ESTIMATION OF DAMPING IN SUSPENSION BRIDGES

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Summary

Since there is little analytical foundation for assigning damping values in dynamic response analyses of long span suspension bridges the process usually reduces to using values derived from published physical test data. Such sources of damping data and the accuracy of measurement methods are examined. Methods of energy dissipation in components of the structure are evaluated to help explain the observations and to provide a basis for rational prediction of values.

Keywords

aerodynamic bridge damping dynamic error friction suspension

INTRODUCTION

What are the mechanisms and factors that limit the amplitude of dynamic response to a given force space-time distribution? Some damping mechanisms that are believed to be effective in suspension bridges are friction effects at bearings and expansion joints, friction at bolted or riveted construction joints, hysteresis in wire ropes (hangers), material damping, aerodynamic damping, radiation at foundations etc..

Some of these effects can be described as structural nonlinearities, and these are in addition to the fundamental geometric nonlinearity. Material (non-conservative) nonlinearity in the major structural elements is not considered here since it is only likely to occur at amplitudes approaching failure state where response mechanisms are increasingly complex.

It is remarkable that while in many cases dynamic analyses are most sensitive to variations in damping, there appears to be little or no analytical basis for the choice. The best information derives from full-scale measurements which are subject to a variety of additional effects and errors that may mask the 'true' values.

This paper is an attempt to rationalise the appropriate choice of damping values for dynamic analysis of suspension bridges, based on consideration of structural effects, values obtained from prototype tests and the measurement processes used.

OBSERVATIONS FROM EXPERIMENTAL DATA

Quite a large body of experimental data has been obtained from full-scale testing but because there are large variations in the scale and construction of suspension bridges, and because some of the data are less reliable it is not straightforward to generalise relationships between structural features, damping and frequency, although several attempts have been made¹⁻⁴. Probably the best data derive from tests where attention has been paid to levels of response and to signal processing effects in ambient vibration surveys (AVSs)⁵⁻¹⁹, or better still where damping has been measured using forced vibration tests (FVTs)^{2,12,20-24}.

When collecting together data for several modes of several bridges an inverse relationship appears^{1,3} between damping and modal frequency. It is tempting to read off a value for a given span length but this approach can be dangerous, partly because it leads to the conclusion that a single simple mechanism is at work. Also many of the data for very long span bridges (span > 1km) are from the potentially less reliable AVSs.

Some of the published data are summarised in Table 1. From FVT data on relatively short Norwegian bridges Selberg²⁰ observed that damping increases: with decreasing span, for timber floored bridges, for torsional modes and for lower amplitudes. In the same bridge damping was almost always lower for higher vertical and torsional modes.

Ito et al.⁴, from FVT data for suspension and cable-stayed bridges observed a wide scatter but that damping is higher: for torsional modes, for shorter spans (both torsional and vertical modes), for higher frequencies (horizontal modes) and at higher amplitudes (in model tests). Other observations were that tall piers and bridges on such tended to have horizontal mode damping clustered around 1.8% and (from model tests) that in the bridge superstructure damping is mostly governed by effects at joints, interfaces and supports. Standard damping values were suggested e.g. for vertical modes $\zeta \approx 0.5\%$.

Most FVT data relate to truss or girder bridges, but the Oshima box deck bridge²¹ gave consistently lower damping values than the truss bridges.

Among the AVSs, with some exceptions⁷, long span bridges show decreased damping for higher modes, also in box decks, for similar frequencies. In most cases torsional damping for long span bridges (however measured) is less than for vertical modes.

Data for lateral modes are relatively scarce because they are of less interest for aerodynamic stability. With few exceptions²³ values derive from AVSs where signal processing problems and errors are acute for the very low frequencies and where higher modes have much lower damping, settling at around 1% in most cases.

Other possible data may derive from the many full-scale tests of cable-stayed bridges²⁵, although only for deck designs common to both bridge types and for lateral modes since the geometric effect should be weaker. Scale models used for dynamic studies may be able to provide information about relationships between damping values for different directions and for different frequencies.

Can it be concluded that torsional damping is higher or lower than vertical damping? Will a long span bridge have lower damping because it has a longer span or due to lower frequencies?. There is a need for even more data from well controlled full-scale tests and to find some physical bases for the measured variations in damping.

CONTRIBUTIONS TO DAMPING

The prototype measured value of equivalent viscous damping (fraction or % of critical) ζ_{me} is the sum total of several effects:

$$\zeta_{me} = \zeta_{ma} + \zeta_{fc} + \zeta_{hy} + \zeta_{ra} + \zeta_{ac} + \zeta_{nl} + \zeta_{ns} + \dots + \zeta_{er} \dots\dots\dots 1)$$

where the components are contributions due to material damping (ζ_{ma}), friction/coulomb damping (ζ_{fc}), hysteretic damping (ζ_{hy}), radiation damping at foundations (ζ_{ra}), aerodynamic damping (ζ_{ac}), geometric nonlinearity (ζ_{nl}), effects of non-stationarity (ζ_{ns}) and measurement error (ζ_{er}). The list is not exhaustive, and there may not necessarily be clear distinctions between different forms.

ζ_{ma} , ζ_{fc} , ζ_{hy} and ζ_{ra} are energy dissipating effects, ζ_{ac} and ζ_{nl} affect response levels in variable ways and ζ_{ns} , ζ_{er} are effects which neither dissipate energy nor control response but erroneously appear as damping when measured.

For welded box-deck structures ζ_{ma} probably will not be more than a fraction of a %. ζ_{fc} should be significant for bridges with many connections in a stiffening truss, where deck bearings restrain motion and due to other interfaces.

The main cables may contribute more than just ζ_{ma} if there is slippage between the strands³ and certainly a large contribution of ζ_{hy} arises from friction in the hangers which are usually spiral wound steel ropes. The losses can be increased if the hangers are inclined.

In terms of implementing these damping mechanisms in an analytical model, the tangent stiffness and damping matrices K^t , C^t used in the nonlinear analyses can provide for various forms of damping. The difficulty is in relating numerical implementations to physical causes and simple scalar values. Appendix A summarises some implementations of damping in a single degree of freedom (SDOF) system.

STRUCTURAL DAMPING

Structural damping ζ_s includes ζ_{ma} , ζ_{fc} , ζ_{hy} and possibly ζ_{ra} .

Strain dependent damping

For ζ_{ma} and ζ_{hy} the rate of energy dissipation ΔE per cycle at a given frequency is proportional to elastic strain energy E i.e. $\Delta E = \psi E$ (Appendix A). The ability to dissipate energy in part of a structure depends on the proportion ν of total energy stored in that sub-structure as strain energy.

For discretised systems ν for mode n with mode shape φ_n is defined as

$$\nu_n = \varphi_n^T \Delta K \varphi_n / \varphi_n^T K \varphi_n \quad \dots \dots \dots 2)$$

where ΔK is the contribution of that region to the global stiffness matrix K .

This approach to quantifying damping has been tried on buildings²⁶, as a means to assess the contribution from inclined hangers²⁷, and more recently on cable stayed bridges²⁸ where in model tests values of ν for sub-structures varied with amplitude.

For a suspension bridge in which the tangent stiffness at equilibrium is a combination of geometric and elastic stiffness i.e. $K = K_g + K_e$ the energy stored at the stationary point in a vibration cycle is a combination of gravity potential and elastic strain energy and that part of energy stored elastically in a sub-structure is found by replacing ΔK in Eq. 2 with ΔK_e .

Fig. 1 shows the lowest antisymmetric and symmetric vertical modes (labelled VA1 and VS1) for Humber, Fatih and Bogazici bridges and Fig. 2a-c shows ν for these and higher modes in the vertical plane. The modes indicated increase in frequency from left to right; upper frequency ($< 0.2\text{Hz}$) is indicated.

Cable systems range from the 'taut string' condition where energy due to lateral deflection is stored elastically, to the 'inelastic chain' condition where storage is as geometric potential energy. Suspension bridges in general and these three in particular approach the inelastic chain condition, for low modes at least. For example in mode VA1 there is significant net vertical movement of the entire cable at twice the modal frequency¹³ as energy switches from gravity to kinetic energy.

In a heavy sagged cable (without deck etc.) energy is stored as geometric potential except for a small fraction stored as elastic stretching in a few of the lowest symmetric modes. Energy dissipation capability of a catenary is clearly reduced when it sags and functions as a chain. Strangely this is contrary to findings from damping measurements on marine cables²⁹ of different tension (hence sag).

For all three bridges the elastic energy proportion increases with increasing modal frequency. Generally the lowest two symmetric modes store significant strain energy

in the cable, and while ν for hangers changes little (and is negligible for Fatih), ν for the deck steadily increases. In the case of the towers ν is significant only when tower vibration predominates.

This suggests that for control of response to higher frequency loads e.g. traffic, earthquake it would pay to effect greater energy dissipation in the deck while for low frequency loads e.g. wind less can be done, except by increasing damping in main cables. Increasing damping in hangers and inclining them (particularly the longer ones) is effective for a wide range of frequencies.

Fig. 2d shows ν for some low frequency lateral (L) and torsional (T) modes of Bogazici obtained from a full three-dimensional model. Typically, most of the energy is stored elastically in the deck and small amounts in the hangers. The mode labelled 'C' is one example of many modes in which lateral motion of the cables, and occasionally the towers, dominates.

Strain dependent damping can be implemented in numerical simulations in several ways, including the forms given in Appendix A for which $E \propto \nu^2$:

Rayleigh damping

The damping matrix often used in finite element analyses is proportional, in the Rayleigh sense:

$$\alpha, \beta \quad \mathbf{C} = \alpha \mathbf{K} + \beta \mathbf{M} \quad \dots \dots \dots 3)$$

This form is commonly used since it is easy to implement, the modes can be decoupled, the mode shapes are real and they are identical to those for the undamped case. Values of equivalent viscous damping ξ vary with frequency but only slowly in the narrow frequency range of a resonance. There cannot be much justification for including geometric stiffness so $\mathbf{K}_e$ should be used in Eq. 3 in place of $\mathbf{K}$. In this case damping is no longer proportional.

According to Ewins³⁰ the stiffness related damping from α in Eq. 3 relates to hysteretic material damping and the mass related damping from β relates to friction damping (but see Appendix A), so Rayleigh damping may be supposed to account for all forms of structural damping including ξ_{ma} , which usually accounts for less than 0.1% of critical³¹.

Viscous damping

Although the form is used as a convenient analytical model, true viscous damping is only likely to arise from elastomeric expansion joints or mechanical viscous dampers for which the effect can be modelled by incorporating additional values at appropriate locations of $\mathbf{C}$.

Hysteresis

Hysteresis is a major source of structural damping, particularly in the hangers.

The ideal view of hysteresis is as a force(f_d)/displacement(v) loop of constant proportion. For harmonic response described in the frequency domain the form $f_d \propto i v$ is appropriate for the damping force and produces an elliptical hysteresis loop of constant loss factor ψ irrespective of frequency. The 90° phase shift operator 'i' cannot be used in time history analysis and an alternative is $f_d = \gamma k |v/\dot{v}| \dot{v}$. One possible model³² for hangers is based on the friction restraining rotation of cable strands during elongation. This is costly to implement and necessarily requires non-zero deflection as the hanger tension varies through the dead load state. Adjusting γ (above) to give values of ψ found in the literature of cable testing³³ is a simpler option. For example:

$$\begin{aligned}\psi &= < 0.4 \quad (\text{upper bound}), \\ \nu &= < 0.2 \quad (\text{Fig. 2c}), \\ \zeta_{hy} &= \nu \times \psi / 4\pi, \\ \therefore \zeta_{hy} &\approx 0.6\% \text{ maximum.}\end{aligned}$$

This is a significant contribution to damping, independent of amplitude, particularly for inclined hanger bridges. If this analysis is correct Fatih (vertical hangers) would show consistently lower damping values for all modes: It does. Even so damping was apparently not a consideration in retaining inclined hangers in the strengthened Severn Bridge³⁴.

Friction³⁵

As well as the friction between hanger strands, there is possibly some type of 'stick-slip' mechanism between longitudinal strands of the main cables³, both ending up being described as hysteresis. Friction at hanger terminations is likely to be ineffective in unmobilised joints but energy must be dissipated at the restraining bearings connecting the span sections with each other and the support points. For high amplitudes more bearings and construction joints will contribute to friction forces.

For the three bridges of Figs 1 and 2 the behaviour of these bearings is fundamental to the response in the lower modes.

In particular, analysis of bridges with inclined hangers shows that mode VA1 can occur at one of two well separated frequencies³⁶, depending on boundary conditions. The lower frequency mode will be present if both bearings do not restrict longitudinal motion; this is the 'sliding-sliding' or 'ss' condition. The higher mode occurs if either bearing is 'seized' to prevent longitudinal motion; this is the 'fixed-sliding' or 'fs' condition.

While Humber exhibits the seized bearing characteristic with high frequency mode VA1, both bearings do in fact move simultaneously due to the various static and dynamic loads, there being no component at VA1 frequency¹⁹. For Bogazici the bearings move somewhat more than Humber, mainly due to very high traffic loads, and there is weak response in the first antisymmetric mode at *both* frequencies.

Several models of this behaviour are possible. The simple model of constant restoring force can cause instability in the numerical integration, so two more robust schemes have been tested:

1: Modified friction

For a deck end node mass m with horizontal velocity $\dot{u}$ and horizontal elastic restoring force f_l , the force required to set $\dot{u}$ to 0 in the integration step Δt is

$f_{stop} = (m\dot{u}_l/\Delta t) - f_l$, where the subscript l refers to left hand bearing (or r for right hand bearing). The friction force (static and kinetic) is f_f and the resultant f_l acting on the node is adjusted as follows to account for friction:

$$\begin{aligned} \epsilon \quad & \text{if } |\dot{u}_l| > \epsilon \text{ at start of step then,} \\ & \quad \text{if } f_{stop} > f_f \text{ then } f_l = f_l - f_f \cdot \text{sign}(f_{stop}) \\ & \quad \text{if } f_{stop} \leq f_f \text{ then } f_l = f_l - f_{stop} \\ & \text{if } |\dot{u}_l| \leq \epsilon \text{ at start of step then,} \\ & \quad \text{if } f_l > f_f \text{ then } f_l = f_l - f_f \cdot \text{sign}(f_l) \\ & \quad \text{if } f_l \leq f_f \text{ then } f_l = 0 \\ & \dots\dots\dots 4) \end{aligned}$$

In other words when the end of the deck is moving, a restraining force not more than the limiting friction will act in opposition to bring it to a halt within the time step, and when the deck end is stationary a force not more than the limiting friction will oppose the elastic force. Ideally $\epsilon = 0$, but practically, because inertia and damping forces also act on the deck end in the integration time step, f_{stop} is only an estimate and the case $\dot{u}_l = 0$ occurs only at the start of the integration so $\epsilon > 0$.

Simulations using this model in random response analyses³⁷ shows that this mechanism provides a small contribution to damping without affecting the bearing fixity. For bearing kinetic friction coefficient μ , normal force $f_{N_{l,r}}$ at bearing r, l and mode shape ϕ , friction damping is:

$$f' \quad \zeta_{fc} = \frac{\Delta E}{4\pi E} = \frac{4\mu f_{N_{l,r}} \phi_{l,r}}{\pi \phi^T K \phi} \dots\dots\dots 5)$$

Assuming $\mu = 1$ and that $f_{N_{l,r}}$ is due to overhung weight, for a spanwise maximum vertical amplitude of 1cm Eq. 5 applied to Bogazici gives $\zeta_{fc} = 0.56\%$ for mode VA1 and $\zeta_{fc} = 0.04\%$ for mode VA2, provided the bearings do actually move.

For large displacements the damping would be reduced since $E/\Delta E \propto \phi^2/\phi$; this is consistent with Selberg's observations²⁰. Longitudinal motion and energy dissipation are, with one exception, lower for higher modes. The minimal effect for symmetric modes was found in experimental investigations by Ito et al.⁴ and is also borne out by numerical simulations using the above mechanism. Practically the contribution of this type of friction will be significant only for small amplitudes.

2: Stiction

Another possible model is 'stiction', illustrated in Fig. 3, which describes a system alternating between seizure and free sliding. This is implemented as follows, with k_{ll} representing the element of K for deck end longitudinal DOF 1, k_{ll0} the reference value in the unstuck state, k_c a grounding (sticking) stiffness and x_l the nodal coordinate for the deck end:

When in slip mode,

if $|\dot{u}_l| < u_{th}$ change to stick: $x_l = x_l + u_l$, $k_{ll} = k_{ll0} + k_c$, $u_l, \dot{u}_l, \ddot{u}_l = 0$.

When in stick mode,

if $|f_l| > f_{th}$ change to slip: $k_{ll} = k_{ll0}$ 6)

The values of threshold velocity and elastic force u_{th} and f_{th} are a difficult choice, but k_c should be quite large. Modified friction could be added to stiction.

Fig. 4 shows auto power spectra of displacement for the Bogazici bridge model with the ss condition, for a value of $u_{th} = 0.5 \text{ mm/sec}$ and $f_{th} = 1 \text{ MN}$ simulated using a 0-10Hz random load applied in anti-phase at two anti-nodal points (approximate quarter span positions). For low loads the bearings are stuck and function as if in the fs condition with the mode appearing at a low frequency. As the load level increases the bearings free until a mode corresponding to the ss condition, at a lower frequency, replaces the higher fs mode. In the transition from fs to fs both variants appear weakly and the response in the antisymmetric mode is a minimum, as if heavily damped. The corresponding full-scale behaviour is shown in Fig. 5 which shows two weak but recurring peaks, one for each type of VA1, at 0.125Hz and 0.182Hz either side of the strong VS1 mode at 0.165Hz.

TOWERS AND PIERS

Judged from Fig. 2 the contribution to ζ_s of towers is minimal except for beam type modes in which the towers are the principal participants. The analysis makes the assumption that the tower bases are fixed and although this is unrealistic, the measurements on the three European bridges^{5,7,8} showed pier vibration levels at least an order of magnitude less than anywhere else on the structures. This makes it seem unlikely that much energy could be lost through ζ_{ra} at the foundations.

Tower damping data are scarce since the opportunity for testing only arises during construction, although parallels could be drawn with other free-standing tower structures. Free decay of the longitudinal cantilever mode of a concrete Norwegian tower³⁸ give $0.03 < \delta < 0.05$ ($0.5\% < \zeta < 0.8\%$), and for the steel towers of Akashi Bridge the figure is believed to be rather lower. Dynamic response in towers is usually measured in AVSs and FVTs^{22,23} but it is the total damping effect which is measured.

AERODYNAMIC DAMPING

Wind induces a 'self-excited' force on the bridge deck proportional to velocity which leads to positive or negative aerodynamic damping ζ_{ac} .

By considering work done by lift force and the constant steady state aerodynamic lift derivative, Davenport³⁹ derives the relation

$$\delta = \frac{dL/d\alpha}{\omega V} \quad \text{or } \zeta_{ac} = \frac{1}{4} \left(\frac{\rho B^2}{m} \right) \left(\frac{dC_L}{d\alpha} \right) \left(\frac{V}{\omega B} \right) \dots\dots\dots 7)$$

where ω = radiant natural frequency, V = mean wind speed, B = chord, ρ = air density m = mass/unit length, α = angle of attack and L and C_L are lift forces and coefficient respectively. For Humber $dC_L/d\alpha \approx 2.82$, $B = 28.5\text{m}$, $m = 14,200\text{kg/m}$ so $\zeta_{ac} \approx 0.00028V/f$, i.e. 4.4% for mode VA1 in 20m/sec wind.

A more appropriate representation is given by

$$\zeta_{ac} = -\rho B^4 l (H_1^* G_n) / 4I_n \dots\dots\dots 8)$$

G_n and I_n for mode n relate to mode shape and mass distribution for span l , while H_1^* is the frequency dependent self-excited aeroelastic force coefficient for vertical velocity of the deck. Usually $-H_1^*$ (hence also ζ_{ac}), increases approximately linearly with the 'reduced velocity' $K = V/B\omega$.

The dynamic coefficient H_1^* is related to static aerodynamic coefficients via

$$KH_1^*(K) = h_1 (dC_L/d\alpha - C_D) \dots\dots\dots 9)$$

when deck rotation is also a variable and h_1 is a frequency dependent factor close to unity linking the static and dynamic parameters⁴¹, hence Eqs. 7 and 8 are equivalent except for the (small) C_D .

Unlike ζ_s it is possible to provide estimates of ζ_{ac} via wind tunnel tests on representative models. Such confidence has been demonstrated in the estimation $\zeta_s = \zeta_{me} - \zeta_{ac}$ for Deer Isle Bridge^{10,40}.

Since aerodynamic effects can be very strong ζ_s should be measured in still air and preferably at different amplitudes, although wind may be the only practical means of generating higher amplitudes. Fig. 6 shows data collected from Humber in 1991⁶ for modes VA1, VS1 and first antisymmetric torsional mode TA1 for different wind speed ranges and Fig. 7 plots, ζ_{me} for various vertical modes against frequency for a number of wind speed ranges. The values for the lowest wind speeds must derive solely from ζ_s , signal processing and stationarity effects.

ζ_{me} obtained from the two Turkish bridges with similar signal processing parameters and in low winds are also plotted on Fig. 7 for the lowest measurable vertical modes.

Damping estimates for all three bridges converge to similar values for the lowest modes.

GEOMETRIC NONLINEARITY

The fundamental nonlinear stiffness characteristic of the suspension bridge, due to the oscillating sag and main cable tension and the inability of hangers to sustain compressive loads, is described in detail elsewhere³⁷. When variable dynamic loads are applied, variable response amplitude and stiffness will result in variable natural frequencies. The large deflections > 2m peak to peak vertically at midspan required for the geometric nonlinear effects to become significant will only occur in very strong wind storms or earthquakes. Not even the Japanese FVTs²¹ achieved such levels, although some amplitude dependency was observed in the testing.

To check the effect in random loading (like wind) the analytical models were driven by randomly distributed Gaussian random vertical loading at deck nodes. Fig. 8 shows auto power spectra of midspan vertical deck displacement for two levels of loading for Bogazici Bridge, the high load level producing peak displacements approaching of such magnitude that the main cable would become unloaded. Fig. 9 shows the effect of attempting the usual frequency domain curve fitting to the data. The 'resonant' frequency generally decreases with increasing load giving a broadening of the response (power) spectrum. As this is not a result of increased energy dissipation so much as an effect of spreading the energy, it is not real damping. Even so it appears as such to curve fitting techniques and may even reduce peak amplitudes under certain types of loading. For all but extreme conditions these nonlinear effects do not contribute in any noticeable way to ζ_{mc} .

MEASUREMENT ERRORS

Damping ζ_{mc} and natural frequency f_{mc} can be measured in several ways^{30,42}, the most popular of which are:

- a) 'step relaxation' or free decay from imposed static deflection
- b) impulsive decay after a transient load
- c) free decay rundown from harmonic oscillation in a pure mode
- d) Multi-DOF curve fitting (may be done in time domain)
- e) half power bandwidth
- f) SDOF curve fitting

a-c apply only to FVTs, d requires knowledge of the forcing function and e,f can be applied without artificial forcing or knowledge of it.

Fig. 10 shows how poor signal to noise ratios can bury weak signals. In the frequency domain (d-f) the discernible part of the response peak is reduced and slightly flattened to noise (Fig. 10a) so that a resonance curve appears to have an increased half power

bandwidth. In time domain (a-c) free decay tails become indistinct (Fig. 10b) or, if amenable to curve fitting, show increased damping. Little can be done to improve noisy data even if the noise level can be measured. To protect the investment of time and money the best equipment and greatest care in selection of signal ranges are required.

FREQUENCY DOMAIN SIGNAL PROCESSING EFFECTS

Potentially the free decay methods a-c (above) should yield the most reliable damping estimates but the AVS in which the forcing function is unknown but assumed to be random with flat power spectrum has many advantages such as low cost, simplicity and the lack of interference with bridge operation. AVS data are usually examined in the frequency domain in which case errors are introduced due to the random character of the load and by failing to meet the requirements of periodicity and stationarity for the frequency analysis.

Stationarity

A precondition for spectral analysis is that the generating process is stationary i.e. the system parameters are constant⁴³. For a suspension bridge the modal parameters can change as temperature, traffic and wind vary during a measurement so that sharp resonant peaks are broadened and errors ζ_{ns} arise.

For example, applying a 10°C temperature rise in Bogazici would increase cable sag by 0.31m. A 1000 tonne distributed traffic load, representing 1% of deck mass would increase cable sag by 0.55m. Table 2 shows the simulated effects for five modes. Fig. 11 shows prototype frequencies varying through the late afternoon 'rush hour' to the cooler early morning.

Fig. 12 illustrates the nonlinear relationship between frequency variations Δf and ζ_{mc} . The $\Delta \zeta_{mc}$ were obtained by curve fitting the (RMS) average of resonance curves generated with natural frequencies varying between f and $f + \Delta f$. The effect on ζ_{mc} is strongly (inversely) related to the true damping value ζ and less strongly (directly) to baseline natural frequency f_0 .

Data for spectral analysis should be obtained from measurements during constant wind, temperature and load conditions, but this is not realistic and some compromise is adopted. Despite Fig. 12 it is easiest is to ignore traffic and temperature and find periods approaching stationarity where wind is constant between reasonable limits since the effect of wind is greatest. The narrower the limits that define stationarity, the fewer and shorter the periods of data left to analyse and the more greater will be the errors from the spectral analysis.

Variance and bias

Commonly the Discrete Fourier Transform ('FFT') is applied to a sequence of data points in the 'stationary' time history of duration T seconds, yielding amplitude values at frequencies separated by the resolution $df = 1/T$. Being a random process it is necessary to average the amplitudes over n records so that the estimates will approach the expected values with low variance as n increases. Even if n is large the limited duration T and the enforced periodicity of the signal will bias the expected value (mean) of the spectral amplitude positively with respect to the true value⁹.

Given values of n and T , variance and bias errors can be quoted⁴³ for estimates of amplitude of individual spectral lines. Estimates of natural frequency and damping ratio obtained by curve fitting to a collection of amplitude estimates will also have bias and variance errors, but these errors will not necessarily be the same as for individual spectral line estimates and will probably (we hope) be smaller.

Bias and variance errors are reduced if T and n respectively are increased. In the practical case of a limited quantity of quasi-stationary data the product nT is fixed and there is a trade off between variance and bias errors. It is worth quantifying these errors which together constitute ζ_{er} so the most accurate estimates and the associated errors can be obtained from the available data.

A simple numerical exercise was set up to investigate bias and variance of measured frequency and damping parameters f_{mc} , ζ_{mc} obtained from fitting an exact SDOF response curve to spectra of stationary displacement response $v(t)$ obtained from a SDOF oscillator with frequency $f_0 = 0.1\text{Hz}$ driven by a random force $f(t)$. The parameters for the exercise were:

(for the oscillator) ζ_0

(for the processing) T , n , type of lag/window (Hanning/rectangular).

Fig. 13 summarises the SDOF oscillator random response simulation. It also shows how the ability to resolve an exact resonance with given f_0 , ζ_0 and df could be quantified in terms of the parameter $W = 2\zeta_0 f_0 / df = 2\zeta_0 f T$. $\Delta f = 2\zeta_0 f_0$ is the half power bandwidth (HPB) between the points where response amplitude is 0.7071 of maximum (resonant) value and W represents the number of spectral lines spanned by HPB. For example $f_0 = 0.12\text{Hz}$, $\zeta_0 = 1\%$ and $df = 0.001\text{Hz}$ gives $\Delta f = 0.0024\text{Hz}$ and $W = 2.4$.

The single parameter W appears to be the major factor in determining the bias of damping values; resonances with constant f_0 but different ζ_0 will be resolved equally well if the resolution change is the inverse of the damping change, and vice-versa. Bias is then characterised by the increase of measured W (W_{me}) from true W arising from the combination of frequency, damping and resolution.

To study the relationship between n and variance errors of damping and frequency, also between W and bias, parameters f_{mc} and ζ_{mc} were obtained for each combination of n and W . Each combination was repeated 64 times to obtain large samples from which to obtain estimates of f_0 and ζ_0 .

Fig. 14 gives, for the Hanning window, the normalised errors of frequency and damping estimates defined respectively as

$$\begin{aligned} \epsilon(f_{mc}) &= 100 \sigma(f_{mc}) / \mu(f_{mc}), \\ \epsilon(\zeta_{mc}) &= 100 \sigma(\zeta_{mc}) / \mu(\zeta_{mc}). \end{aligned} \quad \dots \dots \dots .10)$$

Both $\epsilon(f)$ and $\epsilon(\zeta)$ with increasing n , approximately as $1/n^{1/2}$. $\epsilon(f)$ decreases with increasing W and decreasing ζ . Damping estimates are a little more accurate when obtained using larger W .

Fig. 15 shows, for $n = 16$, the bias error defined as $\frac{\zeta_{mc}}{\zeta_0} - 1$ in which data for different ζ values are collapsed together. On the log-log scale the inverse relationship is quite clear for $W \geq 0.5$. The best fit straight line gives, as an approximation, $\text{bias} \propto W^{-1.5}$.

This suggests a strategy for signal processing i.e. $W \geq 1$ or $df = 2\zeta_0 f_0$; $n \geq .16 f_0$ should be known quite accurately but not ζ_0 , so either ζ_{mc} could be used to start an iteration or a likely minimum ζ_0 could be assumed. Given the improved estimate a confidence interval could be estimated.

POINTERS TO BETTER ASSESSMENT OF STRUCTURAL DAMPING

- 1) Structural damping in the deck has an increasing contribution ν for higher modes, provided the loss factor ψ is significant (generally not the case for welded box decks).
- 2) Deck bearings may provide significant damping but only for low frequency antisymmetric modes at low amplitude levels. Stick-slip behaviour can almost completely inhibit the first antisymmetric bending mode VA1. By their nature, reduction of proportional energy dissipation by friction as amplitude increases may offset activation of more friction sources at high amplitudes.
- 3) Hangers can provide an effective level of damping via hysteresis if they are inclined.
- 4) Main causes of amplitude dependency seem to be friction, geometric nonlinearity and aerodynamics. Frequency dependent effects derive from energy distribution ν according to mode shape, aerodynamics and measurement procedures.
- 5) By reducing the proportion of stored elastic energy the geometric stiffness effect appears to reduce rate of energy dissipation in vertical and some torsional modes as compared to lateral modes.
- 6) Geometric nonlinearity is negligible for the range of vibrations experienced during all but the most severe forms of loading, for which it would be helpfully disruptive.
- 7) In high winds aerodynamic damping could be counted on to reduce amplitude of response to other loading. Wind tunnel studies should be capable of providing a measure of this effect.
- 8) When using frequency domain techniques on ambient response data measurements should be made during conditions of minimal wind, low traffic and modest temperature variations. Given a limited duration of such data a strategy for minimising and even estimating the errors due to signal processing limitations has been suggested.

For design calculations the conservative approach would be to assume only structural damping and an appropriate value is sought. For truss type decks the numerous joints provide significant damping but for a box-deck suspension bridge there can be little damping due to the deck and even estimates of 1% may not be sufficiently conservative in respect of low to moderate amplitude vibrations.

Given reliable data on damping and the distribution factor ν , the next stage of research is to work back and identify the loss factors in the structural components.

Should single amplitudes approach several meters (in a 1km bridge) the combined nonlinear effects and unforeseen structural behaviour such as dramatic change of boundary conditions and mode type make it hard to think in terms of linear damping mechanisms.

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FIGURES

Fig. 1	Fundamental modes for Bogazici, Fatih and Humber
Fig. 2	Energy distribution factor ν for
Humber, vertical plane modes
Fatih, vertical plane modes
Bogazici, vertical plane modes
Bogazici lateral/torsional modes
Fig. 3	Stiction model (for one of two bearings l, r)
Fig. 4	Transition of bearing behaviour in stiction model applied to Bogazici
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(mode VS1 constant at $\approx 0.16\text{Hz}$)
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a) RMS of spread resonant curves
b) effect of frequency shift for different f_0, ξ_0
Fig. 13	Simulation of AVS data processing problem
a) bias effect for damping due to finite resolution and windowing
b) estimation of normalised error and bias for oscillator with damping ratio ξ_0 , frequency f_0 , for n averages

Fig. 14 Effect of resolution on Variance of frequency and damping estimates
a) $e(f)$ error for different W
b) $e(z)$ for different W

Fig. 15 Effect of resolution (as W) on bias of damping estimate

Symbols

c	damping coefficient
f	nodal force; natural frequency
h	hysteresis damping factor
h_1	aeroelastic correction factor
i	$(-1)^{1/2}$, 90° phase operator
k	stiffness; stiffness element
$u, \dot{u}, \ddot{u}$	horizontal displacement, velocity, acceleration
$v, \dot{v}, \ddot{v}$	vertical; nodal displacement, velocity, acceleration
l	span length
m	mass; deck mass/unit length
n	mode number; no. of data segments
p	forcing function
x	longitudinal position
B	deck width
C, C^t	damping matrix, tangent damping matrix
C_d, C_l	drag, lift coefficient
$E; \Delta E$	stored energy; energy loss
G	modal integral
H_i^*	aeroelastic coefficient
I	modal inertia
K	reduced wind velocity $V/B\omega$
K, K^t	stiffness matrix, tangent stiffness matrix
K_e, K_g	elastic and geometric components of K
L	lift force
M	mass matrix
T	segment length (time); vibration period $1/f$
V	wind speed

α	stiffness related damping factor; angle of attack
β	mass related damping factor
δ	logarithmic decrement
ϵ	normalised error; small number
ϕ	mode shape
γ	stiffness related friction damping factor
η	mass related friction damping factor
μ	friction coefficient; mean
ν	energy distribution factor
ρ	air density
ω, ω_n	radiant frequency, natural frequency
ψ	loss factor
ζ	damping (fraction or % critical)

subscripts/superscripts:

ae	aerodynamic
d	damping
e	grounding value; elastic component
er	measurement error
f	friction
fc	friction/coulomb
g	geometric component
hy	hysteretic
l	left bearing
ll	matrix element for left bearing
ma	material
n	for mode n
nl	geometric nonlinearity
ns	nonstationarity
N	normal
o	base value
r	right bearing
ra	radiation
th	threshold
T	transpose

Table 2. Bogazici vertical mode natural frequency shifts for changed deck load and ambient temperature

Mode	Average frequency /Hz	% change /10°C	% change /1000t
VA1	0.1248	-0.22	-1.32
VS1	0.1688	+0.03	-1.92
VS2	0.2253	+0.03	-1.71
VA2	0.2861	-0.16	-1.52
VS3	0.3746	-0.13	-1.66

Bogazici mode VA1: 0.129/0.182Hz

Bogazici mode VS1: 0.160Hz

Fatih mode VA1: 0.125Hz

Fatih mode VS1: 0.155Hz

Humber mode VS1: 0.116Hz

Humber mode VS1: 0.154Hz

Fig. 1 Fundamental modes for Bogazici, Fatih and Humber

Fig. 2 Energy distribution factor ν for a) Humber, vertical plane modes

Fig. 2 Energy distribution factor ν for b) Fatih, vertical plane modes

Fig. 2 Energy distribution factor ν for c) Bogazici, vertical plane modes

Fig. 2 Energy distribution factor ν for d) Bogazici, lateral/torsional modes

a) Slip mode: $\dot{u}_l$ drops below u_{th} .

High k_e 'locks' bearing after slip of u_l .

b) Stick mode: f_l , due to elastic and dynamic forces rises above f_{th} .

k_e set to 0, bearing moves, either way.

Fig. 3 Stiction model (for one of two bearings l, r)

Fig. 4 Transition of bearing behaviour in stiction model applied to Bogazici,

Fig. 5 Bogazici: Observed transition of mode VA1 frequency from upper to lower value for different response levels (mode VS1 constant at $\approx 0.16\text{Hz}$)

Fig. 6 Variation of ζ_{mc} with wind speed for Humber vertical and torsional mode

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Fig. 9

Variations of modal damping and stiffness with amplitude, Bogazici

Fig. 10 Effect of noise on damping estimation in time and frequency domains

Fig. 11 11-hour variation of vertical mode natural frequencies for Bogazici Bridge

a) RMS of spread curves

b) effect of frequency shift for different f_0, ξ_0

Fig. 12 Effect of nonstationary modal frequency on estimated damping

$\Delta f = 2\zeta_o f_o$ = half power bandwidth
 $df = 1/T$ = frequency sampling interval
 $W = \Delta f/df = 2\zeta_o f_o T$

a) Bias effect for damping due to finite resolution and windowing

b) Estimation of normalised error and bias for oscillator with damping ratio ζ_o , frequency f_o , for n averages

Fig. 13 Simulation of AVS data processing problem

a) $e(f)$ error for different W

b) $e(z)$ for different W

Fig. 14

Effect of resolution on Variance of frequency and damping estimates

Fig. 15 Effect of resolution (as W) on bias of damping estimate