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The importance of dialogic processes to conceptual development in mathematics

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1 Introduction: The essential dialogic distinction

Socrates first raises the fundamental dialogic distinction in the context of a critique of the cognitive affordances of writing. Written words, he points out, only pretend to have meaning. They are like paintings of people as opposed to real people in that if you question the painted image of a person it is not able to answer back. Real words, he argues, have a living soul and give answers when you question them (Plato 360 BCE/2006).

Socrates was an oral thinker. His argument might seem odd viewed from within a culture dominated by print literacy but we think that he makes an important distinction for understanding conceptual development in mathematics education. We interpret the essential contrast he is making as not between speech and writing so much as between signs experienced as if outside of any dialogue ("dead words" for Socrates) and signs experienced within a dialogue ("living words" for Socrates). Socrates was pointing out that signs outside of a dialogue are just fixed and finished facts, which cannot lead to understanding whereas signs within a dialogue are generative and persuasive voices that can stimulate new understanding (Plato, 360 BCE/2006).

This essential dialogic distinction between the inside and the outside of a dialogue was taken up and reformulated by Bakhtin, the main reference behind contemporary dialogic theory in education. Bakhtin (1986) defines dialogue as a chain of questions and answers where each answer gives rise to new questions. He adds that: "If an answer does not give rise to a new question from itself, it falls out of the dialogue and enters systemic cognition, which is essentially impersonal" (Bakhtin, 1986, p. 168). Meaning, Bakhtin argues, is not found in "systemic cognition" but only within dialogic relationships.

In this paper we argue that a dialogic perspective is needed if we are to understand the moments of mathematical insight that are essential to conceptual development in mathematics. In our view existing theories of conceptual development derived from both Piagetian and Vygotskian perspectives fail to take into account the important role of dialogic relationships between voices in enabling moments of insight and the growth of understanding. The significance of dialogic processes for the formation of algebraic concepts has been brought out by Radford (2003). In this paper we wish to build on this insight and show that it can lead to a Bakhtinian understanding of social semiotic processes that is distinct from the Vygotskian understanding because of its focus on relationships rather than on signs.

We begin with an outline of some key concepts for a Bakhtin inspired dialogic theory of conceptual development. We follow this with a brief review of some widespread theories currently used in mathematics education, categorising these as either Piagetian in origin or Vygotskian in origin. Inevitably there is some revisiting of ideas that will be familiar to many readers in our expositions of these theories but this is necessary in order to bring out the essential differences between them. We point out that these theories fail to adequately explain some dialogic aspects of conceptual development. To illustrate our claim that existing accounts of conceptual development can be usefully augmented with dialogism we analyse a short episode of classroom talk using Piagetian, Vygotskian and dialogic perspectives. The empirical data used for this micro-genetic analysis of the growth of understanding of two 12-year-old boys learning about probability using *TinkerPlots 2.0* software (Konold & Miller 2011) is part of a larger design based research study (Kazak, Wegerif, & Fujita, 2014).

The dialogic idea of augmentation that we apply should not be understood as the claim that Piaget, Vygotsky and Bakhtin are all conceptually compatible, but rather, quite the opposite, that they have conceptually distinct perspectives but that, despite this, they can engage in dialogue together in a way that can add to our understanding of the object. We follow Matusov (2011) in his argument that Vygotsky's and Bakhtin's ideas are incompatible since Vygotsky's underlying framework is monological whereas Bakhtin's is dialogical.

However, while Matusov implies that these theories cannot usefully be combined in any form, we argue that the creative quality of dialogues often lies in bringing together incommensurate perspectives. This would align us with those who argue for the value of "networking theories" in mathematics education (Kidron, Bikner-Ahsbahs, Monaghan, Radford, & Sensevy, 2012). Their idea is that bringing together different theories can be done in a useful way without disregarding their fundamental conceptual and methodological assumptions.

2 Dialogic theory relevant to the issue of conceptual development

The notion of "dialogic" has been used by some scholars (e.g., Wells, 1999) to characterise a Vygotskian account of conceptual development, but we think that this use ignores an important difference between Vygotsky's understanding of social interaction and Bakhtin's understanding of dialogic (Matusov, 2009; Wegerif, 2005). Vygotsky's account of conceptual development does not distinguish clearly between social mediation by tools and relationships between voices.

In a dialogue, the other that is addressed is located outside of the self but also appears as a voice on the inside of the self shaping utterances from within. Bakhtin (1981) distinguishes between the "persuasive" voice that enters inside my words and changes them from within and contrasts this to the authoritative voice that remains outside and can be either acquiesced to or resisted, but cannot be engaged in dialogue.

The distinction between interaction between things and dialogue between voices implies a switch in ontology from an ontology based on identity to one based on difference (e.g., Wegerif, Boero, Andriessen, & Forman, 2009). Bakhtin (1984) points out that if two voices in dialogue were to coincide with each other then the flow of meaning would cease. A dialogic relationship between two voices requires that there is a gap of incommensurability separating them. This gap, which is essential for meaning, is referred to in this paper as the *dialogic gap*.

Dialogues occur in external space but they generate their own *dialogic space* (Hand, 2012). If we see two children talking in a classroom we can locate their talk in external space and time, and record it in terms of sound waves. At the same time, on the inside of the dialogue, they may invoke different times and spaces and a range of voices and perspectives. Dialogic space is the space that emerges between voices, but that is also a shared space within which voices relate to each other.

A *voice* is a first person perspective in a dialogue. Voices only exist in relationship with other voices within a dialogic space. Bakhtin (1984) refers to voices as embodied but does not usually mean by this that they are all attached to a physical body. After all, Bakhtin elaborated his theory by looking at relationships between voices found within texts and wrote "I hear voices in everything and dialogic relations between them" (Bakhtin, 1986, p. 169).

Bakhtin did not elaborate a theory of education but his writing suggests that the key causal process in conceptual development is seeing or experiencing from an outside point of view. This *switch in perspective* is mediated by a relationship with an outside voice – the voice of the other. While the voice of the other sometimes has a concrete embodiment in a person or a cultural voice represented by a text, it can also take on the more open form of the "superaddressee" (Bakhtin, 1986, p. 126). Bakhtin argues that in any dialogue between two voices there is always also a third voice or a "witness position". This "superaddressee" voice in a dialogue is a kind of personification of otherness in general (Wegerif, 2011).

In order to be able to bring an initially outside point of view into the dialogue it is necessary to be able to switch perspective to see one's initial position as if from that outside point of view. A precondition of such a switch is to be able to step back from or de-identify

with the initial position (Zack & Graves, 2001, p. 235). So "*stepping back*" is another key moment in a dialogic theory of conceptual development.

Bakhtin refers to understanding as something that only emerges in the context of a relationship between two or more voices. In order to see something we need to see it as if from an outside perspective. The process of the emergence of understanding in a dialogue between two outsides is referred to as "*inter-illumination"* (Bakhtin, 1981, p. 12).

Bakhtin's model of conceptual development is not linear but implies an expansion of the range of perspectives that one can bring to bear on any issue. He refers to development through the *augmentation* of an initial point of view with the addition of a new "outside" point of view. So, for example, Bakhtin (1986, p. 7) understood his situation in 20th century Russia better by reading about ancient Greece. It is in this same spirit that we argue in this paper that a Bakhtinian-inspired point of view does not replace Piagetian and Vygotskian theories but can augment them. This dialogic understanding of progress in science through expansion of dialogue should not be confused with the claim that the three theories involved are conceptually compatible. As Wegerif (1999, 2007) argues, both Piaget and Vygotsky appear to endorse and implement Hegelian dialectical thinking that converts apparently different views into unity in a more inclusive rational integration or synthesis. Bakhtin explicitly rejected this dialectic approach as leading to the death of meaning (Bakhtin, 1986, p162).

3 Piagetian accounts of conceptual development

According to Piaget (1970) there is a distinction between physical knowledge and logical mathematical knowledge. While the former refers to the knowledge about objects based on experiences through physical acting (figurative activities), the latter applies to structures that are abstract and constructed through mental operating (operative activities). These lead to three types of abstraction described by Piaget: 1) *empirical abstraction* which is derived from the object that the subject acts upon, 2) *reflective abstraction* which is drawn from the action itself and 3) *pseudo empirical abstraction* which is derived from the properties of the object resulting from subject's actions (von Glasersfeld, 1995). The second one is reflective in a sense that "at the level of thought a reorganization takes place" (Piaget, 1970, p. 18), and it always involves coordinated actions in mental processes. *Reflective abstraction* is central to Piagetian accounts of conceptual development (Simon, Tzur, Heinz, & Kinzel, 2004).

3.1 Piaget's scheme theory

In Piaget's theory there are two key causal processes leading to conceptual development, *assimilation* and *accommodation*. *Assimilation* involves an incorporation of new experiences and perceptions of the world into an existing scheme, while *accommodation* refers to adaptation of the existing scheme to a new structure. These two processes work in a dialectical relationship. When a scheme leads to a *perturbation*, then accommodation takes place in order to maintain or re-establish the *equilibrium*. The causal process of *equilibration* or maintaining *equilibrium*, is assumed as fundamental and lies behind both the process of assimilation and the process of accommodation. For Piaget, as Simon et al. (2004) note, "reflective abstraction was a characterization of the mechanism of equilibration" (p. 309).

3.2 Socio-cognitive conflict theory

Based on Piaget's equilibration of cognitive structures, the role of *socio-cognitive conflict* has been proposed as a causal mechanism leading from social interaction to conceptual development (Mugny & Doise, 1978, p. 191). It is conjectured that social cognitive conflict between different views leads to cognitive reorganization as students try to reach a consensus (e.g., Mugny & Doise, 1978; Silverman & Stone, 1972). The beneficial role of socio-cognitive conflict is attributed to decentralizing from one's own perspective that might initially focus on only certain aspects of the problem to incorporate other possible perspectives.

3.3 Critique of Piagetian accounts of conceptual development from a Bakhtinian dialogic perspective

One way to understand what is not being thought through properly in Piaget's theory is through exploring what is meant by the term "reflection". When it is argued by Piaget and his followers that "reflection on activity" leads to abstraction this sounds as if the process has been explained. While Simon et al. (2004) take the idea of reflective abstraction further and go into more detail they still overlook the fundamental problem of the dialogic nature of reflection. This illusion of explanation is based upon the assumption that we know what reflection is and that it is an unproblematic linear monologic process with inputs (action) leading to outputs (abstraction) (see von Glasersfeld, 1995, p. 104, for Piaget's description of reflective abstraction composed of two essential features, projection of existing structures on to a higher plane of thought and cognitive reorganisation of them). What remains to be explained, however, is the empirical variation we actually find with abstraction occurring in some contexts and not in others. To give one example, girls asked to learn (in the sense of "abstract a scheme") from a route-planning game did much better in a game with honeybears than in a game with pirates, despite the logical isomorphism of the two games (Littleton, Light, Joiner, Messer, & Barnes, 1998). This and many other examples indicate that learning is not just about the abstract logic of evolving schemes but is also about relationship with the content matter.

From a dialogic perspective, to "reflect on" or be more consciously aware of an action is to look at it as if one was taking on the perspective of an outside "other", a process referred to by Bakhtin as taking the "witness" position or engaging in a dialogic relationship with the "superaddressee". Although this is the projection of an outside point of view it is always a socially situated construction since there is no neutral unsituated standpoint outside of society and history from which we can reflect. Given these dialogic assumptions it follows that reflection can be understood, as the "inter-illumination" of two or more perspectives in dialogue.

Children asked to reflect in the mathematics classrooms are really being asked to invoke the perspective of mathematics and engage in dialogue with it. As Walkerdine (1990) and Solomon (2012) bring out, not all of the ways in which individuals invoke the perspective of mathematics are equally educationally productive. Invoking the voice of mathematics in reflection on activity may, for some children in some contexts, result in what Piaget calls "equilibration". But this process is not uniform and automatic because each child will construct the voice of mathematics differently.

This dialogic critique of the Piagetian assumption of equilibration applies, a fortiori, to neo-Piagetian socio-cognitive conflict theory. The assumption behind socio-cognitive conflict theory is that exposure to different views will automatically lead to conceptual

development because of the underlying equilibration process. However, if we re-interpret the underlying process as dialogic inter-illumination we can see that this process is rather more complex than was assumed and will only lead to conceptual development under certain circumstances.

4 Vygotskian accounts of conceptual development

In contrast to Piaget, Vygotsky assumes that higher mental functioning in the individual emerges from the social context. According to Vygotsky's general genetic law of cultural development, any mental function occurs first in the social plane and then in the psychological plane (Vygotsky, 1978).

Vygotsky (1978) argues that higher mental processes are mediated by tools (i.e., technical tools, such as a calculator, graph paper, etc.) and signs (i.e., psychological tools, such as language, numerical system, algebraic symbols, and so on). Development takes place when these different forms of mediation create a transformation in mental functioning. Individuals have access to the mediational means as part of a socio-cultural context, from which individuals "appropriate" them (Wertsch, 1985).

Vygotsky (1978) proposed that education takes place in a *zone of proximal development* (ZPD) where learners are drawn beyond their current understanding by working with a teacher, adult or more competent peer. In the ZPD the spontaneous concepts of the child are taken up by the teacher and grafted onto the scientific concepts that pre-exist in the culture and are represented by tools such as mathematical procedures and concept words (Vygotsky, 1986).

4.1 Vygotsky and instrumentalisation

Through elaborating on Vygotsky's notion of tool mediation, there has been a focus on the role of instruments and instrumented activity, especially in relation to the use of technology. Within the French research tradition of instrumentalisation, which is compatible with Vygotsky, a distinction is made between artefact and instrument involved in the instrumented activity (Vérillon, 2000):

> …the instrument is a psychological construct distinct from the artifact. More exactly, the artifact, as a material or semiotic construct, is only a partial component of instrumented action. The other component is manifested by the complex set of representations, knowledge, mental operations, and motor skills that are brought into play by the user during operation. (p. 7)

Artigue (2002) points out that the artefact is not of an instrumental use at the beginning. The process in which the artefact becomes an instrument is called *instrumental genesis* (Guin & Trouche, 1999). According to Artigue (2002), this process entails "the construction of personal schemes or, more generally, the appropriation of social pre-existing schemes" (p. 250) when learners are working with artefacts which are related to certain mathematical concepts.

Trouche (2004) argues that we need to pay attention to instrumented gestures when learners are using particular tools. This is what Drijvers, Kieran, and Mariotti (2010) define as observable *instrumented techniques*, which are a "stable sequence of interactions between the users and the artifact with particular goal" (p. 109). We will give our own example in latter parts of this paper.

4.2 Vygotsky and language

Working within a Vygotskian tradition Mercer and colleagues developed a programme called Thinking Together for teaching "Exploratory Talk" which was claimed to be an educationally effective kind of talk (Mercer, Wegerif, & Dawes, 1999). This way of talking taken as a whole was understood as a Vygotskian tool supporting social cognition. Studies of the impact of Exploratory Talk suggest that it is effective in improving children's reasoning and problem solving (e.g., Mercer & Sams, 2006; Monaghan, 2005). In a recent paper Mercer (2013) summarises the causal processes that might be behind the link between Exploratory Talk and problem solving as "appropriation, co-construction and transformation". *Appropriation* is where one child shows another how to solve the problems so it is about transmitting knowledge. *Co-construction* is about generating new knowledge together. Mercer's third causal process, *transformation*, refers to the idea that through engaging in dialogue with others children learn to engage more effectively in "intra-mental" dialogue.

4.3 Critique of Vygotskian accounts of conceptual development from a Bakhtinian dialogic perspective

In order for tools, including words and ways of using language, to re-appear transformed on the inside of thought as ways of seeing and ways of thinking, they first need to be experienced as living voices within a dialogue. Children do not only learn how to use tools to get things done, they also learn how to experience the tools as voices to think with within a dialogic space of multiple possible voices.

The same arguments that we have made about the internalisation of tools also apply to the internalisation of ways of using language considered as a tool in concepts such as Exploratory Talk. However nicely children talk together to ask each other questions and give each other reasons this will not automatically translate into insight. In the "aha" moment of insight the language or tool becomes a voice in a dialogue which means that it is experienced as if from an outside position. This switch in perspective from just using a tool to understanding a tool is a dialogic switch, meaning a change in perspective within a dialogue.

5 Description of an empirical research study

In the next section we focus on a short episode involving the talk of two 12-year-old students working together to explore the fairness of some chance games using *TinkerPlots 2.0* software (Konold & Miller, 2011). The data reported here come from the first iteration of a design-based research study that took place in an afterschool club with five volunteering participants at a private secondary school in Exeter, UK. The aim of this study was to examine the conceptual development of 10-12-year-old students' understanding of probability and data analysis through students' talk together and their use of ICT tools.

In the study, the afterschool program comprised a total of 11 one-hour long sessions co-taught by the authors. In the early sessions, we introduced certain expectations for group work to promote a dialogic way of talking in joint activity (Dawes, Mercer, & Wegerif, 2000). These included (1) making sure that each person has an opportunity to contribute ideas, (2) asking each other "why?", listening to the explanation, and trying to understand, (3) asking others what they think, (4) considering alternative ideas or methods and (5) trying to reach an agreement before they do anything on the computer. Throughout the study, our teaching

approach entailed the use of these expectations in small group work to foster dialogic interactions.

Students worked in groups of two to three and each group's work around a computer was recorded by a video camera. The tasks in which students were engaged involved analysing data sets, such as students' reaction times and backpack weights, data modelling tasks, and chance events using *TinkerPlots* software. Prior to modelling chance situations, students used *TinkerPlot*'s modelling tool (Sampler) to build "data factories" that generated 100 objects like trucks with different type, make, year and length by using spinner or mixer (urn) devices for each attribute. Then students were introduced to exploring probability events with the Chips Game Task (see section 6) in the eighth session of the afterschool program. The task involved iterations of making predictions about the fairness of some games, testing initial theories by collecting simulated data from a model built in *TinkerPlots*, and making conclusions based on data.

Our research aim was to develop theory through exploring the different "affordances" of theoretical perspectives in order to explore the unique contribution, if any, of dialogic theory. For analysing this short episode Piagetian, Vygotskian and Dialogic were used as different lenses. Each researcher took the initiative to do the analysis from a theoretical perspective with which they have more experience. These analyses were discussed together and agreed in face-to-face meetings.

6 Analysing a dyad's investigation in the chips game task

In the following subsections we focus on different analyses of the episode where two 12-year-old boys, Chris and Jacob (pseudonyms), investigated their initial predictions about the fairness of the following chance game:

> There are two bags containing game chips of two colours–red and blue. To play the game, you will randomly select a chip from each bag. If the two chips are the same colour, you will win. If they are different colour, teacher will win. Bag one has 4 red chips; bag two has 2 red chips, 2 blue chips. Is this a fair game?

The students were initially quite confident that the game was unfair. First Chris offered an explanation for why they thought the game was unfair: "Bag two is fair. Bag one is all red chips. It is impossible to pick out a blue chip from there. It is all made up of red chips." Right after this, Jacob added, "If I took out a guess here, I'd probably say, uh about, probably about 70% we would win, about 30% Taro."

Chris's response, which seems to be also agreed by Jacob, indicates that students' intuitions lead them to focus on the number of red and blue chips in each bag rather than the combined outcomes of a two-stage experiment. Using the additive reasoning they employed for their predictions in the previous games (e.g., Bag one has 2 red chips, 2 blue chips; Bag two has 2 red chips, 2 blue chips) they pay attention to the imbalance of the total number of red and blue chips in two bags instead of the number of all possible outcomes for getting the same or mixed colour pair of chips (see Kazak et al., 2014). Moreover, Chris uses the word "fair" in relation to the number of red and blue chips in a bag to indicate the equal proportion of red and blue chips in the bag. There seem to be two notions of fairness here, one for the content of each bag (input) and another for the results of the game involving combined outcomes (output). And the former is more easily or spontaneously available to students in this example.

After the initial predictions, students built a model of this game in *TinkerPlots* (see Fig. 1) to test their conjecture. In their model, they built the Sampler on the left in Fig. 1 using

two spinners, one with 100% red and the other with 50% red and 50% blue sections that represent the proportion of red and blue chips in bag 1 and bag 2 respectively. Students said that they preferred spinners when there were fewer items to put in the randomisation device. After the RUN button is hit, each spinner spins once to execute a trial of randomly drawing a chip from each bag. In the example below, students chose to repeat this 1000 times. The results table in the middle displays the repetitions as they occur in the Sampler. The plot on the right shows the percentage of the combined outcomes (the same colour and the mixed colour) in 1000 trials.

Fig. 1 Computer model of the chips game built by Chris and Jacob in *TinkerPlots*.

Below is the excerpt showing the interchange mainly between the two students, Chris and Jacob, after they completed building their model in *TinkerPlots* (Our transcription notations include: (.)=Micro pause; (1.2)=Timed pause; [word]=Overlapping talk; (( ))=A note (in italics) describing action or giving commentary). Next we will analyse this exchange in terms of how students make a conceptual shift from their initial frequency-oriented reasoning to process-oriented reasoning from the three different theoretical perspectives, outlined above. In particular, we are looking at the development of an understanding that the probability of combined events is determined by the number of ways they can occur.

1	Teacher:	((*After students built and explained their model in TinkerPlots*)) Okay.
2		And you think that you guys will win most of the time, huh?
3	Chris:	I think we will actually win most of the time.
4	Jacob:	Actually, I am actually debating now ((*while he presses the run*
5		*button to collect 1000 data*))
6	Jacob:	Oh yes, it is fifty fifty because oh yeah!
7	Chris:	Jeez, we got an entire (.) army on our side!
8	Jacob:	No, no, Chris you don't get it. The first one ((*pointing to the first*
9		*spinner on the screen*)) you always get hundred percent red
10	Chris:	Exactly
11	Jacob:	[Then the next
12	Chris:	[So, then] the next one you could get ((*pointing to the second spinner*
13		*on the screen*))
14	Jacob:	It's a fifty fifty chance of getting either the same (0.5) ((*he is laughing*
15		*and almost speechless*))
16	Chris:	I don't get it!
17	Jacob:	So basically the first time you will get a red, next time you got a fifty
18		fifty chance of either getting the same or something different ((*covering*
19		*his face with his hands, leaning down the desk and laughing*))
20	Chris:	Jacob, I don't get this at all. Why are you laughing?
21	Teacher:	Yeah yeah, I agree, explain
22	Chris:	Jacob, why are you laughing? Just calm down, calm down ((*taking a*
23		*deep breath*))
24	Jacob:	((*now talking to Chris*)) First one you will definitely [get a red]
25	Chris:	[get a red],
26	Jacob:	so the next one you would get either a red or a blue. So basically you
27		can either get fifty percent
28	Chris:	[Yeah]

29 Jacob: you will get [red]
30 Chris: [Red] Yeah. So it is
31 Jacob: Fifty percent you will [get blue] ((*laughing*))
32 Chris: [It is, it is fifty fifty]
33 Jacob: So fifty fifty
34 Teacher: I think I think, so this is the bag one, hundred percent red. Yeah. And
35 Chris: Yeah I think it is fifty (.) I think it is fifty fifty. What Jacob said, it is
36 fifty fifty. That is fair.
37 Jacob: Because in the first one [you will get a red]
38 Chris: [you will get a red]
39 The next one you will, you will probably get a fifty fifty chance of
40 getting a red [or a blue]. So it is either you get [red or blue]
41 Chris: [or a blue] [red or blue]
42 Jacob: So basically if you get red, you get the same. Blue is different. So
43 basically they are fifty fifty. The first bag [(inaudible)
44 Chris: [the first bag is just red.
45 What (.) what (.) the whole competition is in the second bag.
46 Jacob: We believe that this game four [is fair], like [game three]
47 Chris: [is fair] [game three]
48 Jacob: although it does not look fair when you look at it.

6.1 Piagetian approach

When Chris claimed that the game was not fair because bag 1 had only red chips and no blue ones, he was engaging in an assimilation process in which the problem situation was understood with regard to his existing schemes. One scheme he used is that "50-50" (i.e., the ratio of blue and red chips in bag 2) means fair and otherwise unfair. However, he failed to coordinate it with another scheme involving the relationship between single (i.e., red or blue) and combined outcomes (i.e., same or mixed colour) of a random experiment (i.e., drawing a chip randomly from each bag).

After modelling the game in *TinkerPlots*, seeing the results from a sample of 1000 led students to modify their initial conjecture. First, Jacob reinterpreted the whole experiment to explain the conflicting results seen in the graph. According to Piaget's equilibration model, one might argue that Jacob experienced a cognitive perturbation and hence needed an accommodation of his schemes (assuming that because he did not challenge Chris, it is similar to Chris's as explained above). Through this accommodation process Jacob was able to interpret the random experiment modelled in *TinkerPlots* more satisfactorily by seeing the relationship between the single and combined outcomes in the experiment: (lines 37-43) "Because in the first one you will get a red. The next one you will, you will probably get a fifty fifty chance of getting a red or a blue. So it is either you get red or blue. So basically if you get red, you get the same. Blue is different. So basically they are fifty fifty." The results of the experiment led Jacob to see the model they had created in *TinkerPlots* in a new way through reflection by which he could recognise the significance of the two stages, drawing first from one bag and then from the other one. These two stages are clear in the model on the screen but he had not previously anticipated their relevance to his prediction of how fair the game was.

We also see in the data that although Chris did not initially understand Jacob's new interpretation of the problem, he was eventually able to change his understanding of the situation as well through his engagement in the peer interaction (lines 24-45). Especially

significant is the argument that Jacob makes quoted above (lines 37-43) which is picked up by Chris and repeated in his own words (line 45). This exchange could be seen as illustrating the causal impact of socio-cognitive conflict.

6.2 Vygotskian approach

From a Vygotskian perspective the causal processes of learners' conceptual development are mediated by language or technical tools. In the case of Chris and Jacob, their conjectures and explanations about the fairness of the game were mediated by the combination of *TinkerPlots* (technical tools) and their talk (language). From the instrumental genesis point of view, at this point of episode *TinkerPlots* has already become an "instrument" (in the sense of Vérillon, 2000) for Chris and Jacob and they effectively chose to use two spinner devices to model each bag with given proportions of red and blue chips. This two-in-line-spinner model generates the same sort of data as the actual physical game by randomly drawing first from the left spinner and then from the right spinner in Fig. 1. The choice of device type was crucial in students' instrumented activity. Even though the mixer resembles quite closely the actual situation involving bags and chips placed in them (see Fig. 2), students used the spinner device, which shows proportions rather than discrete objects to be counted. Thus, we can argue that the spinner model enabled Jacob to shift his initial focus on the frequencies of red and blue chips in each bag and to pay more attention to the proportions of the combined outcomes, such as mixed and same colour chips, while watching the simulation (lines 24-33).

Fig. 2 On the left a model of the chips game using mixer devices in *TinkerPlots* and on the right the two bags and chips placed in them in the actual game.

The interface representations (Fig. 1) became internalised in students' thinking as the artefacts were integrated into their mental schemes. This was observed by their gestures (instrumental techniques) pointing at the spinners on the screen (e.g., line 8 Jacob and line 12 Chris). This instrumented action then provided an opportunity for Jacob and Chris to talk together, which contributed to their co-construction and appropriation (in the sense of Mercer, 2013) of reasoning to justify why the game was actually fair. Our teaching approach in the afterschool club had previously emphasised the importance of talking together and the ground rules for talk, in particular asking "why?" and stating explicit reasoning. Co-construction can be seen in the language used by students while Chris was echoing and talking with Jacob (e.g., lines 24-25, lines 29-30 and lines 37-41). From the extract above, it is evident that two students, for example, took turns, argued their points of view (e.g., lines 8-9 Jacob and line 16 Chris), and reformulated and elaborated on the reasoning (e.g., line 33 Jacob), which Rojas-Drummond, Albarr'an, and Littleton (2008) suggested as the key features of collaborative talk. The phrase in line 45 "the whole competition is in the second bag" is a new psychological tool that sums up Chris's new insight and enables him to carry it forward (Kozulin, 1998).

6.3 Dialogic approach

When we have shown the video of this episode at conferences people always comment on the laughter. Both Jacob and Chris laugh a lot and at times they laugh loudly. Neither Piagetian

approaches nor Vygotskian approaches tend to mention the significance of socio-emotional phenomena such as laughter. Bakhtin (1986, p. 147), however, was interested in laughter. He introduced the idea of dialogic precisely as a contrast to the abstract disembodied logic of dialectical reasoning. Bakhtin (1968) wrote: "Certain essential aspects of the world are accessible only to laughter." (p. 66).

In our extract Jacob laughs first and Chris then laughs with him even though he then claims that he does not know why Jacob is laughing. Chris asks Jacob "why are you laughing" (line 20) while he himself is laughing so much it is hard for him to speak properly. This fits with the finding of recent neuro-cognitive research that laughter is one of the strongest activators of mirror systems meaning that when you see someone laugh it can be hard not to laugh yourself (Scott, 2013). Dialogic theory suggests that switches in perspective are not automatic but involve being able to take the point of view of the other. The openness to the other that comes with the loss of self-boundaries and the sense of unity across borders during laughter would facilitate this dialogic switch.

When Chris repeats word for word what Jacob is saying this process of ventriloquation (Radford, 2000) is not just about language use but is also made possible by the physical mirroring of the two boys brought together as one physical entity by their shared laughter.

Incongruity humour evokes laughter by revealing that our frames of reference are constructed and potentially absurd (Martienson, 2006). A version of this kind of laughter, laughter that dissolves our social construction of reality, can be found in Jacob's laugh when he first realises the answer to the problem (line 14). The view he had held up until that point now seems absurd to him. The laughter dramatises the fact that this is a major shift in perspective.

Laughter was unleashed by Jacob's sudden shift in perspective, but what process led to the shift? Initially he was unsure of the answer, he said "I am debating now" (line 4). Who was he debating with? This seems to be an internal dialogue in which one voice agreed with Chris that the reds would always win while another voice was less sure. The voice is a version of that outside voice present in every dialogue which Bakhtin (1986) referred to as the "superaddresse" or "witness". Before the experiment was run, this third voice was just a doubt, a question mark for Jacob. After the experiment, it crystallised suddenly around the *TinkerPlots* display of the two stages of the experiment. Before Jacob had looked at this representation and it had not meant very much, after his conversion he points to it and now its meaning is obvious to him. The representation on the computer screen is important here but it only works to persuade Jacob because it can enter into a dialogue.

A dialogic analysis inspired by Bakhtin draws attention to the switch in perspective behind the experience of insight. In this episode we see two such switches. First, Jacob switches perspective mediated by a dialogue with a superaddressee voice. The laughter signals that this is a dialogic switch between voices or ways of seeing or frames of reference. Jacob then explains his insight to Chris. Chris's switch in perspective is facilitated not only by the language used but also by his warm personal relationship with Jacob, which enables him to share his ignorance and be open to seeing the problem in a new way. The laughter they share brings them together and facilitates the step back required and openness to the other required as part of the expectations in our dialogic teaching approach where it was OK to show that one does not understand. In both cases we see that this switch in perspective is not a mechanical or automatic but requires "inter-illumination" or seeing as if from the point of view of the other. Similarly, Mason and Davis (1988) point out such switches in perspective at the moments of new insights students might have in mathematical problem solving and in mathematical abstraction (Mason, 1989). They explain this phenomenon as "a shift of attention", which is essential for learning (Mason & Davis, 1988, p. 488). The current

analysis of such switches in students' perspective from a dialogic approach contributes to the factors leading to such shifts identified by Mason and Davis.

7 Conclusion

We began this paper with the essential dialogic distinction articulated first by Socrates between dead signs outside of dialogue and living signs inside of dialogue. Given a mathematical representation of a problem in a computer programme the question is: How does that "tool" move from being just a part of an inert background to becoming a living voice that helps the child solve the problem with an "aha!" or "eureka!" experience? Our argument is that this transformation of the tool in the computer programme into a voice for the child is not automatic but depends upon a dialogic shift in perspective. What is it that allows some students to relate to the representation so that it comes alive for them while others do not? Logical arguments and representational tools are important but they do not, alone, force students to understand concepts. The crucial step to understanding is a dialogic one where an argument or a tool becomes a voice in a dialogue for the learner. Good relationships facilitate the possibility of that switch in perspectives. This entails not only a good relationship between peers, as we saw in the example, or between students and teacher, but also a good relationship between students and their personal superaddressee "voice of mathematics". Our teaching approach and learning environment with a combination of promoting dialogic talk and the use of *TinkerPlots* played a role in fostering these dialogic shifts as well.

Through arguments, illustrated by the analyses of an extract of classroom data, we have attempted to show the strengths and the limitations of Piagetian and Vygotskian approaches for understanding conceptual development in mathematics. The Bakhtinian inspired dialogic approach that we argue for is not meant to be a complete answer to the question of how we perceive children's mathematical conceptual development, but is offered as an augmentation that enables us to see causally significant processes that the other approaches miss. Piagetian perspectives help us to see the logical structures underlying development in mathematical thinking; Vygotskian approaches enable us to recognize the role of concrete cultural tools. A Bakhtin inspired approach, on the other hand, reveals the role of relationships between voices. We argue that this focus on voices in dialogue offers a distinct and valuable perspective that is not reducible to a focus on either developing schemes or mediating means.

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