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HYBRID MODELLING AND SIMULATION (M&S): DRIVING INNOVATION IN THE THEORY AND PRACTICE OF M&S

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ABSTRACT

Hybrid Simulation (HS) is the application of two or more simulation techniques (e.g., ABS, DES, SD) in a single M&S study. Distinct from HS, Hybrid Modelling (HM) is defined as the combined application of simulation approaches (including HS) with methods and techniques from the broader OR/MS literature and also across disciplines. In this paper, we expand on the unified conceptual representation and classification of hybrid M&S, which includes both HS (Model Types A-C), hybrid OR/MS models (D, D.1) and cross-disciplinary hybrid models (Type E), and assess their innovation potential. We argue that model types associated with HM (D, D.1, E), with its focus on OR/MS and cross-disciplinary research, are particularly well-placed in driving innovation in the theory and practice of M&S. Application of these innovative HM methodologies will lead to innovation in the application space as new approaches in stakeholder engagement, conceptual modelling, system representation, V&V, experimentation, etc. are identified.

1 INTRODUCTION

Innovation is enabled by the interplay between individuals, and the expression of their ideas (Schrage, 1999). To make these ideas actionable, they must be externalized in a representation that stimulates thought and conversation, and creates opportunity; hence, M&S can play a role. In fact, M&S studies have been used to understand how innovation emerges/diffuses (e.g. Kiesling et al. 2012) and as a driver for innovations (e.g. Roukouni et al. 2019, Anderson et al. 2017). Roukouni et al. (2019) used simulation games to improve understanding of complex trade-offs in the transport and logistics sector, and to improve the long-term success of innovations. In the context of Serious Games, Anderson et al. (2017) addressed the interface design for an agent-based model in the domain of global food security to enable effective decision-making among a group of users. These are cross-disciplinary approaches to enabling open innovation, where ideas can come from inside or outside the organization (Tate et al. 2018). Obstacles to open innovation also exist where the need or opportunity is known but resources have not yet been committed to possible solutions. M&S can also effectively support this phase, for example Jia et al. (2016) used M&S to show that 3D printing technology could be a disruptive innovation to chocolate manufacturers.

To maximize the contribution of M&S to active learning and creating opportunities to innovate, methodologies which combine methods and theories are required. However, these methodologies themselves can drive innovation. Hybrid M&S studies which combine simulation with hard or soft Operational Research/Management Science (OR/MS) methods and techniques, and cross-disciplinary M&S studies which combine simulation with methods from other disciplines, are uniquely placed to contribute to the theory and practice of M&S. In this paper, we expand on the conceptual representation for hybrid M&S (Mustafee and Powell, 2018) and introduce a new Model Type (Section 2). Sections 3-5 present examples of the six hybrid model types from literature, and section 6 articulates their innovation potential, both to the theory and application of M&S and also innovation in application space. Section 7 concludes the paper.

2 A UNIFIED CONCEPTUAL REPRESENTATION & CLASSIFICATION OF HYBRID M&S

In a world in flux, there is a constant drive for innovation and a growing commitment to open innovation in all sectors. Open innovation acknowledges that collaborative relationships with stakeholders and partners is needed to access valuable knowledge from a range of external resources. However, surfacing good ideas, and deciding which ideas to pursue, remains a challenge for many organizations (Tate et al. 2018). M&S can effectively support this phase, and hybrid methods enhance the opportunity for engagement with teams who have recourse to knowledge constructs that have not traditionally been applied to M&S studies.

Hybrid M&S supports the best possible representation and analysis of the system under scrutiny, and adds further value to both the conventional and the HS studies and its application to practice. In the paper, *"From Hybrid Simulation to Hybrid Systems Modelling"*, Mustafee and Powell (2018) present a unified conceptual representation of hybrid simulation (HS) and hybrid models (HM), and use this to categorize models into distinct Model Types. Figure 1 illustrates the unified representation of hybrid M&S with the addition of cross-disciplinary HM (Model Type E). Figure 1 terminologies are described in Table 1. In this unified representation, HS are Model Types A, B and C. Most researchers agree that the uptake of HS has increased with the availability of simulation packages, such as AnyLogic, which aid the development of such mixed models in a single modelling environment.

Model Types D, D1 and E are referred to as HM (rather than HS) since only one constituent of the combined model is a computer simulation, and which is used in conjunction with other established methods and techniques in Hard OR, e.g., forecasting and game theory (Type D); soft OR, e.g., SSM and QSD (Type D.1); or cross-disciplinary methods and research approaches, e.g., using formal verification techniques for testing models, use of ethnographic methods for the development of behavioural models, experimental and field-studies (including RCT) to test the efficacy of solutions through simulation experimentation, integrating the concepts of design research into the life cycle of a simulation study and the use of Grid/Cloud computing and Parallel and Distributed Simulation (PADS) with traditional simulation models (Type E).

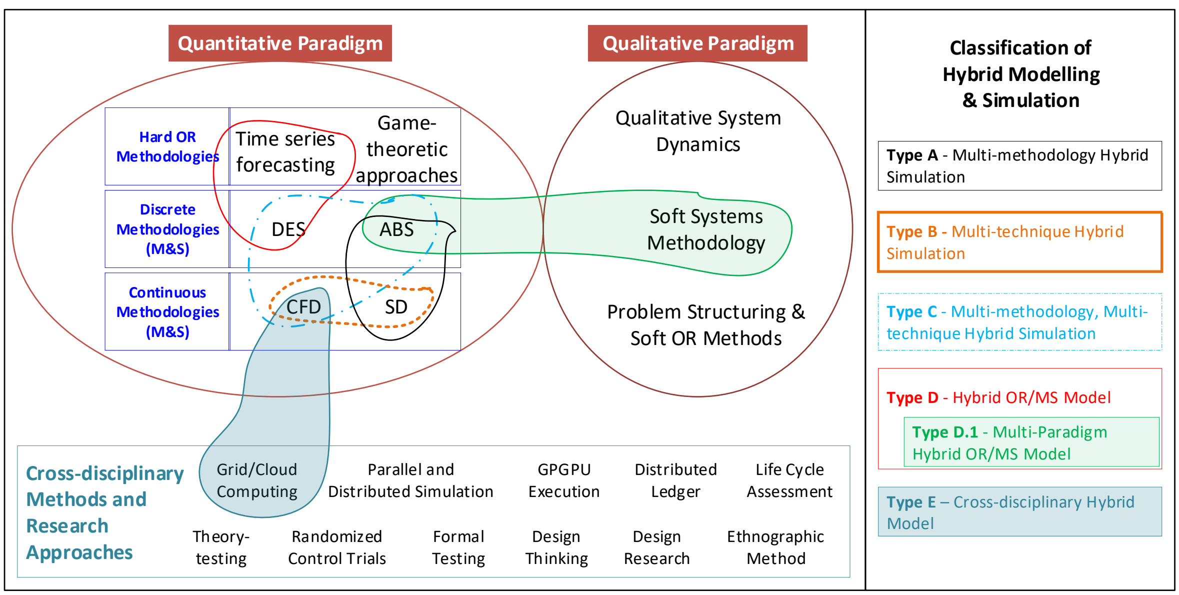

Figure 1: The unified conceptual representation of Hybrid M&S (left) with classification of Model Types (right). Adapted from Mustafee and Powell (2018). Refer to Table 1 for explanation of the terminologies.

The rational of introducing a new model type is as follows: **(a)** In their original paper, all the model types were referred to as HS, with types D and D1 labelled as hybrid systems model, or HSM. It was felt that the addition of the word "systems" (to mean, operational systems) adequately identified the use of such models in OR/MS context. However, since the publication of the original article, it has become apparent that a change in terminology is necessary to distinguish HS studies from those that have used simulation in conjunction with a range of other OR/MS and cross-disciplinary methods (HM studies); **(b)** the focus of the original paper was on OR/MS, and therefore the model types were restricted to analytical models (Hard/Soft OR models including simulation). As a result, the representation of hybridity that incorporated wider methods were ignored. For example, Powell and Mustafee (2017) identified the used of applied computing approaches such as parallel and distributed simulation (PADS), Grid/Cloud-based execution and GPGPU as a means of speeding up simulation execution. Such models cannot be classified using the framework presented in Mustafee and Powell (2018). Based on the aforementioned discussion, Type D model has been relabelled as **Hybrid OR/MS Model** and, in a similar vein, Type D.1 as **Multi-Paradigm Hybrid OR/MS Model**. A new model type has been introduced, and which we refer to as **Type E Cross-disciplinary Hybrid Model**. Type E model encompasses the original definition of hybridity and the use of cross-disciplinary techniques, beyond just OR/MS, in multiple stages of a M&S study (Powell and Mustafee, 2016). The extension of the classification scheme is consistent with the intention of the authors. *"A classification scheme also has the benefit of being extensible, thus allowing the accommodation of new types of hybrid models that may be realized in the future"* (Mustafee and Powell, 2018).

The unified HS-HM representation was developed using the vocabulary of OR/MS, and towards this, the authors revisited Mingers and Brocklesby's (1997) definitions of paradigm, methodology, technique and tool (Table 1). These definitions were adapted to make them relevant to the discussion on HS and HM. It was also felt that the consistent use of these terms would allow the M&S community to better engage with the classification scheme. Prior to the discussion on paradigms and techniques, we must first define what we mean by hybrid simulation (Model Type A, B, C), hybrid OR/MS model (Model Type D and D.1), cross-disciplinary hybrid model (Model Type E) and hybrid M&S study.

- **Hybrid Simulation (HS)** is the application of two or more simulation techniques (e.g., ABS, DES, SD) in a single M&S study. This is our **Model Type A, B and C** (Figure 1).
- The combined application of a single simulation technique (or, indeed, hybrid simulation) with methods and techniques from the wider OR/MS literature is termed as **Hybrid OR/MS Modelling**. This maps to **Type D** (for Hard OR) and **Type D.1** (for soft OR) in Figure 1.
- Combining simulation approaches with established methods and techniques from disciplines such as Applied Computing, Economics, Systems Engineering and Arts and Humanities, is termed as **Cross-disciplinary Hybrid Modelling**. In conjunction with simulation techniques, it is expected that the CD-type hybrid models will employ methods that span across disciplines and are frequently multi-, inter- and transdisciplinary in nature. This is our **Type E model** (Figure 1). In the remainder of the paper, we use the term hybrid modelling or HM to refer to both Hybrid OR/MS Models (Type D, D.1) and Cross-disciplinary Hybrid Models (Type E), unless we need to specifically refer to either type.
- In the context of HM, the conjoined application of simulation with OR/MS and cross-disciplinary methods is relevant not only in the model development/implementation stage of a M&S study (as is commonly the case with HS), but could be applied to other stages in the lifecycle, e.g.,, conceptual modelling, input and output data analysis, V&V, scenario development, experimentation, engaging with the stakeholders in the implementation of the results of a study, model formalization and documentation. In such cases, a conventional M&S study is transformed into a **Hybrid M&S study** with a focus on multiple complementary approaches which could be applied to one or more stages in its lifecycle.

Table 1: Terminologies associated with the Unified Conceptual Representation and Classification of Hybrid M&S (also refer to Figure 1).

Terminology	Description	Alignment with Hybrid M&S **Model Type**
Paradigm	*Soft OR* is qualitative; *Hard OR* is quantitative.	**Type D.1 Multi-paradigm Hybrid OR/MS Models** combine Soft and Hard OR.

Methodology	Methodologies develop within a paradigm and usually embody its philosophical assumptions (Mingers and Brocklesby, 1997). Specific to M&S, in the quantitative paradigm, we distinguish between discrete and continuous methodologies. In addition to simulation, there are a plethora of quantitative methods in OR/MS, and these were represented in their conceptual representation as (other) "Hard OR Methods".	In discrete execution, the system state changes from one event to the next (bound events, conditional events, time-steps). For continuous simulation, the change in system state is continuous (as with SD). In the context of M&S**:** • **Type A Multi-methodology Hybrid Simulation** is one which has both Discrete and Continuous elements, e.g., SD-DES, SD-ABS. • The multi-methodology approach to hybrid simulation also contributes to Type C Hybrid Simulation (see below). The combined application of M&S with (other) Hard OR methods is an example of **Type D Hybrid OR/MS Model** (forecasting & DES).
Technique	As techniques exist within the context of methodologies, Mustafee and Powell (2018) distinguish techniques such as DES (event list/queuing theory) and ABS (time stepped/emergence) under discrete methodology, and SD (stock and flow) and Computational Fluid Dynamics (numerical approach) under continuous methodology. Some broad categories of techniques are also identified under "Hard OR", e.g., time-series forecasting and game-theoretic approaches.	A **Type B Multi-technique Hybrid Simulation** is one which uses two or more techniques under the same methodology (e.g., SD-CFD, DES-ABS). A **Type C Multi-methodology, Multi-technique Hybrid Simulation** is one which uses a combination of techniques from both discrete and continuous approaches, with at least two techniques from either of the two underlying methodologies, e.g., studies that employ SD-DES-ABS. In the qualitative paradigm, Mustafee and Powell (2018) listed Soft Systems Methodology and Qualitative System Dynamics (there are several other structured problem solving techniques), and the combined application of such Soft OR techniques with hard numerical simulations is an example of Type D.1.
Tool	These were defined as M&S packages and other software artefacts for model development.	Discussion of the tool (commercial, bespoke and open-source) was not important for the purposes of the classification scheme.
Cross-Disciplinary (CD) Approaches	The combined application of simulation with methodologies, techniques and research approaches from disciplines that are wider than OR/MS, for example, Arts and Humanities, Computer Science, Engineering and Economics, and which leads to cross-disciplinary research.	The realisation of **Type E Cross-disciplinary Hybrid Model** generally requires expertise that go beyond our field of research (i.e., M&S and OR/MS), and it is therefore imperative that we engage with scholars from other scientific disciplines. For example, Cloud-based execution of CFD simulations; distributed simulation of ABS models; using theoretical constructs to model agent relationships (e.g., Theory of Planned Behaviour).

3 HYBRID SIMULATION MODELS (MODEL TYPES A, B AND C)

Type A Multi-Methodology HS typically focus on the combined use of DES/SD/ABS. With DES/ABS being discrete method and SD continuous method, the definition has gradually gravitated towards the mixed application of simulations developed using the two modelling methods. Thus, DES-SD and ABS–SD qualify as HS. However, Type A models can also include simulation techniques like Monte Carlo, mesoscopic modelling, which places itself between continuous and discrete methods (Reggelin and Tolujew 2011), Petri Nets, and Computational Fluid Dynamics (CFD). Hence, the defining characteristic of Type A models will be the combined application of techniques in which one belongs to discrete method and the other belongs to continuous method. Examples of Type A models are listed in Table 2.

Table 2: Examples of Type A - Multi-Methodology Hybrid Simulation.

Type A		Description with emphasis on use of M&S methods	Reference
Discrete – Continuous with SD	ABM -SD	An integrated ABM-SD model is developed to better understand behavioural diversities associated with multi-type labourers in multinational projects, revealing the associated impacts and improving project management. ABS is used to model the behaviour of the labourers and estimate their performance, and the SD model uses this data to summarize these individual performance and evaluate the deviation in the timelines of the project **(Construction Planning)**	Wu et al. (2019)
	ABM -SD	The authors developed a hybrid model to estimate the market share evolution of electric vehicles (EVs). The agent-based discrete choice models of consumer choice and awareness are combined with macro-level SD elements that model the interdependencies between consumer choice, technology evolution and available infrastructure for EVs **(Transportation)**	Kieckhäfer et al. (2014)
	SD-DES	A hybrid model is developed to analyse 'schedule risk' in infrastructural projects. DES is used to model construction process, resource usage and other micro variables, whereas the SD element of the hybrid model represents the feedbacks associated with work allocation, rework, etc. and provides the systems perspective **(Construction Planning)**	Xu et al., (2018)
	SD-DES	The authors investigate total productive maintenance using SD-DES HS. The problem being modelled involved both maintenance scheduling (DES) and considerations for human factors such as attitude (SD). **(Maintenance)**	Oleghe and Salonitis (2019)
Discrete – Continuous with CFD	ABM -CFD	To demonstrate the feasibility of a hybrid approach for evacuation planning, the authors model the hypothetical case of toxic aerosol release in downtown LA (using CFD), and simulate the response of large spatially-distributed agent population (ABS). **(Evacuation Planning)**	Epstein et al. (2011)
	ABM -CFD	The authors developed a CFD-ABM framework to capture the interaction between the dynamics of blood flow (hemodynamic) in a 3D artery model (CFD simulation) and arterial wall remodelling in atherosclerosis (a disease in which plaque builds up insider arteries). The CFD provides hemodynamic input to 2D ABMs that simulate cellular dynamics **(Biological Modelling)**	Corti et al. (2020)
	DES-CFD	The authors propose a HS consisting of a DES that models the flow of materials through a production line (manufacturing system simulation) with a CFD simulation of compressed air system. This enables the combined evaluation of the aforementioned systems, with the overall objective of optimising energy consumption per unit of production. **(Manufacturing)**	Nagasawa et al. (2017)

Type B Multi-Technique HS employ two or more techniques from the same underlying M&S methodology (Table 1). There is some debate as to whether these could be called hybrid since both techniques conform to either discrete, or continuous method. In our classification, a combined application of ABS-DES and SD-DFD are Type B HS since there are fundamental differences in the execution of the simulation logic, and which makes them agreeable to model a particular category of problems (top-down queuing approach versus bottom-up emergence). Table 3 lists examples of Type B.

Table 3: Examples of Type B - Multi-Technique Hybrid Simulation.

Type B	Description with emphasis on use of M&S methods	Reference

Continuous - Continuous	SD-CFD	The authors present a CFD-SD HM which is applied to the study of transient thermal hydraulic (the study of gas pipeline pressures, temperatures, and flow rates) behaviour of nuclear power plant (note that the SD element is not very well explained). **(Engineering)**	Natesan and Velusamy (2019)
	SD-CFD	With the objective of investigating the effects of traffic volume and toll collection methods on dispersion of pollutants at a toll plaza, the authors used real data to developed a SD model to simulate vehicle movements with different traffic volumes, and a CFD model to simulate the dispersion of pollutants. **(Transportation)**	He et al. (2011)
Discrete-Discrete	DES-ABM	The authors present a case study related to the London emergency medical service in which they implement a Type B model. In their hybrid ABS-DES model, DES simulates the hospital processes, whereas ABS models the ambulance service. **(Healthcare).** Also refer to Table 7 (**Type E**).	Anagnostou and Taylor (2017)
	DES-ABM	The authors implemented a Type B hybrid ABS-DES model for the planning of capacity and patient flow in a post-term pregnancy outpatient clinic. The DES modelled the processes through the clinic, and the ABM modelled pregnant women as agents. **(Healthcare)**	Viana et al. (2020)

Type C Multi-Methodology, Multi-Technique HS is the combined application of three or more simulation techniques, of which the two methods (continuous and discrete) must be represented, for example DES-ABM-SD. Like Type A and Type B HS, we believe that the definition of Type C HS should not be restricted to particular techniques, which is presently the case with DES/ABM/SD being widely used (Brailsford et al., 2019), but instead, allow for the exploration of synergies that could be achieved through assessment of the wider range of M&S approaches (see discussion on Type A HS). Table 4 presents an example where CFD was used in conjunction with DES and ABM. It also list four other examples of Type C HS, where DES and ABM was combined with SD to realize the objective of the simulation study.

Table 4: Examples of Type C - Multi-Methodology, Multi-Technique Hybrid Simulation.

Type C	Description with emphasis on use of M&S methods	Reference
DES-ABM-CFD	The authors developed a hybrid model for evaluating countermeasures for chemical gas emergencies. The gas flow dynamics is modelled in CFD, human movement in ABS and an evacuation model in DEVS. **(Evacuation Planning)**	Seok et al., (2016)
DES-ABM-SD	The authors combined two discrete methods (DES and ABS) and one continuous method (SD) and applied this to an earthmoving operations case study. The DES models the process flow of the earthmoving operation; the trucks and drivers are modelled as agents; and SD was used to model agents' physiological processes and decision behaviors. **(Construction Planning)**	Goh and Ali (2016)
DES-ABM-SD	The authors developed an integrated DES-ABS-SD model to complement the standard lifecycle assessment (LCA) methodology. They validated the model using a case study of drink products (e.g., bottled water). SD was used to model the lifecycle of each beverage (e.g., bottled water production and bottles recycled), distribution and energy use; customer behavior was modelled in ABS. Although the authors claim to have used two discrete methods, there are no inherent queuing structures in the hybrid model. **(Environment)**	Wang et al. (2014)
DES-ABM-SD	A Type C model was developed for the assessment of innovative healthcare technologies, i.e. the evaluation of mobile stroke units and prostate cancer screening. DES was used to represent hospital processes and agents were generated from the SD component of the hybrid model. **(Healthcare)**	Djanatliev and German (2013)

DES-ABM-SD	The authors developed a hybrid model for energy efficiency analysis, using SD to depict energy demand of production processes, and DES/ABS to map the material flows and logistic processes, applied to mechanical processing of die-cast parts. DES provided meso-level workflow perspective and ABS modelled micro-level active processes. **(Manufacturing)**	Roemer and Strassburger (2019)

4 HYBRID OR/MS MODELS (MODEL TYPE D AND D.1)

We define Hybrid OR/MS Modelling as the combined application of a single simulation technique (or, indeed, HS) with methods and techniques from the wider OR/MS literature. Refer to Mustafee and Katsaliaki (2020) for a classification of OR methods and techniques, and which can be potentially used in conjunction with simulation approaches to realize hybrid OR/MS models.

- ***Type D Hybrid OR/MS Models*** combine computer simulation with Hard OR techniques such as forecasting, mathematical programming and optimization, meta-heuristics, game theory, graph theory, inventory models, Multiple-criteria decision-making (MCDM), Data Envelopment Analysis (DEA), process mining and Machine Learning. Table 5 presents examples of Type D models.
- ***Type D.1 Hybrid OR/MS Models*** combine computer simulation with Soft OR techniques such as Soft Systems Methodology (SSM), Qualitative System Dynamics (QSD) and cognitive mapping. Table 6 presents examples of Type D.1 models. Type D.1 bridges the qualitative and quantitative paradigm and should not be seen merely as a sub-set of the Type D model.

Table 5: Examples of Type D Hybrid OR/MS Models employing Hard OR methods.

Type D	**Description with emphasis on M&S and Hard OR**	**Reference**
Forecasting with DES	The authors used demographic projections and regression analysis to forecast demand for diagnostic services, as inputs into a DES to support long-term capacity planning. **(Healthcare)**	Harper et al. (2017)
Optimal Packing Problem with ABS	A hybrid model was developed to analyse trade-offs between loading efficiency (using Container Loading optimisation algorithms) and various important considerations in relation to the cargo, such as its stability, fragility or possible cross-contamination between different types of items over time (ABS). **(Transportation)**	Mustafee and Bischoff (2013)
Optimal coverage problem with ABS	The authors combine ABS and Optimisation model to find the location of wireless sensors that maximises security coverage. The use of ABS is innovative as it allows them to evaluate scenarios in which intruders are intelligent, i.e. they can learn from others. (**Security**)	Karatas and Onggo (2019)
Process Mining with DES	A hybrid model was developed that integrated process mining in the conceptual modelling phase to support the development of DES models. **(Healthcare)**	Abohamad et al. (2017)
Machine Learning with DES	The authors investigated a hybrid modelling approach that integrates simulation modelling with Machine Learning in an attempt to improve the validity of the simulation model outputs. **(Healthcare)**	Elbattah and Molloy (2016)
DES with data mining and GIS	This study combined simulation with data mining, optimisation and GIS-based analytics to model a blood supply chain with a high level of complexity. **(Supply Chain)**	Delen et al. (2011)
Neural Networks with SD	This work improved the conceptual model development process in SD by using neural network to provide modelers with several probable model structures to be considered for refinement. **(Methodology)**	Abdelbari and Shafi (2017)

Table 6: Examples of Type D1.1 Hybrid OR/MS Models employing Soft OR methods.

Type D.1	**Description with emphasis on M&S and Soft OR**	**Reference**

Group support with DES	The authors used a collaborative simulation approach combining group support with DES to enhance convergence of stakeholder viewpoints, acceptance of outcomes and model quality. **(Airline industry)**	den Hengst et al. (2007)
Lean with DES	The authors combined Lean methodology with DES to improve stakeholder engagement and impact through three use cases. **(Healthcare)**	Robinson et al. (2012)
Cognitive mapping with DES	Cognitive mapping was used to elicit staff perspectives to support DES experiments and guide DES model execution for enhancing surgical capacity. **(Healthcare)**	Pessôa et al. (2015)
SSM with DES	PartiSim, a multi-methodology framework to support participative simulation studies, combines DES with SSM to engage stakeholders in the study lifecycle through a set of stages and activities. **(Healthcare)**	Tako and Kotiadis (2015)
QSD with DES	The paper discusses the combined application of QSD with DES, to aid the understanding of the system in the problem formulation/conceptual modelling stage of a hybrid M&S study. **(Healthcare)**	Powell and Mustafee (2017)

5 CROSS-DISCIPLINARY HYBRID MODELS (MODEL TYPE E)

Distinct from Model Types D and Type D.1, which mainly focus on the use of simulation with broader OR/MS methods, ***Type E hybrid models*** necessitate cross-disciplinary engagement between researchers and practitioners from M&S and broader fields of study. From the perspective of our research community, exploration of the extant knowledge in disciplines such as Engineering and Computer Science, Data Science, Arts and Humanities, Medicine and Health Sciences, allow us to identify established research philosophies, methods, techniques and tools, and which could be deployed in conjunction with computer simulation in one or more stages of a M&S study. Table 7 presents some examples of Type E models.

Table 7: Examples of Type E Cross-disciplinary Hybrid Models.

Type E	**Description with emphasis on M&S and Cross-disciplinary methods**	**Reference**
DES with Grid Computing	Two hybrid models were implemented by interfacing a desktop-based grid middleware (WinGrid) with a DES software and a Monte Carlo simulation package, respectively (two case studies). The objective was to demonstrate faster execution of models using both dedicated and non-dedicated PCs. **(Automotive; Financial Services)**	Mustafee and Taylor (2009)
CFD and Cloud Computing	A Cloud-based simulation platform is presented that enables SMEs to use the Platform-as-a-Service solution to execute CFD simulations. **(Engineering)**	Taylor et al. (2018)
ABS-DES with Distributed Simulation	The authors present a distributed simulation framework for linking **Type B Discrete-Discrete models** (refer to Table 3). The framework is deployed in a case study related to the London emergency medical service. In their hybrid model, DES simulated the hospital processes, whereas ABS modelled the ambulance service. **(Healthcare)**	Anagnostou, and Taylor (2017)
DES with Distributed Simulation	The IEEE 1516 HLA standard and DMSO RTI1.3-NG was used to investigate the speed-up of DES models using Time Advance Request and Next Event Request. The DES model was on the supply chain of blood. The objective of this study was to enable faster execution of DES models using distributed simulation. **(Healthcare)**	Mustafee et al. (2009)
ABS with Parallel Computing	The authors developed ABS of population dynamics in demography. To make the simulation faster, they used parallel computing technique. The experiment was done on Marenostrum supercomputer. (**Demography**)	Montañola-Sales et al. (2016)
Symbiotic simulation (Real-time	Symbiotic simulation is a technology that enables interaction between physical systems and their digital twins. The authors propose a hybrid modelling architecture for symbiotic simulation which includes data	Onggo et al. (2018)

data, DES, Machine Learning)	acquisition to receive data from physical system, simulation/ optimisation/ML models, scenario manager and decision maker/ actuator that relays the results to the physical system. **(Methodology)**	
Real-time Simulation (Real-time data, Forecasting and DES)	The authors used *NHSquicker* real-time data on ED/MIU wait times for time-series forecasting, with the objective of triggering real-time DES experiments. The authors investigate proactive service recovery in ED. It is an example of both **Type D** Hybrid OR/MS Model (as it uses forecasting) and also **Type E** (as it employs real-time technologies that are commonly associated with Data Science). **(Healthcare)**	Harper and Mustafee (2019)

6 INNOVATION IN THE THEORY AND PRACTICE OF M&S THROUGH HYBRID M&S

The unified conceptual representation and classification of hybrid M&S identifies six distinct model types (Figure 1). Figure 2 illustrates increasing innovation in M&S theory and practice (T&P) based on the degree of cross-disciplinarity associated with the model types. Why is cross-disciplinarity important for our discussion? We argue that increasing engagement in cross-disciplinary research drives the potential for innovation in the theory and practice of M&S; we learn from disparate disciplines on methods and techniques that have been tried and tested and that have existed as extant knowledge within the respective fields; we absorb the new knowledge constructs (philosophies, methods, techniques) and deploy them co-operatively to complement (rather than supplement) our existing approaches; we operationalize the new learning in one or more stages of the lifecycle of an M&S study; we develop methodological frameworks which fuse the newly acquired knowledge with our established M&S approaches – all of which extends the T&P of M&S (Figure 2 inset). Innovation in the methodology space (T&P), in turn, *drives* innovative use of M&S in the application space. This is not surprising since an extension in methodology enables the development of modelling artefacts which hitherto were not easily implementable. As new possibilities are realized, it serves as a *stimulus* for further innovation in the T&P of M&S, which in turn *articulates the need for* increasing cross-disciplinary engagement. As shown in Figure 2 inset, cross-disciplinary research engagement can directly *enable* innovative use of M&S in the application space. For example, areas such as Circular Economy (CE) emerge from cross-disciplinary research including M&S which promotes the innovative use of M&S to complement existing research approaches from other disciplines. However, with time, as the application of M&S matures in the context of the CE, there is likely to be an increased impetus for innovation in the T&P of M&S to support further M&S innovations in CE.

The degree of cross-disciplinary engagement in the methodology space increases as we move from conventional one-technique models (e.g., DES or SD) to Types A-C Hybrid Simulation (HS). Although HS continues to be inward looking (i.e. it still focusses on expertise available within our community), it has arguably been successful in bridging the world-views associated with DES, ABS and SD modelling and has galvanized researchers working in those distinct M&S sub-communities to collaborate and to learn new techniques. Moving on from HS to Types D and D.1 Hybrid OR/MS Models, we increasingly look outward to the OR/MS community. However, it is arguable that this transition is moderately challenging since, like M&S, OR/MS largely concerns with analytical models for problem solving and decision making. Indeed, simulation is the most widely used OR technique after heuristics, DEA, optimization and integer programming (Mustafee and Katsaliaki, 2020). Moving on from Type D/D.1 models, the realization of Type E Cross-disciplinary Hybrid Model generally requires expertise that goes beyond our field of research (i.e., M&S and OR/MS), and it is therefore imperative that we engage with scholars from broader scientific disciplines. Type E models can be challenging as one has to venture outside their immediate area of expertise; however, these models are arguably also the frontrunners in making a novel methodological contribution to the T&P of M&S. Both hybrid OR/MS models (Type D, D.1) and cross-disciplinary Type E hybrid models are uniquely placed in driving innovation! Figure 2 presents the synthesis.

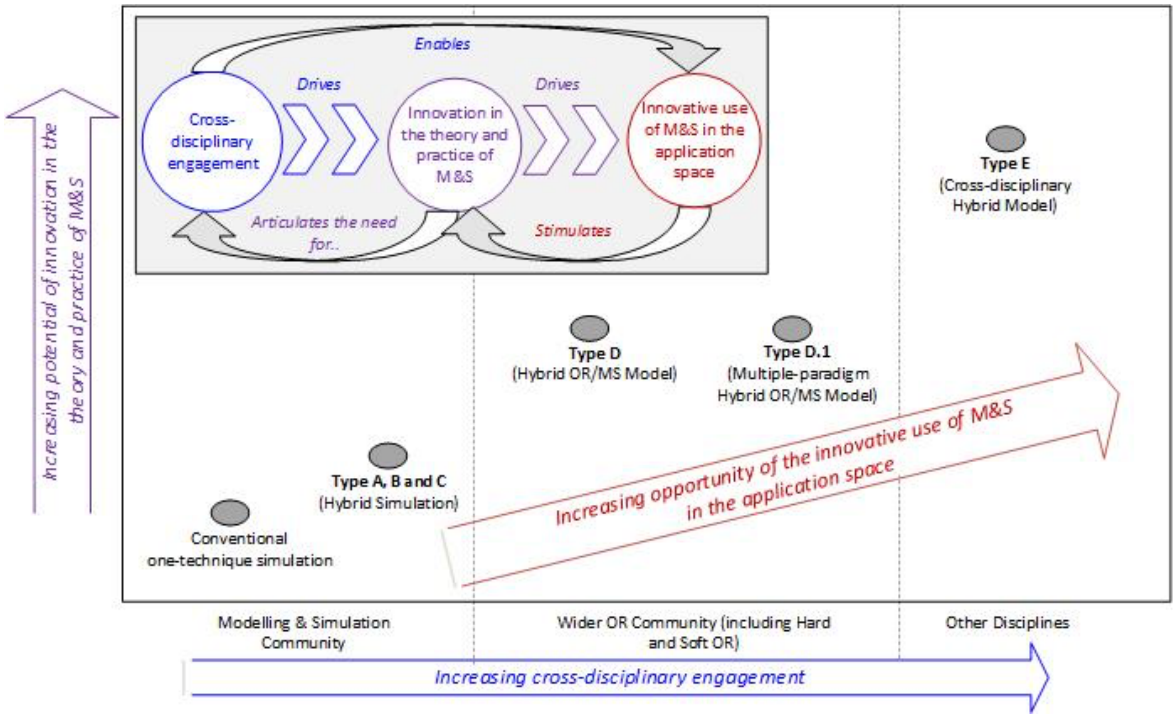

Figure 2: The degree of cross-disciplinary engagement in the methodology space (x-axis) increases with hybrid Model Types D, D.1 and E. This drives innovation in the theory and practice of M&S (y-axis), which in turn increases the opportunity of the innovative use of M&S in the application space (z-axis).

8 CONCLUSION

We have presented a unified conceptual representation and classification of hybrid M&S that identifies six distinct model types (Figure 1). Our classification is extensible, e.g., multi-disciplinary hybrid models could be categorized further (E.1, E.2), when there is an established body of research that employs cross-disciplinary techniques. Open innovation acknowledges that collaborative and participative relationships with stakeholders, and co-operative working with partners is needed to access valuable knowledge from a range of external resources. Our conceptual representation of hybrid M&S, which includes both hybrid simulation (Model Types A-C), hybrid OR/MS models (Model Types D, D.1) and cross-disciplinary hybrid models (Model Type E), illustrates how increasing engagement in cross-disciplinary research increases the potential for innovation in the theory and practice of M&S. This has been evidenced with numerous examples of studies that have applied the Model Types A-E categorized in the framework in Figure 2. However we further argue that innovation in the M&S theory space drives innovation in the application space. As the M&S community embraces hybrid M&S, we would like to emphasize the opportunities that are made possible by the use of interdisciplinary and cross-disciplinary approaches in traditional simulation studies toward enabling and sustaining innovation.

A defining characteristic of cross-disciplinary HM (Type E) is its reliance on cross-disciplinary research in the methodology space. We use the term hybrid teams to emphasize the need for interdisciplinary M&S groups that bring together problem stakeholders, researchers and practitioners. They are essentially composed of individuals specialising in specific fields of study or, as in the case of problem stakeholders, having tacit knowledge of the underlying system of enquiry. When considered as a whole, such hybrid teams will contribute knowledge constructs (theories, methodologies, techniques, applications, etc.) that have not traditionally been applied to M&S studies. Such teams are arguably better poised to address challenges pertinent with hybrid systems as the very constitution of the team allows for opportunities to leverage from the diverse body of knowledge, and individual expertise and skillsets, and make it possible to work towards common end goals. Innovation cannot occur in isolated silos and cannot be accomplished by researchers that are separated from the system under scrutiny. This cross-disciplinary involvement both enables innovative use of HM in practice, and stimulates the need for innovations in the theory and practice of M&S, leveraging the diverse body of knowledge to support open innovation.

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