A bi-objective weighted model for improving the discrimination power in MCDEA

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ABSTRACT

Lack of discrimination power and poor weight dispersion remain major contention issues in Data Envelopment Analysis (DEA) models, which have also hampered the developments in the multiobjective DEA domain. Since the initial multi- criteria DEA (MCDEA) model of Li and Reeves (1999), only one other research by Bal, Örkcü and Çelebioğlu (2010) attempted to solve the MCDEA framework through two goal programming approaches, i.e. GPDEA-CCR and GPDEA-BCC. It was claimed that both models improved upon the discrimination power of DEA by balancing the distribution of input-output weights. It was also claimed that both GPDEA models are major improvements to the original MCDEA of Li and Reeves (1999). In this research we first checked the validity of GPDEA models and found that they do not improve the discrimination power as it has been claimed, we further propose an alternative solution to the formulation using bi-objective linear programming. It is shown that the proposed bi-objective multiple criteria DEA(BiO-MCDEA) performs better than the GPDEA models in the aspects of discrimination power and weight dispersion, as well as requiring less computational codes. An application of energy dependency among 26 European Union member countries is further used to describe the efficacy of our approach.

Key words: Data envelopment analysis, Multiple criteria data envelopment analysis, Goal programming, Discrimination power.

1. Introduction

Data envelopment analysis (DEA) was first proposed by Charnes et al. (Charnes, Cooper, & Rhodes, 1978) and remained the leading technique for measuring the relative efficiency of decision-making units (DMUs) based on their respective multiple inputs and outputs. DEA has been the fastest growing discipline in the past three decades covering easily over a thousand papers in the *Operations Research* and *Management Science* discipline (see (Emrouznejad, Parker, & Tavares, 2008)). The efficiency of a

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DMU is defined as a weighted sum of its outputs divided by the weighted sum of its inputs on a bounded ratio scale.

One of the drawbacks of the DEA is the problem of lack of discrimination among efficient decision making units (DMUs) and hence yielding many DMUs as efficient. The problem is noted when the number of DMUs evaluated is significantly lesser than the number of inputs and outputs used in the evaluation. The weights derived post-hoc from the evaluation exercise may reveal that some inputs or outputs have zero values. This is counter-intuitive especially in a decision making exercise, where one expects to use all the inputs and output values that are rated for the DMUs. Hence, it further implies that some of the variables were not used in the evaluation exercise in achieving the final ranking. On the contrary, unrealistic weight distribution for DEA also occurs when some DMUs are rated as efficient because of extreme large weights in a single output and/or extreme small weights in a single input.

Thompson et al. (1986) are among the first authors to propose the use of weight restriction to increase the discrimination power of DMUs. The issue was immediately picked up by many authors, including Dyson and Thanassoulis (Dyson & Thanassoulis, 1988), Charnes et al. (Charnes, Cooper, Huang, & Sun, 1990a), Thanassoulis et al. (Thanassoulis & Allen, 1998). Hence, several methods such as assurance region (AR) procedure (R. G. Thompson, Langemeier, Lee, & Thrall, 1990), cone ratio envelopment (Charnes, Cooper, Huang, & Sun, 1990b) area adressed in the literature as strategies to solve problems arising from unrealistic weight distribution. Subsequently, other DEA models were introduced in the literature to overcome the discriminant power problems, such as the cross-efficiency evaluation technique (Anderson, Hollingsworth, & Inman, 2002; Doyle & Green, 1994, 1995; Green, Doyle, & Cook, 1996; Sexton, Silkman, & Hogan, 1986), super-efficiency model (Andersen & Petersen, 1993; Y. Chen, 2005), and multiple criteria (or multi-objective) DEA (Y.-W. Chen, Larbani, & Chang, 2009; Li & Reeves, 1999).

The focus in this paper is on the most recent development in the area; that is, to introduce a weighted model for improving the discrimination power and weight dispersion, which focuses on multiple criteria Data Envelopment Analysis (MCDEA). The rest of the paper is organized as follows. Section 2 gives a brief description of the multiple criteria data envelopment analysis (MCDEA) and the more recent goal programming data envelopment analysis (GPDEA) as a procedure for MCDEA. Section 3 highlights some drawbacks on using GPDEA to represent MCDEA analysis. We therefore introduce an alternative bi-objective multiple criteria model (BiO-MCDEA) to improve the discrimination power of MCDEA in Section 4. An application of energy dependency among 26 EU member countries demonstrates the efficacy of the model in Section 5. Concluding remarks are given in Section 6.

2. Improving discrimination power in DEA: Recent developments

Multiple criteria data envelopment analysis (MCDEA)

The MCDEA model; consisting of three objectives, was proposed by Li and Reeves (Li&Reeves, 1999) to improve the discriminating power of classical DEA model. Classical definition of relative efficiency is considered one of the criteria in this model, hence the classical DEA solution is said to be contained in the set of MCDEA solutions. In other words, a wider solution is possible with MCDEA, so as to gain more reasonable input and output weights. Another 2 objectives are the Minimax and Minsum criteria.

In MCDEA, the three objectives are analyzed separately; one at a time, and no preference order was set for those objectives. The solutions derived from each run are considered non-dominated in the multi objective linear programming (MOLP) sense. Li and Reeves (Li & Reeves, 1999) note that generally the Minimax criterion is more restrictive than the Minsum criterion, while the first criterion is the least restrictive of all. Since the Minimax and Minsum criteria tend to provide less number of efficient DMUs as compared to the first criteria, it is said to provide better discriminating power than a classical DEA model. As such, the Minimax and Minsum criteria are helpful when the number of DMUs is significantly larger than the number of inputs and outputs used for evaluation.

Consider in evaluating the relative efficiency of n DMUs which use m inputs ($x_i,\ i = 1, \dots, m$) to produce s outputs ($y_j,\ j = 1, \dots, s$). The MCDEA model proposed by Li and Reeves (Li&Reeves, 1999) which considers three objective functions: i) minimizing d_o(or maximizing θ_o), ii) minimizing the maximum deviation, and iii) minimizing the sum of deviation, is defined as follows in Model 1:

<u>Model 1: Multi criteria data envelopment analysis</u>

$$\min d_o (or \max \theta_o = \sum\nolimits_{r=1}^{s} u_r y_{ro})$$

$$\min M,$$

$$\min \sum\nolimits_{j=1}^{n} d_j$$

$$\sum\nolimits_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \dots, n,$$

$$M - d_j \geq 0, \quad j = 1, \dots, n,$$

$u_r \geq 0, \quad r = 1,...,s,$

$v_i \geq 0, \quad i = 1,...,m,$

$$d_j \geq 0, \quad j = 1,...,n, \tag{1}$$

The quantity d_o in the first objective function is bounded on an interval [0, 1) and is regarded as a measure of inefficiency. Thus, DMU $_o$ is efficient at h_o=1-d_o where h_o is the efficiency measure in a classical DEA. In short, the first objective function (i.e. $\min d_o (or \max \theta_o = \sum_{r=1}^{s} u_r y_{ro})$) is equivalent to the objective function of a classical DEA. The *M* in the second objective function (minmax criterion) represents the maximum quantity of all deviation variables $d_j (j = 1,...,n)$. The third objective function is a Minsum of all deviation variables. Another noteworthy point is the introduction of the $M - d_j \geq 0, (j = 1,...,n)$ constraint in MCDEA, which does not alter the feasible region of the solution but merely to ensure that $\max d_j \geq 0$.

Goal programming DEA models (GPDEA)

Li and Reeves (Li&Reeves, 1999) did not suggest a solution for their proposed MCDEA model that optimizes all objectives simultaneously. The aim of their proposed MCDEA model solution process is not to extract an optimal solution; but instead, to find a series of non-dominated solutions that is left to the analyst in selecting the most preferred one, if need be. Therefore, goal programming as a solution for the MCDEA model can be seen as a natural progression in converting the multiple objective programming in the MCDEA model into a single objective problem.

Goal programming is a type of multi-objective optimization, also known as multiple-criteria decision making which can provide a way of striving toward several such objectives simultaneously. The basic approach of goal programming is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of unwanted deviations of these objective functions from their respective goals.

Bal et al. (Bal, et al., 2010) recently proposed the following goal programming to solve the formulation proposed by Li and Reeves (Li & Reeves, 1999). The former adopted the non-weighted approach in their solution design and claimed to be an equivalent single objective form to the latter's three objectives.

<u>Model 2: Goal programming data envelopment analysis under CRS</u>

$$\min a = \left\{ d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j \right\}$$

$$\sum_{i=1}^m v_i x_{io} + d_1^- - d_1^+ = 1,$$

$$\sum_{r=1}^s u_r y_{ro} + d_2^- - d_2^+ = 1,$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j = 0, \quad j = 1,\ldots,n,$$

$$M - d_j + d_{3j}^- - d_{3j}^+ = 0, \quad j = 1,\ldots,n,$$

$$u_r \geq 0, \quad r = 1,\ldots,s,$$

$$v_i \geq 0, \quad i = 1,\ldots,m,$$

$$d_j \geq 0, \quad j = 1,\ldots,n,$$

$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0,$$

$$d_{3j}^-, d_{3j}^+ \geq 0, \quad j = 1,\ldots,n, \qquad (2)$$

The above model is with the assumption of constant returns to scale (CRS) (Bal, et al., 2010), where d_1^- and d_1^+ are the unwanted deviations for the goal which the weighted sum of inputs equal to unity, d_2^- and d_2^+ are the wanted and unwanted deviation variables which make the weighted sum of outputs less than or equal to one, whereas d_{3j}^- and d_{3j}^+ (j=1,…, n) are the unwanted and wanted deviation variables for the goal $M - d_j \geq 0\,(j = 1,\ldots,n)$. M remains as the maximum deviation d_j, for DMU $_j$ (j=1,…,n), which is also an unwanted deviation. A similar model under the variable returns to scale (VRS) assumption is placed in Appendix 1.

The achievement objective function $\left\{ d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j \right\}$ states that all deviations have been given equal weights. In the GPDEA's case, minimizing the unwanted deviations from the goal values are to be desired (Ignizio, 1976; lee, 1972). However, there are fundamental flaws associated with the GPDEA model, ranging from the interpretation of a goal programming method to the reported results. We highlight some of these issue separately in the next section.

3. Some drawbacks on both GPDEA models

The purpose of this section is to highlight some drawbacks of the GPDEA models, which will help us to further develop the new bi-objective multiple criteria DEA (BiO-MCDEA) model in Section 4.

The validity of GPDEA and the issue of zero weights for all variables in some DMUs

We were initially intrigued by the use of goal programming as a means to achieve greater weight dispersion and discrimination power among criteria in DEA. When attempting to reproduce the analysis in Bal et al. (Bal, et al., 2010)**,** we have noted some methodological and formulation problems. We found some of these problems to be consistent for all datasets in Bal et al (Bal, et al., 2010). However, for the purpose of illustrating the inappropriateness of the GPDEA models, we only explain the solutions of 'dataset 1' and 'university dataset' in Bal et al (Bal, et al., 2010).

Let us first start with the hypothetical dataset consisting of 10 DMUs with four inputs and four outputs (see Table 1- which is reproduced from Bal et al. (Bal, et al., 2010) for ease of reference).

Table 1

Example 1 dataset

DMU	Outputs				Inputs			
	y_1	y_2	y_3	y_4	x_1	x_2	x_3	x_4
1	47	93	54	65	32	50	82	46
2	88	56	92	80	61	56	68	37
3	94	65	80	80	42	58	45	34
4	50	53	93	97	73	39	88	81
5	47	42	70	52	45	38	68	41
6	86	45	100	47	86	62	44	32
7	83	91	62	74	38	74	71	74
8	79	60	72	98	61	54	70	62
9	85	68	51	41	84	52	38	47
10	78	95	70	92	87	47	31	52

We used Model 1 formulation for both CRS and VRS assumptions to reproduce the results as depicted in Table 2 and Table 3. It is easy to observe that the true efficiency values differ significantly from the ones reported in Bal et al. (Bal, et al., 2010). More importantly, we examined the weights and noticed contrary to what had been claimed in Bal et al. (Bal, et al., 2010), the input-output weights and efficiency values for some DMUs could attain zero values for all variables. For example in this case, zero weights assigned to all variables for DMU_1 (under CRS) and DMU_5 (under CRS and VRS). This just disproves the "…improvement of the dispersion of input-output weights and the improvement of discrimination power…" as claimed in Bal et al. (Bal, et al., 2010). This is problematic when some of the efficiency values can be 1 at the same instance, thus confirming the inability for the input and output weights to translate into technical efficiency effectively (see appendix 2 for proof).

It is rather quite simple to reason where the problem lies. As one can easily observe in the next section, we impose some restrictions on the weights to avoid this issue. In an input-oriented model, it is necessary to set the constraint $\sum_{i=1}^{m} v_i x_{io} = 1$, and seek to achieve an output that is as high as possible. This is a fundamental aspect of scaling and benchmarking, where one has to fix either the sum of input or the sum of output to be 1, before proceeding to determine the other. In Bal et al.'s case (Bal, et al., 2010), they chose to set both $\sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1$ and $\sum_{r=1}^{s} u_r y_{ro} + d_2^- - d_2^+ = 1$. It stands to reason that proper scaling cannot be achieved in this manner as the model is neither input nor output oriented. Even if we eliminate $\sum_{r=1}^{s} u_r y_{ro} + d_2^- - d_2^+ = 1$, there is a possibility for $0 \le \sum_{i=1}^{m} v_i x_{io} \le 1$ due to the minimization of $d_1^- - d_1^+$ in the objective function.

Table 2

GPDEA-CCR results based on example 1 dataset

DMU	Output weights				Input weights				Efficiency	Efficiency
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	true values	Provided by Bal
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.968
2	0.00317	0.00434	0.00464	**0**	0.00403	0.0135	0	0	0.948	0.951
3	0.00333	0.00456	0.00488	**0**	0.00424	0.0142	0	0	1	1
4	0	0.00488	0.00797	**0**	0.00336	0.0118	0.0006	0.003	1	1
5	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.95
6	0.00268	0.00367	0.00392	**0**	0.00341	0.0114	0	0	0.788	0.794
7	0.0007	0.00371	0.00564	**0**	0.00245	0.0099	0	0.002	0.745	0.779
8	0.00084	0.00446	0.00679	**0**	0.00295	0.0119	0	0.003	0.823	0.843
9	0.00305	0.00417	0.00446	**0**	0.00388	0.013	0	0	0.771	0.767
10	0.00322	0.00441	0.00471	**0**	0.00409	0.0137	0	0	1	1

Table 3

GPDEA-BCC results based on example 1 dataset

DMU	Output weights				Input weights				Efficiency	Efficiency
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	true values	Provided by Bal
1	0.00762	0	0.00172	**0**	0.00155	0.019	0	0	0.765	0.971
2	0.0034	0.00328	0.00307	**0**	0.00368	0.0138	0	0	0.945	0.951
3	0.00355	0.00343	0.00321	**0**	0.00385	0.0145	0	0	1	1
4	0	0.005	0.00821	**0**	0.00314	0.0119	0.0003	0.003	1	1
5	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.961
6	0.00289	0.00279	0.00261	**0**	0.00313	0.0118	0	0	0.788	0.965
7	0.0052	0	0.00118	**0**	0.00106	0.013	0	0	0.718	0.798
8	0.00349	0.00338	0.00316	**0**	0.00379	0.0142	0	0	0.89	1
9	0.0033	0.00319	0.00298	**0**	0.00358	0.0135	0	0	0.824	0.909
10	0.0035	0.00338	0.00316	**0**	0.00379	0.0143	0	0	1	1

The validity of GPDEA when compared with the results of MCDEA

To explore the results of MCDEA models we further compared the results of GPDEA with MCDEA. We discovered that the GPDEA models do not conduct nor achieve the same purposes as the MCDEA model. MCDEA model uses non-dominated solutions and each objective is handled one at a time; hence unlike the GPDEA models, MCDEA does not attempt to get a global optimal value but more towards generating a series of non-dominated solutions interactively. In other words, MCDEA can be used to achieve either a stricter or more lenient solution set, depending on whether more or less number of efficient DMUs are sought by the analyst in the decision making process.

We recomputed the results of the MCDEA model of Li and Reeves using the Minsum objective function of $\sum_j d_j$ and reproduce them in Table 4 (CRS) and Table 5 (VRS). If one would compare the efficiency values of Table 4 and Table 5 with the GPDEA models of Table 2 and Table 3, the observation would yield similar efficiency values. Again, the comparison has to be made on the corrected values denoted as 'true values' in Table 2 and Table 3 and not the 'values reported in Bal et al. (2010)'. In summary, we found that the GPDEA models' achievement objective function $\{d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j\}$ cannot handle all three criteria of the MCDEA model.

Table 4

Minsum DEA-CCR results based on example 1 dataset

DMU	Output weights				Input weights				Efficiency
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	
1	0.00102	0.00543	0.00827	**0**	0.00359	0.01453	0	0.0034	1
2	0.00317	0.00434	0.00464	**0**	0.00403	0.01347	0	0	0.948
3	0.00333	0.00456	0.00488	**0**	0.00424	0.01417	0	0	1
4	0	0.00488	0.00797	**0**	0.00336	0.01182	0.0006	0.003	1
5	0.00119	0.00636	0.00967	**0**	0.0042	0.01699	0	0.004	1
6	0.00268	0.00367	0.00392	**0**	0.00341	0.0114	0	0	0.788
7	0.0007	0.00371	0.00564	**0**	0.00245	0.0099	0	0.0024	0.745
8	0.00084	0.00446	0.00679	**0**	0.00295	0.01193	0	0.0028	0.823
9	0.00305	0.00417	0.00446	**0**	0.00388	0.01297	0	0	0.771
10	0.00322	0.00441	0.00471	**0**	0.00409	0.0137	0	0	1

The validity of GPDEA when investigating the case of variable returns to scales (VRS)

In classical VRS model (Banker, Charnes, Cooper 1984), c_o is a free variable placed in both the objective function and the inequality constraint. We ran the analysis based on a wrongly formulated VRS model on purpose by considering only c_o in the constraint $\sum_r u_r y_{rj} - \sum_i v_i x_{ij} + d_j = 0$ but not in the objective function of MCDEA Model 1 for the Minsum objective function of $\sum_j d_j$ (see Appendix 3). With the exception of DMU5, we achieved the same efficiency results as Bal et al. (Bal, et al., 2010) with this purposefully intended incorrect formulation! This can be observed by comparing the true values in Table 3 against the efficiency values in Table 5. It can therefore be concluded that GPDEA model under VRS proposed by Bal et al.(Bal, et al., 2010), as seen in Appendix 3, is not an acceptable model as an extension of VRS model (Banker, Charnes, & Cooper, 1984) for MCDEA.

Table 5

Minsum DEA-BCC results based on example 1 dataset

DMU	Output weights				Input weights				Efficiency
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	
1	0.00762	0	0.00172	**0**	0.00155	0.01901	0	0	0.765
2	0.0034	0.00328	0.00307	**0**	0.00368	0.01385	0	0	0.945
3	0.00355	0.00343	0.00321	**0**	0.00385	0.01446	0	0	1
4	0	0.005	0.00821	**0**	0.00314	0.0119	0.0003	0.0034	1
5	0.00491	0.00475	0.00444	**0**	0.00532	0.02001	0	0	1
6	0.00289	0.00279	0.00261	**0**	0.00313	0.01178	0	0	0.788
7	0.0052	0	0.00118	**0**	0.00106	0.01297	0	0	0.718
8	0.00349	0.00338	0.00316	**0**	0.00379	0.01424	0	0	0.89
9	0.0033	0.00319	0.00298	**0**	0.00358	0.01345	0	0	0.824
10	0.0035	0.00338	0.00316	**0**	0.00379	0.01426	0	0	1

The validity of GPDEA and the issue of zero weights on specific variable for all DMUs

Table 6 is the same data as used in Bal et al. (Bal, et al., 2010), which is reproduced here for ease of reference. The data consists of 7 departments (DMUs) in a university with the following input and output variables: number of academic staff (x_1), academic staff salaries in thousands of pounds (x_2), support staff salaries in thousands of pounds (x_3), number of undergraduate students (y_1), number of postgraduate students (y_2), number of research papers (y_3).

Table 6

Example 2 university dataset

DMU	Outputs			Inputs		
	y_1	y_2	y_3	x_1	x_2	x_3
1	60	35	17	12	400	20
2	139	41	40	19	750	70
3	225	68	75	42	1500	70
4	90	12	17	15	600	100
5	253	145	130	45	2000	250
6	132	45	45	19	730	50
7	305	159	97	41	2350	600

When applying the GPDEA model, we first noticed the results reported in Bal et al (Bal, et al., 2010) were incorrect. We therefore reported the correct results in Table 7 to 10. It is easy to observe that the input-output weights do not discriminate well and the GPDEA model cannot be representative of the

MCDEA model. Based on the correct weights reported in Tables 7 to 10 derived from the analysis, it can be noted that the third input is ignored by almost all DMUs in Tables 7 and 9. Also, the first and third outputs are ignored by all DMUs in Tables 8 and 10 (as it be seen all weights are set to zero). That suggests that these variables have no effect in the efficiency of values of the evaluation!. We will see that in the proposed model of Section 4; we would impose some restrictions on the weights to avoid this issue.

Table 7
GPDEA-CCR results of the university dataset

DMU	Output weights			Input weights			Efficiency true values	Efficiency Provided by Bal
	u_1	u_2	u_3	v_1	v_2	v_3		
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	1
2	0.00333	0.00921	0.0029	0.02019	0.00082	**0**	0.9556	0.955
3	0.0016	0.00442	0.0014	0.0097	0.00039	**0**	0.7648	0.764
4	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.576
5	0.0013	0.00361	0.0011	0.00791	0.00032	**0**	1	1
6	0.00339	0.00936	0.0029	0.02053	0.00084	**0**	1	1
7	0.0026	0.00218	0	0	0.00041	**6.4E-05**	1	1

Table 8
GPDEA-BCC results of the university dataset

DMU	Output weights			Input weights			Efficiency true values	Efficiency Provided by Bal
	u_1	u_2	u_3	v_1	v_2	v_3		
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	1
2	0.00834	0.007	**0**	**0**	0.00131	0.00021	1	0.963
3	0.0042	0.00353	**0**	**0**	0.00066	0.0001	0.9603	0.813
4	0.01031	0.00866	**0**	**0**	0.00162	0.00025	0.4796	0.576
5	0.00311	0.00261	**0**	**0**	0.00049	7.7E-05	1	1
6	0.00861	0.00722	**0**	**0**	0.00136	0.00021	1	1
7	0.0026	0.00218	**0**	**0**	0.00041	6.4E-05	1	1

Table 9
Minsum DEA-CCR results of the university dataset

DMU	Output weights			Input weights			Efficiency
	u_1	u_2	u_3	v_1	v_2	v_3	
1	0.00583	0.01612	0.0051	0.03536	0.00144	**0**	1
2	0.00333	0.00921	0.0029	0.02019	0.00082	**0**	0.9556
3	0.0016	0.00442	0.0014	0.0097	0.00039	**0**	0.7648
4	0.00418	0.01157	0.0036	0.02537	0.00103	**0**	0.5769
5	0.0013	0.00361	0.0011	0.00791	0.00032	**0**	1
6	0.00339	0.00936	0.0029	0.02053	0.00084	**0**	1
7	0.00121	0.00334	0.001	0.00732	0.0003	**0**	1

Table 10

Minsum DEA-BCC results of the university dataset

DMU	Output weights			Input weights			Efficiency
	u_1	u_2	u_3	v_1	v_2	v_3	
1	0.01575	0.01322	**0**	**0**	0.00248	0.00039	0.5639
2	0.00834	0.007	**0**	**0**	0.00131	0.00021	1
3	0.0042	0.00353	**0**	**0**	0.00066	0.0001	0.9603
4	0.01031	0.00866	**0**	**0**	0.00162	0.00025	0.4796
5	0.00311	0.00261	**0**	**0**	0.00049	7.7E-05	1
6	0.00861	0.00722	**0**	**0**	0.00136	0.00021	1
7	0.0026	0.00218	**0**	**0**	0.00041	6.4E-05	1

4. A new bi-objective multiple criteria (BiO-MCDEA) model

The aim of this section is to introduce an alternative MCDEA model which is able to provide better weight dispersion and discrimination while allowing multiple criteria to be optimised simultaneously. In our attempt, we seek to avoid the earlier issues raised in the GPDEA models.

Although there are a variety of solution procedures for multi-objective or multi-criteria linear programming (MOLP or MCLP), only goal programming had been suggested for optimizing all objectives simultaneously. The difficulty of a multi-objective problem is not just in finding an optimal solution for each objective function but to find an optimal solution that simultaneously optimizes all objectives. In most cases, no single optimal solution would satisfy all the conditions simultaneously, thus requiring a set of efficient or non-dominated solutions. Further details on MOLP problem can be found in (Cohon, 1987; Dimitris P, 2003).

In Li and Reeves (Li&Reeves, 1999) proposed MCDEA model, they used the ''non-dominated'' solution approach. Bal et al. (Bal, et al., 2010) proposed goal programming as an alternative for achieving all objectives simultaneously in the MCDEA model. It has been pointed out in the previous section that the proposed GPDEA models suffer from serious drawbacks. We are compelled therefore to consider an alternative approach to optimise all objectives simultaneously in a MCDEA model, i.e. a bi-objective weighted formulation.

Recalling Li and Reeves (Li & Reeves, 1999) approach, the MCDEA model's objective functions consistof three parts: min d_o , min M, and min $\sum_j d_j$ as defined in model 1. In a weighted method, the MCDEA's tri-objective function can be restated as follows, $w_1 d_o + w_2 M + w_3 \sum_j d_j$ for the single weighted objective equivalent. The weights $w_i (i = 1,2,3)$ can be varied to obtain different efficient solutions.

However, given that the first objective w_1 is in fact the equivalent to a conventional CCR model, it can be eliminated from the MCDEA in the weighted objective sense. Besides, Li and Reeves had demonstrated that the first objective yields lower discrimination power as compared to the other two objectives. Hence, for our proposed model, we solved the bi-objective weighted problem using both the second and third objectives. The value of w_1 is set equal to zero because whenever $\sum_j d_j$ is minimized, d_o will be minimized as well. Thus, we proposed the following model:

Model 3: A new bi-objective MCDEA (Bio-MCDM) model under CRS

$$\text{Min } h = \left(w_2 M + w_3 \sum_j d_j \right)$$

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1,...,n,$$

$$M - d_j \geq 0, \quad j = 1,...,n,$$

$$u_r \geq \varepsilon, \quad r = 1,...,s,$$

$$v_i \geq \varepsilon, \quad i = 1,...,m,$$

$$d_j \geq 0, \quad j = 1,...,n, \qquad (4)$$

where d_o and d_j (j =1,…,n) are the deviation variables for DMU_o and the j_{th} DMU respectively. DMU_o is efficient if and only if d_o=0, otherwise the efficiency value of DMU_o is $h_o = 1 - d_o$. The effect of constraints $M - d_j, (j = 1, \dots, n)$ which do not change the feasible region of the solution is merely to make M the maximum deviation. The values of u_r and v_r are set to be greater than or equal to ε, thus ensuring that this lower bound specification will avoid inputs or the outputs from being ignored by the $DMUs$.

We analyzed the dataset of 'Example 1' and the 'university dataset' with the proposed approach. The efficiency values in Table 11 and Table 12 perform better when compared against the actual efficiency values of the GPDEA-CCR models (Table 2 and Table 7, respectively).

Table 11

Model (4) results based on example 1 dataset (ε=0.0001)

DMU	Output weights				Input weights				Efficiency	Super Efficiency	Rank
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4			
1	0.0042	0.00481	0.00573	0.0001	0.00453	0.01678	0.0001	0.0002	0.961	0.961	4
2	0.0029	0.00435	0.0048	0.0001	0.00404	0.01324	0.0001	0.0001	0.948	0.948	5
3	0.00358	0.00408	0.00488	0.0001	0.00386	0.01429	0.0001	0.0001	1	1.21	2
4	0.0001	0.00486	0.00782	0.0001	0.00344	0.01191	0.0006	0.0029	1	1.079	3
5	0.0042	0.00624	0.0069	0.0001	0.00576	0.01906	0.0001	0.0002	0.947	0.947	6
6	0.00245	0.00369	0.00408	0.0001	0.00344	0.01123	0.0001	0.0001	0.789	0.789	8
7	0.00116	0.00373	0.00522	0.0001	0.00283	0.01031	0.0001	0.0016	0.767	0.767	9
8	0.00147	0.00445	0.00617	0.0001	0.00339	0.01237	0.0001	0.0019	0.837	0.837	7
9	0.00279	0.00418	0.00463	0.0001	0.00389	0.01275	0.0001	0.0001	0.761	0.761	10
10	0.00294	0.00441	0.00488	0.0001	0.0041	0.01346	0.0001	0.0001	1	1.419	1

Table 12

Model (4) results of the university dataset (ε=0.0001)

DMU	Output weights			Input weights			Efficiency	Super Efficiency	Rank
	u_1	u_2	u_3	v_1	v_2	v_3			
1	0.00584	0.01619	0.00486	0.0371	0.00138	0.0001	1	1.136	3
2	0.00335	0.0093	0.0027	0.022	0.00077	0.0001	0.955	0.955	5
3	0.00162	0.00452	0.0012	0.0115	0.00034	0.0001	0.763	0.763	6
4	0.00419	0.01162	0.00343	0.0271	0.00097	0.0001	0.575	0.575	7
5	0.00133	0.00372	0.00095	0.0098	0.00027	0.0001	1	1.171	2
6	0.00341	0.00947	0.00275	0.0224	0.00078	0.0001	1	1.037	4
7	0.00122	0.00342	0.00086	0.0091	0.00024	0.0001	1	1.241	1

5. An Application of Energy Dependency among EU member countries

We further illustrate our proposed model with a 3-inputs and 2-outputs data of 26 European Union (EU) member countries (except Malta) as presented in Appendix 4. Data were based on the EU Emissions Trading Scheme of more than 10,000 installations that generate an excess of 20^{MW} each within the country. This is believed to capture about half of the CO_2 emissions within EU. We termed the model as energy dependency as the choice of inputs are based on a set of resources that generate carbon emissions and the output will be the extent of those resources in limiting the carbon effects. The operational definition of the 3 inputs and 2 outputs are as follows:

Table 13

Model Variables and Operational Definition

Input Variables	**Definition**
Installation Count (x_1)	An installation is a stationary technical unit where one or more activities are carried out, which could have an effect on emissions and pollution.
Allocated Carbon Allowances (x_2)	It is an allowance distributed each year for free to installations according to the national allocation plan, measured in tonnes of carbon dioxide equivalent.
Gross Inland energy consumption (GIC), by fuel (x_3)	GIC is the quantity of energy, expressed in oil equivalents, consumed within the borders of a country. It is calculated as total domestic energy production plus energy imports and changes in stocks minus energy exports.
Output Variables	
Electricity Generated From Renewable Sources (y_1)	Percentage of gross electricity consumed.
Share of renewable energy in fuel consumption of transport (y_2)	The degree to which conventional fuels have been substituted by biofuels in transportation.

Although the weights of our proposed Model 4 can be varied to obtain a set of efficiency scores according to the decision analyst's preference, we have set equal objectives such that w_2=w_3=0.5. The results are presented in Table 14 and compared against Bal et al.'s GPDEA CCR solution in Table 15. The results show that the proposed Model outperforms the GPDEA model; both, in terms of discrimination power and weight dispersion.

Table 14

Model (4) results of the 26-country dataset (Ɛ = 0.0001)

DMU	Output Weights		Inputs Weight			Efficiency	Rank
	u_1	u_2	v_1	v_2	v_3		
Austria	0.1762	0.0165	0.0001	3.1433	0.0001	0.1887	8
Belgium	0.4010	0.0793	5.5165	0.0001	0.0001	0.0669	14
Bulgaria	0.0737	0.2344	0.0001	0.0001	9.6501	0.0272	23
Cyprus	0.5878	1.8698	0.0001	0.0001	76.9715	0.4355	4
Czech Republic	0.0348	0.1107	0.0001	0.0001	4.5564	0.0473	17
Denmark	0.3558	0.0703	4.8945	0.0001	0.0001	0.1492	10
Estonia	0.2523	0.8026	0.0001	0.0001	33.0405	0.0417	18
Finland	0.2196	0.0434	3.0211	0.0001	0.0001	0.0964	11
France	0.0784	0.0074	0.0001	1.3977	0.0001	0.0210	24
Germany	0.0077	0.0243	0.0001	0.0001	0.9998	0.0180	25
Greece	0.0473	0.1505	0.0001	0.0001	6.1934	0.0279	21
Hungary	0.5376	0.1063	7.3962	0.0001	0.0001	0.0946	12
Ireland	1.1707	0.2314	16.1047	0.0001	0.0001	0.2952	6
Italy	0.0969	0.0091	0.0001	1.7278	0.0001	0.0338	20
Latvia	0.9906	0.0929	0.0001	17.6725	0.0001	0.7431	3
Lithuania	1.2734	0.2517	17.5175	0.0001	0.0001	0.2279	7
Luxembourg	9.6775	1.9132	133.1332	0.0001	0.0001	1.0000	1
Netherlands	0.0357	0.1135	0.0001	0.0001	4.6723	0.0603	15
Poland	0.0148	0.0471	0.0001	0.0001	1.9389	0.0276	22
Portugal	0.3279	0.0307	0.0001	5.8497	0.0001	0.1762	9
Romania	0.0405	0.1287	0.0001	0.0001	5.2964	0.0409	19
Slovakia	0.7222	0.1428	9.9352	0.0001	0.0001	0.3361	5
Slovenia	1.4516	0.2870	19.9699	0.0001	0.0001	0.8628	2
Spain	0.1270	0.0251	1.7470	0.0001	0.0001	0.0592	16
Sweden	0.0981	0.0092	0.0001	1.7505	0.0001	0.0906	13
United Kingdom	0.0138	0.0438	0.0001	0.0001	1.8016	0.0151	26

Note: The results are based on the normalized version of the raw data provided in Appendix 4

Table 15

Bal et al.'s GPDEA-CCR results of the 26-country dataset (Ɛ = 0.0001)

DMU	Output Weights		Inputs Weight			Efficiency
	u_1	u_2	v_1	v_2	v_3	
Austria	0.0001	0.0001	0.00089	0.0001	0.00359	0.00018
Belgium	0.0001	0.0001	0.00089	0.0001	0.00359	0.00005
Bulgaria	0.0001	0.0001	0.00089	0.0001	0.00359	0.00002
Cyprus	0.0001	0.0001	0.00089	0.0001	0.00359	0.00002
Czech Republic	0.0001	0.0001	0.00089	0.0001	0.00359	0.00005
Denmark	0.0001	0.0001	0.00089	0.0001	0.00359	0.00005
Estonia	0.0001	0.0001	0.00089	0.0001	0.00359	0.00001
Finland	0.0001	0.0001	0.00089	0.0001	0.00359	0.00007
France	0.0001	0.0001	0.00089	0.0001	0.00359	0.00009
Germany	0.0001	0.0001	0.00089	0.0001	0.00359	0.00009
Greece	0.0001	0.0001	0.00089	0.0001	0.00359	0.00003
Hungary	0.0001	0.0001	0.00089	0.0001	0.00359	0.00005
Ireland	0.0001	0.0001	0.00089	0.0001	0.00359	0.00004
Italy	0.0001	0.0001	0.00089	0.0001	0.00359	0.00007
Latvia	0.0001	0.0001	0.00089	0.0001	0.00359	0.00009
Lithuania	0.0001	0.0001	0.00089	0.0001	0.00359	0.00006
Luxembourg	0.0001	0.0001	0.00089	0.0001	0.00359	0.00003
Netherlands	0.0001	0.0001	0.00089	0.0001	0.00359	0.00006
Poland	0.0001	0.0001	0.00089	0.0001	0.00359	0.00006
Portugal	0.0001	0.0001	0.00089	0.0001	0.00359	0.00009
Romania	0.0001	0.0001	0.00089	0.0001	0.00359	0.00006
Slovakia	0.0001	0.0001	0.00089	0.0001	0.00359	0.00013
Slovenia	0.0001	0.0001	0.00089	0.0001	0.00359	0.00008
Spain	0.0001	0.0001	0.00089	0.0001	0.00359	0.00008
Sweden	0.0001	0.0001	0.00089	0.0001	0.00359	0.00017
United Kingdom	0.0001	0.0001	0.00089	0.0001	0.00359	0.00004

Note: The analysis above is based on Bal et al.'s GPDEA-CCR model. The weights and efficiency values are close to zero, rendering the model to have poor weight dispersion and discriminant power. The results are based on the normalized version of the raw data provided in Appendix 4.

Comparing the two, it can be easily observed from Table 13 and Table 14 that the efficiency scores from our proposed model could provide easy ranking without any ties. Such is not the case for the GPDEA efficiency scores in Table 14. All the efficiency scores and weights for GPDEA appear to be approximate to zero.

6. Concluding remarks

Since 1999 when Li and Reeves first proposed the MCDEA, there is only but one other proposed solution approach that was suggested for MCDEA, which is the GPDEA. We have shown that the GPDEA models are not alternatives to the MCDEA model. It has major drawbacks in both discrimination power and weight dispersion, aside from the misreported efficiency values of all the tests. Hence, the fair basis of comparison would be between our proposed model and the GPDEA models, given that the MCDEA model merely provided a mathematical formulation with an interactive solution procedure without any emphasis placed on the issues of discrimination power and weight dispersion.

Hence, we have illustrated that our bi-objective multiple criteria DEA model outperforms the GPDEA model in terms of both weight dispersion and discrimination power.

Although we have proposed a bi-objective weighted method for solving the MCDEA model, we stress that there may be other procedures that can be used to extract solutions under multiobjective LP environment. We merely show a procedure that performs better than the GPDEA in terms of ease of formulation and mathematical programming (i.e. less computational codes). We hope that future researchers in DEA will provide better solution procedures for the MCDEA model.

References

Andersen, P., &Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science, 39*, 1261-1264.

Anderson, T. R., Hollingsworth, K., &Inman, L. (2002). The fixed weighting nature of a cross-evaluation model *Journal of productivity analysis, 17*, 249-255.

Bal, H., Örkcü, H. H., &Çelebioglu, S. (2010). Improving the discrimination power and weights dispersion in the data envelopment analysis. *Computers & Operations Research, 37*, 99-107.

Banker, R. D., Charnes, A., &Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*, 1078-1092.

Charnes, A., Cooper, W. W., Huang, Z. M., &Sun, D. B. (1990a). Polyhedral Cone-Ratio DEA Models with an illustrative application to large commercial banks. *Journal of Econometrics, 46*, 73-91.

Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1990b). Polyhedral cone-ratio models with an illustrative application to large commercial banks *Journal of Econometrics, 46*, 73-91.

Charnes, A., Cooper, W. W., &Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429-444.

Chen, Y.-W., Larbani, M., &Chang, Y.-P. (2009). Multiobjective data envelopment analysis. *Journal of the Operational Research Society, 60*, 1556-1566.

Chen, Y. (2005). Measuring super-efficiency in DEA in the presence of infeasibility. *European Journal of Operational Research, 161*, 447-468.

Cohon, J. L. (1987). *Multiobjective Programming and Planning*. New York: Academic Press

Dimitris P, M. (2003). Multiobjective programming methods in the reserve selection problem. *European Journal of Operational Research, 150*, 640-652.

Doyle, J. R., &Green, R. H. (1994). Efficiency an cross-efficiency in DEA: derivation, measuring and uses. *Journal of the Operational Research Society, 45*, 567-578.

Doyle, J. R., &Green, R. H. (1995). Cross-evaluation in DEA: Improving discrimination among DMUs. *INFOR, 33*, 205-222.

Dyson, R. G., &Thanassoulis, E. (1988). Reducing Weight Flexibility in Data Envelopment Analysis. *Journal of the Operational Research Society, 39*, 563-576.

Emrouznejad, A., Parker, B. R., &Tavares, G. (2008). Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA. *Socio-Economic Planning Sciences, 42*, 151-157.

Green, R. H., Doyle, J. R., &Cook, W. D. (1996). Preference voting and project ranking using DEA and cross-evaluation. *European Journal of Operational Research, 90*, 461-472.

Ignizio, J. P. (1976). *Goal programming and extensions*. Lexington: Lexington Books.

lee, S. M. (1972). *Goal programming for decision analysis*. Philadelphia: Auerback.

Li, X.-B., &Reeves, G. R. (1999). A multiple criteria approach to data envelopment analysis. *European Journal of Operational Research, 115*, 507-517.

Sexton, T. R., Silkman, R. H., &Hogan, A. J. (1986). The methodology of data envelopment analysis In R. H. Silkman (Ed.), *Measuring efficiency: An assessment of data envelopment analysis* (pp. 7-29). San Fransisco: Jossey-Bass.

Thanassoulis, E., &Allen, R. (1998). Simulating Weights Restrictions in Data Envelopment Analysis by Means of Unobserved DMUs. *Management Science, 44*, 586-594.

Thompson, R. G., F. D. Singleton Jr., Thrall, R. M., &Smith, B. A. (1986). Comparative Site Evaluations for Locating a High-Energy Physics Lab in Texas. *INTERFACES, 16*, 35-49.

Thompson, R. G., Langemeier, L. N., Lee, C. T., &Thrall, R. M. (1990). The role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics, 46*, 93-108.

Appendix 1:

Goal programming DEA model under variable returns to scale as proposed in Bal et al's (Bal, et al., 2010) ***(see also appendix 3)***

$$\min a = \left\{ d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j \right\}$$

$$\sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1,$$

$$\sum_{r=1}^{s} u_r y_{ro} + c_o + d_2^- - d_2^+ = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + c_o + d_j = 0, \quad j = 1,\dots,n,$$

$$M - d_j + d_{3j}^- - d_{3j}^+ = 0, \quad j = 1,\dots,n,$$

$$u_r \geq 0, \quad r = 1,\dots,s,$$

$$v_i \geq 0, \quad i = 1, \ldots, m,$$

$$d_j \geq 0, \quad j = 1, \ldots, n,$$

$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0,$$

$$d_{3j}^-, d_{3j}^+ \geq 0, \quad j = 1, \ldots, n,$$

$$c_o \ \textit{free in sign} \qquad (3)$$

Appendix 2:

<u>Proof of logical invalidity of GPDEA formulation</u>

From Bal el al.'s GPDEA-CCR model (2):

$$\sum_{r=1}^{s} u_r y_{ro} + d_2^- - d_2^+ = 1 \quad (I)$$

$$\sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1 \quad (II)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad (III)$$

Multiplying equality (II) by -1:

$$-\sum_{i=1}^{m} v_i x_{io} - (d_1^- - d_1^+) = -1 \qquad (IV)$$

adding $(I)$and (IV)yields:

$$\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + d_2^- - d_2^+ - (d_1^- - d_1^+) = 0 \quad (V)$$

Suppose that $j = o$ in equality (III):

$$\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + d_o = 0 \quad (VI)$$

By considering(V), (VI), it can be concluded: $d_o = d_2^- - d_2^+ - (d_1^- - d_1^+) \quad (VII)$

Since the efficiency value for DMU under evaluation, h_o, must be equal to $\sum_{r=1}^{s} u_r y_{ro}$,($I$) can be restated as:

$$h_o = \sum_{r=1}^{s} u_r y_{ro} = 1 - (d_2^- - d_2^+)$$

Since$h_o = 1 - d_o$ in classical DEA, $d_o = (d_2^- - d_2^+)$ in $(VIII)$. Therefore, the value of $d_1^- - d_1^+$ in (VII) must be equal to zero to render correctness. Nonetheless, in Bal el al.'s GPDEA models, the weighted sum of inputs for DMU under evaluation$\sum_{i=1}^{m} v_i x_{io}$, can be zero or less than unity, which is highly problematic. Without loss of generality, the same problem applies to the GPDEA-BCC model.

Appendix 3:

<u>Minsum BCC-DEA model under variable returns to scale, a wrongly formulated VRS model</u>

$$\min \sum_{j=1}^{n} d_j$$

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + c_o + d_j = 0, \quad j = 1, \ldots, n,$$

$$M - d_j \geq 0, \quad j = 1, \ldots, n,$$

$$u_r \geq 0, \quad r = 1, \ldots, s,$$

$$v_i \geq 0, \quad i = 1, \ldots, m,$$

$$d_j \geq 0, \quad j = 1, \ldots, n,$$

$$c_o \ \textit{free in sign} \qquad (4)$$

Appendix 4:

The Energy Dependency Dataset

Dataset of 26 countries

Countries	Outputs		Inputs		
	Y_1	Y_2	X_1	X_2	X_3
Austria	66.793	6.5	225	8810	31887710
Belgium	6.083	3.3	362	2242	56797576
Bulgaria	9.808	0.6	146	1087	40591231
Cyprus	0.073	2.0	13	98	5089082
Czech Republic	6.783	3.4	425	2425	85968002
Denmark	27.390	0.4	408	3242	23912314
Estonia	6.105	0.2	56	717	11855527
Finland	25.777	2.3	661	7887	37069940
France	13.547	6.0	1125	19811	128660709
Germany	16.200	5.7	1997	27693	391714624
Greece	12.276	1.1	162	1861	63246705
Hungary	6.988	3.1	270	1854	23844843
Ireland	13.925	1.9	124	641	19951911
Italy	20.536	3.8	1201	16026	208982856
Latvia	49.232	1.2	111	1567	3532491
Lithuania	5.505	4.2	114	874	7573712
Luxembourg	3.678	2.1	15	121	2488229
Netherlands	9.152	4.2	443	3148	83834170
Poland	5.804	4.8	943	6265	202011597
Portugal	33.267	3.6	280	4734	30902050
Romania	27.916	1.6	275	5270	73956515
Slovakia	17.880	8.6	201	1214	32140581
Slovenia	36.783	1.9	100	887	8216051
Spain	25.747	3.5	1143	12091	150707494
Sweden	56.378	7.3	821	15819	21103878
United Kingdom	6.664	2.7	1140	6214	217404830

Note: The data are taken from three databases: European commision's Eurostat, Carbonmarketdata.com and i-insights.com