A Study of Convection and Dynamo in Rotating Fluid Systems

Submitted by Xiaoya Zhan, to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Mathematics, March 2010.

I certify that all material in this thesis which is not my own work has been identified and that no material has been submitted and approved for the award of a degree by this or any other University.

Contents

List of Figures

List of Tables

Acknowledgements

I would like to thank my supervisor, Professor Keke Zhang, for his instruction, encouragement and help during my study for Ph.D degree.

I am also grateful to the Professor Rixiang Zhu, Professor Xinhao Liao and Dr Kit H. Chan, for their help and support over the past four years.

I wish to thank my roommates for their help and other postgraduate students for making the life enjoyable and easier.

I am also full of gratitude to my parents for their support and encouragement all the time.

Finally, I would like to thank the financial support of the ERS award and the scholarship from the University of Exeter and the School of Engineering, Computing and Mathematics. Without them, it is impossible to finish this thesis.

Abstract

Convection in a Boussinesq fluid confined by a annular channel fast rotating about a vertical axis and uniformly heated from below, is one of our concerns in this thesis. An assumption that the channel has a sufficiently large radius in comparison with its gap-width is employed, so that the curvature effect can be neglected. The aspect ratio of the channel has great influence on the convective flow in it. Guided by the result of the linear stability analysis, we perform three-dimensional numerical simulations to investigate the convective flows under three different types of aspect ratios, which are namely the moderate or large aspect ratios, the very small aspect ratios and the moderately small aspect ratios. Also, we numerically study how convection in the channel is affected by inhomogeneous heat fluxes on sidewalls, which is a simple simulation of the thermal interaction between the Earth's core and mantle.

Convection and dynamo action in a rapidly rotating, self-gravitating, Boussinesq fluid sphere is the other concern. We develop a finite element model for the dynamo problem in a whole sphere. This model is constructed by incorporating dynamo equations with globally implemented magnetic boundary conditions to a whole sphere convection model, which is also presented here. The coordinate singularity at the center usually encountered when applying the spectral method is no longer an obstacle and no nonphysical assumptions (i.e. hyper-diffusivities) are used in our model. A large effort has been made to efficiently parallelize the model. Consequently, it can take the full advantage of modern massively parallel computers. Based on this dynamo model, we investigate the dynamo process in a sphere and find that self-sustaining dynamos are more difficult to obtain in a sphere than in a spherical shell. They are activated at relatively high Rayleigh numbers. Moreover, the magnetic fields generated are not dipole-dominant, different from those generated in most dynamo simulations.