

# Influence of a large-scale field on energy dissipation in magnetohydrodynamic turbulence

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## ABSTRACT

In magnetohydrodynamic (MHD) turbulence, the large-scale magnetic field sets a preferred local direction for the small-scale dynamics, altering the statistics of turbulence from the isotropic case. This happens even in the absence of a total magnetic flux, since MHD turbulence forms randomly oriented large-scale domains of strong magnetic field. It is therefore customary to study small-scale magnetic plasma turbulence by assuming a strong background magnetic field relative to the turbulent fluctuations. This is done, for example, in reduced models of plasmas, such as reduced MHD, reduced-dimension kinetic models, gyrokinetics, etc., which make theoretical calculations easier and numerical computations cheaper. Recently, however, it has become clear that the turbulent energy dissipation is concentrated in the regions of strong magnetic field variations. A significant fraction of the energy dissipation may be localized in very small volumes corresponding to the boundaries between strongly magnetized domains. In these regions the reduced models are not applicable. This has important implications for studies of particle heating and acceleration in magnetic plasma turbulence. The goal of this work is to systematically investigate the relationship between local magnetic field variations and magnetic energy dissipation, and to understand its implications for modeling energy dissipation in realistic turbulent plasmas.

**Key words:** turbulence – plasmas – MHD – magnetic fields

## 1 INTRODUCTION

Large-scale magnetic fields are an essential part of magnetohydrodynamic (MHD) turbulence. Even when a large-scale magnetic field is not imposed on the system externally (as may be done in laboratory devices, for example), it is generated by turbulence due to magnetic dynamo action. Such a magnetic field plays a crucial role in magnetic turbulence at small scales. Indeed, unlike a uniform large-scale velocity field, the large-scale magnetic field cannot be removed by a Galilean transformation, and it mediates the energy cascade at all scales. Turbulent plasmas with high Reynolds numbers are therefore anisotropic at small scales with the eddies stretched along a local strong background field. This allows various reduced models, such as reduced MHD models, gyrokinetics models, or models with reduced spatial dimensionality (e.g., Dmitruk et al. 2005; Perez & Boldyrev 2008; Tobias et al. 2011; Mason et al. 2012; Schekochihin et al. 2009;

Camporeale & Burgess 2011; Wu et al. 2013; Karimabadi et al. 2013; Franci et al. 2015), to accurately describe the local dynamics. Recently, there has been a widespread application of these reduced models to understand the dissipation of the turbulent cascade in the solar wind (Camporeale & Burgess 2011; Wu et al. 2013; Karimabadi et al. 2013; Franci et al. 2015; Howes et al. 2008b,a; Boldyrev et al. 2011; Howes et al. 2011; TenBarge & Howes 2013; TenBarge et al. 2013; Told et al. 2015) and in the solar corona (Einaudi & Velli 1999; Oughton et al. 2001; Dmitruk et al. 2002; Rappazzo et al. 2007, 2008; Wan et al. 2014).

On the other hand, a large fraction of energy is now known to be dissipated in a small fraction of the volume that is often characterized by strong variations in the large-scale magnetic field (e.g., Zhdankin et al. 2016, and references therein). In fact, one may think of MHD turbulence without an imposed large-scale field as consisting of subdomains where the large-scale magnetic field is strong, separated by thin boundaries where the direction of large-scale field changes abruptly. This structure is consistent, for ex-

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ample, with the Borovsky picture of solar wind turbulence as an ensemble of tightly packed flux tubes (Borovsky 2008), and with statistical significance of strong rotational magnetic discontinuities observed in other studies (e.g., Bruno et al. 2001; Li 2008; Zhdankin et al. 2012; Greco et al. 2009). The regions of strong magnetic field variations are very intermittent, i.e., they occupy a small volume and contain only a small fraction of the total energy. However, they may contain a significant fraction of the magnetic energy dissipation. If so, the reduced model may not be used to properly describe turbulent energy dissipation. The question of the extent to which energy dissipation is skewed toward the regions of strong variation of the magnetic field is therefore of principal importance for phenomenological and numerical modeling of MHD turbulence. This work presents the quantitative statistical analysis of this issue.

In this work, we utilize numerical simulations of MHD turbulence to investigate the local relationship between the energy dissipation rate and the relative variation of the magnetic field. We find that a significant fraction of the energy dissipation occurs in regions where this variation is large, although this fraction slowly decreases with increasing size of the inertial range. We therefore argue that caution is required when applying reduced models to systems where the inertial interval is not sufficiently long. For instance, we estimate that in systems where the inertial interval spans less than three orders of magnitude, more than 15% of the energy dissipation occurs in the regions with strong variations of the magnetic field, where the reduced models are not applicable. These regions occupy a very small fraction of the volume, however, the dissipation inside them is very intense. Models that fail to properly account for such regions may lead to an incomplete description of energy dissipation and particle heating in MHD plasma turbulence.

## 2 METHODS

For our analysis we numerically solve the incompressible MHD equations for the plasma velocity  $\mathbf{v}(\mathbf{x}, t)$  and the magnetic field  $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0 + \mathbf{b}(\mathbf{x}, t)$  (where  $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$  is the uniform background field):

$$\begin{aligned} \partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\nabla p + (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{v} + \mathbf{f}_1, \\ \partial_t \mathbf{B} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} + \mathbf{f}_2, \end{aligned} \quad (1)$$

where  $p$  is the plasma pressure, along with  $\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0$ . For simplicity, we take the resistivity to be equal to the viscosity,  $\eta = \nu$ , in the simulations. The turbulence is driven by random large-scale external forces  $\mathbf{f}_1$  and  $\mathbf{f}_2$  that are applied in Fourier space and have amplitudes chosen so that  $v_{rms} \approx 1$ . The forces have no component in the direction of the background field and are solenoidal in the  $xy$ -plane. The Fourier coefficients are non-zero only for wavenumbers  $k_{x,y,z} = \pm 1$  or  $\pm 2$ , and in such cases the coefficients are chosen from a Gaussian distribution and are refreshed on average every  $0.1L/(2\pi v_{rms})$  time units, where  $L$  is the size of the domain (that is, the force is updated approximately 10 times per large-scale turnover time). While the particular choice of the force's statistical properties and correlation time does not affect the spectra of turbulence at smaller

scales (Mason et al. 2006; Mason et al. 2008), this setup allows us to supply energy in large-scale Alfvénic fluctuations in a controlled fashion. The equations are solved on a triply periodic domain using standard pseudospectral methods. Time advancement of the diffusive terms is carried out exactly using the integrating factor method, while the remaining terms are treated using a third-order Runge-Kutta scheme. Further details of the numerical approach can be found in Cattaneo et al. (2003). A background magnetic field with relatively small amplitude  $B_0 = 0.5b_{rms}$  is imposed. For the present analysis, we focus on a simulation with a  $1024^3$  lattice and a Reynolds number  $Re = v_{rms}(L/2\pi)/\nu \sim 5500$ ; simulations with smaller  $Re$  give similar results. We carry out our analysis on 5 snapshots, each separated by 2 eddy turnover times.

We aim to understand how the relative amplitude of the magnetic fluctuations is correlated with the local energy dissipation rate. For that we have designed the following statistical approach. We subdivide the simulation domain into cubes of size  $\Delta x$  and consider the statistical properties of fluctuations in the cubes. Consider a cube of size  $\Delta x$ , which is centered at the point  $\mathbf{x}$ . The local mean magnetic field in this cube is then given by

$$\bar{\mathbf{B}}_{\Delta x}(\mathbf{x}) = \frac{1}{(\Delta x)^3} \int_{\Delta x} d^3 x' \mathbf{B}(\mathbf{x} + \mathbf{x}'), \quad (2)$$

while the root-mean-square (rms) magnetic field fluctuation is given by

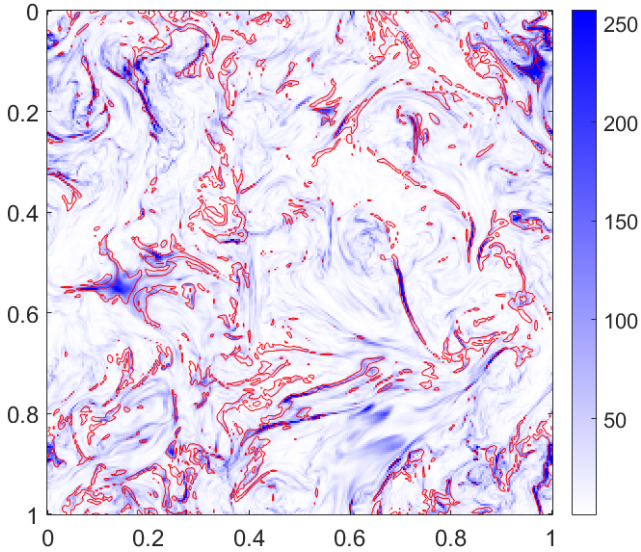
$$B_{rms, \Delta x}(\mathbf{x}) = \left[ \frac{1}{(\Delta x)^3} \int_{\Delta x} d^3 x' |\mathbf{B}(\mathbf{x} + \mathbf{x}') - \bar{\mathbf{B}}_{\Delta x}(\mathbf{x})|^2 \right]^{1/2}, \quad (3)$$

where  $\int_{\Delta x}$  denotes an integral across the volume of the cube. The local strength of the fluctuations is then characterized by the ratio  $R_{\Delta x} = B_{rms, \Delta x} / \bar{B}_{\Delta x}$ . The presence of a strong local magnetic field is implied by  $R_{\Delta x} \ll 1$ . The local energy dissipation rate in a cube centered at point  $\mathbf{x}$  is given by

$$\mathcal{E}_{\Delta x}(\mathbf{x}) = \int_{\Delta x} d^3 x' \left[ \eta |\mathbf{j}(\mathbf{x} + \mathbf{x}')|^2 + 2\nu \sigma(\mathbf{x} + \mathbf{x}') : \sigma(\mathbf{x} + \mathbf{x}') \right], \quad (4)$$

where  $\mathbf{j} = \nabla \times \mathbf{B}$  is the current density and  $\sigma = [\nabla \mathbf{v} + (\nabla \mathbf{v})^T]/2$  is the rate-of-strain tensor. We note that  $\mathcal{E}_{\Delta x}$  includes contributions from both resistive and viscous dissipation, but our results are broadly similar when either dissipation mechanism is considered individually. We measure the above quantities for cubes of varying size  $\Delta x$  in order to understand the scale dependence of the field fluctuations and energy dissipation. Our major object of study is the correlation between the local intensity of fluctuations  $R_{\Delta x}$  and the local energy dissipation rate  $\mathcal{E}_{\Delta x}$ .

For reference, in Fig. 1 we show contours of  $R_{\Delta x} = 1/3$  overlaid on an image of the local energy dissipation rate  $\mathcal{E}_{\Delta x}$  in an arbitrarily chosen 2D plane of the simulation, for  $\Delta x/L = 1/256$ . There is evidently a strong degree of correlation between the two quantities, with both often being concentrated in thin, sheet-like coherent structures.



**Figure 1.** Contour plot of fluctuation-to-mean ratio  $R_{\Delta x}$  (red) overlaid on an image of the local energy dissipation rate  $\mathcal{E}_{\Delta x}$  (blue) for  $\Delta x/L = 1/256$ , in an  $xy$  slice of the simulation. The contours are taken at  $R_{\Delta x} = 1/3$ , and the colorbar indicates  $30 \times \mathcal{E}_{\Delta x} / \langle \mathcal{E}_{\Delta x} \rangle$ . A strong correlation between the magnetic field variations and the energy dissipation is observed.

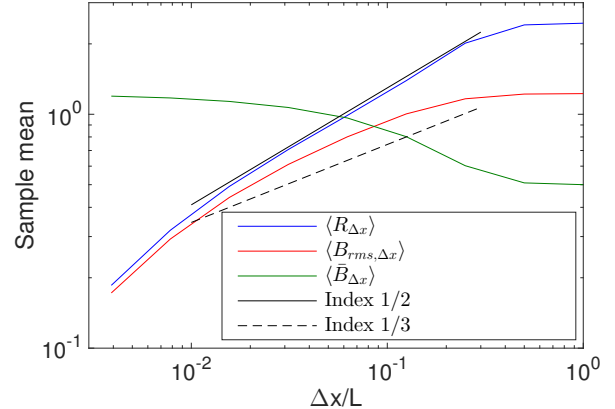
### 3 RESULTS

#### 3.1 The mean field and the fluctuations

We begin by analysing the statistical properties of the *mean* local quantities  $\langle \bar{B}_{\Delta x} \rangle$ ,  $\langle B_{\text{rms},\Delta x} \rangle$  and  $\langle R_{\Delta x} \rangle$ , where the angular brackets denote averaging over all the cubes of size  $\Delta x$  in the simulation domain. Fig. 2 shows the scaling of these quantities versus  $\Delta x$ . We see that the local mean field measured at progressively smaller scales,  $\langle \bar{B}_{\Delta x} \rangle|_{\Delta x \rightarrow 0}$ , approaches the value of the large-scale fluctuations,  $\langle B_{\text{rms},\Delta x} \rangle|_{\Delta x \rightarrow L}$ , confirming that the large-scale magnetic field fluctuations act as a local background field for the small-scale fluctuations. This important fact, quantified in Fig. 2, is behind the applicability of models of MHD turbulence that assume a strong imposed uniform magnetic field.

In addition, we see that, to a very good approximation,  $\langle R_{\Delta x} \rangle \propto (\Delta x)^{1/2}$  in the inertial range, while the fluctuations  $\langle B_{\text{rms},\Delta x} \rangle$  and mean  $\langle \bar{B}_{\Delta x} \rangle$  are not as well fit by power laws. One may, however, roughly approximate  $\langle B_{\text{rms},\Delta x} \rangle \propto (\Delta x)^{1/3}$ , which is broadly consistent with the scaling (1/3) of magnetic field increments in the Goldreich-Sridhar phenomenology (Goldreich & Sridhar 1995), and not far from the scaling (1/4) predicted in the model of scale-dependent dynamic alignment (e.g., Boldyrev 2006; Perez et al. 2012). We however note that these phenomenologies assume the presence of a *strong* and *constant* large-scale magnetic field in the whole domain, whereas in our measurement we instead average over the cubes with all possible values of the mean field.

Finally, we note that the observed scaling of the fluctuations-to-mean ratio,  $\langle R_{\Delta x} \rangle \propto (\Delta x)^{1/2}$ , implies that anisotropy grows significantly with decreasing scale. For example,  $\langle R_{\Delta x} \rangle$  decreases from 1 to roughly 1/10 after



**Figure 2.** The mean ratio  $\langle R_{\Delta x} \rangle = \langle B_{\text{rms},\Delta x} / \bar{B}_{\Delta x} \rangle$  (blue), mean fluctuations  $\langle B_{\text{rms},\Delta x} \rangle$  (red), and mean field  $\langle \bar{B}_{\Delta x} \rangle$  (green) versus scale  $\Delta x$ . The power-law scalings  $\Delta x^{1/2}$  (black, solid) and  $\Delta x^{1/3}$  (black, dashed) are shown for reference.

$\Delta x$  decreases by only two decades. This implies that the reduced models of MHD turbulence should formally be valid for a description of energy distribution in the bulk of the small-scale fluctuations in most space and astrophysical systems.

#### 3.2 The energy dissipation

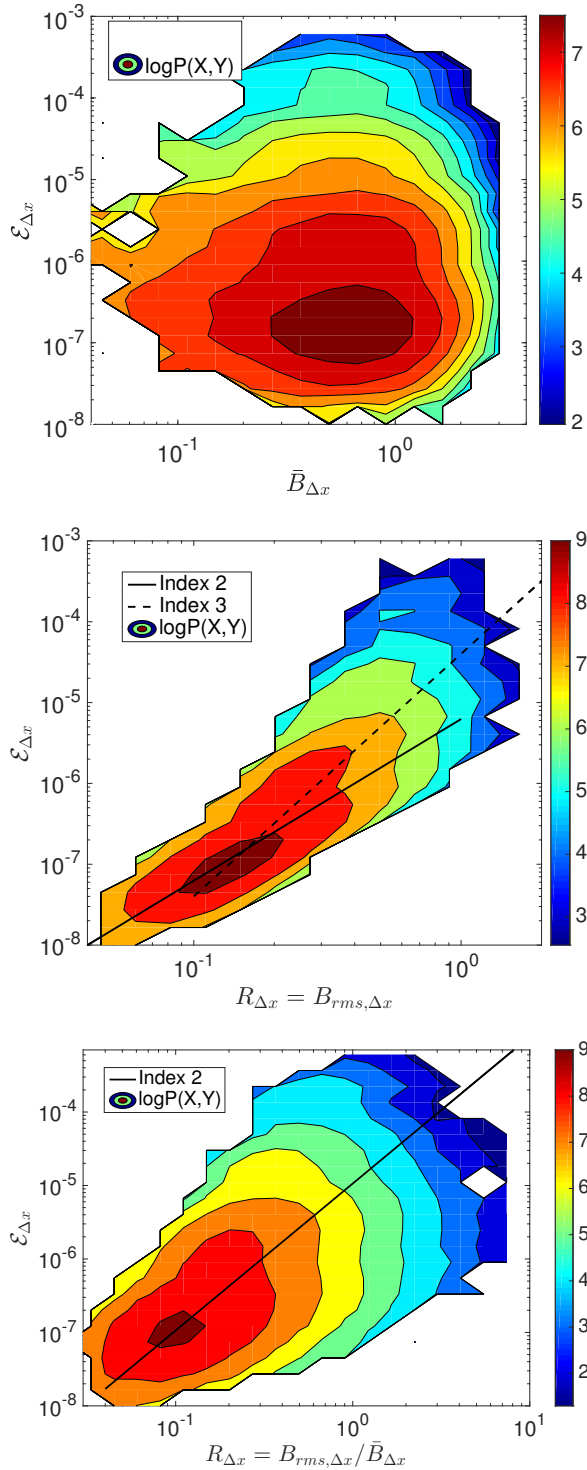
The picture changes significantly when we consider the *energy dissipation*, which is known to be very intermittent, that is, not space filling (e.g., Osman et al. 2012; Zhdankin et al. 2016). We now analyze the correlation of the local energy dissipation rate  $\mathcal{E}_{\Delta x}$  with  $R_{\Delta x}$ .

In Fig. 3, we show 2D joint probability density functions of  $\mathcal{E}_{\Delta x}$  and each of  $\bar{B}_{\Delta x}$ ,  $B_{\text{rms},\Delta x}$ , and  $R_{\Delta x}$  separately. We find a strong correlation between the dissipation and the fluctuations, such that the most intense dissipation indeed takes place in regions of large relative fluctuations in the magnetic field. In particular, we find that the results can be fit rather well by a quadratic scaling,  $\mathcal{E}_{\Delta x} \propto R_{\Delta x}^2$ . The scaling of dissipation with absolute fluctuations can be approximated by  $\mathcal{E}_{\Delta x} \propto B_{\text{rms},\Delta x}^3$  for inertial-range fluctuations and  $\mathcal{E}_{\Delta x} \propto B_{\text{rms},\Delta x}^2$  for weaker fluctuations. On the other hand, there is very little correlation between the dissipation and local mean field  $\bar{B}_{\Delta x}$ , consistent with the mean field being set by the background, large-scale eddies.

While the scaling of  $\mathcal{E}_{\Delta x}$  with  $R_{\Delta x}$  is non-trivial to explain, we note that a cubic scaling of the dissipation with respect to  $B_{\text{rms},\Delta x}$  can be expected on general grounds. This is because the local dissipation should be comparable to the local magnetic energy divided by the cascade time, which can be estimated by the local eddy turnover time,  $\tau_{\Delta x} \sim \Delta x / B_{\text{rms},\Delta x}$  (in simulation units). We then arrive at

$$\mathcal{E}_{\Delta x} \sim (\Delta x)^3 \frac{B_{\text{rms},\Delta x}^2}{\tau_{\Delta x}} \sim (\Delta x)^2 B_{\text{rms},\Delta x}^3. \quad (5)$$

To assess more quantitatively the extent to which dissipation occurs in regions with the large fluctuations-to-mean ratio,  $R_{\Delta x} \gtrsim 1$ , we consider the cumulative distribution of



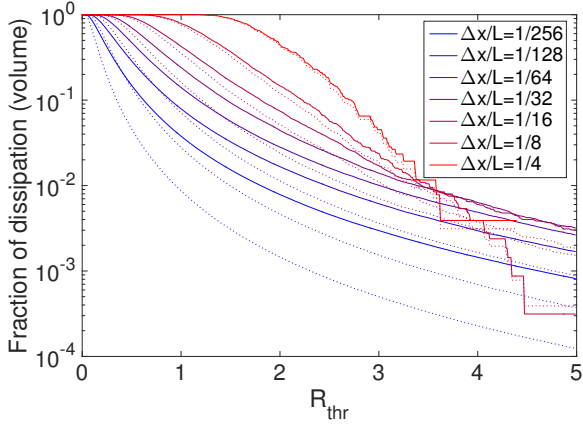
**Figure 3.** The 2D joint probability density function of the local energy dissipation rate  $\mathcal{E}_{\Delta x}$  and mean magnetic field  $\bar{B}_{\Delta x}$  (top panel), rms fluctuations  $B_{rms,\Delta x}$  (center panel), and rms-to-mean ratio  $R_{\Delta x} = B_{rms,\Delta x}/\bar{B}_{\Delta x}$  (bottom panel), for  $\Delta x/L = 1/128$  (the plots for other  $\Delta x$  are similar). A quadratic scaling (solid black line) and cubic scaling (dashed black line) are shown for reference.

energy dissipation conditioned on  $R_{\Delta x}$ , which we denote by  $\mathcal{E}_{cum,R_{\Delta x}}(R_{thr})$ . In particular, we set a threshold  $R_{thr}$  and measure the fraction of the total energy dissipation that occurs in cubes with  $R_{\Delta x} > R_{thr}$ . The results are shown in Fig. 4, along with the volume occupied by cubes exceeding the threshold. The cumulative distributions extend to large values of  $R_{\Delta x}$ , implying that, indeed, a significant fraction of energy dissipation may occur in regions with  $R_{\Delta x} \gtrsim 1$ . However, the tail of the distribution function shifts downwards for decreasing  $\Delta x$ . Hence, at sufficiently small scales, the majority of energy dissipation should occur in regions where  $R_{\Delta x}$  is small. This means that asymptotically in the limit of very large Reynolds number, both the small-scale fluctuations and the energy dissipation are adequately captured by the reduced models.

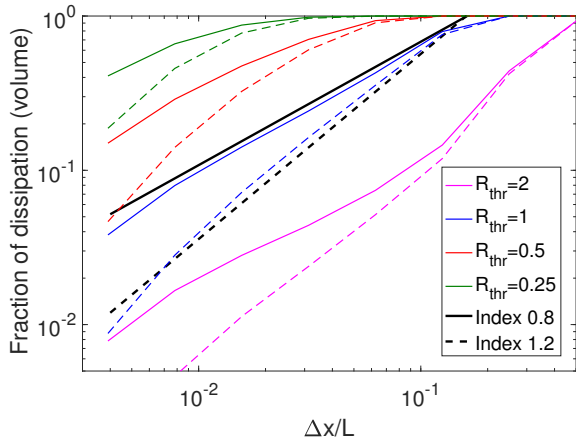
As we now demonstrate, however, this convergence is rather slow. Figure 5 shows the dependence of the cumulative energy dissipation  $\mathcal{E}_{cum,R_{\Delta x}}(R_{thr})$  on the scale  $\Delta x$ , for several values of  $R_{thr}$ . From the plot corresponding to  $R_{thr} = 1$  we estimate a scaling  $\mathcal{E}_{cum,R_{\Delta x}}(1) \propto (\Delta x)^{0.8}$ . The curves with  $R_{thr} < 1$  are similar; they seem to have the same scaling but shifted upward with respect to the curve with  $R_{thr} = 1$ . To understand the implications of this slow convergence we analyze the following example. Consider the curve corresponding to the threshold  $R_{thr} = 1/4$ . Such a threshold approximately separates the cubes where the reduced MHD model provides a good description for the spectrum of MHD turbulence ( $R_{\Delta x} < 1/4$ ) from the cubes where it does not ( $R_{\Delta x} > 1/4$ ) (e.g., Mason et al. 2006; Mason et al. 2008; Mason et al. 2012). Assuming that we may extrapolate the observed scaling to very small  $\Delta x$ , we can estimate for this curve:  $\mathcal{E}_{cum,R_{\Delta x}}(1/4) \sim 40 \times (\Delta x/L)^{0.8}$ . The fraction of the dissipation occurring inside the cubes with  $R_{\Delta x} < 1/4$  will thus exceed 85% if  $\Delta x/L < 10^{-3}$ , and 97% if  $\Delta x/L < 10^{-4}$ . This means, for example, that the reduced MHD model will correctly capture more than 97% of the energy dissipation only if the inertial interval of the turbulence extends to scales smaller than  $\Delta x/L < 10^{-4}$ . In this case, less than 3% of the magnetic energy dissipation will happen inside the cubes where magnetic fluctuation-to-mean ratio exceeds 1/4, that is, where the reduced models are not applicable.

The regions with  $R_{thr} > 1/4$ , corresponding to 3% of all the energy dissipation in the considered example, are, however, extremely intense. From Fig. 5 we estimate the volume occupied by the structures with  $R_{thr} > 1$  as  $V_{R_{\Delta x}}(1) \propto (\Delta x)^{1.2}$ . Assuming that the same scaling holds for smaller values of  $R_{thr}$ , we can estimate from Fig. 5 that  $V_{R_{\Delta x}}(1/4) \sim 150 \times (\Delta x/L)^{1.2}$ . The cubes of the size  $\Delta x/L = 10^{-4}$ , which correspond to  $R_{thr} > 1/4$ , considered in the previous example, will therefore occupy only about 0.2% of the total volume. They include significant variations in the magnetic field direction. In cases where the energy dissipation or particle acceleration effects are strongly skewed toward the environments with strong variations of the magnetic field direction (e.g., Chen et al. 2015; Chasapis et al. 2015; Tessein et al. 2016), these effects will not be adequately captured by the reduced models.





**Figure 4.** The fraction of total energy dissipation occurring in cubes of size  $\Delta x$  with the ratio of local fluctuations-to-mean exceeding a threshold,  $R_{\Delta x} > R_{\text{thr}}$ , for  $\Delta x/L \in \{1/256, 1/128, 1/64, 1/32, 1/16, 1/8, 1/4\}$  (blue to red, solid lines). The corresponding fraction of volume occupied by the cubes is also shown (dotted lines).



**Figure 5.** The fraction of total energy dissipated (solid colored lines) and volume occupied (dashed colored lines) in cubes of size  $\Delta x$  with the ratio of local fluctuations-to-mean exceeding a threshold,  $R_{\Delta x} > R_{\text{thr}}$ , for  $R_{\text{thr}} = 2$  (magenta), 1 (blue), 1/2 (red), and 1/4 (green). Power-law scalings  $\Delta x^{0.8}$  (solid black line) and  $\Delta x^{1.2}$  (dashed black line) are shown for reference (black).

## 4 CONCLUSIONS

Magnetic plasma turbulence is intrinsically anisotropic, meaning that small scale fluctuations experience a large-scale magnetic field that mediates nonlinear interactions. This happens even if the strong magnetic field is not imposed externally, since such a field is self-consistently generated by turbulence itself. It is therefore customary in studies of MHD turbulence to assume the presence of a strong uniform background magnetic field. This assumption, introduced mostly phenomenologically in previous studies, is put on a firmer, quantitative ground in our work. In particular, we argued that the so-called reduced models, that is, models assuming a strong background field and

correspondingly anisotropic fluctuations (reduced MHD, gyrokinetics, models with reduced dimensionality, etc.), should describe the turbulent energy distribution correctly, when their inertial interval is sufficiently long. We however also established that such models may not work as well for describing the energy dissipation in systems where the inertial interval for magnetic fluctuations is not long enough. The reason is that the dissipation is extremely spatially intermittent. It is skewed toward the regions where the magnetic field fluctuations are relatively large compared to the mean field, as happens at the boundaries between nearly uniformly magnetized domains. For example, we estimated that more than 3% of the energy dissipation is not captured by the reduced models if the MHD inertial interval extends over less than four orders of magnitude. Such constraints may be relevant for some natural systems (e.g., solar wind turbulence (Kiyani et al. 2015)), and they may also be essential for laboratory experiments, say liquid metal experiments, where the magnetic Reynolds numbers are not large enough (e.g., Lathrop 2005). Moreover, the regions of strong energy dissipation occupy very small volumes, and, therefore, they may be extremely intense. No matter how large the Reynolds number is, the reduced models always miss a certain fraction of intense dissipation events generated by turbulence, which may become important in phenomena involving higher-order moments of field variations, say, transport phenomena.

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