

## **The Relationship Between Household Life Cycle and Brand Loyalty**

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### **ABSTRACT**

This research investigates changes in brand loyalty as households pass from one stage of the household life cycle to another. Analysing 45 brands in three consumer product categories in the UK, we find that the changes follow a U shape pattern. Brand loyalty declines as households shift from the young single stage to the young couple and the young family stage, remains relatively lower through the older family stage, and then increases at the post family and older single stages.

### **INTRODUCTION**

Brand loyalty is a fundamental concept in brand buying behavior research. It is generally recognized as an asset that firms need to invest in (Aaker, 1984). Therefore developing and maintaining customer loyalty to the company's brand is a critical task for all marketers (Goldberg, 1982). In order to do so, marketers need to understand factors that influence brand loyalty.

Brand loyalty can be classified in either behavioral or attitudinal terms (Chaudhuri and Holbrook, 2001). Behavioral loyalty measures the actual purchases of the brand observed over a particular time period while attitudinal loyalty is based on stated preferences, commitment or purchase intentions of the consumers (Mellens et al., 1996). While many covariates of attitudinal loyalty have been posited, behavioral loyalty has shown few correlates or predictors and this has been controversial. With an attempt to settle some of the current debates, behavioral loyalty is the area of interest for this article.

The relationship between behavioral loyalty and general demographics has been of interest to researchers for decades. For example, Frank's (1967) study of multiple brands in 44 grocery product categories finds virtually no associations between socioeconomic and brand loyalty. As such, he concludes that demographic variables are not effective predictors of brand loyalty. Fennell et al. (2003) and Hammond et al. (1996) examine brands in multiple grocery product categories and find that general demographic and psychographic variables are not useful in explaining brand choice. Uncles and Ehrenberg (1990) and Uncles and Lee (2006) extensively study the relationship between age and brand choice and find that patterns of brand buying are similar between older and younger consumers.

Conversely, other researchers postulate that older consumers' brand choices are different from younger consumers (e.g. Cole et al., 2008; Lambert-Pandraud and Laurent, 2010; Lambert-Pandraud et al., 2005). Lambert-Pandraud and Laurent (2010) and Lambert-Pandraud et al. (2005) provide evidence that younger consumers have a greater propensity to change their preferred brand while older consumers exhibit a propensity to remain attached for a longer duration to the same preferred brand. Lambert-Pandraud and Laurent (2010) argue that innovativeness and attachment mechanisms lead to such results. The innovativeness mechanism suggests that as the consumers get older, they are less willing to explore new options. With the attachment mechanism, the consumers can develop an attachment to their preferred brands over the years, from childhood to older ages. As such, the older the consumers are, the more loyalty the consumers devote to their preferred brands. Yet, it is worth noting that, in contrast to previous

studies of household grocery products, Lambert-Pandraud and Laurent (2010) and Lambert-Pandraud et al. (2005)'s findings are based on high involvement products (perfume and cars).

In summary, studies find that brand loyalty does not vary across age ranges for household consumption products. But for high involvement products, age does correlate with brand loyalty. One possible reason to explain this contradiction is that using age alone as a covariate for household brand loyalty only captures one aspect of the household, while household brand choice might be influenced by multidimensional household characteristics.

To overcome this drawback, we propose using household life cycle rather than age to predict brand loyalty for household grocery products. Research suggests that life cycle is likely to be a more meaningful way of classifying consumers than age (Wells and Gubar, 1966). Additionally, life-changing events strongly correlate with changes in consumer preference (Mathur et al., 2008; 2003). As such, we might expect some effects of household life cycle on brand loyalty.

The advantage of household life cycle over other demographic variables is that it captures multiple characteristics of consumers including age, household size, and life changing events such as marriage, the addition/departure of children, and death of spouse. Thus it reflects many factors that influence brand choice, especially in household grocery products. For example, financial burdens at the young family stage with dependent children might encourage households to switch brands to find cheaper alternatives. In contrast, at the post family stage with no dependent children households might be in a better financial position, hence they can afford to buy their preferred brands, resulting in higher brand loyalty. Moreover, the degree of agreement in purchasing decisions between members of the household could influence brand loyalty over the household life cycle stages. Further, the preferences of husband and wife become more similar over the stages of household life cycle (Cox, 1975). Consequently we could expect less variation in brand choice at the later stages of the household life cycle.

The relationship between household life cycle and consumer behavior is widely studied in marketing and consumer research (e.g. Danko and Schaninger, 1990; Douthitt and Fedyk, 1988; Douthitt and Fedyk, 1990; Du and Kamakura, 2006; Fritzsche, 1981; McLeod and Ellis, 1982; Putler et al., 2007; Redondo-Bellón et al., 2001; Wagner and Hanna, 1983; Schaninger and Danko, 1993; Wells and Gubar, 1966; Wilkes, 1995). Most of these studies examine the effects of household life cycle on product category buying behavior. For example, Wilkes (1995) finds a strong relationship between household life cycle and household spending across a wide range of categories including appliance, furniture, entertainment, travel, childcare and automobile. Danko and Schaninger (1990) report significant differences in food and beverage consumption across different household life cycle stages. Wagner and Hanna (1983) show that life cycle successfully explains the variance in total family clothing expenditures. Fritzsche (1981) finds a significant difference in energy consumption by stage of life cycle. Yet there is a lack of research on household life cycle effects on brand, rather than category, buying behavior.

Therefore, this research explores whether and how household loyalty to a brand varies through different stages in their life cycle. We seek to establish the patterns of brand loyalty that households express at different stages in their life cycle.

## **RESEARCH METHOD**

We use share of category requirements (SCR) to measure brand loyalty. SCR is the ratio of total purchases of the brand to total category purchases among those who buy the brand (Fader and Schmittlein, 1993). SCR indicates how much the buyers of each brand satisfy their product

needs by purchasing a particular brand rather than buying other brands (Uncles et al., 1994). As such, the higher SCR the higher brand loyalty.

Although SCR is one of the most important measures of brand loyalty (Farris et al., 2006), defining brand loyalty in this way is not without its problems (Danaher et al., 2003). For example, a consumer may be seen as being loyal to a brand through repurchase of the same brand when it is on price promotion (Allenby and Rossi 1991). However, comparing different measures of loyalty is a complex topic and it is not the purpose of this paper. We chose SCR because of its widespread use in industry and academia (e.g. in Bhattacharya et al., 1996; Bhattacharya, 1997; Danaher et al., 2003; Ehrenberg et al., 2004; Fader and Schmittlein, 1993; Johnson, 1984; Reibstein, 2002; Stern and Hammond, 2004; Tellis, 1988), making it the most practical measure for behavioral loyalty over the family life cycle.

We use the Dirichlet multinomial negative binomial distribution (known as the Dirichlet model in marketing literature) to estimate SCR. The Dirichlet model was introduced by Goodhardt et al. (1984) to model buyer behavior of multi brands in established competitive markets. We chose the Dirichlet model as past research has shown that the model is very accurate at predicting brand SCR for household grocery products (Bhattacharya, 1997; Danaher et al., 2003; Ehrenberg et al., 2004; Fader and Schmittlein, 1993; Goodhardt et al., 1984; Uncles et al., 1994). The model is also one of the most well-established empirical generalizations in marketing (Uncles et al., 1995), and has successfully characterized brand loyalty across a wide range of categories and conditions (for details about the Dirichlet model and its applications see Goodhardt et al. (1984) and Ehrenberg et al. (2004)). Consequently, the use of a model such as the Dirichlet model corrects any minor disturbances in the observations, allowing purchase behavior to be described in a more accurate way. It also represents a test of the consistency of the findings with prior knowledge. Mathematical expressions of the Dirichlet model are shown in Appendix 1.

We estimate the SCR for six traditional household life stages including young single (age <35), young couple (main shopper age < 35, without children), young family (young couple, main shopper age < 35, with dependent children), older family (older couple, main shopper age: 35+, with dependent children), post family (older couple, main shopper age: 35+, with no children living with them), and single elderly (age: 65+) Although there are different views on how to categorize the life cycle (e.g. Wells and Gubar, 1966; Murphy and Staples, 1979; and Gilly and Ennis, 1982), agreement exists on one central idea that each family progresses through a number of distinct stages from point of formation to death of both spouses (Murphy and Staples, 1979). The classification in this paper demonstrates six distinct phases in the household life cycle, from young single to older single, which is consistent with previous research using traditional household life cycle. The age cut-offs are based on the traditional flow in Murphy and Staples' model of household life cycle, although due to data availability we are not able to separate out single parent households.

We examine brands in three grocery product categories including shampoo, toothpaste and fabric-washing to establish patterns of relationship between household life cycle and brand loyalty. For reasons of data stability, brands with less than 1 percent market share were excluded from the analysis (this criterion has been used in previous research applying the Dirichlet model such as Fader and Schmittlein (1993) and Bhattacharya (1997)). This left us with the total of 45 brands in our analysis. The data came from a one-year panel of approximately 16000 households in the UK. The data was kindly provided by TNS superpanel database. Estimation was performed using the Dirichlet software developed by Kearns (2000).

## RESULTS AND DISCUSSION

We first report the detailed results for the shampoo category. Then we report the overall results for the other categories.

Table 1 shows the actual and estimated values of SCRs for brands in the shampoo category. As we can see from the table, the Dirichlet model predicts SCRs quite well with the overall mean absolute percentage errors (MAPE) of 16%. The actual and estimated SCRs are very close and show the U shape pattern. Brand loyalty declines from the young single stage to the young couple stage and the young family stage; then remains relatively low at the older family stage; and increases at the post family stage and the single elderly stage.

**Table 1. Actual and estimated SCRs by household life cycle for shampoo brands**

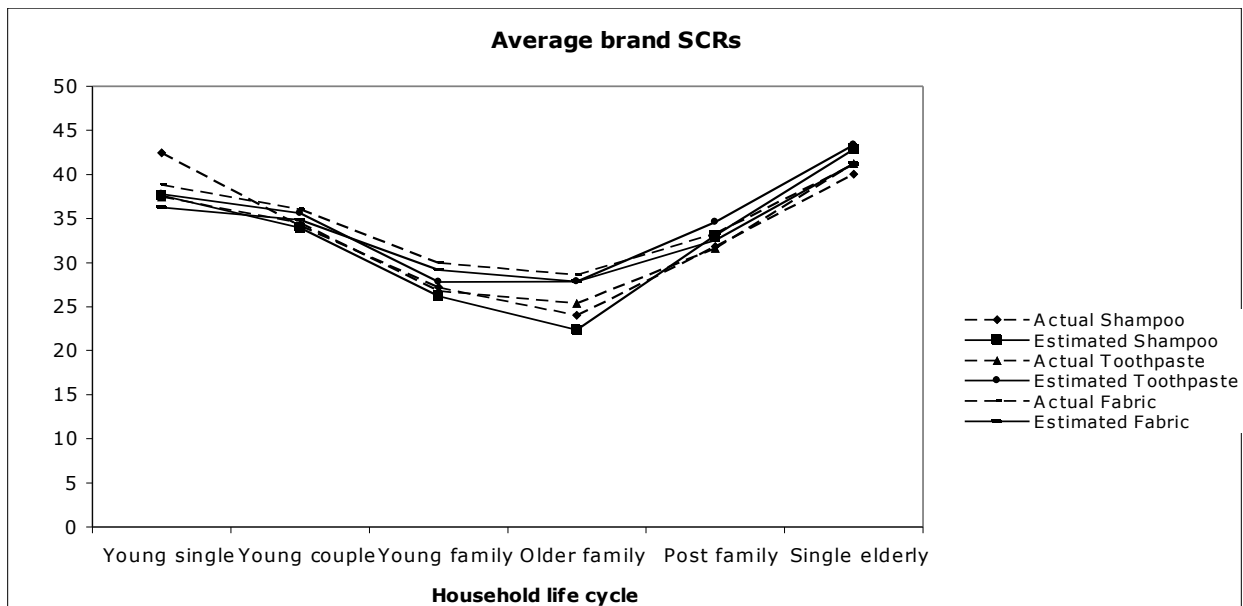
Brand	Young single		Young couple		Young family		Older family		Post family		Single elderly	
	Act	Est	Act	Est	Act	Est	Act	Est	Act	Est	Act	Est
SCR (%)												
Alberto	48	40	32	36	31	30	29	27	34	36	38	45
Head & Shoulders	66	42	54	40	37	30	30	22	41	37	52	45
Pantene	50	41	38	36	33	28	26	21	36	36	45	46
Herbal Essences	40	40	35	37	30	29	27	21	33	35	37	44
LOreal	32	38	26	34	28	29	27	27	36	35	39	43
Dove	21	37	29	34	24	26	21	22	31	34	38	44
Tresemme	40	38	33	34	26	26	22	21	31	33	35	43
Timotei	34	36	30	33	22	26	22	21	28	33	33	42
Tesco	74	37	34	33	30	26	30	27	35	33	47	43
Asda	31	38	34	33	28	26	25	22	32	32	44	42
Wash & Go	51	37	31	32	24	25	22	21	35	33	42	42
Johnsons	37	37	34	33	23	26	21	21	25	32	36	42
Sainsbury	75	36	64	34	34	25	27	27	35	32	48	42
Aussie	31	37	39	33	25	25	24	22	32	32	38	42
Sunsilk	41	36	21	32	22	25	17	21	25	32	31	42
Morrisons	49	36	32	33	26	25	23	21	31	32	38	42
Vosene	37	36	21	32	21	25	21	27	32	32	43	42
Supersoft	27	36	20	32	18	24	16	22	22	31	26	42
T/Gel	23	36	42	33	34	24	26	21	32	31	50	42
<b>Average</b>	<b>42</b>	<b>38</b>	<b>34</b>	<b>34</b>	<b>27</b>	<b>26</b>	<b>24</b>	<b>22</b>	<b>32</b>	<b>34</b>	<b>40</b>	<b>43</b>
<b>MAPE</b>	<b>.30</b>		<b>.19</b>		<b>.13</b>		<b>.13</b>		<b>.09</b>		<b>.14</b>	

Figure 1 graphically describes the average actual and estimated values of SCR for each of the three categories. Again, we find the same pattern as in the shampoo category.

The continuous decline in brand loyalty from the young single stage to the older family stage could be explained by several reasons. First, the decline in brand loyalty from the young single stage to the young couple stage could be due to an increase in joint decision-making by both partners. As each partner might have their own preferred brands, an increase in joint decision-making might reduce their loyalty to their preferred brands by accepting the other partner's preferred brands. Second, the decline in brand loyalty from the young couple stage to the young family stage could be due to more financial burdens at the young family stage compared to the pre family stage, hence households might increase brand switching to find cheaper options. Third, the decline in brand loyalty from the young family stage to the older family stage could be due to household needs become more heterogeneous as household size increases (e.g children develop their own preference).

Brand loyalty rebounds at the post family stage in the household life cycle, possibly due to a better financial position with no dependent children, and greater consensus about brand choice among husbands and wives. Research shows that families in the later stages of the household life cycle demonstrate greater similarities between the preferences of husbands and wives (Cox, 1975). Further, households become more homogenous as children leave home, and are most homogenous once the household reduces to a single elderly person. At this final household life cycle stage, the joint decision is replaced by an individual decision. This could sharpen individual brand loyalty, resulting in an increase in brand loyalty at this stage.

**Figure 1. Brand loyalty by household life cycle stages**



### IMPLICATIONS

Our results suggest that household life cycle succeeds in predicting brand loyalty for packaged grocery products, where age has previously failed for these categories. Brand loyalty follows a regular U shape pattern as households change from one stage to another. This suggests that some changes in life stages have positive impacts on brand loyalty while other changes have negative impact on brand loyalty. Studies that have bi-modal comparisons of young and old consumers may not detect this non-linear evolution of loyalty.

Studies of individual high involvement products may be confounded by this U shape effect. For these products, age could be a good predictor of brand loyalty as the role of declining innovativeness and attachment could lead to the differences in brand choice across age ranges (Lambert-Pandraud and Laurent, 2010). Gender-related high involvement products, such as perfume, may also not be subject to the same degree of joint decision making as products consumed by the whole family. However, the U shape pattern uncovered here highlights the risk that longitudinal effects of innovativeness and attachment may be confounded by disposable income, household composition and joint decision making. We hypothesize that that the U shape is due to the combined effects of five of these factors. If this is so, then changes to loyalty over time will depend on the relative strength of these five factors in a particular context.

As it stands, knowledge of this empirical regularity is important to marketing. It highlights the complexity of changes to loyalty over time, and the risk that investigating just one or two of these factors will allow confounding influences to swamp detection of main effects. It is also helpful to practitioners in their efforts to preserve their existing customer base as well as attract new customers. For example, practitioners should be aware that their customers are at the lowest loyalty once they experience the family stages in the household life cycle. On the other hand, there are also opportunities for practitioners who wish to attract new customers as those who are in these stages are at the highest chance of brand switching.

### **LIMITATIONS AND FUTURE RESEARCH**

Due to data availability, our study only examines the traditional household life cycle; this does not separate out some specific groups, such as single parents. The growth of the non-traditional household has emerged as an important research issue (Schaninger and Danko, 1993), so future research could usefully extend to study the relationship between brand loyalty and the life cycle of non-traditional households.

Our discussion has identified five possible factors that may contribute to the U shape present in the data: innovativeness, attachment, joint decision making, disposable income and household composition. The relative importance of these factors may explain how longitudinal changes in brand loyalty varies between contexts; such as perfume, cars and packaged grocery products. Therefore, future research could replicate this study in other contexts where the mix of effects is likely to be different, such as individual versus household consumption, hedonic versus utilitarian goods, infrequent versus frequent purchases, high versus low involvement products, and subscription versus repertoire markets. The resulting empirical patterns would provide guidelines to practitioners as well as empirical boundaries for any theoretical explanations of the underlying mechanisms.

Future research could seek to directly test the relative strength of these five effects. For example, the impact of household composition could be quantified by conducting comparative studies of household panels with personal impulse purchase panels, where all purchases are directly associated with an individual customer rather than satisfying the needs of an entire household. Alternatively, the impact of household composition could be assessed by the relationship between brand loyalty and the number of members in the household. Such studies would help to determine the relative importance of household heterogeneity, compared with attachment, innovativeness and income, to changes in brand loyalty over time. Similar work could be undertaken for each of the factors that we hypothesize underlying the U shape detected here.

Meanwhile, we hope the striking empirical pattern reported in this paper will motivate more studies in this under-researched area.

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## APPENDIX1. MATHEMATICAL EXPRESSIONS



The Dirichlet model is a mixture of the negative binomial distribution (NBD) of category purchase rate and the Dirichlet multinomial distribution (DMD) of the purchases of the individual brands, conditional on the category purchase rate.

The probability density function of NBD is

$$f(n) = (1+a)^{-k} \frac{\Gamma(n+k)}{n! \Gamma(k)} \left( \frac{a}{1+a} \right)^n \quad (1)$$

Where  $n$  is category purchase rate,  $k$  and  $a$  are the shape and scale parameters of the gamma distribution, respectively.

The probability density function of DMD is

$$f_{\alpha_1, \alpha_2, \dots, \alpha_m}(x_1, x_2, \dots, x_m \mid x_1 + x_2 + \dots + x_m = n) = \frac{\Gamma(s)n!}{\Gamma(s+n)} \prod_{i=1}^m \frac{\Gamma(\alpha_i + x_i)}{x_i! \Gamma(\alpha_i)} \quad (2)$$

Where  $m$  is the number of brands,  $x$  is brand purchase rate,  $\alpha_i$  is parameter of DMD, and  $s$  is the sum of  $\alpha_i$

Combining (1) and (2) the probability density function of the Dirichlet model is

$$f_{k,a,\alpha_1,\alpha_2,\dots,\alpha_m}(x_1, x_2, \dots, x_m) = (1+a)^{-k} \frac{\Gamma(n+k)}{n! \Gamma(k)} \left( \frac{a}{1+a} \right)^n \frac{\Gamma(s)n!}{\Gamma(s+n)} \prod_{i=1}^m \frac{\Gamma(\alpha_i + x_i)}{x_i! \Gamma(\alpha_i)} \quad (3)$$

Share of category requirement for brand  $i$  is

$$SCR_i = \frac{\text{Average purchase rate for brand } i}{\sum f(n)n \left[ 1 - \frac{\Gamma(s)\Gamma(s-\alpha_i+n)}{\Gamma(s+n)\Gamma(s-\alpha_i)} \right]} \quad (4)$$

Goodhardt et al. (1984)