

# **A Study of Convection and Dynamo in Rotating Fluid Systems**

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## Abstract

Convection in a Boussinesq fluid confined by an annular channel fast rotating about a vertical axis and uniformly heated from below, is one of our concerns in this thesis. An assumption that the channel has a sufficiently large radius in comparison with its gap-width is employed, so that the curvature effect can be neglected. The aspect ratio of the channel has great influence on the convective flow in it. Guided by the result of the linear stability analysis, we perform three-dimensional numerical simulations to investigate the convective flows under three different types of aspect ratios, which are namely the moderate or large aspect ratios, the very small aspect ratios and the moderately small aspect ratios. Also, we numerically study how convection in the channel is affected by inhomogeneous heat fluxes on sidewalls, which is a simple simulation of the thermal interaction between the Earth's core and mantle.

Convection and dynamo action in a rapidly rotating, self-gravitating, Boussinesq fluid sphere is the other concern. We develop a finite element model for the dynamo problem in a whole sphere. This model is constructed by incorporating dynamo equations with globally implemented magnetic boundary conditions to a whole sphere convection model, which is also presented here. The coordinate singularity at the center usually encountered when applying the spectral method is no longer an obstacle and no nonphysical assumptions (i.e. hyper-diffusivities) are used in our model. A large effort has been made to efficiently parallelize the model. Consequently, it can take the full advantage of modern massively parallel computers. Based on this dynamo model, we investigate the dynamo process in a sphere and find that self-sustaining dynamos are more difficult to obtain in a sphere than in a spherical shell. They are activated at relatively high Rayleigh numbers. Moreover, the magnetic fields generated are not dipole-dominant, different from those generated in most dynamo simulations.