A performance assessment of dynamic heuristic optimisers on the IEEE CEC 2015 niching competition test problems

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Abstract

We present the performance results of five multi-modal heuristic optimisers on the 20 benchmark functions of the *IEEE CEC 2015 competition on niching methods for multimodal optimisation*. All the algorithms compared here are pre-defined in existing works, are dynamic in their niche maintenance, and exploit 'hill-valley' niche detection. Code and files of output statistics required by the competition are also provided online. We find that although the recently developed Niching Migratory Multi-Swarm Optimiser (NMMSO) algorithm performs best overall, other optimisers perform better on some of the problems, with the main divergence in performance apparent between homogeneous and heterogeneous problem landscapes.

1 Introduction

Given a legal search domain $\mathcal{X}$, the aim of multi-modal optimisation, without loss of generality, is to find those $\mathbf{x} \in \mathcal{X}$, given any equality and inequality constraints, which maximise some $f(\mathbf{x})$. The aim is not simply to identify a single design $\mathbf{x}$ which maximises $f(\mathbf{x})$ given the constraints (as is typical in uni-objective optimisation), but *all* $\mathbf{x}^* \in \mathcal{X}$ which obtain the maximum possible function response, but which inhabit isolated peak regions. By a peak region we mean that the mapped objective values in the immediate vicinity of an $\mathbf{x}^*$ are all lower than $f(\mathbf{x}^*)$ (indicating $\mathbf{x}^*$ lies on a peak), or some points have a lower response and some an equal response (indicating $\mathbf{x}^*$ lies on a ridge). When the function value corresponds to the best obtainable value these are *global* optima. *Local* optima (local modes/peaks) in contrast are locations which are surrounded in the immediate vicinity with locations which map to lower (or equal) responses from $f(\cdot)$, but which do not themselves have the highest possible response obtainable. Regions around a peak are often called *niches*.

There are a number of reasons why finding multiple mode solutions is desirable, over finding a single 'best' solution, including:

1. insight may be gained into the problem domain when identifying a range of different designs which are operationally (functionally) equivalent;

2. if it transpires that some designs are not machinable, due to e.g. $\mathcal{X}$ being misspecified, a range of solutions mitigates against this;

3. $f(\cdot)$ may be in error in certain regions, therefore a wide range of good solutions can be helpful if the 'best' design does not perform as emulated (it is useful to have local optima, not just global optima stored in this case – as there is no guarantee that all the global optima under $f(\cdot)$ are not in error).

Multi-modal optimisers have grown in popularity in recent years in light of this, and a competition comparing the performance of multi-modal optimisers was held in the IEEE Congress on Evolutionary Computation (CEC) in 2013 (where 15 algorithms were compared) [1], and will be held again in 2015 (including the results presented here).

2 The algorithms

In this report we present the results of a number of different algorithms, all of which employ 'hill-valley' approaches to niche maintenance within their search mechanisms. These are:

1. the Multinational Evolutionary Algorithm (MEA) of Ursem [2];

2. the Multi-Sub-Swarm Particle Swarm Optimisation Algorithm (MSSPSO) of Zhang *et al.* [3];

3. the Localised Search Evolutionary Algorithm using a Gaussian Process (LSEA_{GP}) of Fieldsend [4];

4. the Localised Search Evolutionary Algorithm using an Evolutionary Algorithm (LSEA_{EA}) of Fieldsend [5];

5. the Niching Migratory Multi-Swarm Optimiser (NMMSO) of Fieldsend [6].

MATLAB implementations of these algorithms are available online[1], and the interested reader is directed toward the original papers for descriptions of the respective algorithm properties (the works [4, 5, 6] are also available at the author's institutional repository[2]).

The 20 benchmark problems of the IEEE CEC 2015 competition are of varying dimensionality and number of optima, and are derived from 12 base test problems. In the Equal Maxima (F2), Himmelblau (F4), Vincent (F7, F9) and Modified Rastrigin (F10) problems all peaks are global peaks. The Five-Uneven-Peak Trap (F1), Uneven Decreasing Maxima (F3), Shubert (F6, F8), and Composite Functions 1 (F11), 2 (F12), 3 (F13, F14, F16, F18) and 4 (F15, F16, F19, F20) all have local maxima as well as global maxima (with the Shubert and Composite Functions having considerably more

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[1] https://github.com/fieldsend
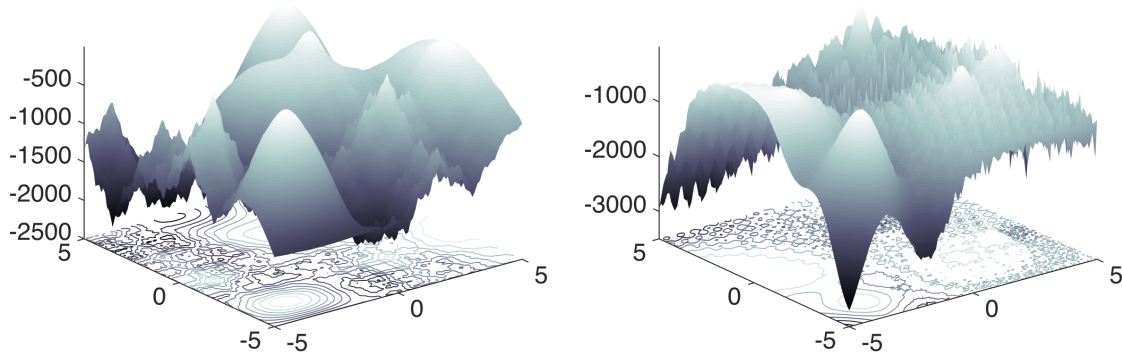[2] https://ore.exeter.ac.uk/repository/

Figure 1: 2D versions of composite functions 1 & 4.

local maxima than global maxima). Figure 1 illustrates the landscape of two of the composite functions. A technical report provides extensive details on the test problems [7].

The IEEE CEC 2015 test suite contains the same problems as the 2013 test suite, and is obtainable online in MATLAB, Java and C++[3]. Alongside the test problem implementations, functions to evaluate the quality of a set of designs returned by an optimiser are also provided by the competition organisers. For the competition, each algorithm must to be run 50 times on each test problem (by implication with a different random seed each time), for a predefined maximum number of function evaluations.

Two different average quality performance characteristics are assessed on the solutions returned by an optimiser. The *success rate* (SR) measures the proportion of successful runs – those the find *all* global optima given a prescribed accuracy level ϵ, and vicinity to a global peak location r. A value of 1.0 indicates that all 50 runs found all global peaks, whereas a value of 0.5 indicates that half of the runs found all global peaks. The *peak ratio* (PR) measure gives the average proportion of global peaks found across runs, i.e. for q runs:

$$PR = \frac{\sum_{i=1}^{q} o_i}{tq} \tag{1}$$

where o_i denotes the number of global optima discovered by the ith run, and t is the total number of global peaks.

Whether or not a peak may be categorised as being located by an optimiser on a particular run is determined by how close a design that has been found is to a peak location. The parameter r gives the maximum distance (in design space) a solution may be from a peak be categorised to have found it – subject to a further *accuracy* level, ϵ, which gives the maximum distance from the global maximum in objective space. For all

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Table 1: Performance measurement parameters and maximum number of function evaluations per optimiser run.

Function	F1	F2	F3	F4	F5
r	0.01	0.01	0.01	0.01	0.5
Peak height	200	1	1	200	1.03163
Max. evaluations	50k	50k	50k	50k	50k
Function	F6	F7	F8	F9	F10
r	0.5	0.2	0.5	0.2	0.01
Peak height	186.731	1	2709.0935	1	-2
Max. evaluations	200k	200k	400k	400k	200k
Function	F11	F12	F13	F14	F15
r	0.01	0.01	0.01	0.01	0.01
Peak height	0	0	0	0	0
Max. evaluations	200k	200k	200k	400k	400k
Function	F16	F17	F18	F19	F20
r	0.01	0.01	0.01	0.01	0.01
Peak height	0	0	0	0	0
Max. evaluations	400k	400k	400k	400k	400k

problems five different accuracy levels are assessed, $\epsilon = \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$. Other problem assessment criteria are detailed in Table 1.

Result files formatted in the competition style, along with MATLAB scripts to reproduce the results presented here, are available online[4]. We use the default algorithm parameterisation described in the original papers, [2, 3, 4, 5, 6]. In the case of LSEA$_{GP}$ however we do not crossover all maintained niches, but instead only crossover a maximum of 100 (selected either at random, or the best ranked, with equal probability). A similar approach is used for niche population evolution stage (this balances the approach for problems with vast numbers of local and global peaks, leveraging the research undertaken in this area in [5, 6]).

3   Results

In this section, for each algorithm, we present box plot figures of the proportion of global peaks found by each run, and tables of mean PR, mean SR and median function evaluations until perfect SR is achieved. We also present a summary comparison at the end.

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[4]https://github.com/fieldsend/IEEE_CEC_2015_niching_competition

Figure 2: Box plots of the proportion of global peaks found by each run for each problem, at each accuracy level, for MEA. The top row displays F1-F5, the second row F6-F10, the third row F11-F15 and the final row F16-F20.

3.1 MEA results

Figure 2 shows box plots of the proportion of global peaks discovered each run across problems and accuracy levels for the Multinational EA. The left part of table 2 presents the mean PR results of 50 runs of the MEA algorithm on each problem, assessed at each accuracy level. The right part of table 2 presents the mean SR results of 50 runs of the MEA algorithm on each problem, assessed at each accuracy level. No runs are seen to find all peaks (for any selected accuracy level) for problems F6-9 and F18-20, or for $\epsilon = 0.01$ or smaller for F11-F17. Table 3 presents the median time to find all peaks at the given accuracy level. A '-' indicates that over half the runs failed to find all peaks at that accuracy level, and therefore no median is provided (as any value would be misleading). This notation is used in all subsequent tables displaying this information.

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Table 2: Mean peak ratios and success rates across problems, MEA.

| | Peak Ratio | | | | | | Success Rate | | | | |
| | ϵ | | | | | | ϵ | | | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}		10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	0.810	0.280	0.050	0.050	0.050		0.700	0.040	0.000	0.000	0.000
F2	1.000	1.000	1.000	0.996	0.980		1.000	1.000	1.000	0.980	0.900
F3	1.000	1.000	1.000	1.000	0.960		1.000	1.000	1.000	1.000	0.960
F4	1.000	0.590	0.055	0.005	0.005		1.000	0.240	0.000	0.000	0.000
F5	1.000	1.000	1.000	0.640	0.070		1.000	1.000	1.000	0.420	0.000
F6	0.188	0.034	0.004	0.000	0.000		0.000	0.000	0.000	0.000	0.000
F7	0.490	0.404	0.402	0.383	0.302		0.000	0.000	0.000	0.000	0.000
F8	0.000	0.000	0.000	0.000	0.000		0.000	0.000	0.000	0.000	0.000
F9	0.195	0.134	0.130	0.112	0.054		0.000	0.000	0.000	0.000	0.000
F10	0.992	0.992	0.990	0.965	0.707		0.920	0.920	0.900	0.760	0.100
F11	0.930	0.593	0.563	0.477	0.380		0.700	0.000	0.000	0.000	0.000
F12	0.355	0.285	0.233	0.138	0.070		0.000	0.000	0.000	0.000	0.000
F13	0.790	0.493	0.410	0.337	0.277		0.180	0.000	0.000	0.000	0.000
F14	0.913	0.390	0.373	0.350	0.313		0.660	0.000	0.000	0.000	0.000
F15	0.840	0.145	0.138	0.135	0.125		0.240	0.000	0.000	0.000	0.000
F16	1.000	0.220	0.010	0.000	0.000		1.000	0.000	0.000	0.000	0.000
F17	0.507	0.037	0.000	0.000	0.000		0.420	0.000	0.000	0.000	0.000
F18	0.000	0.000	0.000	0.000	0.000		0.000	0.000	0.000	0.000	0.000
F19	0.000	0.000	0.000	0.000	0.000		0.000	0.000	0.000	0.000	0.000
F20	0.000	0.000	0.000	0.000	0.000		0.000	0.000	0.000	0.000	0.000

Table 3: Median function evaluations to find all global peaks MEA.

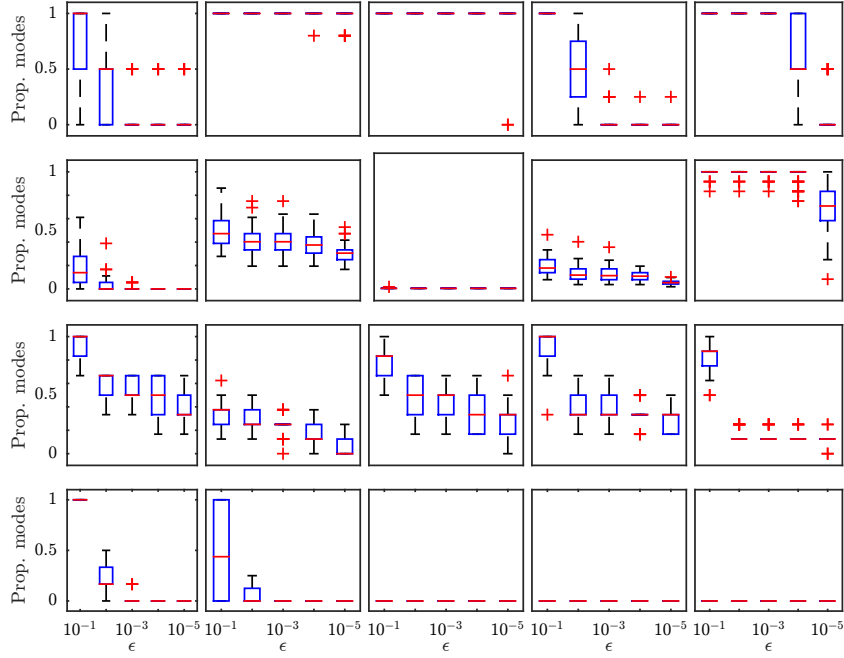| | | | ϵ | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	43583.5	-	-	-	-
F2	6521.5	6521.5	6522.5	20871.5	36702.0
F3	7051.0	7051.0	7051.0	7431.5	24936.0
F4	27828.5	-	-	-	-
F5	5671.0	10460.0	31002.5	-	-
F6	-	-	-	-	-
F7	-	-	-	-	-
F8	-	-	-	-	-
F9	-	-	-	-	-
F10	38857.0	83077.5	131373.5	182406.5	-
F11	83882.5	-	-	-	-
F12	-	-	-	-	-
F13	-	-	-	-	-
F14	162289.0	-	-	-	-
F15	-	-	-	-	-
F16	303476.0	-	-	-	-
F17	-	-	-	-	-
F18	-	-	-	-	-
F19	-	-	-	-	-
F20	-	-	-	-	-

Figure 3: Box plots of the proportion of global peaks found by each run for each problem, at each accuracy level, for MSSPSO. The top row displays F1-F5, the second row F6-F10, the third row F11-F15 and the final row F16-F20.

3.2 MSSPSO results

Figure 3 shows box plots of the proportion of global peaks discovered each run across problems and accuracy levels for the MSSPSO algorithm. The left part of table 4 presents the mean PR results of 50 runs of the MSSPSO algorithm on each problem, assessed at each accuracy level. The right part of table 4 presents the mean SR results of 50 runs of the MSSPSO algorithm on each problem, assessed at each accuracy level. No runs are seen to find all peaks (for any selected accuracy level) for problems F8-9 and F11-20, or for $\epsilon = 0.01$ or smaller for F7 & F10. Table 5 presents the median time to find all peaks at the given accuracy level.

Table 4: Mean peak ratios and success rates across problems, MSSPSO.

| | Peak Ratio | | | | | Success Rate | | | | |
| | ϵ | | | | | ϵ | | | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F2	1.000	1.000	1.000	1.000	0.952	1.000	1.000	1.000	1.000	0.760
F3	1.000	1.000	1.000	1.000	0.940	1.000	1.000	1.000	1.000	0.940
F4	0.685	0.130	0.010	0.005	0.000	0.240	0.000	0.000	0.000	0.000
F5	1.000	0.990	0.650	0.050	0.000	1.000	0.980	0.460	0.000	0.000
F6	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F7	0.989	0.459	0.202	0.030	0.004	0.860	0.000	0.000	0.000	0.000
F8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F9	0.110	0.017	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F10	0.883	0.457	0.048	0.007	0.000	0.240	0.000	0.000	0.000	0.000
F11	0.170	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F12	0.010	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F13	0.043	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 5: Median function evaluations to find all global peaks MSSPSO.

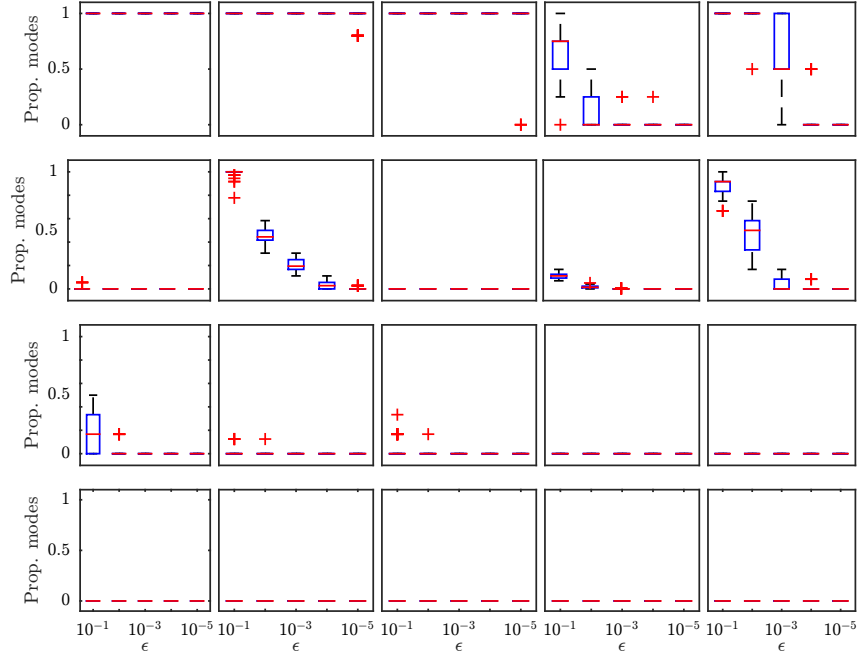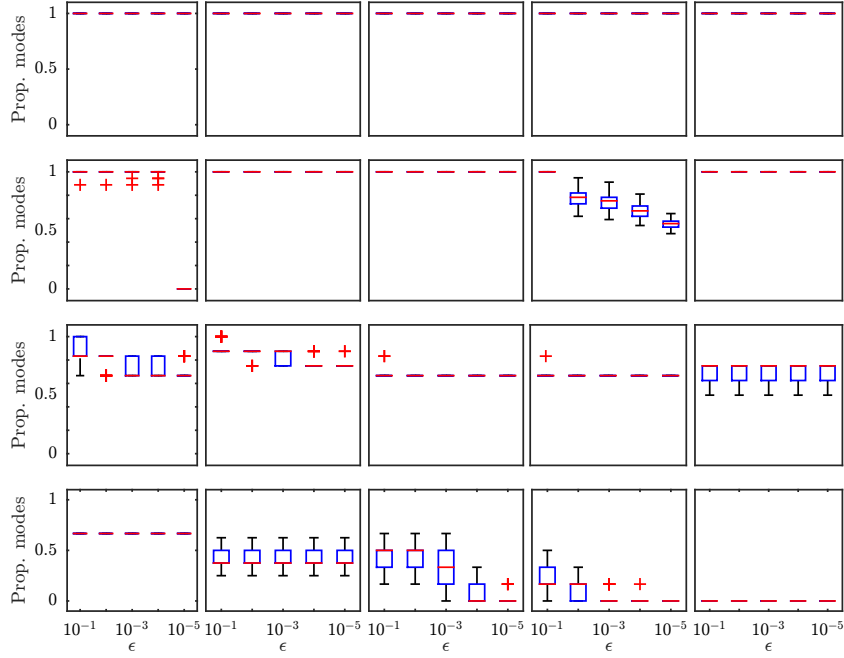| | | | ϵ | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	6728.5	6798.0	7759.5	7759.5	7759.5
F2	2598.0	2598.0	2605.5	8927.0	34242.5
F3	2584.0	2584.0	2589.0	4644.0	8908.0
F4	-	-	-	-	-
F5	2568.0	4639.5	-	-	-
F6	-	-	-	-	-
F7	64765.5	-	-	-	-
F8	-	-	-	-	-
F9	-	-	-	-	-
F10	-	-	-	-	-
F11	-	-	-	-	-
F12	-	-	-	-	-
F13	-	-	-	-	-
F14	-	-	-	-	-
F15	-	-	-	-	-
F16	-	-	-	-	-
F17	-	-	-	-	-
F18	-	-	-	-	-
F19	-	-	-	-	-
F20	-	-	-	-	-

Figure 4: Box plots of the proportion of global peaks found by each run for each problem, at each accuracy level, for LSEA$_{GP}$. The top row displays F1-F5, the second row F6-F10, the third row F11-F15 and the final row F16-F20.

3.3 LSEA$_{GP}$ results

Figure 4 shows box plots of the proportion of global peaks discovered each run across problems and accuracy levels for the LSEA$_{GP}$ algorithm. The left part of table 6 presents the mean PR results of 50 runs of the LSEA$_{GP}$ algorithm on each problem, assessed at each accuracy level. The right part of table 6 presents the mean SR results of 50 runs of the LSEA$_{GP}$ algorithm on each problem, assessed at each accuracy level. No runs are seen to find all peaks (for any selected accuracy level) for problems F13-20, or for $\epsilon = 0.01$ or smaller for F9 and F11-12. Table 7 presents the median time to find all peaks at the given accuracy level.

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Table 6: Mean peak ratios and success rates across problems, LSEA$_{GP}$.

| | Peak Ratio | | | | | | Success Rate | | | | |
| | ϵ | | | | | | ϵ | | | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}		10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F2	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F3	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F4	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F5	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F6	0.998	0.998	0.997	0.996	0.000		0.980	0.980	0.960	0.940	0.000
F7	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F8	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F9	1.000	0.777	0.744	0.668	0.556		1.000	0.000	0.000	0.000	0.000
F10	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F11	0.880	0.803	0.743	0.710	0.697		0.300	0.000	0.000	0.000	0.000
F12	0.895	0.865	0.830	0.767	0.755		0.160	0.000	0.000	0.000	0.000
F13	0.673	0.667	0.667	0.667	0.667		0.000	0.000	0.000	0.000	0.000
F14	0.670	0.667	0.667	0.667	0.667		0.000	0.000	0.000	0.000	0.000
F15	0.708	0.708	0.708	0.708	0.705		0.000	0.000	0.000	0.000	0.000
F16	0.667	0.667	0.667	0.667	0.667		0.000	0.000	0.000	0.000	0.000
F17	0.422	0.417	0.415	0.412	0.412		0.000	0.000	0.000	0.000	0.000
F18	0.477	0.443	0.330	0.087	0.013		0.000	0.000	0.000	0.000	0.000
F19	0.203	0.117	0.040	0.003	0.000		0.000	0.000	0.000	0.000	0.000
F20	0.000	0.000	0.000	0.000	0.000		0.000	0.000	0.000	0.000	0.000

Table 7: Median function evaluations to find all global peaks LSEA$_{GP}$.

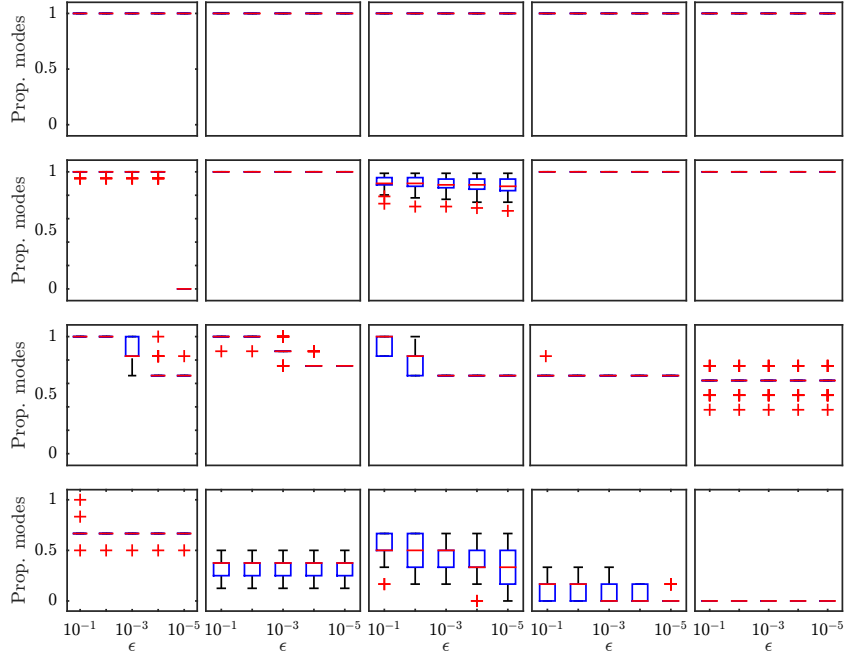| | ϵ | | | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	354.5	428.5	531.5	770.0	1916.0
F2	100.0	252.5	286.5	303.5	331.0
F3	100.0	202.0	252.0	256.0	287.5
F4	358.0	383.5	418.0	465.0	533.5
F5	258.5	320.0	357.0	407.0	454.5
F6	21988.0	22559.5	23854.5	26586.0	-
F7	6323.0	11896.5	13064.0	14059.5	14856.5
F8	121347.0	125296.0	139105.5	151666.5	176533.5
F9	25279.5	-	-	-	-
F10	434.5	523.0	683.0	860.5	1245.0
F11	-	-	-	-	-
F12	-	-	-	-	-
F13	-	-	-	-	-
F14	-	-	-	-	-
F15	-	-	-	-	-
F16	-	-	-	-	-
F17	-	-	-	-	-
F18	-	-	-	-	-
F19	-	-	-	-	-
F20	-	-	-	-	-

Figure 5: Box plots of the proportion of global peaks found by each run for each problem, at each accuracy level, for LSEA$_{EA}$. The top row displays F1-F5, the second row F6-F10, the third row F11-F15 and the final row F16-F20.

3.4   LSEA$_{EA}$ results

Figure 5 shows box plots of the proportion of global peaks discovered each run across problems and accuracy levels for the LSEA$_{EA}$ algorithm. The left part of table 8 presents the mean PR results of 50 runs of the LSEA$_{EA}$ algorithm on each problem, assessed at each accuracy level. The right part of table 8 presents the mean SR results of 50 runs of the LSEA$_{EA}$ algorithm on each problem, assessed at each accuracy level. No runs are seen to find all peaks (for any selected accuracy level) for problems F8, F14-15, and F17-20, or for $\epsilon = 0.01$ or smaller for F16. Table 9 presents the median time to find all peaks at the given accuracy level.

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Table 8: Mean peak ratio and success rate across problems, LSEA$_{EA}$.

	Peak Ratio					Success Rate				
	ϵ					ϵ				
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F6	0.996	0.996	0.996	0.993	0.000	0.920	0.920	0.920	0.880	0.000
F7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F8	0.906	0.900	0.893	0.886	0.879	0.000	0.000	0.000	0.000	0.000
F9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
F11	1.000	1.000	0.853	0.697	0.670	1.000	1.000	0.320	0.020	0.000
F12	0.998	0.998	0.877	0.772	0.750	0.980	0.980	0.200	0.000	0.000
F13	0.957	0.763	0.667	0.667	0.667	0.740	0.060	0.000	0.000	0.000
F14	0.670	0.667	0.667	0.667	0.667	0.000	0.000	0.000	0.000	0.000
F15	0.615	0.615	0.615	0.610	0.610	0.000	0.000	0.000	0.000	0.000
F16	0.673	0.663	0.663	0.663	0.663	0.020	0.000	0.000	0.000	0.000
F17	0.350	0.345	0.345	0.333	0.333	0.000	0.000	0.000	0.000	0.000
F18	0.520	0.500	0.460	0.383	0.300	0.000	0.000	0.000	0.000	0.000
F19	0.123	0.097	0.073	0.057	0.040	0.000	0.000	0.000	0.000	0.000
F20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 9: Median function evaluations to find all global peaks LSEA$_{EA}$.

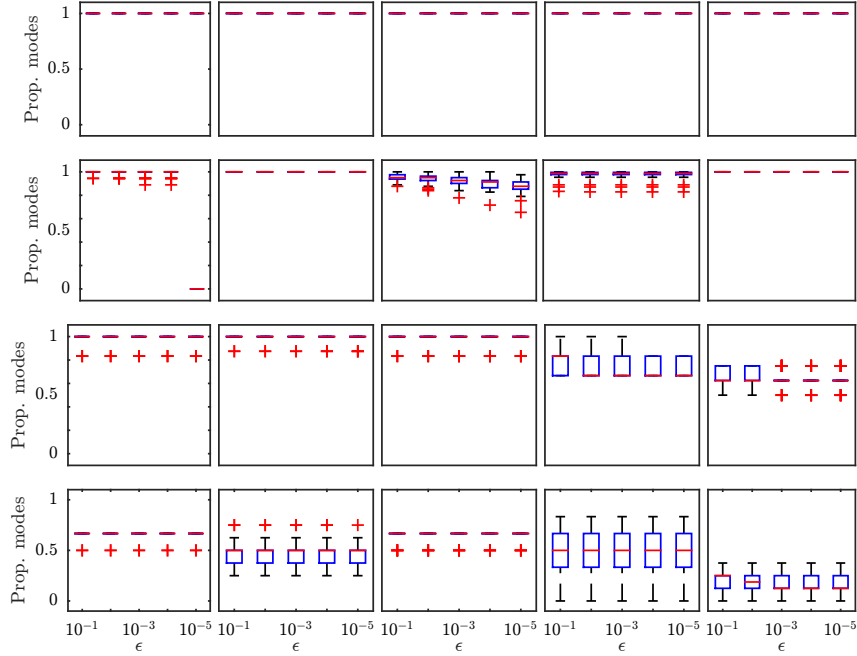|	ϵ				
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	876.5	1264.0	1560.5	1963.0	2413.5
F2	100.0	242.0	407.5	615.0	887.5
F3	100.0	202.0	276.5	484.0	737.5
F4	667.5	981.0	1270.0	1630.0	1922.5
F5	247.0	436.5	655.0	888.0	1239.5
F6	118313.5	121688.0	128724.0	145743.0	-
F7	4881.5	7678.0	8732.5	10902.5	13599.0
F8	-	-	-	-	-
F9	34461.5	73582.0	105190.5	155614.5	206680.5
F10	1109.5	1501.0	2002.0	2560.0	3094.0
F11	6263.5	10466.5	-	-	-
F12	46189.0	67685.0	-	-	-
F13	112831.0	-	-	-	-
F14	-	-	-	-	-
F15	-	-	-	-	-
F16	-	-	-	-	-
F17	-	-	-	-	-
F18	-	-	-	-	-
F19	-	-	-	-	-
F20	-	-	-	-	-

Figure 6: Box plots of the proportion of global peaks found by each run for each problem, at each accuracy level, for NMMSO. The top row displays F1-F5, the second row F6-F10, the third row F11-F15 and the final row F16-F20.

3.5 NMMSO results

Figure 6 shows box plots of the proportion of global peaks discovered each run across problems and accuracy levels for the NMMSO algorithm. The left part of table 10 presents the mean PR results of 50 runs of the NMMSO algorithm on each problem, assessed at each accuracy level. Up to F13 this is seen to be 1.0 (or very close too this) across accuracy levels. Only F6 at $\epsilon = 10^{-5}$ has a PR of 0.0 (this problem and accuracy combination is not obtained by any of the optimisers compared here). The right part of table 10 presents the mean SR results of 50 runs of the NMMSO algorithm on each problem, assessed at each accuracy level. No runs are seen to find all peaks (for any selected accuracy level) for problems F15-20. Table 11 presents the median time to find all peaks at the given accuracy level.

Table 10: Mean peak ratio and success rate across problems, NMMSO.

| | Peak Ratio | | | | | | Success Rate | | | | |
| | ϵ | | | | | | ϵ | | | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}		10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F2	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F3	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F4	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F5	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F6	0.998	0.994	0.992	0.992	0.000		0.960	0.900	0.880	0.880	0.000
F7	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F8	0.954	0.939	0.922	0.899	0.870		0.060	0.040	0.020	0.020	0.000
F9	0.978	0.978	0.978	0.978	0.978		0.120	0.120	0.120	0.120	0.120
F10	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
F11	0.990	0.990	0.990	0.990	0.990		0.940	0.940	0.940	0.940	0.940
F12	0.995	0.995	0.995	0.993	0.990		0.960	0.960	0.960	0.940	0.920
F13	0.990	0.987	0.983	0.983	0.983		0.940	0.920	0.900	0.900	0.900
F14	0.770	0.740	0.723	0.720	0.720		0.020	0.020	0.020	0.000	0.000
F15	0.650	0.647	0.642	0.632	0.632		0.000	0.000	0.000	0.000	0.000
F16	0.660	0.660	0.660	0.660	0.660		0.000	0.000	0.000	0.000	0.000
F17	0.480	0.477	0.470	0.468	0.460		0.000	0.000	0.000	0.000	0.000
F18	0.650	0.650	0.650	0.650	0.650		0.000	0.000	0.000	0.000	0.000
F19	0.460	0.460	0.457	0.450	0.437		0.000	0.000	0.000	0.000	0.000
F20	0.180	0.175	0.172	0.172	0.172		0.000	0.000	0.000	0.000	0.000

Table 11: Median function evaluations to find all global peaks NMMSO.

| | | | ϵ | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	503.5	682.0	820.5	1014.0	1242.0
F2	146.5	229.5	346.5	483.0	630.0
F3	35.5	169.5	236.0	351.5	463.0
F4	938.0	1205.5	1400.0	1630.5	1947.5
F5	82.0	211.0	363.0	558.0	831.0
F6	82344.5	85192.5	93138.0	102185.0	-
F7	17759.0	19998.5	19998.5	19998.5	19998.5
F8	-	-	-	-	-
F9	-	-	-	-	-
F10	1244.0	1671.0	2077.0	2497.5	3011.0
F11	5299.0	6651.0	7821.0	8216.5	8718.0
F12	39833.0	49927.5	60352.5	69583.5	73049.5
F13	50138.0	63474.5	71890.5	77680.5	79687.0
F14	-	-	-	-	-
F15	-	-	-	-	-
F16	-	-	-	-	-
F17	-	-	-	-	-
F18	-	-	-	-	-
F19	-	-	-	-	-
F20	-	-	-	-	-

Table 12: Mean peak ratios: algorithm with best average for each problem/accuracy combination. Algorithms denoted with numeric labels, MEA:1, MSSPSO:2, LSEA$_{GP}$:3, LSEA$_{EA}$:4, NMMSO:5. Statistically significant results indicated with a '*'.

| | Peak Ratio | | | | |
| | | | ϵ | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	2,3,4,5	2,3,4,5	2,3,4,5	2,3,4,5	2,3,4,5
F2	1,2,3,4,5	1,2,3,4,5	1,2,3,4,5	2,3,4,5	3,4,5
F3	1,2,3,4,5	1,2,3,4,5	1,2,3,4,5	1,2,3,4,5	3,4,5
F4	1,3,4,5	3,4,5	3,4,5	3,4,5	3,4,5
F5	1,2,3,4,5	1,3,4,5	1,3,4,5	3,4,5	3,4,5
F6	3,5	3	3	3	-
F7	3,4,5	3,4,5	3,4,5	3,4,5	3,4,5
F8	3*	3*	3*	3*	3*
F9	3,4	4*	4*	4*	4*
F10	3,4,5	3,4,5	3,4,5	3,4,5	3,4,5
F11	4	4	5*	5*	5*
F12	4	4	5*	5*	5*
F13	5	5*	5*	5*	5*
F14	1*	5*	5*	5*	5*
F15	1*	3*	3*	3*	3*
F16	1*	3	3	3	3
F17	1	5*	5*	5*	5
F18	5*	5*	5*	5*	5*
F19	5*	5*	5*	5*	5*
F20	5*	5*	5*	5*	5*

3.6 Summary results

Table 12 lists the best (or joint best) performing algorithm(s) on each of the problem/accuracy combinations. It is interesting to note that no single algorithm is best across all problems. For instance LSEA$_{GP}$ is better suited to F6 and F8, LSEA$_{EA}$ is better suited to F9, NMMSO is better suited to F13, F18-20 and MEA is better suited to the low accuracy optimisation of F14-17. Table 13 highlights which algorithm is on average quickest (in terms of function evaluations) to find all global peaks. Again, no single algorithm is seen to always be best regarding this measure either – although LSEA$_{GP}$ takes the fewest function evaluations on the bulk of the accuracy levels for F1-10, whereas NMMSO is the fastest on F11-F13. An algorithm which is significantly better than the other four is indicated in Tables 12 and 13 with a '*'. Statistical significance is calculated using pairwise Wilcoxon signed ranks tests between algorithms nominally at the 5% level, but modified using a Bonferroni correction (i.e. $\alpha = 0.05/5$) to compensate for multiple simultaneous comparisons being made.

Table 14 presents the summary results averaged across problems and accuracy levels, and may be compared directly to those presented in [7] for a wide range of algorithms. The results for NMMSO are overall the best amongst the algorithms

Table 13: Median function evaluations to find all global peaks: algorithm with best average for each problem/accuracy combination. Algorithms denoted with numeric labels, MEA:1, MSSPSO:2, LSEA$_{GP}$:3, LSEA$_{EA}$:4, NMMSO:5. Statistically significant results indicated with a '*'.

| | Peak Ratio | | | | |
| | | | ϵ | | |
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
F1	3*	3*	3*	3*	5*
F2	3,4	5	3*	3*	3*
F3	5*	5	5	3*	3*
F4	3*	3*	3*	3*	3*
F5	5*	5*	3	3*	3*
F6	3*	3*	3*	3*	-
F7	4*	4*	4*	4*	4
F8	3*	3*	3*	3*	3*
F9	3*	4*	4*	4*	4*
F10	3*	3*	3*	3*	3*
F11	5	5*	5*	5*	5*
F12	5	5	5*	5*	5*
F13	5*	5*	5*	5*	5*
F14	1*	-	-	-	-
F15	-	-	-	-	-
F16	1*	-	-	-	-
F17	-	-	-	-	-
F18	-	-	-	-	-
F19	-	-	-	-	-
F20	-	-	-	-	-

Table 14: Summary algorithm performance across problems, PR. The results of best performing algorithm in the 2013 competition, NEA2, also given (taken from [7]). Best values in bold, second best in italics.

Alg.	Median	Mean	Std. Dev
MEA	0.2076	0.3585	0.3852
MSSPSO	0.0000	0.2188	0.3913
LSEA$_{GP}$	0.7904	0.7302	0.3268
LSEA$_{EA}$	*0.9031*	0.7477	0.3236
NMMSO	**0.9883**	**0.8221**	0.2537
NEA2	0.8513	*0.7940*	0.2332

compared here, and indeed are better than the best performing algorithm of the 2013 competition presented in [7] (NEA2 [8]).

4 Conclusion

This short report presents the performance of five heuristic multi-modal optimisation algorithms, on the IEEE CEC 2015 niching competition problem suite. The recently published NMMSO algorithm [6] is seen to perform the best overall, however on particular problems other algorithms give better average performance. In terms of speed of convergence, the Gaussian Process assisted LEA_{GP} algorithm generally performs best on the first 10 problems, however for the problems with more complex/heterogeneous landscapes NMMSO is seen to perform better. It would be interesting to investigate further the sources of these performance differences, and whether detecting landscape properties during the search could be exploited.

Acknowledgement

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