

# **Benchmark footbridge for vibration serviceability assessment under vertical component of pedestrian load**

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## **ABSTRACT**

Vibration serviceability criteria are governing the design and determining the cost of modern, slender footbridges. Efficient and reliable evaluation of dynamic performance of these structures usually requires a detailed insight into the structural behaviour under human induced dynamic loading. Design procedures are becoming ever more sophisticated and versatile and for their successful use a thorough verification on a range of structures is required. The verification is currently hampered by a lack of experimental data that are presented in the form directly usable in the verification process.

This study presents a comprehensive experimental data set acquired on a box girder footbridge that is lively in the vertical direction. The data are acquired under normal operating conditions, and are presented using a range of descriptors suitable for easy extraction of desired information. This will allow researchers and designers to use this bridge as a benchmark structure for vibration serviceability checks under vertical component of the pedestrian loading. In addition, capabilities of a sophisticated force model (developed for walking over rigid surfaces) to predict vibrations on this lively bridge are investigated. It was found that there are discrepancies between computed and measured responses. It was argued that these differences are a consequence of the pedestrian-structure interaction on this lively bridge. The interaction was then quantified in the form of pedestrian contribution to the overall damping of the human-structure system.

**Keywords:** benchmark footbridge, vibration, walking loading, pedestrian-structure interaction

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## **Introduction**

Increasing slenderness of modern footbridge structures makes them sensitive to dynamic loading. It is now well known that the vibration serviceability under human induced dynamic forces is, most often, the governing criterion for design of slender footbridges. Accuracy of the vibration serviceability checks relies on reliable estimates of expected loading scenarios and dynamic properties of the structure, as well as on the accuracy of the models representing dynamic loading and human response to vibration.

Estimation of the loading scenarios and the criteria for acceptability of vibrations to footbridge users are usually within the remit of the footbridge owners (Sétra 2006), while the dynamic properties of the bridge in the design phase could be determined from a finite element model (with damping estimates based on best engineering judgment and previous experience). The models for human-induced dynamic loading for different traffic scenarios are still under development. In the last couple of years they have advanced from traditional deterministic approach focused on a single pedestrian causing resonance of the structure (HA 2002) towards more comprehensive probabilistic approach focused on a stream of pedestrians characterised by probability distributions of pedestrian arrival time, pacing frequency, step length, and force magnitude (Caetano et al. 2009; Ingólfsson et al. 2007; Pedersen and Frier 2010; Piccardo and Tubino 2009; Racic and Brownjohn 2011; Živanović et al. 2007a). These concepts have already found their place in recent design guidelines, such as a French guideline (Sétra 2006) and the UK National Annex to Eurocode 1 (BSI 2008).

Improving the existing and developing new design models for vibration serviceability assessment of footbridges relies on the availability of experimental data that could be used for verification purposes. At the moment, however, a comprehensive experimental data set that includes dynamic properties of the structure and statistical properties of pedestrian traffic and the corresponding vibration response is not readily available. Those experimental data on vibration performance of footbridges that are available in the public domain (Brownjohn and Middleton 2005; Dallard et al. 2001; Kasperski 2006; Macdonald 2008) are rarely produced in a form that could enable verification of a range of models. A possible reason for this “shortcoming” is that the

experimental campaigns are usually time restricted and concentrated on collection and presentation of a particular data type only, as relevant for an investigated aspect of the footbridge behaviour. In addition, the current design models, some of which are listed in Table 1, produce a range of different outputs and are likely to remain diverse. These issues emphasise the need for provision of experimental data in their most informative form so that they can be used for verification of the range of design models and for effective comparison of their performance. This is especially important in the light of a likely move towards performance based design for vibration serviceability.

Apart from lack of the experimental data that contain information about both the traffic and the vibration response, little is known about how pedestrians interact with the deck oscillating in the vertical direction. There is some limited evidence that walking people could alter the dynamic properties of the vibrating system (Bocian et al. 2011). Using output only identification procedures Willford (2002) and Brownjohn et al. (2004) indicated that pedestrian traffic tend to increase the damping of the system compared with that of the empty structure. However, to quantify this effect, further research is required. Availability of experimental data from lively structures, on which interaction is most likely to occur, would significantly contribute to the modelling process.

This paper aims to provide a comprehensive experimental set of data for a steel box girder footbridge in relation to its vibration in the vertical direction under normal in-service conditions. Since the dynamic properties of the bridge are described in detail elsewhere (Živanović et al. 2006) the emphasis in this study is on providing in-depth information about the statistical properties of pedestrian traffic and the corresponding vibration response. The data are presented in a form compatible with a range of vibration measures used in the existing design models, which will allow this bridge to be used as a benchmark structure for verification of the current and future models. It is hoped that this study could inspire establishing a “standardised” form for data presentation in future vibration serviceability checks.

In addition, capabilities of a sophisticated force model, developed by Živanović et al. (2007a) for walking over rigid surfaces, to predict vibrations on this lively bridge are investigated. The

discrepancies between computed and measured responses were attributed to human – structure interaction and quantified in form of pedestrian contribution to the overall damping of the dynamic system.

The paper starts with a brief description of the footbridge and the experimental data collected before proceeding to statistical description of the traffic and the corresponding vibration response. This is followed by a short correlation analysis between the traffic and the response. Then the capability of numerical simulations to predict the measured structural response and model the human-structure interaction on this lively bridge is briefly analysed, before proceeding to the conclusions.

## **Experimental data**

The experimental campaign was conducted on a steel box girder footbridge (Figure 1a) over the Morača River in Podgorica, capital of Montenegro, on 1 April 2004. This section briefly describes the dynamic properties of the structure and the data collection procedure.

### ***Footbridge structure***

The structure investigated is a 104 m long bridge, with 78 m long main span between the inclined columns. The bridge is situated near the city centre and it mainly responds to pedestrian traffic in the first vertical vibration mode at 2.04 Hz (Živanović et al. 2010) shown in Figure 1b. This mode is most prone to vibration due to having an extremely low damping ratio of about 0.26% and the natural frequency that can be matched by the normal walking frequency. Figure 2a shows the damping ratio estimated from every 10 successive cycles of a free decay response. It can be noticed that the damping ratio is only weakly dependent on the vibration amplitude relevant for in-service conditions (for more than 90% of time the response amplitude is below 0.4 m/s<sup>2</sup>). Therefore the damping ratio of 0.26% can be considered as a representative value. An example of in-service vibration record is shown in Figure 2b. Other dynamic properties of the mode at 2.04 Hz (modal mass, mode shape and natural frequency) are estimated using at least two different methods, and could be considered as quite accurate (Živanović et al. 2006).

## ***Data collection***

The vibration of the footbridge under normal pedestrian traffic was measured at the midspan point during three time slots, each lasting about 44 minutes. A piezoelectric accelerometer Endevco 7754-1000 (with nominal sensitivity of 1000 mV/g) was available for this purpose, and the data were sampled at 40 Hz. To get modal acceleration, the measured signal was band-pass filtered in the range 1.5-2.5 Hz. More than 90% of the energy of the response was concentrated in mode 1. The modal acceleration records from one of the tests and the corresponding 1s root-mean-square (RMS) trend are shown in Figure 2b. Simultaneously with the acceleration response, the pedestrian traffic was monitored using two synchronised video cameras, each focussed at one end of the bridge. This enabled measuring the time that each pedestrian needed to cross the bridge, and calculating the pedestrian's average speed and walking frequency. Also the number of people on the structure at any particular moment could be estimated. The cameras were also synchronised with the acceleration record, so that possible correlation between the traffic loading and the acceleration response could be analysed.

The bridge is situated in a park near Podgorica city centre. The data collection took place during an afternoon on a normal working day when many working people and children cross the bridge on the way from home to work/school and back. First test started at about 3pm, second at 4:30pm and third at about 5:55pm. Pedestrian traffic on this urban and frequently used bridge is normally free flowing and spatially unrestricted, i.e. pedestrians are able to walk at their preferred walking speed. This type of traffic was observed during all three tests, even during a brief spell of quite crowded traffic seen in Test 3.

## **Statistics of pedestrian traffic**

This section describes the statistics of pedestrian traffic during the three monitoring tests. In total 2019 people crossed the bridge. An average pacing frequency for each person was extracted from the video records by measuring time needed to make about 20 steps. In addition, the entry and exit times were also identified leading to an estimate of time spent on crossing the bridge.

Dividing the bridge length of 104 m by the estimated crossing time an average walking speed was

identified for each pedestrian. The average step length was then derived by dividing the walking speed by the walking frequency. 14 outliers were removed from the data before the analysis.

These were related to extremely slow pedestrians (walking speed of less than 0.6 m/s) who were mainly slowed down due to stopping to greet friends met on the bridge.

The relative error in frequency estimates is up to 6% (caused by a maximum error in time estimate of 0.5 s over about 20 steps), while the error in the estimate of the crossing time is maximum 3%, corresponding to about 2 s (1 s from each camera).

The probability density functions for the measured step frequency, walking speed and step length are shown as solid lines in Figure 3. The normal distribution (dashed lines in Figure 3) proves to be a good model for all three parameters. The mean values and standard deviations for these distributions are: 1.87 Hz and 0.19 Hz for step frequency, 1.39 m/s and 0.20 m/s for walking speed and 0.74 m and 0.08 m for step length. The mean speed of 1.39 m/s means that, on average, one person needs about 75 s to cross this 104 m long bridge.

The coefficient of linear correlation between the step frequency and the step length (Figure 4a) is nearly zero, confirming previous research findings that the two parameters are linearly independent (Ricciardelli et al. 2007; Živanović et al. 2007b). On the other hand it is evident that the speed increases with increase in both the step frequency (Figure 4b) and the step length (Figure 4c), with the degree of linear correlation being 0.66 and 0.75, respectively. Best linear models, in least square sense, for the two relationships, are also shown in Figure 4, and they are found to be in line with the models identified by Ricciardelli et al. (2007). However, it should be noticed that the linear relationships for both the speed – step frequency and speed – step length should not be used for traffic modelling purposes because they do not satisfy the physical interrelation between the three parameters (the speed has to be equal to the product between the step frequency and step length). Instead the idea here is to provide a simple model that could be used to get two parameters when experimental data are available for a single parameter only. For example, if the distribution of the step frequency on a particular bridge is known, then the most likely speed for each pedestrian can be obtained using the relationship from Figure 4b, while the

third parameter (step length in this case) is always to be obtained from the physical relationship (dividing the speed by the step frequency).

Number of people arriving on the bridge during 75 s time window (i.e. average time frame needed to cross the bridge once) in the three tests is shown in Figure 5. Solid lines in the first column represent the arrival of people going to the city centre while the second column shows the arrival of people walking in the opposite direction. The third column shows fluctuations of the pedestrian flow over time – the number of people in the flow was calculated at discrete time points spaced by 1 s. Traffic seen during first two tests can be considered as being normal/usual traffic on the bridge, while traffic during Test 3 is rush hour traffic, and it includes quite a busy period between 2100 and 2300s. The traffic was free flowing during all the tests.

The dashed lines in first two columns in Figure 5 represent the corresponding Poisson distributions, which are reasonably good fits for the data in most tests. The only exception is the large discrepancy between the experimental and theoretical Poisson distributions in Test 3 in the traffic directed towards the city. The theoretical distribution underestimates both tails of the experimental distribution. This discrepancy is probably due to large variations in the traffic numbers in this direction (Table 2).

The amount of time the bridge was occupied by a particular number of people in the three tests is shown in Figure 6a and it demonstrates that the traffic was busiest in Test 3. Statistics for the traffic in each direction, as well as for the total traffic during each test is shown in Table 2. The table also presents slightly different pedestrian parameters identified in each test, most significant difference being a slightly higher mean walking frequency in Test 1 compared with the other two tests.

## **Statistics of vibration response**

A convenient way to represent the response to normal traffic is to use statistical means of describing the measured response, such as cumulative distribution functions (CDFs) and/or probability density functions (PDFs). These forms of data presentation allow using a single graph

to extract the response with any desired probability of exceedance, therefore covering a range of responses which can be of interest to designers. The choice of vibration measures to present here is driven by their use in practice (Table 1), and these are: the instantaneous acceleration, peak acceleration per cycle, 1s RMS trend, and peak per 150 s, i.e. the peak response over time frame required for an average pedestrian to cross the bridge twice. In addition, the peak per 75 was added since it represents the acceleration level that an “average” pedestrian would experience while on the bridge.

The crest factors for the return period of 44 minutes (the peak value of the modal acceleration divided by the RMS value over total duration of the signal) for the vibration of this bridge are 5.4, 4.5 and 5.0, in the three tests. There is not much data about the crest factor typical of footbridge vibrations under pedestrian traffic. Collecting more information across a range of footbridge structures would allow statistically reliable conversion between the RMS and the peak response, which is a useful means routinely used in data analysis in wind engineering (Simiu and Scanlan 1996). The crest factor value depends on the return period chosen. Figure 6b shows the change in the crest factor with increase of the return period from 1 s to 40 minutes.

PDFs for instantaneous acceleration (that accounts for every data point sampled) in the three tests are shown as solid lines in Figure 7a. The corresponding normal distributions are also shown in Figure 7a, as dashed lines. It can be concluded that this narrow-band vibration response does not follow the normal distribution closely in the first two tests. This is an interesting finding that should be taken into account when statistical models for prediction of vibration response are being developed and it suggests that more crowded traffic or longer monitoring time is required to achieve better fits. In addition Figure 7b shows the distributions of the 1s RMS vibration measure.

The CDFs for peak acceleration per cycle, per 75s and per 150s, as well as 1s RMS trend are presented in Figure 8, and are of interest when evaluating the predictive capability of design models (such as those listed in Table 1). The statistics of these responses is shown in Table 3.



The response of the structure is of narrow band nature. It is known that for Gaussian narrow band processes, the resulting probability density function  $y(x)$  for peak per cycle response  $x$  follows the Rayleigh distribution (Newland 2005):

$$y(x) = \frac{x}{b^2} e^{-\left(\frac{x^2}{2b^2}\right)} \quad (1)$$

where  $b$  is the variance of the original narrow band random signal. Given that the data from Test 3 best agree with the normal distribution (Figure 7a), it is expected that the PDF for the peak per cycle response in this test would agree best with the corresponding Rayleigh distribution. This is confirmed in Figure 9a, where the solid lines are related to the experimental data while the dashed lines are the corresponding Rayleigh PDFs. It can be seen that in the first two tests the Rayleigh distribution tends to underestimate the probability of lowest and largest responses, suggesting that the Weibull distribution:

$$y(x) = ba^{-b} x^{b-1} e^{-\left(\frac{x}{b}\right)^b} \quad (2)$$

where  $a$  and  $b$  are the distribution parameters, might be more suitable for data description (Newland 2005).

Figure 9b shows that the Weibull distribution (dashed lines) does provide a better description of the peak response for this narrow band process. This is confirmed by the better quality of the fitted models as shown on probability (Figure 9c) and quantile (Figure 9d) plots, given for Test 1 only (the resulting plots are similar for Test 2 data, and show improved fit quality for Test 3 data). The model is considered a good fit if the points on the graph lie on the diagonal line. It is clear that the Weibull distribution is better of the two models. Its loss of quality for acceleration values above  $0.5 \text{ m/s}^2$  (Figure 9d) is probably caused by much smaller number of experimental data points in this region. The parameters and their 95% confidence intervals for the Rayleigh and Weibull distribution models in the three tests are obtained by using the maximum likelihood estimation, and are summarized in Table 4.

## **Correlation between traffic and vibration response**

The number of people on the bridge in each second of the monitoring time is shown in Figure 5. The corresponding modal response can be extracted from the peak acceleration envelope of the measured acceleration. Figure 10 shows the relationship between the two parameters taking into account the data from all three tests. It can be noticed that there is no clear correlation between the two parameters. This could be expected given that, for example, number of people crossing the bridge in Test 3 is almost two times larger than the number of people in Test 2, yet the properties of the vibration responses are quite similar (Table 3). The mean response (solid line in Figure 10 on 6-35 interval on which at least 40 data points were available) shows only a slight increase of the response with increasing number of people. The lack of stronger correlation between the response and the number of people, as well as large scatter in the data, might be a consequence of human-structure interaction occurring on this lively bridge (including occasional stopping by some test subjects to greet their friends met on the bridge) and/or (difficult to monitor) intra- and inter-subject variations of the traffic parameters along the bridge and in crowds of different size.

## **Numerical prediction of the vibration response**

In a previous publication the author and her co-workers conducted numerical simulations to generate the force representing traffic in Test 2, and concluded that modelling the force as for walking on rigid surfaces leads to an overestimation of the measured vibration response (Živanović et al. 2010). It seems that walking on this lively bridge, where about a third of 100 interviewed pedestrians were found to be unhappy about the perceived vibrations of the structure (Živanović and Pavić 2009), the walking force is being attenuated possibly due to people's interaction with vibrating deck. Currently there is no model that can account for this attenuation. An alternative and simple way to simulate it is through an increase in the damping of the human-structure system. In Test 2 the damping of the joint system was found to be more than two times larger than the damping of the empty structure (Živanović et al. 2010). Here, the analysis is expanded and repeated for other two tests to check consistency of the results. Test 3 is analysed

initially since it represents the busiest traffic, with up to 78 pedestrians on the bridge at the same time.

A comprehensive probabilistic force model for a single person walking on rigid surfaces is again used (Živanović et al. 2007a). The model has been verified on a footbridge structure with lower vibration levels where the interaction is less likely to occur (Živanović et al. 2010). The model covers the frequency content of the force up to the fifth harmonic (note that only the first harmonic is of interest on this bridge) taking into account the narrow band nature of the force around each harmonic. Individual properties of pedestrians featuring the model – walking frequency and step length– are generated from previously described distributions (Figure 3a and c). The pedestrian speed is then calculated as the product of the step frequency and step length. Dividing the bridge length of 104 m by the pedestrian speed, the duration of the simulation force for the pedestrian considered is then obtained. The peak magnitude of the first harmonic  $F_1$  is modelled using the empirical equation defined by Kerr (1998):

$$F_1 = (-0.2649f_p^3 + 1.3206f_p^2 - 1.7597f_p + 0.7613)W \quad (3)$$

where  $f_p$  is the walking frequency in Hz, and  $W$  is the pedestrian weight. The coefficient of variation in the magnitude is taken as 16% (Kerr 1998). Data about the pedestrian weight could not be collected during the experiments. For estimating this parameter, data related for Montenegrin population living in two regions in neighbouring Serbia were used (Pavlica 2009). The mean weight is taken as 750 N and the standard deviation as 150 N, and the weight is assumed to follow the normal distribution.

The forces are generated for each pedestrian taking into account the time each one needs to cross the bridge, and weighted by the mode shape to get individual modal forces. The pedestrian arrival time is then generated from the appropriate Poisson distribution, and the forces are summed up to obtain the modal force representing the 44 minute pedestrian traffic.

A typical power spectral density (PSD) function of the force is shown in Figure 11a. This force, multiplied by the square of the acceleration frequency response function (FRF) of the system, results in the PSD of the response (Newland 2005). The human-structure system is considered to

have natural frequency of 2.00 Hz since this frequency dominates the measured response of the structure (Živanović et al. 2010) and the initial damping ratio of the empty bridge (0.26%). The pedestrian contribution to the modal mass is taken by recalculating the modal mass of 58000 kg (Figure 1b) to accommodate the shift in the natural frequency from 2.04 Hz to 2.00 Hz assuming that the modal stiffness remains unchanged. This leads to the modal mass estimate of 60343 kg. The accelerance FRF of this human-structure system is represented as the solid line in Figure 11b. The resulting PSD of the response is shown as solid line in Figure 11c. The RMS of the calculated response (i.e. the square root of the area enclosed by the PSD function) clearly overestimates the measured response represented by the dashed line in the same figure. On average, over a range of simulations the ratio between simulated and measured RMS responses was about 1.75 in Test 3 (while in Test 1 it was about 1.30 and in Test 2 about 1.40). If, however, the damping of the human-structure system is increased to 1.02% in this particular simulation (dot-dashed line in Figure 11b), then the corresponding PSD of the response (dot-dashed line in Figure 11c) is very close to that measured.

To get statistically reliable results for the damping estimate, the simulations were repeated 60 times. The average damping identified for the human-structure system relevant for traffic conditions from Test 3 was 1.00%, with the coefficient of variation (COV) 8.7%. The equivalent damping ratios found in each simulation are shown as dots in Figure 12, while crosses represent the averaged damping ratio with increasing number of simulations, resulting in a stable trend after about 40 simulations.

It is interesting to see how much the weight of people would have to be reduced to achieve good match between the numerical and the experimental responses without increasing the damping of the system. This reduction was found to be from 750 N to 451 N. The new estimate is clearly inappropriate for description of the actual population observed during the experiments. Similarly, the mean walking frequency in the simulations was being reduced until a good match with the experimental data was achieved. This required the mean frequency to decrease from 1.86 Hz to 1.68 Hz – unrealistically slow traffic for this bridge. This is an important conclusion given that the measurement error for walking frequency was quite high – up to 6%. If we assume a highly

unlikely case that the error consistently led to an overestimate of the walking frequency for each pedestrian by 0.1 Hz, the corrected measured frequency would be 1.76 Hz – still higher than 1.68 Hz required and the measured response would still be overestimated.

Finally, following the same procedure as for Test 3, the equivalent damping ratio required to match the experimentally measured response in other two tests was found to be 0.56% (COV 16.0%) in Test 1, and 0.65% (COV 14.9%) in Test 2. Since the damping ratio of the empty structure is 0.26% (or 3900 Ns/m), the pedestrians' contribution is equal to 0.30% (4500 Ns/m) in Test 1, 0.39% (5850 Ns/m) in Test 2 and 0.74% (11100 Ns/m) in Test 3. The average number of people on the bridge in the three tests is 14.9, 15.7 and 26.1 (Table 2). Assuming that people are uniformly distributed along the bridge, and taking into account the mode shape effect, the effective average number becomes 4.0, 4.2 and 7.0, respectively. Therefore, for the three tests the average added damping is 0.075% (1125 Ns/m), 0.093% (1395 Ns/m) and 0.106% (1590 Ns/m) per person for this footbridge. The smallest damping contribution per pedestrian occurred in Test 1 when the mean walking frequency was closest to the structural natural frequency, while it was very similar in tests 2 and 3 which, although representing quite different traffic density, were characterised with the same mean walking frequency. To generalise this finding, however, the analysis of structures of different dynamic properties and level of liveliness is required. A more sophisticated analysis should also account for variation in the damping contribution over time. In addition, laboratory experiments involving pedestrians walking on a vertically vibrating surface under controlled conditions are required to progress modelling pedestrian behaviour on lively structures. This could be done in a similar way to the research work already conducted successfully for walking on laterally oscillating surfaces (Ingólfsson et al. 2011).

## **Conclusions**

This paper presents a detailed insight into the experimentally acquired properties of the pedestrian traffic and the corresponding vibration response of a lively footbridge structure. It was found that the walking frequency, step length and pedestrian speed in normal traffic follow normal distribution, while the pedestrian arrival time can be approximated by Poisson

distribution. The statistical properties of the structural acceleration measured at the midspan were presented for the instantaneous, 1s RMS, peak per cycle, peak per 75 s and peak per 150 s response. It was found that there was no obvious correlation between the number of people on the structure and the structural vibration response. However, it was also suggested that even more detailed monitoring of pedestrians and possibly longer monitoring time would further contribute to learning about structural in-service behaviour.

In addition, it was shown that a force model defined for walking on rigid surfaces is not appropriate for modelling walking on structures that are lively, such as the footbridge presented in this paper. The presence of pedestrians seems to damp out vibrations, resulting in lower than expected vibration levels. This is equivalent to adding damping to the human-structure system compared with the damping of the empty structure. It was found that, on average, a pedestrian adds between 0.075% (1125 Ns/m) and 0.106% (1395 Ns/m) of damping on this footbridge.

Finally, the thorough description of the traffic and the response on the Podgorica bridge is hoped to establish this bridge as a benchmark structure for vibration serviceability checks in the vertical direction. It is also hoped that this study could inspire establishing a “standardised” form for data presentation in future vibration serviceability checks. The key pieces of information to be included in the standardised form, in the author’s opinion, are: distribution of pedestrian arrival times (which determines the traffic density), distribution of walking frequencies (since the response analysis is most sensitive to this traffic parameter) and the distribution of the chosen vibration measures. Vibration measures of choice should be in agreement with established practice, which currently normally demands vibration estimates in form of either peak or 1s RMS response (sometimes with a pre-specified probability of non-exceedance). Above all, however, availability of the data in electronic format could offer most in terms of comparison of theoretical and experimental responses. To help this process the electronic format of the data used in this paper is available at [www.warwick.ac.uk/go/civileng/crg/structures/publications/data](http://www.warwick.ac.uk/go/civileng/crg/structures/publications/data).

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## **Table captions:**

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## Tables

Table 1: Vibration measures in four frequently used design guidelines.

| <b>Design guideline</b>         | <b>Vibration measure for acceleration response</b> |
|---------------------------------|--|
| Setra (2006)                    | Peak* with 95% probability of nonexceedance        |
| UK NA to Eurocode 1 (BSI, 2008) | Peak with 79% probability of nonexceedance         |
| BD 37/01 (HA, 2002)             | Absolute peak                                      |
| ISO 10137 (ISO, 2007)           | 1s root mean square (1s RMS)                       |

\*Time required by an average pedestrian to cross the bridge twice is used to calculate the peak value.

Table 2: Traffic statistics.

| Traffic          | Parameter                                | Test 1       | Test 2       | Test 3       |
|------------------|--|--------------|--------------|--------------|
| Towards the city | Mean no of people on the bridge          | 6.0          | 8.1          | 16.7         |
|                  | STD                                      | 3.5          | 3.7          | 12.0         |
|                  | Mean crowd density [ped/m <sup>2</sup> ] | 0.019        | 0.026        | 0.054        |
|                  | Min and max                              | [0; 16]      | [0; 20]      | [0; 67]      |
|                  | Total number of pedestrians              | 230          | 290          | 570          |
| From the city    | Mean no of people on the bridge          | 8.9          | 7.7          | 9.4          |
|                  | STD                                      | 3.2          | 3.7          | 4.1          |
|                  | Mean crowd density [ped/m <sup>2</sup> ] | 0.029        | 0.025        | 0.030        |
|                  | Min and max                              | [0; 21]      | [1; 24]      | [1; 21]      |
|                  | Total number of pedestrians              | 324          | 263          | 342          |
| Total traffic    | Mean no of people on the bridge          | 14.9         | 15.7         | 26.1         |
|                  | STD                                      | 4.3          | 5.9          | 13.6         |
|                  | Mean crowd density [ped/m <sup>2</sup> ] | 0.048        | 0.050        | 0.084        |
|                  | Min and max                              | [0; 29]      | [2; 38]      | [2; 78]      |
|                  | Step frequency (mean, std) [Hz]          | (1.89, 0.19) | (1.86, 0.19) | (1.86, 0.18) |
|                  | Speed (mean, std) [m/s]                  | (1.42, 0.20) | (1.38, 0.21) | (1.38, 0.19) |
|                  | Step length (mean, std) [m]              | (0.75, 0.08) | (0.74, 0.08) | (0.74, 0.08) |

Table 3: Response statistics [m/s<sup>2</sup>].

| Vibration type | Test 1 |      |               | Test 2 |      |               | Test 3 |      |               |
|----------------|--------|------|---------------|--------|------|---------------|--------|------|---------------|
|                | Mean   | STD  | [Min, Max]    | Mean   | STD  | [Min, Max]    | Mean   | STD  | [Min, Max]    |
| Instantaneous  | 0.00   | 0.15 | [-0.80, 0.80] | 0      | 0.13 | [-0.59, 0.59] | 0      | 0.14 | [-0.69, 0.69] |
| Peak per cycle | 0.18   | 0.12 | [0, 0.80]     | 0.16   | 0.10 | [0, 0.59]     | 0.17   | 0.10 | [0, 0.69]     |
| 1s RMS         | 0.12   | 0.08 | [0, 0.57]     | 0.11   | 0.07 | [0, 0.42]     | 0.12   | 0.07 | [0, 0.49]     |
| Peak per 75s   | 0.40   | 0.14 | [0.20, 0.80]  | 0.33   | 0.11 | [0.10, 0.59]  | 0.38   | 0.10 | [0.21, 0.69]  |
| Peak per 150s  | 0.49   | 0.14 | [0.29, 0.80]  | 0.39   | 0.10 | [0.22, 0.59]  | 0.44   | 0.10 | [0.32, 0.69]  |

Table 4: Estimates of distribution parameters and their 95% confidence intervals.

| <b>Distributions</b> | <b>Test 1</b>        | <b>Test 2</b>        | <b>Test 3</b>        |
|----------------------|----------------------|----------------------|----------------------|
| Rayleigh: b          | 0.150 [0.148, 0.152] | 0.132 [0.130, 0.134] | 0.141 [0.139, 0.143] |
| Weibull: a           | 0.198 [0.195, 0.202] | 0.173 [0.170, 0.176] | 0.196 [0.193, 0.199] |
| b                    | 1.588 [1.557, 1.620] | 1.550 [1.518, 1.582] | 1.859 [1.822, 1.896] |



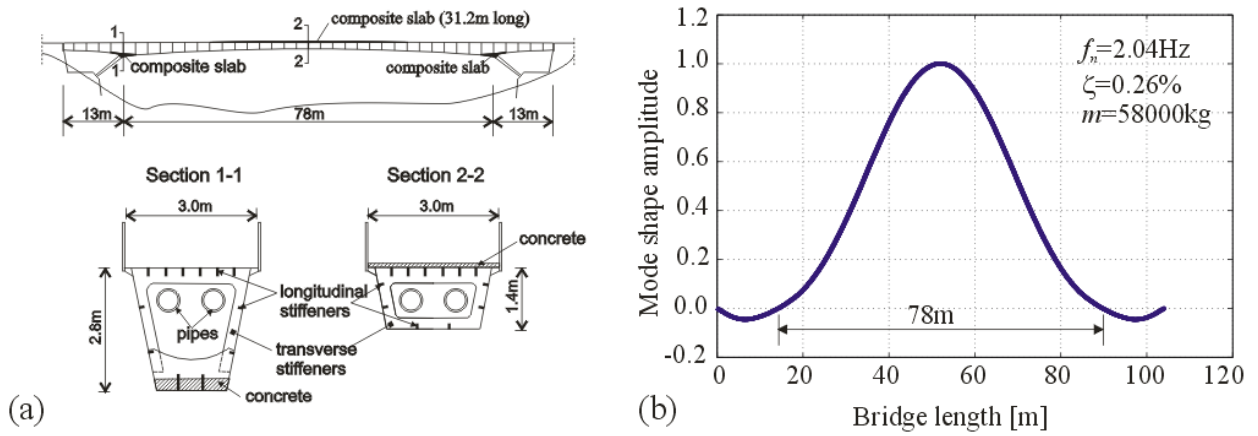


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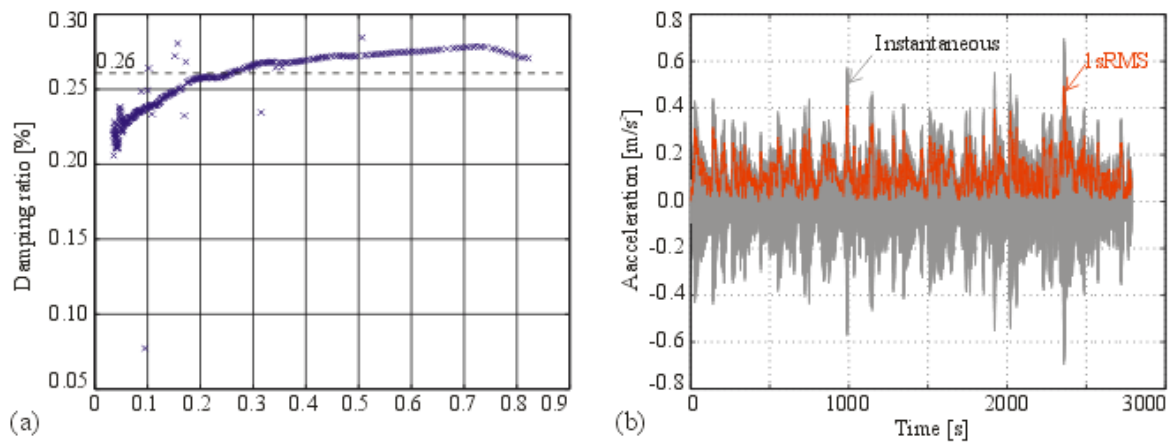


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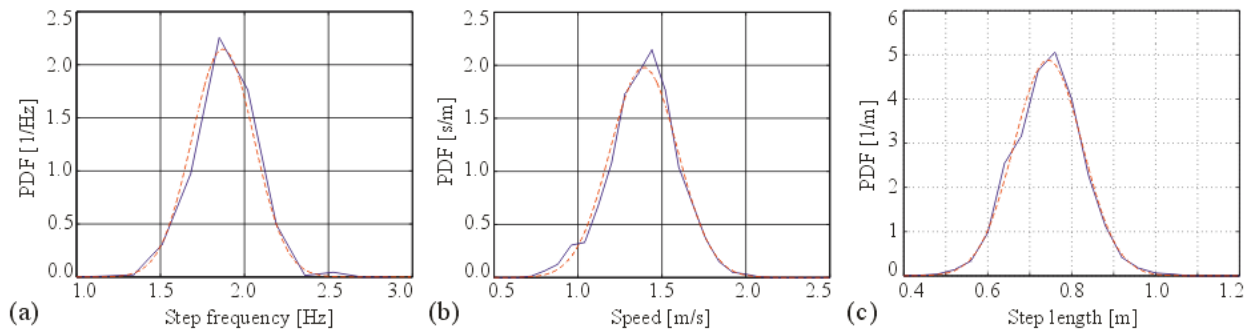


Figure 3: Probability density functions for (a) step frequency, (b) speed and (c) step length.

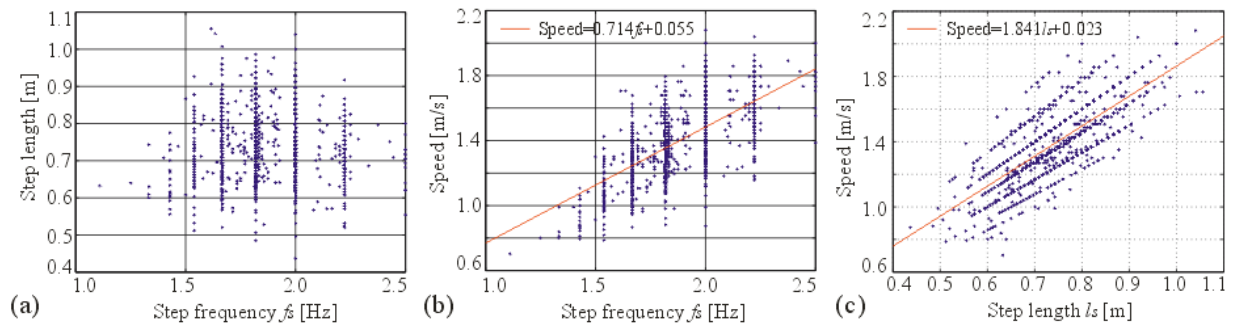
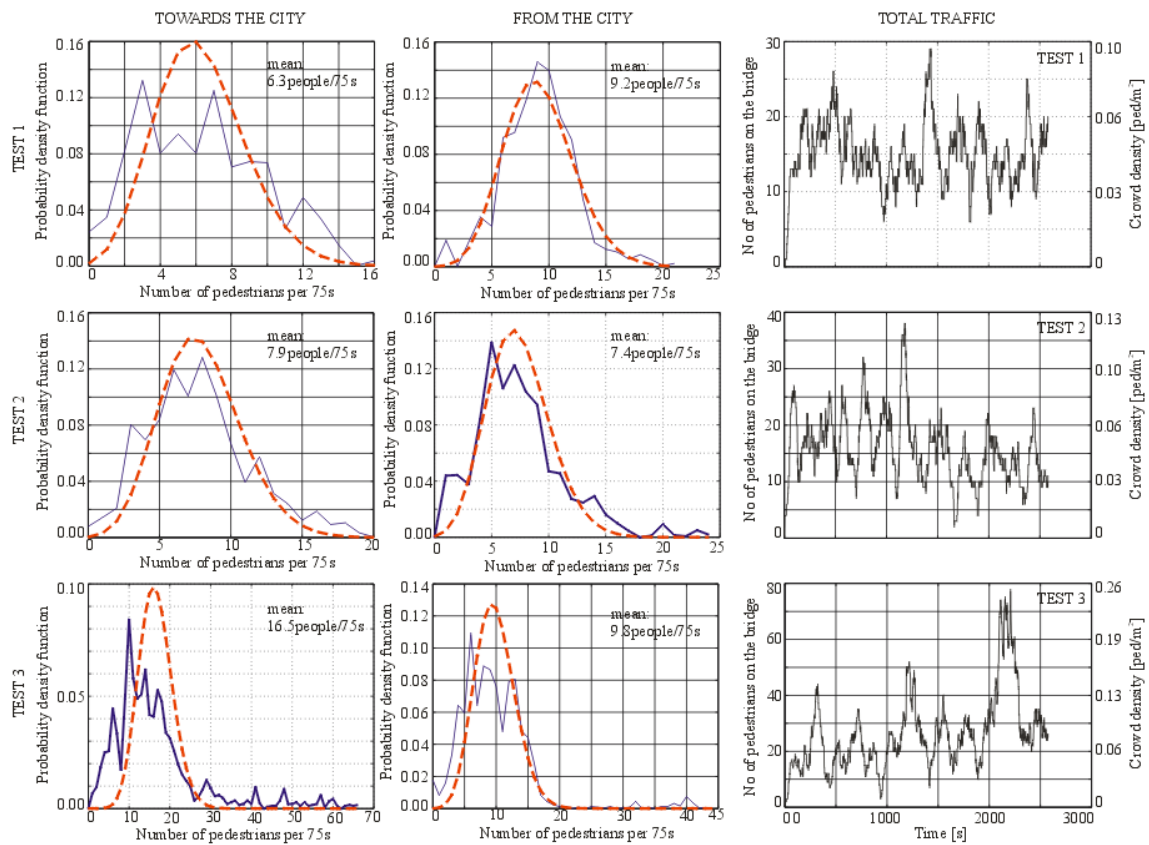


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Figure

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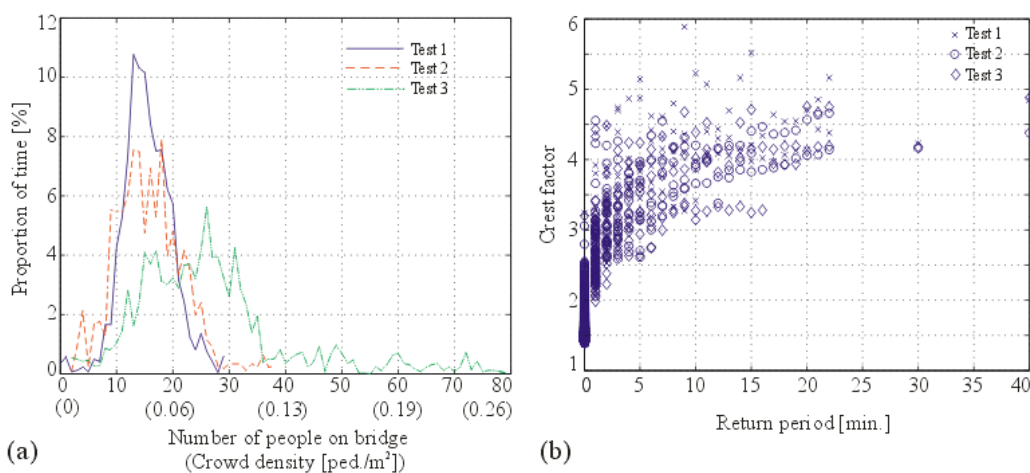


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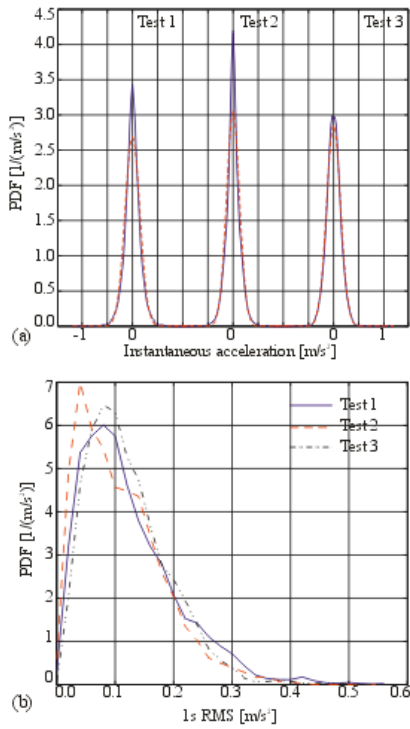


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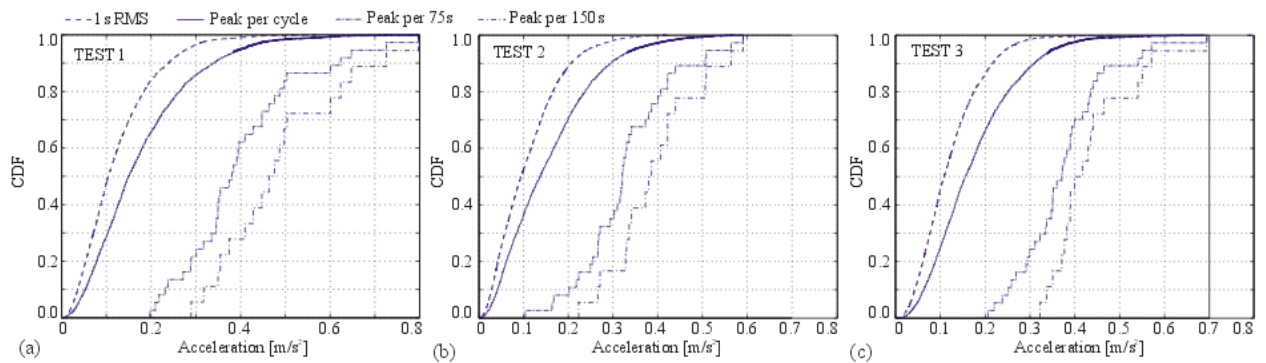


Figure 8: CDFs for acceleration response in the three tests.

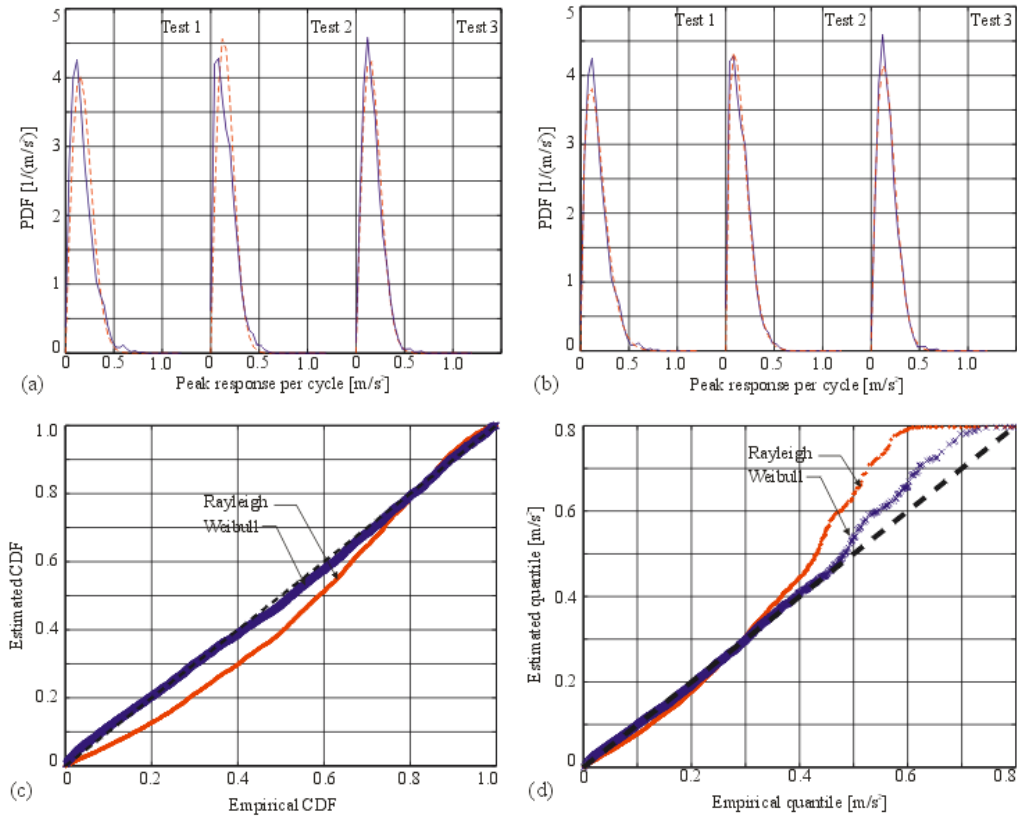


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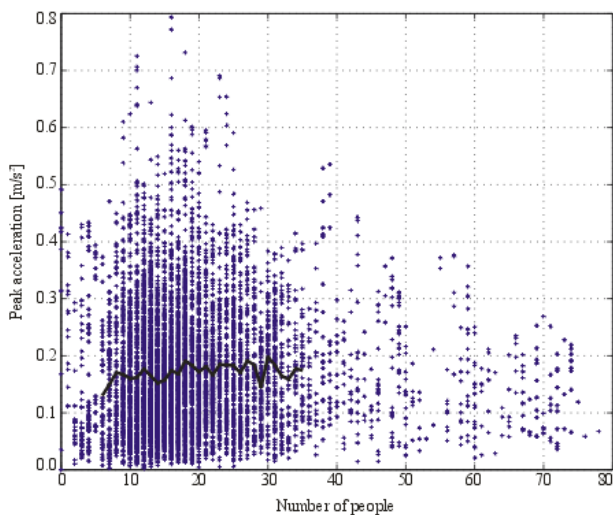


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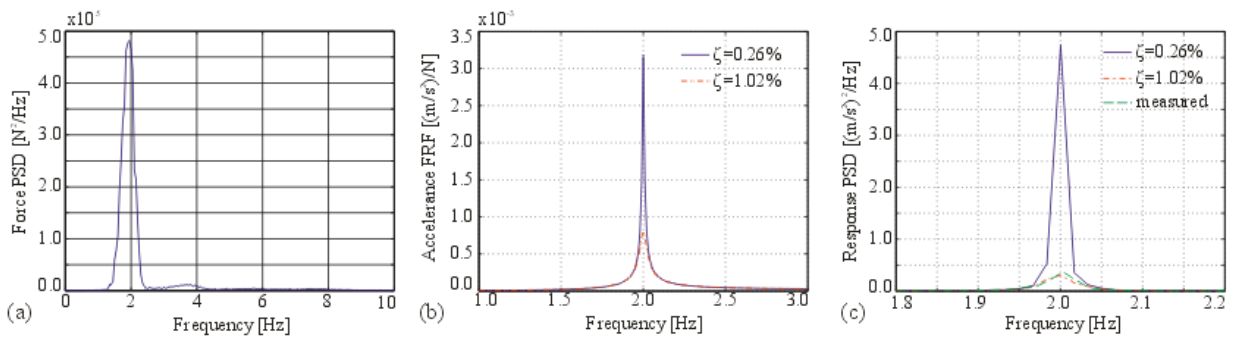


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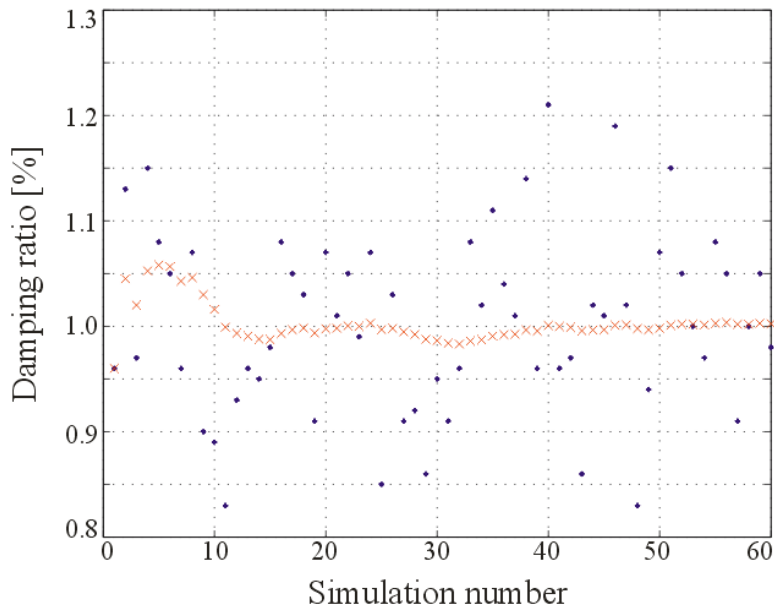


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