compared with the benchmark performance in the period from November 1994 to May 2010. Return is the average realised excess return, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Both VaR and CVaR are measured on the empirical distribution. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk. In the construction of the portfolio, both VaR and CVaR are estimated by the parametric method with assumption of normal distribution ('N') and t-distribution ('t') at confidence levels of 99%, 95% and 90%. The implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem. The MVaR-BL portfolio is constructed by achieving maximal return to VaR ratio in the optimisation problem. The MCVaR-BL portfolio is constructed by achieving maximal return to CVaR ratio in the optimisation problem.

		Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR Ratio	Reward to CVaR Ratio
Benchmark		0.0005	0.0435	-0.9217	4.8813	0.0114	-	0.1314	0.1692	0.0038	0.0029
Implied BL	DCC	0.0161	0.0741	1.6980	13.5684	0.2170	0.2825	0.1550	0.1973	0.1037	0.0815
	EWMA	0.0118	0.0663	1.1784	14.2414	0.1773	0.2397	0.1945	0.2347	0.0605	0.0501
	RW50	0.0078	0.0537	0.9112	10.3663	0.1459	0.2530	0.1215	0.1631	0.0645	0.0480
SR-BL	DCC	0.0168	0.1067	2.9898	33.3293	0.1578	0.1773	0.2241	0.4052	0.0751	0.0416
	EWMA	0.0080	0.0776	-3.8259	38.7728	0.1030	0.1366	0.1723	0.4573	0.0464	0.0175
	RW50	0.0069	0.0506	-0.0394	6.6242	0.1359	0.2731	0.1338	0.1769	0.0514	0.0389
99% Confidence Level:											
MVaR-BL N	DCC	0.0168	0.1002	2.6574	25.9971	0.1673	0.1901	0.2273	0.3604	0.0737	0.0465
	EWMA	0.0081	0.0744	-2.1733	16.6340	0.1087	0.1325	0.3252	0.4441	0.0249	0.0182
	RW50	0.0055	0.0492	-0.0741	6.7730	0.1122	0.2340	0.1357	0.1746	0.0406	0.0316
MVaR-BL t	DCC	0.0177	0.0914	1.7811	14.2931	0.1934	0.2181	0.2006	0.3001	0.0881	0.0589
	EWMA	0.0115	0.0651	0.1921	10.4777	0.1769	0.2345	0.2024	0.2694	0.0569	0.0427
	RW50	0.0052	0.0475	-0.3752	5.9277	0.1099	0.2538	0.1347	0.1733	0.0388	0.0301
MCVaR-BL N	DCC	0.0172	0.1019	2.1356	23.3166	0.1688	0.1890	0.2271	0.3907	0.0758	0.0440
	EWMA	0.0097	0.0685	-1.9666	17.1389	0.1420	0.2013	0.2123	0.3725	0.0458	0.0261
	RW50	0.0053	0.0487	-0.1731	6.3070	0.1089	0.2342	0.1350	0.1738	0.0393	0.0305
MCVaR-BL t	DCC	0.0167	0.0840	2.2125	14.9555	0.1988	0.2292	0.2041	0.2253	0.0819	0.0741
	EWMA	0.0102	0.0672	-0.3290	10.9595	0.1518	0.1990	0.2717	0.2953	0.0376	0.0345
	RW50	0.0050	0.0462	-0.4476	5.6863	0.1086	0.2632	0.1342	0.1694	0.0374	0.0296

Table 5.1.18 (continued)

95% Confidence Level:											
MVaR-BL N	DCC	0.0156	0.0965	2.6794	31.1439	0.1618	0.1861	0.2163	0.3654	0.0722	0.0427
	EWMA	0.0112	0.0705	-1.1705	14.0331	0.1588	0.2184	0.2100	0.3558	0.0533	0.0315
	RW50	0.0061	0.0509	0.2809	8.5933	0.1194	0.2289	0.1373	0.1764	0.0442	0.0344
MVaR-BL t	DCC	0.0178	0.1031	2.7070	25.3649	0.1731	0.1958	0.2139	0.3685	0.0834	0.0484
	EWMA	0.0077	0.0753	-2.2108	16.3470	0.1026	0.1245	0.3465	0.4426	0.0223	0.0174
	RW50	0.0056	0.0496	0.0294	7.2542	0.1132	0.2297	0.1360	0.1749	0.0413	0.0321
MCVaR-BL N	DCC	0.0180	0.1042	2.7141	25.3831	0.1726	0.1951	0.2181	0.3707	0.0825	0.0485
	EWMA	0.0082	0.0744	-2.0069	14.9794	0.1097	0.1333	0.3463	0.4255	0.0235	0.0192
	RW50	0.0057	0.0498	0.0707	7.4625	0.1147	0.2301	0.1361	0.1750	0.0419	0.0326
MCVaR-BL t	DCC	0.0171	0.0923	1.9659	17.3624	0.1849	0.2113	0.2008	0.3127	0.0850	0.0546
	EWMA	0.0118	0.0657	0.1737	10.5876	0.1802	0.2411	0.2119	0.2747	0.0559	0.0431
	RW50	0.0052	0.0479	-0.3091	6.0802	0.1083	0.2422	0.1344	0.1732	0.0385	0.0299
90% Confidence Level:											
MVaR-BL N	DCC	0.0164	0.0970	2.8621	29.8258	0.1689	0.1941	0.2157	0.3477	0.0760	0.0471
	EWMA	0.0115	0.0729	-0.3907	14.7680	0.1583	0.2096	0.1917	0.3409	0.0602	0.0339
	RW50	0.0060	0.0500	-0.1221	7.0089	0.1209	0.2479	0.1404	0.1830	0.0430	0.0330
MVaR-BL t	DCC	0.0164	0.0962	2.9324	31.1876	0.1700	0.1952	0.2132	0.3465	0.0768	0.0472
	EWMA	0.0110	0.0714	-1.1526	14.2593	0.1545	0.2109	0.2326	0.3752	0.0474	0.0294
	RW50	0.0063	0.0517	0.4229	9.7389	0.1226	0.2282	0.1379	0.1802	0.0459	0.0352
MCVaR-BL N	DCC	0.0157	0.0966	2.5736	30.4959	0.1620	0.1857	0.2215	0.3680	0.0707	0.0425
	EWMA	0.0109	0.0700	-1.3407	14.3504	0.1554	0.2149	0.2253	0.3679	0.0483	0.0296
	RW50	0.0061	0.0511	0.4030	9.2824	0.1197	0.2248	0.1369	0.1759	0.0446	0.0347
MCVaR-BL t	DCC	0.0165	0.0984	2.6333	25.7018	0.1676	0.1910	0.2272	0.3542	0.0726	0.0466
	EWMA	0.0074	0.0761	-2.2782	16.2389	0.0971	0.1174	0.3593	0.4421	0.0206	0.0167
	RW50	0.0054	0.0492	-0.0412	6.8735	0.1101	0.2279	0.1354	0.1742	0.0400	0.0311

Table 5.1.19 Unconstrained BL Portfolio Performance in a Sub-period (Aug 98 – May 10)

This table shows realised unconstrained BL portfolio performance compared with the benchmark performance in the sub-period from August 1998 to May 2010. Return is the average realised excess return, risk is the standard deviation, SR is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Both VaR and CVaR are measured on the empirical distribution. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk. In construction of portfolio, both VaR and CVaR is estimated by the parametric method with assumption of normal distribution ('N') and t-distribution ('t') at confidence level of 99%, 95%,90%. The implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem. MVaR-BL portfolio is constructed by achieving maximal return to VaR ratio in the optimisation problem. The MCVaR-BL portfolio is constructed by achieving maximal return to conditional VaR ratio in the optimisation problem.

Aug 98- May 10		Return	Risk	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR Ratio	Reward to CVaR Ratio
Benchmark		-0.0014	0.0467	-0.8860	4.5499	-0.0305	-	0.1459	0.1780	-0.0098	-0.0080
Implied BL	DCC	0.0130	0.0752	1.6127	14.4827	0.1730	0.2638	0.1675	0.2093	0.0777	0.0622
	EWMA	0.0087	0.0693	1.1489	14.8658	0.1259	0.2082	0.2300	0.2333	0.0379	0.0374
	RW50	0.0051	0.0530	0.0356	6.4232	0.0972	0.2918	0.1284	0.1778	0.0401	0.0290
SR-BL	DCC	0.0150	0.1174	2.9202	30.0047	0.1277	0.1601	0.2821	0.4538	0.0532	0.0330
	EWMA	0.0046	0.0844	-3.9208	35.7595	0.0540	0.0994	0.2217	0.5566	0.0205	0.0082
	RW50	0.0048	0.0527	-0.2759	6.0001	0.0904	0.2818	0.1467	0.1890	0.0325	0.0252
99% Confidence Level:											
MVaR-BL N	DCC	0.0153	0.1104	2.5860	23.2672	0.1385	0.1752	0.2674	0.3975	0.0572	0.0385
	EWMA	0.0044	0.0799	-2.3766	15.6055	0.0549	0.0932	0.4008	0.4556	0.0110	0.0096
	RW50	0.0028	0.0498	-0.6159	5.1348	0.0561	0.2482	0.1484	0.1848	0.0188	0.0151
MVaR-BL t	DCC	0.0169	0.1004	1.7206	12.8620	0.1684	0.2088	0.2218	0.3326	0.0762	0.0508
	EWMA	0.0093	0.0697	0.1540	10.1775	0.1331	0.2097	0.2440	0.2765	0.0380	0.0336
	RW50	0.0027	0.0488	-0.7334	5.1057	0.0550	0.2646	0.1468	0.1838	0.0183	0.0146
MCVaR-BL N	DCC	0.0160	0.1127	2.0535	20.5940	0.1423	0.1764	0.2711	0.4391	0.0591	0.0365
	EWMA	0.0066	0.0733	-2.1778	16.4459	0.0905	0.1642	0.2645	0.4151	0.0251	0.0160
	RW50	0.0027	0.0498	-0.6023	5.1435	0.0544	0.2442	0.1473	0.1843	0.0184	0.0147
MCVaR-BL t	DCC	0.0154	0.0914	2.1906	13.9749	0.1689	0.2157	0.2104	0.2313	0.0733	0.0667
	EWMA	0.0075	0.0725	-0.3494	10.3838	0.1036	0.1673	0.2906	0.2955	0.0259	0.0254
	RW50	0.0025	0.0476	-0.7460	5.0924	0.0525	0.2719	0.1455	0.1789	0.0172	0.0140

Table 5.1.19 (continued)

95% Confidence Level:											
MVaR-BL N	DCC	0.0136	0.1055	2.6546	28.7592	0.1293	0.1678	0.2657	0.4046	0.0514	0.0337
	EWMA	0.0079	0.0737	-1.6426	13.7627	0.1072	0.1851	0.2610	0.3927	0.0302	0.0201
	RW50	0.0031	0.0502	-0.6427	5.1037	0.0623	0.2647	0.1510	0.1863	0.0207	0.0168
MVaR-BL t	DCC	0.0166	0.1137	2.6157	22.5851	0.1465	0.1831	0.2689	0.4070	0.0619	0.0409
	EWMA	0.0039	0.0809	-2.3933	15.2673	0.0477	0.0835	0.4016	0.4553	0.0096	0.0085
	RW50	0.0029	0.0499	-0.6234	5.1297	0.0572	0.2513	0.1489	0.1851	0.0192	0.0154
MCVaR-BL N	DCC	0.0168	0.1150	2.6199	22.5656	0.1462	0.1824	0.2694	0.4104	0.0624	0.0410
	EWMA	0.0043	0.0797	-2.2161	14.0565	0.0540	0.0918	0.3983	0.4323	0.0108	0.0099
	RW50	0.0029	0.0499	-0.6274	5.1388	0.0588	0.2545	0.1491	0.1852	0.0197	0.0158
MCVaR-BL t	DCC	0.0161	0.1014	1.9091	15.6466	0.1584	0.2002	0.2238	0.3497	0.0718	0.0459
	EWMA	0.0097	0.0704	0.1320	10.2760	0.1372	0.2172	0.2529	0.2802	0.0382	0.0345
	RW50	0.0026	0.0490	-0.7131	5.0452	0.0539	0.2578	0.1464	0.1838	0.0180	0.0144
90% Confidence Level:											
MVaR-BL N	DCC	0.0144	0.1057	2.8404	27.7814	0.1364	0.1757	0.2648	0.3794	0.0544	0.0380
	EWMA	0.0076	0.0738	-1.2859	13.7266	0.1033	0.1762	0.2373	0.3822	0.0321	0.0199
	RW50	0.0035	0.0507	-0.7004	5.3330	0.0683	0.2762	0.1562	0.1933	0.0222	0.0179
MVaR-BL t	DCC	0.0146	0.1051	2.9071	28.8546	0.1391	0.1787	0.2634	0.3783	0.0555	0.0386
	EWMA	0.0076	0.0743	-1.6961	13.9033	0.1024	0.1771	0.2976	0.4030	0.0256	0.0189
	RW50	0.0033	0.0505	-0.6850	5.2637	0.0650	0.2712	0.1523	0.1911	0.0215	0.0172
MCVaR-BL N	DCC	0.0137	0.1057	2.5404	28.0441	0.1297	0.1676	0.2655	0.4090	0.0516	0.0335
	EWMA	0.0077	0.0736	-1.7507	14.0583	0.1051	0.1826	0.2834	0.3994	0.0273	0.0194
	RW50	0.0031	0.0500	-0.6348	5.1242	0.0627	0.2655	0.1503	0.1859	0.0209	0.0169
MCVaR-BL t	DCC	0.0150	0.1084	2.5662	23.0522	0.1381	0.1756	0.2661	0.3893	0.0563	0.0385
	EWMA	0.0035	0.0821	-2.4120	14.9481	0.0429	0.0770	0.4015	0.4559	0.0088	0.0077
	RW50	0.0027	0.0498	-0.6076	5.1347	0.0545	0.2438	0.1480	0.1846	0.0183	0.0147

Table 5.2.1 Weights in the VaR-Constrained BL Portfolio in August 1998

This table reports weights allocated to each index in the VaR-constrained BL portfolio in August 1998. The standard deviation is estimated by a conditional covariance matrix of DCC, EWMA and RW50 models. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

Aug 98	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.1005	0.1447	0.0341	0.0875	0.0450	0.0327
UK CONSUMER GDS	0.0481	0.1288	0.0339	0.0387	0.0146	0.0312
UK CONSUMER SVS	-0.0197	-0.0327	0.0155	-0.0144	-0.0272	0.0142
UK FINANCIALS	0.0220	0.0581	0.0411	0.0180	0.0035	0.0393
UK HEALTH CARE	-0.0407	-0.0763	0.0111	-0.0268	-0.0191	0.0109
UK TECHNOLOGY	-0.0220	-0.0833	-0.0109	-0.0268	-0.1211	-0.0173
UK INDUSTRIALS	0.0073	0.0533	0.0176	0.0128	-0.0119	0.0156
UK OIL & GAS	0.0657	0.1019	0.0267	0.0490	0.0092	0.0258
UK TELECOM	-0.0549	-0.2828	-0.0362	-0.0515	-0.1174	-0.0355
UK UTILITIES	-0.0564	-0.1297	-0.0248	-0.0478	-0.0636	-0.0244
USA BASIC MATS	0.0437	0.0626	0.0248	0.0359	0.0158	0.0234
USA CONSUMER GDS	-0.0488	-0.0707	-0.0014	-0.0382	-0.0517	-0.0026
USA CONSUMER SVS	0.0174	-0.0484	0.0546	0.0270	-0.0011	0.0531
USA FINANCIALS	0.0263	-0.0031	0.0813	0.0407	0.0157	0.0788
USA HEALTH CARE	0.0473	0.0162	0.1121	0.0655	0.0703	0.1115
USA INDUSTRIALS	0.0070	-0.0255	0.0416	0.0163	0.0083	0.0400
USA OIL & GAS	0.0789	0.1073	0.0527	0.0652	0.0291	0.0517
USA TECHNOLOGY	0.0541	0.0184	0.0842	0.0607	0.0466	0.0824
USA TELECOM	-0.0599	-0.1684	-0.0269	-0.0485	-0.0805	-0.0268
USA UTILITIES	-0.0216	-0.0805	0.0118	-0.0095	-0.0282	0.0111
JAPAN BASIC MATS	0.0849	0.1174	0.0422	0.0796	0.1847	0.0476
JAPAN CONSUMER GDS	0.1016	0.0960	0.0526	0.0878	0.0767	0.0540
JAPAN CONSUMER SVS	0.0829	0.1281	0.0503	0.0748	0.1213	0.0535
JAPAN FINANCIALS	0.1133	0.1557	0.0691	0.1129	0.1911	0.0712
JAPAN HEALTH CARE	0.0436	0.1187	0.0366	0.0409	0.0990	0.0389
JAPAN INDUSTRIALS	0.1061	0.1623	0.0461	0.0896	0.1162	0.0495
JAPAN OIL & GAS	0.0895	0.0935	0.0339	0.1018	0.2010	0.0388
JAPAN TECHNOLOGY	0.1229	0.2634	0.0649	0.1035	0.1346	0.0663
JAPAN TELECOM	0.0814	0.1812	0.0489	0.0658	0.0911	0.0509
JAPAN UTILITIES	-0.0204	-0.0062	0.0122	-0.0106	0.0481	0.0143

Table 5.2.2 Weights in the VaR-Constrained BL Portfolio in November 1998

This table reports weights allocated to each index in the VaR-constrained BL portfolio in November 1998. In the portfolio construction, the standard deviation is estimated by the conditional covariance matrix of DCC, EWMA and RW50 models. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

Nov 98	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	-0.0248	-0.0436	-0.0117	-0.0521	-0.1409	-0.0059
UK CONSUMER GDS	-0.0275	-0.0372	-0.0161	0.0102	0.0327	-0.0067
UK CONSUMER SVS	-0.0434	-0.0636	0.0179	0.0928	-0.1104	0.0233
UK FINANCIALS	-0.0020	-0.0158	0.0242	-0.0117	0.0301	0.0241
UK HEALTH CARE	0.0784	0.1018	0.0447	0.0616	0.1844	0.0461
UK TECHNOLOGY	-0.0126	-0.0203	0.0014	-0.0177	-0.0236	0.0005
UK INDUSTRIALS	-0.0192	-0.0330	-0.0095	-0.0238	0.1126	-0.0024
UK OIL & GAS	-0.0357	-0.0488	-0.0126	-0.0653	-0.1192	-0.0091
UK TELECOM	0.0586	0.0713	0.0464	0.0336	-0.0173	0.0477
UK UTILITIES	0.0705	0.0755	0.0404	0.0725	0.1411	0.0431
USA BASIC MATS	-0.0211	-0.0372	0.0045	0.0513	0.0230	0.0049
USA CONSUMER GDS	-0.0147	-0.0301	-0.0110	-0.0477	-0.0267	-0.0134
USA CONSUMER SVS	0.1162	0.1350	0.0749	0.1256	0.2004	0.0719
USA FINANCIALS	0.0761	0.0665	0.0849	0.0083	0.0143	0.0765
USA HEALTH CARE	0.1765	0.1923	0.1514	0.1377	0.1603	0.1475
USA INDUSTRIALS	0.0112	-0.0047	0.0449	-0.0379	-0.1689	0.0411
USA OIL & GAS	-0.0127	-0.0250	0.0300	0.0492	0.0826	0.0330
USA TECHNOLOGY	0.1326	0.1455	0.1027	0.0512	0.0680	0.0928
USA TELECOM	0.0659	0.0830	0.0422	0.0446	0.0355	0.0389
USA UTILITIES	0.1199	0.1428	0.0934	0.1410	0.1650	0.0980
JAPAN BASIC MATS	0.0391	0.0473	0.0400	-0.0310	0.0359	0.0409
JAPAN CONSUMER GDS	-0.0297	-0.0329	0.0263	0.0431	-0.0749	0.0256
JAPAN CONSUMER SVS	0.0667	0.0709	0.0404	0.1593	-0.0094	0.0408
JAPAN FINANCIALS	0.0050	-0.0013	0.0078	-0.0277	-0.0852	0.0010
JAPAN HEALTH CARE	0.0727	0.0651	0.0322	0.1233	-0.0335	0.0324
JAPAN INDUSTRIALS	0.0587	0.0694	0.0286	0.1408	0.3961	0.0298
JAPAN OIL & GAS	0.0263	0.0321	0.0219	-0.0020	0.0312	0.0180
JAPAN TECHNOLOGY	-0.0167	-0.0264	0.0030	-0.0706	-0.0971	0.0000
JAPAN TELECOM	0.0466	0.0626	0.0161	0.0384	0.0785	0.0162
JAPAN UTILITIES	0.0389	0.0590	0.0404	0.0029	0.1153	0.0434

Table 5.2.3 VaR-Constrained BL Portfolio Performance in the Single Period

This table reports the VaR-constrained BL portfolio performance evaluated by realised return, CSR, PT, reward to CVaR ratio in August 1998 and November 1998. The standard deviation is calculated by a dynamic covariance matrix of DCC, EWMA and RW50 models. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period. Note that I follow Israelsen's (2003) method to adjust the CSR and the reward to CVaR ratio in August 1998 because the negative realised excess return would lead to invalid SR measures for portfolio evaluation.

Panel A: Aug 1998

	Normal Distribution				t-Distribution			
	Realised Excess Return	Adjusted CSR	PT	Adjusted Reward to CVaR	Realised Return	Adjusted CSR	PT	Adjusted Reward to CVaR
DCC	-0.1166	-0.5372	3.2104	-2.7915	-0.0661	-0.1850	1.7834	-0.9305
EWMA	-0.1236	-1.1429	5.1285	-4.5738	-0.0737	-0.4143	3.0409	-1.6472
RW50	-0.0953	-0.2717	N/A	-1.6323	-0.0343	-0.0653	N/A	-0.2918

Panel B: Nov 1998

	Normal Distribution				t-Distribution			
	Realised Excess Return	CSR	PT	Reward to CVaR	Realised Return	CSR	PT	Reward to CVaR
DCC	0.0444	0.9132	1.5942	0.5212	0.0326	0.9909	1.2594	0.5918
EWMA	0.0448	0.9129	2.4943	0.5210	0.0288	0.9001	1.4859	0.5099
RW50	0.0458	1.3896	0.7923	1.0894	0.0382	1.7210	1.4397	1.8226

Table 5.2.4 VaR-Constrained BL Portfolio Performance in Multiple Periods

This table shows realised VaR-constrained BL portfolio performance in the period from November 1994 to May 2010, and the sub-period from August 1998 to May 2010. Return is the average realised excess return, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
Panel A: Normal Distribution (Nov 94-May 10)										
DCC	0.0132	0.0689	1.2655	13.2598	0.1922	0.2560	0.1895	0.2278	0.0699	0.0581
EWMA	0.0099	0.0553	-0.0008	7.3906	0.1787	0.2665	0.1485	0.1980	0.0665	0.0499
RW50	0.0056	0.0462	0.2770	7.1026	0.1212	0.2290	0.1069	0.1469	0.0523	0.0381
Panel B: t-Distribution (Nov 94-May 10)										
	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0079	0.0477	0.9542	10.7419	0.1655	0.2087	0.1327	0.1609	0.0595	0.0491
EWMA	0.0060	0.0388	1.2181	14.9013	0.1559	0.1731	0.1166	0.1415	0.0519	0.0427
RW50	0.0014	0.0383	0.5419	6.7292	0.0365	0.0311	0.0979	0.1115	0.0143	0.0125
Panel C: Normal Distribution (Aug 98-May 10)										
	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0105	0.0717	1.2366	13.8451	0.1467	0.2314	0.2183	0.2290	0.0482	0.0459
EWMA	0.0073	0.0575	-0.1498	7.4068	0.1265	0.2390	0.1634	0.2118	0.0445	0.0343
RW50	0.0036	0.0478	-0.0211	6.1351	0.0751	0.2476	0.1160	0.1597	0.0310	0.0225
Panel D: t-Distribution (Aug 98-May 10)										
	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0055	0.0488	0.8070	11.1746	0.1124	0.1917	0.1556	0.1610	0.0352	0.0341
EWMA	0.0044	0.0408	1.1484	15.1532	0.1078	0.1734	0.1389	0.1404	0.0316	0.0313
RW50	0.0003	0.0402	0.0377	4.3037	0.0081	0.0615	0.1085	0.1117	0.0030	0.0029

Table 5.2.5 Effects on the VaR-Constrained BL Portfolio Optimisation (Aug 1998)

This table reports the statistics inputted into the VaR-constrained SR-BL model such as estimated expected BL return (μ) and standard deviation (based on the DCC model) and the results of ECSR, expected excess return to VaR ratio (μ / VaR) and expected excess return to CVaR ratio ($\mu / CVaR$) in the optimisation process. VaR and CVaR are estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. The VaR constraint (VaR_0) is set to be equal to the scaling factor (VaR Factor) multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

Panel A: Normal Distribution

99% Confidence Level:

VaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	ECSR
0.99	0.0047	0.0461	0.1025	0.1181	0.0461	0.0400	0.1025
0.95	0.0045	0.0442	0.0983	0.1133	0.0459	0.0399	0.1022
0.90	0.0043	0.0419	0.0931	0.1073	0.0457	0.0396	0.1016
0.80	0.0037	0.0372	0.0828	0.0954	0.0447	0.0388	0.0995
0.70	0.0031	0.0325	0.0724	0.0834	0.0426	0.0370	0.0950
0.60	0.0023	0.0277	0.0621	0.0715	0.0374	0.0325	0.0839
0.50	0.0011	0.0247	0.0565	0.0648	0.0195	0.0170	0.0445

95% Confidence Level:

VaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0059	0.0568	0.0876	0.1114	0.0669	0.0527	0.1032
0.95	0.0059	0.0568	0.0876	0.1114	0.0669	0.0527	0.1032
0.90	0.0059	0.0568	0.0876	0.1114	0.0669	0.0527	0.1032
0.80	0.0055	0.0537	0.0828	0.1052	0.0669	0.0526	0.1031
0.70	0.0048	0.0470	0.0724	0.0921	0.0666	0.0524	0.1027
0.60	0.0041	0.0402	0.0621	0.0789	0.0654	0.0515	0.1010
0.50	0.0032	0.0334	0.0517	0.0657	0.0621	0.0489	0.0962

90% Confidence Level:

VaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.95	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.90	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.80	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.70	0.0052	0.0507	0.0724	0.1214	0.0720	0.0430	0.1030
0.60	0.0044	0.0434	0.0621	0.1040	0.0713	0.0425	0.1020
0.50	0.0036	0.0361	0.0517	0.0866	0.0688	0.0411	0.0987

Table 5.2.5 (continued)

Panel B: t-Distribution

99% Confidence Level:

VaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0024	0.0280	0.1025	0.1437	0.0232	0.0165	0.0850
0.95	0.0021	0.0268	0.0983	0.1378	0.0218	0.0156	0.0800
0.90	0.0018	0.0253	0.0931	0.1305	0.0190	0.0136	0.0699
0.80	0.0012	0.0252	0.0932	0.1304	0.0133	0.0095	0.0492
0.70	N/A	N/A	N/A	N/A	N/A	N/A	N/A

95% Confidence Level:

VaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0052	0.0505	0.1025	0.1565	0.0508	0.0332	0.1030
0.95	0.0050	0.0485	0.0983	0.1502	0.0507	0.0332	0.1028
0.90	0.0047	0.0459	0.0931	0.1423	0.0505	0.0331	0.1025
0.80	0.0041	0.0408	0.0828	0.1265	0.0498	0.0326	0.1012
0.70	0.0035	0.0356	0.0724	0.1106	0.0484	0.0317	0.0984
0.60	0.0028	0.0304	0.0621	0.0947	0.0449	0.0294	0.0916
0.50	0.0017	0.0251	0.0517	0.0786	0.0325	0.0214	0.0672

90% Confidence Level:

VaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.95	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.90	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.80	0.0059	0.0568	0.0813	0.1362	0.0722	0.0431	0.1032
0.70	0.0052	0.0507	0.0724	0.1214	0.0720	0.0430	0.1030
0.60	0.0044	0.0434	0.0621	0.1040	0.0713	0.0425	0.1020
0.50	0.0036	0.0361	0.0517	0.0866	0.0688	0.0411	0.0987

Table 5.2.6 Effects on Weights of the VaR-Constrained BL Portfolio (Aug 98)

This table shows the positions of each asset in a VaR-constrained BL portfolio in August 1998 under normal distribution and t-distribution at a confidence level of 99% (Panel A), 95% (Panel B), and 90% (Panel C). Note that the covariance matrix applied to the VaR-constrained BL model is the DCC covariance matrix in this table.

Panel A: 99% Confidence Level	Normal Distribution					t-Distribution				
VaR Factor:	0.99	0.95	0.90	0.80	0.70	0.99	0.95	0.90	0.80	0.70
UK BASIC MATS	0.0301	0.0238	0.0158	-0.0011	-0.0198	-0.0414	-0.0485	-0.0599	-0.1182	N/A
UK CONSUMER GDS	0.0499	0.0479	0.0452	0.0397	0.0335	0.0264	0.0241	0.0204	0.0269	N/A
UK CONSUMER SVS	-0.0159	-0.0045	0.0101	0.0407	0.0747	0.1138	0.1266	0.1474	0.2200	N/A
UK FINANCIALS	0.1184	0.1138	0.1079	0.0954	0.0816	0.0657	0.0605	0.0521	0.0109	N/A
UK HEALTH CARE	-0.0391	-0.0330	-0.0252	-0.0090	0.0090	0.0298	0.0366	0.0475	0.0756	N/A
UK TECHNOLOGY	-0.0325	-0.0327	-0.0330	-0.0335	-0.0341	-0.0348	-0.0350	-0.0354	-0.0307	N/A
UK INDUSTRIALS	-0.0404	-0.0396	-0.0386	-0.0365	-0.0343	-0.0316	-0.0307	-0.0294	-0.0119	N/A
UK OIL & GAS	0.0381	0.0295	0.0185	-0.0046	-0.0301	-0.0595	-0.0692	-0.0848	-0.0899	N/A
UK TELECOM	-0.0362	-0.0364	-0.0367	-0.0373	-0.0380	-0.0388	-0.0391	-0.0395	-0.0435	N/A
UK UTILITIES	-0.0421	-0.0325	-0.0202	0.0055	0.0340	0.0669	0.0777	0.0951	0.1202	N/A
USA BASIC MATS	0.0965	0.0928	0.0882	0.0783	0.0674	0.0548	0.0507	0.0441	0.0634	N/A
USA CONSUMER GDS	-0.0706	-0.0698	-0.0688	-0.0667	-0.0643	-0.0616	-0.0607	-0.0592	-0.0282	N/A
USA CONSUMER SVS	0.0501	0.0625	0.0785	0.1121	0.1494	0.1923	0.2064	0.2291	0.2195	N/A
USA FINANCIALS	0.0130	0.0085	0.0028	-0.0093	-0.0227	-0.0381	-0.0432	-0.0513	-0.0414	N/A
USA HEALTH CARE	0.0757	0.0818	0.0897	0.1063	0.1247	0.1459	0.1528	0.1640	0.1285	N/A
USA INDUSTRIALS	-0.0585	-0.0637	-0.0704	-0.0844	-0.0999	-0.1178	-0.1237	-0.1331	-0.0653	N/A
USA OIL & GAS	0.1655	0.1688	0.1731	0.1820	0.1919	0.2032	0.2070	0.2130	0.2222	N/A
USA TECHNOLOGY	0.0264	0.0219	0.0161	0.0040	-0.0095	-0.0250	-0.0301	-0.0383	-0.1088	N/A
USA TELECOM	-0.0360	-0.0302	-0.0228	-0.0074	0.0096	0.0293	0.0357	0.0461	0.0891	N/A
USA UTILITIES	-0.0115	-0.0069	-0.0011	0.0111	0.0246	0.0402	0.0454	0.0536	-0.0112	N/A
JAPAN BASIC MATS	0.1019	0.1017	0.1014	0.1007	0.1000	0.0992	0.0989	0.0985	0.1650	N/A
JAPAN CONSUMER GDS	0.1034	0.0994	0.0943	0.0834	0.0714	0.0576	0.0530	0.0457	0.0220	N/A
JAPAN CONSUMER SVS	0.1273	0.1275	0.1277	0.1281	0.1285	0.1291	0.1292	0.1295	0.0963	N/A
JAPAN FINANCIALS	0.0488	0.0410	0.0312	0.0106	-0.0123	-0.0386	-0.0472	-0.0612	-0.0688	N/A
JAPAN HEALTH CARE	0.0593	0.0488	0.0354	0.0072	-0.0239	-0.0599	-0.0717	-0.0907	-0.0641	N/A
JAPAN INDUSTRIALS	0.1293	0.1326	0.1368	0.1457	0.1556	0.1669	0.1707	0.1767	0.0836	N/A
JAPAN OIL & GAS	0.0217	0.0161	0.0089	-0.0063	-0.0231	-0.0425	-0.0488	-0.0591	-0.0930	N/A
JAPAN TECHNOLOGY	0.0497	0.0418	0.0318	0.0107	-0.0126	-0.0395	-0.0484	-0.0626	-0.0814	N/A
JAPAN TELECOM	0.1035	0.1037	0.1040	0.1047	0.1055	0.1064	0.1067	0.1072	0.1612	N/A
JAPAN UTILITIES	-0.0259	-0.0147	-0.0003	0.0298	0.0631	0.1016	0.1142	0.1345	0.1521	N/A

Table 5.2.6 (continued)

Panel B: 95% Confidence Level	Normal Distribution					t-Distribution				
VaR Factor:	0.99	0.95	0.90	0.80	0.7	0.99	0.95	0.90	0.80	0.70
UK BASIC MATS	0.0649	0.0649	0.0649	0.0550	0.0331	0.0447	0.0380	0.0295	0.0119	-0.0070
UK CONSUMER GDS	0.0614	0.0614	0.0614	0.0581	0.0509	0.0547	0.0525	0.0498	0.0440	0.0377
UK CONSUMER SVS	-0.0790	-0.0790	-0.0790	-0.0610	-0.0214	-0.0424	-0.0303	-0.0149	0.0171	0.0515
UK FINANCIALS	0.1441	0.1441	0.1441	0.1368	0.1206	0.1292	0.1243	0.1180	0.1050	0.0911
UK HEALTH CARE	-0.0725	-0.0725	-0.0725	-0.0630	-0.0420	-0.0531	-0.0467	-0.0385	-0.0215	-0.0033
UK TECHNOLOGY	-0.0314	-0.0314	-0.0314	-0.0317	-0.0324	-0.0321	-0.0323	-0.0325	-0.0331	-0.0337
UK INDUSTRIALS	-0.0446	-0.0446	-0.0446	-0.0434	-0.0407	-0.0421	-0.0413	-0.0403	-0.0381	-0.0358
UK OIL & GAS	0.0857	0.0857	0.0857	0.0721	0.0422	0.0580	0.0489	0.0373	0.0133	-0.0126
UK TELECOM	-0.0349	-0.0349	-0.0349	-0.0353	-0.0361	-0.0357	-0.0359	-0.0362	-0.0369	-0.0376
UK UTILITIES	-0.0951	-0.0951	-0.0951	-0.0800	-0.0467	-0.0643	-0.0541	-0.0412	-0.0144	0.0145
USA BASIC MATS	0.1168	0.1168	0.1168	0.1110	0.0983	0.1050	0.1011	0.0962	0.0859	0.0749
USA CONSUMER GDS	-0.0751	-0.0751	-0.0751	-0.0738	-0.0710	-0.0724	-0.0716	-0.0706	-0.0683	-0.0659
USA CONSUMER SVS	-0.0192	-0.0192	-0.0192	0.0006	0.0440	0.0210	0.0343	0.0512	0.0861	0.1239
USA FINANCIALS	0.0379	0.0379	0.0379	0.0308	0.0152	0.0234	0.0187	0.0126	0.0000	-0.0135
USA HEALTH CARE	0.0415	0.0415	0.0415	0.0512	0.0727	0.0613	0.0679	0.0762	0.0935	0.1121
USA INDUSTRIALS	-0.0297	-0.0297	-0.0297	-0.0379	-0.0560	-0.0465	-0.0520	-0.0590	-0.0736	-0.0893
USA OIL & GAS	0.1470	0.1470	0.1470	0.1523	0.1639	0.1578	0.1613	0.1658	0.1751	0.1851
USA TECHNOLOGY	0.0515	0.0515	0.0515	0.0443	0.0286	0.0369	0.0321	0.0260	0.0133	-0.0003
USA TELECOM	-0.0676	-0.0676	-0.0676	-0.0586	-0.0387	-0.0492	-0.0431	-0.0354	-0.0193	-0.0021
USA UTILITIES	-0.0367	-0.0367	-0.0367	-0.0295	-0.0137	-0.0220	-0.0172	-0.0111	0.0017	0.0154
JAPAN BASIC MATS	0.1033	0.1033	0.1033	0.1029	0.1020	0.1025	0.1022	0.1019	0.1012	0.1005
JAPAN CONSUMER GDS	0.1258	0.1258	0.1258	0.1194	0.1054	0.1128	0.1085	0.1031	0.0918	0.0796
JAPAN CONSUMER SVS	0.1265	0.1265	0.1265	0.1267	0.1272	0.1269	0.1271	0.1273	0.1278	0.1282
JAPAN FINANCIALS	0.0912	0.0912	0.0912	0.0791	0.0524	0.0665	0.0584	0.0480	0.0265	0.0034
JAPAN HEALTH CARE	0.1173	0.1173	0.1173	0.1008	0.0644	0.0836	0.0725	0.0584	0.0290	-0.0026
JAPAN INDUSTRIALS	0.1109	0.1109	0.1109	0.1162	0.1277	0.1217	0.1251	0.1296	0.1389	0.1488
JAPAN OIL & GAS	0.0530	0.0530	0.0530	0.0441	0.0244	0.0348	0.0288	0.0212	0.0054	-0.0116
JAPAN TECHNOLOGY	0.0931	0.0931	0.0931	0.0807	0.0535	0.0679	0.0596	0.0490	0.0270	0.0034
JAPAN TELECOM	0.1020	0.1020	0.1020	0.1024	0.1033	0.1028	0.1031	0.1035	0.1042	0.1050
JAPAN UTILITIES	-0.0879	-0.0879	-0.0879	-0.0702	-0.0313	-0.0519	-0.0400	-0.0249	0.0065	0.0403

Table 5.2.6 (continued)

Panel C: 90% Confidence Level	Normal Distribution					t-Distribution				
VaR Factor:	0.99	0.95	0.90	0.80	0.70	0.99	0.95	0.90	0.80	0.70
UK BASIC MATS	0.0649	0.0649	0.0649	0.0649	0.0649	0.0649	0.0649	0.0649	0.0649	0.0452
UK CONSUMER GDS	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0549
UK CONSUMER SVS	-0.0790	-0.0790	-0.0790	-0.0790	-0.0790	-0.0790	-0.0790	-0.0790	-0.0790	-0.0433
UK FINANCIALS	0.1441	0.1441	0.1441	0.1441	0.1441	0.1441	0.1441	0.1441	0.1441	0.1296
UK HEALTH CARE	-0.0725	-0.0725	-0.0725	-0.0725	-0.0725	-0.0725	-0.0725	-0.0725	-0.0725	-0.0535
UK TECHNOLOGY	-0.0314	-0.0314	-0.0314	-0.0314	-0.0314	-0.0314	-0.0314	-0.0314	-0.0314	-0.0321
UK INDUSTRIALS	-0.0446	-0.0446	-0.0446	-0.0446	-0.0446	-0.0446	-0.0446	-0.0446	-0.0446	-0.0422
UK OIL & GAS	0.0857	0.0857	0.0857	0.0857	0.0857	0.0857	0.0857	0.0857	0.0857	0.0587
UK TELECOM	-0.0349	-0.0349	-0.0349	-0.0349	-0.0349	-0.0349	-0.0349	-0.0349	-0.0349	-0.0356
UK UTILITIES	-0.0951	-0.0951	-0.0951	-0.0951	-0.0951	-0.0951	-0.0951	-0.0951	-0.0951	-0.0651
USA BASIC MATS	0.1168	0.1168	0.1168	0.1168	0.1168	0.1168	0.1168	0.1168	0.1168	0.1053
USA CONSUMER GDS	-0.0751	-0.0751	-0.0751	-0.0751	-0.0751	-0.0751	-0.0751	-0.0751	-0.0751	-0.0725
USA CONSUMER SVS	-0.0192	-0.0192	-0.0192	-0.0192	-0.0192	-0.0192	-0.0192	-0.0192	-0.0192	0.0200
USA FINANCIALS	0.0379	0.0379	0.0379	0.0379	0.0379	0.0379	0.0379	0.0379	0.0379	0.0238
USA HEALTH CARE	0.0415	0.0415	0.0415	0.0415	0.0415	0.0415	0.0415	0.0415	0.0415	0.0608
USA INDUSTRIALS	-0.0297	-0.0297	-0.0297	-0.0297	-0.0297	-0.0297	-0.0297	-0.0297	-0.0297	-0.0461
USA OIL & GAS	0.1470	0.1470	0.1470	0.1470	0.1470	0.1470	0.1470	0.1470	0.1470	0.1575
USA TECHNOLOGY	0.0515	0.0515	0.0515	0.0515	0.0515	0.0515	0.0515	0.0515	0.0515	0.0373
USA TELECOM	-0.0676	-0.0676	-0.0676	-0.0676	-0.0676	-0.0676	-0.0676	-0.0676	-0.0676	-0.0496
USA UTILITIES	-0.0367	-0.0367	-0.0367	-0.0367	-0.0367	-0.0367	-0.0367	-0.0367	-0.0367	-0.0224
JAPAN BASIC MATS	0.1033	0.1033	0.1033	0.1033	0.1033	0.1033	0.1033	0.1033	0.1033	0.1025
JAPAN CONSUMER GDS	0.1258	0.1258	0.1258	0.1258	0.1258	0.1258	0.1258	0.1258	0.1258	0.1131
JAPAN CONSUMER SVS	0.1265	0.1265	0.1265	0.1265	0.1265	0.1265	0.1265	0.1265	0.1265	0.1269
JAPAN FINANCIALS	0.0912	0.0912	0.0912	0.0912	0.0912	0.0912	0.0912	0.0912	0.0912	0.0671
JAPAN HEALTH CARE	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173	0.0844
JAPAN INDUSTRIALS	0.1109	0.1109	0.1109	0.1109	0.1109	0.1109	0.1109	0.1109	0.1109	0.1214
JAPAN OIL & GAS	0.0530	0.0530	0.0530	0.0530	0.0530	0.0530	0.0530	0.0530	0.0530	0.0353
JAPAN TECHNOLOGY	0.0931	0.0931	0.0931	0.0931	0.0931	0.0931	0.0931	0.0931	0.0931	0.0685
JAPAN TELECOM	0.1020	0.1020	0.1020	0.1020	0.1020	0.1020	0.1020	0.1020	0.1020	0.1028
JAPAN UTILITIES	-0.0879	-0.0879	-0.0879	-0.0879	-0.0879	-0.0879	-0.0879	-0.0879	-0.0879	-0.0527

Table 5.2.7 Effects on VaR-Constrained SR-BL Portfolio Performance Evaluation (Aug 98)

This table reports VaR-constrained BL portfolio performance results including realised return, CSR, PT and reward to CVaR ratio in August 1998. The standard deviation is estimated by a conditional covariance matrix of the DCC model. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. The VaR constraint (VaR_0) is set to be equal to the scaling factor k multiplied by the estimated VaR of the implied BL portfolio. The scaling factor k is called VaR factor. Following Israelsen's (2003) method, the adjusted CSR is equal to the product of negative realised excess return and the standard deviation multiplied by 100. Using this idea, I adjust the reward to CVaR ratio equal to the product of negative realised excess return and CVaR multiplied by 100.

Panel A: 99% Confidence Level

	Normal Distribution				t-Distribution			
VaR Factor	Realised Excess Return	Adjusted CSR	Portfolio Turnover	Adjusted Reward to CVaR	Realised Excess Return	Adjusted CSR	Portfolio Turnover	Adjusted Reward to CVaR
0.99	-0.1166	-0.5372	3.2104	-2.7915	-0.0661	-0.1850	1.7834	-0.9305
0.95	-0.1122	-0.4958	3.1625	-2.5794	-0.0611	-0.1639	1.7147	-0.8108
0.90	-0.1065	-0.4458	3.0055	-2.3220	-0.0531	-0.1345	1.6541	-0.6402
0.80	-0.0946	-0.3516	2.6077	-1.8313	-0.0507	-0.1279	1.4237	-0.5984
0.70	-0.0814	-0.2641	2.1825	-1.3660	N/A	N/A	N/A	N/A
0.60	-0.0650	-0.1799	1.7735	-0.9016	N/A	N/A	N/A	N/A
0.50	-0.0528	-0.1307	1.8450	-0.6273	N/A	N/A	N/A	N/A

Panel B: 95% Confidence Level

	Normal Distribution				t-Distribution			
VaR Factor	Realised Excess Return	Adjusted CSR	Portfolio Turnover	Adjusted Reward to CVaR	Realised Excess Return	Adjusted CSR	Portfolio Turnover	Adjusted Reward to CVaR
0.99	-0.1412	-0.8023	3.6582	-4.1311	-0.1269	-0.6408	3.3383	-3.3179
0.95	-0.1412	-0.8023	3.6582	-4.1311	-0.1222	-0.5920	3.2790	-3.0706
0.90	-0.1412	-0.8023	3.6582	-4.1311	-0.1162	-0.5333	3.2059	-2.7714
0.80	-0.1342	-0.7204	3.4873	-3.7197	-0.1038	-0.4231	2.9162	-2.2046
0.70	-0.1187	-0.5577	3.2360	-2.8960	-0.0904	-0.3220	2.4730	-1.6753
0.60	-0.1024	-0.4118	2.8767	-2.1461	-0.0750	-0.2282	1.9896	-1.1702
0.50	-0.0842	-0.2812	2.2801	-1.4578	-0.0512	-0.1283	1.6679	-0.6038

Panel C: 90% Confidence Level

	Normal Distribution				t-Distribution			
VaR Factor	Realised Excess Return	Adjusted CSR	Portfolio Turnover	Adjusted Reward to CVaR	Realised Excess Return	Adjusted CSR	Portfolio Turnover	Adjusted Reward to CVaR
0.99	-0.1412	-0.8023	3.6582	-4.1311	-0.1412	-0.8023	3.6582	-4.1311
0.95	-0.1412	-0.8023	3.6582	-4.1311	-0.1412	-0.8023	3.6582	-4.1311
0.90	-0.1412	-0.8023	3.6582	-4.1311	-0.1412	-0.8023	3.6582	-4.1311
0.80	-0.1412	-0.8023	3.6582	-4.1311	-0.1412	-0.8023	3.6582	-4.1311
0.70	-0.1412	-0.8023	3.6582	-4.1311	-0.1272	-0.6445	3.3435	-3.3367
0.60	-0.1319	-0.6948	3.4317	-3.5910	-0.1102	-0.4781	3.1413	-2.4884
0.50	-0.1114	-0.4886	3.1539	-2.5425	-0.0916	-0.3304	2.5219	-1.7199

Table 5.2.8 Effects on VaR-Constrained SR-BL Portfolio Performance Evaluation (Nov 98)

This table reports VaR-constrained BL portfolio performance results including realised excess return, CSR, PT and reward to CVaR ratio in November 1998. The standard deviation is estimated by a conditional covariance matrix of the DCC model. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. The VaR constraint (VaR_0) is set to be equal to the scaling factor k multiplied by the estimated VaR of the implied BL portfolio. The scaling factor k is called VaR factor. August 1998 is the first period to construct a portfolio in the use of the RW method with a 50 window length, therefore, there are no results of PT shown for RW50. Following Israelsen's (2003) method, the adjusted CSR is equal to the product of negative realised excess return and the standard deviation multiplied by 100. Using this idea, I adjust the reward to CVaR ratio equal to the product of negative realised excess return and CVaR multiplied by 100.

Panel A: 99% Confidence Level

	Normal Distribution				t-Distribution			
VaR Factor	Realised Excess Return	Conditional Sharpe Ratio	Portfolio Turnover	Reward to CVaR	Realised Excess Return	Conditional Sharpe Ratio	Portfolio Turnover	Reward to CVaR
0.99	0.0444	0.9132	1.5942	0.5212	0.0326	0.9909	1.2594	0.5918
0.95	0.0444	0.9132	1.5743	0.5212	0.0314	0.9940	1.2451	0.5947
0.90	0.0444	0.9132	1.5414	0.5212	0.0297	0.9938	1.2447	0.5946
0.80	0.0407	0.9407	1.4109	0.5455	0.0254	0.9570	1.2892	0.5603
0.70	0.0367	0.9694	1.3099	0.5716	N/A	N/A	N/A	N/A
0.60	0.0322	0.9924	1.2531	0.5932	N/A	N/A	N/A	N/A
0.50	0.0260	0.9658	1.2768	0.5683	N/A	N/A	N/A	N/A

Panel B: 95% Confidence Level

	Normal Distribution				t-Distribution			
VaR Factor	Realised Excess Return	Conditional Sharpe Ratio	Portfolio Turnover	Reward to CVaR	Realised Excess Return	Conditional Sharpe Ratio	Portfolio Turnover	Reward to CVaR
0.99	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.95	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.90	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.80	0.0444	0.9132	1.5942	0.5212	0.0435	0.9201	1.5043	0.5272
0.70	0.0444	0.9132	1.5942	0.5212	0.0393	0.9506	1.3687	0.5544
0.60	0.0428	0.9250	1.4793	0.5315	0.0348	0.9812	1.2861	0.5826
0.50	0.0372	0.9656	1.3168	0.5682	0.0293	0.9926	1.2457	0.5934

Panel C: 90% Confidence Level

	Normal Distribution				t-Distribution			
VaR Factor	Realised Excess Return	Conditional Sharpe Ratio	Portfolio Turnover	Reward to CVaR	Realised Excess Return	Conditional Sharpe Ratio	Portfolio Turnover	Reward to CVaR
0.99	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.95	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.90	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.80	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.70	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.60	0.0444	0.9132	1.5942	0.5212	0.0444	0.9132	1.5942	0.5212
0.50	0.0444	0.9132	1.5942	0.5212	0.0394	0.9501	1.3711	0.5540

Table 5.2.9 Effects on VaR-Constrained BL Portfolio Performance in Multiple Periods (Nov 94-May 10)

This table shows realised VaR-constrained BL portfolio performance in the period from November 1994 to May 2010. The conditional covariance matrix applied to the portfolio construction is the DCC model. Return is the average realised excess return, risk is the standard deviation, SR is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, VaR is estimated by the parametric method with assumption of normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. The VaR constraint (VaR_0) is set to be equal to the scaling factor k multiplied by the estimated VaR of the implied BL portfolio. The scaling factor k is called VaR factor.

Panel A: Normal Distribution (Nov 94 - May 10)

VaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0132	0.0689	1.2655	13.2598	0.1922	0.2560	0.1895	0.2278	0.0699	0.0581
0.95	0.0131	0.0676	1.3673	13.9462	0.1935	0.2575	0.1835	0.2211	0.0713	0.0592
0.90	0.0127	0.0654	1.3925	14.0490	0.1944	0.2590	0.1761	0.2127	0.0722	0.0598
0.80	0.0116	0.0600	1.3177	13.1911	0.1936	0.2585	0.1611	0.1957	0.0721	0.0594
0.70	0.0101	0.0539	1.1980	12.3143	0.1867	0.2465	0.1459	0.1782	0.0690	0.0565
0.60	0.0081	0.0478	1.0340	11.1983	0.1696	0.2123	0.1325	0.1599	0.0690	0.0565
0.50	0.0060	0.0415	0.8318	9.9352	0.1440	0.1608	0.1215	0.1398	0.0491	0.0427
95% Confidence Level:										
0.99	0.0143	0.0776	1.0805	12.0869	0.1922	0.2560	0.2243	0.2773	0.0637	0.0516
0.95	0.0142	0.0767	1.0595	11.9605	0.1846	0.2377	0.2243	0.2725	0.0631	0.0520
0.90	0.0140	0.0757	1.0384	11.8600	0.1848	0.2397	0.2243	0.2664	0.0624	0.0525
0.80	0.0136	0.0734	1.0356	11.9931	0.1856	0.2444	0.2134	0.2546	0.0638	0.0535
0.70	0.0133	0.0695	1.2360	13.0337	0.1917	0.2551	0.1925	0.2308	0.0692	0.0577
0.60	0.0124	0.0639	1.4576	14.4292	0.1945	0.2581	0.1710	0.2066	0.0727	0.0602
0.50	0.0105	0.0557	1.3362	13.1450	0.1885	0.2477	0.1492	0.1817	0.0704	0.0578
90% Confidence Level:										
0.99	0.0150	0.0843	1.3622	14.6555	0.1776	0.2181	0.2243	0.3150	0.0668	0.0475
0.95	0.0149	0.0831	1.3002	14.0508	0.1787	0.2209	0.2243	0.3087	0.0662	0.0481
0.90	0.0147	0.0817	1.2277	13.3753	0.1800	0.2244	0.2243	0.3008	0.0656	0.0489
0.80	0.0145	0.0788	1.1149	12.3723	0.1834	0.2327	0.2243	0.2849	0.0644	0.0507
0.70	0.0140	0.0760	1.0429	11.8938	0.1846	0.2388	0.2243	0.2690	0.0626	0.0522
0.60	0.0136	0.0729	1.0659	12.0696	0.1867	0.2461	0.2108	0.2509	0.0645	0.0542
0.50	0.0131	0.0672	1.3978	14.0736	0.1942	0.2579	0.1828	0.2196	0.0714	0.0595

Table 5.2.9 (continued)

Panel B: t-Distribution (Nov 94 - May 10)

VaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0079	0.0477	0.9542	10.7419	0.1655	0.2087	0.1327	0.1609	0.0595	0.0491
0.95	0.0075	0.0462	0.9046	10.4302	0.1617	0.1994	0.1300	0.1562	0.0575	0.0479
0.90	0.0067	0.0445	0.8444	9.9797	0.1516	0.1804	0.1267	0.1503	0.0532	0.0449
0.80	0.0054	0.0407	0.7032	9.1800	0.1330	0.1464	0.1200	0.1376	0.0451	0.0394
0.70	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
95% Confidence Level:										
0.99	0.0135	0.0716	1.1062	12.3292	0.1882	0.2495	0.2035	0.2434	0.0662	0.0554
0.95	0.0134	0.0704	1.1715	12.6750	0.1902	0.2529	0.1971	0.2362	0.0680	0.0567
0.90	0.0132	0.0688	1.2759	13.3155	0.1889	0.2563	0.1889	0.2271	0.0700	0.0583
0.80	0.0125	0.0643	1.3888	13.9640	0.1946	0.2593	0.1726	0.2087	0.0724	0.0599
0.70	0.0112	0.0582	1.3092	13.0604	0.1919	0.2554	0.1561	0.1899	0.0715	0.0588
0.60	0.0090	0.0516	1.1623	11.8980	0.1751	0.2267	0.1394	0.1706	0.0648	0.0530
0.50	0.0070	0.0447	0.9439	10.5598	0.1573	0.1849	0.1268	0.1497	0.0554	0.0470
90% Confidence Level:										
0.99	0.0145	0.0792	1.1315	12.4880	0.1830	0.2316	0.2243	0.2869	0.0646	0.0505
0.95	0.0144	0.0783	1.1009	12.2380	0.1839	0.2341	0.2243	0.2817	0.0642	0.0511
0.90	0.0142	0.0771	1.0694	12.0231	0.1845	0.2367	0.2243	0.2751	0.0634	0.0517
0.80	0.0138	0.0749	1.0263	11.8364	0.1848	0.2410	0.2225	0.2622	0.0622	0.0528
0.70	0.0135	0.0717	1.1115	12.3108	0.1884	0.2496	0.2042	0.2438	0.0662	0.0555
0.60	0.0130	0.0669	1.4175	14.2426	0.1942	0.2579	0.1811	0.2180	0.0717	0.0596
0.50	0.0115	0.0592	1.4214	13.8357	0.1938	0.2566	0.1578	0.1915	0.0727	0.0599

Table 5.2.10 Effects on VaR-Constrained BL Portfolio Performance in Sub-period (Aug 98-May 10)

This table shows realised VaR-constrained BL portfolio performance in the period from November 1994 to May 2010. The conditional covariance matrix applied to the portfolio construction is the DCC model. Return is the average realised excess return, risk is the standard deviation, SR is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, VaR is estimated by the parametric method with assumption of normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. The VaR constraint (VaR_0) is set to be equal to the scaling factor k multiplied by the estimated VaR of the implied BL portfolio. The scaling factor k is called VaR factor.

Panel A: Normal Distribution (Aug 98 - May 10)

VaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0105	0.0717	1.2366	13.8451	0.1467	0.2314	0.2183	0.2290	0.0482	0.0459
0.95	0.0104	0.0703	1.3455	14.6298	0.1475	0.2331	0.2120	0.2222	0.0489	0.0466
0.90	0.0100	0.0677	1.3580	14.8033	0.1471	0.2346	0.2040	0.2137	0.0489	0.0466
0.80	0.0089	0.0615	1.2011	13.8472	0.1443	0.2369	0.1880	0.1964	0.0472	0.0452
0.70	0.0075	0.0550	1.0521	12.9342	0.1363	0.2285	0.1717	0.1785	0.0436	0.0420
0.60	0.0056	0.0486	0.8846	11.7044	0.1147	0.1933	0.1552	0.1597	0.0359	0.0350
0.50	0.0038	0.0425	0.6634	10.0369	0.0904	0.1505	0.1382	0.1388	0.0278	0.0277
95% Confidence Level:										
0.99	0.0117	0.0818	1.0355	12.1711	0.1425	0.2113	0.2631	0.2798	0.0443	0.0417
0.95	0.0115	0.0807	1.0103	12.0984	0.1422	0.2128	0.2624	0.2732	0.0437	0.0420
0.90	0.0112	0.0794	0.9838	12.0669	0.1416	0.2144	0.2615	0.2649	0.0430	0.0424
0.80	0.0108	0.0766	0.9774	12.3486	0.1406	0.2181	0.2450	0.2557	0.0440	0.0421
0.70	0.0106	0.0723	1.2044	13.6178	0.1462	0.2301	0.2221	0.2317	0.0476	0.0456
0.60	0.0096	0.0660	1.4200	15.3028	0.1461	0.2335	0.1990	0.2072	0.0485	0.0466
0.50	0.0078	0.0567	1.1913	13.8416	0.1381	0.2283	0.1756	0.1818	0.0446	0.0431
90% Confidence Level:										
0.99	0.0125	0.0902	1.3456	14.3381	0.1391	0.1951	0.2687	0.3311	0.0467	0.0379
0.95	0.0124	0.0888	1.2799	13.8133	0.1396	0.1976	0.2678	0.3224	0.0463	0.0384
0.90	0.0122	0.0870	1.2018	13.2262	0.1402	0.2007	0.2666	0.3116	0.0457	0.0391
0.80	0.0119	0.0834	1.0757	12.3831	0.1422	0.2085	0.2642	0.2901	0.0449	0.0409
0.70	0.0113	0.0799	0.9898	12.0767	0.1417	0.2137	0.2618	0.2684	0.0432	0.0422
0.60	0.0107	0.0760	1.0098	12.4810	0.1414	0.2198	0.2424	0.2514	0.0443	0.0427
0.50	0.0103	0.0697	1.3765	14.8561	0.1474	0.2327	0.2122	0.2199	0.0484	0.0468

Table 5.2.10 (continued)

Panel B: t-Distribution

VaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0055	0.0488	0.8070	11.1746	0.1124	0.1917	0.1556	0.1610	0.0352	0.0341
0.95	0.0052	0.0473	0.7542	10.8036	0.1105	0.1864	0.1516	0.1562	0.0345	0.0335
0.90	0.0047	0.0455	0.6803	10.2378	0.1043	0.1745	0.1464	0.1500	0.0324	0.0316
0.80	0.0034	0.0418	0.5508	9.3063	0.0817	0.1383	0.1360	0.1367	0.0251	0.0250
0.70	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
95% Confidence Level:										
0.99	0.0107	0.0747	1.0609	12.7539	0.1432	0.2240	0.2335	0.2448	0.0458	0.0437
0.95	0.0107	0.0734	1.1338	13.1677	0.1452	0.2281	0.2266	0.2375	0.0471	0.0449
0.90	0.0105	0.0715	1.2475	13.9164	0.1468	0.2316	0.2179	0.2283	0.0482	0.0460
0.80	0.0097	0.0664	1.3433	14.7439	0.1465	0.2348	0.2004	0.2096	0.0485	0.0464
0.70	0.0084	0.0594	1.1769	13.7059	0.1420	0.2347	0.1828	0.1904	0.0462	0.0443
0.60	0.0064	0.0525	1.0086	12.4986	0.1226	0.2078	0.1649	0.1705	0.0390	0.0377
0.50	0.0049	0.0456	0.7758	10.8697	0.1077	0.1755	0.1465	0.1491	0.0335	0.0330
90% Confidence Level:										
0.99	0.0119	0.0839	1.0946	12.4771	0.1420	0.2075	0.2645	0.2927	0.0450	0.0407
0.95	0.0118	0.0828	1.0594	12.2779	0.1425	0.2098	0.2637	0.2856	0.0447	0.0413
0.90	0.0116	0.0813	1.0223	12.1332	0.1423	0.2120	0.2627	0.2767	0.0440	0.0418
0.80	0.0110	0.0785	0.9683	12.0972	0.1407	0.2150	0.2584	0.2602	0.0427	0.0424
0.70	0.0107	0.0748	1.0649	12.7630	0.1433	0.2238	0.2349	0.2447	0.0456	0.0438
0.60	0.0102	0.0694	1.3979	15.0241	0.1474	0.2330	0.2100	0.2186	0.0487	0.0468
0.50	0.0087	0.0605	1.3103	14.6303	0.1443	0.2357	0.1849	0.1917	0.0472	0.0456

Table 5.3.1 Weights in the CVaR-Constrained BL Portfolio in August 1998

This table reports weights allocated to each index in a CVaR-constrained BL portfolio in August 1998. The standard deviation is estimated by a conditional covariance matrix of the DCC, EWMA and RW50 models. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

Aug-98	Normal distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0302	0.0416	-0.0195	-0.0562	-0.1406	-0.2946
UK CONSUMER GDS	0.0500	0.1427	0.0411	0.0188	0.1020	0.0452
UK CONSUMER SVS	-0.0157	-0.2198	-0.0949	0.1832	-0.1806	-0.3185
UK FINANCIALS	0.1180	0.2251	0.1251	0.0137	0.1686	0.2424
UK HEALTH CARE	-0.0390	-0.0913	0.0253	0.0900	0.0857	0.1859
UK TECHNOLOGY	-0.0325	-0.0755	-0.0150	-0.0376	-0.0546	-0.0394
UK INDUSTRIALS	-0.0405	0.0093	0.0507	-0.0430	0.1926	0.3389
UK OIL & GAS	0.0380	0.1357	0.0544	-0.1410	0.0164	0.0556
UK TELECOM	-0.0361	-0.1851	-0.0621	-0.0356	-0.1923	-0.2133
UK UTILITIES	-0.0420	-0.1099	-0.0085	0.1456	0.0572	0.0761
USA BASIC MATS	0.0966	0.2175	0.1132	0.0460	0.1427	0.2263
USA CONSUMER GDS	-0.0709	-0.0934	0.0059	-0.0604	-0.0270	0.1360
USA CONSUMER SVS	0.0494	0.0477	0.1358	0.2703	0.2262	0.5171
USA FINANCIALS	0.0135	-0.0425	0.0489	-0.0545	-0.0949	-0.0799
USA HEALTH CARE	0.0756	0.0178	0.0500	0.2050	0.0818	-0.1411
USA INDUSTRIALS	-0.0581	-0.2707	-0.1153	-0.1580	-0.3845	-0.5865
USA OIL & GAS	0.1656	0.2735	0.0590	0.2200	0.2200	-0.1085
USA TECHNOLOGY	0.0263	-0.0104	0.0659	-0.0615	-0.0119	0.0090
USA TELECOM	-0.0359	-0.1159	-0.0309	0.0751	-0.0298	-0.0561
USA UTILITIES	-0.0116	-0.1141	0.1179	0.0431	0.0512	0.5270
JAPAN BASIC MATS	0.1018	0.1126	0.0142	0.0685	0.0506	-0.1029
JAPAN CONSUMER GDS	0.1033	0.1030	0.0081	0.0304	-0.0822	-0.2268
JAPAN CONSUMER SVS	0.1271	0.1268	0.0332	0.1483	0.0033	-0.0800
JAPAN FINANCIALS	0.0487	0.0558	-0.0054	-0.0914	-0.0643	-0.2280
JAPAN HEALTH CARE	0.0594	0.2153	0.1009	-0.1004	0.1320	0.2389
JAPAN INDUSTRIALS	0.1299	0.4258	0.2192	0.2127	0.6433	0.8130
JAPAN OIL & GAS	0.0217	0.0827	0.0523	-0.0773	0.0378	0.1352
JAPAN TECHNOLOGY	0.0496	0.0705	0.0224	-0.1107	-0.0463	-0.0644
JAPAN TELECOM	0.1035	0.1615	0.0025	0.1041	0.0650	-0.1662
JAPAN UTILITIES	-0.0258	-0.1366	0.0057	0.1530	0.0329	0.1598

Table 5.3.2 Weights in the CVaR-Constrained BL Portfolio in November 1998

This table reports weights allocated to each index in a CVaR-constrained BL portfolio in November 1998. The standard deviation is estimated by a conditional covariance matrix of DCC, EWMA and RW50 models. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

Nov-98	Normal distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	-0.0247	-0.0429	-0.0185	-0.0670	-0.1780	-0.3185
UK CONSUMER GDS	-0.0276	-0.0369	-0.0157	0.0308	0.0593	0.0313
UK CONSUMER SVS	-0.0442	-0.0644	-0.0142	0.1679	-0.1287	-0.2175
UK FINANCIALS	-0.0021	-0.0167	0.0127	-0.0171	0.0477	0.1555
UK HEALTH CARE	0.0785	0.1026	0.0633	0.0523	0.2157	0.2093
UK TECHNOLOGY	-0.0125	-0.0205	-0.0028	-0.0206	-0.0249	-0.0315
UK INDUSTRIALS	-0.0189	-0.0321	-0.0094	-0.0264	0.1683	0.3897
UK OIL & GAS	-0.0354	-0.0488	-0.0215	-0.0815	-0.1452	-0.1046
UK TELECOM	0.0584	0.0705	0.0444	0.0200	-0.0508	-0.1164
UK UTILITIES	0.0705	0.0775	0.0380	0.0736	0.1655	0.1722
USA BASIC MATS	-0.0211	-0.0386	-0.0039	0.0909	0.0453	0.2999
USA CONSUMER GDS	-0.0147	-0.0299	-0.0065	-0.0658	-0.0252	-0.0475
USA CONSUMER SVS	0.1162	0.1376	0.1101	0.1307	0.2253	0.4690
USA FINANCIALS	0.0760	0.0668	0.0929	-0.0290	-0.0058	-0.0133
USA HEALTH CARE	0.1765	0.1941	0.1699	0.1165	0.1477	0.1631
USA INDUSTRIALS	0.0115	-0.0067	0.0290	-0.0648	-0.2302	-0.5718
USA OIL & GAS	-0.0129	-0.0264	0.0102	0.0832	0.1224	-0.0328
USA TECHNOLOGY	0.1325	0.1457	0.1320	0.0066	0.0391	0.0109
USA TELECOM	0.0660	0.0808	0.0529	0.0329	0.0173	-0.1211
USA UTILITIES	0.1203	0.1430	0.0965	0.1525	0.1738	0.3331
JAPAN BASIC MATS	0.0387	0.0466	0.0296	-0.0694	0.0312	-0.0471
JAPAN CONSUMER GDS	-0.0297	-0.0334	0.0032	0.0833	-0.0907	-0.1279
JAPAN CONSUMER SVS	0.0673	0.0713	0.0425	0.2100	-0.0392	-0.0768
JAPAN FINANCIALS	0.0051	-0.0020	0.0144	-0.0457	-0.1174	-0.1494
JAPAN HEALTH CARE	0.0728	0.0647	0.0376	0.1511	-0.0713	-0.0751
JAPAN INDUSTRIALS	0.0589	0.0697	0.0362	0.1856	0.5208	0.6112
JAPAN OIL & GAS	0.0264	0.0333	0.0169	-0.0175	0.0311	0.1534
JAPAN TECHNOLOGY	-0.0168	-0.0269	-0.0042	-0.1002	-0.1243	-0.1325
JAPAN TELECOM	0.0462	0.0642	0.0312	0.0340	0.0845	0.0961
JAPAN UTILITIES	0.0388	0.0577	0.0331	-0.0169	0.1367	0.0890

Table 5.3.3 CVaR-Constrained BL Portfolio Performance in a Single Period

This table reports the CVaR-constrained BL portfolio performance evaluated by realised return, CSR, PT, and reward to CVaR ratio in August 1998 and November 1998. The standard deviation is estimated by a conditional covariance matrix of DCC, EWMA and RW50 models. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period. Note that I follow Israelsen's (2003) method to adjust the CSR and the reward to CVaR ratio in August 1998 because the negative realised excess return would lead to invalid SR measures for portfolio evaluation.

Panel A: Aug 1998

	Normal Distribution				t-Distribution			
	Realised Return	Adjusted CSR	PT	Adjusted Reward to CVaR	Realised Return	Adjusted CSR	PT	Adjusted Reward to CVaR
DCC	-0.1166	-0.5371	3.2105	-2.7905	-0.0362	-0.0880	1.6783	-0.3654
EWMA	-0.1237	-1.1435	5.1408	-4.5773	-0.0596	-0.2752	2.4236	-1.0881
RW50	-0.1118	-0.3505	N/A	-2.1833	-0.0024	-0.0037	N/A	-0.0103

Panel B: Nov 1998

	Normal Distribution				t-Distribution			
	Realised Return	CSR	PT	Reward to CVaR	Realised Return	CSR	PT	Reward to CVaR
DCC	0.0444	0.9132	1.5992	0.5213	0.0262	0.9685	1.2714	0.5708
EWMA	0.0448	0.9124	2.5124	0.5206	0.0228	0.8739	1.2002	0.4878
RW50	0.0479	1.3257	0.8673	0.9898	0.0350	1.9181	1.9260	2.5674

Table 5.3.4 CVaR-Constrained BL Portfolio Performance in Multiple Periods

This table shows realised CVaR-constrained BL portfolio performance in the period from November 1994 to May 2010, and the sub-period from August 1998 to May 2010. Return is the average realised excess return, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
Panel A: Normal Distribution (Nov 94-May 10)										
DCC	0.0133	0.0690	1.2940	13.4657	0.1921	0.2557	0.1895	0.2278	0.0700	0.0582
EWMA	0.0099	0.0553	-0.0018	7.3818	0.1785	0.2664	0.1492	0.1986	0.0662	0.0498
RW50	0.0064	0.0486	0.0745	6.7634	0.1322	0.2668	0.1164	0.1605	0.0552	0.0401
Panel B: t-Distribution (Nov 94-May 10)										
	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0058	0.0411	0.7205	9.2884	0.1409	0.1562	0.1209	0.1393	0.0479	0.0415
EWMA	0.0043	0.0316	1.5039	16.6840	0.1377	0.1197	0.0941	0.1149	0.0462	0.0378
RW50	-0.0004	0.0373	0.3149	5.6752	-0.0114	-0.0262	0.0933	0.1041	-0.0046	-0.0041
Panel C: Normal Distribution (Aug 98-May 10)										
	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0105	0.0719	1.2701	14.0619	0.1468	0.2311	0.2183	0.2291	0.0483	0.0460
EWMA	0.0073	0.0576	-0.1505	7.3944	0.1263	0.2388	0.1645	0.2121	0.0442	0.0343
RW50	0.0043	0.0505	-0.1882	6.0535	0.0856	0.2825	0.1253	0.1752	0.0345	0.0247
Panel D: t-Distribution (Aug 98-May 10)										
	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0041	0.0421	0.5516	9.4172	0.0962	0.1564	0.1373	0.1386	0.0295	0.0293
EWMA	0.0028	0.0331	1.4235	17.1082	0.0859	0.1265	0.1132	0.1138	0.0251	0.0250
RW50	-0.0010	0.0401	-0.0934	3.6951	-0.0247	0.0120	0.0971	0.1069	-0.0102	-0.0093

Table 5.3.5 Effects on CVaR-Constrained BL Portfolio Optimisation (Aug 98)

This table reports the statistics inputted into the CVaR-constrained SR-BL model, such as estimated expected BL return (μ) and standard deviation (based on the DCC model) and the results of ECSR, Reward to VaR ratio (μ / VaR) and Reward to CVaR ratio ($\mu / CVaR$). VaR and CVaR are estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at confidence levels of 99%, 95% and 90%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor (CVaR Factor) multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

Panel A: Normal Distribution

99% Confidence Level

CVaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0047	0.0461	0.1025	0.1181	0.0461	0.0400	0.1025
0.95	0.0045	0.0442	0.0983	0.1133	0.0459	0.0399	0.1022
0.90	0.0043	0.0419	0.0931	0.1073	0.0457	0.0396	0.1016
0.80	0.0037	0.0372	0.0828	0.0954	0.0447	0.0388	0.0995
0.70	0.0031	0.0325	0.0724	0.0834	0.0426	0.0370	0.0950
0.60	0.0023	0.0277	0.0621	0.0715	0.0374	0.0325	0.0839
0.50	0.0011	0.0247	0.0565	0.0648	0.0195	0.0170	0.0445

95% Confidence Level

CVaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0059	0.0568	0.0876	0.1114	0.0669	0.0527	0.1032
0.95	0.0059	0.0568	0.0876	0.1114	0.0669	0.0527	0.1032
0.90	0.0059	0.0568	0.0876	0.1114	0.0669	0.0527	0.1032
0.80	0.0055	0.0537	0.0828	0.1052	0.0669	0.0526	0.1031
0.70	0.0048	0.0470	0.0724	0.0921	0.0666	0.0524	0.1027
0.60	0.0041	0.0402	0.0621	0.0789	0.0654	0.0515	0.1010
0.50	0.0032	0.0334	0.0517	0.0657	0.0621	0.0489	0.0962

90% Confidence Level

CVaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0059	0.0568	0.0670	0.0939	0.0876	0.0625	0.1032
0.95	0.0059	0.0568	0.0670	0.0939	0.0876	0.0625	0.1032
0.90	0.0059	0.0568	0.0670	0.0939	0.0876	0.0625	0.1032
0.80	0.0059	0.0568	0.0670	0.0939	0.0876	0.0625	0.1032
0.70	0.0059	0.0568	0.0670	0.0939	0.0876	0.0625	0.1032
0.60	0.0054	0.0527	0.0621	0.0870	0.0875	0.0624	0.1031
0.50	0.0045	0.0439	0.0517	0.0725	0.0866	0.0618	0.1021

Table 5.3.5 (continued)

Panel B: t-Distribution

99% Confidence Level

CVaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0012	0.0251	0.0928	0.1192	0.0134	0.0104	0.0496
0.95	0.0012	0.0250	0.0926	0.1189	0.0134	0.0104	0.0494
0.90	0.0012	0.0245	0.0905	0.1163	0.0132	0.0102	0.0487
0.80	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.70	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.60	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.50	N/A	N/A	N/A	N/A	N/A	N/A	N/A

95% Confidence Level

CVaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0038	0.0381	0.0773	0.1181	0.0492	0.0322	0.1000
0.95	0.0036	0.0365	0.0742	0.1133	0.0487	0.0319	0.0990
0.90	0.0034	0.0346	0.0703	0.1073	0.0479	0.0314	0.0975
0.80	0.0028	0.0307	0.0626	0.0954	0.0451	0.0296	0.0921
0.70	0.0021	0.0267	0.0549	0.0835	0.0388	0.0255	0.0796
0.60	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.50	N/A	N/A	N/A	N/A	N/A	N/A	N/A

90% Confidence Level

CVaR Factor	Expected Return	Standard Deviation	VaR	CVaR	μ / VaR	$\mu / CVaR$	CSR
0.99	0.0051	0.0493	0.0705	0.1181	0.0719	0.0429	0.1029
0.95	0.0049	0.0473	0.0676	0.1133	0.0718	0.0428	0.1027
0.90	0.0046	0.0448	0.0641	0.1073	0.0715	0.0427	0.1023
0.80	0.0040	0.0398	0.0570	0.0954	0.0704	0.0420	0.1008
0.70	0.0034	0.0348	0.0499	0.0835	0.0680	0.0407	0.0976
0.60	0.0027	0.0297	0.0429	0.0716	0.0624	0.0374	0.0900
0.50	0.0014	0.0244	0.0360	0.0596	0.0386	0.0233	0.0570

Table 5.3.6 Effects on Weights of the CVaR-Constrained BL Portfolio (Aug 98)

This table shows positions of each asset in a CVaR-constrained BL portfolio in August 1998 under normal distribution and t-distribution at confidence levels of 99% (Panel A), 95% (Panel B), and 90% (Panel C). Note that the covariance matrix applied to the CVaR-constrained BL model is the DCC covariance matrix in this table.

Panel A: 99% Confidence Level	Normal Distribution					t-Distribution			
CVaR Factor	0.99	0.95	0.90	0.80	0.70	0.99	0.95	0.90	0.80
UK BASIC MATS	0.0302	0.0239	0.0158	-0.0010	-0.0196	-0.0562	-0.0906	-0.0688	N/A
UK CONSUMER GDS	0.0500	0.0480	0.0454	0.0398	0.0335	0.0188	0.0239	0.0054	N/A
UK CONSUMER SVS	-0.0157	-0.0043	0.0103	0.0409	0.0747	0.1832	0.2772	0.1839	N/A
UK FINANCIALS	0.1180	0.1133	0.1073	0.0951	0.0816	0.0137	0.0905	0.0496	N/A
UK HEALTH CARE	-0.0390	-0.0330	-0.0253	-0.0090	0.0090	0.0900	-0.0103	0.0621	N/A
UK TECHNOLOGY	-0.0325	-0.0327	-0.0330	-0.0335	-0.0341	-0.0376	-0.0249	-0.0338	N/A
UK INDUSTRIALS	-0.0405	-0.0398	-0.0388	-0.0366	-0.0343	-0.0430	-0.0383	-0.0154	N/A
UK OIL & GAS	0.0380	0.0294	0.0184	-0.0047	-0.0301	-0.1410	-0.1413	-0.1026	N/A
UK TELECOM	-0.0361	-0.0363	-0.0366	-0.0373	-0.0380	-0.0356	-0.0209	-0.0513	N/A
UK UTILITIES	-0.0420	-0.0324	-0.0201	0.0056	0.0339	0.1456	0.1379	0.1381	N/A
USA BASIC MATS	0.0966	0.0929	0.0882	0.0781	0.0672	0.0460	-0.0045	0.0019	N/A
USA CONSUMER GDS	-0.0709	-0.0702	-0.0692	-0.0669	-0.0644	-0.0604	-0.0719	-0.0415	N/A
USA CONSUMER SVS	0.0494	0.0620	0.0781	0.1119	0.1491	0.2703	0.2843	0.2262	N/A
USA FINANCIALS	0.0135	0.0090	0.0033	-0.0088	-0.0224	-0.0545	-0.1382	-0.0549	N/A
USA HEALTH CARE	0.0756	0.0818	0.0896	0.1062	0.1246	0.2050	0.1831	0.1681	N/A
USA INDUSTRIALS	-0.0581	-0.0632	-0.0699	-0.0839	-0.0997	-0.1580	-0.1450	-0.0791	N/A
USA OIL & GAS	0.1656	0.1690	0.1732	0.1821	0.1920	0.2200	0.2683	0.2070	N/A
USA TECHNOLOGY	0.0263	0.0218	0.0160	0.0039	-0.0095	-0.0615	-0.0359	-0.0736	N/A
USA TELECOM	-0.0359	-0.0302	-0.0229	-0.0077	0.0095	0.0751	0.0651	0.0690	N/A
USA UTILITIES	-0.0116	-0.0071	-0.0013	0.0109	0.0245	0.0431	0.0809	0.0502	N/A
JAPAN BASIC MATS	0.1018	0.1016	0.1014	0.1007	0.1000	0.0685	0.1394	0.0790	N/A
JAPAN CONSUMER GDS	0.1033	0.0993	0.0942	0.0835	0.0716	0.0304	-0.0336	0.0417	N/A
JAPAN CONSUMER SVS	0.1271	0.1273	0.1275	0.1280	0.1285	0.1483	0.0884	0.0998	N/A
JAPAN FINANCIALS	0.0487	0.0409	0.0311	0.0106	-0.0122	-0.0914	-0.0603	-0.1012	N/A
JAPAN HEALTH CARE	0.0594	0.0489	0.0354	0.0072	-0.0239	-0.1004	-0.1278	-0.0662	N/A
JAPAN INDUSTRIALS	0.1299	0.1331	0.1372	0.1458	0.1554	0.2127	0.1613	0.1531	N/A
JAPAN OIL & GAS	0.0217	0.0161	0.0089	-0.0062	-0.0230	-0.0773	-0.0481	-0.0582	N/A
JAPAN TECHNOLOGY	0.0496	0.0418	0.0317	0.0106	-0.0125	-0.1107	-0.0663	-0.0552	N/A
JAPAN TELECOM	0.1035	0.1038	0.1042	0.1048	0.1056	0.1041	0.0742	0.1173	N/A
JAPAN UTILITIES	-0.0258	-0.0146	-0.0003	0.0299	0.0631	0.1530	0.1832	0.1491	N/A

Table 5.3.6 (continued)

Panel B: 95% Confidence Level	Normal Distribution					t-Distribution				
CVaR Factor	0.99	0.95	0.90	0.80	0.70	0.99	0.95	0.90	0.80	0.70
UK BASIC MATS	0.0648	0.0648	0.0582	0.0389	0.0183	0.0022	-0.0036	-0.0111	-0.0277	-0.0489
UK CONSUMER GDS	0.0614	0.0614	0.0593	0.0527	0.0462	0.0408	0.0389	0.0364	0.0309	0.0239
UK CONSUMER SVS	-0.0789	-0.0789	-0.0664	-0.0313	0.0059	0.0351	0.0456	0.0593	0.0892	0.1277
UK FINANCIALS	0.1436	0.1436	0.1387	0.1244	0.1091	0.0975	0.0933	0.0877	0.0757	0.0601
UK HEALTH CARE	-0.0723	-0.0723	-0.0663	-0.0473	-0.0276	-0.0121	-0.0065	0.0007	0.0167	0.0371
UK TECHNOLOGY	-0.0313	-0.0313	-0.0317	-0.0323	-0.0329	-0.0334	-0.0336	-0.0338	-0.0344	-0.0351
UK INDUSTRIALS	-0.0447	-0.0447	-0.0439	-0.0415	-0.0392	-0.0370	-0.0362	-0.0353	-0.0333	-0.0307
UK OIL & GAS	0.0857	0.0857	0.0769	0.0498	0.0218	-0.0003	-0.0082	-0.0185	-0.0411	-0.0701
UK TELECOM	-0.0350	-0.0350	-0.0352	-0.0358	-0.0365	-0.0372	-0.0374	-0.0377	-0.0383	-0.0391
UK UTILITIES	-0.0954	-0.0954	-0.0853	-0.0551	-0.0238	0.0006	0.0095	0.0210	0.0462	0.0785
USA BASIC MATS	0.1171	0.1171	0.1133	0.1018	0.0896	0.0800	0.0766	0.0722	0.0625	0.0501
USA CONSUMER GDS	-0.0747	-0.0747	-0.0741	-0.0720	-0.0695	-0.0673	-0.0666	-0.0656	-0.0634	-0.0607
USA CONSUMER SVS	-0.0202	-0.0202	-0.0074	0.0321	0.0732	0.1055	0.1170	0.1321	0.1651	0.2074
USA FINANCIALS	0.0381	0.0381	0.0336	0.0197	0.0050	-0.0066	-0.0107	-0.0161	-0.0281	-0.0434
USA HEALTH CARE	0.0414	0.0414	0.0481	0.0672	0.0872	0.1030	0.1087	0.1161	0.1325	0.1533
USA INDUSTRIALS	-0.0294	-0.0294	-0.0349	-0.0510	-0.0678	-0.0813	-0.0861	-0.0923	-0.1064	-0.1241
USA OIL & GAS	0.1471	0.1471	0.1503	0.1609	0.1719	0.1804	0.1835	0.1875	0.1962	0.2075
USA TECHNOLOGY	0.0515	0.0515	0.0469	0.0324	0.0177	0.0063	0.0021	-0.0033	-0.0153	-0.0305
USA TELECOM	-0.0674	-0.0674	-0.0613	-0.0437	-0.0251	-0.0106	-0.0053	0.0015	0.0168	0.0362
USA UTILITIES	-0.0368	-0.0368	-0.0322	-0.0179	-0.0030	0.0086	0.0128	0.0182	0.0303	0.0457
JAPAN BASIC MATS	0.1022	0.1022	0.1023	0.1020	0.1015	0.1008	0.1006	0.1004	0.0997	0.0989
JAPAN CONSUMER GDS	0.1255	0.1255	0.1211	0.1087	0.0957	0.0856	0.0819	0.0771	0.0665	0.0529
JAPAN CONSUMER SVS	0.1257	0.1257	0.1262	0.1268	0.1274	0.1279	0.1281	0.1283	0.1287	0.1293
JAPAN FINANCIALS	0.0916	0.0916	0.0836	0.0593	0.0341	0.0145	0.0075	-0.0018	-0.0219	-0.0479
JAPAN HEALTH CARE	0.1178	0.1178	0.1066	0.0738	0.0395	0.0126	0.0029	-0.0096	-0.0373	-0.0727
JAPAN INDUSTRIALS	0.1125	0.1125	0.1158	0.1255	0.1360	0.1441	0.1471	0.1510	0.1596	0.1707
JAPAN OIL & GAS	0.0531	0.0531	0.0472	0.0295	0.0111	-0.0033	-0.0085	-0.0153	-0.0302	-0.0493
JAPAN TECHNOLOGY	0.0928	0.0928	0.0845	0.0604	0.0348	0.0147	0.0075	-0.0020	-0.0225	-0.0490
JAPAN TELECOM	0.1019	0.1019	0.1023	0.1032	0.1041	0.1047	0.1049	0.1052	0.1059	0.1068
JAPAN UTILITIES	-0.0876	-0.0876	-0.0762	-0.0413	-0.0047	0.0241	0.0345	0.0479	0.0773	0.1152

Table 5.3.6 (continued)

Panel C: 90% Confidence Level	Normal Distribution					t-Distribution				
CVaR Factor	0.99	0.95	0.90	0.80	0.70	0.99	0.95	0.90	0.80	0.70
UK BASIC MATS	0.0648	0.0648	0.0648	0.0648	0.0448	0.0408	0.0342	0.0259	0.0084	-0.0104
UK CONSUMER GDS	0.0614	0.0614	0.0614	0.0614	0.0547	0.0533	0.0513	0.0486	0.0430	0.0367
UK CONSUMER SVS	-0.0789	-0.0789	-0.0789	-0.0789	-0.0422	-0.0348	-0.0229	-0.0078	0.0237	0.0579
UK FINANCIALS	0.1436	0.1436	0.1436	0.1436	0.1288	0.1258	0.1209	0.1147	0.1020	0.0883
UK HEALTH CARE	-0.0723	-0.0723	-0.0723	-0.0723	-0.0531	-0.0491	-0.0429	-0.0349	-0.0181	0.0000
UK TECHNOLOGY	-0.0313	-0.0313	-0.0313	-0.0313	-0.0321	-0.0322	-0.0324	-0.0326	-0.0332	-0.0338
UK INDUSTRIALS	-0.0447	-0.0447	-0.0447	-0.0447	-0.0422	-0.0417	-0.0410	-0.0401	-0.0378	-0.0354
UK OIL & GAS	0.0857	0.0857	0.0857	0.0857	0.0581	0.0525	0.0435	0.0321	0.0083	-0.0174
UK TELECOM	-0.0350	-0.0350	-0.0350	-0.0350	-0.0356	-0.0358	-0.0360	-0.0363	-0.0369	-0.0376
UK UTILITIES	-0.0954	-0.0954	-0.0954	-0.0954	-0.0643	-0.0580	-0.0481	-0.0353	-0.0089	0.0198
USA BASIC MATS	0.1171	0.1171	0.1171	0.1171	0.1053	0.1029	0.0990	0.0941	0.0837	0.0727
USA CONSUMER GDS	-0.0747	-0.0747	-0.0747	-0.0747	-0.0727	-0.0722	-0.0714	-0.0704	-0.0681	-0.0657
USA CONSUMER SVS	-0.0202	-0.0202	-0.0202	-0.0202	0.0200	0.0282	0.0414	0.0581	0.0929	0.1305
USA FINANCIALS	0.0381	0.0381	0.0381	0.0381	0.0240	0.0211	0.0163	0.0104	-0.0021	-0.0155
USA HEALTH CARE	0.0414	0.0414	0.0414	0.0414	0.0613	0.0653	0.0717	0.0799	0.0969	0.1153
USA INDUSTRIALS	-0.0294	-0.0294	-0.0294	-0.0294	-0.0461	-0.0494	-0.0548	-0.0616	-0.0760	-0.0916
USA OIL & GAS	0.1471	0.1471	0.1471	0.1471	0.1576	0.1598	0.1634	0.1679	0.1771	0.1871
USA TECHNOLOGY	0.0515	0.0515	0.0515	0.0515	0.0367	0.0338	0.0291	0.0231	0.0108	-0.0027
USA TELECOM	-0.0674	-0.0674	-0.0674	-0.0674	-0.0491	-0.0454	-0.0395	-0.0320	-0.0163	0.0008
USA UTILITIES	-0.0368	-0.0368	-0.0368	-0.0368	-0.0222	-0.0192	-0.0145	-0.0085	0.0040	0.0176
JAPAN BASIC MATS	0.1022	0.1022	0.1022	0.1022	0.1021	0.1021	0.1019	0.1017	0.1011	0.1004
JAPAN CONSUMER GDS	0.1255	0.1255	0.1255	0.1255	0.1125	0.1099	0.1058	0.1005	0.0895	0.0776
JAPAN CONSUMER SVS	0.1257	0.1257	0.1257	0.1257	0.1266	0.1267	0.1270	0.1272	0.1277	0.1282
JAPAN FINANCIALS	0.0916	0.0916	0.0916	0.0916	0.0668	0.0617	0.0536	0.0433	0.0221	-0.0008
JAPAN HEALTH CARE	0.1178	0.1178	0.1178	0.1178	0.0839	0.0770	0.0661	0.0521	0.0231	-0.0083
JAPAN INDUSTRIALS	0.1125	0.1125	0.1125	0.1125	0.1225	0.1245	0.1279	0.1321	0.1409	0.1506
JAPAN OIL & GAS	0.0531	0.0531	0.0531	0.0531	0.0350	0.0313	0.0253	0.0178	0.0023	-0.0146
JAPAN TECHNOLOGY	0.0928	0.0928	0.0928	0.0928	0.0679	0.0628	0.0546	0.0442	0.0225	-0.0010
JAPAN TELECOM	0.1019	0.1019	0.1019	0.1019	0.1030	0.1031	0.1034	0.1037	0.1044	0.1052
JAPAN UTILITIES	-0.0876	-0.0876	-0.0876	-0.0876	-0.0521	-0.0447	-0.0330	-0.0181	0.0129	0.0465

Table 5.3.7 Effects on CVaR-Constrained SR-BL Portfolio Performance Evaluation (Aug 98)

This table reports CVaR-constrained BL portfolio performance results including realised excess return, adjusted CSR, PT and adjusted reward to CVaR ratio in August 1998. The standard deviation is estimated by a conditional covariance matrix of DCC, EWMA and RW50 models. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period. Following Israelsen's (2003) method, adjusted conditional Sharpe Ratio is equal to the product of negative realised excess return and the standard deviation multiplied by 100. Using this idea, I adjust the reward to CVaR ratio equal to the product of negative realised excess return and CVaR multiplied by 100.

Panel A: 99% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realised Excess Return	Adjusted CSR	PT	Adjusted Reward to CVaR	Realised Excess Return	Adjusted CSR	PT	Adjusted Reward to CVaR
0.99	-0.1166	-0.5371	3.2105	-2.7905	-0.0362	-0.0880	1.6783	-0.3654
0.95	-0.1121	-0.4957	3.1616	-2.5783	-0.0291	-0.0737	1.7066	-0.2811
0.90	-0.1065	-0.4457	3.0049	-2.3212	-0.0401	-0.0982	1.2276	-0.4228
0.80	-0.0946	-0.3518	2.6077	-1.8326	N/A	N/A	N/A	N/A
0.70	-0.0814	-0.2645	2.1826	-1.3683	N/A	N/A	N/A	N/A
0.60	-0.0651	-0.1806	1.7732	-0.9056	N/A	N/A	N/A	N/A
0.50	-0.0283	-0.0741	1.6500	-0.2772	N/A	N/A	N/A	N/A

Panel B: 95% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realised Excess Return	Adjusted CSR	PT	Adjusted Reward to CVaR	Realised Excess Return	Adjusted CSR	PT	Adjusted Reward to CVaR
0.99	-0.1412	-0.8025	3.6583	-4.1322	-0.0969	-0.3686	2.6799	-1.9208
0.95	-0.1412	-0.8025	3.6583	-4.1322	-0.0928	-0.3387	2.5447	-1.7638
0.90	-0.1366	-0.7482	3.5458	-3.8598	-0.0875	-0.3024	2.3711	-1.5713
0.80	-0.1227	-0.5973	3.2863	-3.0970	-0.0758	-0.2324	2.0056	-1.1938
0.70	-0.1082	-0.4606	3.0684	-2.3977	-0.0608	-0.1625	1.7150	-0.8028
0.60	-0.0926	-0.3375	2.5474	-1.7572	N/A	N/A	N/A	N/A
0.50	-0.0744	-0.2250	1.9736	-1.1531	N/A	N/A	N/A	N/A

Panel C: 90% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realised Excess Return	Adjusted CSR	PT	Adjusted Reward to CVaR	Realised Excess Return	Adjusted CSR	PT	Adjusted Reward to CVaR
0.99	-0.1412	-0.8025	3.6583	-4.1322	-0.1240	-0.6112	3.3031	-3.1677
0.95	-0.1412	-0.8025	3.6583	-4.1322	-0.1194	-0.5645	3.2456	-2.9301
0.90	-0.1412	-0.8025	3.6583	-4.1322	-0.1135	-0.5082	3.1765	-2.6428
0.80	-0.1412	-0.8025	3.6583	-4.1322	-0.1013	-0.4029	2.8317	-2.0994
0.70	-0.1270	-0.6416	3.3398	-3.3222	-0.0880	-0.3060	2.3931	-1.5906
0.60	-0.1099	-0.4758	3.1313	-2.4765	-0.0725	-0.2154	1.9167	-1.1000
0.50	-0.0914	-0.3292	2.5130	-1.7136	-0.0449	-0.1096	1.6752	-0.4938

Table 5.3.8 Effects on CVaR-Constrained SR-BL Portfolio Performance Evaluation (Nov 98)

This table reports CVaR-constrained BL portfolio performance results including realised return, CSR, PT and reward to CVaR ratio in August 1998. The standard deviation is estimated by a conditional covariance matrix of DCC, EWMA and RW50 models. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

Panel A: 99% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realised Return	CSR	PT	Reward to CVaR	Realised Return	CSR	PT	Reward to CVaR
0.99	0.0444	0.9132	1.5992	0.5213	0.0262	0.9685	1.2714	0.5708
0.95	0.0444	0.9132	1.5759	0.5213	0.0242	0.9338	1.3167	0.5394
0.90	0.0445	0.9141	1.5397	0.5220	0.0202	0.8088	1.4906	0.4357
0.80	0.0407	0.9401	1.4123	0.5449	N/A	N/A	N/A	N/A
0.70	0.0367	0.9691	1.3101	0.5714	N/A	N/A	N/A	N/A
0.60	0.0322	0.9930	1.2498	0.5938	N/A	N/A	N/A	N/A
0.50	0.0260	0.9664	1.2752	0.5689	N/A	N/A	N/A	N/A

Panel B: 95% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realised Return	CSR	PT	Reward to CVaR	Realised Return	CSR	PT	Reward to CVaR
0.99	0.0444	0.9132	1.5992	0.5213	0.0415	0.9344	1.4383	0.5399
0.95	0.0444	0.9132	1.5992	0.5213	0.0402	0.9436	1.3977	0.5481
0.90	0.0444	0.9132	1.5992	0.5213	0.0386	0.9560	1.3423	0.5593
0.80	0.0444	0.9132	1.5992	0.5213	0.0351	0.9794	1.2873	0.5809
0.70	0.0444	0.9131	1.5411	0.5211	0.0312	0.9948	1.2394	0.5956
0.60	0.0399	0.9461	1.3899	0.5503	N/A	N/A	N/A	N/A
0.50	0.0345	0.9832	1.2793	0.5845	N/A	N/A	N/A	N/A

Panel C: 90% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realised Return	CSR	PT	Reward to CVaR	Realised Return	CSR	PT	Reward to CVaR
0.99	0.0444	0.9132	1.5992	0.5213	0.0444	0.9132	1.5992	0.5213
0.95	0.0444	0.9132	1.5992	0.5213	0.0444	0.9132	1.5992	0.5213
0.90	0.0444	0.9132	1.5992	0.5213	0.0444	0.9132	1.5905	0.5213
0.80	0.0444	0.9132	1.5992	0.5213	0.0428	0.9261	1.4762	0.5325
0.70	0.0444	0.9132	1.5992	0.5213	0.0386	0.9557	1.3429	0.5591
0.60	0.0444	0.9132	1.5486	0.5213	0.0341	0.9855	1.2741	0.5867
0.50	0.0394	0.9502	1.3726	0.5540	0.0285	0.9897	1.2452	0.5907

Table 5.3.9 Effects on CVaR-Constrained BL Portfolio Performance in Multiple Periods (Nov 94-May 10)

This table shows realised CVaR-constrained BL portfolio performance in the period November 1994 to May 2010. The conditional covariance matrix applied to the portfolio construction is the DCC model. Return is the average realised excess return, risk is the standard deviation, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor k multiplied by the estimated CVaR of the implied BL portfolio. The scaling factor k is called CVaR factor.

Panel A: Normal Distribution (Nov 94 - May 10)

CVaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0133	0.0690	1.2940	13.4657	0.1921	0.2557	0.1895	0.2278	0.0700	0.0582
0.95	0.0131	0.0676	1.3657	13.9287	0.1936	0.2576	0.1835	0.2211	0.0713	0.0592
0.90	0.0127	0.0654	1.3945	14.0568	0.1944	0.2590	0.1760	0.2126	0.0723	0.0598
0.80	0.0116	0.0601	1.3165	13.1659	0.1932	0.2580	0.1611	0.1956	0.0721	0.0593
0.70	0.0101	0.0540	1.1860	12.2390	0.1873	0.2474	0.1459	0.1782	0.0693	0.0568
0.60	0.0079	0.0476	1.0575	11.3082	0.1650	0.2056	0.1324	0.1599	0.0594	0.0492
0.50	0.0060	0.0415	0.8347	9.8482	0.1440	0.1583	0.1215	0.1399	0.0493	0.0428
95% Confidence Level:										
0.99	0.0140	0.0757	1.0567	11.9643	0.1850	0.2398	0.2241	0.2659	0.0625	0.0527
0.95	0.0139	0.0750	1.0457	11.9473	0.1849	0.2411	0.2219	0.2617	0.0625	0.0530
0.90	0.0137	0.0739	1.0472	12.0392	0.1854	0.2433	0.2154	0.2566	0.0636	0.0534
0.80	0.0134	0.0707	1.1906	12.7840	0.1902	0.2526	0.1977	0.2369	0.0680	0.0568
0.70	0.0128	0.0660	1.3818	13.9654	0.1943	0.2588	0.1786	0.2151	0.0719	0.0596
0.60	0.0115	0.0594	1.3642	13.4418	0.1936	0.2575	0.1588	0.1928	0.0724	0.0596
0.50	0.0091	0.0514	1.2047	12.1321	0.1766	0.2272	0.1389	0.1697	0.0654	0.0535
90% Confidence Level:										
0.99	0.0145	0.0793	1.1465	12.5640	0.1831	0.2317	0.2241	0.2864	0.0647	0.0507
0.95	0.0144	0.0783	1.1172	12.3226	0.1840	0.2342	0.2241	0.2811	0.0643	0.0513
0.90	0.0142	0.0772	1.0865	12.1161	0.1846	0.2368	0.2241	0.2746	0.0636	0.0519
0.80	0.0139	0.0750	1.0450	11.9517	0.1848	0.2410	0.2221	0.2618	0.0624	0.0529
0.70	0.0135	0.0718	1.1340	12.4662	0.1885	0.2495	0.2037	0.2436	0.0664	0.0555
0.60	0.0130	0.0672	1.4957	14.8471	0.1940	0.2569	0.1808	0.2176	0.0721	0.0599
0.50	0.0114	0.0591	1.4048	13.6892	0.1931	0.2559	0.1575	0.1912	0.0724	0.0596

Table 5.3.9 (continued)

Panel B: t-Distribution (Nov 94 - May 10)

CVaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0058	0.0411	0.7205	9.2884	0.1409	0.1562	0.1209	0.1393	0.0479	0.0415
0.95	0.0055	0.0398	0.6663	9.0208	0.1375	0.1472	0.1186	0.1350	0.0462	0.0406
0.90	0.0053	0.0387	0.5530	8.3054	0.1375	0.1445	0.1142	0.1293	0.0466	0.0412
0.80	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.70	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
95% Confidence Level:										
0.99	0.0118	0.0609	1.2966	13.1027	0.1940	0.2596	0.1638	0.1988	0.0722	0.0595
0.95	0.0114	0.0591	1.2656	12.8286	0.1934	0.2589	0.1588	0.1932	0.0719	0.0591
0.90	0.0109	0.0565	1.2071	12.4541	0.1923	0.2569	0.1526	0.1860	0.0713	0.0585
0.80	0.0091	0.0515	1.1090	11.6381	0.1769	0.2303	0.1400	0.1713	0.0651	0.0532
0.70	0.0073	0.0462	0.9837	10.8418	0.1583	0.1941	0.1300	0.1559	0.0562	0.0469
0.60	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
90% Confidence Level:										
0.99	0.0135	0.0710	1.1660	12.6676	0.1894	0.2514	0.1995	0.2391	0.0674	0.0563
0.95	0.0133	0.0698	1.2374	13.0856	0.1911	0.2542	0.1932	0.2319	0.0691	0.0575
0.90	0.0132	0.0681	1.3608	13.9065	0.1931	0.2569	0.1855	0.2232	0.0709	0.0590
0.80	0.0123	0.0631	1.3642	13.7105	0.1945	0.2595	0.1694	0.2051	0.0724	0.0599
0.70	0.0110	0.0570	1.2672	12.8013	0.1930	0.2569	0.1533	0.1867	0.0718	0.0589
0.60	0.0089	0.0504	1.1286	11.7628	0.1761	0.2267	0.1370	0.1676	0.0648	0.0529
0.50	0.0066	0.0439	0.9146	10.2978	0.1507	0.1752	0.1254	0.1470	0.0527	0.0450

Table 5.3.10 Effects on CVaR-Constrained BL Portfolio Performance in Sub-period (Aug 98-May 10)

This table shows realised CVaR-constrained BL portfolio performance in the sub-period August 1998 to May 2010. The conditional covariance matrix applied to the portfolio construction is the DCC model. Return is the average realised excess return, risk is the standard deviation, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor k multiplied by the estimated CVaR of the implied BL portfolio. The scaling factor k is called CVaR factor.

Panel A: Normal Distribution (Aug 98 - May 10)

CVaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0105	0.0719	1.2701	14.0619	0.1468	0.2311	0.2183	0.2291	0.0483	0.0460
0.95	0.0104	0.0703	1.3434	14.6084	0.1476	0.2332	0.2120	0.2223	0.0489	0.0467
0.90	0.0100	0.0678	1.3604	14.8106	0.1472	0.2346	0.2039	0.2137	0.0489	0.0467
0.80	0.0088	0.0615	1.1980	13.8126	0.1439	0.2363	0.1879	0.1964	0.0471	0.0450
0.70	0.0076	0.0551	1.0379	12.8324	0.1372	0.2297	0.1717	0.1785	0.0440	0.0423
0.60	0.0055	0.0485	0.9027	11.8704	0.1137	0.1912	0.1552	0.1596	0.0355	0.0345
0.50	0.0041	0.0423	0.6693	10.1693	0.0971	0.1549	0.1383	0.1389	0.0297	0.0296
95% Confidence Level:										
0.99	0.0113	0.0795	1.0048	12.1774	0.1418	0.2146	0.2612	0.2642	0.0431	0.0427
0.95	0.0111	0.0786	0.9908	12.2146	0.1409	0.2152	0.2576	0.2599	0.0430	0.0426
0.90	0.0109	0.0773	0.9913	12.3731	0.1407	0.2171	0.2482	0.2571	0.0438	0.0423
0.80	0.0107	0.0737	1.1557	13.2933	0.1453	0.2277	0.2276	0.2380	0.0470	0.0450
0.70	0.0101	0.0684	1.3499	14.7139	0.1472	0.2341	0.2069	0.2159	0.0487	0.0466
0.60	0.0088	0.0607	1.2429	14.1445	0.1442	0.2365	0.1857	0.1933	0.0471	0.0453
0.50	0.0065	0.0523	1.0470	12.7664	0.1243	0.2089	0.1644	0.1696	0.0395	0.0383
90% Confidence Level:										
0.99	0.0119	0.0839	1.1114	12.5543	0.1421	0.2076	0.2643	0.2921	0.0451	0.0408
0.95	0.0118	0.0828	1.0777	12.3649	0.1426	0.2100	0.2635	0.2850	0.0448	0.0414
0.90	0.0116	0.0813	1.0417	12.2297	0.1425	0.2122	0.2625	0.2761	0.0441	0.0420
0.80	0.0111	0.0785	0.9901	12.2195	0.1408	0.2151	0.2579	0.2600	0.0429	0.0425
0.70	0.0107	0.0749	1.0916	12.9263	0.1434	0.2239	0.2344	0.2445	0.0458	0.0439
0.60	0.0103	0.0698	1.4901	15.6753	0.1475	0.2321	0.2095	0.2182	0.0491	0.0472
0.50	0.0087	0.0603	1.2853	14.4462	0.1435	0.2349	0.1845	0.1914	0.0469	0.0452

Table 5.3.10 (continued)

Panel B: t-Distribution (Aug 98 - May 10)

CVaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0041	0.0421	0.5516	9.4172	0.0962	0.1564	0.1373	0.1386	0.0295	0.0293
0.95	0.0038	0.0407	0.5075	9.2533	0.0929	0.1482	0.1338	0.1339	0.0283	0.0283
0.90	0.0037	0.0397	0.3942	8.3715	0.0933	0.1464	0.1272	0.1289	0.0291	0.0288
0.80	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
95% Confidence Level:										
0.99	0.0091	0.0625	1.1884	13.7405	0.1450	0.2375	0.1907	0.1998	0.0475	0.0454
0.95	0.0087	0.0604	1.1371	13.4229	0.1441	0.2381	0.1854	0.1939	0.0470	0.0449
0.90	0.0083	0.0578	1.0693	13.0334	0.1430	0.2381	0.1787	0.1866	0.0462	0.0443
0.80	0.0065	0.0525	0.9588	12.1954	0.1239	0.2106	0.1652	0.1715	0.0394	0.0379
0.70	0.0051	0.0471	0.8200	11.2611	0.1086	0.1837	0.1516	0.1556	0.0338	0.0329
0.60	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
90% Confidence Level:										
0.99	0.0107	0.0741	1.1291	13.1419	0.1446	0.2264	0.2293	0.2405	0.0467	0.0446
0.95	0.0106	0.0727	1.2086	13.6368	0.1461	0.2296	0.2225	0.2332	0.0477	0.0456
0.90	0.0104	0.0709	1.3411	14.5805	0.1473	0.2323	0.2140	0.2243	0.0488	0.0465
0.80	0.0095	0.0650	1.3005	14.4668	0.1460	0.2356	0.1969	0.2059	0.0482	0.0461
0.70	0.0084	0.0582	1.1284	13.4193	0.1437	0.2380	0.1797	0.1871	0.0465	0.0447
0.60	0.0063	0.0513	0.9764	12.3625	0.1223	0.2068	0.1622	0.1675	0.0386	0.0374
0.50	0.0044	0.0447	0.7598	10.7040	0.0995	0.1646	0.1443	0.1463	0.0308	0.0304

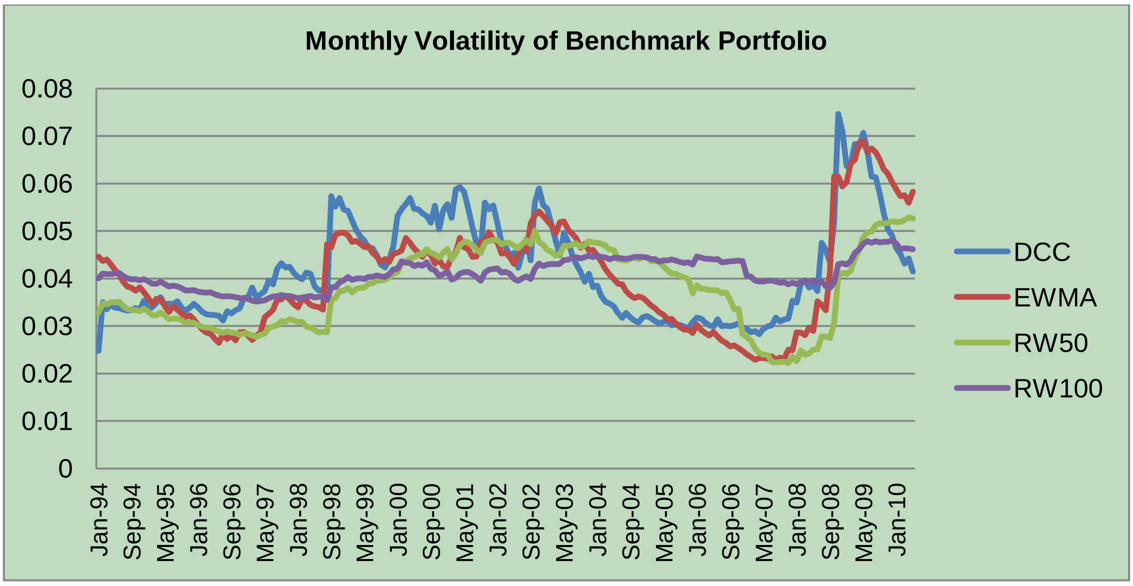


Figure 5.1.1 Monthly Volatility of the Benchmark Portfolio

This figure plots the time-varying standard deviation for the benchmark portfolio from January 1994 to May 2010. The time-varying standard deviation is calculated by the DCC model (blue line), the EWMA model (red line), the RW method (green line) with a window length of 50; the RW method (purple line) with a window length of 100. Note that a simple simulation technique is used to estimate volatilities for 50 missing values and 100 missing values in the RW method. For a window length of M, σ_1^2 is the sample historical variance. $\sigma_t^2 = \frac{1}{M}\left[(M-1)\sigma_{t-1}^2 + \sum_{i=1}^{t-1} r_{t-i}^2\right]$, where $t = 2,\ldots,M$.

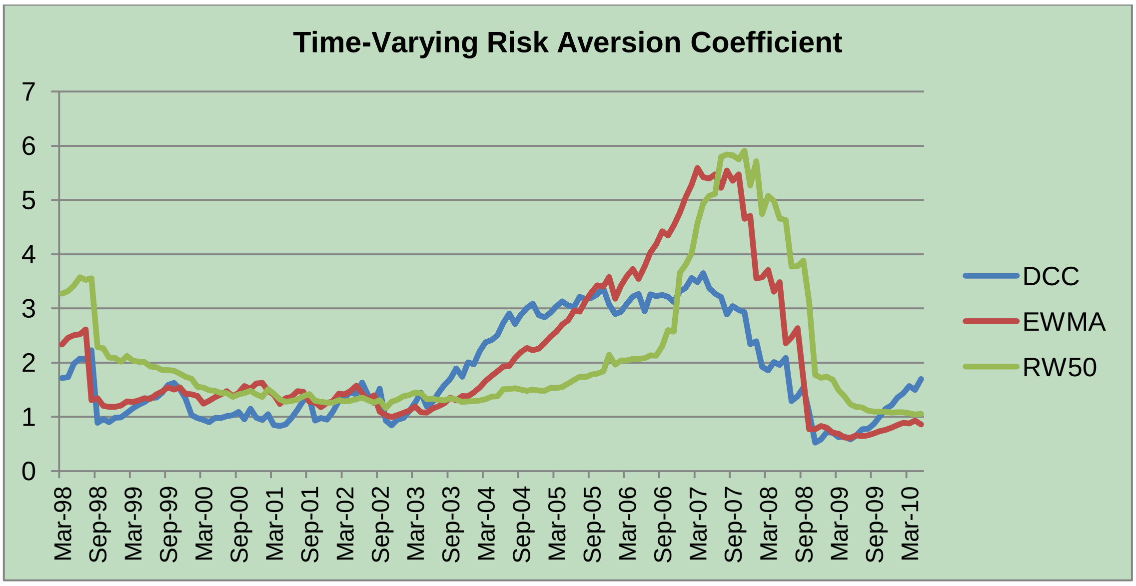


Figure 5.1.2 Time-Varying Risk Aversion Coefficient

This figure plots the time-varying risk aversion coefficient from March 1998 to May 2010. The risk aversion coefficient is calculated by the monthly world risk premium divided by monthly time-varying market variance. The monthly world risk premium is set at 0.29% (=3.5%/12). The time-varying standard deviation is calculated by the DCC model (blue line), the EWMA model (red line), and the RW method (green line) with a window length of 50. To avoid the noise from the simulated data of conditional variance in the RW method and to make a comparable analysis, I report the risk aversion coefficient from the 51st period (March 1998).

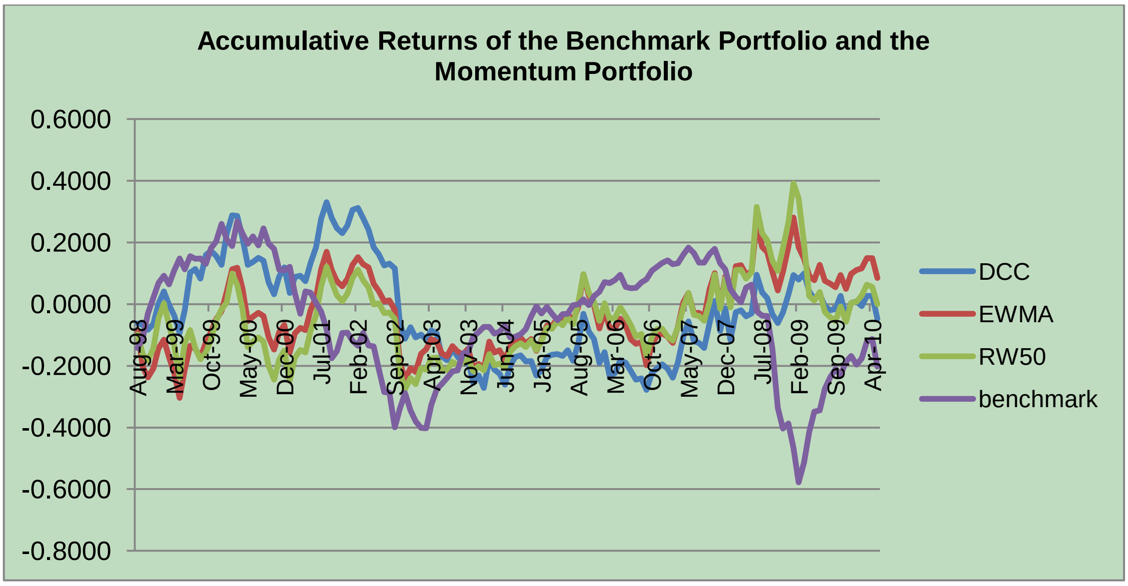


Figure 5.1.3 **Accumulative Returns of the Benchmark Portfolio and the Momentum Portfolio**

This figure plots the accumulative returns of the benchmark portfolio and the momentum portfolio from August 1998 to May 2010. Based on the time-varying standard deviation, which is estimated by the DCC model (blue line), the EWMA model (red line), and the RW method (green line) with a window length of 50, the momentum portfolio is constructed by the method of Fabozzi et al. (2006).

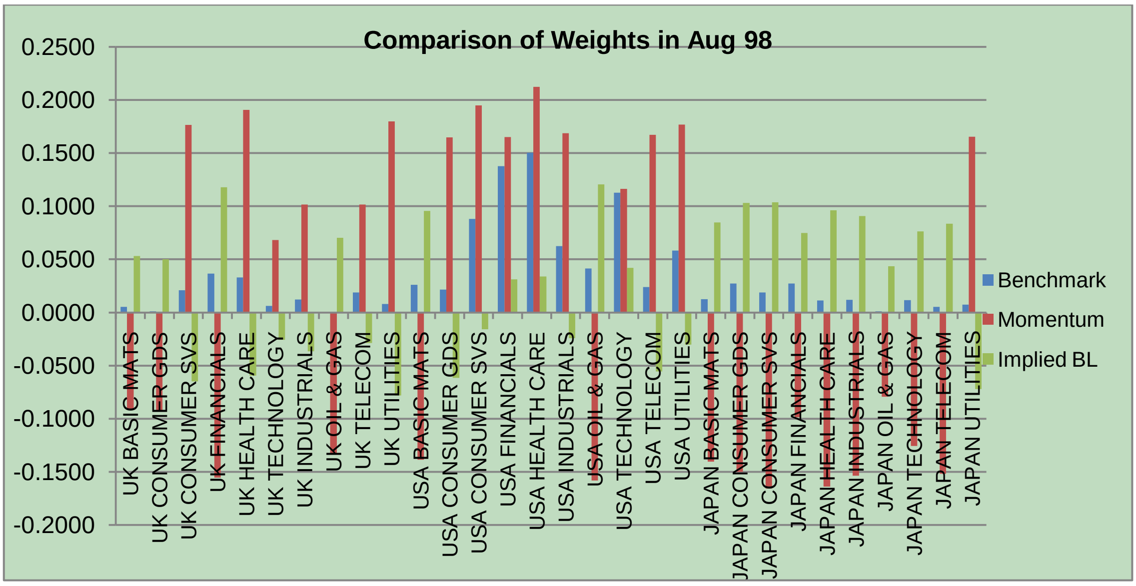


Figure 5.1.4 Comparison of Weights in August 1998

This figure plots the weights of each asset in the benchmark portfolio, the momentum portfolio and the implied BL portfolio in August 1998.

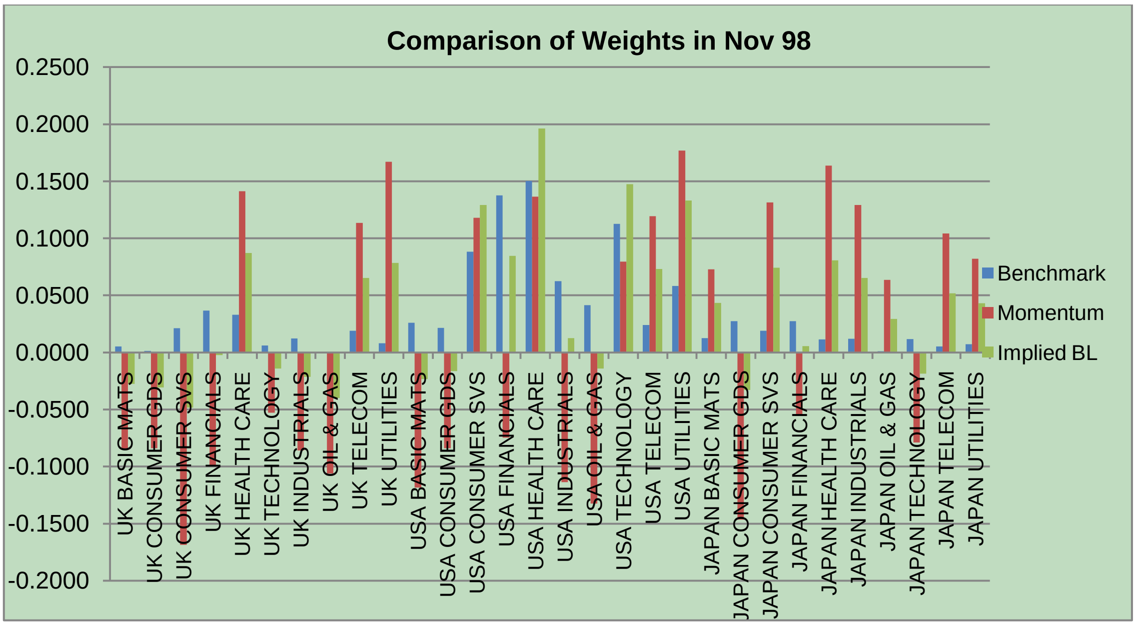


Figure 5.1.5 Comparison of Weights in November 1998

This figure plots the weights of each asset in the benchmark portfolio, the momentum portfolio and the implied BL portfolio in November 1998.

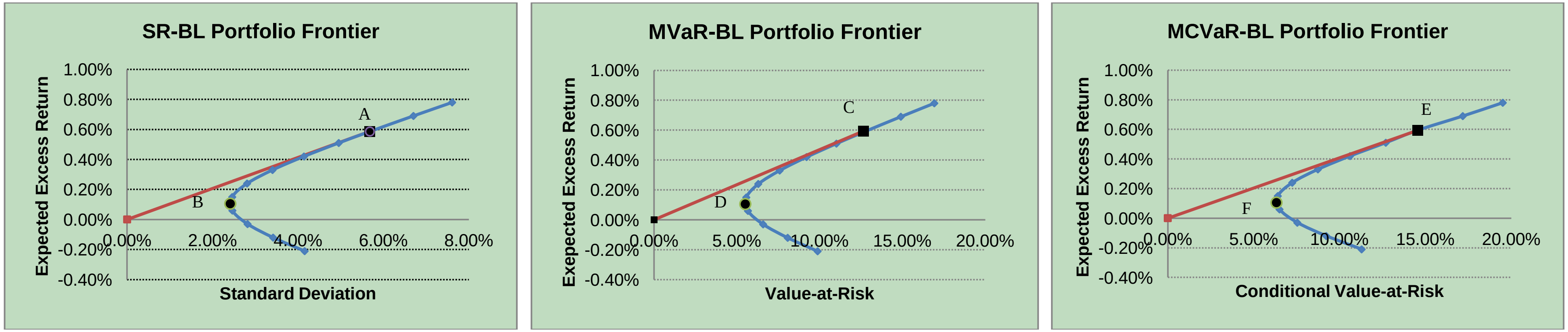


Figure 5.1.6 (a) Figure 5.1.6 (b) Figure 5.1.6 (c)

Figure 5.1.6 The Unconstrained Portfolio Frontier

This figure plots unconstrained portfolio frontier for three different optimisation models. Point A in Figure 5.1.6 (a) is the tangent portfolio that has the highest Sharpe Ratio (SR). Point B is the minimum variance portfolio. The curve above point B is the efficient frontier in the SR-BL model. Point C in Figure 5.1.6 (b) is the tangent portfolio that has the highest reward to VaR ratio, and Point D is the minimum VaR portfolio; the curve above Point D is the efficient frontier in the MVaR-BL model. Point E in Figure 5.1.6 (c) is the tangent portfolio that has the highest reward to CVaR ratio and Point F is the minimum CVaR portfolio; the curve above Point F is the efficient frontier in the MCVaR-BL model. Note that all of results are based on excess return, so the starting point of the tangent line is zero.

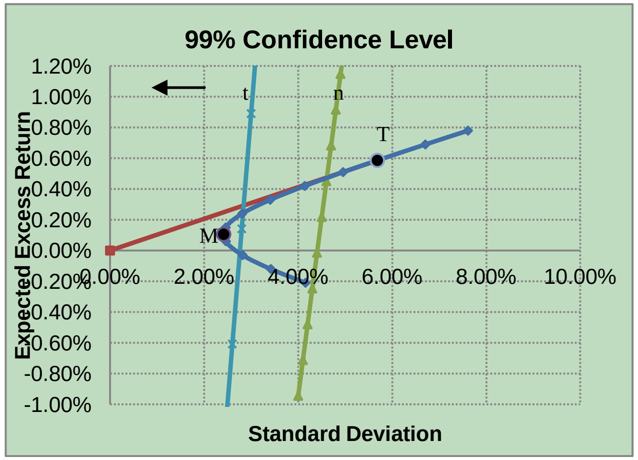


Figure 5.2.1 (a)

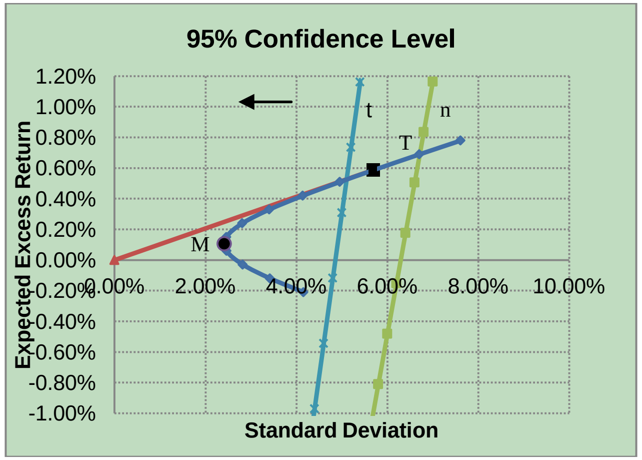


Figure 5.2.1 (b)

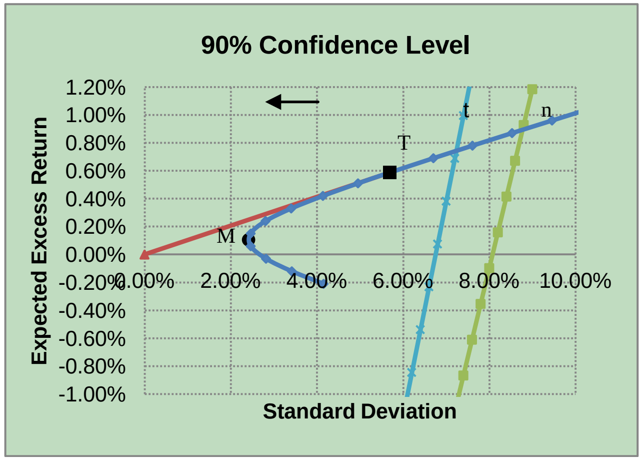


Figure 5.2.1 (c)

Figure 5.2.1 The VaR-Constrained BL Portfolio Frontier

This figure plots the VaR-constrained BL portfolio frontier with different distribution assumptions (normal distribution and t-distribution) and confidence levels (99%, 95%, and 90%) when scaling VaR factor k is equal to 0.99. Note that the constraint is equal to $k * VaR_0$, where VaR_0 is the estimated VaR of the implied BL portfolio. VaR_0 is equal to -10.35% in August 1998. The light blue line represents the VaR constraints in t-distribution, the green line represents the VaR constraints in normal distribution. The black point M is the minimum variance portfolio, and the red square point T is the tangent portfolio that has the maximum SR. The left direction of the black arrow means the VaR constraints tighten as the VaR constraints line move to left.

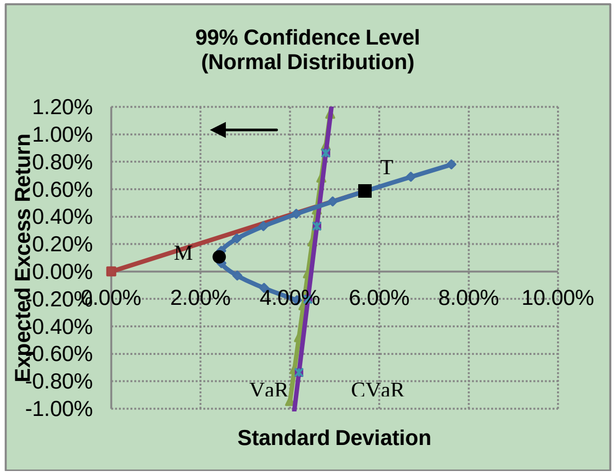


Figure 5.3.1 (a)

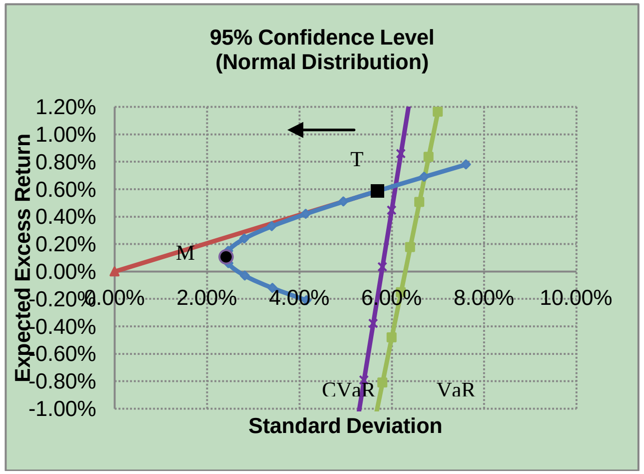


Figure 5.3.1 (b)

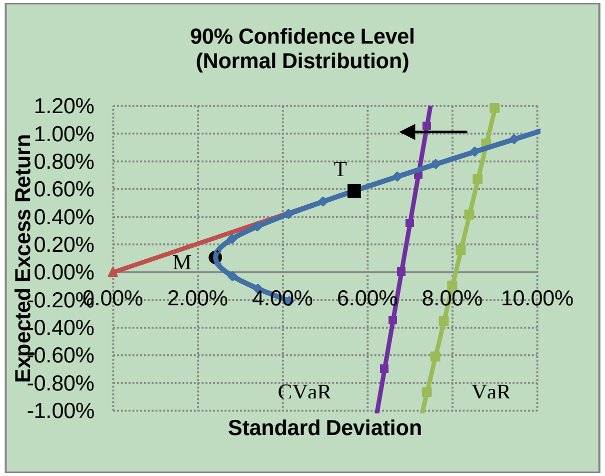


Figure 5.3.1 (c)

Figure 5.3.1 Comparison between VaR Constraints and CVaR Constraints on the BL Portfolio Frontier (Normal Distribution)

This figure plots the VaR constraints and the CVaR constraints on the BL portfolio frontier with a normality assumption at different confidence levels (99%, 95%, 90%) when scaling CVaR factor k is equal to 0.99. Note that the constraint is equal to $k * CVaR_0$, where $CVaR_0$ is the estimated CVaR of the implied BL portfolio. $CVaR_0$ is equal to -11.93% in August 1998. The green line represents the VaR constraints, and the purple line represents the CVaR constraints. The black point M is the minimum variance portfolio, and the red square point T is the tangent portfolio that has the maximum SR. The left direction of the black arrow means the VaR constraints and CVaR constraints tighten as constraints lines move to left.

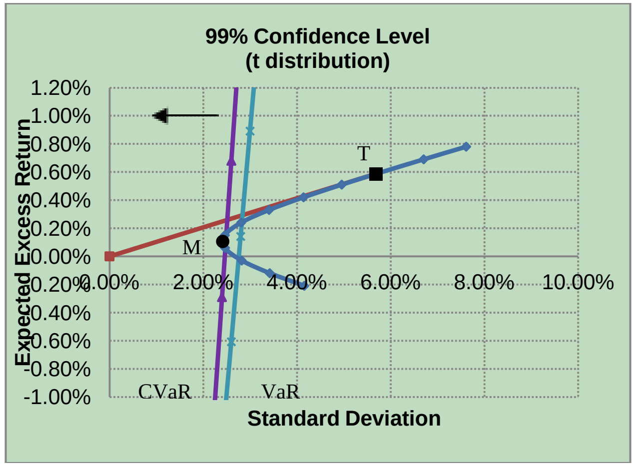


Figure 5.3.2 (a)

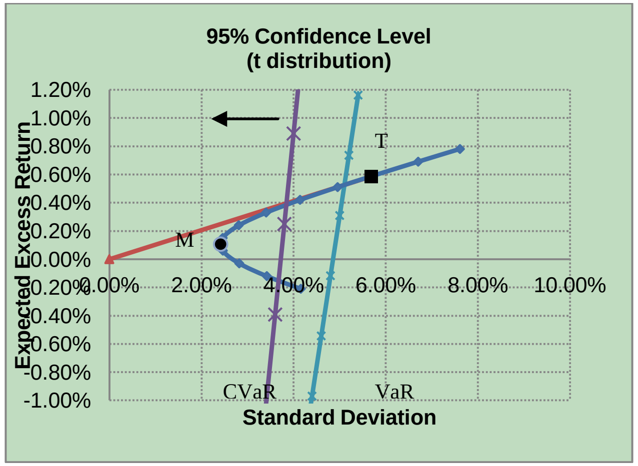


Figure 5.3.2 (b)

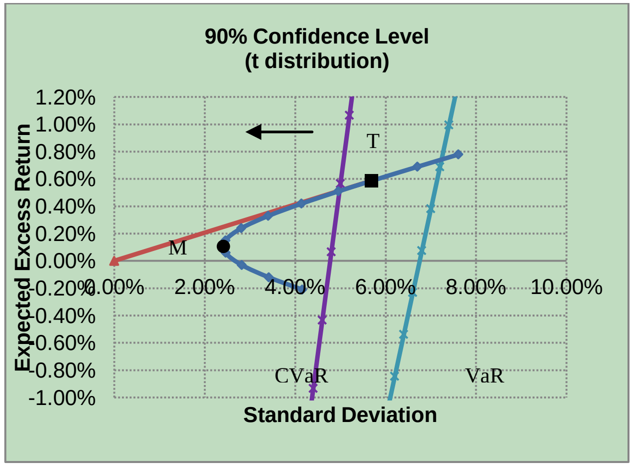


Figure 5.3.2 (c)

Figure 5.3.2 Comparison between VaR Constraints and CVaR Constraints on the BL Portfolio Frontier (t-Distribution)

This figure plots the VaR constraints and CVaR constraints on the BL portfolio frontier with a t distribution assumption at different confidence levels (99%, 95%, 90%) when scaling CVaR factor k is equal to 0.99. Note that the constraint is equal to $k * CVaR_0$, where $CVaR_0$ is the estimated CVaR of the implied BL portfolio. $CVaR_0$ is equal to -11.93% in August 1998. The light blue line represents the VaR constraints, and the purple line represents the CVaR constraints. The black point M is the minimum variance portfolio, and the red square point T is the tangent portfolio that has the maximum SR. The left direction of the black arrow means the VaR constraints and the CVaR constraints tighten as constraints lines move to left.

Appendix 5.1.1 Risk Aversion Coefficient and Implied Equilibrium Return in November 1998

This appendix reports the risk aversion coefficient δ (Panel A) and implied equilibrium return of each index $\boldsymbol{\pi}$ (Panel B) in August 1998. $\delta = \frac{E(r_M) - E(r_f)}{\sigma_M^2}$, the numerator is market risk premium and the denominator is market variance. $\boldsymbol{\pi} = \delta \mathbf{H} \mathbf{w}$, where δ is the risk aversion coefficient, $\mathbf{H}$ is the conditional covariance matrix in the use of the RW model with a window length of 50, the EWMA model and the DCC model, $\mathbf{w}$ is the market capitalisation weight of each index.

Panel A: Risk Aversion Coefficient

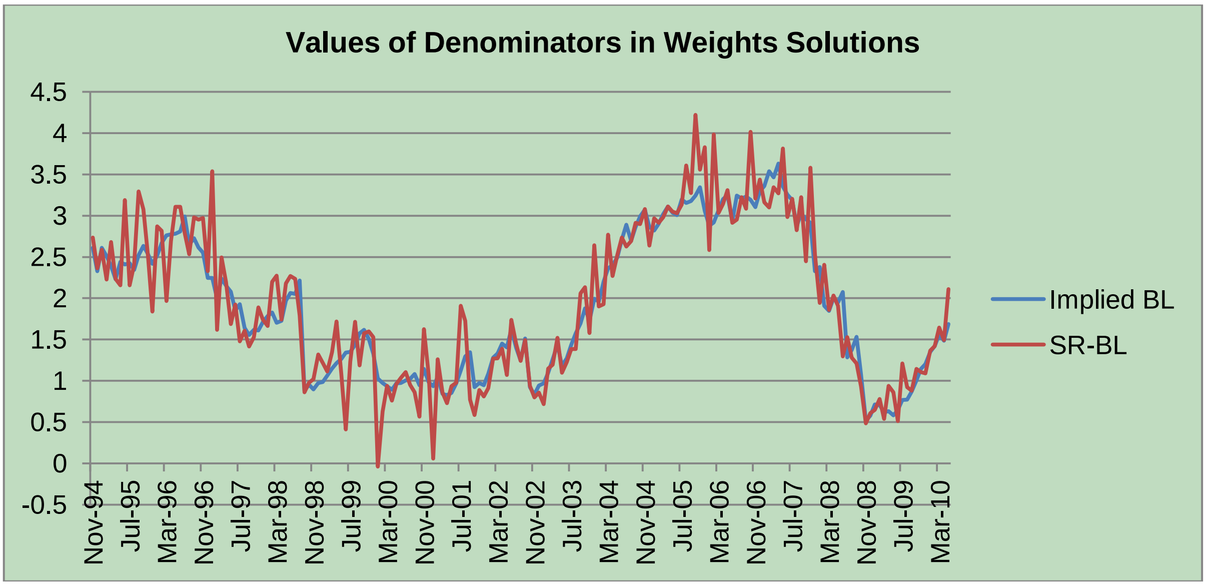
	DCC	EWMA	RW50
Risk Aversion Coefficient	0.8949	1.1774	2.0794

Panel B: Implied Equilibrium Return

	DCC	EWMA	RW50
UK BASIC MATS	0.0027	0.0018	0.0020
UK CONSUMER GDS	0.0025	0.0011	0.0015
UK CONSUMER SVS	0.0017	0.0014	0.0016
UK FINANCIALS	0.0032	0.0026	0.0027
UK HEALTH CARE	0.0015	0.0017	0.0018
UK TECHNOLOGY	0.0039	0.0010	0.0019
UK INDUSTRIALS	0.0030	0.0022	0.0018
UK OIL & GAS	0.0023	0.0020	0.0022
UK TELECOM	0.0020	0.0017	0.0014
UK UTILITIES	0.0011	0.0009	0.0012
USA BASIC MATS	0.0026	0.0026	0.0028
USA CONSUMER GDS	0.0040	0.0035	0.0033
USA CONSUMER SVS	0.0031	0.0031	0.0029
USA FINANCIALS	0.0048	0.0044	0.0042
USA HEALTH CARE	0.0023	0.0029	0.0028
USA INDUSTRIALS	0.0032	0.0031	0.0032
USA OIL & GAS	0.0019	0.0019	0.0022
USA TECHNOLOGY	0.0041	0.0042	0.0041
USA TELECOM	0.0023	0.0023	0.0022
USA UTILITIES	0.0011	0.0006	0.0008
JAPAN BASIC MATS	0.0029	0.0023	0.0023
JAPAN CONSUMER GDS	0.0017	0.0024	0.0027
JAPAN CONSUMER SVS	0.0015	0.0020	0.0023
JAPAN FINANCIALS	0.0039	0.0044	0.0043
JAPAN HEALTH CARE	0.0011	0.0019	0.0022
JAPAN INDUSTRIALS	0.0020	0.0019	0.0022
JAPAN OIL & GAS	0.0023	0.0028	0.0031
JAPAN TECHNOLOGY	0.0032	0.0031	0.0034
JAPAN TELECOM	0.0017	0.0021	0.0025
JAPAN UTILITIES	0.0010	0.0011	0.0014

Appendix 5.1.2 Denominators in Weights Solutions (Nov 94 – May 10)

This appendix shows the time-varying denominators in the weights calculation of the implied BL portfolio and the SR-BL portfolio. Weights in the unconstrained implied BL portfolio are calculated by $\mathbf{w}^*_{BL,t} = \frac{1}{\delta_t}\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$. The SR-BL portfolios allocate assets to achieve the maximal SR in the optimisation problem, weights can be calculated by $\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}$.



Appendix 5.1.3 Weights in the Traditional Mean-Variance Portfolio (Nov 94 – May 10)

This table reports the weights allocated to each asset in the use of traditional mean-variance model with short-selling and without short-selling in the period from November 1994 to May 2010.

	Short	Non-Short
UK BASIC MATS	0.3258	0
UK CONSUMER GDS	0.5419	0
UK CONSUMER SVS	-0.6925	0
UK FINANCIALS	-0.4961	0
UK HEALTH CARE	0.0405	0
UK TECHNOLOGY	-0.5110	0
UK INDUSTRIALS	-0.1783	0
UK OIL & GAS	-0.0733	0
UK TELECOM	0.1016	0
UK UTILITIES	0.6328	0
USA BASIC MATS	-0.9184	0
USA CONSUMER GDS	-0.7857	0
USA CONSUMER SVS	1.3015	0
USA FINANCIALS	-0.2253	0
USA HEALTH CARE	1.1630	0.4796
USA INDUSTRIALS	1.4963	0
USA OIL & GAS	1.2291	0.4687
USA TECHNOLOGY	0.4165	0.0517
USA TELECOM	-1.0000	0
USA UTILITIES	-0.4064	0
JAPAN BASIC MATS	0.2738	0
JAPAN CONSUMER GDS	0.1870	0
JAPAN CONSUMER SVS	-0.2913	0
JAPAN FINANCIALS	-0.7356	0
JAPAN HEALTH CARE	0.2024	0
JAPAN INDUSTRIALS	-0.0141	0
JAPAN OIL & GAS	-0.3372	0
JAPAN TECHNOLOGY	-0.2026	0
JAPAN TELECOM	0.3094	0
JAPAN UTILITIES	-0.3539	0

Appendix 5.1.4 Average Value of Weights in the Unconstrained Implied BL Portfolio and the SR-BL Portfolio (Nov 94 – May 10)

This table reports the average value of weights assigned in each index in the unconstrained implied BL portfolio and the SR-BL portfolio in the period from November 1994 to May 2010. An implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem.

	DCC		EWMA		RW50	
	Implied BL	SR-BL	Implied BL	SR-BL	Implied BL	SR-BL
UK BASIC MATS	0.0055	0.0045	0.0111	0.0135	0.0087	0.0092
UK CONSUMER GDS	0.0092	0.0090	0.0115	0.0134	0.0120	0.0108
UK CONSUMER SVS	0.0120	0.0162	0.0125	0.0158	0.0168	0.0161
UK FINANCIALS	0.0390	0.0368	0.0429	0.0434	0.0384	0.0368
UK HEALTH CARE	0.0322	0.0164	0.0338	0.0214	0.0358	0.0328
UK TECHNOLOGY	0.0023	0.0046	0.0024	0.0037	0.0062	0.0065
UK INDUSTRIALS	0.0179	0.0211	0.0225	0.0269	0.0232	0.0231
UK OIL & GAS	0.0025	0.0009	0.0083	0.0106	0.0011	0.0021
UK TELECOM	0.0167	0.0224	0.0156	0.0162	0.0188	0.0184
UK UTILITIES	0.0067	-0.0086	0.0058	-0.0004	0.0097	0.0085
USA BASIC MATS	0.0318	0.0289	0.0380	0.0411	0.0301	0.0297
USA CONSUMER GDS	0.0385	0.0323	0.0430	0.0369	0.0384	0.0362
USA CONSUMER SVS	0.0982	0.1088	0.0955	0.0949	0.0891	0.0878
USA FINANCIALS	0.1212	0.1172	0.1262	0.1210	0.1203	0.1196
USA HEALTH CARE	0.1249	0.1140	0.1253	0.1153	0.1268	0.1239
USA INDUSTRIALS	0.0808	0.0828	0.0753	0.0736	0.0714	0.0697
USA OIL & GAS	0.0510	0.0458	0.0544	0.0515	0.0494	0.0501
USA TECHNOLOGY	0.1055	0.1162	0.1037	0.1060	0.1074	0.1076
USA TELECOM	0.0331	0.0309	0.0245	0.0230	0.0279	0.0270
USA UTILITIES	0.0405	0.0321	0.0366	0.0352	0.0402	0.0395
JAPAN BASIC MATS	0.0115	0.0165	0.0068	0.0120	0.0124	0.0139
JAPAN CONSUMER GDS	0.0149	0.0272	0.0196	0.0279	0.0213	0.0225
JAPAN CONSUMER SVS	0.0263	0.0344	0.0214	0.0193	0.0189	0.0200
JAPAN FINANCIALS	0.0341	0.0401	0.0333	0.0373	0.0353	0.0361
JAPAN HEALTH CARE	0.0096	0.0071	0.0108	0.0075	0.0098	0.0108
JAPAN INDUSTRIALS	0.0117	0.0204	0.0117	0.0194	0.0135	0.0158
JAPAN OIL & GAS	-0.0006	-0.0052	-0.0002	-0.0012	0.0011	0.0020
JAPAN TECHNOLOGY	0.0093	0.0172	0.0084	0.0146	0.0083	0.0100
JAPAN TELECOM	0.0101	0.0175	0.0073	0.0057	0.0066	0.0083
JAPAN UTILITIES	0.0036	-0.0074	0.0017	-0.0056	0.0044	0.0054
Absolute Position Range	0.1255	0.1259	0.1263	0.1266	0.1257	0.1219

Appendix 5.1.5 Standard Deviation of Weights in the Unconstrained Implied BL Portfolio and the SR-BL Portfolio (Nov 94 – May 10)

This table reports standard deviation of weights assigned in each index in the unconstrained implied BL portfolio and the SR-BL portfolio in the period from November 1994 to May 2010. An implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem.

	DCC		EWMA		RW50	
	Implied BL	SR-BL	Implied BL	SR-BL	Implied BL	SR-BL
UK BASIC MATS	0.0772	0.1025	0.0701	0.0734	0.0438	0.0378
UK CONSUMER GDS	0.0757	0.0963	0.0776	0.0891	0.0396	0.0344
UK CONSUMER SVS	0.1023	0.1347	0.0960	0.1055	0.0546	0.0478
UK FINANCIALS	0.0827	0.1184	0.0726	0.0804	0.0425	0.0365
UK HEALTH CARE	0.1239	0.1577	0.1091	0.1183	0.0570	0.0495
UK TECHNOLOGY	0.0444	0.0538	0.0439	0.0485	0.0224	0.0201
UK INDUSTRIALS	0.0773	0.0969	0.0754	0.0875	0.0435	0.0370
UK OIL & GAS	0.0901	0.1226	0.0812	0.0906	0.0447	0.0383
UK TELECOM	0.0812	0.1108	0.0762	0.0866	0.0442	0.0381
UK UTILITIES	0.1189	0.1543	0.1068	0.1130	0.0528	0.0478
USA BASIC MATS	0.0831	0.1051	0.0772	0.0844	0.0465	0.0394
USA CONSUMER GDS	0.0990	0.1257	0.0985	0.1036	0.0562	0.0488
USA CONSUMER SVS	0.1039	0.1495	0.0949	0.1023	0.0569	0.0482
USA FINANCIALS	0.0880	0.0945	0.0791	0.0751	0.0498	0.0439
USA HEALTH CARE	0.1304	0.1451	0.1186	0.1122	0.0672	0.0565
USA INDUSTRIALS	0.0958	0.1347	0.0884	0.0906	0.0537	0.0458
USA OIL & GAS	0.1010	0.1227	0.0909	0.0926	0.0547	0.0480
USA TECHNOLOGY	0.0724	0.1231	0.0720	0.0854	0.0480	0.0473
USA TELECOM	0.0938	0.1243	0.0849	0.0966	0.0509	0.0434
USA UTILITIES	0.1108	0.1340	0.1058	0.1156	0.0589	0.0517
JAPAN BASIC MATS	0.0823	0.1133	0.0683	0.0743	0.0345	0.0319
JAPAN CONSUMER GDS	0.0999	0.1390	0.0901	0.1029	0.0448	0.0409
JAPAN CONSUMER SVS	0.1114	0.1552	0.1046	0.1179	0.0484	0.0441
JAPAN FINANCIALS	0.0671	0.0908	0.0611	0.0693	0.0381	0.0365
JAPAN HEALTH CARE	0.1133	0.1534	0.0957	0.1027	0.0483	0.0428
JAPAN INDUSTRIALS	0.0895	0.1237	0.0768	0.0893	0.0402	0.0371
JAPAN OIL & GAS	0.0603	0.0842	0.0532	0.0604	0.0271	0.0243
JAPAN TECHNOLOGY	0.0626	0.0783	0.0629	0.0703	0.0321	0.0290
JAPAN TELECOM	0.0714	0.0848	0.0724	0.0858	0.0375	0.0331
JAPAN UTILITIES	0.1103	0.1400	0.0957	0.1066	0.0486	0.0429
Average Standard Deviation	0.0907	0.1190	0.0833	0.0910	0.0462	0.0408

Appendix 5.1.6 Average Value of Weights in the Unconstrained MVaR-BL Portfolio (Nov 94 – May 10)

This table reports average value of weights allocated to each index in the unconstrained MVaR-BL portfolio in the period from November 1994 to May 2010. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0102	0.0115	0.0085	0.0148	0.0052	0.0084
UK CONSUMER GDS	0.0111	0.0115	0.0093	0.0103	0.0086	0.0083
UK CONSUMER SVS	0.0132	0.0146	0.0183	0.0124	0.0142	0.0176
UK FINANCIALS	0.0265	0.0365	0.0379	0.0314	0.0357	0.0372
UK HEALTH CARE	0.0189	0.0257	0.0291	0.0300	0.0300	0.0279
UK TECHNOLOGY	0.0025	-0.0003	0.0044	-0.0117	-0.0018	0.0035
UK INDUSTRIALS	0.0231	0.0250	0.0212	0.0186	0.0237	0.0220
UK OIL & GAS	-0.0022	0.0059	-0.0006	0.0039	0.0011	0.0000
UK TELECOM	0.0203	0.0128	0.0177	0.0156	0.0105	0.0169
UK UTILITIES	-0.0017	0.0119	0.0092	0.0079	0.0123	0.0098
USA BASIC MATS	0.0245	0.0296	0.0243	0.0226	0.0261	0.0236
USA CONSUMER GDS	0.0293	0.0316	0.0333	0.0289	0.0313	0.0318
USA CONSUMER SVS	0.0928	0.0852	0.0874	0.0853	0.0877	0.0863
USA FINANCIALS	0.1189	0.1281	0.1279	0.1240	0.1314	0.1281
USA HEALTH CARE	0.1147	0.1230	0.1264	0.1235	0.1265	0.1264
USA INDUSTRIALS	0.0718	0.0703	0.0685	0.0667	0.0695	0.0674
USA OIL & GAS	0.0485	0.0582	0.0511	0.0524	0.0515	0.0524
USA TECHNOLOGY	0.1113	0.1008	0.1088	0.1012	0.1010	0.1088
USA TELECOM	0.0297	0.0219	0.0243	0.0272	0.0228	0.0237
USA UTILITIES	0.0241	0.0393	0.0400	0.0342	0.0368	0.0412
JAPAN BASIC MATS	0.0190	0.0144	0.0134	0.0196	0.0143	0.0151
JAPAN CONSUMER GDS	0.0352	0.0249	0.0266	0.0287	0.0247	0.0266
JAPAN CONSUMER SVS	0.0251	0.0196	0.0183	0.0227	0.0224	0.0189
JAPAN FINANCIALS	0.0403	0.0372	0.0381	0.0442	0.0370	0.0387
JAPAN HEALTH CARE	0.0114	0.0126	0.0119	0.0171	0.0167	0.0128
JAPAN INDUSTRIALS	0.0280	0.0173	0.0164	0.0210	0.0168	0.0182
JAPAN OIL & GAS	-0.0004	-0.0004	0.0014	0.0074	-0.0001	0.0011
JAPAN TECHNOLOGY	0.0252	0.0090	0.0105	0.0118	0.0099	0.0100
JAPAN TELECOM	0.0228	0.0119	0.0083	0.0146	0.0171	0.0081
JAPAN UTILITIES	0.0057	0.0105	0.0083	0.0137	0.0171	0.0093
Absolute Position Range	0.1211	0.1285	0.1285	0.1357	0.1332	0.1282

Appendix 5.1.7 Standard Deviation of Weights in the Unconstrained MVaR-BL Portfolio (Nov 94 – May 10)

This table reports standard deviation of weights allocated to each index in the unconstrained MVaR-BL portfolio in the period from November 1994 to May 2010. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0854	0.1123	0.0301	0.0898	0.1303	0.0247
UK CONSUMER GDS	0.0879	0.0704	0.0328	0.0771	0.0718	0.0272
UK CONSUMER SVS	0.0730	0.0613	0.0274	0.0605	0.0519	0.0207
UK FINANCIALS	0.0900	0.0700	0.0397	0.0809	0.0614	0.0301
UK HEALTH CARE	0.1026	0.0762	0.0333	0.1197	0.0621	0.0251
UK TECHNOLOGY	0.1027	0.0743	0.0241	0.1550	0.0708	0.0252
UK INDUSTRIALS	0.0789	0.0827	0.0317	0.0667	0.0784	0.0233
UK OIL & GAS	0.0873	0.1035	0.0367	0.0937	0.1125	0.0285
UK TELECOM	0.1131	0.0939	0.0369	0.1014	0.0856	0.0304
UK UTILITIES	0.0919	0.0642	0.0299	0.0959	0.0610	0.0233
USA BASIC MATS	0.0862	0.0775	0.0271	0.0750	0.0724	0.0222
USA CONSUMER GDS	0.0818	0.0583	0.0371	0.0740	0.0568	0.0319
USA CONSUMER SVS	0.0853	0.0613	0.0284	0.0854	0.0510	0.0232
USA FINANCIALS	0.1067	0.0669	0.0391	0.0994	0.0720	0.0362
USA HEALTH CARE	0.1086	0.0712	0.0440	0.1036	0.0660	0.0396
USA INDUSTRIALS	0.0621	0.0448	0.0251	0.0588	0.0379	0.0200
USA OIL & GAS	0.1016	0.0894	0.0432	0.0933	0.1000	0.0337
USA TECHNOLOGY	0.1169	0.0897	0.0424	0.1328	0.0804	0.0405
USA TELECOM	0.0893	0.0717	0.0337	0.0759	0.0621	0.0268
USA UTILITIES	0.1213	0.0760	0.0300	0.1100	0.0683	0.0257
JAPAN BASIC MATS	0.0913	0.0493	0.0285	0.0899	0.0491	0.0240
JAPAN CONSUMER GDS	0.1129	0.0579	0.0241	0.1107	0.0514	0.0212
JAPAN CONSUMER SVS	0.0701	0.0676	0.0260	0.0673	0.0731	0.0221
JAPAN FINANCIALS	0.0986	0.0823	0.0386	0.1054	0.0797	0.0381
JAPAN HEALTH CARE	0.0793	0.0846	0.0268	0.0782	0.0749	0.0213
JAPAN INDUSTRIALS	0.0993	0.0613	0.0316	0.1115	0.0574	0.0258
JAPAN OIL & GAS	0.1168	0.0622	0.0287	0.1219	0.0666	0.0287
JAPAN TECHNOLOGY	0.1296	0.0842	0.0331	0.1587	0.0815	0.0320
JAPAN TELECOM	0.0946	0.1063	0.0366	0.1011	0.1064	0.0351
JAPAN UTILITIES	0.0882	0.1051	0.0352	0.0983	0.0951	0.0256
Average Standard Deviation	0.0951	0.0759	0.0327	0.0964	0.0729	0.0277

Appendix 5.1.8 Average Effect of Distribution Assumptions and Confidence Levels on MVaR-BL Portfolio Weights

This table shows average value of weights in each index and average standard deviation in the unconstrained MVaR-BL portfolio in the period from November 1994 to May 2010. Note that the covariance matrix applied to the MVaR-BL model is the DCC covariance matrix in this table.

	Normal Distribution			t-Distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
UK BASIC MATS	0.0102	0.0070	0.0064	0.0148	0.0095	0.0072
UK CONSUMER GDS	0.0111	0.0131	0.0142	0.0103	0.0113	0.0136
UK CONSUMER SVS	0.0132	0.0129	0.0127	0.0124	0.0130	0.0125
UK FINANCIALS	0.0265	0.0293	0.0305	0.0314	0.0265	0.0298
UK HEALTH CARE	0.0189	0.0195	0.0205	0.0300	0.0188	0.0198
UK TECHNOLOGY	0.0025	0.0041	0.0031	-0.0117	0.0015	0.0039
UK INDUSTRIALS	0.0231	0.0210	0.0207	0.0186	0.0209	0.0205
UK OIL & GAS	-0.0022	-0.0036	-0.0039	0.0039	-0.0029	-0.0035
UK TELECOM	0.0203	0.0233	0.0239	0.0156	0.0209	0.0229
UK UTILITIES	-0.0017	-0.0025	-0.0022	0.0079	-0.0027	-0.0013
USA BASIC MATS	0.0245	0.0274	0.0289	0.0226	0.0258	0.0284
USA CONSUMER GDS	0.0293	0.0340	0.0358	0.0289	0.0295	0.0344
USA CONSUMER SVS	0.0928	0.0975	0.0988	0.0853	0.0943	0.0977
USA FINANCIALS	0.1189	0.1224	0.1218	0.1240	0.1193	0.1221
USA HEALTH CARE	0.1147	0.1178	0.1203	0.1235	0.1159	0.1194
USA INDUSTRIALS	0.0718	0.0771	0.0770	0.0667	0.0729	0.0765
USA OIL & GAS	0.0485	0.0493	0.0486	0.0524	0.0487	0.0495
USA TECHNOLOGY	0.1113	0.1143	0.1144	0.1012	0.1122	0.1144
USA TELECOM	0.0297	0.0323	0.0345	0.0272	0.0302	0.0333
USA UTILITIES	0.0241	0.0254	0.0258	0.0342	0.0243	0.0262
JAPAN BASIC MATS	0.0190	0.0152	0.0152	0.0196	0.0191	0.0148
JAPAN CONSUMER GDS	0.0352	0.0330	0.0312	0.0287	0.0354	0.0318
JAPAN CONSUMER SVS	0.0251	0.0225	0.0222	0.0227	0.0249	0.0219
JAPAN FINANCIALS	0.0403	0.0369	0.0362	0.0442	0.0413	0.0361
JAPAN HEALTH CARE	0.0114	0.0099	0.0079	0.0171	0.0109	0.0087
JAPAN INDUSTRIALS	0.0280	0.0246	0.0231	0.0210	0.0277	0.0238
JAPAN OIL & GAS	-0.0004	-0.0060	-0.0068	0.0074	-0.0012	-0.0059
JAPAN TECHNOLOGY	0.0252	0.0196	0.0183	0.0118	0.0247	0.0192
JAPAN TELECOM	0.0228	0.0191	0.0176	0.0146	0.0227	0.0186
JAPAN UTILITIES	0.0057	0.0036	0.0033	0.0137	0.0047	0.0039
Absolute Position Range	0.1211	0.1284	0.1286	0.1357	0.1222	0.1280
Average Standard Deviation	0.0951	0.0941	0.0964	0.0964	0.0995	0.0947

Appendix 5.1.9 Average Value of Weights in the Unconstrained MCVaR-BL Portfolio (Nov 94 – May 10)

This table reports average value of weights allocated to each index in the unconstrained MCVaR-BL portfolio in the period from November 1994 to May 2010. The weight in the MCVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to CVaR ratio. CVaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0111	0.0160	0.0085	0.0139	0.0012	0.0075
UK CONSUMER GDS	0.0122	0.0107	0.0088	0.0104	0.0087	0.0071
UK CONSUMER SVS	0.0134	0.0162	0.0183	0.0113	0.0158	0.0170
UK FINANCIALS	0.0285	0.0385	0.0377	0.0335	0.0372	0.0357
UK HEALTH CARE	0.0228	0.0239	0.0290	0.0331	0.0329	0.0267
UK TECHNOLOGY	-0.0022	0.0019	0.0042	-0.0102	-0.0012	0.0032
UK INDUSTRIALS	0.0220	0.0256	0.0217	0.0157	0.0240	0.0219
UK OIL & GAS	-0.0011	0.0068	-0.0004	0.0073	-0.0040	0.0008
UK TELECOM	0.0195	0.0132	0.0171	0.0109	0.0093	0.0162
UK UTILITIES	0.0010	0.0120	0.0096	0.0102	0.0132	0.0087
USA BASIC MATS	0.0238	0.0314	0.0240	0.0235	0.0237	0.0231
USA CONSUMER GDS	0.0296	0.0310	0.0330	0.0272	0.0341	0.0312
USA CONSUMER SVS	0.0909	0.0864	0.0868	0.0809	0.0901	0.0860
USA FINANCIALS	0.1206	0.1293	0.1279	0.1254	0.1353	0.1282
USA HEALTH CARE	0.1173	0.1218	0.1264	0.1263	0.1295	0.1262
USA INDUSTRIALS	0.0707	0.0701	0.0679	0.0654	0.0700	0.0673
USA OIL & GAS	0.0476	0.0580	0.0515	0.0551	0.0461	0.0535
USA TECHNOLOGY	0.1083	0.1031	0.1088	0.0974	0.1027	0.1090
USA TELECOM	0.0285	0.0224	0.0238	0.0238	0.0220	0.0240
USA UTILITIES	0.0253	0.0388	0.0404	0.0373	0.0365	0.0417
JAPAN BASIC MATS	0.0198	0.0142	0.0139	0.0214	0.0134	0.0163
JAPAN CONSUMER GDS	0.0337	0.0258	0.0268	0.0262	0.0247	0.0263
JAPAN CONSUMER SVS	0.0249	0.0172	0.0186	0.0212	0.0230	0.0194
JAPAN FINANCIALS	0.0416	0.0357	0.0384	0.0481	0.0365	0.0385
JAPAN HEALTH CARE	0.0124	0.0095	0.0120	0.0184	0.0191	0.0133
JAPAN INDUSTRIALS	0.0262	0.0178	0.0172	0.0203	0.0138	0.0192
JAPAN OIL & GAS	0.0027	-0.0011	0.0008	0.0128	-0.0023	0.0025
JAPAN TECHNOLOGY	0.0210	0.0090	0.0106	0.0103	0.0083	0.0106
JAPAN TELECOM	0.0191	0.0077	0.0082	0.0113	0.0166	0.0087
JAPAN UTILITIES	0.0086	0.0070	0.0084	0.0117	0.0198	0.0102
Absolute Position Range	0.1227	0.1304	0.1284	0.1366	0.1393	0.1274

Appendix 5.1.10 Standard Deviation of Weights in the Unconstrained MCVaR-BL Portfolio (Nov 94 – May 10)

This table reports standard deviation of weights allocated to each index in the unconstrained MVaR-BL portfolio in the period from November 1994 to May 2010. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0870	0.0837	0.0278	0.0872	0.1443	0.0229
UK CONSUMER GDS	0.0856	0.0701	0.0319	0.0728	0.0653	0.0239
UK CONSUMER SVS	0.0702	0.0534	0.0259	0.0594	0.0463	0.0170
UK FINANCIALS	0.0900	0.0593	0.0365	0.0775	0.0627	0.0233
UK HEALTH CARE	0.1108	0.0679	0.0298	0.1108	0.0760	0.0194
UK TECHNOLOGY	0.1297	0.0628	0.0241	0.1322	0.0733	0.0264
UK INDUSTRIALS	0.0773	0.0797	0.0286	0.0697	0.0744	0.0210
UK OIL & GAS	0.0888	0.1031	0.0343	0.0866	0.1308	0.0224
UK TELECOM	0.1126	0.0919	0.0349	0.0932	0.0847	0.0256
UK UTILITIES	0.0903	0.0638	0.0272	0.0863	0.0586	0.0193
USA BASIC MATS	0.0829	0.0629	0.0248	0.0669	0.0732	0.0197
USA CONSUMER GDS	0.0806	0.0563	0.0357	0.0715	0.0674	0.0297
USA CONSUMER SVS	0.0836	0.0516	0.0264	0.0800	0.0552	0.0206
USA FINANCIALS	0.1094	0.0653	0.0376	0.0948	0.0906	0.0352
USA HEALTH CARE	0.1121	0.0683	0.0421	0.0917	0.0741	0.0379
USA INDUSTRIALS	0.0605	0.0420	0.0227	0.0543	0.0396	0.0178
USA OIL & GAS	0.0990	0.0886	0.0395	0.0828	0.1156	0.0281
USA TECHNOLOGY	0.1267	0.0796	0.0420	0.1252	0.0772	0.0393
USA TELECOM	0.0864	0.0682	0.0305	0.0670	0.0597	0.0215
USA UTILITIES	0.1218	0.0709	0.0288	0.0982	0.0648	0.0233
JAPAN BASIC MATS	0.0912	0.0494	0.0273	0.0890	0.0540	0.0215
JAPAN CONSUMER GDS	0.1176	0.0526	0.0231	0.0995	0.0468	0.0199
JAPAN CONSUMER SVS	0.0689	0.0538	0.0245	0.0669	0.0718	0.0207
JAPAN FINANCIALS	0.0992	0.0747	0.0385	0.1130	0.0770	0.0377
JAPAN HEALTH CARE	0.0783	0.0668	0.0252	0.0728	0.0782	0.0192
JAPAN INDUSTRIALS	0.1043	0.0599	0.0296	0.1062	0.0713	0.0232
JAPAN OIL & GAS	0.1212	0.0627	0.0289	0.1237	0.0869	0.0264
JAPAN TECHNOLOGY	0.1502	0.0836	0.0325	0.1507	0.0765	0.0303
JAPAN TELECOM	0.1078	0.0786	0.0358	0.0975	0.1070	0.0324
JAPAN UTILITIES	0.0870	0.0807	0.0322	0.0853	0.0988	0.0213
Average Standard Deviation	0.0977	0.0684	0.0310	0.0904	0.0767	0.0249

Appendix 5.1.11 Average Effect of Distribution Assumptions and Confidence Levels on MCVaR-BL Portfolio Weights

This table shows average value of weights in each index and average standard deviation in the unconstrained MCVaR-BL portfolio in the period from November 1994 to May 2010. Note that the covariance matrix applied to the MCVaR-BL model is the DCC covariance matrix in this table.

MCVaR-BL Portfolio Weights	Normal Distribution			t-Distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
UK BASIC MATS	0.0111	0.0096	0.0075	0.0139	0.0135	0.0097
UK CONSUMER GDS	0.0122	0.0125	0.0127	0.0104	0.0114	0.0112
UK CONSUMER SVS	0.0134	0.0122	0.0128	0.0113	0.0132	0.0126
UK FINANCIALS	0.0285	0.0270	0.0290	0.0335	0.0299	0.0264
UK HEALTH CARE	0.0228	0.0184	0.0190	0.0331	0.0274	0.0201
UK TECHNOLOGY	-0.0022	0.0004	0.0048	-0.0102	-0.0070	0.0028
UK INDUSTRIALS	0.0220	0.0215	0.0204	0.0157	0.0200	0.0223
UK OIL & GAS	-0.0011	-0.0022	-0.0035	0.0073	0.0013	-0.0017
UK TELECOM	0.0195	0.0209	0.0239	0.0109	0.0194	0.0194
UK UTILITIES	0.0010	-0.0026	-0.0018	0.0102	0.0066	0.0004
USA BASIC MATS	0.0238	0.0260	0.0272	0.0235	0.0223	0.0235
USA CONSUMER GDS	0.0296	0.0294	0.0333	0.0272	0.0289	0.0290
USA CONSUMER SVS	0.0909	0.0943	0.0970	0.0809	0.0872	0.0912
USA FINANCIALS	0.1206	0.1202	0.1227	0.1254	0.1228	0.1192
USA HEALTH CARE	0.1173	0.1156	0.1170	0.1263	0.1223	0.1158
USA INDUSTRIALS	0.0707	0.0741	0.0769	0.0654	0.0680	0.0709
USA OIL & GAS	0.0476	0.0491	0.0489	0.0551	0.0509	0.0488
USA TECHNOLOGY	0.1083	0.1123	0.1142	0.0974	0.1050	0.1106
USA TELECOM	0.0285	0.0308	0.0334	0.0238	0.0294	0.0293
USA UTILITIES	0.0253	0.0239	0.0251	0.0373	0.0333	0.0257
JAPAN BASIC MATS	0.0198	0.0182	0.0145	0.0214	0.0189	0.0195
JAPAN CONSUMER GDS	0.0337	0.0351	0.0327	0.0262	0.0287	0.0347
JAPAN CONSUMER SVS	0.0249	0.0242	0.0233	0.0212	0.0225	0.0247
JAPAN FINANCIALS	0.0416	0.0407	0.0366	0.0481	0.0414	0.0403
JAPAN HEALTH CARE	0.0124	0.0105	0.0090	0.0184	0.0161	0.0121
JAPAN INDUSTRIALS	0.0262	0.0280	0.0244	0.0203	0.0211	0.0277
JAPAN OIL & GAS	0.0027	-0.0019	-0.0058	0.0128	0.0033	0.0010
JAPAN TECHNOLOGY	0.0210	0.0253	0.0203	0.0103	0.0140	0.0240
JAPAN TELECOM	0.0191	0.0224	0.0195	0.0113	0.0153	0.0218
JAPAN UTILITIES	0.0086	0.0038	0.0048	0.0117	0.0129	0.0069
Absolute Position Range	0.1227	0.1228	0.1285	0.1366	0.1298	0.1209
Average Standard Deviation	0.0977	0.1010	0.0941	0.0904	0.0973	0.0929

Appendix 5.2.1 Average Value of Weights in the VaR-Constrained BL Portfolio (Nov 94 – May 10)

This table reports average value of weights allocated to each index in the VaR-constrained BL portfolio in the period from November 1994 to May 2010. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0059	0.0051	-0.0081	0.0140	-0.0596	-0.0632
UK CONSUMER GDS	0.0109	0.0142	0.0108	0.0184	0.0257	0.0050
UK CONSUMER SVS	0.0153	0.0172	0.0177	0.0498	0.0443	0.0124
UK FINANCIALS	0.0361	0.0396	0.0428	-0.0080	0.0148	0.0616
UK HEALTH CARE	0.0317	0.0273	0.0228	0.0586	0.0152	-0.0179
UK TECHNOLOGY	0.0003	0.0013	-0.0001	-0.0240	-0.0155	-0.0207
UK INDUSTRIALS	0.0185	0.0210	0.0362	0.0296	0.0111	0.0891
UK OIL & GAS	-0.0046	0.0042	-0.0134	-0.0563	-0.0168	-0.0575
UK TELECOM	0.0169	0.0154	0.0222	0.0261	0.0133	0.0365
UK UTILITIES	0.0098	0.0154	0.0267	0.0515	0.0817	0.0777
USA BASIC MATS	0.0262	0.0350	0.0285	-0.0231	-0.0072	0.0294
USA CONSUMER GDS	0.0333	0.0384	0.0282	0.0052	0.0326	0.0021
USA CONSUMER SVS	0.1045	0.1016	0.1064	0.1346	0.1772	0.1746
USA FINANCIALS	0.1119	0.1115	0.1045	0.0653	0.0381	0.0505
USA HEALTH CARE	0.1286	0.1317	0.1388	0.1465	0.2245	0.1762
USA INDUSTRIALS	0.0706	0.0616	0.0437	0.0205	-0.0114	-0.0431
USA OIL & GAS	0.0563	0.0616	0.0711	0.0940	0.1273	0.1257
USA TECHNOLOGY	0.0984	0.0946	0.0855	0.0427	0.0184	0.0200
USA TELECOM	0.0340	0.0221	0.0177	0.0493	0.0000	-0.0144
USA UTILITIES	0.0442	0.0384	0.0549	0.0976	0.0544	0.1178
JAPAN BASIC MATS	0.0112	0.0109	0.0197	-0.0047	0.0096	0.0360
JAPAN CONSUMER GDS	0.0235	0.0256	0.0368	0.0600	0.0630	0.0828
JAPAN CONSUMER SVS	0.0422	0.0310	0.0406	0.1360	0.0959	0.1195
JAPAN FINANCIALS	0.0265	0.0271	0.0190	-0.0388	-0.0366	-0.0391
JAPAN HEALTH CARE	0.0029	0.0065	-0.0081	-0.0549	-0.0114	-0.0716
JAPAN INDUSTRIALS	0.0192	0.0254	0.0264	0.0572	0.0844	0.0663
JAPAN OIL & GAS	-0.0021	-0.0052	-0.0021	-0.0086	-0.0335	-0.0086
JAPAN TECHNOLOGY	0.0044	0.0045	0.0000	-0.0407	-0.0338	-0.0268
JAPAN TELECOM	0.0109	0.0075	0.0071	0.0171	0.0062	-0.0005
JAPAN UTILITIES	0.0126	0.0095	0.0236	0.0853	0.0883	0.0801
Absolute Position Range	0.1333	0.1370	0.1521	0.2027	0.2842	0.2479

Appendix 5.2.2 Standard Deviation of Weights in the VaR-Constrained BL Portfolio (Nov 94 – May 10)

This table reports standard deviation of weights allocated to each index in the VaR-constrained BL portfolio in the period from November 1994 to May 2010. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0696	0.0625	0.0410	0.0888	0.0876	0.0788
UK CONSUMER GDS	0.0678	0.0692	0.0352	0.0456	0.0638	0.0761
UK CONSUMER SVS	0.0907	0.0910	0.0548	0.1079	0.0877	0.1272
UK FINANCIALS	0.0836	0.0671	0.0406	0.0774	0.0778	0.0923
UK HEALTH CARE	0.1100	0.0972	0.0540	0.0773	0.0850	0.1268
UK TECHNOLOGY	0.0401	0.0397	0.0185	0.0309	0.0309	0.0311
UK INDUSTRIALS	0.0708	0.0700	0.0466	0.0872	0.0791	0.1435
UK OIL & GAS	0.0835	0.0816	0.0491	0.0757	0.1161	0.1139
UK TELECOM	0.0746	0.0725	0.0402	0.0504	0.0619	0.0774
UK UTILITIES	0.1032	0.0902	0.0440	0.0618	0.0758	0.0780
USA BASIC MATS	0.0768	0.0715	0.0454	0.0812	0.0897	0.1072
USA CONSUMER GDS	0.0890	0.0882	0.0513	0.0928	0.0777	0.0841
USA CONSUMER SVS	0.1086	0.0958	0.0548	0.1259	0.1372	0.1528
USA FINANCIALS	0.0878	0.0772	0.0631	0.1332	0.1036	0.1453
USA HEALTH CARE	0.1103	0.0902	0.0547	0.1036	0.0887	0.1000
USA INDUSTRIALS	0.0911	0.0879	0.0664	0.0993	0.1378	0.1656
USA OIL & GAS	0.0866	0.0821	0.0490	0.0833	0.1107	0.1110
USA TECHNOLOGY	0.0680	0.0692	0.0417	0.0521	0.0631	0.0611
USA TELECOM	0.0834	0.0768	0.0454	0.0726	0.0734	0.0794
USA UTILITIES	0.0965	0.0961	0.0742	0.0881	0.1324	0.2182
JAPAN BASIC MATS	0.0777	0.0617	0.0358	0.0622	0.0720	0.0748
JAPAN CONSUMER GDS	0.0987	0.0804	0.0443	0.0787	0.1151	0.0899
JAPAN CONSUMER SVS	0.1185	0.0880	0.0558	0.0932	0.1084	0.1477
JAPAN FINANCIALS	0.0598	0.0526	0.0294	0.0444	0.0649	0.0671
JAPAN HEALTH CARE	0.1023	0.0832	0.0496	0.0806	0.0972	0.1345
JAPAN INDUSTRIALS	0.0839	0.0852	0.0540	0.0719	0.1373	0.1397
JAPAN OIL & GAS	0.0569	0.0525	0.0290	0.0442	0.0528	0.0685
JAPAN TECHNOLOGY	0.0535	0.0520	0.0282	0.0501	0.0600	0.0595
JAPAN TELECOM	0.0649	0.0694	0.0374	0.0495	0.0630	0.0690
JAPAN UTILITIES	0.0955	0.0811	0.0451	0.0703	0.0946	0.0825
Average Standard Deviation	0.0835	0.0761	0.0460	0.0760	0.0882	0.1034

Appendix 5.3.1 Average Value of Weights in the CVaR-Constrained BL Portfolio (Nov 94 – May 10)

This table reports average value of weights allocated to each index in the CVaR-constrained BL portfolio in the period from November 1994 to May 2010. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0058	0.0053	0.0047	0.0133	-0.0873	-0.0865
UK CONSUMER GDS	0.0109	0.0140	0.0111	0.0195	0.0309	-0.0013
UK CONSUMER SVS	0.0153	0.0172	0.0172	0.0566	0.0615	0.0082
UK FINANCIALS	0.0361	0.0393	0.0386	-0.0178	0.0062	0.0704
UK HEALTH CARE	0.0317	0.0275	0.0306	0.0623	0.0115	-0.0441
UK TECHNOLOGY	0.0003	0.0014	0.0045	-0.0288	-0.0207	-0.0299
UK INDUSTRIALS	0.0185	0.0210	0.0260	0.0372	0.0052	0.1161
UK OIL & GAS	-0.0046	0.0043	-0.0026	-0.0678	-0.0232	-0.0760
UK TELECOM	0.0169	0.0154	0.0193	0.0286	0.0105	0.0464
UK UTILITIES	0.0097	0.0154	0.0145	0.0643	0.1035	0.0983
USA BASIC MATS	0.0263	0.0348	0.0287	-0.0324	-0.0207	0.0315
USA CONSUMER GDS	0.0333	0.0387	0.0337	-0.0011	0.0262	-0.0124
USA CONSUMER SVS	0.1045	0.1012	0.0926	0.1436	0.2051	0.2053
USA FINANCIALS	0.1118	0.1118	0.1154	0.0420	0.0124	0.0231
USA HEALTH CARE	0.1288	0.1315	0.1290	0.1569	0.2578	0.1952
USA INDUSTRIALS	0.0705	0.0621	0.0631	0.0039	-0.0372	-0.0792
USA OIL & GAS	0.0562	0.0615	0.0566	0.1053	0.1529	0.1507
USA TECHNOLOGY	0.0985	0.0949	0.1009	0.0231	-0.0121	-0.0102
USA TELECOM	0.0340	0.0218	0.0243	0.0492	-0.0091	-0.0284
USA UTILITIES	0.0442	0.0385	0.0437	0.1060	0.0544	0.1500
JAPAN BASIC MATS	0.0111	0.0107	0.0153	-0.0044	0.0134	0.0442
JAPAN CONSUMER GDS	0.0234	0.0256	0.0257	0.0745	0.0822	0.1060
JAPAN CONSUMER SVS	0.0421	0.0308	0.0249	0.1665	0.1208	0.1598
JAPAN FINANCIALS	0.0265	0.0272	0.0313	-0.0549	-0.0556	-0.0670
JAPAN HEALTH CARE	0.0030	0.0064	0.0057	-0.0682	-0.0213	-0.1033
JAPAN INDUSTRIALS	0.0194	0.0253	0.0194	0.0712	0.0992	0.0800
JAPAN OIL & GAS	-0.0022	-0.0050	0.0003	-0.0149	-0.0455	-0.0129
JAPAN TECHNOLOGY	0.0044	0.0047	0.0065	-0.0541	-0.0467	-0.0370
JAPAN TELECOM	0.0109	0.0074	0.0083	0.0170	0.0070	-0.0053
JAPAN UTILITIES	0.0125	0.0097	0.0106	0.1032	0.1183	0.1082
Absolute Position Range	0.1333	0.1365	0.1316	0.2347	0.3451	0.3087

Appendix 5.3.2 Standard Deviation of Weights in the CVaR-Constrained BL Portfolio (Nov 94 – May 10)

This table reports standard deviation of weights allocated to each index in the CVaR-constrained BL portfolio in the period from November 1994 to May 2010. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
	DCC	EWMA	RW50	DCC	EWMA	RW50
UK BASIC MATS	0.0696	0.0625	0.0375	0.1005	0.1158	0.1043
UK CONSUMER GDS	0.0678	0.0701	0.0336	0.0424	0.0691	0.1050
UK CONSUMER SVS	0.0909	0.0923	0.0491	0.1109	0.1000	0.1703
UK FINANCIALS	0.0836	0.0673	0.0364	0.0770	0.0929	0.1251
UK HEALTH CARE	0.1101	0.0975	0.0497	0.0755	0.0899	0.1754
UK TECHNOLOGY	0.0401	0.0397	0.0187	0.0279	0.0312	0.0401
UK INDUSTRIALS	0.0708	0.0703	0.0388	0.1020	0.0927	0.1962
UK OIL & GAS	0.0835	0.0821	0.0416	0.0777	0.1439	0.1506
UK TELECOM	0.0747	0.0727	0.0385	0.0466	0.0666	0.1087
UK UTILITIES	0.1034	0.0907	0.0440	0.0531	0.0772	0.1044
USA BASIC MATS	0.0768	0.0719	0.0397	0.0844	0.1089	0.1421
USA CONSUMER GDS	0.0891	0.0892	0.0490	0.0992	0.0816	0.1064
USA CONSUMER SVS	0.1085	0.0948	0.0471	0.1344	0.1707	0.2037
USA FINANCIALS	0.0879	0.0777	0.0499	0.1561	0.1193	0.1922
USA HEALTH CARE	0.1104	0.0909	0.0538	0.1124	0.1001	0.1307
USA INDUSTRIALS	0.0911	0.0886	0.0521	0.1132	0.1733	0.2206
USA OIL & GAS	0.0867	0.0823	0.0447	0.0912	0.1387	0.1469
USA TECHNOLOGY	0.0680	0.0692	0.0436	0.0522	0.0726	0.0805
USA TELECOM	0.0835	0.0767	0.0434	0.0739	0.0806	0.1017
USA UTILITIES	0.0966	0.0967	0.0558	0.0863	0.1554	0.2941
JAPAN BASIC MATS	0.0779	0.0618	0.0327	0.0652	0.0860	0.0999
JAPAN CONSUMER GDS	0.0990	0.0803	0.0407	0.0856	0.1368	0.1184
JAPAN CONSUMER SVS	0.1190	0.0880	0.0458	0.0894	0.1375	0.2041
JAPAN FINANCIALS	0.0597	0.0526	0.0333	0.0409	0.0729	0.0946
JAPAN HEALTH CARE	0.1022	0.0835	0.0435	0.0768	0.1135	0.1855
JAPAN INDUSTRIALS	0.0846	0.0848	0.0429	0.0787	0.1611	0.1834
JAPAN OIL & GAS	0.0570	0.0525	0.0254	0.0441	0.0628	0.0895
JAPAN TECHNOLOGY	0.0535	0.0523	0.0266	0.0549	0.0744	0.0779
JAPAN TELECOM	0.0649	0.0697	0.0345	0.0508	0.0675	0.0906
JAPAN UTILITIES	0.0956	0.0810	0.0422	0.0632	0.1172	0.1074
Average Standard Deviation	0.0835	0.0763	0.0412	0.0789	0.1037	0.1383

CHAPTER 6 OUT-OF-SAMPLE DYNAMIC BLACK-LITTERMAN PORTFOLIOS

According to the empirical results from the Chapter 5, the dynamic BL portfolio using the Sharpe ratio optimisation method does not always perform well with the sample. Resorting to other reward to risk ratio optimisations, such as reward to VaR and reward to CVaR optimisation, the in-sample portfolio performance could be enhanced. Adding moderate VaR constraints or CVaR constraints would also improve the portfolio performance. The aim of this chapter is to study whether the performance of the out-of-sample risk-constrained dynamic BL portfolio could be improved by these methods. The new method of Giacometti et al. (2007) will also be applied in order to estimate the tail risk-adjusted BL expected return and this will be compared to the performance of the out-of-sample BL portfolio.

Section 6.1 details the procedure of construction and performance evaluation of the out-of-sample dynamic unconstrained BL portfolio, while Section 6.2 and Section 6.3 focus on the construction of the out-of-sample dynamic VaR-constrained BL portfolio and the out-of-sample dynamic CVaR-constrained BL portfolio with performance evaluation, respectively. Section 6.4 applies the Giacometti et al. (2007) method to construct a new BL portfolio with a risk-adjusted BL expected return and evaluates the performance.

6.1 Out-of-sample Dynamic Unconstrained BL Portfolios

It is important to examine if the unconstrained BL portfolio and risk constrained BL portfolio can generate a better performance than the benchmark within the out-of-sample framework. Therefore, an out-of-sample analysis for these BL portfolios will be conducted. In their out-of-sample analysis, Giacometti et al. (2007) use a window length of 110 in the rolling window method, thus initial estimates for each of the three volatility models will be made using the first 110 observations (from January 1994 to February 2003) in order to generate a one-month ahead out-of-sample forecast of the conditional covariance matrix for month 111 (March 2003). The estimation sample is then rolled forward by one month and the models are re-estimated and used to generate out-of-sample forecasts for month 112 (April 2003) and so on until the end of the sample (May

2010). The starting parameter values for every iteration of each model are set to the values estimated in the previous iteration and this procedure results in a total of 88 out-of-sample estimates. A momentum portfolio can then be constructed using a holding period of six months to input as the view portfolio into the BL portfolio. Thus, the first period for the construction of the BL portfolio is August 2003 and the total number of out-of-sample estimates is reduced to 82. In order to show the results of the portfolio turnover, the results from September 2003 are reported in this chapter.

Based on the one-step ahead forecast for the covariance matrix at time $t+1$, the same procedure as introduced in Sections 4.2.3 to 4.2.5 is used to construct an out-of-sample unconstrained BL portfolio, a VaR-constrained BL portfolio and a CVaR-constrained BL portfolio. In addition, the out-of-sample BL portfolio's performance is evaluated using the same methods as discussed in Section 4.2.6.2. Comparisons are made between the unconstrained BL portfolios and the risk-constrained BL portfolios and these are discussed in Sections 6.1, 6.2 and 6.3. In Section 6.4, the method of Giacometti et al. (2007) introduced in Section 3.3.2 is followed in order to construct risk-adjusted BL portfolios which are then compared with the previous unconstrained BL portfolio and risk-constrained BL portfolio.

6.1.1 Construction of Out-of-Sample Unconstrained BL Portfolio

6.1.1.1 Estimation of Implied Equilibrium Return

The first important task in the estimation of the out-of-sample implied equilibrium return is to forecast the one-step ahead covariance matrix. As described in Section 4.2.1, the rolling window method can be used in conjunction with equation (4.2), the EWMA model with equation (4.3), and the DCC model with equation (4.6), in order to forecast the 30×30 vector of the covariance matrix $\mathbf{H}_{t+1}$.

Derived from the equation (4.9), the one-step ahead conditional equilibrium return vector $\boldsymbol{\pi}_{t+1}$ can be forecasted using:

$$\boldsymbol{\pi}_{t+1} = \delta_{t+1}\mathbf{H}_{t+1}\mathbf{w}_t \tag{6.1}$$

where $\mathbf{w}_t$ is the $N \times 1$ vector of market capitalisation weights at time t, δ_{t+1} is the risk aversion coefficient at time $t+1$, which is equal to the global market risk premium divided by the market variance $\mathbf{w}_t^{'}\mathbf{H}_{t+1}\mathbf{w}_t$.

According to Table 6.1.1 Panel A, the risk aversion coefficient in the DCC model was the lowest at 1.4798, followed by 1.5714 in the RW110 model and 1.6713 in the EWMA model. In Table 6.1.1 Panel B, the implied equilibrium returns in both the EWMA model and the RW110 model were the same for each asset, whilst the implied equilibrium returns in the DCC model were a little higher.

6.1.1.2 Estimation of Views Portfolio

Different to the in-sample momentum portfolio, when the momentum portfolio was constructed using the out-of-sample framework, then the normalised six-month return can be expressed as:

$$z_{t+1,i} = \frac{p_{t,i} - p_{t-5,i}}{p_{t-5,i}\sigma_{t+1,i}} \tag{6.2}$$

Where, $p_{t,i}$ is the price of country index i at time t, $p_{t-5,i}$ is the price of country industrial index i five months before t, and $\sigma_{t+1,i}$ is the one step ahead volatility of the country industrial index i at time $t+1$.

The top and bottom half of the country industrial indexes are allocated weights of $\omega_{i,t+1} = \frac{1}{\sigma_{t+1,i}c}$ and $\omega_{i,t+1} = -\frac{1}{\sigma_{t+1,i}c}$, respectively. Similar to the in-sample calculation, c is set equal to 35. The method for obtaining the $N \times 1$ vector of the view weights matrix $\mathbf{P}_{t+1}$ at time $t+1$, the view expected return vector $\mathbf{q}_{t+1}$ at time $t+1$, and the confidence level $\mathbf{\Omega}_{t+1}$ in the views at time $t+1$, are same as that of Fabozzi et al. (2006) and the results are shown in Table 6.1.2. The view weights matrix $\mathbf{P}_{t+1}$, view expected return vector $\mathbf{q}_{t+1}$, and the confidence level $\mathbf{\Omega}_{t+1}$ in the out-of-sample view portfolio based on the EWMA model were close to the corresponding results based on the RW110 model; however, the DCC model generated more different results. The choice of volatility models did not affect the direction of the long or short of an asset but instead affected the specific positions within the momentum portfolio.

Table 6.1.3 compares the performance of the momentum portfolio with the benchmark portfolio from September 2003 to May 2010. It was evident from this that the momentum portfolio based on the rolling window method performed worst with a negative average return of -0.04%. Both the momentum portfolio based on the DCC model and the EWMA model with Sharpe ratios equal to 3.71% and 4.42%, respectively, overcame the benchmark portfolio when the Sharpe ratio was equal to 1.06%.

6.1.1.3 Estimation of BL Expected Return in out of sample

When considering the momentum portfolio as the only view portfolio inputted into the BL model, the $N \times 1$ vector of conditional expected returns $\boldsymbol{\mu}_{BL,t+1}$ in the out-of-sample can be forecast as detailed in Section 4.2.2.3, and this can be denoted by:

$$\boldsymbol{\mu}_{BL,t+1} = \boldsymbol{\pi}_{t+1} + \tau \mathbf{H}_{t+1} \mathbf{P}_{t+1} (\mathbf{P}_{t+1}' \mathbf{H}_{t+1} \mathbf{P}_{t+1} \tau + \boldsymbol{\Omega}_{t+1})^{-1} (\mathbf{q}_{t+1} - \mathbf{P}_{t+1}' \boldsymbol{\pi}_{t+1}), \qquad (6.3)$$

The estimated $N \times N$ vector of covariance matrix $\mathbf{V}_{t+1}$ in the out-of-sample can be expressed as:

$$\mathbf{V}_{t+1} = \mathbf{H}_{t+1} + ((\tau \mathbf{H}_{t+1})^{-1} \boldsymbol{\pi}_{t+1} + \mathbf{P}_{t+1}' \boldsymbol{\Omega}_{t+1}^{-1} \mathbf{P}_{t+1})^{-1} \qquad (6.4)$$

Table 6.1.4 reports the BL expected return for each asset in September 2003. As can be seen, the BL expected return based on the EWMA model and the RW110 model were almost the same with all of positive estimates. In contrast, the BL expected return based on the DCC model showed significantly different results and negative estimates were obtained for several of the assets.

6.1.1.4 Construction of Out-of-Sample Implied BL Portfolios and SR-BL Portfolios

Implied BL portfolio

Since the BL expected return $\boldsymbol{\mu}_{BL,t+1}$ and the corresponding covariance matrix $\mathbf{V}_{t+1}$ have been estimated, according to equation (4.13) in Section 4.2.3, the implied weights $\mathbf{w}_{BL,t+1}^{*}$ at time t+1 in the BL model can be calculated by:

$$\mathbf{w}^{*}_{BL,t+1} = \frac{1}{\delta_{t+1}} \mathbf{V}^{-1}_{t+1} \boldsymbol{\mu}_{BL,t+1} \quad (6.5)$$

SR-BL portfolio

In addition, the unconstrained BL portfolio (the SR-BL portfolio) can be constructed by solving the maximisation of the Sharpe ratio (SR) optimisation problem, as shown below:

$$\max \frac{\mathbf{w}^{'}_{BL,t+1} \boldsymbol{\mu}_{BL,t+1}}{\sqrt{\mathbf{w}^{'}_{BL,t+1} \mathbf{V}_{t+1} \mathbf{w}_{BL,t+1}}} \quad (6.6)$$

$$\text{subject to } -1 \leq \mathbf{w}_{BL,t+1} \leq 2, \mathbf{w}^{'}_{BL,t+1} \mathbf{1} = 1$$

where $\boldsymbol{\mu}_{BL,t+1}$ is the expected return of the BL portfolio and $\sqrt{\mathbf{w}^{'}_{BL,t+1} \mathbf{V}_{t+1} \mathbf{w}_{BL,t+1}}$ is the conditional portfolio standard deviation, $\mathbf{w}_{BL,t+1}$ is the $N \times 1$ vector of portfolio weights. The vector of optimal portfolio positions can be solved as:

$$\mathbf{w}^{*}_{BL,t+1} = \frac{\mathbf{V}^{-1}_{t+1} \boldsymbol{\mu}_{BL,t+1}}{\mathbf{1}' \mathbf{V}^{-1}_{t+1} \boldsymbol{\mu}_{BL,t+1}} \quad (6.7)$$

Table 6.1.5 reports the weights allocated in the implied BL portfolio and the SR-BL portfolio based on the DCC, EWMA and RW110 models in September 2003. The SR-BL portfolio had smaller positions for both the long and short assets than the implied BL portfolio, regardless of which volatility model was chosen. Based on each volatility model, the rank of the asset from the largest negative weight to the largest positive weight was nearly the same for the implied BL portfolio and the SR-BL portfolio; however, the position ranges of the implied BL portfolio were obviously wider than those of the SR-BL portfolio. Specifically, in the DCC model which had the widest position range, the position in the implied portfolio ranged between -11.46% (Japan Consumer Goods) and 33.91% (USA Health Care), whilst the position range in the SR-BL portfolio was narrower, between -7.95% (Japan Consumer Goods) and 23.45% (USA Health Care). In the EWMA model, which had the narrowest position range, the position range in the implied BL portfolio was within the interval -4.20% (UK Basic Materials) to 21.76% (USA Health Care), and the position range in the SR-BL portfolio with a reduced width was within the interval -3.65% (UK Consumer Services) to 19.20%

(USA Health Care). Slightly wider than the EWMA model, the position range in the implied BL portfolio was within the interval -4.31% (UK Basic Materials) to 21.83% (USA Health Care), whereas in the RW110 model, the position range in the SR-BL portfolio was narrower and within the interval -3.77% (UK Basic Materials) to 19.44% (USA Health Care). Therefore, the use of the DCC model was found to construct the implied BL portfolio with the most aggressive weight allocation in September 2003. It should be noted that the assets with short selling in the implied BL portfolio and the SR-BL portfolio were not consistent with the view portfolio, but instead demonstrated a converse relationship between long and short positions. This was due to the expected return of the view portfolio being negative, which led to the inverse effect observed for the direction of long or short assets. In addition, as explained in section 5.1.8.1, the value of δ_{t+1} in equation (6.5) and the value of $\mathbf{1'V}_{t+1}^{-1}\boldsymbol{\mu}_{BL,t+1}$ in equation (6.7), are the determiner of the different weights solutions in the implied BL portfolio and the SR-BL portfolio.

Appendix 6.1.1 reports average value of weights assigned in each index in the out-of-sample unconstrained implied BL portfolio and the SR-BL portfolio and Appendix 6.1.2 reports standard deviation of weights in the period from September 2003 to May 2010. It can be concluded that the use of the DCC model could construct the implied BL portfolio and the SR-BL portfolio with widest average absolute position range and with most volatile weight solutions compared with other volatility models. In addition, the DCC-SR-BL portfolio could have narrower average absolute position range and less volatile weight solutions than the implied DCC-BL portfolio.

6.1.1.5 Construction of the Out-of-Sample Unconstrained MVaR-BL Portfolios

When the SR-BL portfolio is constructed, the optimisation function in the optimisation problem is to optimise the Sharpe ratio, which in turn adjusts the excess return with the risk measured by the standard deviation. To construct the MVaR-BL portfolio, the forecasted VaR is used to measure the risk within the portfolio, and the optimisation problem here is to solve the weights in order to satisfy the maximal ratio between the excess expected BL return and the expected VaR in the BL portfolio. The optimisation problem can be written as:

$$\max \frac{\mathbf{w}'_{BL,t+1}\boldsymbol{\mu}_{BL,t+1}}{VaR_{\beta,t+1}} \tag{6.8}$$

$$\text{subject to } -1 \le \mathbf{w}_{BL,t+1} \le 2, \mathbf{w}'_{BL,t+1}\mathbf{1} = 1$$

As explained in Section 4.2.3, by changing equation (4.17), the VaR of the BL portfolio at time $t+1$ can be expressed as:

$$VaR_{\beta,t+1} = \xi_{\beta}\sqrt{\mathbf{w}'_{BL,t+1}\mathbf{V}_{t+1}\mathbf{w}_{BL,t+1}} - \mathbf{w}'_{BL,t+1}\boldsymbol{\mu}_{BL,t+1} \tag{6.9}$$

where $\xi_{\beta} = -F^{-1}(1-\beta)$ and $F(.)$ is the cumulative distribution. β is the confidence level equal to 99%, 95% and 90%.

Table 6.1.6 shows weight solutions in the unconstrained MVaR-BL portfolio based on the DCC, EWMA and RW110 models with the normal distribution and the t-distribution assumption in September 2003. For the normal distribution, the position range in the DCC-MVaR-BL portfolio was the widest between -12.69% (Japan Consumer Goods) and 27.85% (USA Health Care), while the RW110-MVaR-BL portfolio had a much narrower position range between -3.93% (UK Consumer Services) and 19.53% (USA Health Care). The position range in the EWMA-MVaR-BL portfolio was the narrowest between -3.73% (UK Basic Mats) and 19.18% (USA Health Care). For the t-distribution, the position range in the DCC-MVaR-BL portfolio was the widest between -13.00% (Japan Consumer Goods) and 28.30% (USA Health Care), while the RW110-MVaR-BL portfolio had a much narrower position range between -3.82% (UK Consumer Services) and 19.15% (USA Health Care). The position range in the EWMA-MVaR-BL portfolio was the narrowest between -3.99% (UK Basic Mats) and 19.60% (USA Health Care). It was found that the position range for the t-distribution was wider than the position range for the normal distribution, and the change to the t-distribution had a greater effect on widening the position range in the DCC-MVaR-BL portfolio. In addition, the average of the absolute value of the weights difference between the normal distribution and the t-distribution was calculated at 40bp for the DCC model, 3bp for the EWMA model and 4bp for the RW110 model. Therefore it can be said that the weight solutions in the EWMA-MVaR-BL portfolio and the RW110-MVaR-BL portfolio were not sensitive to the change to the t-distribution. The position range in the MVaR-BL portfolio was narrower than the implied BL portfolio but wider than the SR-BL portfolio when using the same volatility model.

Appendix 6.1.3 reports average value of weights assigned in each index in the out-of-sample unconstrained MVaR-BL portfolio and Appendix 6.1.4 reports standard deviation of weights in the period from September 2003 to May 2010. It can be concluded that the use of the DCC model could construct the out-of-sample unconstrained MVaR-BL portfolio with the widest average absolute position range and with most volatile weight solutions compared with other volatility models. In addition, the average absolute position range and average standard deviation of weight in the out-of-sample unconstrained MVaR-BL portfolio seemed to be insensitive to the change to the t-distribution in the use of three volatility models. Moreover, the average absolute position range in the DCC-MVaR-BL portfolio was slightly wider than that in the implied BL portfolio and the SR-BL portfolio, and the average standard deviation in the DCC-MVaR-BL portfolio was slightly smaller than that in the implied BL portfolio and slightly bigger than that in the SR-BL portfolio.

Effect of the Distribution Assumption and Confidence Levels on the out-of-sample DCC-MVaR-BL Portfolio

Table 6.1.6 demonstrates that the change to the t-distribution would impose a greater effect by widening the position range in the DCC-MVaR-BL portfolio. Therefore the effect of the distribution assumption and confidence levels on the weight solutions in the out-of-sample DCC-MVaR-BL portfolio will be investigated. Table 6.1.7 reports the position of each asset in the MVaR-BL portfolio in September 2003 for the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. For the normal distribution the weights allocated to each asset at a confidence level of 99% are very different to the weights allocated to each asset for the confidence levels of 95% and 90%. The position range at a confidence level of 99% was about 9.12% wider than the position range for the confidence levels of 95% and 90% which was within the interval between -7.95% (Japan Consumer Goods) and 23.46% (USA Health Care). For the t-distribution, the position range at a confidence level of 99% was approximately 0.87% wider than the position range at a confidence level of 95% and was within the interval between -12.66% (Japan Consumer Goods) and 27.76% (USA Health Care). In addition, it was 1.29% wider than the position range at a confidence level of 90% which was within the interval between -12.54% (Japan Consumer Goods) and 27.47% (USA Health Care). Therefore, it can be concluded that the weight solutions are not sensitive to the choice of a lower confidence level between 95% and 90% within the normal distribution. Nevertheless, a higher confidence level would lead to a wider position range in both distributions, and the change from the normal distribution to the t-distribution would widen the position range. These conclusions can also apply to Appendix 6.1.5. Appendix 6.1.5 also reflects that a confidence level of 99% would lead to most volatile weight solutions over the out of sample.

Moreover, when ranking the position of each asset from the largest short position to the largest long position, it was found that the choice of the distribution assumptions and the confidence levels had no impact on the ranks of the assets in Table 6.1.7.

6.1.1.6 Construction of Out-of-Sample MCVaR-BL Portfolios

To build the out-of-sample MCVaR-BL portfolio, another essential task is to forecast the CVaR in order to measure the tail risk in the portfolio. The optimisation problem in this case is to solve the weights to satisfy the maximal ratio between the excess expected BL return and the expected CVaR in the BL portfolio. The optimisation problem can be expressed as:

$$\max \frac{\mathbf{w}'_{BL,t+1}\boldsymbol{\mu}_{BL,t+1}}{CVaR_{\beta,t+1}} \tag{6.10}$$

$$\text{subject to } -1 \le \mathbf{w}_{BL,t+1} \le 2, \mathbf{w}'_{BL,t+1}\mathbf{1} = 1$$

As explained in Section 4.2.3, by changing equation (4.19), the CVaR of the BL portfolio at time $t+1$ can be expressed as:

$$CVaR_{\beta,t+1} = \zeta_{\beta,t+1}\sqrt{\mathbf{w}'_{BL,t+1}\mathbf{V}_{t+1}\mathbf{w}_{BL,t+1}} - \mathbf{w}'_{BL,t+1}\boldsymbol{\mu}_{BL,t+1} \tag{6.11}$$

where $\zeta_{\beta,t+1} = \frac{-\int_{-\infty}^{-F^{-1}(1-\beta)} gf(g)dg}{1-\beta}$, g is denoted by $-\mathbf{w}'_{BL,t+1}\boldsymbol{\mu}_{BL,t+1} - VaR_{\beta,t+1}$.

Table 6.1.8 shows the weights solution in the unconstrained MCVaR-BL portfolio based on the DCC, EWMA and RW110 models with the normal distribution and the t-distribution at a confidence level of 99% in September

2003. For the normal distribution, the DCC-MCVaR-BL portfolio had the widest range of between -12.96% (Japan Consumer Goods) and 28.22% (USA Health Care), much wider than the range between -3.76% (UK Basic Mats) and 19.18% (USA Health Care) as determined for the EWMA-MCVaR-BL portfolio and the range between -3.93% (UK Consumer Services) and 19.53% (USA Health Care) for the RW110-MCVaR-BL portfolio. For the t-distribution, the position range of assets in the DCC-MCVaR-BL portfolio increased the width from -12.94% (Japan Consumer Goods) to 32.43% (USA Health Care). The interval was much wider than the slightly widened range between -4.02% (UK Basic Mats) and 19.10% (USA Health Care) found in the EWMA-MCVaR-BL portfolio, and the slightly widened range between -3.93% (UK Consumer Services) and 19.53% (USA Health Care) in the RW110-MCVaR-BL portfolio. Thus, it can be concluded that the DCC-MCVaR-BL portfolio invested assets more aggressively with a widest position range than the other two MCVaR-BL portfolios, and these portfolios had weights solutions with a wider range for the t-distribution than for the normal distribution. Compared with the out-of-sample MVaR-BL portfolio, the out-of-sample MCVaR-BL portfolio had a much wider position range when using the DCC model, but nearly the same position range when the EWMA and RW110 models were used, additionally, the MVaR-BL portfolio and the MCVaR-BL portfolio consistently chose the same assets to long or short when using the same volatility model. The position range in the MCVaR-BL portfolio was narrower than the implied BL portfolio but wider than the SR-BL portfolio when using the same volatility model.

Appendix 6.1.6 reports average value of weights assigned in each index in the out-of-sample unconstrained MCVaR-BL portfolio and Appendix 6.1.7 reports standard deviation of weights in the period from September 2003 to May 2010. It can be concluded that the use of the DCC model could construct the out-of-sample unconstrained MCVaR-BL portfolio with the widest average absolute position range and with most volatile weight solutions compared with other volatility models. In addition, the average absolute position range and average standard deviation of weight in the out-of-sample unconstrained MCVaR-BL portfolio seemed to be insensitive to the change to the t-distribution in the use of three volatility models. Moreover, the average absolute position range in the DCC-MCVaR-BL portfolio was close to that in the DCC-MVaR-BL portfolio, and the average standard deviation in the DCC-MCVaR-BL portfolio was much smaller than that in the DCC-MVaR-BL portfolio.

Effect of Distribution Assumption and Confidence Levels on out-of-sample DCC-MCVaR-BL Portfolio

Table 6.1.8 has demonstrated that a change to the t-distribution would have a greater effect on widening the position range in the DCC-MCVaR-BL portfolio. Therefore the effect of the distribution assumption and confidence levels on weight solutions in the out-of-sample DCC-MCVaR-BL portfolio was investigated. Table 6.1.9 reports the position of each asset in the DCC-MCVaR-BL portfolio in September 2003 with the normal distribution and the t-distribution assumptions for confidence levels of 99%, 95% and 90%. For the normal distribution, when the confidence level was reduced to 95% the position range narrowed to between -12.65% (Japan Consumer Goods) and 27.73% (USA Health Care) compared to the interval between -12.96% and 28.22% for the corresponding assets at the 99% confidence level. When the confidence level was further reduced to 90% then the position range further narrowed to between -12.59% (Japan Consumer Goods) and 27.59% (USA Health Care). When the normal distribution was changed to the t-distribution there was no change in allocating the largest short position in Japan Consumer Goods and the largest long position in USA Health Care, although the position range substantially widened to between -12.94% and 32.43% at the 99% confidence level, moderately widened to between -12.98% and 28.27% at the 95% confidence level and only slightly widened at the 90% confidence level to between -12.71% and 27.91%. In both distributions, the higher the confidence level, the wider the position range in the DCC-MCVaR-BL portfolio. However, this effect was not obvious on average absolute position range over the out of sample in Appendix 6.1.8, it showed the effect that the higher the confidence level, the more volatile weight solutions. Moreover, ranking the position of each asset from the largest short position to the largest long position, it was found that the choice of the distribution assumptions and the confidence levels had no impact on the ranking of assets in Table 6.1.7. Compared with the DCC-MVaR-BL portfolio shown in Table 6.1.7, the DCC-MCVaR-BL portfolio ranked each asset similarly, from the largest short position to the largest long position, to the DCC-MVaR-BL portfolio. The DCC-MCVaR-BL portfolio always had a wider position range than the DCC-MVaR-BL portfolio for each confidence level regardless of the distribution, this conclusion also applied to Appendix 6.1.8. In addition, the DCC-MCVaR-BL portfolio could allocate much less volatile weights than the DCC-MVaR-BL portfolio could at the confidence level of 99% according to Appendix 6.1.8.

6.1.2 Single Period Out-of-Sample Performance

The evaluation of the out-of-sample unconstrained BL portfolio performance for the single period of September 2003 is shown in Table 6.1.10.

6.1.2.1 Out-of-sample Implied BL portfolio and SR-BL portfolio

The implied DCC-BL portfolio performed best with the highest conditional Sharpe ratio of 88.05% and a reward to CVaR ratio of 49.34% at the cost of the highest portfolio turnover of 2.95, followed by the implied EWMA-BL portfolio with a Sharpe ratio equal to 73.51% and a reward to CVaR ratio of 38.08%. The implied RW110-BL portfolio performed the worst.

In the SR-BL portfolios, the DCC-SR-BL portfolio still demonstrated a better performance than the EWMA-SR-BL and RW110-SR-BL portfolios, with a conditional Sharpe ratio of 88.16% and a reward to CVaR ratio equal to 49.42%, even better than the implied DCC-BL portfolio. In addition, the portfolio turnover decreased to 1.7944, much less than that of the implied DCC-BL portfolio.

Both the implied BL and the SR-BL portfolios overcame the benchmark portfolio for September 2003.

6.1.2.2 Out-of-sample MVaR-BL portfolio

At each confidence level the DCC-MVaR-BL portfolio always performed the best followed by the EWMA-MVaR-BL portfolio, whilst the RW110-MVaR-BL portfolio performed the worst. In addition, the portfolio turnover of over 2.33 in the DCC-MVaR-BL portfolio was much greater than that of the RW110-MVaR-BL portfolio which was close to 0.71, while the EWMA-MVaR-BL portfolio possessed the lowest portfolio turnover of around 0.67.

For the normal distribution and the t-distribution, the DCC-MVaR-BL portfolio demonstrated an improved performance as the confidence level decreased from 99% to 90%. Concretely, the conditional Sharpe ratio increased from 84.46%

to 90.68%, the reward to CVaR ratio increased from 46.39% to 51.58%, and the portfolio turnover reduced from 2.3908 to 2.3390 for the normal distribution. For the t-distribution, the conditional Sharpe ratio increased from 83.69% to 90.93%, the reward to CVaR ratio increased from 45.78% to 51.79%, and the portfolio turnover reduced from 2.3989 to 2.3279. In contrast, the performance of the EWMA-MVaR-BL and the RW110-MVaR-BL portfolios were barely sensitive to any change in the confidence level, exhibiting only a slight decrease in their evaluation ratios of less than 0.2%.

For each confidence level, when the normal distribution was changed to the t-distribution, then both the EWMA-MVaR-BL and the RW110-MVaR-BL portfolios demonstrated a slightly improved performance and an increased portfolio turnover. However, the performance of the DCC-MVaR-BL portfolio failed to improve exhibiting decreased evaluation ratios and an increased portfolio turnover for confidence levels of 99% and 95%. At the 90% confidence level, the DCC-MVaR-BL portfolio behaved better with an improved conditional Sharpe ratio and a reward to CVaR ratio, whilst the portfolio turnover was reduced.

The MVaR-BL portfolio successfully overcame the benchmark portfolio in September 2003. Compared with the implied BL and SR-BL portfolios under both distribution assumptions, the DCC-MVaR-BL portfolio only outperformed at the confidence levels of 95% and 90%, whilst the EWMA-MVaR-BL portfolio only outperformed at confidence levels of 99% and 95%; however, the RW110-MVaR-BL portfolio outperformed at all of the confidence levels. In addition, the portfolio turnover in the MVaR-BL portfolio was much less than that in the implied BL portfolio, but slightly higher than that of the SR-BL portfolio.

6.1.2.3 Out-of-sample MCVaR-BL portfolio

For each confidence level the DCC-MCVaR-BL portfolio always performed the best, the RW110-MCVaR-BL portfolio performed the worst, and the EWMA-MCVaR-BL portfolio was ranked in the middle. In addition, the portfolio turnover in the DCC-MCVaR-BL portfolio was over 2.33, much higher than the RW110-MCVaR-BL portfolio which was close to 0.71, whilst the EWMA-MCVaR-BL portfolio had the lowest portfolio turnover of around 0.67.

Under both distribution assumptions, as the confidence level was reduced from 99% to 90%, the portfolio turnover was reduced for the MCVaR-BL portfolio, while only the DCC-MCVaR-BL portfolio demonstrated a gradually enhanced performance. In contrast, the performance of the RW110-MCVaR-BL portfolio was not subject to any impact resulting from a decrease in the confidence level. For the normal distribution, the EWMA-MCVaR-BL portfolio performed a little worse and was further decreased by a lower confidence level. For the t-distribution, the EWMA-MCVaR-BL portfolio gave the best performance which was reflected by the highest Sharpe ratio (73.69%) and reward to CVaR ratio (38.21%) when the confidence level decreased to 95%, although the evaluation ratios dropped slightly at the confidence level of 90%.

At each confidence level, when the normal distribution was changed to the t-distribution, then the portfolio turnover in the MCVaR-BL portfolio increased and the DCC-MCVaR-BL portfolio performed worse. However, the EWMA-MCVaR-BL portfolio showed a slightly improved performance for the 95% and 90% confidence levels, and the RW110-MCVaR portfolio demonstrated a slightly improved performance at the 99% and 90% confidence levels but showed no change at a 95% confidence level.

The MCVaR-BL portfolio significantly outperformed the benchmark portfolio in September 2003, and exhibited a superior performance to the implied BL portfolio with a much lower portfolio turnover based on the EWMA and RW110 models. In addition, the performance of the RW110-MCVaR-BL portfolio was better than that of the RW110-SR-BL and EWMA-MCVaR-BL portfolios. It was better at the 99% confidence level for the normal distribution and at the 95% and 90% confidence levels for the t-distribution. The DCC-MCVaR-BL portfolio also performed better than the implied BL portfolio and the SR-BL portfolio under both distribution assumptions at a confidence level of 90% and for the normal distribution at a confidence level of 95%.

Compared with the MVaR-BL portfolio, the MCVaR-BL portfolio demonstrated a similar performance for the normal distribution; however, for the t-distribution, the DCC-MCVaR-BL portfolio behaved worse whilst the other two MCVaR-BL portfolios behaved slightly better at relative lower confidence levels.

6.1.3 Multiple Period Out-of-Sample Performance

In the dynamic asset allocation process, the portfolio performance is time-varying for a single period. However, the previous single-period performance indeed provides some evidences that the out-of-sample MVaR-BL portfolio and the MCVaR-BL portfolio could perform better than the out-of-sample implied BL portfolio and the SR-BL portfolio with a choice of a certain volatility model at an acceptable confidence level. Additionally, the MCVaR-BL portfolio could beat the MVaR-BL portfolio in certain circumstances. It is necessary to evaluate the average performance of the out-of-sample unconstrained BL portfolios in the multiple periods to check the validity of conclusions from the single-period performance. Table 6.1.11 displays the average performance of the out-of-sample unconstrained BL portfolios.

As can be seen in Table 6.1.11, the benchmark portfolio had the biggest negative skewness of 1.4455, which reflects the left tail risk. Therefore, it was not surprising to see that the empirical VaR and empirical CVaR were highest in the benchmark portfolio at 16.55% and 17.94%, respectively.

6.1.3.1 Out-of-sample Implied BL portfolio and SR-BL portfolio

According to Table 6.1.11, in the implied BL portfolio the risk-adjusted performance in the implied DCC-BL portfolio was best, with the Sharpe ratio, reward to VaR ratio and reward to CVaR ratio equal to 16.04%, 6.57% and 6.50%, respectively. The active performance in the implied DCC-BL portfolio was also the best with a value of 26.73% for the information ratio. The risk-adjusted performance in the implied EWMA-BL portfolio was better than that of the implied RW110-BL portfolio, but the implied RW110-BL portfolio had a better active performance than the implied EWMA-BL portfolio.

Ranking by the Sharpe, reward to VaR and reward to CVaR ratios from the highest to the lowest value in the SR-BL portfolio and the implied BL portfolio, the DCC-SR-BL portfolio performed the best, followed by the EWMA-SR-BL and RW110-SR-BL portfolios. When compared to the active portfolio performance, the DCC-SR-BL portfolio remained the best, but the RW110-SR-BL portfolio outperformed the EWMA-SR-BL portfolio.

For multiple periods, all of the SR-BL portfolios underperformed compared to the implied BL portfolios. Both the implied BL portfolio and the SR-BL portfolio beat the benchmark portfolio and exhibited less empirical VaR and CVaR than the benchmark portfolio.

6.1.3.2 Out-of-sample MVaR-BL portfolio

From Table 6.1.11 it can be seen that at each confidence level the risk-adjusted performance and active performance of the DCC-MVaR-BL portfolio was always better than the other two MVaR-BL portfolios. In addition, compared to the RW110-MVaR-BL portfolio, the EWMA-MVaR-BL portfolio demonstrated a better risk-adjusted performance but worse active portfolio performance.

Under both distribution assumptions, the DCC-MVaR-BL portfolio behaved better at a confidence level of 99% than at confidence levels of 95% and 90%, and the choice of confidence level did not affect the out-of-sample performance of both the EWMA-MVaR-BL and RW110-MVaR-BL portfolios.

At each confidence level the DCC-MVaR-BL portfolio showed a slightly better out-of-sample performance for the t-distribution than for the normal distribution, while the performance of the EWMA-MVaR-BL and RW110-MVaR-BL portfolios were not sensitive to a change in distribution assumption.

For multiple periods, the MVaR-BL portfolio overtook the benchmark portfolio. While the performance of the DCC-MVaR-BL portfolio was substantially better than the performance of the DCC-SR-BL portfolio, the performance of the MVaR-BL portfolio based on the EWMA model showed only a limited improvement and the performance of the RW110-MVaR-BL portfolio was nearly even with those of the corresponding SR-BL portfolios. The MVaR-BL portfolio was unable to perform better than the implied BL portfolio.

6.1.3.3 Out-of-sample MCVaR-BL portfolio

Table 6.1.11 compares the MCVaR-BL portfolios. The DCC-MCVaR-BL portfolio showed impressive risk-adjusted and active performances, whilst the risk-adjusted performance in the EWMA-MCVaR-BL portfolio was inferior. This was followed by the RW110-MCVaR-BL portfolio, where the active performance was better than that of the EWMA-MCVaR-BL portfolio. The performance rank was same for the MVaR-BL, implied BL and SR-BL portfolios.

For the normal distribution, when the confidence level was reduced from 99% to 95%, the performance of the DCC-MCVaR-BL portfolio slightly deteriorated due to the risk-adjusted performance and the active performance, however when the confidence level was further reduced to 90%, then the DCC-MCVaR-BL portfolio had a slightly improved performance. The EWMA-MCVaR-BL and RW110-MCVaR-BL portfolios exhibited a barely altered performance as the confidence level changed. For the t-distribution at a confidence level of 99%, the DCC-MCVaR-BL portfolio achieved the best performance with the Sharpe, information, reward to VaR and reward to CVaR ratios improving to 14.17%, 21.64%, 5.54% and 5.39%, respectively. When the confidence level was lowered to 95% and then further to 90%, the MCVaR-BL portfolio at first performed worse and then the performance was enhanced a little.

For each confidence level the change from the normal distribution to the t-distribution had an obviously positive effect on improving the performance of the DCC-MCVaR-BL portfolio, but only an extremely small positive effect was evident on the performance of both the EWMA-MCVaR-BL and RW110-MCVaR-BL portfolios.

For multiple periods, the MCVaR-BL portfolio performed dramatically better than the benchmark portfolio. The DCC-MCVaR-BL portfolio apparently outperformed the SR-BL portfolio; however, the EWMA-MCVaR-BL portfolio showed a tiny improvement and the RW110-MCVaR-BL portfolio hardly overtook it. None of the MCVaR-BL portfolios could beat the implied BL portfolio.

There was no large difference observed between the out-of-sample performance of the MVaR-BL portfolio and the MCVaR-BL portfolio at lower confidence levels of 95% and 90%. For the t-distribution and a confidence level of 99%, the DCC-MCVaR-BL portfolio performed better than the DCC-MVaR-BL portfolio.

6.1.4 Conclusions

Several findings concerning the unconstrained BL portfolios can be concluded through this out-of-sample analysis. Firstly, within the out-of-sample framework, the dynamic unconstrained BL portfolios demonstrate a superior performance to the benchmark portfolio for both a single period and for multiple periods. In addition, the weight solutions for the out-of-sample unconstrained BL portfolios are more balanced and reasonable than those obtained using the traditional mean-variance method.

Secondly, the dynamic implied BL portfolio and the SR-BL portfolio have different weights solutions but same directions for long or short assets. Different volatility models have different influences on the dynamic implied BL portfolio and the SR-BL portfolio. The use of the DCC model to construct the implied BL portfolio and SR-BL portfolios results in the most aggressive weight allocations with the widest range, and demonstrates a better performance for both a single period and for multiple periods than when using the two other models (EWMA and RW110). It is worth noting that the SR-BL portfolio had a bigger empirical VaR and empirical CVaR for multiple periods, especially when employing the DCC model.

Thirdly, both the MVaR-BL portfolio and the MCVaR-BL portfolio at certain confidence levels are able to outperform the implied-BL and SR-BL portfolios for a single period; however, for multiple periods the MVaR-BL portfolio and the MCVaR-BL portfolio could only overcome the SR-BL portfolio. Although the MCVaR-BL portfolio could not perform better than the MVaR-BL portfolio for a single period, the MCVaR-BL portfolio actually could outperform the MVaR-BL portfolio for multiple periods. The use of different volatility models, distribution assumptions and confidence levels are key elements which affect the weights solutions and performance of the MVaR-BL and MCVaR-BL portfolios. For a single period, it is found that changing to the t-distribution has a greater effect on widening the position range in the MVaR-BL and MCVaR-BL portfolios when using the DCC model, whilst the weight solutions in the MVaR-BL and MCVaR-BL portfolios when employing the EWMA and RW110 models are not sensitive to the change to the t-distribution. The higher confidence level leads to a wider position range for both distributions, and the change from the normal distribution to the t-distribution widen the position range in the MVaR-BL and MCVaR-BL portfolios when the DCC model is used. The out-of-sample MCVaR-BL portfolio has a much wider position range than the MVaR-BL portfolio, however both are consistent in choosing the same assets to long or short under the same volatility model. For the out-of-sample single-period performance for the normal distribution and the t-distribution, the DCC-MVaR-BL and DCC-MCVaR-BL portfolios demonstrate a better performance than when utilising the other two volatility models, and the performance improves as the confidence level decreases from 99% to 90%. Both the MVaR-BL and MCVaR-BL portfolios based upon the EWMA and RW110 models demonstrate a slightly improved performance at a higher confidence level, whilst the MVaR-BL and MCVaR-BL portfolios employing the DCC model behave better at a lower confidence level when the normal distribution is changed to the t-distribution. Regarding the multiple-period out-of-sample performance, the MVaR-BL and MCVaR-BL portfolios based upon the DCC model perform better at a higher confidence level with the t-distribution. The DCC-MCVaR-BL portfolio could achieve the best performance for the t-distribution at a confidence level of 99%, significantly better than other MVaR-BL portfolios and MCVaR-BL portfolios.

6.2 Out-of-sample Dynamic VaR-Constrained BL Portfolios

Function (4.20) represents the optimisation function with VaR constraints for the in-sample portfolio. Similarly, for the out-of-sample portfolio, the optimisation function can be denoted as:

$$\max \frac{\mathbf{w}'_{BL,t+1}\boldsymbol{\mu}_{BL,t+1}}{\sqrt{\mathbf{w}'_{BL,t+1}\mathbf{V}_{t+1}\mathbf{w}_{BL,t+1}}} \tag{6.12}$$

$$\text{subject to } VaR_{\beta,t+1} \le VaR_0, -1 \le \mathbf{w}_{BL,t+1} \le 2, \mathbf{w}'_{BL,t+1}\mathbf{1} = 1$$

where VaR_0 is the target VaR, and $VaR_{\beta,t+1}$ is calculated as equation (6.9).

In Table 6.1.11, it can be seen that the empirical VaR in the SR-BL portfolio were larger than the implied BL portfolio; therefore, the VaR constraint (VaR_0) was initially set equal to the scaling factor 0.99 and multiplied by the estimated VaR of the implied BL portfolio in the corresponding period. Since the optimisation process with VaR constraint has been discussed in detail in Chapter 5, Section 5.2.1.1, it will not be discussed again here.

6.2.1 Construction of VaR-Constrained BL Portfolios

The weight solutions obtained using function (6.12) is used to form the VaR-constrained BL portfolio. Table 6.2.1 reports the weights allocated to each index in the VaR-constrained BL portfolio in September 2003. Comparing Table 6.2.1 with Table 6.1.5, it can be observed that the VaR-BL portfolio weight solutions for the normal distribution were nearly the same as those for the SR-BL portfolio. The reason for this was that the loose VaR limit was not sufficiently large enough to change the weight solution for the normal distribution. When the distribution was changed from the normal distribution to the t-distribution, the VaR constraint was tightened and the position range in the DCC-VaR-BL portfolio changed from between -7.94% (Japan Consumer Goods) and 23.49% (USA Health Care) to between -3.70% (USA Industrials) and 25.49% (USA Health Care). The absolute value of the position range actually decreased to about 2.24%. Both the EWMA-VaR-BL and RW110-VaR-BL portfolios were unbounded and arose as a result of the VaR constraint being too tight and the t-distribution assumption.

Appendix 6.2.1 reports average value of weights allocated to each index in the out-of-sample VaR-constrained BL portfolio in the period from September 2003 to May 2010 and Appendix 6.2.2 reports the standard deviation of time-varying weights in each index. It can be concluded that the change from the normal distribution to the t-distribution has the effect of widening the average absolute position range and increasing the average standard deviation of weights on the out-of-sample VaR-constrainted BL portfolio. In addition, for the t-distribution, the VaR-BL portfolio using the DCC model has narrowest average absolute position range and most volatile weight solutions than using the EWMA model and the RW110 model.

6.2.2 Single Period Out-of-Sample VaR-Constrained BL Performance

Table 6.2.2 shows the out-of-sample VaR-constrained BL portfolio performance in September 2003. For the normal distribution, the DCC-VaR-BL portfolio performed best with much higher conditional Sharpe and reward to CVaR ratios equal to 88.04% and 49.33%, respectively, and the cost of a greater portfolio turnover was 1.7957. The EWMA-VaR-BL portfolio performed slightly better than the RW110-VaR-BL portfolio with a small increase in the conditional evaluation ratios and a lower portfolio turnover. For the t-distribution, the tighter VaR limit improved the DCC-VaR-BL performance and resulted in an increased conditional Sharpe ratio (91.58%) and reward to CVaR ratio (52.35%). Both the

EWMA-VaR-BL and RW110-VaR-BL portfolios were unbounded, and so the results of their performance were not reported. This single-period performance indeed provided some evidence that the VaR-BL portfolio using the DCC model performed better than using other two volatility models. However, it is still necessary to evaluate average performances over multiple periods to get more reliable conclusion.

6.2.3 Multiple Periods Out-of-Sample VaR-Constrained BL Performance

Table 6.2.3 reports the results of the out-of-sample VaR-constrained BL portfolio performance for multiple periods from September 2003 to May 2010. It should be noted that if the VaR constraint was too tight to bind the SR-BL portfolio, then the positions in the minimum variance portfolio were used to replace the missing weight solutions for some single periods. The risk-adjusted performance and the active performance of the DCC-VaR-BL portfolio were best for both the normal distribution and the t-distribution and gave the highest evaluation ratios compared to the other two VaR-BL portfolios. For the normal distribution, the EWMA-VaR-BL portfolio performed better than the RW110-VaR-BL portfolio, but it was outperformed by the RW110-VaR-BL portfolio when the normal distribution was changed to the t-distribution. In addition, the performance of the VaR-BL portfolio for the t-distribution was better than the performance of the VaR-BL portfolio for the normal distribution. This indicates that the tighter VaR constraints improve the multi-period out-of-sample performance.

Compared with the SR-BL portfolio performance in Table 6.1.11, it was found that the VaR-constrained BL portfolio could perform better for both the normal distribution and the t-distribution. Moreover, when comparison were made with the MVaR-BL and MCVaR-BL portfolios, the VaR-constrained BL portfolio based on the EWMA and RW110 models demonstrated a better risk-adjusted performance for both distributions and a better active performance with the normal distribution in multiple periods. From the perspective of contrasting risk-adjusted performance, the DCC-VaR-BL portfolio with the t-distribution at a 99% confidence level was able to outperform the DCC-MVaR-BL portfolio for both distributions; however, it was not better than the performance of the DCC-MCVaR-BL portfolio with the t-distribution. In addition, the active performance of the DCC-VaR-BL portfolio was always worse than that of the DCC-MVaR-BL and DCC-MCVaR-BL portfolios. With regards to portfolio risk measurement, all of the VaR-constrained BL portfolios had smaller risks than the SR-BL portfolios as measured by standard deviation, empirical VaR and empirical CVaR. Furthermore, all the VaR-constrained BL portfolios outperformed the benchmark portfolio. Nevertheless, the implied DCC-BL portfolio still performed the best. It should be noted that adding the VaR constraint onto the EWMA-SR-BL and RW110-SR-BL portfolios resulted in a multi-period performance that was even better than the implied BL portfolio.

6.2.4 Effects of Distributions and Confidence Levels

There are three main tasks in this section. The first task is to determine the weight solutions of the out-of-sample VaR-constrained BL portfolio at decreasing level of VaR constraints with different distribution assumptions and alter confidence levels. Secondly, the effects on the out-of-sample VaR-constrained BL portfolio performance for a single period will be investigated (September 2003), and finally, these effects on the out-of-sample VaR-constrained BL portfolio performance for multiple periods (from September 2003 to May 2010) will be determined. Since Sections 6.2.2 and 6.2.3 concluded that the DCC-VaR-BL portfolio always performed better than the other two VaR-constrained BL portfolios for a single period and for multiple periods, only the effects on the DCC-VaR-BL portfolio will be investigated.

6.2.4.1 Effects on Weights of the Out-of-sample VaR-Constrained BL Portfolio

Table 6.2.4 displays the positions of each asset within the VaR-constrained BL portfolio in September 2003 for the normal distribution and the t-distribution at a confidence level of 99%. It should be noted that the expected VaR of the implied DCC-BL portfolio was 14.45%, whilst the expected VaR of the DCC-SR-BL was much lower at 10.01%. Therefore, when adding the scaling factors multiplied by the VaR of the implied BL portfolio as the VaR_0, these VaR constraints were not sufficiently tight enough to bind the DCC-SR-BL portfolio for the normal distribution, but when altered to the t-distribution then this increased the VaR constraints. As can be seen in Table 6.2.4, the positions did not change for the normal distribution and resulted from unbounded VaR constraints; however, for the t-distribution at a confidence level of 99%, as the VaR bounds tightened the position range gradually became wider. When the VaR factors decreased from 0.99 to 0.80, it was found that the largest short position in the USA Industrials increased from 8.52% to 12.69%, and the largest long position in the USA Health Care increased from 25.49% to 28.47%.

Weights were not reported for each asset for confidence levels of 95% and 90% with both distributions. It was because the VaR constraints at the lower confidence level were not sufficiently tight enough to bind the DCC-SR-BL portfolio. This reason can also explain why weight solutions for a confidence level of 99% in the normal distribution are same.

6.2.4.2 Effects on the Out-of-sample VaR-Constrained BL Portfolios Performance in the Single Period

Table 6.2.5 reports the results for the performance of the out-of-sample VaR-constrained BL portfolios in September 2003 for the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%, as the VaR factor fell to 0.7. As explained earlier, the loose VaR bound failed to constrain the SR-BL portfolio at a lower confidence level for both distributions, so the results of the DCC-VaR-BL portfolio were same for 95% and 90% confidence levels. The performance of the DCC-VaR-BL portfolio at a confidence level of 99% was provided in Table 6.2.5, Panel A. For the normal distribution, it was found that the portfolio turnover increased as the VaR constraints tightened, although the performance did not change. The explanation for this was that the weights changes became greater during the previous period when the VaR factor fell to 0.7. When the distribution was changed to the t-distribution, the single period out-of-sample performance was gradually improved as the VaR bounds became tighter.

6.2.4.3 Effects on the Out-of-sample VaR-Constrained BL Portfolios Performance in Multiple Periods

Since the single period out-of-sample performance has demonstrated a preference for tighter VaR constraints, what is the out-of-sample performance for multiple periods? Table 6.2.6 displays the performance of the DCC-VaR-BL portfolios for multiple periods from September 2003 to May 2010.

For the normal distribution at a 99% confidence level, the mean return of the DCC-VaR-BL remained at nearly the same level of 44bp as the VaR factor fell to 0.7. The risk, as measured by standard deviation, empirical VaR and empirical CVaR, demonstrated a decreasing trend which led to an increasing trend for the risk-adjusted performance evaluation ratios. However, the active portfolio performance was not strictly consistent with the increasing trend of the risk-adjusted performance. Different evaluation ratios generated slightly different ranking for the risk-adjusted performance of the DCC-VaR-BL portfolio. At a confidence level of 95%, as the VaR constraint tightened, the negative skewness of the portfolio became smaller and this led to less tail risks as reflected by the decreased empirical VaR and CVaR. As the risk-adjusted performance of the DCC-VaR-BL portfolio improved, the same ranking was generated by the different evaluation ratios, and the active performance was consistent with the increasing trend. At a confidence level of 90%, as the VaR factor decreased from 0.99 to 0.90, both the risk-adjusted performance and the active performance deteriorated with slightly decreasing evaluation ratios. However, when the VaR factor fell first to 0.8 and then further to 0.7, both the risk-adjusted performance and the active performance demonstrated a better performance. In addition, it was found that a higher confidence level resulted in a better performance for both distributions.

For the t-distribution at a confidence level of 99%, while the decreasing VaR factor reflected the increasing VaR constraint, the mean return and risk were much reduced. Each risk-adjusted performance ratio fell from the highest point to a relatively low point then rallied to a relatively high point before dropping to a much lower level, while the information ratio decreased gradually. At a confidence level of 95%, there was a general trend of increasing evaluation ratios as the VaR factor decreased; however, the performance rankings based on different evaluation ratios were slightly different. At a confidence level of 90%, the mean return slightly increased as the VaR constraint tightened, the negative skewness of the portfolio became smaller and the empirical VaR and CVaR decreased. Both the risk-adjusted and the active performance of the DCC-VaR-BL portfolio were consistent with an improving performance and were denoted the same rank by different evaluation ratios. The risk-adjusted performance in the DCC-VaR-BL portfolio was best at a 99% confidence level, but worsened as the confidence level was reduced. The active portfolio performance of the DCC-VaR-BL portfolio was best at a moderate confidence level of 95%.

At each confidence level, the risk adjusted performance of the DCC-VaR-BL portfolio was better for the t-distribution than the normal distribution. In addition, the active performance for the t-distribution was better than for the normal distribution at confidence levels of 95% and 90%.

When thinking about the effect of the confidence level and distribution on the VaR limit, it is known that a higher confidence level leads to tighter VaR constraints, and these are further increased for the t-distribution. Therefore, it can be summarised that tighter VaR constraints for higher confidence levels improve the performance of the SR-BL portfolio.

Compared with the results shown in Table 6.1.11, all of the DCC-VaR-BL portfolios outperformed the benchmark portfolio. Most of DCC-VaR-BL portfolio constrained at an intermediate level performed better than the DCC-SR-BL portfolio. In addition, the risk-adjusted performance of the DCC-VaR-constrained BL portfolio at a confidence level of 99% for the t-distribution outperformed the DCC-MVaR-BL and DCC-MCVaR-BL portfolios. It should be noted that although the DCC-MCVaR-BL portfolio achieved a higher Sharpe ratio of 14.17% at a confidence level of 99% for the t-distribution, when considering the greater negative skewness of -0.6143 and higher kurtosis of 3.7680, it was not appropriate to use the Sharpe ratio to rank performance, and the reward to CVaR ratio would be a better choice. When the information ratio was used to rank the active portfolio performance, the DCC-MVaR-BL and DCC-MCVaR-BL portfolios were superior to the DCC VaR-constrained BL portfolio. None of DCC-VaR-BL portfolios were able to perform better than the implied BL portfolio.

6.2.4 Conclusions

The superior performance of VaR-constrained BL portfolio provides evidence that adding an intermediate level of the VaR constraint to the SR-BL portfolio improves the SR-BL portfolio performance for a single period and for multiple periods using an out-of-sample framework. Employing the DCC model to construct a dynamic VaR-constrained BL portfolio results in a better performance than using the EWMA and RW110 models for both a single period and for multiple periods. Although the risk-adjusted performance DCC-VaR-BL portfolio outperforms that of the MVaR-BL and MCVaR-BL portfolios in some circumstances, the DCC-VaR-BL portfolio is not the best choice to achieve the best active portfolio performance. The implied BL portfolio that has the best performance for multiple periods is the VaR-constrained BL portfolio based on the DCC model; however, the use of the EWMA and RW110 models for the VaR-constrained BL portfolio could outperform the corresponding implied BL portfolio.

After I study the effect of the distribution assumptions and confidence levels on the DCC-VaR-BL portfolio, I can conclude that the out-of-sample DCC-VaR-BL portfolio performance could be improved by adding tighter VaR constraints. However, if the VaR constraints were too tight, the performance would deteriorate. It shows a diminishing effect of adding tighter VaR constraints.

6.3 Out-of-sample Dynamic CVaR-Constrained BL Portfolios

In Section 6.2, it was illustrated that adding the VaR constraint at intermediate levels significantly improved the performance of the out-of-sample SR-BL portfolio. In addition, the empirical CVaR in the SR-BL portfolio was relative higher than in the implied BL portfolio, as can be seen in Table 6.1.11. One of research questions in this section is to examine whether adding the CVaR constraint could improve the out-of-sample SR-BL portfolio performance and the out-of-sample VaR-constrained BL portfolio performance for a single period and for multiple periods and this question will be studied in the following three sub-sections. Another research task is to investigate the effects of the level of CVaR constraints, distribution assumptions, and confidence levels on the CVaR-constrained BL portfolio performance, and these will be discussed in Section 6.3.4.

6.3.1 Construction of Out-of-sample CVaR-Constrained BL Portfolios

The first task before evaluating the CVaR-constrained BL portfolio is to construct it. As proposed in the method in Section 4.2.5, the out-of-sample CVaR-constrained BL portfolio optimisation problem can be rewritten as:

$$\max \frac{\mathbf{w}'_{BL,t+1}\boldsymbol{\mu}_{BL,t+1}}{\sqrt{\mathbf{w}'_{BL,t+1}\mathbf{V}_{t+1}\mathbf{w}_{BL,t+1}}} \qquad (6.13)$$

$$\text{subject to } CVaR_{\beta,t+1} \leq CVaR_0, -1 \leq \mathbf{w}_{BL,t+1} \leq 2, \mathbf{w}'_{BL,t+1}\mathbf{1} = 1$$

Similar to set VaR_0, I set the value of $CVaR_0$ equal to decreasing scaling factor k multiplied by CVaR of unconstrained implied BL portfolio at each time t, k could be equal to 0.99, 0.95, 0.90 and reduces sequentially. $CVaR_0$ is not constant during the whole period.

The weight solutions from the optimisation problem are used to construct the CVaR-constrained BL portfolio and Table 6.3.1 reports the weights allocated to each asset for September 2003. Comparing Table 6.3.1 with Table 6.1.5, similar to the VaR-BL portfolio, the CVaR-BL portfolio weight solutions for the normal distribution were nearly the same as those for the SR-BL portfolio. This was because that the CVaR limit was not large enough to change the weights solution for the normal distribution. However, when the distribution was changed to the t-distribution, the CVaR limit was tightened, and the position range in the DCC-CVaR-BL portfolio was changed from between -7.95% (Japan Consumer Goods) and 23.45% (USA Health Care) to a wider of between -12.35% (USA Industrials) and 28.25% (USA Health Care). The absolute value of the position range was actually widened about 9.20%. The position range in the DCC-CVaR-BL portfolio was approximately 11.41% wider than the position range in the DCC-VaR-BL portfolio for the t-distribution compared with the results shown in Table 6.2.1. As the VaR-constrained BL portfolio is based upon the EWMA and RW110 models, both EWMA-CVaR-BL portfolio and the RW110-CVaR-BL portfolio were unbounded because the CVaR constraints were too tight for the t-distribution assumption, thus no weights solution were reported in the table.

Appendix 6.3.1 reports average value of weights allocated to each index in the out-of-sample CVaR-constrained BL portfolio in the period from September 2003 to May 2010 and Appendix 6.3.2 reports the standard deviation of time-varying weights in each index. Similar to the out-of-sample VaR-constrained BL portfolio, it can be concluded that the change from the normal distribution to the t-distribution has the effect of widening the average absolute position range and increasing the average standard deviation of weights on the out-of-sample

CVaR-constrainted BL portfolio. In addition, for the t-distribution, the CVaR-BL portfolio using the DCC model has narrowest average absolute position range and most volatile weight solutions than using the EWMA model and the RW110 model. Moreover, for the t-distribution, the CVaR-BL portfolio has wider average absolute position range and more volatile weight solutions than the VaR-BL portfolio over the out of sample.

6.3.2 Single Period Out-of-Sample CVaR-Constrained BL Portfolio Performance

Table 6.3.2 reports the out-of-sample CVaR-constrained BL portfolio performance in September 2003. For the normal distribution, the DCC-CVaR-BL portfolio performed best with a much higher conditional Sharpe ratio and reward to CVaR ratio equal to 88.16% and 49.42%, respectively, and the price for the greater portfolio turnover was 1.7944. The EWMA-CVaR-BL portfolio performed slightly better than the RW110-CVaR-BL portfolio, with a marginally higher conditional evaluation ratios and a lower portfolio turnover. For the t-distribution, the tighter CVaR constraint improved the DCC-CVaR-BL performance resulting in a higher conditional Sharpe ratio (95.95%) and a reward to CVaR ratio (56.25%). Both the EWMA-CVaR-BL and RW110-CVaR-BL portfolios were unbounded and so the performance results were not reported.

In contrast to the VaR-constrained BL portfolio shown in Table 6.2.2, the single-period performance of the CVaR-constrained BL portfolio as detailed in Table 6.3.2 demonstrated a slightly better performance but at a cost of a higher portfolio turnover. This single-period performance indeed provided some evidence that the CVaR-BL portfolio using the DCC model performed better than using other two volatility models and the CVaR-BL portfolio could perform better than the VaR-BL portfolio. However, it is still necessary to evaluate average performances over multiple periods to get more reliable conclusion.

6.3.3 Multiple Period Out-of-Sample Performance CVaR-Constrained BL Portfolio Performance

Table 6.3.3 reports the results of the out-of-sample VaR-constrained BL portfolio performance for multiple periods from September 2003 to May 2010. It should be noted that if the CVaR constraints were too tight for binding the SR-BL portfolio, then the positions in the minimum variance portfolio were used to replace the missing weight solutions for some single periods.

For the normal distribution, the risk-adjusted performance and the active performance of the DCC-CVaR-BL portfolio were best with the highest evaluation ratios compared to the other two VaR-BL portfolios. The EWMA-CVaR-BL portfolio performed better than the RW110-VaR-BL portfolio. However, for the t-distribution, the EWMA-CVaR-BL portfolio had the best risk-adjusted performance, followed by the DCC-CVaR-BL portfolio and then the RW110-VaR-BL portfolio. If the CVaR-BL portfolios were ranked using the information ratio then the DCC-CVaR-BL portfolio showed the most outstanding active performance with an information ratio equal to 10.71%, followed by the RW110-CVaR-BL and the EWMA-CVaR-BL portfolios with information ratios equal to 7.08% and 2.95%, respectively. In addition, performance of the CVaR-BL portfolio for a multiple period and the t-distribution was better than the performance of the CVaR-BL portfolio with the normal distribution. This reflected that tighter CVaR constraints could improve the multi-period out-of-sample performance.

Compared to the SR-BL portfolio performance shown in Table 6.1.11, the CVaR-constrained BL portfolio performed better for both the normal distribution and the t-distribution. Moreover, when compared to the MVaR-BL and MCVaR-BL portfolios, the CVaR-constrained BL portfolio based upon the EWMA and RW110 models demonstrated a better risk-adjusted performance for both distributions and a better active performance for multiple periods with the normal distribution. The DCC-CVaR-BL portfolio for the t-distribution, where the CVaR constraint was much tighter than for the normal distribution, outperformed the DCC-MVaR-BL portfolio and the DCC-MCVaR-BL with the normal distribution; however, this performance was not observed for the normal distribution. Moreover, the active performance for the DCC-CVaR-BL portfolio was always worse than that of the DCC-MVaR-BL portfolio and the DCC-MCVaR-BL for both distributions. With regards to portfolio risk measurement, all of the CVaR-constrained BL portfolios had smaller risks than the SR-BL portfolios, the MVaR-BL portfolios and the MCVaR-BL portfolios as measured by standard deviation, empirical VaR and empirical CVaR. Furthermore, all of the CVaR-constrained BL portfolios outperformed the benchmark portfolio. Although the DCC-CVaR-BL portfolio was inferior to the implied DCC-BL portfolio which always gave the best risk-adjusted performance and active performance for multiple periods, the CVaR-BL portfolio based on the EWMA and RW110 models with the t-distribution, outperformed the risk-adjusted performance for the implied BL portfolio.

Compared to the VaR-BL portfolio performance shown in Table 6.2.3, the CVaR-BL portfolio with the normal distribution did not demonstrate any differences, but the EWMA-CVaR-BL portfolio had a better risk-adjusted performance with the t-distribution.

6.3.4 Effects on Out-of-sample CVaR-Constrained BL Portfolios Performance

Since the DCC-CVaR-BL portfolio performed relatively better than the other two VaR-constrained BL portfolios for a single period and for multiple periods, as explained in sections 6.3.2 and 6.3.3, the effects of the level of CVaR constraints, distribution assumptions, and confidence level on the DCC-VaR-BL portfolio would be investigated.

6.3.4.1 Effects on Weights of the Out-of-sample CVaR-Constrained BL Portfolio

Table 6.3.4 shows the positions of each asset in the DCC-CVaR-BL portfolio in September 2003 for the normal distribution and the t-distribution at a confidence level of 99% as the CVaR factor decreases. It should be noted that the expected CVaR of the implied DCC-BL portfolio was 16.65%, whilst the expected VaR of the DCC-SR-BL was much lower at 11.52%. Therefore, when adding the scaling factors multiplied by the CVaR of the implied BL portfolio as the $CVaR_0$, these CVaR constraints were insufficiently tight to bind the DCC SR-BL portfolio for the normal distribution, but when altered to the t-distribution then the CVaR constraints were increased. Since the CVaR constraints at a lower confidence level were not tight enough to bind the DCC-SR-BL portfolio, and the weights allocated were the same as those at a confidence level of 99% for the normal distribution, the weights for each asset for confidence levels of 95% and 90% for both distributions are not reported.

As can be seen in Table 6.3.4, the positions barely changed for the normal distribution due to the unbounded CVaR constraints; however, for the t-distribution the CVaR constraint tightened at a confidence level of 99%, and the position range gradually became wider. Specifically, when the CVaR factors decreased from 0.99 to 0.80, it was found that the largest short position in the USA Industrials increased from 12.35% to 17.02%, and the largest long position in USA Health Care increased from 28.25% to 31.55%.

Compared with the weight solutions for the VaR-constrained BL portfolio, the CVaR-constrained BL portfolio allocated assets with the same positions for the normal distribution; however, when this was altered to the t-distribution, then the position range in the CVaR-constrained BL portfolio was much wider for each level of constraint factor.

6.3.4.2 Effects on the out-of-sample CVaR-Constrained BL portfolios performance in the single period

Table 6.3.5 reports the out-of-sample CVaR-constrained BL portfolios performance results for September 2003 for the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90% as the CVaR factor decreased to 0.7. As explained earlier, the CVaR limit was not sufficiently tight to constrain the SR-BL portfolio at a lower confidence level for both distributions; consequently, the results for the DCC-CVaR-BL portfolio were nearly same at both 95% and 90% confidence levels. The performance of the DCC-CVaR-BL portfolio at a confidence level of 99% is shown in Table 6.3.5, Panel A. For the normal distribution, the portfolio turnover increased smoothly as the CVaR constraints tightened, while the performance barely changed. A reasonable explanation for the increasing portfolio turnover was that the weights changes became greater in the previous period when the CVaR factor was reduced to 0.7. Unlike the performance for the normal distribution, the single period out-of-sample performance showed obvious signs of improvement. For example, as the CVaR factor reduced from 0.99 to 0.90, the conditional evaluation ratios gradually increased and portfolio turnover gradually decreased.

6.3.4.3 Effects on the out-of-sample CVaR-Constrained BL portfolios performance in multiple periods

For a single period there were some evidences to support the statement that the performance of the DCC-SR-BL portfolio can be improved by adding an intermediate level of CVaR constraints, although the effects of distribution assumptions and confidence levels need to be considered. However, it was still necessary to do investigate multiple periods to get reliable conclusions. Table 6.3.6 reports the performance of the DCC-CVaR-BL portfolios for multiple periods from September 2003 to May 2010.

For the normal distribution at a 99% confidence level, the mean return of the DCC-CVaR-BL portfolio remained at nearly the same level of 44bp when the CVaR factor was reduced to 0.7, and risk as measured by the standard deviation, empirical VaR and empirical CVaR showed a decreasing trend which led to an increasing trend for the risk-adjusted performance evaluation ratios. However, the active portfolio performance was not strictly consistent with the increasing trend for the risk-adjusted performance. Different evaluation ratios generated slightly different rankings for the risk-adjusted performance of the DCC-CVaR-BL portfolio. At confidence levels of 95% and 90%, as the CVaR constraint tightened, the negative skewness of the portfolio became smaller and this led to less tail risks as reflected in the decreased empirical VaR and CVaR. The risk-adjusted performance of the DCC-CVaR-BL portfolio improved with same ranking being denoted by the different evaluation ratios, and the active performance was consistent with the enhancing trend. In addition, at a higher confidence level, risk-adjusted performance and active performance was better with tighter CVaR constraints.

For the t-distribution at a confidence level of 99%, while the CVaR factor decreased, the mean return and risk decreased smoothly. At first, the risk-adjusted performance ratios and the information ratio improved to the highest values but then deteriorated. At confidence levels of 95% and 90%, there was an increasing trend in the risk-adjusted performance evaluation ratios as the CVaR factor was reduced; however, the active performance rank based on the information ratio was inconsistent. The risk-adjusted performance for the DCC-

CVaR-BL portfolio was best at a 99% confidence level, but worsened at lower confidence levels. The active portfolio performance for the DCC-VaR-BL portfolio was best at a moderate confidence level of 95% and a CVaR constraints factor of 0.80.

For each confidence level, the risk adjusted performance of the DCC-CVaR-BL portfolio was better for the t-distribution than the normal distribution. The active performance for the t-distribution was better than for the normal distribution at confidence levels of 95% and 90%.

Comparisons with Table 6.1.11 indicated that all of the DCC-CVaR-BL portfolios could outperform the benchmark portfolio. In contrast to the risk-adjusted performance, all of DCC-CVaR-BL portfolios performed better than the DCC-SR-BL portfolio. When evaluating the active performance, most of the DCC-CVaR-BL portfolios performed better for the normal distribution at all three confidence levels and for the t-distribution with confidence levels of 95% and 90%. In addition, the risk-adjusted performance of the DCC-CVaR-constrained BL portfolio for the t-distribution with certain constraints was able to outperform the DCC-MVaR-BL and DCC-MCVaR-BL portfolios. When the information ratio was used to rank the active portfolio performance, then the DCC-MVaR-BL and DCC-MCVaR-BL portfolios were better than the DCC CVaR-constrained BL portfolio. None of DCC-CVaR-BL portfolios performed better than the implied BL portfolio.

Comparisons with the DCC-VaR-BL portfolio shown in Table 6.2.5 revealed that the DCC-CVaR-BL portfolio demonstrated the same performance for normal distribution at a confidence level of 99%, whilst at a lower confidence level the DCC-CVaR-BL portfolio performed better. For the t-distribution, the DCC-CVaR-BL portfolio also had a superior performance to the DCC-VaR-BL portfolio for each confidence level.

A higher confidence level and the t-distribution led to tighter CVaR constraints, and the CVaR constraints were relatively tighter than the VaR constraints for the same level of confidence. The main finding for the CVaR-constrained BL portfolio was that tighter CVaR constraints at intermediate levels resulted in a better performance of the constrained SR-BL portfolio for multiple periods.

6.3.5 Conclusions

Similar to the out-of-sample VaR-constrained BL portfolio, the out-of-sample CVaR-constrained BL portfolio also exhibits an attractive performance for a single period and for multiple periods, thereby supporting the argument that imposing an intermediate level of CVaR constraint enhances the performance of the SR-BL portfolio. In addition, using the DCC model to construct a dynamic CVaR-constrained BL portfolio results in a better performance than when the EWMA and RW110 models are employed for both a single period and for multiple periods. The risk-adjusted performance of the DCC-CVaR-BL portfolio is better than that of the MVaR-BL and MCVaR-BL portfolios under certain circumstances but they have a better active performance. The CVaR-constrained BL portfolio outperforms the implied BL portfolio based upon the EWMA model and RW110 models. In addition, the CVaR-constrained BL portfolio even demonstrates a better performance than the VaR-constrained BL portfolio for the t-distribution for a single period and multiple periods.

When investigating the effect of distribution assumptions and confidence levels on the DCC-CVaR-BL portfolio, it is found that a tighter CVaR constraint generates a positive effect by improving the performance if the constraint line excludes the maximal Sharpe ratio point. However, this effect diminishes as the CVaR constraint tightens. Note that the change to t-distribution assumption and a higher confidence level could result in a tighter CVaR constraint. Another trend noted is that the tighter the constraint, the wider the positions range within the portfolio.

6.4 Out-of-sample Risk-Adjusted BL Portfolio

Giacometti et al. (2007) proposed a revision to the equilibrium returns for VaR and CVaR corresponding to different distributions and Section 3.3.2 reviewed the method of their estimation of the risk-adjusted equilibrium return. The main aim of this section is to construct the out-of-sample risk-adjusted BL portfolio based on the RW110 model and to evaluate the portfolio performance for a single period and for multiple periods. Comparisons will also be made with other dynamic BL portfolios built in the previous three sections.

6.4.1 Construction of the Risk-Adjusted BL Portfolio

The main difference between forming the BL portfolio and the risk-adjusted BL portfolio is the different method used in estimating the equilibrium return. The first step in building the risk-adjusted BL portfolio is to adjust the equilibrium returns with VaR and CVaR for different confidence levels and different distributions. The second step is to input the view portfolio constructed by the momentum portfolio into the BL model. The final step is to combine the risk-adjusted equilibrium returns with the view portfolio in order to form the BL portfolio. The following sections provide more details about each procedure.

6.4.1.1 Estimation of Risk-Adjusted Implied Equilibrium Return

Following the method of Giacometti et al. (2007) as reviewed Section 3.3.2, in order to present the equation consistent with those in the previous sections, the revised equilibrium returns can be denoted as follows:

$$\boldsymbol{\pi}_{t+1} = \delta_{t+1}\mathbf{H}_{t+1}\mathbf{w}_t \qquad (2.20)$$

$$\boldsymbol{\pi}_{t+1} = \delta_{t+1}\left(VaR_{t+1}\frac{\mathbf{H}_{t+1}\mathbf{w}_t}{\sqrt{\mathbf{w}_t^{'}\mathbf{H}_{t+1}\mathbf{w}_t}} - E(\mathbf{r})\right) \qquad (2.21)$$

$$\boldsymbol{\pi}_{t+1} = \delta_{t+1}\left(CVaR_{t+1}\frac{\mathbf{H}_{t+1}\mathbf{w}_t}{\sqrt{\mathbf{w}_t^{'}\mathbf{H}_{t+1}\mathbf{w}_t}} - E(\mathbf{r})\right) \qquad (2.22)$$

where $VaR_{t+1} = \xi_\beta\sqrt{\mathbf{w}_t^{'}\mathbf{H}_{t+1}\mathbf{w}_t} - E(\mathbf{w}'\mathbf{r})$, $\xi_\beta = -F^{-1}(1-\beta)$, $F(.)$ is the cumulative distribution function, $CVaR_{t+1} = \zeta_\beta\sqrt{\mathbf{w}_t^{'}\mathbf{H}_{t+1}\mathbf{w}_t} - E(\mathbf{w}'\mathbf{r})$, $\zeta_\beta = \frac{-\int_{-\infty}^{-F^{-1}(1-\beta)} gf(g)dg}{1-\beta}$, and g is denoted by $-E(\mathbf{w}'\mathbf{r}) - VaR_{t+1}$. β indicates the confidence level which is equal to 99%, 95% and 90%. $E(\mathbf{r})$ is the expected returns of each asset. It should be noted that the risk aversion coefficients δ_{t+1} are equal to the solution of an optimisation problem, which minimises the sum of the squared error between the neutral equilibrium returns $\boldsymbol{\pi}_{t+1}$ and the day after calculating the return for 82 consecutive months for a rolling window of 110 months. The same values for all of the out-of-sample periods are therefore fixed equal to the solution for the first period.

Table 6.4.1 reports the results of the risk aversion coefficients δ (Panel A and Panel C) and the implied equilibrium return for each index $\boldsymbol{\pi}$ (Panel B and Panel D), for the variance-adjusted, VaR-adjusted and CVaR-adjusted BL portfolios for September 2003, with assumptions of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. Panel A shows that the risk aversion coefficients δ for equation (6.1) was equal to 0.5, which is the same as the value reported by Giacometti et al. (2007). When VaR was considered in the equilibrium return, the risk aversion coefficients δ for equation (6.15) differed depending on the distribution assumption and the confidence level. In the cases detailed by Giacometti et al. (2007), the risk aversion coefficients were set to be equal to 0.30, which was close to the value for the normal distribution at a confidence level of 90% (Panel A). The risk aversion coefficients solved for other cases were lower than 0.30, and were smaller for the t-distribution than for the normal distribution. In addition, the CVaR-adjusted risk aversion coefficients in Panel C were a little smaller than the VaR-adjusted risk aversion coefficients presented in Panel A for the same level of confidence and same distribution. Compared with the risk aversion coefficients in Table 6.1.1, the alternative risk aversion coefficients of 0.5 was approximately three times smaller than the value, around 1.5714, and the VaR and CVaR risk aversion coefficients were nearly 10 times smaller. Therefore, it was expected that the equilibrium returns would also be smaller. As can be seen in Table 6.4.1 Panel B and Panel D, the implied equilibrium returns for each asset were much smaller than the implied equilibrium returns shown in Table 6.1.1. The implied equilibrium return for each asset less than 20bp was not quite sensitive to the choice of risk measure, and the effect of the distribution assumption and confidence level were small.

6.4.1.2 Estimation of Risk-Adjusted BL Expected Return

After estimating the implied equilibrium returns, the view portfolio was inputted into the BL model. The parameters for the view portfolio were shown in Table 6.1.2, and the risk-adjusted BL expected return can be estimated using equation (6.3).

Table 6.4.2 reports the BL expected returns for the variance-adjusted, VaR-adjusted and CVaR-adjusted BL portfolios for September 2003, with assumptions of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. After combining the view portfolio with the market portfolio, several negative expected BL returns were estimated in the assets, including UK Basic Materials, UK Consumer Goods, UK Technology, UK Industrials and Japan Technology, similar to the BL portfolio expected returns based on the DCC model shown in Table 6.1.4. However, the values of the BL expected returns were much smaller and were caused by small values for the risk aversion coefficients. Moreover, the expected returns of assets that were less than 12bp were not quite sensitive to the choice of risk measurement, and the effects of the distribution assumption and confidence level were small.

6.4.1.3 Construction of Unconstrained Risk-Adjusted BL Portfolios

According to equations (6.5) and (6.7), the weight solutions for the variance-adjusted, VaR-adjusted and CVaR-adjusted BL portfolios for the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90% can be solved. Table 6.4.3 reports the positions allocated for each asset under these different conditions for September 2003. The common observation was that all of the BL portfolios possessed the same decision regarding the long or short position for a selected asset, but the positions of assets were different. Specifically, the largest short position was allocated to UK Consumer Services and the largest long position was allocated to USA Health Care.

The position range in the implied variance-adjusted BL portfolio was between -16.19% (UK Consumer Services) and 36.2% (USA Health Care); however, the position range in the variance-adjusted SR-BL portfolio narrowed to between -11.73% (UK Consumer Services) and 25.94% (USA Health Care).

In the VaR-adjusted SR-BL portfolio[9], the position range was slightly wider at a confidence level of 99% with an absolute value of 42.80%, than at confidence levels of 95% and 90% with an absolute value of 42.54% for the normal distribution. For the t-distribution, the absolute value of the position range at confidence levels of 99%, 95% and 90% was 39.31%, 40.20% and 41.31%, respectively. The position range became wider as the confidence level decreased, and the position range for the t-distribution was slightly narrower than for the normal distribution. This conclusion also applied to Appendix 6.1.4, Panel A, which reports average value of weights assigned in each index in the implied variance-adjusted BL portfolio and the VaR-adjusted SR-BL portfolio in the period from September 2003 to May 2010.

In the CVaR-adjusted SR-BL portfolio[10], when the confidence level was reduced from 99% to 90%, the largest short position was UK Consumer Services which increased from 12.90% to 13.42% and the largest long position was USA Health Care which increased from 26.92% to 27.23% for the normal distribution, leading to a wider position range. For the t-distribution, the largest short position was UK Consumer Services which increased from 12.53% to 12.99% and the largest long position was USA Health Care which increased from 26.53% to 26.79%, again leading to a wider position range. The position range became wider as the confidence level decreased, and the position range for the t-distribution was slightly narrower than for the normal distribution. Compared with the VaR-adjusted SR-BL portfolio for the same distribution level of confidence, the position range of the CVaR-adjusted SR-BL portfolio was a little narrower. These conclusions also applied to Appendix 6.1.4, Panel B, which reported average value of weights assigned in each index in the CVaR-adjusted SR-BL portfolio in the period from September 2003 to May 2010.

In addition, according to Appendix 6.1.5, which reports standard deviation of weights assigned in each index in the out-of-sample unconstrained risk-adjusted BL portfolio in the period from September 2003 to May 2010, it can be found that the weight solutions became more volatile as the confidence level

[9] Weights in the VaR-adjusted implied BL portfolio were not reasonable so the maximised Sharpe ratio optimisation model was employed.
[10] Weights in CVaR-adjusted implied BL portfolio were are not reasonable and so the maximised Sharpe ratio optimisation model was employed.

decreased. Moreover, the average standard deviation of weight solutions for the t-distribution was smaller than for the normal distribution. Compared with the VaR-adjusted SR-BL portfolio for the same distribution level of confidence, the average standard deviation of weight solutions of the CVaR-adjusted SR-BL portfolio was smaller. The variance-adjusted SR-BL portfolio allocated asset more volatile than the implied variance-adjusted BL portfolio did.

6.4.2 Single Period Out-of-Sample Risk-Adjusted BL Portfolio Performance

After constructing the risk-adjusted BL portfolios their performance for a single period will now be evaluated. Table 6.4.4 shows the performance of the risk-adjusted BL portfolios evaluated by their excess return, conditional Sharpe ratio, portfolio turnover and reward to CVaR ratio for September 2003. Surprisingly, all of the risk-adjusted BL portfolios outperformed all of the BL portfolios constructed within the previous sections with much higher evaluation ratios for September 2003.

The implied variance-adjusted BL portfolio had the highest excess return of 8.20% and performed better than the variance-adjusted SR-BL portfolio which had a bigger conditional Sharpe ratio and a reward to CVaR ratio at the price of the highest portfolio turnover which was equal to 2.6756.

For the normal distribution, as the confidence level decreased the VaR-adjusted SR-BL portfolio performed worse with gradually decreasing evaluation ratios; however, for the t-distribution the VaR-adjusted SR-BL portfolio performed better with gradually increasing evaluation ratios at the cost of an increasing portfolio turnover. For the same level of confidence, the VaR-adjusted SR-BL portfolio with the t-distribution always performed better than with the normal distribution and produced a higher portfolio turnover with the t-distribution. Compared to the variance-adjusted BL portfolio, the VaR-adjusted SR-BL portfolio with the t-distribution performed better.

For the normal distribution and the t-distribution, the performance of the CVaR-adjusted SR-BL portfolio improved with gradually increasing evaluation ratios, as the confidence level was decreased. For the same level of confidence, the CVaR-adjusted SR-BL portfolio with the normal distribution always demonstrated a better performance than with the t-distribution which produced a higher portfolio turnover. Compared with the variance-adjusted BL portfolio, all of the CVaR-adjusted SR-BL portfolios behaved better for either distribution. In contrast to the VaR-adjusted SR-BL portfolios, the performance of the CVaR-adjusted SR-BL portfolios was better with the normal distribution at each confidence level at the cost of a larger portfolio turnover; however, when the distribution was altered to the t-distribution, the VaR-adjusted SR-BL portfolios outperformed the CVaR-adjusted SR-BL portfolios.

This single-period performance indeed provided some evidence that the implied variance-adjusted portfolio performed better than the variance-adjusted SR-BL portfolio, and the CVaR-adjusted SR-BL portfolios could beat the VaR-adjusted SR-BL portfolios with the normal distribution. However, it is still necessary to evaluate average performances over multiple periods to get more reliable conclusion.

6.4.3 Multiple-Period Out-of-Sample Risk-Adjusted BL Portfolio Performance

Table 6.4.5 shows the performance of the out-of-sample risk-adjusted unconstrained BL portfolios for the period from September 2003 to May 2010. All of the risk-adjusted unconstrained BL portfolios demonstrated a negative skewness for multiple periods.

Compared to the other risk-adjusted unconstrained BL portfolios, the implied variance-adjusted BL portfolio not only showed the best risk-adjusted performance as evaluated by the risk-adjusted evaluation ratios, including the Sharpe ratio, reward to VaR ratio and reward to CVaR ratio, but also exhibited the best active portfolio performance as evaluated by the information ratio. Compared to the out-of-sample unconstrained implied BL portfolio shown in Table 6.1.11, the implied variance-adjusted BL portfolio performed worse than the implied DCC-BL portfolio but better than the implied RW110-BL portfolio with regards to the risk-adjusted performance. In addition, the risk-adjusted performance of the implied variance-adjusted BL portfolio outperformed other unconstrained SR-BL, MVaR-BL and MCVaR-BL portfolios, as well as the VaR-constrained BL portfolio (see Table 6.2.3) and the CVaR-constrained BL portfolio (see Table 6.3.3). The performance of the variance-adjusted SR-BL portfolio was atrocious with extreme negative skewness and high kurtosis, leading to quite small risk-adjusted evaluation ratios.

Furthermore, the VaR-adjusted SR-BL portfolio performed even worse with the normal distribution although this was partially improved by reducing the lower confidence level. When the normal distribution was changed to the t-distribution, the VaR-adjusted SR-BL portfolio performance notably increased to a stunning level with a Sharpe ratio and information ratio rocketing from 2.80% to 13.04% and from 1.97% to 13.47%, respectively at a confidence level of 99%. As the confidence level decreased, the VaR-adjusted SR-BL portfolio performance decreased slightly. Compared to the out-of-sample unconstrained implied BL portfolio (see Table 6.1.11), the VaR-adjusted SR-BL portfolio with the t-distribution could only outperform the implied EWMA-BL portfolio and the risk-adjusted performance of the implied RW110-BL portfolio rather than the active performance. When comparisons were made with the out-of-sample unconstrained SR-BL portfolios, then the VaR-adjusted SR-BL portfolio with the t-distribution always outperformed. Moreover, the VaR-adjusted SR-BL portfolio with the t-distribution behaved better than most of the MVaR-BL portfolios and the MCVaR-BL portfolios for a risk-adjusted performance as evaluated by the Sharpe ratio and active performance. However, compared to the VaR-constrained BL portfolio (see Tables 6.2.3 and 6.2.5), the VaR-adjusted SR-BL portfolio with the t-distribution could only outperform the EWMA-VaR-BL portfolio, and failed to be better than the DCC-VaR-BL and RW110-VaR-BL portfolios with the t-distribution at an intermediate level of VaR constraints. In contrast to the CVaR-constrained BL portfolio (see Tables 6.3.3 and 6.3.5), the VaR-adjusted SR-BL portfolio could only outperform the RW110-CVaR-BL portfolio with the t-distribution at a confidence level of 99% rather than the other CVaR-BL portfolio with the t-distribution at an intermediate level of CVaR constraints.

The risk-adjusted performance and the active performance for the CVaR-adjusted BL portfolios became worse as the confidence level was reduced for the normal distribution and the t-distribution. In addition, the CVaR-adjusted BL portfolios demonstrated a better risk-adjusted performance and active performance for the t-distribution than the normal distribution. At each confidence level the CVaR-adjusted BL portfolios outperformed the VaR-adjusted BL portfolios for both the normal distribution and the t-distribution. The use of the CVaR-adjusted equilibrium return in the BL portfolios also significantly improved the performance of the variance-adjusted SR-BL portfolio. Compared with the out-of-sample unconstrained implied BL portfolio shown in Table 6.1.11, the CVaR-adjusted SR-BL portfolio with the t-distribution outperformed the implied BL portfolio based on the EWMA and RW110 models for the risk-adjusted performance rather than the active performance, and also performed better than all of the unconstrained SR-BL portfolios. In contrast to the MVaR-BL portfolios and the MCVaR-BL portfolios, the CVaR-adjusted portfolio demonstrated a better risk-adjusted performance as evaluated by the Sharpe ratio but a worse active performance. If performance was evaluated by the reward to VaR and reward to CVaR ratios, the CVaR-adjusted portfolio was inferior to the DCC-MVaR-BL and DCC-MCVaR-BL portfolios. In comparison to the VaR-constrained BL portfolio (see Tables 6.2.3 and 6.2.5), the CVaR-adjusted SR-BL portfolio with the t-distribution could only outperform the EWMA-VaR-BL portfolio, and not the DCC-VaR-BL and RW110-VaR-BL portfolios with the t-distribution at an intermediate level of VaR constraints. Furthermore, in contrast to the CVaR-constrained BL portfolio (see Tables 6.3.3 and 6.3.5), the CVaR-adjusted SR-BL portfolio only outperformed the RW110-CVaR-BL portfolio with the t-distribution at a confidence level of 99%, and demonstrated a limited ability to perform better than other CVaR-BL portfolios with the t-distribution at an intermediate level of CVaR constraints.

6.4.4 Conclusions

The two main procedures utilised in the construction of the risk-adjusted BL portfolio from the unconstrained BL portfolio are the estimation of the risk aversion coefficients and the risk-adjusted equilibrium return. It is found that using the method of Giacometti et al. (2007) produces much smaller values of the estimated risk aversion coefficients and the equilibrium return than those of the unconstrained BL portfolio. After inputting the same view portfolio into the risk-adjusted BL model, the estimated expected returns are also smaller. It is found that the reverse optimisation employed in the BL model would be invalid when the VaR-adjusted and CVaR-adjusted expected returns are used, and the weights solutions are unrealistic; however, using the maximal Sharpe ratio optimiser could remedy this problem. Therefore, comparisons are made with the other unconstrained BL portfolios using the VaR-adjusted SR-BL and CVaR-adjusted SR-BL portfolios.

The out-of-sample risk-adjusted BL portfolio demonstrates an impressive single-period performance, which is superior to the unconstrained BL portfolios and the risk constrained BL portfolios, as illustrated in the previous three sections, however, this conclusion may not be reliable without further evaluation of the average performance. In addition, both the VaR-adjusted BL portfolio and the CVaR-adjusted BL portfolio perform better than the variance-adjusted BL portfolio. The CVaR-adjusted BL portfolio outperforms the VaR-adjusted BL portfolio under certain circumstances. The effects of the distribution assumption and confidence level are inconsistent for the VaR-adjusted BL and the CVaR-adjusted BL portfolios.

For multiple periods, the implied variance-adjusted BL portfolio exhibits the best risk-adjusted performance and active portfolio performance of the risk-adjusted BL portfolios. The implied variance-adjusted BL portfolio outperforms all of the unconstrained BL portfolios and the risk-constrained BL portfolios except for the implied DCC-BL portfolio. The risk-adjusted performance of both the VaR-adjusted BL portfolio and the CVaR-adjusted BL portfolio was better than most of the unconstrained BL portfolios, but the active performance fails to be better than that of the MVaR-BL and the MCVaR-BL portfolio. In addition, the VaR-adjusted BL portfolio and the CVaR-adjusted BL portfolio demonstrate a limited ability to outperform the VaR-constrained BL portfolio and CVaR-constrained BL portfolio for the t-distribution at an intermediate level of constraints. Finally, the CVaR-adjusted BL portfolio performs better than the VaR-adjusted BL portfolio at a lower confidence level.

Table 6.1.1 Out-of-sample Risk Aversion Coefficient and Implied Equilibrium Return in September 2003

This table reports the risk aversion coefficient δ (Panel A) and implied equilibrium return of each index $\boldsymbol{\pi}$ (Panel B) in September 2003. $\delta = \frac{E(r_M) - E(r_f)}{\sigma_M^2}$, the numerator is market risk premium and the denominator is market variance. $\boldsymbol{\pi} = \delta \mathbf{H} \mathbf{w}$, where δ is the risk aversion coefficient, **H** is the conditional covariance matrix in the use of the RW model with a window length of 110, the EWMA model and the DCC model, **w** is the market capitalisation weight of each index.

Pannel A: Risk Aversion Coefficient

	DCC	EWMA	RW110
Risk Aversion Coefficient	1.4798	1.6713	1.5714

Pannel B: Implied Eqiulibrium Return

	DCC	EWMA	RW110
UK BASIC MATS	0.0030	0.0025	0.0025
UK CONSUMER GDS	0.0027	0.0022	0.0023
UK CONSUMER SVS	0.0026	0.0023	0.0023
UK FINANCIALS	0.0031	0.0030	0.0030
UK HEALTH CARE	0.0011	0.0012	0.0012
UK TECHNOLOGY	0.0041	0.0048	0.0048
UK INDUSTRIALS	0.0042	0.0033	0.0033
UK OIL & GAS	0.0017	0.0023	0.0023
UK TELECOM	0.0023	0.0027	0.0027
UK UTILITIES	0.0009	0.0009	0.0009
USA BASIC MATS	0.0031	0.0029	0.0029
USA CONSUMER GDS	0.0034	0.0031	0.0031
USA CONSUMER SVS	0.0037	0.0032	0.0032
USA FINANCIALS	0.0034	0.0034	0.0034
USA HEALTH CARE	0.0019	0.0019	0.0019
USA INDUSTRIALS	0.0034	0.0032	0.0032
USA OIL & GAS	0.0019	0.0019	0.0019
USA TECHNOLOGY	0.0040	0.0052	0.0052
USA TELECOM	0.0041	0.0028	0.0028
USA UTILITIES	0.0015	0.0013	0.0013
JAPAN BASIC MATS	0.0020	0.0018	0.0018
JAPAN CONSUMER GDS	0.0023	0.0023	0.0023
JAPAN CONSUMER SVS	0.0016	0.0016	0.0016
JAPAN FINANCIALS	0.0028	0.0026	0.0026
JAPAN HEALTH CARE	0.0013	0.0013	0.0012
JAPAN INDUSTRIALS	0.0024	0.0024	0.0024
JAPAN OIL & GAS	0.0018	0.0017	0.0016
JAPAN TECHNOLOGY	0.0038	0.0037	0.0037
JAPAN TELECOM	0.0025	0.0028	0.0028
JAPAN UTILITIES	0.0007	0.0007	0.0007

Table 6.1.2 Out-of-Sample Views Portfolio Weights, Expected Return and Confidence Variance in September 2003

This table reports the view portfolio weights (**P**), the view portfolio expected return (**q**), and the confidence variance (**Ω**) in December based on three volatility models including the DCC model, the EWMA model and the rolling window model with a window length of 110. The view portfolio is constructed by the momentum strategy and translated into BL model following the method of Fabozzi et al. (2006).

Panel A: The View Portfolio Weights (**P**)

	DCC	EWMA	RW110
UK BASIC MATS	0.0984	0.1319	0.1306
UK CONSUMER GDS	0.0752	0.0987	0.0998
UK CONSUMER SVS	0.1246	0.1641	0.1592
UK FINANCIALS	0.1439	0.1397	0.1374
UK HEALTH CARE	-0.1995	-0.1941	-0.1882
UK TECHNOLOGY	0.0576	0.0597	0.0580
UK INDUSTRIALS	0.0702	0.1047	0.1024
UK OIL & GAS	-0.1784	-0.1444	-0.1402
UK TELECOM	-0.1024	-0.1170	-0.1142
UK UTILITIES	-0.1819	-0.1864	-0.1814
USA BASIC MATS	0.1239	0.1401	0.1363
USA CONSUMER GDS	-0.1178	-0.1323	-0.1309
USA CONSUMER SVS	0.1253	0.1539	0.1521
USA FINANCIALS	-0.1300	0.1408	0.1367
USA HEALTH CARE	-0.1930	-0.1938	-0.1901
USA INDUSTRIALS	0.1399	0.1539	0.1527
USA OIL & GAS	-0.1834	-0.1696	-0.1710
USA TECHNOLOGY	-0.0855	-0.0841	-0.0823
USA TELECOM	-0.0956	-0.1336	-0.1299
USA UTILITIES	-0.1248	-0.1641	-0.1593
JAPAN BASIC MATS	0.1220	0.1036	0.1133
JAPAN CONSUMER GDS	0.1337	-0.1305	-0.1325
JAPAN CONSUMER SVS	-0.1497	-0.1517	-0.1547
JAPAN FINANCIALS	0.0837	0.0858	0.0902
JAPAN HEALTH CARE	-0.1463	-0.1594	-0.1563
JAPAN INDUSTRIALS	0.1276	0.1261	0.1311
JAPAN OIL & GAS	-0.0786	-0.0913	-0.0889
JAPAN TECHNOLOGY	0.0838	0.0791	0.0826
JAPAN TELECOM	0.1046	0.0912	0.0919
JAPAN UTILITIES	-0.1510	-0.1580	-0.1541

Panel B: Expected Return of the View Portfolio (**q**)

	DCC	EWMA	RW110
Expected Return	-0.0451	-0.0239	-0.0242

Panel C: Confidence Variance of the View Portfolio (**Ω**)

	DCC	EWMA	RW110
Confidence Variance	0.0025	0.0035	0.0037

Table 6.1.3 Out-of-sample Portfolio Performance of the Momentum Portfolio and Benchmark Portfolio

This table shows the average return, standard deviation and Sharpe Ratio (SR) of the constructed momentum portfolio and the benchmark portfolio from September 2003 to May 2010. Note that the initial period for constructing the momentum portfolio is in August 2003 in the out-of-sample analysis.

	DCC	EWMA	RW110	Benchmark
Average Return	0.0016	0.0019	-0.0004	0.0005
Standard Deviation	0.0443	0.0433	0.0465	0.0436
Sharpe Ratio	0.0371	0.0442	-0.0090	0.0106

Table 6.1.4 The Out-of-sample BL Expected Returns for Each Index in September 2003

This table reports the BL expected return $\boldsymbol{\mu}_{BL}$ for each index in September 2003 in the use of three volatility models to forecast corresponding covariance matrices. $\boldsymbol{\mu}_{BL,t} = \boldsymbol{\pi}_t + \tau \mathbf{H}_t \mathbf{P}_t (\mathbf{P}_t{}' \mathbf{H}_t \mathbf{P}_t \tau + \boldsymbol{\Omega}_t)^{-1} (\mathbf{q}_t - \mathbf{P}_t^{'} \boldsymbol{\pi}_t)$, where τ is set to be 0.1.

Sep-03	DCC	EWMA	RW110
UK BASIC MATS	-0.0004	0.0016	0.0015
UK CONSUMER GDS	-0.0019	0.0007	0.0008
UK CONSUMER SVS	0.0004	0.0016	0.0016
UK FINANCIALS	0.0018	0.0024	0.0023
UK HEALTH CARE	0.0022	0.0018	0.0015
UK TECHNOLOGY	-0.0006	0.0031	0.0028
UK INDUSTRIALS	-0.0011	0.0018	0.0018
UK OIL & GAS	0.0012	0.0021	0.0021
UK TELECOM	0.0017	0.0025	0.0022
UK UTILITIES	0.0022	0.0015	0.0014
USA BASIC MATS	0.0012	0.0020	0.0021
USA CONSUMER GDS	0.0020	0.0023	0.0024
USA CONSUMER SVS	0.0019	0.0024	0.0025
USA FINANCIALS	0.0036	0.0031	0.0031
USA HEALTH CARE	0.0030	0.0021	0.0021
USA INDUSTRIALS	0.0021	0.0026	0.0026
USA OIL & GAS	0.0021	0.0019	0.0019
USA TECHNOLOGY	0.0018	0.0041	0.0041
USA TELECOM	0.0049	0.0027	0.0028
USA UTILITIES	0.0035	0.0017	0.0016
JAPAN BASIC MATS	0.0001	0.0018	0.0016
JAPAN CONSUMER GDS	-0.0006	0.0019	0.0017
JAPAN CONSUMER SVS	0.0003	0.0018	0.0015
JAPAN FINANCIALS	0.0001	0.0027	0.0021
JAPAN HEALTH CARE	0.0017	0.0019	0.0017
JAPAN INDUSTRIALS	-0.0008	0.0017	0.0017
JAPAN OIL & GAS	0.0015	0.0024	0.0020
JAPAN TECHNOLOGY	-0.0019	0.0023	0.0021
JAPAN TELECOM	-0.0004	0.0021	0.0020
JAPAN UTILITIES	0.0015	0.0014	0.0011

Table 6.1.5 Weights in the Out-of-sample Unconstrained Implied BL Portfolio and the SR-BL Portfolio in September 2003

This table reports the weights assigned in each index in September 2003. Weights in the unconstrained implied BL portfolio are calculated by $\mathbf{w}^*_{BL,t} = \frac{1}{\delta_t}\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$. The SR-BL portfolio allocates asset to achieve the maximal SR in the optimisation problem, weights can be calculated by $\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}$.

Sep-03	DCC		EWMA		RW110	
	Implied BL	SR-BL	Implied BL	SR-BL	Implied BL	SR-BL
UK BASIC MATS	-0.0940	-0.0652	-0.0420	-0.0364	-0.0431	-0.0377
UK CONSUMER GDS	-0.0732	-0.0507	-0.0325	-0.0287	-0.0341	-0.0304
UK CONSUMER SVS	-0.1075	-0.0747	-0.0410	-0.0365	-0.0411	-0.0369
UK FINANCIALS	-0.1064	-0.0737	-0.0121	-0.0108	-0.0129	-0.0117
UK HEALTH CARE	0.2244	0.1555	0.0967	0.0852	0.0965	0.0860
UK TECHNOLOGY	-0.0543	-0.0375	-0.0172	-0.0152	-0.0173	-0.0154
UK INDUSTRIALS	-0.0474	-0.0329	-0.0138	-0.0123	-0.0142	-0.0127
UK OIL & GAS	0.1787	0.1248	0.0542	0.0476	0.0542	0.0481
UK TELECOM	0.1182	0.0818	0.0593	0.0524	0.0595	0.0532
UK UTILITIES	0.1863	0.1290	0.0732	0.0647	0.0734	0.0655
USA BASIC MATS	-0.1024	-0.0710	-0.0282	-0.0252	-0.0284	-0.0256
USA CONSUMER GDS	0.1287	0.0892	0.0600	0.0528	0.0609	0.0541
USA CONSUMER SVS	-0.0244	-0.0166	0.0464	0.0410	0.0453	0.0404
USA FINANCIALS	0.2781	0.1927	0.1004	0.0884	0.1003	0.0894
USA HEALTH CARE	0.3391	0.2345	0.2176	0.1920	0.2183	0.1944
USA INDUSTRIALS	-0.0825	-0.0574	0.0028	0.0025	0.0014	0.0013
USA OIL & GAS	0.2237	0.1542	0.1032	0.0912	0.1055	0.0940
USA TECHNOLOGY	0.1932	0.1341	0.1395	0.1231	0.1397	0.1245
USA TELECOM	0.1161	0.0803	0.0698	0.0617	0.0698	0.0622
USA UTILITIES	0.1530	0.1064	0.0886	0.0782	0.0886	0.0790
JAPAN BASIC MATS	-0.1089	-0.0753	-0.0235	-0.0207	-0.0282	-0.0250
JAPAN CONSUMER GDS	-0.1146	-0.0795	0.0655	0.0579	0.0675	0.0603
JAPAN CONSUMER SVS	0.1616	0.1120	0.0682	0.0604	0.0708	0.0635
JAPAN FINANCIALS	-0.0618	-0.0428	-0.0081	-0.0071	-0.0107	-0.0095
JAPAN HEALTH CARE	0.1562	0.1085	0.0689	0.0600	0.0694	0.0610
JAPAN INDUSTRIALS	-0.1058	-0.0736	-0.0229	-0.0205	-0.0262	-0.0236
JAPAN OIL & GAS	0.0799	0.0553	0.0355	0.0314	0.0356	0.0318
JAPAN TECHNOLOGY	-0.0726	-0.0503	-0.0165	-0.0146	-0.0187	-0.0167
JAPAN TELECOM	-0.0947	-0.0657	-0.0222	-0.0196	-0.0236	-0.0209
JAPAN UTILITIES	0.1567	0.1085	0.0642	0.0568	0.0644	0.0575

Table 6.1.6 Weights in the Out-of-sample Unconstrained MVaR-BL portfolio in September 2003

This table reports weights allocated to each index in the unconstrained MVaR-BL portfolio in September 2003. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by parametric method with the assumption of the normal distribution and the t-distribution at the confidence level of 99%.

Sep-03	Normal Distribution			t-Distribution		
	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	-0.0769	-0.0373	-0.0359	-0.0844	-0.0382	-0.0356
UK CONSUMER GDS	-0.0671	-0.0287	-0.0300	-0.0681	-0.0286	-0.0304
UK CONSUMER SVS	-0.1227	-0.0362	-0.0393	-0.1252	-0.0360	-0.0399
UK FINANCIALS	-0.1146	-0.0104	-0.0109	-0.1180	-0.0104	-0.0109
UK HEALTH CARE	0.1633	0.0853	0.0860	0.1622	0.0850	0.0857
UK TECHNOLOGY	-0.0474	-0.0152	-0.0155	-0.0325	-0.0152	-0.0151
UK INDUSTRIALS	-0.0088	-0.0119	-0.0131	-0.0064	-0.0116	-0.0129
UK OIL & GAS	0.1405	0.0485	0.0474	0.1398	0.0489	0.0476
UK TELECOM	0.0835	0.0523	0.0533	0.0875	0.0523	0.0539
UK UTILITIES	0.1211	0.0646	0.0651	0.1199	0.0645	0.0647
USA BASIC MATS	-0.1110	-0.0250	-0.0257	-0.1133	-0.0244	-0.0255
USA CONSUMER GDS	0.1152	0.0532	0.0543	0.1185	0.0530	0.0545
USA CONSUMER SVS	-0.0319	0.0410	0.0401	-0.0291	0.0405	0.0402
USA FINANCIALS	0.2535	0.0881	0.0885	0.2566	0.0882	0.0885
USA HEALTH CARE	0.2785	0.1918	0.1953	0.2830	0.1915	0.1960
USA INDUSTRIALS	-0.0869	0.0031	0.0004	-0.0882	0.0039	-0.0002
USA OIL & GAS	0.1834	0.0897	0.0946	0.1840	0.0887	0.0947
USA TECHNOLOGY	0.1817	0.1232	0.1250	0.1943	0.1231	0.1247
USA TELECOM	0.1066	0.0611	0.0624	0.1148	0.0614	0.0622
USA UTILITIES	0.0961	0.0786	0.0797	0.0942	0.0785	0.0794
JAPAN BASIC MATS	-0.1045	-0.0207	-0.0252	-0.1121	-0.0206	-0.0253
JAPAN CONSUMER GDS	-0.1269	0.0577	0.0617	-0.1300	0.0578	0.0627
JAPAN CONSUMER SVS	0.1397	0.0600	0.0652	0.1366	0.0600	0.0658
JAPAN FINANCIALS	-0.0334	-0.0071	-0.0101	-0.0413	-0.0071	-0.0106
JAPAN HEALTH CARE	0.1218	0.0614	0.0608	0.1183	0.0620	0.0607
JAPAN INDUSTRIALS	-0.1107	-0.0205	-0.0245	-0.1131	-0.0210	-0.0247
JAPAN OIL & GAS	0.0833	0.0310	0.0317	0.0765	0.0308	0.0326
JAPAN TECHNOLOGY	-0.0345	-0.0148	-0.0169	-0.0326	-0.0146	-0.0172
JAPAN TELECOM	-0.0978	-0.0193	-0.0208	-0.0959	-0.0195	-0.0216
JAPAN UTILITIES	0.1072	0.0565	0.0566	0.1041	0.0569	0.0562

Table 6.1.7 Effect of Distribution Assumptions and Confidence Levels on out-of-sample MVaR-BL Portfolio Weights

This table shows positions of each asset in the MVaR-BL portfolio in September 2003 under the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. Note that the covariance matrix applied to the MVaR-BL model is the DCC covariance matrix in this table.

MVaR-BL Portfolio Weights	Normal Distribution			t-Distribution		
	0.99	0.95	0.9	0.99	0.95	0.9
UK BASIC MATS	-0.0769	-0.0654	-0.0652	-0.0844	-0.0759	-0.0725
UK CONSUMER GDS	-0.0671	-0.0507	-0.0507	-0.0681	-0.0668	-0.0656
UK CONSUMER SVS	-0.1227	-0.0747	-0.0747	-0.1252	-0.1224	-0.1205
UK FINANCIALS	-0.1146	-0.0733	-0.0737	-0.1180	-0.1141	-0.1121
UK HEALTH CARE	0.1633	0.1549	0.1555	0.1622	0.1629	0.1621
UK TECHNOLOGY	-0.0474	-0.0376	-0.0376	-0.0325	-0.0478	-0.0489
UK INDUSTRIALS	-0.0088	-0.0329	-0.0329	-0.0064	-0.0084	-0.0075
UK OIL & GAS	0.1405	0.1254	0.1248	0.1398	0.1404	0.1400
UK TELECOM	0.0835	0.0817	0.0818	0.0875	0.0832	0.0828
UK UTILITIES	0.1211	0.1287	0.1290	0.1199	0.1208	0.1203
USA BASIC MATS	-0.1110	-0.0706	-0.0710	-0.1133	-0.1107	-0.1093
USA CONSUMER GDS	0.1152	0.0891	0.0892	0.1185	0.1150	0.1141
USA CONSUMER SVS	-0.0319	-0.0167	-0.0166	-0.0291	-0.0319	-0.0319
USA FINANCIALS	0.2535	0.1932	0.1927	0.2566	0.2531	0.2520
USA HEALTH CARE	0.2785	0.2346	0.2345	0.2830	0.2776	0.2747
USA INDUSTRIALS	-0.0869	-0.0578	-0.0574	-0.0882	-0.0866	-0.0855
USA OIL & GAS	0.1834	0.1532	0.1542	0.1840	0.1830	0.1821
USA TECHNOLOGY	0.1817	0.1341	0.1341	0.1943	0.1808	0.1774
USA TELECOM	0.1066	0.0804	0.0803	0.1148	0.1061	0.1043
USA UTILITIES	0.0961	0.1067	0.1064	0.0942	0.0959	0.0960
JAPAN BASIC MATS	-0.1045	-0.0749	-0.0753	-0.1121	-0.1040	-0.1029
JAPAN CONSUMER GDS	-0.1269	-0.0795	-0.0795	-0.1300	-0.1266	-0.1254
JAPAN CONSUMER SVS	0.1397	0.1128	0.1119	0.1366	0.1397	0.1396
JAPAN FINANCIALS	-0.0334	-0.0431	-0.0428	-0.0413	-0.0328	-0.0314
JAPAN HEALTH CARE	0.1218	0.1081	0.1085	0.1183	0.1217	0.1211
JAPAN INDUSTRIALS	-0.1107	-0.0736	-0.0735	-0.1131	-0.1104	-0.1097
JAPAN OIL & GAS	0.0833	0.0552	0.0553	0.0765	0.0834	0.0830
JAPAN TECHNOLOGY	-0.0345	-0.0503	-0.0502	-0.0326	-0.0343	-0.0339
JAPAN TELECOM	-0.0978	-0.0659	-0.0656	-0.0959	-0.0979	-0.0988
JAPAN UTILITIES	0.1072	0.1089	0.1085	0.1041	0.1070	0.1063

Table 6.1.8 Weights in an Out-of-sample Unconstrained MCVaR-BL portfolio in September 2003

This table reports weights allocated to each index in an unconstrained MCVaR-BL portfolio in September 2003. The weight in the MCVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to CVaR. Correspondingly, CVaR is also estimated by the parametric method with the assumption of the normal distribution and the t-distribution at the confidence level of 99%.

Sep-03	Normal Distribution			t-Distribution		
	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	-0.0837	-0.0376	-0.0359	-0.1087	-0.0402	-0.0356
UK CONSUMER GDS	-0.0677	-0.0287	-0.0300	-0.0880	-0.0287	-0.0304
UK CONSUMER SVS	-0.1252	-0.0362	-0.0393	-0.1223	-0.0354	-0.0400
UK FINANCIALS	-0.1179	-0.0104	-0.0109	-0.1212	-0.0106	-0.0110
UK HEALTH CARE	0.1618	0.0853	0.0860	0.2097	0.0840	0.0857
UK TECHNOLOGY	-0.0332	-0.0152	-0.0155	-0.0691	-0.0147	-0.0151
UK INDUSTRIALS	-0.0060	-0.0118	-0.0131	-0.0622	-0.0114	-0.0128
UK OIL & GAS	0.1397	0.0486	0.0474	0.1639	0.0501	0.0477
UK TELECOM	0.0869	0.0523	0.0533	0.1034	0.0513	0.0541
UK UTILITIES	0.1194	0.0646	0.0651	0.1715	0.0642	0.0647
USA BASIC MATS	-0.1132	-0.0249	-0.0257	-0.1172	-0.0231	-0.0255
USA CONSUMER GDS	0.1182	0.0532	0.0543	0.1139	0.0527	0.0545
USA CONSUMER SVS	-0.0294	0.0409	0.0401	-0.0392	0.0398	0.0402
USA FINANCIALS	0.2561	0.0881	0.0885	0.2633	0.0890	0.0886
USA HEALTH CARE	0.2822	0.1918	0.1953	0.3243	0.1910	0.1962
USA INDUSTRIALS	-0.0882	0.0032	0.0003	-0.0973	0.0054	-0.0003
USA OIL & GAS	0.1836	0.0895	0.0946	0.2090	0.0877	0.0947
USA TECHNOLOGY	0.1936	0.1232	0.1250	0.1784	0.1226	0.1246
USA TELECOM	0.1140	0.0612	0.0624	0.1013	0.0622	0.0621
USA UTILITIES	0.0937	0.0786	0.0797	0.1382	0.0787	0.0793
JAPAN BASIC MATS	-0.1113	-0.0206	-0.0252	-0.1236	-0.0200	-0.0254
JAPAN CONSUMER GDS	-0.1296	0.0577	0.0617	-0.1294	0.0578	0.0629
JAPAN CONSUMER SVS	0.1369	0.0600	0.0652	0.1468	0.0598	0.0659
JAPAN FINANCIALS	-0.0402	-0.0070	-0.0101	-0.0766	-0.0078	-0.0107
JAPAN HEALTH CARE	0.1184	0.0616	0.0608	0.1415	0.0630	0.0606
JAPAN INDUSTRIALS	-0.1127	-0.0206	-0.0245	-0.1206	-0.0216	-0.0248
JAPAN OIL & GAS	0.0772	0.0309	0.0317	0.0651	0.0314	0.0326
JAPAN TECHNOLOGY	-0.0319	-0.0148	-0.0170	-0.0874	-0.0140	-0.0172
JAPAN TELECOM	-0.0956	-0.0194	-0.0209	-0.1095	-0.0201	-0.0217
JAPAN UTILITIES	0.1040	0.0566	0.0566	0.1419	0.0572	0.0561

Table 6.1.9 Effect of Distribution Assumptions and Confidence Levels on out-of-sample MCVaR-BL Portfolio Weights

This table shows positions of each asset in the MVaR-BL portfolio in September 2003 under the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. Note that the covariance matrix applied to the MCVaR-BL model is the DCC covariance matrix in this table.

MCVaR-BL Portfolio Weights	Normal Distribution			t-Distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
UK BASIC MATS	-0.0837	-0.0756	-0.0739	-0.1087	-0.0841	-0.0776
UK CONSUMER GDS	-0.0677	-0.0667	-0.0661	-0.0880	-0.0679	-0.0674
UK CONSUMER SVS	-0.1252	-0.1222	-0.1214	-0.1223	-0.1252	-0.1230
UK FINANCIALS	-0.1179	-0.1140	-0.1130	-0.1212	-0.1180	-0.1149
UK HEALTH CARE	0.1618	0.1628	0.1623	0.2097	0.1620	0.1636
UK TECHNOLOGY	-0.0332	-0.0480	-0.0485	-0.0691	-0.0328	-0.0471
UK INDUSTRIALS	-0.0060	-0.0083	-0.0077	-0.0622	-0.0062	-0.0092
UK OIL & GAS	0.1397	0.1403	0.1401	0.1639	0.1398	0.1407
UK TELECOM	0.0869	0.0831	0.0829	0.1034	0.0873	0.0837
UK UTILITIES	0.1194	0.1207	0.1203	0.1715	0.1197	0.1214
USA BASIC MATS	-0.1132	-0.1105	-0.1099	-0.1172	-0.1133	-0.1112
USA CONSUMER GDS	0.1182	0.1149	0.1145	0.1139	0.1184	0.1154
USA CONSUMER SVS	-0.0294	-0.0319	-0.0319	-0.0392	-0.0292	-0.0319
USA FINANCIALS	0.2561	0.2530	0.2525	0.2633	0.2564	0.2537
USA HEALTH CARE	0.2822	0.2773	0.2759	0.3243	0.2827	0.2791
USA INDUSTRIALS	-0.0882	-0.0865	-0.0860	-0.0973	-0.0882	-0.0871
USA OIL & GAS	0.1836	0.1829	0.1824	0.2090	0.1838	0.1836
USA TECHNOLOGY	0.1936	0.1805	0.1789	0.1784	0.1940	0.1823
USA TELECOM	0.1140	0.1059	0.1050	0.1013	0.1145	0.1070
USA UTILITIES	0.0937	0.0959	0.0958	0.1382	0.0940	0.0963
JAPAN BASIC MATS	-0.1113	-0.1039	-0.1032	-0.1236	-0.1118	-0.1049
JAPAN CONSUMER GDS	-0.1296	-0.1265	-0.1259	-0.1294	-0.1298	-0.1271
JAPAN CONSUMER SVS	0.1369	0.1397	0.1397	0.1468	0.1367	0.1397
JAPAN FINANCIALS	-0.0402	-0.0326	-0.0318	-0.0766	-0.0408	-0.0340
JAPAN HEALTH CARE	0.1184	0.1216	0.1214	0.1415	0.1183	0.1219
JAPAN INDUSTRIALS	-0.1127	-0.1104	-0.1100	-0.1206	-0.1129	-0.1109
JAPAN OIL & GAS	0.0772	0.0835	0.0834	0.0651	0.0768	0.0830
JAPAN TECHNOLOGY	-0.0319	-0.0342	-0.0339	-0.0874	-0.0323	-0.0348
JAPAN TELECOM	-0.0956	-0.0980	-0.0983	-0.1095	-0.0958	-0.0977
JAPAN UTILITIES	0.1040	0.1069	0.1066	0.1419	0.1041	0.1074

Table 6.1.10 Out-of-Sample Unconstrained BL Portfolio Performance Evaluation in the Single Period

This table reports the results of out-of-sample unconstrained BL portfolios and the benchmark portfolio for the portfolio evaluation criteria including realised excess return, Conditional Sharpe Ratio (CSR), Portfolio Turnover (PT) and return to CVaR ratio in September 2003. The standard deviation is estimated by conditional covariance matrix of three volatility models. An implied BL portfolio is constructed by reverse optimisation of the utility function. SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem. The MVaR-BL portfolio is constructed by achieving maximal return to VaR ratio in the optimisation problem. MCVaR-BL portfolio is constructed by achieving maximal return to CVaR ratio in the optimisation problem. Both VaR and CVaR are estimated by the parametric method in the optimisation model with assumption of the normal distribution ('N') and the t-distribution ('t') at confidence levels of 99%, 95% and 90%.

Sep-03	Realized Excess Return			Conditional Sharpe Ratio			Portfolio Turnover			Reward to CVaR Ratio		
	DCC	EWMA	RW110	DCC	EWMA	RW110	DCC	EWMA	RW110	DCC	EWMA	RW110
Benchmark	0.0026	0.0026	0.0026	0.0579	0.0626	0.0612	N/A	N/A	N/A	0.0238	0.0258	0.0252
Implied BL	0.0571	0.0297	0.0296	0.8805	0.7351	0.7285	2.9584	0.8064	0.8466	0.4934	0.3808	0.3762
SR-BL	0.0396	0.0262	0.0264	0.8816	0.7358	0.7295	1.7944	0.6633	0.7077	0.4942	0.3814	0.3769
99% Confidence Level:												
MVaR-BL N	0.0436	0.0262	0.0265	0.8446	0.7361	0.7321	2.3908	0.6661	0.7143	0.4639	0.3816	0.3787
MVaR-BL t	0.0434	0.0263	0.0266	0.8369	0.7371	0.7343	2.3989	0.6698	0.7197	0.4578	0.3823	0.3803
MCVaR-BL N	0.0436	0.0262	0.0265	0.8446	0.7365	0.7321	2.3908	0.6679	0.7145	0.4639	0.3818	0.3787
MCVaR-BL t	0.0479	0.0261	0.0266	0.7671	0.7330	0.7349	2.9584	0.6690	0.7206	0.4041	0.3793	0.3807
95% Confidence Level:												
MVaR-BL N	0.0460	0.0262	0.0264	0.9023	0.7354	0.7298	2.3529	0.6661	0.7077	0.5119	0.3811	0.3771
MVaR-BL t	0.0459	0.0262	0.0265	0.9012	0.7359	0.7319	2.3559	0.6661	0.7138	0.5109	0.3814	0.3786
MCVaR-BL N	0.0460	0.0262	0.0265	0.9023	0.7358	0.7322	2.3529	0.6660	0.7141	0.5119	0.3814	0.3788
MCVaR-BL t	0.0435	0.0263	0.0265	0.8401	0.7369	0.7322	2.3955	0.6678	0.7152	0.4603	0.3821	0.3788
90% Confidence Level:												
MVaR-BL N	0.0460	0.0262	0.0264	0.9068	0.7351	0.7297	2.3390	0.6653	0.7077	0.5157	0.3809	0.3770
MVaR-BL t	0.0460	0.0262	0.0264	0.9093	0.7353	0.7301	2.3279	0.6661	0.7078	0.5179	0.3810	0.3773
MCVaR-BL N	0.0460	0.0262	0.0264	0.9068	0.7356	0.7296	2.3390	0.6661	0.7076	0.5157	0.3812	0.3769
MCVaR-BL t	0.0458	0.0262	0.0265	0.8949	0.7362	0.7320	2.3715	0.6661	0.7142	0.5055	0.3817	0.3786

Table 6.1.11 Out-of-sample Unconstrained BL Portfolio Performance in Multiple Periods (Sep 03 – May 10)

This table shows realised unconstrained BL portfolios performance compared with the benchmark performance in the period from September 2003 to May 2010. Return is the average realised excess return, risk is the standard deviation, Sharpe Ratio is the average excess realized return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Both VaR and CVaR are measured on the empirical distribution. Return to VaR ratio and Return to CVaR ratio evaluate the excess return per unit of tail risk. In the construction of portfolio, both VaR and CVaR are estimated by the parametric method with assumption of the normal distribution ('N') and the t-distribution ('t') at confidence levels of 99%, 95% and 90%.

		Return	Risk	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR Ratio	Reward to CVaR Ratio
Benchmark		0.0005	0.0436	-1.4455	7.0937	0.0106	N/A	0.1655	0.1794	0.0028	0.0026
Implied BL	DCC	0.0073	0.0458	-0.3187	4.1294	0.1604	0.2673	0.1119	0.1131	0.0657	0.0650
	EWMA	0.0030	0.0384	-0.7122	3.6712	0.0788	0.1585	0.1008	0.1057	0.0300	0.0287
	RW110	0.0029	0.0403	-0.8634	4.4114	0.0720	0.1739	0.1208	0.1301	0.0240	0.0223
SR-BL	DCC	0.0041	0.0442	-1.1515	5.5083	0.0922	0.1336	0.1494	0.1603	0.0273	0.0254
	EWMA	0.0025	0.0386	-0.7854	3.8574	0.0655	0.1215	0.1070	0.1080	0.0236	0.0234
	RW110	0.0022	0.0406	-0.9703	4.4691	0.0550	0.1238	0.1222	0.1290	0.0183	0.0173
99% Confidence Level:											
MVaR-BL N	DCC	0.0053	0.0431	-0.6852	3.9800	0.1234	0.2006	0.1142	0.1169	0.0466	0.0455
	EWMA	0.0025	0.0386	-0.7840	3.8577	0.0660	0.1224	0.1069	0.1079	0.0238	0.0236
	RW110	0.0022	0.0406	-0.9696	4.4699	0.0549	0.1233	0.1221	0.1290	0.0183	0.0173
MVaR-BL t	DCC	0.0055	0.0429	-0.6505	3.8958	0.1273	0.2086	0.1091	0.1123	0.0500	0.0486
	EWMA	0.0025	0.0386	-0.7834	3.8574	0.0661	0.1227	0.1068	0.1078	0.0239	0.0237
	RW110	0.0022	0.0406	-0.9689	4.4651	0.0549	0.1230	0.1220	0.1287	0.0183	0.0173
MCVaR-BL N	DCC	0.0053	0.0431	-0.6875	3.9868	0.1228	0.2000	0.1142	0.1169	0.0463	0.0453
	EWMA	0.0025	0.0386	-0.7845	3.8571	0.0660	0.1224	0.1069	0.1079	0.0238	0.0236
	RW110	0.0022	0.0406	-0.9698	4.4691	0.0549	0.1233	0.1221	0.1290	0.0183	0.0173
MCVaR-BL t	DCC	0.0060	0.0427	-0.6143	3.7680	0.1417	0.2164	0.1091	0.1123	0.0554	0.0539
	EWMA	0.0026	0.0386	-0.7799	3.8594	0.0664	0.1234	0.1068	0.1077	0.0240	0.0238
	RW110	0.0022	0.0406	-0.9679	4.4671	0.0551	0.1235	0.1220	0.1287	0.0183	0.0174

Table 6.1.11 (continued)

95% Confidence Level:											
MVaR-BL N	DCC	0.0053	0.0431	-0.6795	3.9827	0.1229	0.1995	0.1144	0.1207	0.0463	0.0440
	EWMA	0.0025	0.0386	-0.7844	3.8577	0.0659	0.1222	0.1069	0.1080	0.0238	0.0236
	RW110	0.0022	0.0406	-0.9700	4.4693	0.0550	0.1237	0.1221	0.1290	0.0183	0.0173
MVaR-BL t	DCC	0.0053	0.0431	-0.6839	3.9780	0.1231	0.2001	0.1142	0.1204	0.0465	0.0441
	EWMA	0.0025	0.0386	-0.7842	3.8578	0.0660	0.1225	0.1069	0.1079	0.0238	0.0236
	RW110	0.0022	0.0406	-0.9696	4.4696	0.0550	0.1233	0.1221	0.1290	0.0183	0.0173
MCVaR-BL N	DCC	0.0053	0.0431	-0.6894	3.9848	0.1224	0.1987	0.1142	0.1204	0.0461	0.0438
	EWMA	0.0025	0.0386	-0.7842	3.8581	0.0660	0.1225	0.1069	0.1079	0.0238	0.0236
	RW50	0.0022	0.0406	-0.9698	4.4695	0.0550	0.1237	0.1221	0.1290	0.0183	0.0173
MCVaR-BL t	DCC	0.0053	0.0427	-0.6671	3.9321	0.1231	0.2014	0.1091	0.1123	0.0481	0.0468
	EWMA	0.0026	0.0386	-0.7838	3.8573	0.0661	0.1227	0.1069	0.1078	0.0239	0.0237
	RW110	0.0022	0.0406	-0.9700	4.4704	0.0548	0.1229	0.1221	0.1290	0.0182	0.0172
90% Confidence Level:											
MVaR-BL N	DCC	0.0053	0.0431	-0.6795	3.9827	0.1229	0.1995	0.1144	0.1207	0.0463	0.0440
	EWMA	0.0025	0.0386	-0.7861	3.8620	0.0657	0.1218	0.1071	0.1083	0.0237	0.0234
	RW110	0.0022	0.0406	-0.9703	4.4695	0.0551	0.1239	0.1221	0.1290	0.0183	0.0173
MVaR-BL t	DCC	0.0053	0.0432	-0.6792	3.9756	0.1234	0.2006	0.1143	0.1205	0.0466	0.0442
	EWMA	0.0025	0.0386	-0.7857	3.8616	0.0657	0.1219	0.1071	0.1083	0.0237	0.0234
	RW110	0.0022	0.0406	-0.9701	4.4693	0.0551	0.1238	0.1221	0.1290	0.0183	0.0173
MCVaR-BL N	DCC	0.0053	0.0431	-0.6801	3.9753	0.1232	0.2001	0.1143	0.1204	0.0465	0.0441
	EWMA	0.0025	0.0386	-0.7854	3.8617	0.0658	0.1220	0.1071	0.1082	0.0237	0.0235
	RW110	0.0022	0.0406	-0.9704	4.4700	0.0550	0.1235	0.1221	0.1290	0.0183	0.0173
MCVaR-BL t	DCC	0.0053	0.0431	-0.6858	3.9801	0.1235	0.2008	0.1142	0.1203	0.0466	0.0442
	EWMA	0.0025	0.0386	-0.7842	3.8578	0.0660	0.1225	0.1069	0.1079	0.0238	0.0236
	RW110	0.0022	0.0406	-0.9699	4.4695	0.0549	0.1233	0.1221	0.1290	0.0183	0.0173

Table 6.2.1 Weights in the Out-of-sample VaR-Constrained BL Portfolio in September 2003

This table reports weights allocated to each index in the out-of-sample VaR-constrained BL portfolio in September 2003. The standard deviation is estimated by the conditional covariance matrix of DCC, EWMA and RW110 models. VaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at confidence level of 99%. The VaR constraints (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	-0.0651	-0.0370	-0.0384	-0.0716	N/A	N/A
UK CONSUMER GDS	-0.0507	-0.0286	-0.0303	-0.0230	N/A	N/A
UK CONSUMER SVS	-0.0744	-0.0362	-0.0366	-0.0631	N/A	N/A
UK FINANCIALS	-0.0737	-0.0106	-0.0115	-0.0667	N/A	N/A
UK HEALTH CARE	0.1555	0.0852	0.0859	0.1533	N/A	N/A
UK TECHNOLOGY	-0.0376	-0.0151	-0.0154	-0.0270	N/A	N/A
UK INDUSTRIALS	-0.0329	-0.0122	-0.0126	-0.0370	N/A	N/A
UK OIL & GAS	0.1239	0.0478	0.0482	0.1164	N/A	N/A
UK TELECOM	0.0818	0.0523	0.0530	0.0864	N/A	N/A
UK UTILITIES	0.1291	0.0646	0.0654	0.1179	N/A	N/A
USA BASIC MATS	-0.0710	-0.0249	-0.0253	-0.0320	N/A	N/A
USA CONSUMER GDS	0.0892	0.0529	0.0542	0.0837	N/A	N/A
USA CONSUMER SVS	-0.0169	0.0408	0.0403	-0.0196	N/A	N/A
USA FINANCIALS	0.1926	0.0886	0.0894	0.1358	N/A	N/A
USA HEALTH CARE	0.2349	0.1919	0.1944	0.2549	N/A	N/A
USA INDUSTRIALS	-0.0571	0.0024	0.0013	-0.0852	N/A	N/A
USA OIL & GAS	0.1549	0.0911	0.0939	0.1542	N/A	N/A
USA TECHNOLOGY	0.1338	0.1230	0.1244	0.1213	N/A	N/A
USA TELECOM	0.0804	0.0616	0.0622	0.0475	N/A	N/A
USA UTILITIES	0.1060	0.0781	0.0789	0.0958	N/A	N/A
JAPAN BASIC MATS	-0.0754	-0.0207	-0.0252	-0.0673	N/A	N/A
JAPAN CONSUMER GDS	-0.0794	0.0578	0.0602	-0.0360	N/A	N/A
JAPAN CONSUMER SVS	0.1119	0.0602	0.0632	0.1152	N/A	N/A
JAPAN FINANCIALS	-0.0428	-0.0072	-0.0096	-0.0404	N/A	N/A
JAPAN HEALTH CARE	0.1083	0.0607	0.0618	0.0859	N/A	N/A
JAPAN INDUSTRIALS	-0.0733	-0.0202	-0.0233	-0.0216	N/A	N/A
JAPAN OIL & GAS	0.0554	0.0313	0.0317	0.0309	N/A	N/A
JAPAN TECHNOLOGY	-0.0503	-0.0145	-0.0167	-0.0761	N/A	N/A
JAPAN TELECOM	-0.0656	-0.0196	-0.0210	-0.0401	N/A	N/A
JAPAN UTILITIES	0.1086	0.0567	0.0574	0.1076	N/A	N/A

Table 6.2.2 Out-of-sample VaR-Constrained BL Portfolio Performance in the Single Period

This table reports the out-of-sample VaR-constrained BL portfolio performance evaluated by realized return, Conditional Sharpe Ratio (CSR), Portfolio Turnover (PT), reward to CVaR ratio in September 2003. The standard deviation is forecasted by the dynamic covariance matrix of DCC, EWMA, and RW110 models. VaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at a confidence level of 99%. The VaR constraints (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

Sep-03	Normal Distribution				t-Distribution			
	Realized Excess Return	CSR	Portfolio Turnover	Reward to CVaR	Realized Excess Return	CSR	Portfolio Turnover	Reward to CVaR
DCC	0.0395	0.8804	1.7957	0.4933	0.0359	0.9158	2.2581	0.5235
EWMA	0.0262	0.7351	0.6401	0.3808	N/A	N/A	N/A	N/A
RW110	0.0264	0.7286	0.6784	0.3762	N/A	N/A	N/A	N/A

Table 6.2.3 Out-of-sample VaR-Constrained BL portfolio Performance in Multiple Periods

This table shows realised out-of-sample VaR-constrained BL portfolio performance in the period from September 2003 to May 2010. Return is the average realised excess return, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, VaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at a confidence level of 99%. The VaR constraints (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

Panel A: Normal Distribution (Sep 03 - May 10)

	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0044	0.0404	-0.6667	3.8162	0.1094	0.1681	0.1009	0.1070	0.0438	0.0413
EWMA	0.0028	0.0371	-0.7160	3.6565	0.0766	0.1382	0.0982	0.1045	0.0289	0.0272
RW110	0.0024	0.0395	-0.9436	4.5374	0.0608	0.1356	0.1198	0.1286	0.0201	0.0187

Panel B: t-Distribution (Sep 03 - May 10)

	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0043	0.0317	-0.3232	3.6576	0.1364	0.1452	0.0789	0.0796	0.0548	0.0543
EWMA	0.0019	0.0186	-0.6901	3.7181	0.0998	0.0494	0.0496	0.0512	0.0374	0.0362
RW110	0.0037	0.0248	-0.7243	3.9740	0.1481	0.1041	0.0773	0.0823	0.0475	0.0447

Table 6.2.4 Effects on Weights of VaR-Constrained BL Portfolio (Sep 03)

This table shows positions of each asset in the VaR-constained BL portfolio in September 2003 under the normal distribution and the t-distribution at confidence level of 99%. Note that the covariance matrix applied to VaR-constained BL model is the DCC covariance matrix in this table.

99% Confidence Level:

	Normal Distribution				t-Distribution			
VaR Factor:	0.99	0.95	0.90	0.80	0.99	0.95	0.90	0.80
UK BASIC MATS	-0.0651	-0.0651	-0.0651	-0.0651	-0.0716	-0.0735	-0.0759	-0.0812
UK CONSUMER GDS	-0.0507	-0.0507	-0.0507	-0.0507	-0.0230	-0.0149	-0.0045	0.0182
UK CONSUMER SVS	-0.0744	-0.0744	-0.0744	-0.0744	-0.0631	-0.0598	-0.0555	-0.0463
UK FINANCIALS	-0.0737	-0.0737	-0.0737	-0.0737	-0.0667	-0.0647	-0.0621	-0.0564
UK HEALTH CARE	0.1555	0.1555	0.1555	0.1555	0.1533	0.1527	0.1518	0.1501
UK TECHNOLOGY	-0.0376	-0.0376	-0.0376	-0.0376	-0.0270	-0.0239	-0.0200	-0.0113
UK INDUSTRIALS	-0.0329	-0.0329	-0.0329	-0.0329	-0.0370	-0.0382	-0.0398	-0.0432
UK OIL & GAS	0.1239	0.1239	0.1239	0.1239	0.1164	0.1142	0.1114	0.1054
UK TELECOM	0.0818	0.0818	0.0818	0.0818	0.0864	0.0877	0.0895	0.0932
UK UTILITIES	0.1291	0.1291	0.1291	0.1291	0.1179	0.1147	0.1105	0.1013
USA BASIC MATS	-0.0710	-0.0710	-0.0710	-0.0710	-0.0320	-0.0207	-0.0060	0.0259
USA CONSUMER GDS	0.0892	0.0892	0.0892	0.0892	0.0837	0.0821	0.0800	0.0756
USA CONSUMER SVS	-0.0169	-0.0169	-0.0169	-0.0169	-0.0196	-0.0204	-0.0214	-0.0236
USA FINANCIALS	0.1926	0.1926	0.1926	0.1926	0.1358	0.1193	0.0978	0.0514
USA HEALTH CARE	0.2349	0.2349	0.2349	0.2349	0.2549	0.2607	0.2683	0.2847
USA INDUSTRIALS	-0.0571	-0.0571	-0.0571	-0.0571	-0.0852	-0.0934	-0.1040	-0.1269
USA OIL & GAS	0.1549	0.1549	0.1549	0.1549	0.1542	0.1539	0.1536	0.1529
USA TECHNOLOGY	0.1338	0.1338	0.1338	0.1338	0.1213	0.1176	0.1129	0.1026
USA TELECOM	0.0804	0.0804	0.0804	0.0804	0.0475	0.0379	0.0255	-0.0014
USA UTILITIES	0.1060	0.1060	0.1060	0.1060	0.0958	0.0929	0.0890	0.0807
JAPAN BASIC MATS	-0.0754	-0.0754	-0.0754	-0.0754	-0.0673	-0.0650	-0.0619	-0.0553
JAPAN CONSUMER GDS	-0.0794	-0.0794	-0.0794	-0.0794	-0.0360	-0.0234	-0.0070	0.0285
JAPAN CONSUMER SVS	0.1119	0.1119	0.1119	0.1119	0.1152	0.1161	0.1173	0.1200
JAPAN FINANCIALS	-0.0428	-0.0428	-0.0428	-0.0428	-0.0404	-0.0397	-0.0387	-0.0367
JAPAN HEALTH CARE	0.1083	0.1083	0.1083	0.1083	0.0859	0.0794	0.0710	0.0527
JAPAN INDUSTRIALS	-0.0733	-0.0733	-0.0733	-0.0733	-0.0216	-0.0066	0.0129	0.0552
JAPAN OIL & GAS	0.0554	0.0554	0.0554	0.0554	0.0309	0.0238	0.0145	-0.0055
JAPAN TECHNOLOGY	-0.0503	-0.0503	-0.0503	-0.0503	-0.0761	-0.0836	-0.0934	-0.1145
JAPAN TELECOM	-0.0656	-0.0656	-0.0656	-0.0656	-0.0401	-0.0327	-0.0231	-0.0022
JAPAN UTILITIES	0.1086	0.1086	0.1086	0.1086	0.1076	0.1074	0.1070	0.1063

Table 6.2.5 Effects on out-of-sample VaR-constrained BL Portfolio Performance Evaluation (Sep 03)

This table reports the out-of-sample VaR-constrained BL portfolio performance results including realized excess return, Conditional Sharpe Ratio (CSR), Portfolio Turnover (PT) and conditional reward to CVaR ratio in September 2003. The standard deviation is forecasted by the conditional covariance matrix of the DCC model. VaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. The VaR constraint (VaR_0) is set to be equal to the scaling factor k multiplied by the estimated VaR of the implied BL portfolio. The scaling factor k is called VaR factor.

Panel A: 99% Confidence Level

VaR Factor	Normal Distribution Realized Return	CSR	PT	Reward to CVaR	t-Distribution Realized Return	CSR	PT	Reward to CVaR
0.99	0.0395	0.8804	1.7957	0.4933	0.0359	0.9158	2.2581	0.5235
0.95	0.0395	0.8804	1.8060	0.4933	0.0348	0.9263	2.3090	0.5327
0.90	0.0395	0.8804	1.8236	0.4933	0.0335	0.9394	2.4309	0.5443
0.80	0.0395	0.8805	1.9773	0.4933	0.0305	0.9636	2.6427	0.5663
0.70	0.0395	0.8805	2.2679	0.4933	N/A	N/A	N/A	N/A

Panel B: 95% Confidence Level

VaR Factor	Normal Distribution Realized Return	CSR	PT	Reward to CVaR	t-Distribution Realized Return	CSR	PT	Reward to CVaR
0.99	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933
0.95	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933
0.90	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933
0.80	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.8487	0.4933
0.70	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	2.0694	0.4933

Panel C: 90% Confidence Level

VaR Factor	Normal Distribution Realized Return	CSR	PT	Reward to CVaR	t-Distribution Realized Return	CSR	PT	Reward to CVaR
0.99	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933
0.95	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933
0.90	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933
0.80	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933
0.70	0.0395	0.8804	1.7957	0.4933	0.0395	0.8804	1.7957	0.4933

Table 6.2.6 Effects on out-of-sample VaR-Constrained BL Portfolio Performance in Multiple Periods (Sep 03-May 10)

This table shows realised VaR-constrained BL portfolio performance in the period from September 2003 to May 2010. The conditional covariance matrix applied to the portfolio construction is the DCC model. Return is the average realised excess return, risk is the standard deviation, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In construction of portfolio, VaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. The VaR constraint (VaR_0) is set to be equal to the scaling factor k (VaR factor) multiplied by the estimated VaR of the implied BL portfolio.

Panel A: Normal Distribution

VaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level :										
0.99	0.0044	0.0404	-0.6667	3.8162	0.1094	0.1681	0.1009	0.1070	0.0438	0.0413
0.95	0.0044	0.0399	-0.6569	3.8444	0.1099	0.1672	0.1005	0.1071	0.0436	0.0409
0.90	0.0044	0.0388	-0.6268	3.8484	0.1129	0.1683	0.0982	0.1035	0.0446	0.0424
0.80	0.0044	0.0364	-0.5280	3.8137	0.1196	0.1665	0.0892	0.0901	0.0489	0.0484
0.70	0.0043	0.0341	-0.3906	3.7911	0.1267	0.1573	0.0825	0.0828	0.0523	0.0522
95% Confidence Level :										
0.99	0.0042	0.0426	-0.9241	4.4914	0.0975	0.1455	0.1286	0.1416	0.0323	0.0294
0.95	0.0042	0.0424	-0.8831	4.3356	0.0991	0.1490	0.1242	0.1347	0.0338	0.0312
0.90	0.0043	0.0420	-0.8354	4.1690	0.1012	0.1534	0.1187	0.1260	0.0358	0.0338
0.80	0.0044	0.0414	-0.7454	3.9146	0.1062	0.1632	0.1076	0.1085	0.0408	0.0405
0.70	0.0044	0.0405	-0.6713	3.8050	0.1094	0.1685	0.1013	0.1069	0.0437	0.0415
90% Confidence Level :										
0.99	0.0040	0.0441	-1.1519	5.5102	0.0916	0.1324	0.1492	0.1741	0.0271	0.0232
0.95	0.0040	0.0441	-1.1529	5.5201	0.0907	0.1305	0.1492	0.1741	0.0268	0.0230
0.90	0.0040	0.0440	-1.1352	5.4382	0.0902	0.1297	0.1478	0.1718	0.0269	0.0231
0.80	0.0041	0.0430	-0.9757	4.7028	0.0955	0.1411	0.1337	0.1497	0.0307	0.0274
0.70	0.0042	0.0421	-0.8422	4.1919	0.1008	0.1526	0.1195	0.1273	0.0355	0.0333

Table 6.2.6 (continued)

Panel B: t-Distribution

VaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0043	0.0317	-0.3232	3.6576	0.1364	0.1452	0.0789	0.0796	0.0548	0.0543
0.95	0.0040	0.0310	-0.2834	3.5863	0.1293	0.1296	0.0780	0.0789	0.0514	0.0508
0.90	0.0041	0.0303	-0.2731	3.4587	0.1337	0.1264	0.0767	0.0780	0.0529	0.0520
0.80	0.0036	0.0287	-0.2176	3.2909	0.1248	0.1022	0.0735	0.0761	0.0487	0.0470
95% Confidence Level:										
0.99	0.0044	0.0410	-0.7134	3.8477	0.1075	0.1658	0.1044	0.1062	0.0422	0.0415
0.95	0.0044	0.0408	-0.6912	3.8182	0.1091	0.1685	0.1028	0.1066	0.0433	0.0418
0.90	0.0044	0.0403	-0.6641	3.8188	0.1095	0.1681	0.1007	0.1070	0.0438	0.0413
0.80	0.0044	0.0382	-0.6056	3.8363	0.1146	0.1686	0.0961	0.1002	0.0456	0.0437
0.70	0.0044	0.0356	-0.4844	3.8001	0.1223	0.1645	0.0858	0.0859	0.0508	0.0507
90% Confidence Level:										
0.99	0.0041	0.0432	-1.0078	4.8403	0.0944	0.1388	0.1367	0.1544	0.0298	0.0264
0.95	0.0041	0.0429	-0.9586	4.6306	0.0962	0.1426	0.1321	0.1471	0.0312	0.0280
0.90	0.0042	0.0425	-0.9012	4.4032	0.0984	0.1473	0.1262	0.1378	0.0331	0.0303
0.80	0.0043	0.0418	-0.7978	4.0546	0.1032	0.1574	0.1143	0.1191	0.0377	0.0362
0.70	0.0044	0.0410	-0.7096	3.8404	0.1076	0.1660	0.1042	0.1063	0.0424	0.0415

Table 6.3.1 Weights in the Out-of-sample CVaR-Constrained BL Portfolio in September 2003

This table reports weights allocated to each index in a CVaR-constrained BL portfolio in September 2003. The standard deviation is estimated by a conditional covariance matrix of the DCC, EWMA and RW110 models. CVaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	-0.0652	-0.0364	-0.0377	-0.0807	N/A	N/A
UK CONSUMER GDS	-0.0507	-0.0287	-0.0304	0.0151	N/A	N/A
UK CONSUMER SVS	-0.0747	-0.0365	-0.0369	-0.0474	N/A	N/A
UK FINANCIALS	-0.0737	-0.0108	-0.0117	-0.0573	N/A	N/A
UK HEALTH CARE	0.1555	0.0852	0.0860	0.1504	N/A	N/A
UK TECHNOLOGY	-0.0375	-0.0152	-0.0154	-0.0124	N/A	N/A
UK INDUSTRIALS	-0.0329	-0.0123	-0.0127	-0.0427	N/A	N/A
UK OIL & GAS	0.1248	0.0476	0.0481	0.1066	N/A	N/A
UK TELECOM	0.0818	0.0524	0.0532	0.0926	N/A	N/A
UK UTILITIES	0.1290	0.0647	0.0655	0.1025	N/A	N/A
USA BASIC MATS	-0.0710	-0.0252	-0.0256	0.0217	N/A	N/A
USA CONSUMER GDS	0.0892	0.0528	0.0541	0.0763	N/A	N/A
USA CONSUMER SVS	-0.0166	0.0410	0.0404	-0.0234	N/A	N/A
USA FINANCIALS	0.1927	0.0884	0.0894	0.0576	N/A	N/A
USA HEALTH CARE	0.2345	0.1920	0.1944	0.2825	N/A	N/A
USA INDUSTRIALS	-0.0574	0.0025	0.0013	-0.1235	N/A	N/A
USA OIL & GAS	0.1542	0.0912	0.0940	0.1527	N/A	N/A
USA TECHNOLOGY	0.1341	0.1231	0.1245	0.1039	N/A	N/A
USA TELECOM	0.0803	0.0617	0.0622	0.0022	N/A	N/A
USA UTILITIES	0.1064	0.0782	0.0790	0.0818	N/A	N/A
JAPAN BASIC MATS	-0.0753	-0.0207	-0.0250	-0.0561	N/A	N/A
JAPAN CONSUMER GDS	-0.0795	0.0579	0.0603	0.0233	N/A	N/A
JAPAN CONSUMER SVS	0.1120	0.0604	0.0635	0.1192	N/A	N/A
JAPAN FINANCIALS	-0.0428	-0.0071	-0.0095	-0.0370	N/A	N/A
JAPAN HEALTH CARE	0.1085	0.0600	0.0610	0.0556	N/A	N/A
JAPAN INDUSTRIALS	-0.0736	-0.0205	-0.0236	0.0498	N/A	N/A
JAPAN OIL & GAS	0.0553	0.0314	0.0318	-0.0029	N/A	N/A
JAPAN TECHNOLOGY	-0.0503	-0.0146	-0.0167	-0.1115	N/A	N/A
JAPAN TELECOM	-0.0657	-0.0196	-0.0209	-0.0050	N/A	N/A
JAPAN UTILITIES	0.1085	0.0568	0.0575	0.1064	N/A	N/A

Table 6.3.2 Out-of-sample CVaR-Constrained BL Portfolio Performance in the Single Period

This table reports the out-of-sample CVaR-constrained BL portfolio performance evaluated by realized return, Conditional Sharpe Ratio (CSR), Portfolio Turnover (PT), Reward to CVaR ratio in September 2003. The standard deviation is estimated by a conditional covariance matrix of the DCC, EWMA and RW110 models. CVaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

Sep-03	Normal Distribution				t-Distribution			
	Realised Return	CSR	PT	Reward to CVaR	Realised Return	CSR	PT	Reward to CVaR
DCC	0.0396	0.8816	1.7944	0.4942	0.0308	0.9595	2.9216	0.5625
EWMA	0.0262	0.7358	0.6387	0.3814	N/A	N/A	N/A	N/A
RW110	0.0264	0.7295	0.6766	0.3769	N/A	N/A	N/A	N/A

Table 6.3.3 Out-of-sample CVaR-Constrained BL Portfolio Performance in Multiple Periods

This table shows realised out-of-sample CVaR-constrained BL portfolio performance in the period from September 2003 to May 2010. Return is the average realised excess return, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In the construction of the portfolio, CVaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

Panel A: The normal distribution (Sep 03 - May 10)

	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0044	0.0404	-0.6656	3.8163	0.1094	0.1679	0.1010	0.1070	0.0437	0.0413
EWMA	0.0028	0.0371	-0.7162	3.6564	0.0765	0.1381	0.0981	0.1045	0.0289	0.0271
RW110	0.0024	0.0395	-0.9443	4.5393	0.0608	0.1355	0.1199	0.1287	0.0200	0.0187

Panel B: the t-distribution (Sep 03 - May 10)

	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
DCC	0.0037	0.0290	-0.2504	3.3095	0.1270	0.1071	0.0740	0.0763	0.0497	0.0482
EWMA	0.0015	0.0116	-0.2190	3.6936	0.1335	0.0295	0.0294	0.0299	0.0527	0.0517
RW110	0.0030	0.0247	-0.4627	3.5157	0.1205	0.0708	0.0724	0.0789	0.0412	0.0378

Table 6.3.4 Effects on Weights of CVaR-Constrained BL Portfolio (Sep 03)

This table shows positions of each asset in the CVaR-constained BL portfolio in September 2003 under the normal distribution and the t-distribution at confidence level of 99%. Note that the covariance matrix applied to CVaR-constained BL model is the DCC covariance matrix in this table.

99% Confidence Level:

	Normal Distribution				t-Distribution			
CVaR Factor:	0.99	0.95	0.90	0.80	0.99	0.95	0.90	0.80
UK BASIC MATS	-0.0652	-0.0652	-0.0652	-0.0655	-0.0807	-0.0825	-0.0851	-0.0913
UK CONSUMER GDS	-0.0507	-0.0507	-0.0507	-0.0507	0.0151	0.0231	0.0340	0.0611
UK CONSUMER SVS	-0.0747	-0.0747	-0.0746	-0.0744	-0.0474	-0.0441	-0.0397	-0.0287
UK FINANCIALS	-0.0737	-0.0737	-0.0736	-0.0735	-0.0573	-0.0552	-0.0525	-0.0456
UK HEALTH CARE	0.1555	0.1555	0.1554	0.1552	0.1504	0.1497	0.1488	0.1467
UK TECHNOLOGY	-0.0375	-0.0375	-0.0376	-0.0376	-0.0124	-0.0094	-0.0053	0.0050
UK INDUSTRIALS	-0.0329	-0.0329	-0.0329	-0.0328	-0.0427	-0.0439	-0.0456	-0.0497
UK OIL & GAS	0.1248	0.1248	0.1248	0.1247	0.1066	0.1044	0.1015	0.0940
UK TELECOM	0.0818	0.0818	0.0818	0.0817	0.0926	0.0940	0.0958	0.1002
UK UTILITIES	0.1290	0.1290	0.1290	0.1291	0.1025	0.0993	0.0950	0.0841
USA BASIC MATS	-0.0710	-0.0710	-0.0709	-0.0708	0.0217	0.0329	0.0481	0.0861
USA CONSUMER GDS	0.0892	0.0892	0.0891	0.0890	0.0763	0.0747	0.0726	0.0672
USA CONSUMER SVS	-0.0166	-0.0166	-0.0167	-0.0168	-0.0234	-0.0242	-0.0253	-0.0278
USA FINANCIALS	0.1927	0.1927	0.1928	0.1932	0.0576	0.0411	0.0190	-0.0365
USA HEALTH CARE	0.2345	0.2345	0.2345	0.2345	0.2825	0.2883	0.2960	0.3155
USA INDUSTRIALS	-0.0574	-0.0574	-0.0574	-0.0575	-0.1235	-0.1317	-0.1426	-0.1702
USA OIL & GAS	0.1542	0.1542	0.1542	0.1540	0.1527	0.1525	0.1523	0.1516
USA TECHNOLOGY	0.1341	0.1341	0.1342	0.1341	0.1039	0.1003	0.0954	0.0831
USA TELECOM	0.0803	0.0803	0.0803	0.0803	0.0022	-0.0073	-0.0201	-0.0523
USA UTILITIES	0.1064	0.1064	0.1064	0.1063	0.0818	0.0788	0.0749	0.0650
JAPAN BASIC MATS	-0.0753	-0.0753	-0.0753	-0.0752	-0.0561	-0.0538	-0.0507	-0.0429
JAPAN CONSUMER GDS	-0.0795	-0.0795	-0.0795	-0.0796	0.0233	0.0359	0.0529	0.0955
JAPAN CONSUMER SVS	0.1120	0.1120	0.1120	0.1123	0.1192	0.1202	0.1215	0.1248
JAPAN FINANCIALS	-0.0428	-0.0428	-0.0429	-0.0429	-0.0370	-0.0363	-0.0353	-0.0328
JAPAN HEALTH CARE	0.1085	0.1085	0.1085	0.1083	0.0556	0.0490	0.0402	0.0183
JAPAN INDUSTRIALS	-0.0736	-0.0736	-0.0735	-0.0737	0.0498	0.0647	0.0849	0.1351
JAPAN OIL & GAS	0.0553	0.0553	0.0553	0.0553	-0.0029	-0.0099	-0.0195	-0.0433
JAPAN TECHNOLOGY	-0.0503	-0.0503	-0.0502	-0.0501	-0.1115	-0.1190	-0.1291	-0.1543
JAPAN TELECOM	-0.0657	-0.0657	-0.0657	-0.0658	-0.0050	0.0023	0.0122	0.0372
JAPAN UTILITIES	0.1085	0.1085	0.1086	0.1089	0.1064	0.1062	0.1059	0.1049

Table 6.3.5 Effects on Out-of-sample CVaR-Constrained SR-BL Portfolio Performance Evaluation (Sep 03)

This table reports out-of-sample CVaR-constrained BL portfolio performance results including realized return, Conditional Sharpe Ratio (CSR), Portfolio Turnover (PT) and reward to CVaR ratio in September 2003. The standard deviation is forecasted by a conditional covariance matrix of the DCC model. CVaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor k multiplied by the estimated CVaR of the implied BL portfolio. The scaling factor k is called CVaR factor.

Panel A: 99% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realized Return	CSR	PT	Reward to CVaR	Realized Return	CSR	PT	Reward to CVaR
0.99	0.0396	0.8816	1.7944	0.4942	0.0308	0.9595	2.9216	0.5625
0.95	0.0396	0.8816	1.8051	0.4942	0.0298	0.9662	2.4386	0.5687
0.90	0.0396	0.8814	1.8239	0.4941	0.0284	0.9720	2.3684	0.5740
0.80	0.0396	0.8818	1.9788	0.4944	0.0249	0.9595	1.5720	0.5625
0.70	0.0396	0.8811	2.2690	0.4938	N/A	N/A	N/A	N/A

Panel B: 95% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realized Return	CSR	PT	Reward to CVaR	Realized Return	CSR	PT	Reward to CVaR
0.99	0.0396	0.8816	1.7944	0.4942	0.0396	0.8817	1.9319	0.4943
0.95	0.0396	0.8816	1.7944	0.4942	0.0396	0.8817	1.9319	0.4943
0.90	0.0396	0.8816	1.7944	0.4942	0.0396	0.8812	2.1283	0.4940
0.80	0.0396	0.8816	1.7944	0.4942	0.0383	0.8927	2.2570	0.5036
0.70	0.0396	0.8816	1.8183	0.4943	0.0348	0.9270	2.3363	0.5333

Panel C: 90% Confidence Level

	Normal Distribution				t-Distribution			
CVaR Factor	Realized Return	CSR	PT	Reward to CVaR	Realized Return	CSR	PT	Reward to CVaR
0.99	0.0396	0.8816	1.7944	0.4942	0.0396	0.8816	1.7944	0.4942
0.95	0.0396	0.8816	1.7944	0.4942	0.0396	0.8816	1.7944	0.4942
0.90	0.0396	0.8816	1.7944	0.4942	0.0396	0.8816	1.8015	0.4942
0.80	0.0396	0.8816	1.7944	0.4942	0.0396	0.8814	1.8790	0.4941
0.70	0.0396	0.8816	1.7944	0.4942	0.0396	0.8813	2.1235	0.4940

Table 6.3.6 Effects on Out-of-sample CVaR-Constrained BL Portfolio Performance in Multiple Periods (Sep 03-May 10)

This table shows realised CVaR-constrained BL portfolio performance in the period from September 2003 to May 2010. The conditional covariance matrix applied to the portfolio construction is the DCC model. Return is the average realised excess return, risk is the standard deviation, Sharpe Ratio is the average excess realised return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Reward to VaR ratio and Reward to CVaR ratio evaluate the excess return per unit of tail risk on the empirical distribution. In construction of portfolio, CVaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor k (CVaR factor) multiplied by the estimated CVaR of the implied BL portfolio.

Panel A: Normal Distribution

CVaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level :										
0.99	0.0044	0.0404	-0.6656	3.8163	0.1094	0.1679	0.1010	0.1070	0.0437	0.0413
0.95	0.0044	0.0399	-0.6563	3.8446	0.1100	0.1672	0.1005	0.1071	0.0436	0.0409
0.90	0.0044	0.0388	-0.6255	3.8481	0.1129	0.1684	0.0981	0.1034	0.0447	0.0424
0.80	0.0044	0.0364	-0.5261	3.8140	0.1198	0.1668	0.0891	0.0901	0.0490	0.0485
0.70	0.0043	0.0341	-0.3893	3.7924	0.1268	0.1575	0.0825	0.0828	0.0524	0.0522
95% Confidence Level :										
0.99	0.0042	0.0421	-0.8348	4.1694	0.1010	0.1532	0.1187	0.1260	0.0358	0.0337
0.95	0.0043	0.0418	-0.8011	4.0660	0.1029	0.1568	0.1148	0.1198	0.0375	0.0359
0.90	0.0044	0.0415	-0.7615	3.9556	0.1051	0.1612	0.1097	0.1118	0.0398	0.0390
0.80	0.0044	0.0408	-0.6915	3.8197	0.1088	0.1678	0.1029	0.1066	0.0431	0.0416
0.70	0.0044	0.0391	-0.6353	3.8476	0.1121	0.1681	0.0990	0.1048	0.0442	0.0418
90% Confidence Level :										
0.99	0.0041	0.0432	-1.0061	4.8363	0.0943	0.1385	0.1367	0.1543	0.0298	0.0264
0.95	0.0041	0.0429	-0.9564	4.6247	0.0961	0.1424	0.1319	0.1468	0.0312	0.0280
0.90	0.0042	0.0425	-0.8990	4.3977	0.0983	0.1472	0.1260	0.1376	0.0331	0.0303
0.80	0.0043	0.0418	-0.7963	4.0520	0.1031	0.1572	0.1142	0.1189	0.0377	0.0362
0.70	0.0044	0.0410	-0.7075	3.8398	0.1076	0.1659	0.1042	0.1063	0.0423	0.0415

Table 6.3.6 (continued)

Panel B: t-Distribution

CVaR Factor	Return	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR	Reward to CVaR
99% Confidence Level:										
0.99	0.0037	0.0290	-0.2504	3.3095	0.1270	0.1071	0.0740	0.0763	0.0497	0.0482
0.95	0.0041	0.0281	-0.2538	3.3486	0.1476	0.1211	0.0726	0.0757	0.0570	0.0547
0.90	0.0042	0.0277	-0.3252	3.3333	0.1502	0.1200	0.0713	0.0746	0.0584	0.0558
0.80	0.0036	0.0269	-0.3554	3.1870	0.1325	0.0982	0.0672	0.0699	0.0530	0.0509
0.70	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
95% Confidence Level:										
0.99	0.0044	0.0369	-0.5507	3.8250	0.1182	0.1675	0.0912	0.0928	0.0479	0.0470
0.95	0.0044	0.0361	-0.5107	3.8127	0.1207	0.1660	0.0878	0.0883	0.0497	0.0494
0.90	0.0044	0.0352	-0.4559	3.7998	0.1239	0.1630	0.0844	0.0851	0.0516	0.0512
0.80	0.0042	0.0332	-0.3393	3.7916	0.1274	0.1493	0.0809	0.0810	0.0522	0.0522
0.70	0.0041	0.0309	-0.2939	3.5893	0.1313	0.1309	0.0778	0.0787	0.0522	0.0515
90% Confidence Level:										
0.99	0.0044	0.0409	-0.6994	3.8296	0.1083	0.1671	0.1035	0.1065	0.0428	0.0416
0.95	0.0044	0.0406	-0.6785	3.8095	0.1095	0.1689	0.1019	0.1068	0.0437	0.0416
0.90	0.0044	0.0400	-0.6605	3.8351	0.1096	0.1672	0.1005	0.1071	0.0437	0.0410
0.80	0.0044	0.0377	-0.5854	3.8308	0.1159	0.1685	0.0943	0.0974	0.0464	0.0449
0.70	0.0044	0.0352	-0.4582	3.7962	0.1238	0.1633	0.0844	0.0852	0.0516	0.0512

Table 6.4.1 Out-of-sample Risk Aversion Coefficient and Risk-Adjusted Implied Equilibrium Return in September 2003

Following Giacometti et al. (2007) method, this table reports the results of risk aversion coefficients δ (Panel A and Panel C) and implied equilibrium return of each index $\boldsymbol{\pi}$ (Panel B and Panel D) of variance-adjusted, VaR-adjusted and CVaR-adjusted BL portfolios in September 2003, with assumptions of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%.

Panel A: Risk Aversion Coefficient							
	Variance	VaR					
		Normal Distribution			t-Distribution		
		99%	95%	90%	99%	95%	90%
Risk Aversion Coefficient	0.5000	0.1633	0.2417	0.3247	0.0975	0.1800	0.2623
Panel B: Implied Eqiulibrium Return							
UK BASIC MATS	0.0008	0.0010	0.0010	0.0006	0.0007	0.0007	0.0007
UK CONSUMER GDS	0.0007	0.0008	0.0008	0.0006	0.0007	0.0006	0.0006
UK CONSUMER SVS	0.0007	0.0009	0.0009	0.0006	0.0007	0.0006	0.0006
UK FINANCIALS	0.0009	0.0012	0.0012	0.0008	0.0009	0.0008	0.0008
UK HEALTH CARE	0.0004	0.0005	0.0005	0.0003	0.0003	0.0003	0.0003
UK TECHNOLOGY	0.0015	0.0019	0.0019	0.0012	0.0014	0.0013	0.0013
UK INDUSTRIALS	0.0011	0.0013	0.0013	0.0008	0.0010	0.0009	0.0009
UK OIL & GAS	0.0007	0.0010	0.0010	0.0006	0.0007	0.0006	0.0006
UK TELECOM	0.0009	0.0011	0.0011	0.0007	0.0008	0.0008	0.0007
UK UTILITIES	0.0003	0.0004	0.0004	0.0002	0.0003	0.0003	0.0002
USA BASIC MATS	0.0009	0.0012	0.0012	0.0007	0.0009	0.0008	0.0008
USA CONSUMER GDS	0.0010	0.0013	0.0013	0.0008	0.0009	0.0009	0.0008
USA CONSUMER SVS	0.0010	0.0013	0.0013	0.0008	0.0009	0.0009	0.0008
USA FINANCIALS	0.0011	0.0014	0.0014	0.0009	0.0010	0.0009	0.0009
USA HEALTH CARE	0.0006	0.0008	0.0008	0.0005	0.0005	0.0005	0.0005
USA INDUSTRIALS	0.0010	0.0013	0.0013	0.0008	0.0009	0.0009	0.0008
USA OIL & GAS	0.0006	0.0008	0.0008	0.0005	0.0006	0.0005	0.0005
USA TECHNOLOGY	0.0016	0.0021	0.0021	0.0013	0.0015	0.0014	0.0014
USA TELECOM	0.0009	0.0011	0.0011	0.0007	0.0008	0.0008	0.0007
USA UTILITIES	0.0004	0.0005	0.0005	0.0003	0.0004	0.0004	0.0003
JAPAN BASIC MATS	0.0006	0.0007	0.0007	0.0005	0.0005	0.0005	0.0005
JAPAN CONSUMER GDS	0.0007	0.0009	0.0009	0.0006	0.0007	0.0006	0.0006
JAPAN CONSUMER SVS	0.0005	0.0006	0.0006	0.0004	0.0005	0.0004	0.0004
JAPAN FINANCIALS	0.0008	0.0010	0.0010	0.0007	0.0008	0.0007	0.0007
JAPAN HEALTH CARE	0.0004	0.0005	0.0005	0.0003	0.0004	0.0003	0.0003
JAPAN INDUSTRIALS	0.0008	0.0009	0.0009	0.0006	0.0007	0.0007	0.0006
JAPAN OIL & GAS	0.0005	0.0006	0.0006	0.0004	0.0005	0.0005	0.0004
JAPAN TECHNOLOGY	0.0012	0.0014	0.0014	0.0009	0.0011	0.0010	0.0010
JAPAN TELECOM	0.0009	0.0010	0.0011	0.0007	0.0008	0.0008	0.0007
JAPAN UTILITIES	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002

Table 6.4.1 (continued)

Panel C: Risk Aversion Coefficient						
	CVaR					
	Normal Distribution			t-Distribution		
	99%	95%	90%	99%	95%	90%
Risk Aversion Coefficient	0.1406	0.1867	0.2243	0.0687	0.1153	0.1509
Panel D: Implied Eqiulibrium Return						
UK BASIC MATS	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
UK CONSUMER GDS	0.0006	0.0006	0.0006	0.0007	0.0006	0.0006
UK CONSUMER SVS	0.0006	0.0006	0.0006	0.0007	0.0007	0.0006
UK FINANCIALS	0.0008	0.0008	0.0008	0.0009	0.0009	0.0008
UK HEALTH CARE	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
UK TECHNOLOGY	0.0014	0.0013	0.0013	0.0014	0.0014	0.0014
UK INDUSTRIALS	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009
UK OIL & GAS	0.0006	0.0006	0.0006	0.0007	0.0006	0.0006
UK TELECOM	0.0008	0.0007	0.0007	0.0008	0.0008	0.0008
UK UTILITIES	0.0003	0.0003	0.0002	0.0003	0.0003	0.0003
USA BASIC MATS	0.0008	0.0008	0.0008	0.0009	0.0008	0.0008
USA CONSUMER GDS	0.0009	0.0009	0.0008	0.0009	0.0009	0.0009
USA CONSUMER SVS	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
USA FINANCIALS	0.0010	0.0009	0.0009	0.0010	0.0010	0.0010
USA HEALTH CARE	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
USA INDUSTRIALS	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
USA OIL & GAS	0.0005	0.0005	0.0005	0.0006	0.0006	0.0005
USA TECHNOLOGY	0.0015	0.0014	0.0014	0.0015	0.0015	0.0015
USA TELECOM	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
USA UTILITIES	0.0004	0.0004	0.0003	0.0004	0.0004	0.0004
JAPAN BASIC MATS	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
JAPAN CONSUMER GDS	0.0007	0.0006	0.0006	0.0007	0.0007	0.0007
JAPAN CONSUMER SVS	0.0004	0.0004	0.0004	0.0005	0.0004	0.0004
JAPAN FINANCIALS	0.0007	0.0007	0.0007	0.0008	0.0007	0.0007
JAPAN HEALTH CARE	0.0004	0.0003	0.0003	0.0004	0.0004	0.0004
JAPAN INDUSTRIALS	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
JAPAN OIL & GAS	0.0005	0.0004	0.0004	0.0005	0.0005	0.0005
JAPAN TECHNOLOGY	0.0010	0.0010	0.0010	0.0011	0.0010	0.0010
JAPAN TELECOM	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
JAPAN UTILITIES	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002

Table 6.4.2 The Out-of-sample Risk-Adjusted BL Expected Returns for Each Index in September 2003

Based on risk-adjusted equilibrium returns (Giacometti et al., 2007), this table reports the BL expected returns of variance-adjusted, VaR-adjusted and CVaR-adjusted Black-Litterman portfolios in September 2003, with assumptions of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%.

	Variance	VaR					
		Normal Distribution			t-Distribution		
Panel A:		99%	95%	90%	99%	95%	90%
UK BASIC MATS	-0.00020	-0.00003	-0.00002	-0.00001	-0.00027	-0.00030	-0.00033
UK CONSUMER GDS	-0.00074	-0.00062	-0.00061	-0.00060	-0.00080	-0.00083	-0.00086
UK CONSUMER SVS	0.00001	0.00019	0.00020	0.00020	-0.00005	-0.00008	-0.00011
UK FINANCIALS	0.00030	0.00058	0.00059	0.00061	0.00021	0.00017	0.00014
UK HEALTH CARE	0.00073	0.00086	0.00087	0.00087	0.00070	0.00068	0.00066
UK TECHNOLOGY	-0.00043	-0.00010	-0.00009	-0.00007	-0.00056	-0.00063	-0.00069
UK INDUSTRIALS	-0.00043	-0.00019	-0.00018	-0.00017	-0.00052	-0.00057	-0.00061
UK OIL & GAS	0.00057	0.00081	0.00082	0.00083	0.00051	0.00048	0.00045
UK TELECOM	0.00037	0.00060	0.00061	0.00062	0.00029	0.00025	0.00022
UK UTILITIES	0.00076	0.00085	0.00086	0.00086	0.00073	0.00072	0.00071
USA BASIC MATS	0.00010	0.00037	0.00038	0.00040	0.00002	-0.00002	-0.00006
USA CONSUMER GDS	0.00032	0.00060	0.00061	0.00062	0.00023	0.00019	0.00014
USA CONSUMER SVS	0.00032	0.00060	0.00061	0.00063	0.00023	0.00019	0.00015
USA FINANCIALS	0.00075	0.00108	0.00109	0.00110	0.00065	0.00061	0.00056
USA HEALTH CARE	0.00080	0.00103	0.00104	0.00104	0.00075	0.00072	0.00070
USA INDUSTRIALS	0.00046	0.00075	0.00076	0.00077	0.00037	0.00033	0.00029
USA OIL & GAS	0.00061	0.00081	0.00082	0.00083	0.00055	0.00053	0.00050
USA TECHNOLOGY	0.00059	0.00103	0.00105	0.00107	0.00045	0.00038	0.00031
USA TELECOM	0.00089	0.00113	0.00114	0.00116	0.00081	0.00077	0.00073
USA UTILITIES	0.00073	0.00086	0.00087	0.00087	0.00069	0.00067	0.00066
JAPAN BASIC MATS	0.00040	0.00048	0.00049	0.00049	0.00035	0.00033	0.00030
JAPAN CONSUMER GDS	0.00018	0.00033	0.00033	0.00034	0.00011	0.00008	0.00005
JAPAN CONSUMER SVS	0.00041	0.00050	0.00050	0.00051	0.00037	0.00034	0.00032
JAPAN FINANCIALS	0.00035	0.00049	0.00050	0.00051	0.00028	0.00024	0.00021
JAPAN HEALTH CARE	0.00080	0.00090	0.00090	0.00091	0.00076	0.00074	0.00073
JAPAN INDUSTRIALS	0.00009	0.00022	0.00022	0.00023	0.00002	-0.00002	-0.00005
JAPAN OIL & GAS	0.00087	0.00099	0.00100	0.00100	0.00082	0.00080	0.00078
JAPAN TECHNOLOGY	-0.00032	-0.00011	-0.00010	-0.00009	-0.00043	-0.00047	-0.00052
JAPAN TELECOM	0.00012	0.00027	0.00028	0.00029	0.00004	0.00000	-0.00004
JAPAN UTILITIES	0.00062	0.00066	0.00066	0.00066	0.00060	0.00059	0.00058

Table 6.4.2 (continued)

	CVaR					
	Normal Distribution			t-Distribution		
Panel B:	99%	95%	90%	99%	95%	90%
UK BASIC MATS	-0.00029	-0.00030	-0.00032	-0.00026	-0.00028	-0.00029
UK CONSUMER GDS	-0.00082	-0.00083	-0.00084	-0.00079	-0.00081	-0.00082
UK CONSUMER SVS	-0.00007	-0.00008	-0.00010	-0.00004	-0.00006	-0.00007
UK FINANCIALS	0.00019	0.00017	0.00015	0.00023	0.00021	0.00019
UK HEALTH CARE	0.00069	0.00068	0.00067	0.00070	0.00069	0.00069
UK TECHNOLOGY	-0.00060	-0.00063	-0.00066	-0.00054	-0.00058	-0.00061
UK INDUSTRIALS	-0.00055	-0.00057	-0.00059	-0.00051	-0.00053	-0.00055
UK OIL & GAS	0.00049	0.00047	0.00046	0.00052	0.00050	0.00049
UK TELECOM	0.00027	0.00025	0.00023	0.00030	0.00028	0.00027
UK UTILITIES	0.00073	0.00072	0.00072	0.00074	0.00073	0.00073
USA BASIC MATS	0.00000	-0.00002	-0.00004	0.00003	0.00001	0.00000
USA CONSUMER GDS	0.00020	0.00018	0.00016	0.00024	0.00022	0.00020
USA CONSUMER SVS	0.00021	0.00019	0.00017	0.00025	0.00022	0.00021
USA FINANCIALS	0.00063	0.00061	0.00059	0.00067	0.00064	0.00063
USA HEALTH CARE	0.00074	0.00072	0.00071	0.00076	0.00074	0.00073
USA INDUSTRIALS	0.00035	0.00033	0.00031	0.00039	0.00036	0.00035
USA OIL & GAS	0.00054	0.00053	0.00051	0.00056	0.00055	0.00054
USA TECHNOLOGY	0.00041	0.00037	0.00034	0.00047	0.00043	0.00040
USA TELECOM	0.00079	0.00077	0.00075	0.00082	0.00080	0.00078
USA UTILITIES	0.00068	0.00067	0.00066	0.00070	0.00069	0.00068
JAPAN BASIC MATS	0.00034	0.00032	0.00031	0.00036	0.00034	0.00033
JAPAN CONSUMER GDS	0.00010	0.00008	0.00007	0.00012	0.00011	0.00009
JAPAN CONSUMER SVS	0.00035	0.00034	0.00033	0.00037	0.00036	0.00035
JAPAN FINANCIALS	0.00026	0.00024	0.00022	0.00029	0.00027	0.00025
JAPAN HEALTH CARE	0.00075	0.00074	0.00073	0.00077	0.00076	0.00075
JAPAN INDUSTRIALS	0.00000	-0.00002	-0.00003	0.00003	0.00001	0.00000
JAPAN OIL & GAS	0.00081	0.00080	0.00079	0.00083	0.00082	0.00081
JAPAN TECHNOLOGY	-0.00045	-0.00048	-0.00050	-0.00041	-0.00044	-0.00046
JAPAN TELECOM	0.00002	0.00000	-0.00002	0.00005	0.00003	0.00001
JAPAN UTILITIES	0.00060	0.00059	0.00059	0.00061	0.00060	0.00060

Table 6.4.3 Weights in the Out-of-sample Risk-Adjusted Unconstrained BL Portfolio in September 2003

This table reports the weights assigned in each index in September 2003, with assumptions of the normal distribution and the t-distribution at confidence levels of 99%, 95% and 90%. Weights in the unconstrained variance-adjusted implied BL portfolio are calculated by $\mathbf{w}^*_{BL,t} = \frac{1}{\delta_t}\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$. The variance-adjusted SR-BL portfolio allocates asset to achieve the maximal Sharpe ratio in the optimisation problem, weights can be calculated by $\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}$. Note that weights in the VaR-adjusted and the CVaR-adjusted BL portfolios are allocated by solving maximal Sharpe ratio optimisation problem.

			VaR					
	Variance		Normal Distribution			t-Distribution		
Panel A:	Implied BL	SR-BL	99%	95%	90%	99%	95%	90%
UK BASIC MATS	-0.1424	-0.1015	-0.1140	-0.1144	-0.1141	-0.1088	-0.1123	-0.1164
UK CONSUMER GDS	-0.1102	-0.0795	-0.0751	-0.0739	-0.0738	-0.0851	-0.0878	-0.0913
UK CONSUMER SVS	-0.1619	-0.1173	-0.1350	-0.1332	-0.1333	-0.1265	-0.1314	-0.1385
UK FINANCIALS	-0.1173	-0.0852	-0.0442	-0.0417	-0.0417	-0.0934	-0.0968	-0.1001
UK HEALTH CARE	0.2370	0.1724	0.1687	0.1667	0.1665	0.1825	0.1851	0.1896
UK TECHNOLOGY	-0.0620	-0.0444	-0.0357	-0.0355	-0.0351	-0.0478	-0.0496	-0.0515
UK INDUSTRIALS	-0.0922	-0.0660	-0.0421	-0.0418	-0.0409	-0.0720	-0.0757	-0.0785
UK OIL & GAS	0.1585	0.1142	0.1528	0.1495	0.1504	0.1217	0.1238	0.1340
UK TELECOM	0.1443	0.1045	0.1067	0.1053	0.1056	0.1104	0.1133	0.1162
UK UTILITIES	0.2087	0.1491	0.1003	0.0993	0.0989	0.1584	0.1650	0.1691
USA BASIC MATS	-0.1319	-0.0940	-0.0590	-0.0581	-0.0577	-0.1019	-0.1069	-0.1122
USA CONSUMER GDS	0.1583	0.1139	0.1219	0.1216	0.1214	0.1205	0.1256	0.1293
USA CONSUMER SVS	-0.0702	-0.0491	0.0091	0.0050	0.0092	-0.0593	-0.0686	-0.0712
USA FINANCIALS	-0.0036	-0.0046	-0.0384	-0.0356	-0.0361	-0.0156	-0.0204	-0.0264
USA HEALTH CARE	0.3602	0.2594	0.2930	0.2922	0.2920	0.2666	0.2706	0.2746
USA INDUSTRIALS	-0.1145	-0.0831	-0.0571	-0.0566	-0.0560	-0.0926	-0.0962	-0.1019
USA OIL & GAS	0.2330	0.1676	0.1376	0.1398	0.1382	0.1761	0.1832	0.1797
USA TECHNOLOGY	0.2005	0.1444	0.1386	0.1384	0.1378	0.1468	0.1481	0.1497
USA TELECOM	0.1664	0.1194	0.0781	0.0792	0.0773	0.1259	0.1295	0.1325
USA UTILITIES	0.2073	0.1486	0.1249	0.1232	0.1228	0.1569	0.1604	0.1691
JAPAN BASIC MATS	-0.1145	-0.0846	-0.1058	-0.1041	-0.1047	-0.0912	-0.0942	-0.0948
JAPAN CONSUMER GDS	0.1661	0.1185	0.0888	0.0905	0.0892	0.1251	0.1297	0.1347
JAPAN CONSUMER SVS	0.1861	0.1322	0.1186	0.1211	0.1175	0.1401	0.1443	0.1515
JAPAN FINANCIALS	-0.0796	-0.0568	-0.0500	-0.0502	-0.0489	-0.0623	-0.0647	-0.0696
JAPAN HEALTH CARE	0.1859	0.1354	0.1787	0.1741	0.1761	0.1439	0.1468	0.1513
JAPAN INDUSTRIALS	-0.1258	-0.0876	-0.0935	-0.0929	-0.0925	-0.0950	-0.0998	-0.1071
JAPAN OIL & GAS	0.1014	0.0739	0.0774	0.0759	0.0764	0.0787	0.0804	0.0828
JAPAN TECHNOLOGY	-0.0819	-0.0602	-0.0422	-0.0421	-0.0416	-0.0652	-0.0673	-0.0698
JAPAN TELECOM	-0.0937	-0.0672	-0.0726	-0.0724	-0.0716	-0.0726	-0.0750	-0.0782
JAPAN UTILITIES	0.1792	0.1277	0.0695	0.0707	0.0688	0.1356	0.1409	0.1435

Table 6.4.3 (continued)

	CVaR					
	Normal Distribution			t-Distribution		
Panel B:	99%	95%	90%	99%	95%	90%
UK BASIC MATS	-0.1102	-0.1124	-0.1152	-0.1080	-0.1092	-0.1106
UK CONSUMER GDS	-0.0866	-0.0881	-0.0895	-0.0841	-0.0859	-0.0871
UK CONSUMER SVS	-0.1290	-0.1320	-0.1342	-0.1253	-0.1274	-0.1299
UK FINANCIALS	-0.0957	-0.0971	-0.0996	-0.0915	-0.0945	-0.0965
UK HEALTH CARE	0.1841	0.1855	0.1894	0.1801	0.1834	0.1857
UK TECHNOLOGY	-0.0485	-0.0498	-0.0506	-0.0474	-0.0480	-0.0487
UK INDUSTRIALS	-0.0742	-0.0759	-0.0770	-0.0711	-0.0727	-0.0744
UK OIL & GAS	0.1224	0.1236	0.1292	0.1217	0.1221	0.1221
UK TELECOM	0.1119	0.1137	0.1153	0.1095	0.1110	0.1124
UK UTILITIES	0.1618	0.1655	0.1658	0.1565	0.1597	0.1619
USA BASIC MATS	-0.1038	-0.1072	-0.1088	-0.1011	-0.1027	-0.1036
USA CONSUMER GDS	0.1232	0.1263	0.1262	0.1195	0.1214	0.1234
USA CONSUMER SVS	-0.0634	-0.0699	-0.0668	-0.0570	-0.0606	-0.0642
USA FINANCIALS	-0.0174	-0.0211	-0.0241	-0.0136	-0.0165	-0.0185
USA HEALTH CARE	0.2692	0.2710	0.2723	0.2653	0.2676	0.2697
USA INDUSTRIALS	-0.0948	-0.0964	-0.0999	-0.0906	-0.0937	-0.0955
USA OIL & GAS	0.1792	0.1845	0.1814	0.1733	0.1773	0.1810
USA TECHNOLOGY	0.1467	0.1483	0.1483	0.1461	0.1469	0.1473
USA TELECOM	0.1277	0.1298	0.1310	0.1246	0.1266	0.1280
USA UTILITIES	0.1580	0.1604	0.1648	0.1570	0.1571	0.1580
JAPAN BASIC MATS	-0.0932	-0.0950	-0.0953	-0.0889	-0.0922	-0.0948
JAPAN CONSUMER GDS	0.1277	0.1300	0.1301	0.1236	0.1260	0.1286
JAPAN CONSUMER SVS	0.1423	0.1443	0.1468	0.1391	0.1409	0.1427
JAPAN FINANCIALS	-0.0628	-0.0649	-0.0666	-0.0613	-0.0628	-0.0627
JAPAN HEALTH CARE	0.1454	0.1472	0.1504	0.1424	0.1447	0.1460
JAPAN INDUSTRIALS	-0.0974	-0.0996	-0.1019	-0.0944	-0.0957	-0.0977
JAPAN OIL & GAS	0.0794	0.0808	0.0819	0.0772	0.0794	0.0802
JAPAN TECHNOLOGY	-0.0662	-0.0677	-0.0681	-0.0634	-0.0657	-0.0670
JAPAN TELECOM	-0.0738	-0.0751	-0.0773	-0.0721	-0.0731	-0.0741
JAPAN UTILITIES	0.1377	0.1412	0.1420	0.1341	0.1366	0.1383

Table 6.4.4 Out-of-Sample Risk-Adjusted Unconstrained BL Portfolio Performance Evaluation in the Single Period

This table reports the results of out-of-sample risk-adjusted unconstrained BL portfolios for the portfolio evaluation criterion including realized excess return, Conditional Sharpe Ratio (CSR), and Portfolio Turnover (PT) and reward to CVaR ratio in September 2003. The standard deviation is estimated rolling window method with window length of 110. The implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal Sharpe ratio in the optimisation problem. Both VaR and CVaR is estimated by the parametric method in the optimisation model with assumption of the normal distribution ('N') and the t-distribution ('t') at confidence levels of 99%, 95% and 90%.

	Realized Excess Return	CSR	PT	Reward to CVaR
Implied BL	0.0820	1.3655	2.6756	1.0507
SR-BL	0.0589	1.3641	1.4859	1.0485
99% Confidence Level:				
π -VaR N	0.0521	1.2677	1.4789	0.9071
π -VaR t	0.0626	1.3904	2.0551	1.0906
π -CVaR N	0.0637	1.3980	2.1273	1.1033
π -CVaR t	0.0620	1.3872	2.0062	1.0855
95% Confidence Level:				
π -VaR N	0.0517	1.2633	1.4623	0.9011
π -VaR t	0.0646	1.4019	2.1992	1.1097
π -CVaR N	0.0647	1.4023	2.2122	1.1103
π -CVaR t	0.0630	1.3930	2.0850	1.0950
90% Confidence Level:				
π -VaR N	0.0515	1.2607	1.4746	0.8977
π -VaR t	0.0667	1.4136	2.3476	1.1294
π -CVaR N	0.0656	1.4075	2.2713	1.1190
π -CVaR t	0.0640	1.4009	2.1468	1.1080

Table 6.4.5 Out-of-sample Risk-Adjusted Unconstrained BL Portfolios Performance in Multiple Periods (Sep 03 – May 10)

This table shows realised out-of-sample risk-adjusted unconstrained BL portfolios performance in the period from September 2003 to May 2010. Return is the average realized excess return, risk is the standard deviation, Sharpe Ratio is the average excess realized return divided by the standard deviation. Information Ratio is the average active return divided by the standard deviation of active return. Both VaR and CVaR are measured on the empirical distribution. Return to VaR ratio and Return to CVaR ratio evaluate the excess return per unit of tail risk. In construction of portfolio, both VaR and CVaR are estimated by the parametric method with assumption of the normal distribution ('N') and the t-distribution ('t') at confidence levels of 99%, 95% and 90%.

	Return	Risk	Skewness	Kurtosis	Sharpe Ratio	Information Ratio	Empirical VaR	Empirical CVaR	Reward to VaR Ratio	Reward to CVaR Ratio
implied BL	0.0074	0.0485	-0.7874	6.3160	0.1531	0.1667	0.1644	0.2032	0.0452	0.0365
SR-BL	0.0026	0.0632	-3.4104	23.3042	0.0415	0.0372	0.3142	0.4289	0.0083	0.0061
99% Confidence Level:										
π -VaR N	0.0020	0.0717	-2.7274	20.0491	0.0280	0.0197	0.3745	0.4376	0.0054	0.0046
π -VaR t	0.0064	0.0487	-0.6489	5.4425	0.1304	0.1347	0.1536	0.1771	0.0413	0.0359
π -CVaR N	0.0064	0.0496	-0.7582	6.0132	0.1285	0.1309	0.1651	0.1950	0.0386	0.0327
π -CVaR t	0.0063	0.0482	-0.5959	5.1651	0.1308	0.1364	0.1468	0.1665	0.0429	0.0379
95% Confidence Level:										
π -VaR N	0.0022	0.0733	-2.4920	19.0190	0.0303	0.0219	0.3767	0.4330	0.0059	0.0051
π -VaR t	0.0064	0.0507	-0.8891	6.7230	0.1265	0.1270	0.1769	0.2137	0.0362	0.0300
π -CVaR N	0.0065	0.0510	-0.8886	6.8550	0.1269	0.1274	0.1791	0.2172	0.0362	0.0298
π -CVaR t	0.0064	0.0491	-0.6892	5.6538	0.1297	0.1332	0.1582	0.1842	0.0402	0.0345
90% Confidence Level:										
π -VaR N	0.0025	0.0756	-2.1156	18.0422	0.0335	0.0250	0.3797	0.4281	0.0067	0.0059
π -VaR t	0.0064	0.0537	-1.3017	9.1521	0.1189	0.1157	0.2080	0.2628	0.0307	0.0243
π -CVaR N	0.0063	0.0521	-1.0815	7.8338	0.1210	0.1194	0.1924	0.2382	0.0328	0.0265
π -CVaR t	0.0064	0.0499	-0.7872	6.1809	0.1281	0.1300	0.1680	0.1997	0.0380	0.0320

Figure 6.1.1 Out-of-sample Monthly Volatility of Benchmark Portfolio

This figure plots the time-varying standard deviation for the benchmark portfolio from March 2003 to May 2010. The time-varying standard deviation is calculated by the DCC model (blue line), the EWMA model (red line), and the RW method (green line) with a window length of 110.

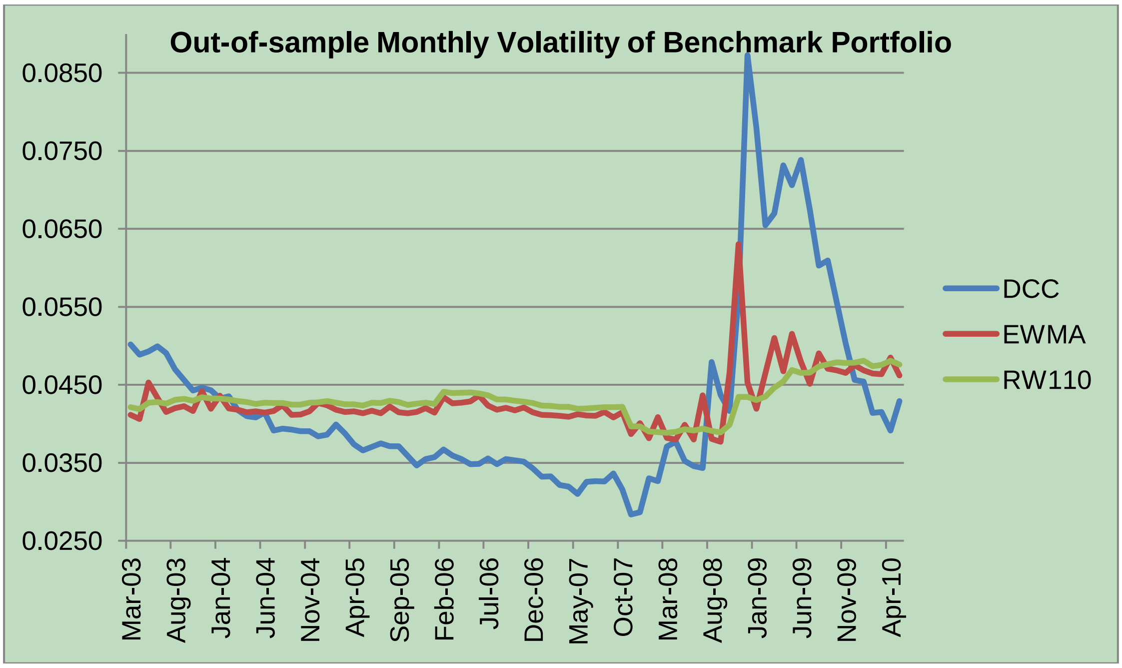
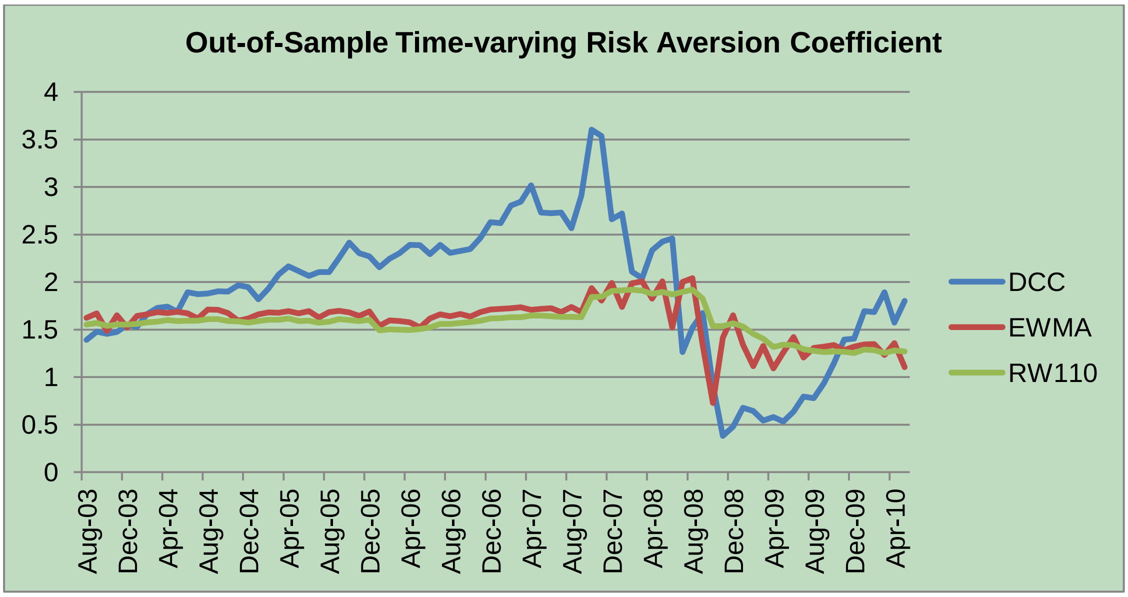


Figure 6.1.2 Out-of-Sample Time-Varying Risk Aversion Coefficient



This figure plots the time-varying risk aversion coefficient from August 2003 to May 2010. The risk aversion coefficient is calculated by the monthly world risk premium divided by monthly time-varying market variance. The monthly world risk premium is set at 0.29% (=3.5%/12). The time-varying standard deviation is calculated by the DCC model (blue line), the EWMA model (red line), the RW method (green line) with a window length of 110.

Appendix 6.1.1 Average Value of Weights in the Out-of-sample Unconstrained Implied BL Portfolio and the SR-BL Portfolio (Sep 03 – May 10)

This table reports average value of weights assigned in each index in the out-of-sample unconstrained implied BL portfolio and the SR-BL portfolio in the period from September 2003 to May 2010. An implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem.

	DCC		EWMA		RW110	
	Implied BL	SR-BL	Implied BL	SR-BL	Implied BL	SR-BL
UK BASIC MATS	0.0030	0.0065	0.0043	0.0046	0.0017	0.0019
UK CONSUMER GDS	0.0030	0.0010	0.0118	0.0122	0.0134	0.0139
UK CONSUMER SVS	-0.0061	-0.0008	-0.0008	-0.0008	-0.0023	-0.0023
UK FINANCIALS	0.0177	0.0190	0.0231	0.0229	0.0225	0.0222
UK HEALTH CARE	0.0210	0.0133	0.0175	0.0162	0.0168	0.0149
UK TECHNOLOGY	0.0000	0.0042	0.0077	0.0082	0.0058	0.0066
UK INDUSTRIALS	0.0192	0.0218	0.0298	0.0299	0.0288	0.0296
UK OIL & GAS	0.0067	0.0040	0.0116	0.0114	0.0114	0.0117
UK TELECOM	0.0181	0.0125	0.0159	0.0158	0.0192	0.0186
UK UTILITIES	0.0262	0.0177	0.0203	0.0184	0.0198	0.0183
USA BASIC MATS	0.0110	0.0125	0.0147	0.0145	0.0110	0.0111
USA CONSUMER GDS	0.0424	0.0350	0.0449	0.0446	0.0462	0.0455
USA CONSUMER SVS	0.0778	0.0750	0.0789	0.0779	0.0778	0.0776
USA FINANCIALS	0.1295	0.1246	0.1216	0.1206	0.1219	0.1206
USA HEALTH CARE	0.1183	0.1089	0.1078	0.1051	0.1054	0.1029
USA INDUSTRIALS	0.0727	0.0666	0.0631	0.0628	0.0641	0.0629
USA OIL & GAS	0.0941	0.0844	0.0761	0.0753	0.0795	0.0787
USA TECHNOLOGY	0.0952	0.0984	0.1033	0.1030	0.1015	0.1017
USA TELECOM	0.0512	0.0463	0.0321	0.0320	0.0355	0.0352
USA UTILITIES	0.0598	0.0517	0.0422	0.0416	0.0475	0.0468
JAPAN BASIC MATS	-0.0020	0.0050	0.0089	0.0099	0.0052	0.0059
JAPAN CONSUMER GDS	0.0010	0.0046	0.0189	0.0190	0.0178	0.0179
JAPAN CONSUMER SVS	0.0373	0.0355	0.0203	0.0207	0.0205	0.0208
JAPAN FINANCIALS	0.0311	0.0280	0.0314	0.0317	0.0342	0.0343
JAPAN HEALTH CARE	0.0338	0.0258	0.0210	0.0209	0.0206	0.0201
JAPAN INDUSTRIALS	0.0085	0.0152	0.0169	0.0173	0.0158	0.0165
JAPAN OIL & GAS	0.0097	0.0127	0.0118	0.0122	0.0110	0.0120
JAPAN TECHNOLOGY	-0.0010	0.0054	0.0094	0.0101	0.0061	0.0069
JAPAN TELECOM	0.0232	0.0229	0.0144	0.0146	0.0167	0.0169
JAPAN UTILITIES	0.0499	0.0424	0.0280	0.0274	0.0308	0.0303
Absolute Position Range	0.1355	0.1254	0.1224	0.1214	0.1242	0.1229

Appendix 6.1.2 Standard Deviation of Weights in the Out-of-sample Unconstrained Implied BL Portfolio and the SR-BL Portfolio (Sep 03 – May 10)

This table reports standard deviation of weights assigned in each index in the out-of-sample unconstrained implied BL portfolio and the SR-BL portfolio in the period from September 2003 to May 2010. An implied BL portfolio is constructed by reverse optimisation of the utility function. The SR-BL portfolio is constructed by achieving maximal SR in the optimisation problem.

	DCC		EWMA		RW110	
	Implied BL	SR-BL	Implied BL	SR-BL	Implied BL	SR-BL
UK BASIC MATS	0.0446	0.0432	0.0268	0.0267	0.0283	0.0284
UK CONSUMER GDS	0.0638	0.0554	0.0292	0.0290	0.0325	0.0328
UK CONSUMER SVS	0.0650	0.0592	0.0352	0.0346	0.0390	0.0389
UK FINANCIALS	0.0486	0.0442	0.0307	0.0298	0.0331	0.0328
UK HEALTH CARE	0.0930	0.0810	0.0474	0.0463	0.0537	0.0533
UK TECHNOLOGY	0.0405	0.0367	0.0175	0.0176	0.0196	0.0200
UK INDUSTRIALS	0.0595	0.0544	0.0273	0.0273	0.0304	0.0312
UK OIL & GAS	0.0847	0.0728	0.0338	0.0334	0.0392	0.0392
UK TELECOM	0.0681	0.0570	0.0284	0.0279	0.0312	0.0311
UK UTILITIES	0.0964	0.0906	0.0469	0.0463	0.0513	0.0516
USA BASIC MATS	0.0569	0.0525	0.0324	0.0321	0.0356	0.0358
USA CONSUMER GDS	0.0832	0.0704	0.0470	0.0463	0.0509	0.0501
USA CONSUMER SVS	0.0722	0.0634	0.0401	0.0381	0.0458	0.0455
USA FINANCIALS	0.0587	0.0520	0.0435	0.0421	0.0464	0.0448
USA HEALTH CARE	0.1065	0.0912	0.0634	0.0592	0.0701	0.0675
USA INDUSTRIALS	0.0643	0.0556	0.0370	0.0360	0.0412	0.0391
USA OIL & GAS	0.0783	0.0638	0.0354	0.0344	0.0405	0.0401
USA TECHNOLOGY	0.0574	0.0638	0.0231	0.0245	0.0260	0.0284
USA TELECOM	0.0723	0.0601	0.0327	0.0325	0.0363	0.0366
USA UTILITIES	0.0767	0.0683	0.0365	0.0357	0.0392	0.0393
JAPAN BASIC MATS	0.0597	0.0557	0.0299	0.0301	0.0341	0.0347
JAPAN CONSUMER GDS	0.0736	0.0670	0.0346	0.0342	0.0407	0.0409
JAPAN CONSUMER SVS	0.0988	0.0867	0.0422	0.0416	0.0476	0.0478
JAPAN FINANCIALS	0.0575	0.0493	0.0246	0.0248	0.0270	0.0274
JAPAN HEALTH CARE	0.0927	0.0814	0.0411	0.0404	0.0475	0.0477
JAPAN INDUSTRIALS	0.0585	0.0555	0.0316	0.0318	0.0355	0.0364
JAPAN OIL & GAS	0.0512	0.0448	0.0220	0.0218	0.0257	0.0257
JAPAN TECHNOLOGY	0.0540	0.0500	0.0230	0.0232	0.0263	0.0269
JAPAN TELECOM	0.0627	0.0567	0.0253	0.0251	0.0288	0.0290
JAPAN UTILITIES	0.0846	0.0774	0.0385	0.0382	0.0433	0.0439
Average Standard Deviation	0.0695	0.0620	0.0342	0.0337	0.0382	0.0382

Appendix 6.1.3 Average Value of Weights in the Out-of-sample Unconstrained MVaR-BL Portfolio (Sep 03 – May 10)

This table reports average value of weights allocated to each index in the out-of-sample unconstrained MVaR-BL portfolio in the period from September 2003 to May 2010. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0067	0.0048	0.0019	0.0053	0.0048	0.0019
UK CONSUMER GDS	0.0015	0.0121	0.0139	0.0016	0.0122	0.0139
UK CONSUMER SVS	-0.0075	-0.0009	-0.0022	-0.0073	-0.0008	-0.0021
UK FINANCIALS	0.0196	0.0228	0.0222	0.0191	0.0227	0.0222
UK HEALTH CARE	0.0157	0.0161	0.0149	0.0163	0.0161	0.0149
UK TECHNOLOGY	0.0009	0.0081	0.0066	0.0006	0.0082	0.0066
UK INDUSTRIALS	0.0196	0.0299	0.0296	0.0194	0.0298	0.0296
UK OIL & GAS	0.0038	0.0112	0.0117	0.0041	0.0113	0.0117
UK TELECOM	0.0145	0.0158	0.0186	0.0154	0.0158	0.0187
UK UTILITIES	0.0208	0.0186	0.0183	0.0216	0.0188	0.0184
USA BASIC MATS	0.0105	0.0143	0.0111	0.0104	0.0140	0.0111
USA CONSUMER GDS	0.0392	0.0447	0.0455	0.0400	0.0447	0.0455
USA CONSUMER SVS	0.0754	0.0779	0.0776	0.0760	0.0779	0.0777
USA FINANCIALS	0.1288	0.1208	0.1205	0.1291	0.1209	0.1205
USA HEALTH CARE	0.1125	0.1051	0.1030	0.1137	0.1053	0.1031
USA INDUSTRIALS	0.0726	0.0628	0.0629	0.0723	0.0630	0.0630
USA OIL & GAS	0.0903	0.0754	0.0787	0.0908	0.0754	0.0787
USA TECHNOLOGY	0.0937	0.1030	0.1017	0.0940	0.1029	0.1016
USA TELECOM	0.0477	0.0320	0.0352	0.0487	0.0319	0.0352
USA UTILITIES	0.0549	0.0415	0.0468	0.0557	0.0414	0.0467
JAPAN BASIC MATS	-0.0015	0.0100	0.0059	-0.0027	0.0100	0.0060
JAPAN CONSUMER GDS	-0.0008	0.0190	0.0179	-0.0010	0.0189	0.0180
JAPAN CONSUMER SVS	0.0340	0.0209	0.0207	0.0341	0.0208	0.0206
JAPAN FINANCIALS	0.0319	0.0317	0.0343	0.0308	0.0317	0.0343
JAPAN HEALTH CARE	0.0298	0.0207	0.0200	0.0300	0.0207	0.0199
JAPAN INDUSTRIALS	0.0090	0.0172	0.0165	0.0078	0.0172	0.0165
JAPAN OIL & GAS	0.0101	0.0122	0.0119	0.0087	0.0122	0.0119
JAPAN TECHNOLOGY	0.0007	0.0100	0.0069	-0.0005	0.0101	0.0069
JAPAN TELECOM	0.0217	0.0146	0.0169	0.0210	0.0146	0.0169
JAPAN UTILITIES	0.0439	0.0274	0.0303	0.0448	0.0275	0.0304
Absolute Position Range	0.1362	0.1216	0.1228	0.1364	0.1217	0.1226

Appendix 6.1.4 Standard Deviation of Weights in the Out-of-sample Unconstrained MVaR-BL Portfolio (Sep 03 – May 10)

This table reports standard deviation of weights allocated to each index in the out-of-sample unconstrained MVaR-BL portfolio in the period from September 2003 to May 2010. The weight in the MVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to VaR ratio. VaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0416	0.0266	0.0284	0.0437	0.0265	0.0285
UK CONSUMER GDS	0.0620	0.0290	0.0327	0.0615	0.0290	0.0326
UK CONSUMER SVS	0.0685	0.0345	0.0389	0.0684	0.0344	0.0386
UK FINANCIALS	0.0481	0.0298	0.0327	0.0495	0.0299	0.0326
UK HEALTH CARE	0.0855	0.0463	0.0533	0.0871	0.0460	0.0534
UK TECHNOLOGY	0.0394	0.0175	0.0200	0.0414	0.0175	0.0200
UK INDUSTRIALS	0.0603	0.0273	0.0313	0.0603	0.0274	0.0312
UK OIL & GAS	0.0842	0.0333	0.0392	0.0828	0.0332	0.0392
UK TELECOM	0.0601	0.0279	0.0311	0.0617	0.0279	0.0311
UK UTILITIES	0.0887	0.0463	0.0513	0.0888	0.0462	0.0512
USA BASIC MATS	0.0561	0.0320	0.0358	0.0568	0.0321	0.0359
USA CONSUMER GDS	0.0790	0.0463	0.0502	0.0800	0.0465	0.0501
USA CONSUMER SVS	0.0742	0.0377	0.0455	0.0733	0.0372	0.0456
USA FINANCIALS	0.0561	0.0421	0.0448	0.0545	0.0420	0.0449
USA HEALTH CARE	0.1012	0.0589	0.0673	0.1031	0.0586	0.0671
USA INDUSTRIALS	0.0637	0.0361	0.0391	0.0626	0.0362	0.0391
USA OIL & GAS	0.0699	0.0346	0.0402	0.0711	0.0347	0.0402
USA TECHNOLOGY	0.0609	0.0245	0.0284	0.0615	0.0246	0.0284
USA TELECOM	0.0659	0.0323	0.0365	0.0672	0.0324	0.0364
USA UTILITIES	0.0682	0.0356	0.0393	0.0696	0.0355	0.0391
JAPAN BASIC MATS	0.0615	0.0304	0.0347	0.0632	0.0305	0.0346
JAPAN CONSUMER GDS	0.0780	0.0342	0.0408	0.0782	0.0341	0.0406
JAPAN CONSUMER SVS	0.0937	0.0414	0.0477	0.0943	0.0410	0.0476
JAPAN FINANCIALS	0.0536	0.0247	0.0274	0.0533	0.0248	0.0274
JAPAN HEALTH CARE	0.0850	0.0405	0.0475	0.0856	0.0404	0.0472
JAPAN INDUSTRIALS	0.0603	0.0312	0.0363	0.0618	0.0307	0.0362
JAPAN OIL & GAS	0.0519	0.0218	0.0256	0.0533	0.0218	0.0255
JAPAN TECHNOLOGY	0.0548	0.0232	0.0270	0.0567	0.0233	0.0270
JAPAN TELECOM	0.0606	0.0251	0.0291	0.0601	0.0252	0.0291
JAPAN UTILITIES	0.0725	0.0382	0.0440	0.0751	0.0382	0.0441
Average Standard Deviation	0.0669	0.0336	0.0382	0.0676	0.0336	0.0381

Appendix 6.1.5 Average Effect of Distribution Assumptions and Confidence Levels on out-of-sample unconstrained MVaR-BL Portfolio Weights

This table shows average value of weights in each index and average standard deviation in the out-of-sample unconstrained MVaR-BL portfolio in the period from September 2003 to May 2010. Note that the covariance matrix applied to the MVaR-BL model is the DCC covariance matrix in this table.

	Normal Distribution			t-Distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
UK BASIC MATS	0.0102	0.0065	0.0065	0.0148	0.0069	0.0076
UK CONSUMER GDS	0.0111	0.0011	0.0010	0.0103	0.0016	0.0017
UK CONSUMER SVS	0.0132	-0.0008	-0.0008	0.0124	-0.0072	-0.0071
UK FINANCIALS	0.0265	0.0191	0.0190	0.0314	0.0198	0.0199
UK HEALTH CARE	0.0189	0.0132	0.0133	0.0300	0.0158	0.0155
UK TECHNOLOGY	0.0025	0.0042	0.0042	-0.0117	0.0012	0.0018
UK INDUSTRIALS	0.0231	0.0217	0.0218	0.0186	0.0197	0.0199
UK OIL & GAS	-0.0022	0.0040	0.0040	0.0039	0.0038	0.0034
UK TELECOM	0.0203	0.0126	0.0126	0.0156	0.0147	0.0140
UK UTILITIES	-0.0017	0.0177	0.0177	0.0079	0.0209	0.0201
USA BASIC MATS	0.0245	0.0125	0.0125	0.0226	0.0102	0.0101
USA CONSUMER GDS	0.0293	0.0349	0.0350	0.0289	0.0389	0.0385
USA CONSUMER SVS	0.0928	0.0750	0.0750	0.0853	0.0752	0.0747
USA FINANCIALS	0.1189	0.1246	0.1246	0.1240	0.1284	0.1282
USA HEALTH CARE	0.1147	0.1089	0.1089	0.1235	0.1121	0.1111
USA INDUSTRIALS	0.0718	0.0666	0.0666	0.0667	0.0724	0.0726
USA OIL & GAS	0.0485	0.0845	0.0844	0.0524	0.0901	0.0893
USA TECHNOLOGY	0.1113	0.0984	0.0984	0.1012	0.0936	0.0934
USA TELECOM	0.0297	0.0462	0.0463	0.0272	0.0476	0.0466
USA UTILITIES	0.0241	0.0516	0.0517	0.0342	0.0549	0.0539
JAPAN BASIC MATS	0.0190	0.0050	0.0050	0.0196	-0.0015	-0.0003
JAPAN CONSUMER GDS	0.0352	0.0046	0.0046	0.0287	-0.0007	-0.0004
JAPAN CONSUMER SVS	0.0251	0.0356	0.0356	0.0227	0.0341	0.0341
JAPAN FINANCIALS	0.0403	0.0280	0.0280	0.0442	0.0319	0.0329
JAPAN HEALTH CARE	0.0114	0.0258	0.0258	0.0171	0.0299	0.0298
JAPAN INDUSTRIALS	0.0280	0.0152	0.0152	0.0210	0.0090	0.0101
JAPAN OIL & GAS	-0.0004	0.0127	0.0127	0.0074	0.0102	0.0111
JAPAN TECHNOLOGY	0.0252	0.0053	0.0054	0.0118	0.0008	0.0023
JAPAN TELECOM	0.0228	0.0229	0.0229	0.0146	0.0218	0.0226
JAPAN UTILITIES	0.0057	0.0424	0.0423	0.0137	0.0439	0.0427
Absolute Position Range	0.1211	0.1254	0.1254	0.1357	0.1355	0.1353
Average Standard Deviation	0.0951	0.0620	0.0620	0.0964	0.0666	0.0658

Appendix 6.1.6 Average Value of Weights in the Out-of-sample Unconstrained MCVaR-BL Portfolio (Sep 03 – May 10)

This table reports average value of weights allocated to each index in the out-of-sample unconstrained MCVaR-BL portfolio in the period from September 2003 to May 2010. The weight in the MCVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to CVaR ratio. CVaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0065	0.0048	0.0020	0.0035	0.0048	0.0019
UK CONSUMER GDS	0.0015	0.0121	0.0139	0.0014	0.0122	0.0139
UK CONSUMER SVS	-0.0076	-0.0009	-0.0022	-0.0073	-0.0007	-0.0020
UK FINANCIALS	0.0195	0.0228	0.0222	0.0183	0.0226	0.0222
UK HEALTH CARE	0.0157	0.0161	0.0149	0.0180	0.0160	0.0149
UK TECHNOLOGY	0.0011	0.0081	0.0066	-0.0006	0.0081	0.0065
UK INDUSTRIALS	0.0197	0.0298	0.0296	0.0178	0.0299	0.0296
UK OIL & GAS	0.0038	0.0112	0.0117	0.0047	0.0113	0.0117
UK TELECOM	0.0146	0.0158	0.0186	0.0161	0.0158	0.0187
UK UTILITIES	0.0207	0.0186	0.0183	0.0235	0.0189	0.0185
USA BASIC MATS	0.0105	0.0142	0.0111	0.0100	0.0139	0.0111
USA CONSUMER GDS	0.0394	0.0447	0.0455	0.0403	0.0447	0.0455
USA CONSUMER SVS	0.0756	0.0779	0.0777	0.0759	0.0777	0.0777
USA FINANCIALS	0.1289	0.1208	0.1205	0.1292	0.1213	0.1205
USA HEALTH CARE	0.1127	0.1052	0.1030	0.1154	0.1053	0.1031
USA INDUSTRIALS	0.0726	0.0629	0.0629	0.0719	0.0630	0.0630
USA OIL & GAS	0.0904	0.0754	0.0787	0.0919	0.0754	0.0786
USA TECHNOLOGY	0.0940	0.1030	0.1017	0.0936	0.1030	0.1015
USA TELECOM	0.0478	0.0320	0.0352	0.0488	0.0319	0.0351
USA UTILITIES	0.0549	0.0415	0.0468	0.0573	0.0412	0.0466
JAPAN BASIC MATS	-0.0017	0.0100	0.0059	-0.0036	0.0100	0.0060
JAPAN CONSUMER GDS	-0.0009	0.0190	0.0179	-0.0010	0.0189	0.0181
JAPAN CONSUMER SVS	0.0340	0.0208	0.0207	0.0345	0.0206	0.0206
JAPAN FINANCIALS	0.0318	0.0317	0.0343	0.0294	0.0315	0.0342
JAPAN HEALTH CARE	0.0296	0.0207	0.0200	0.0308	0.0209	0.0199
JAPAN INDUSTRIALS	0.0089	0.0172	0.0165	0.0072	0.0173	0.0165
JAPAN OIL & GAS	0.0099	0.0122	0.0119	0.0080	0.0122	0.0119
JAPAN TECHNOLOGY	0.0007	0.0101	0.0069	-0.0025	0.0102	0.0069
JAPAN TELECOM	0.0217	0.0146	0.0169	0.0208	0.0145	0.0169
JAPAN UTILITIES	0.0438	0.0274	0.0303	0.0464	0.0276	0.0304
Absolute Position Range	0.1365	0.1216	0.1228	0.1364	0.1220	0.1225

Appendix 6.1.7 Standard Deviation of Weights in the Out-of-sample Unconstrained MCVaR-BL Portfolio (Sep 03 – May 10)

This table reports standard deviation of weights allocated to each index in the out-of-sample unconstrained MCVaR-BL portfolio in the period from September 2003 to May 2010. The weight in the MCVaR-BL portfolio is the solution to the optimisation problem with the target of maximal expected excess return to CVaR ratio. CVaR is estimated by the parametric method with the assumption of normal distribution and t-distribution at the confidence level of 99%.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0419	0.0266	0.0283	0.0462	0.0266	0.0285
UK CONSUMER GDS	0.0620	0.0290	0.0327	0.0626	0.0290	0.0326
UK CONSUMER SVS	0.0687	0.0345	0.0389	0.0683	0.0341	0.0385
UK FINANCIALS	0.0484	0.0298	0.0327	0.0487	0.0298	0.0327
UK HEALTH CARE	0.0854	0.0463	0.0534	0.0904	0.0461	0.0533
UK TECHNOLOGY	0.0392	0.0175	0.0200	0.0429	0.0176	0.0199
UK INDUSTRIALS	0.0603	0.0273	0.0313	0.0619	0.0275	0.0311
UK OIL & GAS	0.0841	0.0333	0.0392	0.0841	0.0329	0.0393
UK TELECOM	0.0602	0.0279	0.0311	0.0628	0.0280	0.0311
UK UTILITIES	0.0886	0.0463	0.0513	0.0917	0.0460	0.0509
USA BASIC MATS	0.0563	0.0320	0.0359	0.0577	0.0322	0.0359
USA CONSUMER GDS	0.0792	0.0463	0.0502	0.0805	0.0465	0.0500
USA CONSUMER SVS	0.0744	0.0376	0.0456	0.0735	0.0366	0.0456
USA FINANCIALS	0.0564	0.0421	0.0448	0.0550	0.0421	0.0451
USA HEALTH CARE	0.1013	0.0589	0.0673	0.1051	0.0584	0.0670
USA INDUSTRIALS	0.0638	0.0361	0.0391	0.0633	0.0363	0.0392
USA OIL & GAS	0.0700	0.0346	0.0402	0.0727	0.0346	0.0401
USA TECHNOLOGY	0.0611	0.0246	0.0284	0.0616	0.0246	0.0283
USA TELECOM	0.0660	0.0323	0.0365	0.0667	0.0322	0.0364
USA UTILITIES	0.0683	0.0356	0.0393	0.0711	0.0357	0.0389
JAPAN BASIC MATS	0.0618	0.0304	0.0347	0.0646	0.0306	0.0343
JAPAN CONSUMER GDS	0.0782	0.0342	0.0408	0.0784	0.0341	0.0403
JAPAN CONSUMER SVS	0.0939	0.0414	0.0477	0.0952	0.0406	0.0475
JAPAN FINANCIALS	0.0537	0.0247	0.0275	0.0556	0.0248	0.0271
JAPAN HEALTH CARE	0.0848	0.0405	0.0475	0.0869	0.0402	0.0469
JAPAN INDUSTRIALS	0.0604	0.0311	0.0363	0.0630	0.0302	0.0361
JAPAN OIL & GAS	0.0520	0.0219	0.0256	0.0541	0.0217	0.0254
JAPAN TECHNOLOGY	0.0549	0.0232	0.0269	0.0588	0.0232	0.0269
JAPAN TELECOM	0.0605	0.0252	0.0290	0.0611	0.0251	0.0291
JAPAN UTILITIES	0.0725	0.0382	0.0440	0.0768	0.0382	0.0440
Average Standard Deviation	0.0669	0.0336	0.0382	0.0687	0.0335	0.0381

Appendix 6.1.8 Average Effect of Distribution Assumptions and Confidence Levels on out-of-sample unconstrained MCVaR-BL Portfolio Weights

This table shows average value of weights in each index and average standard deviation in the out-of-sample unconstrained MVaR-BL portfolio in the period from September 2003 to May 2010. Note that the covariance matrix applied to the MVaR-BL model is the DCC covariance matrix in this table.

	Normal Distribution			t-Distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
UK BASIC MATS	0.0065	0.0070	0.0072	0.0035	0.0063	0.0067
UK CONSUMER GDS	0.0015	0.0017	0.0017	0.0014	0.0015	0.0015
UK CONSUMER SVS	-0.0076	-0.0071	-0.0071	-0.0073	-0.0073	-0.0075
UK FINANCIALS	0.0195	0.0198	0.0198	0.0183	0.0198	0.0196
UK HEALTH CARE	0.0157	0.0156	0.0156	0.0180	0.0159	0.0157
UK TECHNOLOGY	0.0011	0.0014	0.0017	-0.0006	0.0011	0.0009
UK INDUSTRIALS	0.0197	0.0199	0.0198	0.0178	0.0194	0.0196
UK OIL & GAS	0.0038	0.0037	0.0034	0.0047	0.0039	0.0038
UK TELECOM	0.0146	0.0146	0.0142	0.0161	0.0149	0.0145
UK UTILITIES	0.0207	0.0207	0.0202	0.0235	0.0211	0.0208
USA BASIC MATS	0.0105	0.0102	0.0101	0.0100	0.0106	0.0105
USA CONSUMER GDS	0.0394	0.0388	0.0387	0.0403	0.0396	0.0393
USA CONSUMER SVS	0.0756	0.0752	0.0750	0.0759	0.0758	0.0755
USA FINANCIALS	0.1289	0.1282	0.1284	0.1292	0.1296	0.1288
USA HEALTH CARE	0.1127	0.1119	0.1115	0.1154	0.1131	0.1126
USA INDUSTRIALS	0.0726	0.0724	0.0726	0.0719	0.0728	0.0726
USA OIL & GAS	0.0904	0.0899	0.0895	0.0919	0.0905	0.0904
USA TECHNOLOGY	0.0940	0.0936	0.0936	0.0936	0.0940	0.0937
USA TELECOM	0.0478	0.0475	0.0470	0.0488	0.0481	0.0477
USA UTILITIES	0.0549	0.0546	0.0542	0.0573	0.0552	0.0549
JAPAN BASIC MATS	-0.0017	-0.0013	-0.0008	-0.0036	-0.0022	-0.0015
JAPAN CONSUMER GDS	-0.0009	-0.0006	-0.0004	-0.0010	-0.0011	-0.0008
JAPAN CONSUMER SVS	0.0340	0.0342	0.0341	0.0345	0.0337	0.0340
JAPAN FINANCIALS	0.0318	0.0321	0.0326	0.0294	0.0311	0.0319
JAPAN HEALTH CARE	0.0296	0.0298	0.0297	0.0308	0.0296	0.0298
JAPAN INDUSTRIALS	0.0089	0.0092	0.0096	0.0072	0.0085	0.0089
JAPAN OIL & GAS	0.0099	0.0103	0.0106	0.0080	0.0092	0.0101
JAPAN TECHNOLOGY	0.0007	0.0011	0.0019	-0.0025	0.0000	0.0006
JAPAN TELECOM	0.0217	0.0218	0.0222	0.0208	0.0213	0.0217
JAPAN UTILITIES	0.0438	0.0438	0.0432	0.0464	0.0439	0.0439
Absolute Position Range	0.1365	0.1354	0.1355	0.1364	0.1370	0.1363
Average Standard Deviation	0.0669	0.0665	0.0661	0.0687	0.0671	0.0669

Appendix 6.2.1 Average Value of Weights in the Out-of-sample VaR-Constrained BL Portfolio (Sep 03 – May 10)

This table reports average value of weights allocated to each index in the out-of-sample VaR-constrained BL portfolio in the period from September 2003 to May 2010. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0031	0.0006	-0.0026	-0.0115	-0.1063	-0.1113
UK CONSUMER GDS	0.0045	0.0139	0.0158	0.0340	0.0744	0.0770
UK CONSUMER SVS	-0.0009	0.0021	0.0007	0.0139	0.0658	0.0670
UK FINANCIALS	0.0183	0.0228	0.0224	0.0132	0.0320	0.0329
UK HEALTH CARE	0.0178	0.0170	0.0160	0.0512	0.0133	0.0105
UK TECHNOLOGY	0.0026	0.0061	0.0044	-0.0002	-0.0263	-0.0286
UK INDUSTRIALS	0.0203	0.0269	0.0264	-0.0088	-0.0328	-0.0339
UK OIL & GAS	0.0039	0.0082	0.0082	-0.0147	-0.0802	-0.0826
UK TELECOM	0.0140	0.0173	0.0202	0.0292	0.0600	0.0640
UK UTILITIES	0.0250	0.0240	0.0234	0.0555	0.1245	0.1284
USA BASIC MATS	0.0129	0.0139	0.0106	0.0097	0.0055	0.0025
USA CONSUMER GDS	0.0368	0.0437	0.0448	0.0223	0.0358	0.0336
USA CONSUMER SVS	0.0795	0.0800	0.0797	0.1195	0.1020	0.0998
USA FINANCIALS	0.1149	0.1148	0.1145	-0.0371	-0.0465	-0.0484
USA HEALTH CARE	0.1144	0.1129	0.1117	0.1664	0.2689	0.2743
USA INDUSTRIALS	0.0646	0.0577	0.0581	0.0167	-0.0665	-0.0676
USA OIL & GAS	0.0853	0.0798	0.0832	0.0728	0.2019	0.2124
USA TECHNOLOGY	0.0925	0.0987	0.0972	0.0676	-0.0016	-0.0048
USA TELECOM	0.0449	0.0307	0.0333	0.0357	-0.0100	-0.0107
USA UTILITIES	0.0542	0.0409	0.0454	0.1300	0.0238	0.0214
JAPAN BASIC MATS	0.0040	0.0084	0.0049	0.0141	0.0010	0.0015
JAPAN CONSUMER GDS	0.0121	0.0251	0.0247	0.0662	0.1555	0.1593
JAPAN CONSUMER SVS	0.0345	0.0253	0.0253	0.0721	0.1097	0.1089
JAPAN FINANCIALS	0.0257	0.0284	0.0307	-0.0141	-0.0343	-0.0343
JAPAN HEALTH CARE	0.0261	0.0173	0.0166	-0.0014	-0.0533	-0.0556
JAPAN INDUSTRIALS	0.0134	0.0214	0.0210	0.0135	0.1246	0.1257
JAPAN OIL & GAS	0.0090	0.0092	0.0084	-0.0161	-0.0578	-0.0601
JAPAN TECHNOLOGY	0.0013	0.0068	0.0036	-0.0206	-0.0567	-0.0597
JAPAN TELECOM	0.0196	0.0132	0.0153	0.0352	0.0116	0.0131
JAPAN UTILITIES	0.0457	0.0328	0.0359	0.0855	0.1619	0.1652
Absolute Position Range	0.1158	0.1143	0.1171	0.2035	0.3751	0.3856

Appendix 6.2.2 Standard Deviation of Weights in the Out-of-sample VaR-Constrained BL Portfolio (Sep 03 – May 10)

This table reports standard deviation of weights allocated to each index in the out-of-sample VaR-constrained BL portfolio in the period from September 2003 to May 2010. VaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The VaR constraint (VaR_0) is set to be equal to the scaling factor 0.99 multiplied by the estimated VaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0379	0.0252	0.0273	0.0915	0.0412	0.0346
UK CONSUMER GDS	0.0490	0.0284	0.0315	0.0495	0.0279	0.0273
UK CONSUMER SVS	0.0553	0.0334	0.0376	0.1010	0.0637	0.0641
UK FINANCIALS	0.0410	0.0303	0.0324	0.0436	0.0391	0.0390
UK HEALTH CARE	0.0721	0.0443	0.0500	0.0622	0.0315	0.0297
UK TECHNOLOGY	0.0323	0.0156	0.0175	0.0448	0.0159	0.0181
UK INDUSTRIALS	0.0462	0.0254	0.0287	0.0523	0.0246	0.0239
UK OIL & GAS	0.0650	0.0329	0.0380	0.0624	0.0703	0.0752
UK TELECOM	0.0508	0.0270	0.0290	0.0556	0.0245	0.0206
UK UTILITIES	0.0728	0.0401	0.0461	0.0550	0.0409	0.0362
USA BASIC MATS	0.0485	0.0320	0.0351	0.0479	0.0436	0.0419
USA CONSUMER GDS	0.0657	0.0447	0.0477	0.0758	0.0263	0.0224
USA CONSUMER SVS	0.0571	0.0355	0.0419	0.1122	0.0612	0.0580
USA FINANCIALS	0.0515	0.0417	0.0448	0.0743	0.0471	0.0466
USA HEALTH CARE	0.0813	0.0529	0.0599	0.0791	0.0389	0.0375
USA INDUSTRIALS	0.0567	0.0385	0.0405	0.1422	0.0701	0.0667
USA OIL & GAS	0.0533	0.0329	0.0371	0.0659	0.0749	0.0736
USA TECHNOLOGY	0.0524	0.0215	0.0255	0.0806	0.0358	0.0321
USA TELECOM	0.0550	0.0319	0.0348	0.0684	0.0359	0.0360
USA UTILITIES	0.0636	0.0341	0.0359	0.0974	0.0309	0.0264
JAPAN BASIC MATS	0.0496	0.0284	0.0323	0.0653	0.0465	0.0467
JAPAN CONSUMER GDS	0.0577	0.0333	0.0377	0.0735	0.0380	0.0347
JAPAN CONSUMER SVS	0.0825	0.0448	0.0498	0.0867	0.0752	0.0786
JAPAN FINANCIALS	0.0445	0.0227	0.0245	0.0403	0.0199	0.0171
JAPAN HEALTH CARE	0.0739	0.0376	0.0436	0.1002	0.0491	0.0440
JAPAN INDUSTRIALS	0.0485	0.0317	0.0355	0.0935	0.0375	0.0320
JAPAN OIL & GAS	0.0386	0.0199	0.0228	0.0309	0.0259	0.0207
JAPAN TECHNOLOGY	0.0417	0.0204	0.0245	0.0893	0.0309	0.0259
JAPAN TELECOM	0.0503	0.0228	0.0258	0.0514	0.0294	0.0297
JAPAN UTILITIES	0.0697	0.0383	0.0433	0.0590	0.0504	0.0468
Average Standard Deviation	0.0555	0.0323	0.0360	0.0717	0.0416	0.0395

Appendix 6.3.1 Average Value of Weights in the Out-of-sample CVaR-Constrained BL Portfolio (Sep 03 – May 10)

This table reports average value of weights allocated to each index in the out-of-sample CVaR-constrained BL portfolio in the period from September 2003 to May 2010. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0032	0.0007	-0.0025	-0.0129	-0.1359	-0.1337
UK CONSUMER GDS	0.0045	0.0140	0.0158	0.0468	0.0916	0.0953
UK CONSUMER SVS	-0.0009	0.0021	0.0008	0.0147	0.0789	0.0792
UK FINANCIALS	0.0183	0.0227	0.0222	0.0104	0.0282	0.0331
UK HEALTH CARE	0.0177	0.0169	0.0159	0.0613	0.0133	0.0089
UK TECHNOLOGY	0.0025	0.0061	0.0043	0.0015	-0.0368	-0.0364
UK INDUSTRIALS	0.0203	0.0269	0.0264	-0.0172	-0.0516	-0.0542
UK OIL & GAS	0.0038	0.0084	0.0084	-0.0250	-0.1017	-0.1064
UK TELECOM	0.0140	0.0174	0.0203	0.0370	0.0739	0.0753
UK UTILITIES	0.0251	0.0241	0.0235	0.0683	0.1555	0.1543
USA BASIC MATS	0.0128	0.0138	0.0106	0.0093	0.0027	0.0007
USA CONSUMER GDS	0.0367	0.0437	0.0448	0.0123	0.0292	0.0306
USA CONSUMER SVS	0.0795	0.0799	0.0797	0.1413	0.1054	0.1022
USA FINANCIALS	0.1151	0.1149	0.1146	-0.1009	-0.0926	-0.0952
USA HEALTH CARE	0.1144	0.1130	0.1117	0.1950	0.3242	0.3289
USA INDUSTRIALS	0.0646	0.0579	0.0583	-0.0107	-0.1103	-0.1134
USA OIL & GAS	0.0854	0.0796	0.0829	0.0706	0.2346	0.2417
USA TECHNOLOGY	0.0926	0.0988	0.0972	0.0529	-0.0293	-0.0323
USA TELECOM	0.0448	0.0306	0.0332	0.0270	-0.0203	-0.0228
USA UTILITIES	0.0541	0.0409	0.0455	0.1576	0.0182	0.0177
JAPAN BASIC MATS	0.0040	0.0085	0.0049	0.0175	-0.0002	-0.0011
JAPAN CONSUMER GDS	0.0121	0.0250	0.0246	0.0956	0.2042	0.2080
JAPAN CONSUMER SVS	0.0346	0.0255	0.0254	0.0933	0.1493	0.1421
JAPAN FINANCIALS	0.0257	0.0283	0.0306	-0.0325	-0.0549	-0.0555
JAPAN HEALTH CARE	0.0260	0.0173	0.0167	-0.0142	-0.0831	-0.0805
JAPAN INDUSTRIALS	0.0134	0.0213	0.0210	0.0178	0.1572	0.1570
JAPAN OIL & GAS	0.0090	0.0092	0.0084	-0.0277	-0.0780	-0.0800
JAPAN TECHNOLOGY	0.0012	0.0068	0.0036	-0.0321	-0.0814	-0.0784
JAPAN TELECOM	0.0197	0.0132	0.0153	0.0428	0.0077	0.0083
JAPAN UTILITIES	0.0457	0.0327	0.0358	0.1002	0.2020	0.2067
Absolute Position Range	0.1159	0.1143	0.1171	0.2959	0.4600	0.4627

Appendix 6.3.2 Standard Deviation of Weights in the Out-of-sample CVaR-Constrained BL Portfolio (Sep 03 – May 10)

This table reports standard deviation of weights allocated to each index in the out-of-sample CVaR-constrained BL portfolio in the period from September 2003 to May 2010. CVaR is estimated by the parametric method in the optimisation model with assumption of normal distribution and t-distribution at a confidence level of 99%. The CVaR constraint ($CVaR_0$) is set to be equal to the scaling factor 0.99 multiplied by the estimated CVaR of the implied BL portfolio in the corresponding period.

	Normal Distribution			t-Distribution		
99% Confidence Level:	DCC	EWMA	RW110	DCC	EWMA	RW110
UK BASIC MATS	0.0380	0.0253	0.0274	0.1294	0.0511	0.0408
UK CONSUMER GDS	0.0490	0.0285	0.0316	0.0443	0.0330	0.0325
UK CONSUMER SVS	0.0554	0.0335	0.0375	0.1337	0.0820	0.0745
UK FINANCIALS	0.0410	0.0302	0.0324	0.0549	0.0511	0.0484
UK HEALTH CARE	0.0721	0.0443	0.0500	0.0639	0.0389	0.0319
UK TECHNOLOGY	0.0323	0.0156	0.0175	0.0588	0.0220	0.0222
UK INDUSTRIALS	0.0462	0.0254	0.0287	0.0699	0.0308	0.0302
UK OIL & GAS	0.0652	0.0328	0.0380	0.0748	0.0956	0.0965
UK TELECOM	0.0508	0.0270	0.0290	0.0686	0.0324	0.0274
UK UTILITIES	0.0728	0.0402	0.0462	0.0573	0.0485	0.0444
USA BASIC MATS	0.0486	0.0321	0.0350	0.0573	0.0591	0.0559
USA CONSUMER GDS	0.0657	0.0447	0.0477	0.0871	0.0271	0.0262
USA CONSUMER SVS	0.0571	0.0357	0.0421	0.1514	0.0974	0.0941
USA FINANCIALS	0.0515	0.0418	0.0448	0.0813	0.0515	0.0525
USA HEALTH CARE	0.0812	0.0530	0.0599	0.0913	0.0403	0.0385
USA INDUSTRIALS	0.0568	0.0385	0.0405	0.1979	0.0860	0.0867
USA OIL & GAS	0.0532	0.0329	0.0371	0.0885	0.0954	0.0909
USA TECHNOLOGY	0.0525	0.0215	0.0255	0.1012	0.0347	0.0320
USA TELECOM	0.0550	0.0320	0.0349	0.0738	0.0392	0.0391
USA UTILITIES	0.0637	0.0341	0.0359	0.1257	0.0422	0.0410
JAPAN BASIC MATS	0.0496	0.0282	0.0323	0.0827	0.0590	0.0607
JAPAN CONSUMER GDS	0.0577	0.0333	0.0377	0.0842	0.0489	0.0415
JAPAN CONSUMER SVS	0.0825	0.0449	0.0499	0.0984	0.1026	0.1066
JAPAN FINANCIALS	0.0445	0.0227	0.0245	0.0416	0.0271	0.0223
JAPAN HEALTH CARE	0.0739	0.0376	0.0435	0.1244	0.0652	0.0566
JAPAN INDUSTRIALS	0.0486	0.0320	0.0357	0.1334	0.0438	0.0397
JAPAN OIL & GAS	0.0386	0.0199	0.0227	0.0334	0.0255	0.0229
JAPAN TECHNOLOGY	0.0418	0.0204	0.0245	0.1253	0.0483	0.0415
JAPAN TELECOM	0.0502	0.0228	0.0258	0.0706	0.0453	0.0431
JAPAN UTILITIES	0.0697	0.0383	0.0434	0.0664	0.0505	0.0520
Average Standard Deviation	0.0555	0.0323	0.0361	0.0890	0.0525	0.0497

Appendix 6.4.1 Average Value of Weights in the Out-of-sample Risk-Adjusted Unconstrained BL Portfolio (Sep 03 – May 10)

This table reports average value of weights assigned in each index in the period from September 2003 to May 2010, with assumptions of normal distribution ('N') and t-distribution ('t') at confidence levels of 99%, 95% and 90%. Note that weights in the unconstrained variance-adjusted implied BL portfolio are calculated by $\mathbf{w}^*_{BL,t} = \frac{1}{\delta_t}\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$, the variance-adjusted SR-BL portfolio allocates asset to achieve the maximal Sharpe ratio in the optimisation problem, weights can be calculated by $\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}$, weights in the VaR-adjusted and the CVaR-adjusted BL portfolio are allocated by solving maximal Sharpe ratio optimisation problem.

	Variance		VaR					
			Normal Distribution			t-Distribution		
Panel A:	Implied BL	SR-BL	99%	95%	90%	99%	95%	90%
UK BASIC MATS	-0.0174	-0.0142	-0.0536	-0.0557	-0.0584	-0.0224	-0.0240	-0.0253
UK CONSUMER GDS	0.0133	0.0168	0.0168	0.0174	0.0177	0.0213	0.0223	0.0235
UK CONSUMER SVS	-0.0386	-0.0430	-0.0458	-0.0474	-0.0483	-0.0465	-0.0491	-0.0526
UK FINANCIALS	-0.0057	-0.0109	-0.0303	-0.0328	-0.0357	-0.0145	-0.0168	-0.0197
UK HEALTH CARE	0.0117	-0.0036	-0.0128	-0.0159	-0.0196	-0.0098	-0.0123	-0.0149
UK TECHNOLOGY	0.0035	0.0107	0.0115	0.0115	0.0110	0.0097	0.0103	0.0111
UK INDUSTRIALS	0.0283	0.0329	0.0494	0.0515	0.0537	0.0395	0.0405	0.0417
UK OIL & GAS	0.0281	0.0321	0.0680	0.0682	0.0679	0.0303	0.0312	0.0325
UK TELECOM	0.0249	0.0215	0.0210	0.0223	0.0236	0.0279	0.0290	0.0302
UK UTILITIES	0.0411	0.0304	0.0295	0.0314	0.0333	0.0180	0.0161	0.0135
USA BASIC MATS	-0.0038	-0.0017	-0.0093	-0.0122	-0.0152	-0.0136	-0.0153	-0.0177
USA CONSUMER GDS	0.0437	0.0378	0.0490	0.0492	0.0498	0.0442	0.0446	0.0451
USA CONSUMER SVS	0.0656	0.0638	0.0583	0.0578	0.0576	0.0680	0.0672	0.0667
USA FINANCIALS	0.1018	0.0932	0.1035	0.1015	0.0992	0.0935	0.0917	0.0899
USA HEALTH CARE	0.1154	0.0962	0.1011	0.0997	0.0988	0.1009	0.0999	0.0986
USA INDUSTRIALS	0.0515	0.0412	0.0345	0.0315	0.0280	0.0391	0.0379	0.0363
USA OIL & GAS	0.1065	0.1027	0.0964	0.0984	0.0999	0.0998	0.1004	0.1006
USA TECHNOLOGY	0.1067	0.1122	0.1192	0.1192	0.1198	0.1110	0.1121	0.1137
USA TELECOM	0.0507	0.0485	0.0468	0.0494	0.0526	0.0559	0.0571	0.0580
USA UTILITIES	0.0873	0.0869	0.0667	0.0694	0.0725	0.0883	0.0896	0.0907
JAPAN BASIC MATS	-0.0109	-0.0063	-0.0373	-0.0385	-0.0396	-0.0135	-0.0146	-0.0155
JAPAN CONSUMER GDS	-0.0043	-0.0086	0.0122	0.0112	0.0104	-0.0110	-0.0132	-0.0157
JAPAN CONSUMER SVS	0.0311	0.0387	0.0281	0.0287	0.0286	0.0395	0.0419	0.0448
JAPAN FINANCIALS	0.0417	0.0464	0.0631	0.0645	0.0663	0.0475	0.0483	0.0491
JAPAN HEALTH CARE	0.0361	0.0338	0.0533	0.0531	0.0523	0.0439	0.0463	0.0500
JAPAN INDUSTRIALS	-0.0005	0.0083	-0.0213	-0.0229	-0.0244	0.0041	0.0042	0.0041
JAPAN OIL & GAS	0.0232	0.0316	0.0363	0.0384	0.0409	0.0348	0.0365	0.0383
JAPAN TECHNOLOGY	-0.0028	0.0040	0.0087	0.0075	0.0061	0.0022	0.0027	0.0037
JAPAN TELECOM	0.0300	0.0349	0.0410	0.0433	0.0461	0.0380	0.0393	0.0408
JAPAN UTILITIES	0.0691	0.0638	0.0962	0.1004	0.1050	0.0740	0.0762	0.0785
Absolute Position Range	0.1540	0.1552	0.1728	0.1749	0.1782	0.1575	0.1612	0.1663

Appendix 6.4.1 (continued)

	CVaR					
	Normal Distribution			t-Distribution		
Panel B:	99%	95%	90%	99%	95%	90%
UK BASIC MATS	-0.0231	-0.0241	-0.0240	-0.0219	-0.0227	-0.0233
UK CONSUMER GDS	0.0218	0.0224	0.0227	0.0209	0.0215	0.0219
UK CONSUMER SVS	-0.0480	-0.0489	-0.0516	-0.0456	-0.0471	-0.0484
UK FINANCIALS	-0.0158	-0.0171	-0.0183	-0.0138	-0.0149	-0.0160
UK HEALTH CARE	-0.0109	-0.0124	-0.0141	-0.0090	-0.0103	-0.0113
UK TECHNOLOGY	0.0100	0.0103	0.0109	0.0095	0.0098	0.0101
UK INDUSTRIALS	0.0401	0.0405	0.0409	0.0393	0.0397	0.0403
UK OIL & GAS	0.0307	0.0314	0.0331	0.0299	0.0305	0.0309
UK TELECOM	0.0284	0.0293	0.0295	0.0275	0.0281	0.0285
UK UTILITIES	0.0170	0.0159	0.0146	0.0186	0.0176	0.0167
USA BASIC MATS	-0.0146	-0.0156	-0.0171	-0.0131	-0.0141	-0.0148
USA CONSUMER GDS	0.0445	0.0450	0.0446	0.0442	0.0444	0.0446
USA CONSUMER SVS	0.0678	0.0662	0.0680	0.0681	0.0679	0.0679
USA FINANCIALS	0.0928	0.0918	0.0907	0.0941	0.0932	0.0925
USA HEALTH CARE	0.1006	0.0997	0.1000	0.1012	0.1008	0.1006
USA INDUSTRIALS	0.0385	0.0379	0.0376	0.0396	0.0388	0.0384
USA OIL & GAS	0.1002	0.1004	0.0995	0.0996	0.0999	0.1002
USA TECHNOLOGY	0.1115	0.1121	0.1129	0.1107	0.1112	0.1116
USA TELECOM	0.0563	0.0572	0.0571	0.0554	0.0561	0.0564
USA UTILITIES	0.0889	0.0896	0.0905	0.0879	0.0886	0.0891
JAPAN BASIC MATS	-0.0141	-0.0148	-0.0152	-0.0131	-0.0136	-0.0142
JAPAN CONSUMER GDS	-0.0120	-0.0137	-0.0149	-0.0103	-0.0115	-0.0123
JAPAN CONSUMER SVS	0.0406	0.0422	0.0441	0.0387	0.0398	0.0408
JAPAN FINANCIALS	0.0479	0.0486	0.0489	0.0472	0.0477	0.0480
JAPAN HEALTH CARE	0.0451	0.0463	0.0485	0.0430	0.0444	0.0455
JAPAN INDUSTRIALS	0.0042	0.0044	0.0041	0.0043	0.0040	0.0041
JAPAN OIL & GAS	0.0356	0.0366	0.0372	0.0342	0.0352	0.0358
JAPAN TECHNOLOGY	0.0026	0.0026	0.0029	0.0022	0.0022	0.0027
JAPAN TELECOM	0.0386	0.0396	0.0403	0.0375	0.0383	0.0388
JAPAN UTILITIES	0.0749	0.0764	0.0768	0.0732	0.0744	0.0752
Absolute Position Range	0.1595	0.1610	0.1644	0.1563	0.1583	0.1600

Appendix 6.4.2 Standard Deviation of Weights in the Out-of-sample Risk-Adjusted Unconstrained BL Portfolio (Sep 03 – May 10)

This table reports standard deviation of weights assigned in each index in the period from September 2003 to May 2010, with assumptions of normal distribution ('N') and t-distribution ('t') at confidence levels of 99%, 95% and 90%. Note that weights in the unconstrained variance-adjusted implied BL portfolio are calculated by $\mathbf{w}^*_{BL,t} = \frac{1}{\delta_t}\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}$, the variance-adjusted SR-BL portfolio allocates asset to achieve the maximal Sharpe ratio in the optimisation problem, weights can be calculated by $\mathbf{w}^*_{BL,t} = \frac{\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}{\mathbf{1}'\mathbf{V}_t^{-1}\boldsymbol{\mu}_{BL,t}}$, weights in the VaR-adjusted and the CVaR-adjusted BL portfolio are allocated by solving maximal Sharpe ratio optimisation problem.

			VaR					
	Variance		Normal Distribution			t-Distribution		
Panel A:	Implied BL	SR-BL	99%	95%	90%	99%	95%	90%
UK BASIC MATS	0.0836	0.0926	0.1349	0.1396	0.1459	0.0941	0.0989	0.1041
UK CONSUMER GDS	0.0864	0.0950	0.1052	0.1118	0.1196	0.1064	0.1117	0.1178
UK CONSUMER SVS	0.1091	0.1174	0.1687	0.1775	0.1856	0.1303	0.1368	0.1449
UK FINANCIALS	0.0935	0.0986	0.1364	0.1427	0.1500	0.1060	0.1110	0.1176
UK HEALTH CARE	0.1512	0.1625	0.1789	0.1889	0.2012	0.1782	0.1867	0.1954
UK TECHNOLOGY	0.0540	0.0613	0.0772	0.0816	0.0866	0.0682	0.0719	0.0763
UK INDUSTRIALS	0.0843	0.0930	0.1359	0.1433	0.1508	0.1061	0.1119	0.1189
UK OIL & GAS	0.1100	0.1205	0.1898	0.1973	0.2062	0.1336	0.1402	0.1476
UK TELECOM	0.0891	0.0979	0.1273	0.1345	0.1421	0.1094	0.1151	0.1220
UK UTILITIES	0.1470	0.1657	0.2090	0.2189	0.2306	0.1809	0.1906	0.2009
USA BASIC MATS	0.1018	0.1147	0.1404	0.1468	0.1546	0.1205	0.1264	0.1341
USA CONSUMER GDS	0.1212	0.1292	0.1638	0.1717	0.1834	0.1469	0.1537	0.1612
USA CONSUMER SVS	0.1258	0.1332	0.1801	0.1899	0.2006	0.1516	0.1597	0.1677
USA FINANCIALS	0.1137	0.1171	0.1770	0.1838	0.1918	0.1262	0.1315	0.1376
USA HEALTH CARE	0.1769	0.1871	0.2532	0.2668	0.2812	0.2073	0.2174	0.2262
USA INDUSTRIALS	0.1185	0.1199	0.1493	0.1563	0.1645	0.1339	0.1403	0.1481
USA OIL & GAS	0.1128	0.1280	0.1830	0.1897	0.1985	0.1358	0.1425	0.1501
USA TECHNOLOGY	0.0717	0.0899	0.1255	0.1307	0.1373	0.0981	0.1044	0.1122
USA TELECOM	0.1031	0.1149	0.1559	0.1636	0.1729	0.1287	0.1354	0.1431
USA UTILITIES	0.1117	0.1271	0.1645	0.1726	0.1833	0.1405	0.1484	0.1572
JAPAN BASIC MATS	0.0960	0.1107	0.1414	0.1488	0.1562	0.1196	0.1262	0.1340
JAPAN CONSUMER GDS	0.1147	0.1251	0.1412	0.1510	0.1628	0.1403	0.1471	0.1544
JAPAN CONSUMER SVS	0.1337	0.1490	0.1808	0.1917	0.2035	0.1667	0.1754	0.1843
JAPAN FINANCIALS	0.0763	0.0878	0.1253	0.1317	0.1388	0.0960	0.1010	0.1065
JAPAN HEALTH CARE	0.1340	0.1502	0.1650	0.1750	0.1865	0.1647	0.1730	0.1822
JAPAN INDUSTRIALS	0.1023	0.1154	0.1496	0.1566	0.1641	0.1250	0.1311	0.1385
JAPAN OIL & GAS	0.0727	0.0794	0.0890	0.0945	0.1003	0.0874	0.0920	0.0972
JAPAN TECHNOLOGY	0.0734	0.0835	0.1230	0.1295	0.1367	0.0943	0.0993	0.1044
JAPAN TELECOM	0.0801	0.0891	0.1144	0.1209	0.1284	0.1018	0.1073	0.1131
JAPAN UTILITIES	0.1215	0.1397	0.1951	0.2054	0.2170	0.1560	0.1647	0.1746
Average Standard Deviation	0.1057	0.1165	0.1527	0.1604	0.1694	0.1285	0.1351	0.1424

Appendix 6.4.2 (continued)

	CVaR					
	Normal Distribution			t-Distribution		
Panel B:	99%	95%	90%	99%	95%	90%
UK BASIC MATS	0.0962	0.0993	0.1010	0.0925	0.0950	0.0968
UK CONSUMER GDS	0.1086	0.1123	0.1146	0.1043	0.1076	0.1093
UK CONSUMER SVS	0.1338	0.1370	0.1412	0.1280	0.1316	0.1347
UK FINANCIALS	0.1087	0.1116	0.1146	0.1045	0.1069	0.1093
UK HEALTH CARE	0.1820	0.1875	0.1900	0.1750	0.1799	0.1830
UK TECHNOLOGY	0.0698	0.0724	0.0738	0.0669	0.0689	0.0703
UK INDUSTRIALS	0.1094	0.1125	0.1156	0.1045	0.1072	0.1101
UK OIL & GAS	0.1368	0.1410	0.1447	0.1317	0.1349	0.1376
UK TELECOM	0.1121	0.1160	0.1184	0.1074	0.1105	0.1128
UK UTILITIES	0.1853	0.1915	0.1957	0.1778	0.1828	0.1865
USA BASIC MATS	0.1236	0.1270	0.1299	0.1186	0.1217	0.1244
USA CONSUMER GDS	0.1496	0.1549	0.1575	0.1445	0.1482	0.1504
USA CONSUMER SVS	0.1554	0.1621	0.1626	0.1490	0.1532	0.1564
USA FINANCIALS	0.1284	0.1323	0.1341	0.1244	0.1273	0.1292
USA HEALTH CARE	0.2114	0.2186	0.2221	0.2037	0.2094	0.2126
USA INDUSTRIALS	0.1371	0.1407	0.1437	0.1315	0.1352	0.1379
USA OIL & GAS	0.1396	0.1430	0.1454	0.1336	0.1372	0.1404
USA TECHNOLOGY	0.1013	0.1053	0.1086	0.0960	0.0993	0.1021
USA TELECOM	0.1316	0.1361	0.1392	0.1261	0.1301	0.1325
USA UTILITIES	0.1442	0.1491	0.1529	0.1378	0.1421	0.1451
JAPAN BASIC MATS	0.1230	0.1269	0.1303	0.1176	0.1208	0.1237
JAPAN CONSUMER GDS	0.1434	0.1481	0.1510	0.1380	0.1417	0.1443
JAPAN CONSUMER SVS	0.1704	0.1762	0.1804	0.1637	0.1683	0.1714
JAPAN FINANCIALS	0.0981	0.1017	0.1039	0.0943	0.0970	0.0988
JAPAN HEALTH CARE	0.1688	0.1734	0.1776	0.1619	0.1664	0.1699
JAPAN INDUSTRIALS	0.1278	0.1315	0.1351	0.1227	0.1264	0.1287
JAPAN OIL & GAS	0.0891	0.0924	0.0946	0.0855	0.0883	0.0897
JAPAN TECHNOLOGY	0.0959	0.0999	0.1018	0.0922	0.0952	0.0965
JAPAN TELECOM	0.1044	0.1080	0.1105	0.0998	0.1030	0.1051
JAPAN UTILITIES	0.1600	0.1655	0.1691	0.1531	0.1577	0.1610
Average Standard Deviation	0.1315	0.1358	0.1387	0.1262	0.1298	0.1324

CHAPTER 7 CONCLUSIONS

7.1 Conclusions

Overall, based on the in-sample and out-of-sample analyses, the dynamic BL portfolios starting from a conditional estimation of the equilibrium returns combined with the view portfolios generated from dynamic momentum strategies based on three volatility models outperform the benchmark portfolio for multiple periods and for some single periods. In addition, the dynamic BL portfolios demonstrate the superiority of the models employed over the traditional mean-variance model through more balanced and reasonable weights solutions. Specifically, dynamic BL portfolios using the DCC model always give the best in-sample and out-of-sample performance, better than the EWMA and RW models for multiple periods. For a single period, some single-period performance indeed show which asset allocation model could generate better performance. However, the conclusion, that which volatility model should be selected to show best single-period performance, is not robust. Studying the multiple-period performance combined with the single period performance is important to get a thorough overlook of the dynamic BL portfolio performance and the effect of the choice of volatility models, distribution assumptions and confidence levels, and eventually get conclusions that are more reliable.

In this thesis, the dynamic BL portfolio contains the implied BL portfolio formed by the implied reverse optimisation of the BL model, the SR-BL portfolio with maximal Sharpe ratio, the MVaR-BL portfolio with maximal reward to VaR ratio, the MCVaR-BL portfolio with maximal reward to CVaR ratio, the VaR-constrained BL portfolio, and the CVaR-constrained BL portfolio. In each portfolio, three volatility models are used to estimate the variances and covariances. Following the method of Giamouridis and Vrontos (2007) for the out-of-sample analysis, four risk-adjusted BL portfolios, including implied the variance-adjusted BL portfolio, variance-adjusted SR-BL portfolio, VaR-adjusted SR-BL portfolio, and CVaR-adjusted SR-BL portfolio, form the dynamic BL portfolios. The single-period and multiple-period performances of these dynamic BL portfolios through in-sample and out-of-sample analyses are compared within this thesis. Different performance measures give different ranks to these dynamic BL portfolios. The thesis also discusses the effect of the choice of volatility model, distribution assumptions, confidence levels, constraints on these dynamic portfolios on weights solutions, and portfolio performance.

Implied BL Portfolio and SR-BL Portfolio

In both the in-sample and out-of-sample analyses, the implied BL portfolio and the SR-BL portfolio alternate to give the best performance when evaluating the single period performance, while the performance of the implied BL portfolio is always better than that of the SR-BL portfolio for multiple periods. The SR-BL portfolio has a bigger empirical VaR and empirical CVaR for multiple periods, with tail risks also reflect in the negative skewness and high kurtosis. The ranking for the risk-adjusted portfolio performance and active portfolio performance is inconsistent following the use of different volatility models. For a single period, the implied BL portfolio and the SR-BL portfolio have different weight solutions but the same directions for long or short assets. Most of the average value of weight over the full sample and the out of sample are positive; the average absolute position range seems insensitive to the choice of volatility models and the choice of asset allocation model over the full sample. Over the out of sample, the implied DCC-BL portfolio has the widest average absolute position range and most volatile weight solutions.

MVaR-BL Portfolio and MCVaR-BL Portfolio

In the in-sample analysis, both the MVaR-BL and MCVaR-BL portfolios outperform the implied BL and the SR-BL portfolios for a single period and multiple periods at a moderate level of confidence; however, they only perform better than the SR-BL portfolio in the out-of-sample analysis. Although performance evaluation for some single periods is unable to demonstrate that the MCVaR-BL portfolio perform overwhelmingly better than the MVaR-BL portfolio, there is some evidence for multiple periods that the MCVaR-BL portfolio outperform the MVaR-BL portfolio, especially with a t-distribution and at a confidence level of 99%. Both the MVaR-BL portfolio and the MCVaR-BL portfolio perform better with a t-distribution than a normal distribution based on the DCC and EWMA models in the in-sample analysis, although this positive effect of a t-distribution is only significant in the out-of-sample analysis when employing the DCC model. The ranking of the risk-adjusted portfolio performance and the active portfolio performance is inconsistent and dependent upon the volatility model utilised. Over the full sample, changing from a normal distribution to a t-distribution widen the average absolute position range and increase the volatility of weight solutions in the MVaR-BL and the MCVaR-BL portfolios based upon the DCC model.

VaR-Constrained BL Portfolio

In both the in-sample and out-of-sample analyses, the main finding is that adding a moderate level of the VaR constraint to the SR-BL portfolio improves the performance of the SR-BL portfolio for both a single period and multiple periods. The VaR-constrained BL portfolio even outperforms the implied BL, MVaR-BL and MCVaR-BL portfolios for a single period and multiple periods in the in-sample analysis and in the out-of-sample analysis. The risk-adjusted performance of the DCC-VaR-BL portfolio is better than that of the MVaR-BL and MCVaR-BL portfolios under some circumstances, but is not a better choice for active portfolio performance. Although the implied BL portfolio has the best performance for multiple periods, better than that of the VaR-constrained BL portfolio based on the DCC model, the use of the EWMA and RW110 models in the VaR-constrained BL portfolio is better than the corresponding implied BL portfolio in the out-of-sample analysis. According to the study of the effect of distribution assumptions and confidence levels on the DCC-VaR-BL portfolio through in-sample analysis and out-of-sample analysis, it can be concluded that any element that gives rise to tighter VaR constraints could improve performance, until the diminishing effect happens. The 'diminishing effect' on improving the multiple-period performance indicates that at first it improves with tighter limits but then deteriorates as the limits begin to be too tight. In the out-of-sample analysis, the performance of the RW110-VaR-BL with the t-distribution is impressive and much better than the performance of the RW50-VaR-BL with the t-distribution. Over the full sample, changing from a normal distribution to a t-distribution widens the average absolute position range and increases the volatility of weight solutions in the VaR-BL portfolio using three volatility models.

CVaR-Constrained BL Portfolio

In both the in-sample and out-of-sample analyses, the CVaR constrained BL portfolio also exhibites an attractive performance for a single period and multiple periods, thereby supporting the argument that imposing an intermediate level of the CVaR constraint could enhance the performance of the SR-BL portfolio. Similar to the VaR-constrained BL portfolio, most of findings for the CVaR-constrained BL portfolio are consistent. Furthermore, several CVaR-constrained BL portfolios demonstrate an even better performance than the VaR-constrained BL portfolio for a single period and multiple periods at an intermediate level of the CVaR constraint. Moreover, over the full sample, similar to the VaR-BL portfolio, changing from a normal distribution to a t-distribution could also widen the average absolute position range and increase the volatility of weight solutions in the CVaR-BL portfolio using three volatility models. For the t-distribution, the CVaR-BL portfolio has wider average absolute position range and more volatile weight solutions than the VaR-BL portfolio.

Risk-Adjusted BL Portfolio

The estimated expected returns in the risk-adjusted BL portfolio are smaller than the BL portfolio due to the much lower value of the risk aversion coefficients. The out-of-sample analysis finds that the reverse optimisation used in the BL model is invalid when the VaR-adjusted and the CVaR-adjusted expected returns are used and the weights solutions are unrealistic. Use of the maximal Sharpe ratio optimiser addresses this problem when constructing the reasonable VaR-adjusted SR-BL and CVaR-adjusted SR-BL portfolios.

The out-of-sample risk-adjusted BL portfolio shows a much superior single-period performance, significantly better than for any unconstrained BL portfolio and risk-constrained BL portfolio. In addition, both the VaR-adjusted BL portfolio and the CVaR-adjusted BL portfolio perform better than the variance-adjusted BL portfolio. The CVaR-adjusted BL portfolio outperforms the VaR-adjusted BL portfolio under certain circumstances, but effects of the distribution assumption and confidence levels are inconsistent for the VaR-adjusted BL and the CVaR-adjusted BL portfolios.

For multiple periods the implied variance-adjusted BL portfolio demonstrates the best risk-adjusted performance and active portfolio performance of the risk-adjusted BL portfolios. The implied variance-adjusted BL portfolio outperforms all of the unconstrained BL portfolios and the risk-constrained BL portfolios except for the implied DCC-BL portfolio. The risk-adjusted performances of both the VaR-adjusted BL portfolio and the CVaR-adjusted BL portfolio are better than most of the unconstrained BL portfolios, but the active performance is worse than that of the MVaR-BL and MCVaR-BL portfolios. In addition, the VaR-adjusted BL portfolio and the CVaR-adjusted BL portfolio have only a limited ability to outperform the VaR-constrained BL portfolio and CVaR-constrained BL portfolio for the t-distribution at an intermediate level of constraints.

In the VaR-adjusted SR-BL portfolio and CVaR-adjusted SR-BL portfolio, the position range becomes wider as the confidence level decreased and the position range for the t-distribution is slightly narrower than for the normal distribution. In addition, the weight solutions become more volatile as the confidence level decreased. And the average standard deviation of weight solutions for the t-distribution is smaller than for the normal distribution. Compared with the VaR-adjusted SR-BL portfolio for the same distribution level of confidence, the average standard deviation of weight solutions of the CVaR-adjusted SR-BL portfolio is smaller. The variance-adjusted SR-BL portfolio allocates asset more volatile than the implied variance-adjusted BL portfolio does.

7.2 Limitations

There are several limitations to this research, and the first is that this thesis only choose three volatility models with which to conduct the dynamic asset allocation research. Although the dynamic BL portfolios based on the DCC model perform the best, this does not guarantee that the DCC model would be better than other volatility models in achieving the best performance. In addition, the window length in the rolling window estimator may have affected the estimation of the covariance matrix, and this thesis does not include a sensitivity test of window length on the dynamic BL portfolio performance.

The parameters in the BL model, such as scale τ and risk aversion coefficient δ, may also affect the equilibrium returns as the starting point and further affect the performance of the dynamic BL portfolio, and this research does not investigate these possible effects.

Next, this thesis does not impose trading restrictions on the dynamic BL portfolio, for example, some financial institutions might not allow short selling and long only constraints need to be added. In addition, Lejeune (2011b) deals with further trading restrictions in the VaR constrained BL model, including the cardinality constraint, round-lot constraints, and buy-in threshold constraints.

Moreover, the asset class in this thesis is limited to the industry indices. Other asset classes such as bonds, currencies are not within the scope of research. However, in the industry, some financial institutions need to invest in multiple asset classes to satisfy the investor's requirement.

Finally, this research estimates VaR and CVaR using the parametric method without including a forecast performance evaluation and estimation errors might affect the optimisation process for the dynamic BL portfolios.

7.3 Future Research

The methodology and findings of the thesis suggest some directions for future research. First, the multi-period performance of the MVaR-BL and MCVaR-BL portfolios are found to be superior at the cost of higher kurtosis and greater empirical VaR and CVaR in the in-sample analysis. Future research could address this problem by adding risk constraints to these portfolios.

Another future research direction would be to change the maximal performance measures, for example, Biglova et al. (2004) suggest using new performance measures, including the Rachev ratio and Rachev generalised ratio to maximise the portfolio optimisation in order to give the best performance. In addition, Rachev et al. (2007) utilise the reward to CVaR ratio and Rachev ratio to form a momentum portfolio, and they determine that the Rachev ratio could result in the best risk-adjusted performance, thereby confirming the advantage of using the Rachev ratio. Therefore, the future research could employ Rachev ratio to evaluate the dynamic BL portfolio performance.

The BL model could be utilsed as a useful tool in tactical asset allocation with investors' views inputted. This thesis has found some evidence of a better active performance by the dynamic BL portfolio; however, the tracking error variance needs to be minimised in the active portfolio management framework. Adding tracking error constraints to improve the active portfolio performance following the method of Palomba (2008) would be one direction for future research.

Additionally, by expanding the choice of volatility models, the future research could study the effect of the application of other complicated volatility models such as multivariate stochastic volatility models (Harvey et al., 1994) and long memory volatility models (Harris and Nguyen, 2013) on the dynamic BL portfolio performance.

Finally, as discussed in previous section, the thesis focuses only on industry indices as assets; however, Black and Litterman (1992) have show the example that using the Black-Litterman model to construct the global portfolio with equities, bonds and currencies is feasible, even generate weights that are more reasonable. Therefore, future research could make attempt to apply the proposed dynamic BL model in global asset allocation with multiple asset classes such as bonds, currencies and options. It should be noted that, when imposing the VaR constraint on the asset allocation model, using the parametric method for the estimation of the VaR of the portfolio with options would be inappropriate because of its non-linear feature; non-parametric method such as Monte Carlo simulation method could be used.

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