QUANTUM RINGS IN ELECTROMAGNETIC FIELDS

Submitted by Arseny M. Alexeev to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Physics.

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I certify that all material in this thesis which is not my own work has been identified and that no material is included for which a degree has previously been conferred upon me.

Abstract

This thesis is devoted to optical properties of Aharonov-Bohm quantum rings in external electromagnetic fields. It contains two problems.

The first problem deals with a single-electron Aharonov-Bohm quantum ring pierced by a magnetic flux and subjected to an in-plane (lateral) electric field. We predict magneto-oscillations of the ring electric dipole moment. These oscillations are accompanied by periodic changes in the selection rules for inter-level optical transitions in the ring allowing control of polarization properties of the associated terahertz radiation.

The second problem treats a single-mode microcavity with an embedded Aharonov-Bohm quantum ring, which is pierced by a magnetic flux and subjected to a lateral electric field. We show that external electric and magnetic fields provide additional means of control of the emission spectrum of the system. In particular, when the magnetic flux through the quantum ring is equal to a half-integer number of the magnetic flux quantum, a small change in the lateral electric field allows tuning of the energy levels of the quantum ring into resonance with the microcavity mode, providing an efficient way to control the quantum ring-microcavity coupling strength. Emission spectra of the system are calculated for several combinations of the applied magnetic and electric fields.

To my grandmother,

Valentina,

without whose will to live

I would not be born.

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Glossary

2LE	Two-Level Emitter
CEF	Classical Electromagnetic Field
MC	Microcavity
Q-factor	Quality factor
QD(s)	Quantum Dot(s)
QEF	Quantized Electromagnetic Field
QHO	Quantim Harmonic Oscillator
QR(s)	Quantum Ring(s)
SPE	Single-Photon Emitter
SS	Steady State
THz	Terahertz

Introductory notes

Please note that throughout this thesis, when it is clear from the context that an operator is used, the operator symbol $\hat{}$ is omitted for reading ease.

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