Variable versus Fixed Rate Mortgages and Optimal Monetary Policy

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

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Abstract

The overall aim of the research presented in this thesis is threefold: To empirically examine monetary transmission to UK retail mortgage rates; to examine why fixed versus variable rate mortgage lending differs across EU-15 countries; and to build a DSGE model which can be used for analysing optimal monetary policy in economies with different proportions of fixed or variable rate mortgage contracts. Chapter 2 investigates the transmission from UK policy and a range of wholesale money market rates to retail mortgage rates using a single-equation error correction model (SEECM) framework, from 1995 to 2009. The results add to previous studies by showing that the UK retail banking sector is imperfectly competitive at the aggregate level. More specifically, discounted rates, and to a lesser extent fixed rates behave competitively, whilst standard variable rates do not, which can be interpreted as evidence of exploitation of inert borrower behaviour. A snap-shot of the relative levels of variable rate lending across EU-15 countries is taken in the next Chapter 3, illustrating general cross-country differences. Risk simulations show that economies more conducive to variable rate mortgages include those with relatively volatile, persistent, and low inflation; low and stable real interest rates; high real income growth; and low correlation between inflation and real interest rate shocks. Regressions show that macroeconomic histories may indeed be important determinants of variable rate mortgage prevalence. The final Chapter 4 integrates a quantity optimising banking sector that lends under either a fixed or variable rate, into a model with borrowing constrained households. This provides a framework that can be used to investigate relationships between the structure of debt contracts and monetary policy. In particular, the propagation of a productivity shock in the non-durable sector under Ramsey monetary policy is presented, and it is demonstrated that the introduction of overlapping debt contracts tempers the effect of the financial multiplier. An appropriate design of the composition of fixed versus variable rate debt contracts, both their length and interest rate composition, could therefore reduce the volatility of key economic variables, and so there are important policy implications.

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Chapter 1

Introduction

1.1 Aims and Objectives

The overall aim of the research presented in this thesis is threefold: To empirically examine monetary transmission to UK retail mortgage rates; to examine why fixed versus variable rate mortgage lending differs across EU-15 countries; and to build a DSGE model which can be used for analysing optimal monetary policy in economies with different levels of fixed or variable rate mortgage contracts.

The first chapter aims to provide a detailed account of the transmission from UK policy and a range of wholesale money market rates to retail mortgage rates from 1995 to 2009. At the same time, another objective is to document the economy-wide effect of financial market turmoil since August 2007, and show how this has altered long- and short-term relationships between retail mortgage rates and wholesale money market rates of the same maturity. In so doing, this chapter aims to show the extent to which factors such as collusion, exploitation of consumer intertia, menu costs, and endogenous default risk hinder an otherwise competitive rate-setting process.

A snap-shot of the relative levels of variable rate lending across EU-15 countries is taken in the next chapter, with the aim of illustrating general cross-country differences, and to consider whether macroeconomic histories could help to explain them. Some of the previously presented theory forms the basis of simulations which attempt to show how the risks associated with either contract, and hence their relative desirability, vary across different macroeconomic environments. The final objective is to measure these relative risks with European-wide interest and inflation rate data, to establish the relative importance of macroeconomic histories, compared to cultural and other factors in explaining the wide range of variable rate lending levels observed.

In the final chapter, a standard methodological approach is used (DSGE modelling), with the aim of establishing the qualitative differences between economies with predominantly variable, and those with predominately fixed rate mortgage contracts. The main objective is to construct a model that is rich enough to capture realistic features of mortgage and housing markets, yet simple enough to allow comparison of optimal monetary policy between countries with predominantly variable or fixed rate contracts.

1.2 Policy Background

Developments in housing markets have received a great deal of attention from monetary policy-makers, especially since many developed countries experienced a significant rise in house prices over the last decade. With growth in the availability of retail financial products this has allowed easier access to equity, and hence spending, fuelling further house price rises, and so on. This channel, from house prices to consumption via collateral and borrowing, is considered to have been an important source of macroeconomic instability, particularly in the UK and the US. There is, however, an institutional characteristic that is strikingly different: the majority of mortgage interest payments in the US are negotiated at long term, nominally fixed rates, whereas in the UK they are mainly variable. It has been proposed (see for example (Maclennan, Muellbauer & Stephens 1998), (Treasury 2003), (Miles 2004) and (Calza, Monacelli

& Stracca 2009)) that countries with variable rate mortgages may be more exposed to monetary policy changes. Policy messages regarding the desirability of fixed versus variable rate mortgages though, have been mixed across the Atlantic. In a speech given at the Credit Union National Association 2004 Governmental Affairs Conference, Alan Greenspan stated: *'American consumers might benefit if lenders provided greater mortgage product alternatives to the traditional fixed-rate mortgage. To the degree that households are driven by fears of payment shocks but are willing to manage their own interest rate risks, the traditional fixed-rate mortgage may be an expensive method of financing a home.'* In contrast, during a speech given in February 2008, the UK chancellor stated: *'...we also want to see greater availability of affordable long-term fixed rate mortgages. For many households, particularly those on low incomes, fixing the level of mortgage repayments for several years makes real sense; and it can also contribute to wider macroeconomic stability.'* So although the absence of long-term fixed rate mortgages in the UK was one of the main reasons proposed for delaying UK entry into European Monetary Union, there also appear to be other reasons: Alleviating the welfare of low income households who cannot smooth consumption, and promoting macroeconomic stability.

The empirical summary given by (Britton & Whitley 1997) shows how difficult it is to draw firm conclusions about how fixed versus variable rate mortgages may affect the transmission mechanism. It clearly acknowledged that households in France and Germany were using significantly more fixed rate mortgages, but emphasised that this could only be a structural difference in the presense of credit constraints, which were likely to be lower in the UK.

Since the Miles Review, it appears that UK government policy has reached a consensus on three points: First, due to financial industry structure, there is a market failure that causes a short supply of fixed rate mortgage products; Second, that some households, presumably borrowing constrained (see (Graham & Wright 2007)) who are tied to variable rate mortgages suffer from unpredictably variable cash-flows; and finally, that this increases macroeconomic instability. Although dealing with the new financial/economic crisis soon took priority after the publication of the Miles Review, all three points appear to have been acknowledged in the Chancellor's speech in February 2008.

Against this policy background, the chapters presented in this thesis aim to contribute to any debates regarding the merits of fixed rate mortgage promotion. They do this by contributing to a better understanding of transmission from policy rates to retail rates in the UK; by exploring why fixed rate lending may be higher in some European macroeconomies than others; and by providing a model that can be used to investigate the fixed-variable rate contract structure that is optimal for social welfare.

1.3 Methodology

1.3.1 Overview

The methodology applied to the research presented in this thesis is current mainstream applied macroeconomics. To some extent, it is an inter-disciplinary mix of the evolving tools available to the economics profession, including the sub-disciplines of econometrics and microeconomics. The term 'applied' reflects the only clear link between chapters: that they all stem from an attempt to create frameworks that can be developed and used to help policy-makers who may be concerned about whether fixed rate or variable rate mortgage lending should be promoted. The terms 'current' and 'mainstream' indicate that the analyses and techniques used reflect how macroeconomics is applied in academic research currently. The concepts taken from applied time-series econometrics; and dynamic stochastic general equilibrium modelling are very much contemporary. It is likely that had this research been undertaken ten years in the past or future it would be done differently, and so to some extent the way questions are framed and addressed are influenced by current consensuses in the academic literature.

For Chapter 2, the methodology is carefully developed in section 2.1. The applied econometric strategy of (Liu, Margaritis & Tourani-Rad 2008) is used, with a single-equation error correction model (SEECM) to estimate long (equilibrium) and short term relationships. As is always the case in applied studies of this kind, there is a trade-off between loosening assumptions (which may be erroneously restrictive and lead to invalid inference) about the true data generating process on the one hand, and losing inferential power on the other. For example, the specification employed here assumes that changes in policy and wholesale money market rates give rise to changes in retail mortgage rates, but not vice versus. In other words, inference may be invalid to the extent that retail mortgage rates affect consumer behaviour, which in turn affects monetary policy and hence wholesale rates. For this reason, the term 'strategy' is often used, and the judgement of the researcher is important in deciding how likely alternative underlying assumptions are. Here, as in (Liu et al. 2008), it was judged that reverse feedback of this kind is unlikely to be strong enough to warrant using a less restricted VAR model.

Chapter 3 includes simulations of macroeconomic environments, specified in terms of inflation, interest rate and income processes that are compatible with real data. The specification of macroeconomic processes follows (Campbell & Cocco 2003). By adjusting parameters it is possible to analyse what type of macroeconomies are linked to risks associated with variable or fixed rate contracts, and hence it is possible to describe which are more conducive to either type. Once the factors that should encourage the establishment of either contract have been identified, they are proxied from real data across EU-15 countries to provide an indirect description of how 'variable-rate conducive' their environments are. An attempt is made to simplify the diverse nature of mortgage lending across Europe by constructing a weighted proportion of variable rate lending, based on data collected by (Maclennan et al. 1998). There are inevitably large variations in behaviour and structure, so it is not possible to take this index too far, however it is used as a dependent variable in a simple regression to see whether general levels can be explained by the general economic environments observed across Europe.

1.3.2 DSGE Modelling

A substantial part of this thesis is the construction of a DSGE model in Chapter 4. This now common methodology is usefully summarised in (Kremer, Lombardo, von Thadden & Werner 2006). Contemporary macroeconomic method is very much the product of a long conversation between competing ideas. The sometimes fierce battles, and self-labeling of different schools has encouraged the preservation of the terms 'Classical' and 'Keynesian' right up to the present, and perhaps surprisingly, DSGE modelling can be thought of as not the first, but a second synthesis of the two paradigms.

In fairly general philosophical terms, the difference between Classical and Keynesian economics has centered on an explanation of economic fluctuations, in which the former have always had more faith than the latter that nominal prices adjust perfectly in response to real changes. (Goodfriend & King 1997) argues that there was in fact a 1960s synthesis, popularised by Paul Samuelson, in which it was accepted that wage and price stickiness caused business cycles in aggregate models, but at the same time microeconomic analysis was guided by neoclassical principles.

Later, the 1970s were a time of complete disarray, as described in (Mankiw 1990). Here it is argued that the consensus broke down altogether, thanks to a simultaneous empirical conundrum (sustained rising unemployment and inflation was incompatible with the Phillips curve), and a theoretical gap between microeconomic principles and macroeconomic practise. The sealing of this gap equated to the introduction of rational expectations, which effectively paved the way for what (Goodfriend & King 1997) refers to as the 'New Neoclassical Synthesis'. Although the New Classical agenda known as the 'Real Business Cycle' research programme may not have produced a satisfactory explanation of economic fluctuations, more or less elaborate ways of incorporating price rigidities by New Keynesians into their models have been very successful. According to (Goodfriend & King 1997), the key to the success of the new synthesis is their shared belief that macro-models should always be properly micro-founded, an issue which is particularly important with respect to expectations.

In essence, DSGE models are a more general case of Real Business Cycle models, and the simple addition of a New-Keynesian Phillips curve equation is enough to produce a model of the new theoretical synthesis. This description however does not do justice to the effort made by New-Keynesians to base their ideas on as firm as possible microfoundations, allowing them to enter the same framework.

The general procedure runs as follows. First make your economic assumptions, deriving (inter-temporal) equations based on individual behaviour. Next find the first order conditions implied by this (rational optimising) behaviour. Finally, numerical values have to be assigned to the parameters (calibration), which allows matrices associated with the system's recursive form to be determined, before studying its qualitative features.

The agreement on the importance of internal consistency has dressed macro-modelling in a more scientific coat. The main advantage of this consensus approach to model building seems to be one of a shared language within which microeconomic and econometric developments can be understood. The microeconomic assumptions made at the beginning can be explicitly altered, and since the final specification of the recursive system is the same as an econometric VAR, they can be confronted with real data. The DSGE model is also like a computational scientific laboratory, where impulse reponses to external shocks can be studied, and variable moments can be simulated and compared to the real world.

Whilst comforting to a pure scientist, it is important to remember that although the conclusions of each DSGE model are internally consistent, they are also inward looking. Although they can be brought to the data in the form of a VAR, (Juselius & Franchi 2007) shows that the assumptions made at the first stage may impose unacceptably severe restrictions on VAR coefficients. This it could be argued, is a good example of why econometrics and macroeconomics can be thought of not just as separate sciences, but competing philosophies, where the former prefers to let the real world 'speak for itself', and the latter prefers to confront falsifiable theories with data. The abstractions made at each stage may illustrate this conflict:

At the first stage, a DSGE analyst should be aware of how their assumptions relate to the real world, particularly the emphasis placed on rational decision making, and the difference between individual and aggregate behaviour. It may therefore be important for applied scientists to combine this standard methodology with other, more external procedures.

A clear exposition of the differences that can occur between individual and aggregate behaviour when heterogeneity is introduced is provided in (Caballero & Engel 1991). Although valid near the steady state, the typically log-linearised equations are only approximations of the underlying assumptions, so practitioners should also consider implications of the non-linear model. The use of the term 'steady state' instead of 'equilibrium' reflects the importance of the dimension of time (and why the models are called 'dynamic'). The language however may also reflect the problematic nature of defining what equilibrium actually means. At a philosophical level, equilibrium is a dicey concept, not least because, like utility, it cannot be directly observed. The DSGE modelling procedure sheds light on this issue, in the sense that in the linearised world it is possible to establish whether or not the model has a unique steady state or not. When models have multiple equilibria, external shocks force strange jumps, like a ball on a roulette wheel, and with no equilibrium the system explodes. In either of these cases, analysis is not possible, but at least a description of the nature of the system is.

(Kremer et al. 2006) points out that DSGE modelling is not immune from the Lucas critique, which essentially states that policy experiments are valid only when micro-founded behaviour is unaffected by a policy regime change. Economic behaviour can only be guaranteed to be independent for policy changes which are totally unexpected and unprecedented, and such changes have zero probability of happening. A similar problem is that the linearised parts of DSGE models also abstract from risk and uncertainty, despite the fact that impulse response analyses are based on totally unpredictable shocks.

Nevertheless, despite these issues, across all areas of economics, the DSGE model has become a workhorse of analysis. To describe optimal monetary policy, Chapter 4 follows recent literature assuming that the central bank is able to fully pre-commit to a policy plan that all the agents in the economy understand, and that they believe is credible. Whilst the real world may be different, it is still useful to examine this benchmark, especially since the language of central banks often suggests that at least some degree of credible commitment is indeed feasible.

1.4 Summary of Contributions and Suggested Extensions

Chapter 2 makes a contribution to the empirical literature that examines monetary transmission from UK policy and wholesale money market rates, and retail mortgage rates. It documents both long-term and short-term relationships, as well as the specific changes that have taken place since the onset of the financial crisis in August 2007. Consistent with previous studies, it shows that the retail banking system in the UK is subject to imprefect competition, but also that there are clear differences between different products. Specifically, the dynamics of discounted variable rates are consistent with perfect compeition, as are, to a lesser extent, fixed rates, whereas standard variable rates are not. Interpretting this feature, it may be that factors such as oligopolistic collusion, sunk and menu costs, and endogenous default risk may be less important than the exploitation of consumer inertia that has been suggested in the literature. This analysis could be extended to include a richer econometric framework, for example allowing the error correction term to vary smoothly over time, although at the loss of inferential power. The same analysis could also be applied to other countries, to see how similar transmission channels differ.

The analysis of Chapter 3 contributes to the understanding of what types of macroeconomic environment are more likely to encourage the general prevalence of variable rate lending compared to fixed. The trade-offs between macroeconomic parameters and mortgage risk are of theoretical interest in their own right. They show in particular, that economies more conducive to variable rate mortgages are those with relatively: volatile, persistent, and low inflation; low and stable real interest rates; high real income growth; and low correlation between inflation and real interest rate shocks. An index of variable rate prevalence across Europe is constructed, and consistent with the theoretical risk simulations, it is actually positively and negatively correlated with inflation and interest rate volatility respectively. A natural way to investigate these issues further would be to adopt the same methodology as (Campbell & Cocco 2003), with a life-cycle model with stochastic shocks on a representative household solved by backwards induction on a discrete grid. By repeating simulations of this kind, conclusions could be made on the types of environment that should favour fixed versus variable mortgages. This extension is left to future work.

In the final chapter, the qualitative monetary policy differences between economies with predominantly variable, and those with predominately fixed rate mortgage contracts is demonstrated. A contribution is made by constructing a model that combines features from two others ((Graham & Wright 2007) and (Monacelli 2007)), and then showing that the extent of fixed versus variable rate lending may be important for welfare. In particular, the financial sector, by adopting fixed rate mortgages, can play a stabilising role in the economy. In the future, the parameters of this model can be adjusted to analyse this issue across countries, and to specifically determine how the proportion of fixed rate contracts that optimises social welfare varies.

Chapter 2

Monetary Transmission to UK Retail Rates

It is widely acknowledged (see for example (Goodhart 2008) and (Buiter 2007)) that 9th August 2007 was the beginning of the global financial crisis. In the UK interbank money market spreads jumped (see figure 2.0.1), and although Northern Rock did not approach the Bank of England as lender of last resort until September, its sustainability was already in serious doubt. As time passed and more events unfolded, many previously stable interest rate relationships were affected in the UK, and it became almost impossible for net borrowers to raise funds on the wholesale money markets

Against this background, this paper analyses how these events have altered the transmission from UK policy and a range of wholesale money rates to retail mortgage rates. Although many previous studies have examined the channel from central bank policy to consumer mortgage payment flows, and hence to the real economy, none have yet documented the post-August 2007 effect (henceforth referred to as the crisis period).

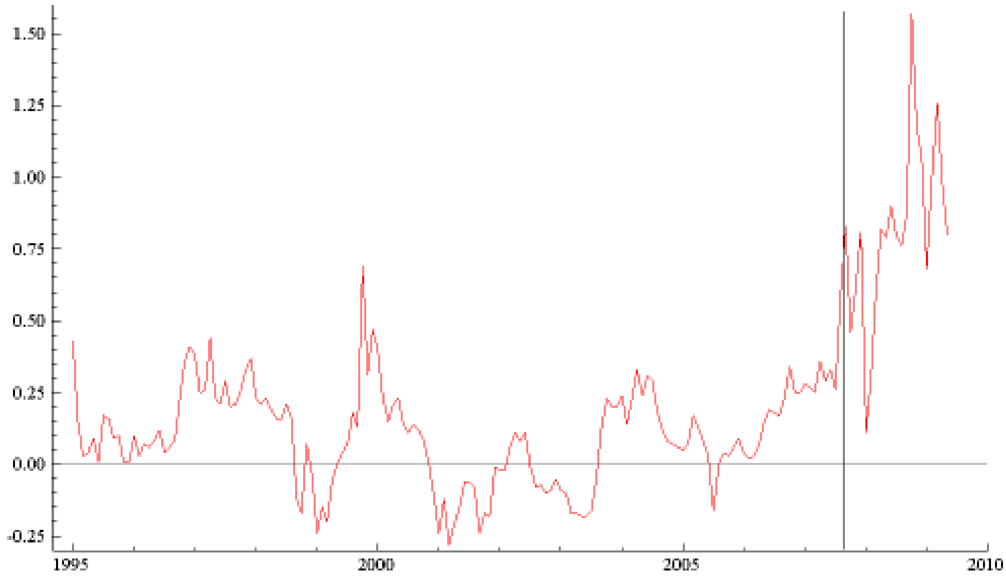


Figure 2.0.1: Spread of LIBOR (3 month) over Policy Rate. Line indicates August 2007.

I contribute to the literature by applying an econometric strategy whose theory is established in (Phillips & Loretan 1991), and which is applied in (Liu et al. 2008). Whereas (Liu et al. 2008) looks at how relationships have changed since the introduction of inflation targeting in New Zealand, their paper provides a strong rationale for using this method (henceforth referred to as the P-L method) in a similar setting. Specifically, we have strong *a priori* evidence for three features that I maintain as working assumptions: Firstly, although imperfections in the banking sector may cause sluggish adjustment to wholesale changes, future expectations of wholesale rates are also likely to be important determinants of retail rates (see appendix A.6 outlining a conceptual model proposed by (Mizen, Hoffman & Street 2004)). If so then estimates of long-run relationships are biased using the standard Engle-Granger procedure, an issue that is resolved by the P-L method. Secondly, interdependence in general equilibrium implies that retail bank rate changes should eventually affect the real economy, and hence feed back to policy and money market rates. This reverse feedback however is likely to be neither relatively strong, nor relatively fast, and so a natural approach to estimating short-run relationships in the literature has been to assume a partial equilibrium with a single-equation error correction model (SEECM). The assumptions behind this approach are also covered by (Liu et al. 2008).

Many studies have already looked at how retail bank rates react to monetary policy in the UK. Early work in this area focused on the question of whether the central bank was able to alter the cost of retail credit in the long-run. (Dale & Haldane 1998) found that changes in bank rates actually overshot changes in the policy rate, whilst (Mariscal & Howells 2002) used an error correction model to examine the impact of monetary policy on *relative* borrowing and lending rates. On finding solid long-run relationships between policy and individual rates, but not on differentials, they argued that monetary policy was therefore unlikely to be effective in altering economic activity through this channel.

The most prolific contributor to this literature is Shelagh Heffernan, with a series of papers which take as their starting point the observation that retail banking is subject to imperfections which delay and obscure transmission. In theory, profit maximising financial intermediaries working under perfect competition with complete information and no adjustment costs should fully pass changes on, symmetrically and instantaneously. Instead there may be oligopolistic collusion between banks; dynamic exploitation of consumer inertia; sunk and menu costs; and the possibility of endogenous default (where an increase in mortgage payments may lead to an increase in the probability of default, and hence alter revenue as a function of rates). Any one of these factors alone could lead to sluggish and asymmetric adjustment.

In (Heffernan 1997) an error correction approach is applied to monthly data from 1986-93 on generic deposit and loan rates, showing that there are considerable differences in rate-setting behaviour between banks, which exhibit many features of imperfect competition. Similar studies, with similar methodologies and findings are later applied in different contexts: (Heffernan 2006) analyses small business loans, and (Fuertes & Heffernan 2006) disaggregates both between and within banks. The possibility of a *non*-linear re-adjustment process is allowed for in (Fuertes, Heffernan, Kalotychou & Row 2006) and (Fuertes, Heffernan, Kalotychou & Row 2008). These studies consider the increasingly popular relaxation of the classic, constant and linear error correction model, into richer specifications that allow the error correction term to vary over time. A comprehensive background to these approaches is provided by (Tong 1993) and (Franses & Van Dijk 2000). For threshold autoregressive models (TAR), once a chosen variable crosses a threshold, the error correction mechanism may switch. When the error-correction term itself is the threshold variable, they are known as self-exciting threshold autoregressive models, or SETARs (for an evaluation of their performance see (Clements & Smith 1999)). The assumption of a discrete switch can also be relaxed so that it may smoothly move across a threshold in smooth transition autoregressive models, STARs (for an application see (Liu 2001)).

A very large sample (662 different rate histories between 1993-2004) is analysed in (Fuertes & Heffernan 2009) in a linear framework, again finding that there are considerable differences in pass-through and re-adjustment speed between banks. They also link this heterogeneity to other bank-specific variables that capture performance, and find that limited branch networks and managerial accountability to shareholders have a positive effect on adjustment speed.

In contrast to all previous work, I focus on the impact of the post-crisis disruption on economy-wide links between policy and money market rates, and retail mortgage rates, which are themselves important determinants of household income flows and generally acknowledged to provide an important monetary policy channel (see (Britton

& Whitley 1997)). I analyse 17 pair-wise relationships between money market and retail mortgage rates of the same maturity using aggregate monthly data from the Bank of England database for the period from January 1995 to May 2009. In the long-run I find evidence of a contrast between the discounted mortgage rates that banks use to initially attract customers, and standard variable rates, with pass-through complete for the former but not for the latter. For fixed rate mortgages, pass-through is generally complete. Since the crisis, for eight of the estimated relationships I find strong evidence in the long-run of both a significant jump in equilibrium spreads, and a fall in pass-through, whilst in the short-run there is a considerable weakening of the process that re-adjusts retail rates back towards their equilibrium with the money market. Although I do not find strong statistical evidence for an asymmetric re-adjustment process before August 2007, retail mortgage rates generally take considerably longer to move back towards their equilibrium with wholesale rates during times when they are relatively expensive. These results add to previous studies by showing the extent to which the UK retail banking sector is imperfectly competitive at the aggregate level, and also suggest that discounted rates are used as a highly competitive loss-leader product. The following section describes the methodology, followed by analytical results in section 3, and section 4 concludes.

2.1 Methodology

I closely follow the procedures of (Liu et al. 2008), which draw on theoretical results derived by (Phillips & Loretan 1991). Since interest rates are generally not found to behave significantly different from unit root processes, the P-L method starts with a long-run equation, as used in the familiar Engle-Granger procedure. Crucially however, the possibility of a more general process is allowed for with the following triangular system of long-run equations:

$$r_t = \alpha_0 + \alpha_1 i_t + \varepsilon_{1t} \tag{2.1.1}$$

$$i_t = i_{t-1} + \varepsilon_{2t}, \tag{2.1.2}$$

where α_0 is the long-run mark-up of retail over wholesale rates and α_1 captures the proportion of (unexpected) changes in wholesale rates that eventually 'slip' or pass-through to retail rates. Pass-through is said to be complete when α_1 is not significantly different from 1, whilst $\alpha_1 < 1$ suggests an imperfectly competitive financial sector. If ε_{1t} is stationary, this implies cointegrated interest rates, so that direct OLS estimates of the parameters are super-consistent, although standard inference has to be adjusted for the non-standard parameter distributions. An often overlooked problem though, is that *a priori* it is reasonably likely that ε_{1t} and ε_{2t} are correlated, because retail bank profitability may depend on anticipating future expected changes in wholesale rates, especially if there are costs associated with adjusting rates. Appendix A.6 provides a conceptual model of forward-looking rate-setting which would cause such a correlation.

The P-L method is hence a happy medium between an unrestricted VAR-type model whose coefficients are difficult to interpret, and the Engle-Granger estimates which are super-consistent, but likely to suffer from small sample bias and non-standard asymptotic parameter distributions. More specifically, the P-L method adds lagged error correction terms to counteract autocorrelation of ε_{1t}, and present, past and future first differences of i_t, to remove the effect of correlation between ε_{1t} and ε_{2t}. At the same time, past policy surprises, and future policy settings are accounted for, and the framework provides a simple way of testing for structural breaks. The long run (P-L) parameters with their associated shifts at the beginning of the crisis period, are derived by non-linear least squares estimation of the following equation:

$$r_t = \alpha_0 + \alpha_1 i_t + \alpha_2 D + \alpha_3 D i_t + \sum_{k=1}^{K} \theta_k \varepsilon_{t-k} + \sum_{i=-l}^{L} \mu_i \Delta i_{t-i} + v_t, \qquad (2.1.3)$$

where D is a dummy variable equal to zero before the crisis period, and one thereafter; and ε_t is the disequilibrium of the long-run relationship calculated from $r_t - \alpha_0 - \alpha_1 i_t - \alpha_2 D - \alpha_3 D i_t$. The parameters α_2 and a_3 capture any long-run mark-up and pass-through shifts that occurred at the start of the crisis period. After establishing the long-run relationships, we can then focus on the short-run dynamics that govern the way rates re-equilibrate. I estimate the SEECM with retail rates as the dependent, endogenous variable:

$$\Delta r_t = \beta_0 \Delta i_t + \beta_{00} D \Delta i_t + \delta \widehat{\varepsilon}_{t-1} + \sum_{i=1}^{x} \beta_i \Delta i_{t-i} + \sum_{i=1}^{y} \gamma_i \Delta r_{t-i} + v_t, \qquad (2.1.4)$$

where β_0 and $\beta_0 + \beta_{00}$ capture how much of a change in wholesale rates instantaneously passes through to retail rates before and after the crisis respectively. The error correction parameter δ indicates the strength of the re-adjustment mechanism, whilst the lagged differences of retail and wholesale rates capture all other short-run dynamics.

After establishing how parameters shifted after the crisis, I then follow (Liu et al. 2008) again by searching for asymmetry of the re-adjustment mechanism. This is achieved by running a similar short-run equation, but with a dummy variable λ, equal to 1 when mortgage rates are above their equilibrium level, that is when the lagged disequilibrium term is positive, and zero when they are below equilibrium:

$$\Delta r_t = \beta_0 \Delta i_t + \beta_{00} \Delta i_t + \delta_1 \lambda \widehat{\varepsilon}_{t-1} + \delta_2 (1-\lambda) \widehat{\varepsilon}_{t-1} + \sum_{i=1}^{x} \beta_i \Delta i_{t-i} + \sum_{i=1}^{y} \gamma_i \Delta r_{t-i} + v_t. \qquad (2.1.5)$$

I am hence able to investigate whether or not the mechanism that moves retail mortgage rates towards their equilibrium with wholesale rates on impact, is stronger or weaker, depending on whether retail rates have (temporarily) moved above or below their equilibrium position relative to wholesale rates.

The linear error correction process provides a simple, parsimonious way of documenting the effects of the relatively small sample of observations from the post-crisis period. However, no allowance is made for the possibility of varying parameters such as the error correction terms over time. Plots of the disequilibrium terms, together with a reflection on the series of events during the financial crisis suggest there may have been increases in volatility, and perhaps a weakening of the process that re-adjusts rates towards their (new) equilibrium. In all cases the post-crisis period was associated only with *positive* disequilibria. A non-linear error correction process, driven by variables that reflect the new mortgage market risk premia of the period (perhaps related to unemployment and house price forecasts) may more closely describe the true underlying data-generating process. At the same time, with so many new events, and not a very long run of observations, I did not consider it worth applying any of the growing number of non-linear specifications in the literature [1]. Instead, the asymmetric analysis is restricted to the more reliable and consistent pre-crisis period. Evidence for asymmetry can be revealed by testing the null hypothesis that $\delta_2 = \delta_3$ with a standard parameter restriction Wald test. Additionally, given potentially low power for this test, a comparison of the short-run adjustment processes is made by comparing what (Hendry 1995) refers to as the Mean Adjustment Lag (MAL) of the two different (above and below equilibrium) regimes. It works by treating the estimated relationship as if it were deterministic. Once stripped down into levels, the MAL is a weighted average of all the lags of the underlying processes, in this case capturing the average length of time it takes retail rates to respond to movements in

[1] The bilinear model proposed by (Peel & Davidson 1998) may be appropriate, given that during the crisis it is likely that risk-premia would not be constant. For applications of non-linear models see (Fuertes et al. 2006) and (Fuertes et al. 2008).

wholesale rates. Appendix A.5 provides a full description of how Hendry's formula is applied in this study.

Although many previous studies outlined in the introduction have searched for and found asymmetry in this context, none have done so at an aggregate level for the UK, nor specifically looked at how the setting of Discounted Variable Rate (DVR) differs from Standard Variable Rate Mortgages (SVR). I follow previous studies which interpret relatively long re-adjustment times for above-equilibrium regimes as indicative of imperfectly competitive bank behaviour. To the extent that the financial sector as a whole is slower to shift rates when they are uncompetitively priced at high levels, this is consistent with the collusive oligopoly and exploitation of consumer inertia arguments proposed in the literature. I am able to more carefully scrutinise this latter claim in particular, since after the two-year period during which DVRs are discounted (and re-set each period), they automatically revert back to SVRs. Evidence outlined in (Callaghan, Fribbance & Higginson 2007) and elsewhere suggests that many borrowers do face significant problems obtaining the time, information, and psychological energy required for continuously searching and re-mortgaging towards the most competitive financial products. Given also that complex penalties, risks, and both financial and subliminal loyalties may be built in to contracts, DVRs could plausibly be used to entice borrowers, in much the same way that many businesses use 'loss-leaders' to physically draw customers into their selling-environment. I therefore interpret more uncompetitive behaviour in the setting of SVRs compared to DVRs as supportive evidence for these features, whilst documenting their extent during the pre-crisis period from January 1995 to August 2007.

2.2 Data and Analysis

I use monthly data provided on the Bank of England website for 18 different interest rate series containing 173 observations from January 1995 to May 2009, plus one series of corporate bond rates from Thomson Datastream. All series are complete except for the DVRs, which only became available from April 1998 (and which therefore contain 134 observations). The whole series covers a timespan of more than 14 years, including more than 3 years of the post-crisis period. Table 2.2.1 provides a summary of the variables.

Table 2.2.1: Summary of Data

Rate	Description	Code
Short Rates		
DVR	Discounted Variable Mortgage Rate	IUMBV48
SVR	Standard Variable Mortgage Rate	IUMTLMV
P1	Official Policy Rate	IUMBEDR
G1	Government Bond Rate	1
LIBOR	Interbank Rate (3 month)	IUMAMIJ
B1	Interbank Rate (2 month)	1
Long Rates		
F2	Fixed Mortgage Rate (2 year)	IUMBV34
F3	Fixed Mortgage Rate (3 year)	IUMBV37
F5	Fixed Mortgage Rate (5 year)	IUMBV42
F10	Fixed Mortgage Rate (10 year)	IUMBV45
G2	Government Bond Rate (2 year)	1
G3	Government Bond Rate (3 year)	1
G5	Government Bond Rate (5 year)	1
G10	Government Bond Rate (10 year)	1
B2	Interbank Rate (2 year)	1
B3	Interbank Rate (3 year)	1
B5	Interbank Rate (5 year)	1
B10	Interbank Rate (10 year)	1
C10	Corporate Bond Rate (10-15 year)	2

1. Extracted from the Bank of England commercial bank and government liability yield curves provided on the financial market data section of their w ebsite.
2. 'UK Corporate Bond Yield' taken from Thomson Datastream based on the Sterling Aggregate Index compiled by Lehman Brothers

Since DVRs and SVRs are re-priced by banks each month, the analysis compares them with short wholesale rates, including the average official policy rate, and short gilt and interbank yields. For fixed mortgage rates, I compare their behaviour to gilt and commercial interbank lending rates of the same maturity. Whilst the structure of such yields are not directly comparable to a mortgage that is nominally fixed for a period, before reverting to a standard variable rate, these analyses offer a guide to the cost of funding such mortgages from wholesale money markets. It is postulated here that by capturing elements of the term structure contained over the same time horizon, any differences in the dynamic behaviour of retail rates are more likely to reflect imperfections in the financial sector. To the extent that the appropriate expectations of inflation and risk premia over the same time horizon are captured and thus conditioned, this analysis sheds light on financial sector frictions, rather than term structure features.

2.3 Equilibrium Relationships

I follow (Liu et al. 2008) in drawing on the theoretical results of (Phillips & Loretan 1991) to derive the long-run equilibria between wholesale money market and retail mortgage rates. First of all, I check the two assumptions that have usually been found in the literature, that the underlying interest rates behave in a non-stationary (unit-root) way, and that they cointegrate in the long-run. Appendix A.1 shows time-series plots of all 17 pairs of interest rates, with their associated spreads. In all cases there is clearly a very strong degree of co-movement, and a maintained economic assumption of this paper is that there is a firm long-run relationship between retail mortgage rates and the associated money market rates that financial institutions borrow and lend at. Even when there are imperfections in the banking sector, it is hard to imagine rates wandering away for too long, and competitive forces not eventually intervening to restore some kind of equilibrium. The results and a summary of the

formal stationarity and cointegration tests are presented in appendices B and C. Without emphatic evidence that the interest rates cannot be treated as non-stationary, and that their relationships cannot be treated as cointegrated, I again follow (Liu et al. 2008) by proceeding with the P-L method. It can be argued that the imposition of discete values for orders of integration may be unrealistic anyway, in which case there are obviously problems with these tests. (Beechey, Hjalmarsson &
"Osterholm 2009) for example develops procedures for testing interest rates when they are *near* integrated (so the cointegrating vector is not restricted to $[1, -1]$). (Fuertes & Heffernan 2009) also discusses the issue of comovement and cointegration, which we do not explore further here.

(Phillips & Loretan 1991) showed how asymptotically median unbiased and efficient estimators could be derived with a range of equivalent methods, namely full systems maximum likelihood; fully modified OLS; systems estimation in the frequency domain; single-equation band spectral estimation; and finally non-linear ECM estimation. Their study systematically analysed the latter method, and stressed the importance of including *leads* of first differences of the regressors in single-equation ECMs (SEECMs). Without such augmentation, the efficiency, unbiasedness, and retention of standard properties for valid inference fail. At the same time they also showed that, given non-stationarity of the underlying (cointegrated) processes, lagged first differences of the variables (in our case lags of Δr_t and Δi_t) did not adequately remove the effects of the past history of errors on the long-run relationship (in our case ε_{1t}). Whilst OLS on linear SEECMs with differenced lags of both variables would be computationally simpler, a non-linear least squares (NLS) procedure is necessary. To estimate the long-run equations (3), (4), and (5) presented in section 2, I also follow their prescription of NLS estimation applied to the non-linear-in-parameters equations that include lagged disequilibrium terms.

Given the structure and assumptions of (Phillips & Loretan 1991), the P-L estimator has asymptotic equivalence with the more popular (full systems maximum likelihood) Johansen procedure, but with the added benefit that we can more easily interpret its natural endogenous-exogenous structure. The next practical problem we face is that of choosing the number of lags and leads to include in the equations presented in section 2. Although other work at the time (see (Saikkonen 1991) and (Stock & Watson 1993)) developed the asymptotic theory of regressions with leads and lags, only more recently have practical procedures been devised for their selection. In the context of lag selection for unit-root testing, (Ng & Perron 1995) explores how to trade-off between parsimonious models with size distortions and over-parameterised models which suffer from low power. They showed that general-to-specific procedures with sequential t- and F-tests outperformed the minimisation of information criteria. For cointegrated models with *leads* however, (Kejriwal & Perron 2008) have extended the analysis of (Saikkonen 1991) and found the opposite: Akaike and Bayesian information criteria (AIC and BIC) can be validly used for the leads and lags regression (which they refer to as 'dynamic OLS') under slightly weaker assumptions; and simulations find smaller mean squared errors and better coverage rates for confidence intervals. (Choi & Kurozumi 2008) also includes analysis of the less well known Cp criterion which minimises the expected squared sum of forecast errors (see (Mallows 2000)). Their simulations show that a simplified approximation of this criterion is the best at reducing bias, whilst BIC is the most successful at reducing mean squared errors. In this study I use the Cp criterion, but also undertake a sensitivity analysis using AIC and BIC (for a description of how these criteria are applied see Appendix A.4.1). The following two sections summarise the estimation results for the 17 pair-wise relationships analysed between variable rate mortgages and short money market yields, and between fixed rate mortgages and money market yields fixed for the same maturity.

2.3.1 Variable Mortgage Rates and Short Yields

Table 2.3.1 presents the estimated long-run relationships between SVRs and wholesale rates. The slope coefficients under the second column show that SVRs are most strongly linked to gilt yields, with 87% of adjustments passing through to SVRs. For interbank rates there is 80% pass-through, whilst only 77% of changes in the policy rate are fully reflected by SVRs in the long-run. Generally speaking, we can say that the way banks set SVRs is not very closely tied to conditions in wholesale markets, with column 7 showing that there is strong evidence in the data to reject the null of complete pass-through.

This contrasts strongly with the long-run relationships for DVRs observed since April 1998, presented in Table 2.3.2. Between 93% and 103% of adjustments to wholesale rates are eventually fully reflected in DVRs, and there is no strong evidence to reject the hypothesis that in the long-run, there is a one-to-one link from policy, gilt, and interbank yields through to DVRs. Compared to SVRs this suggests that banks set DVRs in a more efficient and competitive way.

Another contrasting feature between Tables 2.3.1 and 2.3.2 is the estimated level of equilibrium spreads. Whilst the equilibrium wholesale mark-up for SVRs is around 2.5 percentage points (and 285bps above policy rate), the cost of DVRs is insignificantly different from any of the interbank, government and policy rates.

Moving to the post-crisis period, column 4 shows that the pass-through from policy to SVRs fell significantly (by 6 percentage points down to 71%). In the long-run, the evidence for a long-term break in the link between wholesale and mortgage rates was strongest for DVRs and LIBOR. Pass-through fell by 10 percentage points, and the null of complete pass-through was rejected post-crisis, but not pre-crisis.

There were also significant shifts in the long-run mark-up from policy to SVRs (up 380bps), and even larger jumps in the spreads from gilt and interbank rates to DVRs (up 550 and 570bps). To the extent that the DVR mortgage market is more competitive, this latter result could be interpreted as reflecting increases in mortgage market risk premia caused by mortgage-specific factors such as worsening unemployment and house-price forecasts.

Table 2.3.1: Long Run Pass Through to Standard Variable Mortgage Rates

Short Rate	Pre-Crisis[1]		Post-Crisis Effect		R^2	DW	Ho: Complete Pass Through[2]	
	Constant	Slope	Constant	Slope			Pre-Crisis	Post-Crisis
Policy	2.85^{c}	0.77^{c}	$+0.38^{c}$	-0.06^{c}	1.00	2.02	0.000	0.000
	(8.3)	(12.1)	(5.3)	(-4.8)				
T-Bill	2.42^{c}	0.87^{c}	+0.18	-0.02	0.99	1.96	0.007	0.002
(3-month)	(9.5)	(17.8)	(p=0.105)	(p=0.357)				
LIBOR	2.61^{c}	0.80^{c}	-0.25	+0.03	0.99	1.97	0.000	0.005
(3-month)	(8.8)	(14.8)	(p=0.219)	(p=0.339)				
Interbank	2.55^{c}	0.80^{c}	-0.17	+0.02	0.99	2.00	0.000	0.002
(2-month)	(8.7)	(15.2)	(p=0.245)	(p=0.338)				

1. Pre-crisis includes monthly observations from Jan 1995 to and including July 2007, post-crisis from August 2007 to May 2009.
2. P-values for Chi-Squared Test, Slope Coefficient restricted =1 (small value indicates evidence for incomplete pass through)
Significance levels of 5%, 1%, and 0.1% are indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

Table 2.3.2: Long Run Pass Through to Discounted Variable Mortgage Rates

Short Rate	Pre-Crisis[1]		Post-Crisis Effect		R^2	DW	Ho: Complete Pass Through[2]	
	Constant	Slope	Constant	Slope			Pre-Crisis	Post-Crisis
Policy	0.41	0.93^{c}	$+0.60^{a}$	-0.09	0.97	2.02	0.478	0.072
	(p=0.399)	(9.5)	(2.4)	(p=0.057)				
T-Bill	0.04	1.03^{c}	$+0.55^{b}$	-0.08^{a}	0.99	2.05	0.724	0.478
(3-month)	(p=0.918)	(12.7)	(3.07)	(-2.4)				
LIBOR	0.31	0.93^{c}	$+0.57^{b}$	-0.10^{b}	0.99	2.04	0.174	0.001
(3-month)	(p=0.206)	(19.3)	(3.0)	-2.9				
Interbank	0.15	0.96^{c}	$+0.35^{a}$	-0.06	0.99	2.06	0.540	0.117
(2-month)	(p=0.653)	(14.6)	(2.1)	(p=0.071)				

1. Pre-crisis includes monthly observations from Jan 1995 to and including July 2007, post-crisis from August 2007 to May 2009.
2. P-values for Chi-Squared Test, Slope Coefficient restricted =1 (small value indicates evidence for incomplete pass through)
Significance levels of 5%, 1%, and 0.1% are indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

2.3.2 Fixed Mortgage Rates and Long Yields

The long-run links between fixed mortgage rates, and gilt, interbank, and corporate bond yields of the same maturity are shown in Tables 2.3.3 and 2.3.4. In all cases, the links are very tight, with pass-through ranging from 95% to 105% (except from corporate bonds with 92% pass-through), and no rejections of the complete pass-through hypothesis other than from 3-year gilts to 3-year fixed rates. Another characteristic that indicates a competitive fixed rate mortgage market is the fairly tight long-run spreads, which for the interbank rates are insignificantly different from zero in all cases except for at 3-year maturities. Equilibrium spreads over gilts range from 0.61 percentage points for 2-year rates, up to 1 percentage point for 3-year maturities, in all cases significantly cheaper than the SVR spreads. At this point it is worth pointing out that, just as DVRs revert back to SVRs after the pre-agreed 2-year period, so do fixed rates revert back to SVRs at maturity. The fixed rate mortgage market appears to be more competitive than SVRs, and again this is consistent with the idea that banks use fixed rates (though to a lesser extent than DVRs) as loss-leader products to entice borrowers into deals which later become less competitive.

After the crisis, the fall in pass-through and rise in equilibrium spread is even more clear-cut: For all but the 10-year fixed rates link with 10-year gilts, there was both a significant reduction in pass-through (from 6 to 14 percentage points), and increase in equilibrium spreads (from 44 to 80 percentage points). Another feature of the crisis period is that, for 3-year interbank rates, the hypothesis of complete pass-through to fixed rates is rejected (but not before the crisis).

Table 2.3.3: Long Run Pass Through from Government Bond to Fixed Mortgage Rates

Maturity	Pre-Crisis[1]		Post-Crisis Effect		R^2	DW	Ho: Complete Pass Through[2]	
	Constant	Slope	Constant	Slope			Pre-Crisis	Whole-Sample
Two Year	0.61^{a}	0.98^{c}	$+0.60^{c}$	-0.08^{c}	0.98	2.00	0.674	0.034
	(2.4)	(20.8)	(4.1)	(-2.7)				
Three Year	0.91^{c}	0.97^{c}	$+0.56^{c}$	-0.07	0.98	2.01	0.352	0.023
	(4.7)	(27.0)	(3.3)	(p=0.056)				
Five Year	0.83^{b}	1.00^{c}	+0.36	-0.05	0.99	1.97	0.986	0.362
	(2.8)	(18.5)	(0.064)	(p=0.192)				
Ten Year	1.20^{b}	0.98^{c}	+0.39	-0.08	0.99	2.03	0.849	0.323
	(2.7)	(12.4)	(p=0.178)	(p=0.231)				

1. Pre-crisis includes monthly observations from Jan 1995 to and including July 2007, post-crisis from August 2007 to May 2009.
2. P-values for Chi-Squared Test, Slope Coefficient restricted =1 (small value indicates evidence for incomplete pass through)

Significance levels of 5%, 1%, and 0.1% are indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

Table 2.3.4: Long Run Pass Through from Interbank Loan to Fixed Mortgage Rates

Maturity	Pre-Crisis[1]		Post-Crisis Effect		R^2	DW	Ho: Complete Pass Through[2]	
	Constant	Slope	Constant	Slope			Pre-Crisis	Post-Crisis
Two Year	0.35	0.96^{c}	$+0.44^{b}$	-0.06^{a}	0.98	1.99	0.518	0.052
	(p=0.272)	(17.6.)	(4.7)	(-3.6)				
Three Year	0.60^{a}	0.95^{c}	$+0.48^{b}$	-0.07^{a}	0.99	2.00	0.291	0.007
	(2.3)	(21.2)	(3.0)	(-2.5)				
Five Year	0.18	1.04^{c}	$+0.64^{c}$	-0.10^{c}	0.99	1.97	0.333	0.121
	(p=0.414)	(27.2)	(4.0)	(-3.4)				
Ten Year	0.55	1.02^{c}	$+0.70^{a}$	-0.13	0.99	1.91	0.751	0.165
	(p=0.078)	(19.1)	(2.0)	(p=0.066)				
Ten Year	0.58	0.92^{c}	$+0.65^{a}$	-0.11^{b}	0.99	2.02	0.587	0.158
(Corporate)	(p=0.51)	(6.7)	(2.4)	(-2.8)				

1. Pre-crisis includes monthly observations from Jan 1995 to and including July 2007, post-crisis from August 2007 to May 2009.
2. P-values for Chi-Squared Test, Slope Coefficient restricted =1 (small value indicates evidence for incomplete pass through)

Significance levels of 5%, 1%, and 0.1% are indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

2.4 Short Run Adjustment

2.4.1 Estimation

To estimate the short-run equation 2.1.4, the number of differenced lags of the exogenous (x) and endogenous (y) variable were selected with Bayesian information criteria

(BIC). Using a maximum lag-length of 4, I optimised BIC across all possible combinations of x and y, to derive the chosen $SR(x, y)$ models whose results are presented in Tables 2.4.1-2.4.4. The first and fourth columns of these Tables give the values of β_0 and β_{00} from equation 2.1.4, the proportion of a change in wholesale rates that instantaneously slips through to retail rates before the crisis, and the change in this parameter after the crisis. The re-adjustment coefficient δ is shown in columns 2 and 5 (before the crisis, and the change after the crisis respectively), and indicates the strength of the re-adjustment process towards equilibrium (the proportion of a disequilibrium that is immediately 'corrected'). We would normally expect this parameter to be negative, given that there is a long-term equilibrium, since positive values imply an error-exploding, rather than self-correcting process. Finally, columns 3 and 6 show the results of calculating the Mean Adjustment Lag (MAL), before and after the crisis (see Appendix A.5). The MAL is an average measure of how long it takes for a disequilibrium between wholesale and retail rates to disappear. Since there are both error-correction terms and differenced lags in the final models, this calculation is important in clarifying the general speed of the short-term adjustment of retail rates following changes in wholesale rates.

2.4.2 Variable Mortgage Rates and Short Yields

Table 2.4.1 shows the estimated short-run relationships for SVRs and DVRs respectively. The instantaneous re-adjustment in column 2 is considerably stronger in all cases for DVRs compared to SVRs, indicative of more competitive behaviour. Immediate pass-through was not significant for interbank rates, but again the larger values for DVRs with respect to policy and gilt yields suggest a more immediately competitive DVR-setting process than for SVRs. The MALs on the other hand are longer for DVRs in all cases. This suggests that early on, banks are quicker to pass on the rate changes to DVRs, but then it takes longer on average for the whole change to be reflected in DVRs. Columns 4-6 show how short-run adjustment changed after the crisis. In all cases, the change in instantaneous pass-through was insignificant, and all re-adjustment parameters became weaker (less negative). Caution is required in interpreting the MALs, where they are based on a relatively small number of observations (and in some cases were negative with no meaningful interpretation). Generally speaking, MALs are much longer in the post-crisis period, which probably illustrates a looser link between wholesale and retail rates after the crisis.

Table 2.4.1: Short Run Pass Through to Variable Mortgage Rates

Short Rate	Pre-Crisis[1]			Post-Crisis[1]		
	Impact		MAL[4]	Impact		MAL[4]
	Pass Through[2]	Re-Adjustment[3]		Pass Through[2]	Re-Adjustment[3]	
SVR						
Policy	0.18^c	-0.09^c	0.8	-0.08	$+0.15^b$	1.9
	(4.0)	(-3.5)		(p=0.183)	(2.7)	
T-Bill	0.12^b	-0.14^c	3.3	-0.09	$+0.08^c$	4.9
(3-month)	(2.6)	(3.7)		(p=0.221)	(2.0)	
LIBOR	0.08	-0.17^c	2.8	+0.06	$+0.22^a$	-
(3-month)	(p=0.217)	(-4.6)		(p=0.395)	(2.4)	
Interbank	0.01	-0.13^c	4.8	+0.07	+0.03	2.9
(2-month)	(p=0.868)	(-3.5)		(p=0.322)	(p=0.602)	
DVR						
Policy	0.60^c	-0.19^b	3.0	-0.15	$+0.20^b$	-
	(6.7)	(-3.2)		(p=0.146)	(-3.2)	
T-Bill	0.33^c	-0.20^b	3.7	-0.14	$+0.18^a$	22.7
(3-month)	(4.0)	(-3.1)		(p=0.149)	(2.6)	
LIBOR	0.40^c	-0.29^c	4.0	-0.03	$+0.21^c$	7.7
(3-month)	(4.9)	(-5.7)		(p=0.729)	(3.3)	
Interbank	-0.12	-0.27^c	5.3	+0.14	$+0.18^c$	7.9
(2-month)	(p=0.151)	(-7.0)		(p=0.157)	(4.3)	

1. Pre-crisis includes monthly observations, Jan 1995 to and including July 2007, post-crisis from Aug 2007 to May 2009.
2. Impact pass through indicates how much of a change in wholesale rates is immediately passed through to retail rates.
3. Impact Re-Adjustment is the size of the equilibrium correction parameter.
4. Mean Adjustment Lags, indicate how long it takes retail rates to adjust to changes in wholesale rates (see App A.5).
5. Significance levels of 5%, 1%, and 0.1% indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

2.4.3 Fixed Mortgage Rates and Long Yields

The re-adjustment parameters in Table 2.4.2 suggest that the fixed rate mortgage market link to treasury bill, as opposed to interbank lending, is slightly stronger. Re-adjustment parameters are marginally stronger for gilts at all maturities, and MALs shorter at 3, 5, and 10-year maturities. As for short rates, the post-crisis changes cannot be measured with accuracy, however in all cases there was a significant weakening of the re-adjustment process.

Table 2.4.2: Short Run Pass Through to Fixed Mortgage Rates

Maturity	Pre-Crisis[1]			Post-Crisis[1]		
	Impact		MAL[4]	Impact		MAL[4]
	Pass Through[2]	Re-Adjustment[3]		Pass Through[2]	Re-Adjustment[3]	
T-Bill						
Two Year	0.10	-0.24^{c}	4.4	+0.10	0.22^{c}	17.7
	(p=0.051)	(-5.4)		(p=0.315)	(4.6)	
Three Year	0.09	-0.27^{c}	4.0	+0.13	$+0.23^{c}$	10.2
	(p=0.061)	(-5.7)		(p=0.183)	(4.5)	
Five Year	0.15^{c}	-0.17^{c}	4.6	+0.14	$+0.19^{c}$	-
	(3.7)	(-4.7)		(p=0.109)	(4.6)	
Ten Year	0.17^{b}	-0.14^{c}	5.1	-0.03	+0.05	4.6
	(3.0)	(-4.7)		(p=0.819)	(p=0.499)	
Interbank						
Two Year	0.13^{b}	-0.23^{c}	4.3	+0.21	+0.24	-
	(2.7)	(-5.6)		(2.4)	(4.7)	
Three Year	0.12^{b}	-0.22^{c}	4.4	+0.04	$+0.17^{c}$	9.1
	(2.4)	(-4.8)		(p=0.687)	(3.3)	
Five Year	0.18^{c}	-0.17^{c}	4.5	+0.06	$+0.18^{c}$	-
	(4.4)	(-4.9)		(p=0.498)	(4.2)	
Ten Year	0.18^{b}	-0.10^{c}	6.1	+0.07	$+0.12^{b}$	-
	(3.0)	(-3.4)		(p=0.597)	(2.6)	
Ten Year	0.05	-0.10^{c}	11.4	-0.05	$+0.11^{c}$	-
(Corporate)	(p=0.502)	(-4.6)		(p=0.652)	(4.1)	

1. Pre-crisis includes monthly observations, Jan 1995 to and including July 2007, post-crisis from Aug 2007 to May 2009.
2. Impact pass through indicates how much of a change in wholesale rates is immediately passed through to retail rates.
3. Impact Re-Adjustment is the size of the equilibrium correction parameter.
4. Mean Adjustment Lags, indicate how long it takes retail rates to adjust to changes in wholesale rates (see App A.5).
5. Significance levels of 5%, 1%, and 0.1% indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

2.4.4 Asymmetric Adjustment

Now we turn to the question of whether bank rate-setting behaviour varies according to whether retail rates are above or below their long-term equilibrium with wholesale rates. Some of the literature discussed in the introduction links oligopolistic competition between banks, and other inertia to asymmetric adjustment of rates. Such imperfections may mean that banks are relatively slow to re-adjust during 'expensive' (above equilibrium) periods than during the 'cheap' (below equilibrium) regimes. If so, then we interpret evidence for asymmetry as evidence for an imperfectly competitive financial sector. A time series plot of all rates is provided in appendix A.1. In all cases except for the fixed rate relationship with corporate bonds, the post-crisis period almost exclusively associated with *positive* disequilibria. We also know that this was a time when many events were unfolding. As discussed in previous sections, I do not relax equations 2, 4, and 5 to allow for time-varying error-correction (which is likely to have weakened), and volatility (which is likely to have increased). There are many possible candidate, non-linear models that could capture these features, but the limited observations may not warrant such an approach. For the asymmetry analysis in particular however, there is a clear danger that including the post-crisis period may distort the estimation, possibly biasing the above-equilibrium parameters upwards, as well as causing invalid inference. At the cost of less observations, I adopt a conservative approach by excluding the post-crisis sample from this analysis, thus avoiding the danger that we have not appropriately captured the changes after August 2007.

Table 2.4.3: Asymmetric Adjustment of Variable Mortgage Rates

Short Rate	Impact			MAL[4]		Ho: Asymmetry[6]
	Pass Through[2]	Re-Adjustment[3] Above[5]	Re-Adjustment[3] Below[5]	Above[5]	Below[5]	
SVRs						
Policy	0.36^{c}	-0.31^{c}	-0.23^{c}	1.6	-0.5	0.340
	(4.9)	(-5.0)	(-4.3)			
T-Bill	0.11^{a}	0.17^{c}	-0.12^{a}	3.1	1.5	0.171
(3-month)	(2.4)	(-3.5)	(-2.6)			
LIBOR	0.16^{b}	-0.17^{c}	-0.09	2.7	1.4	0.178
(3-month)	(2.7)	(-3.4)	(p=0.064)			
Interbank	0.20	-0.17^{c}	-0.11^{a}	4.2	3.6	0.142
(2-month)	(p=0.728)	(-3.5)	(-2.4)			
DVRs						
Policy	0.49	-0.19^{b}	-0.13	2.8	1.2	0.576
	(5.7)	(-2.6)	(p=0.125)			
T-Bill	0.26^{c}	-0.23^{c}	-0.30^{c}	4.6	3.0	0.319
(3-month)	(3.7)	(-5.4)	(-4.3)			
LIBOR	0.37^{c}	-0.26^{c}	-0.23^{c}	3.7	3.4	0.713
(3-month)	(5.1)	(-4.7)	(-3.5)			
Interbank	-0.08	-0.30^{c}	-0.27^{c}	5.1	4.7	0.604
(2-month)	(p=0.255)	(-8.0)	(-5.1)			

1. Pre-crisis includes monthly observations from Jan 1995 up to and including July 2007.
2. Impact pass through indicates how much of a change in wholesale rates is immediately passed through to retail rates
3. Impact Re-Adjustment is the size of the equilibrium correction parameter
4. Mean Adjustment Lags indicate how long it takes retail rates to adjust to changes in wholesale rates (see App A.5)
5. 'Above' and 'below' indicate regimes when deviation from long-run equilibrium is positive and negative respectively.
6. P-values for Chi-Squared Test on general restriction that re-adjustment parameters for the two regimes are equal (small values indicate evidence of asymmetry).
7. Significance levels of 5%, 1%, and 0.1% indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

Table 2.4.4: Asymmetric Adjustment of Fixed Mortgages

Maturity	Impact			MAL[4]		Ho: Asymmetry[6]
	Pass Through[2]	Re-Adjustment[3] Above[5]	Re-Adjustment[3] Below[5]	Above[5]	Below[5]	
T-bills						
Two Year	0.10^{a}	-0.21^{c}	-0.25^{c}	4.4	2.0	0.616
	(2.0)	(-4.0)	(-3.3)			
Three Year	0.12^{a}	-0.12^{c}	-0.56^{c}	5.4	0.7	0.005
	(2.6)	(-3.5)	(-3.4)			
Five Year	0.14^{c}	-0.16^{c}	-0.21^{b}	4.9	2.3	0.454
	(3.6)	(-3.7)	(-3.2)			
Ten Year	0.15^{b}	-0.16^{c}	-0.19^{c}	5.5	5.9	0.625
	(2.7)	(-5.1)	(-3.3)			
Interbank						
Two Year	0.13^{a}	-0.19^{c}	-0.22^{b}	4.3	2.0	0.705
	(2.6)	(-3.9)	(3.0)			
Three Year	0.11^{a}	-0.22^{c}	-0.20^{b}	4.0	2.3	0.702
	(2.5)	(-4.0)	(-3.0)			
Five Year	0.20^{c}	-0.24^{c}	-2.1^{b}	4.0	2.4	0.538
	(5.1)	(-4.7)	(-3.2)			
Ten Year	0.19^{c}	-0.26^{c}	-0.24^{c}	4.9	3.1	0.817
	(3.3)	(-6.3)	(-4.2)			
Ten Year	0.08	-0.09^{c}	-0.08	8.7	14.6	0.945
(corporate)	(p=0.205)	(-3.9)	(p=0.060)			

1. Pre-crisis includes monthly observations from Jan 1995 up to and including July 2007.
2. Impact pass through indicates how much of a change in wholesale rates is immediately passed through to retail rates
3. Impact Re-Adjustment is the size of the equilibrium correction parameter
4. Mean Adjustment Lags indicate how long it takes retail rates to adjust to changes in wholesale rates (see App A.5)
5. 'Above' and 'below' indicate regimes when deviation from long-run equilibrium is positive and negative respectively.
6. P-values for Chi-Squared Test on general restriction that re-adjustment parameters for the two regimes are equal (small values indicate evidence of asymmetry).
7. Significance levels of 5%, 1%, and 0.1% indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

The p-values given in the last column of Table 2.4.3 suggest that there is slightly stronger statistical evidence of asymmetry (and hence imperfect competition) for SVRs, although this evidence is not significant. The MALs shown in columns 4 and 5 however, suggest that banks take longer to re-adjust rates during above equilibrium regimes. Pass-through from both LIBOR and treasury bill rates to SVRs takes on average, twice as long when SVRs are relatively expensive, compared to when they are relatively cheap, which can be interpreted as evidence of imperfect competition.

A similar story is given for fixed rate mortgages shown in Table 2.4.4, although strong statistical evidence for asymmetry was found for the 3-year rate relationship with gilt yields. It takes nearly 8 times longer for 3-year fixed rates to re-adjust when they are above equilibrium than when they are below.

2.5 Conclusion

In this paper I have investigated the dynamic relationships between a range of wholesale money markets, and comparable mortgage rates of the same maturity, to show how competitive forces differ between different types of mortgage product, and to see how much these relationships have been altered by the post-August 2007 financial market turmoil. I show the extent to which frictions (whether through a lack of competition or otherwise) prevent fully competitive rate-setting for three types of mortgage: discounted variable rate mortgages (DVRs), standard variable rate mortgages (SVRs), and fixed rate mortgages (FRs). Across three types of dynamic analysis (long-run, short-run, and asymmetry) I find evidence that supports the hypothesis that banks may exploit consumer inertia by using DVRs, and to a lesser extent FRs, to entice borrowers in to 'cheap' deals that later may revert back to SVRs.

Firstly, using the P-L method to investigate long-run relationships I find that banks tend to pass rate changes fully on to DVRs, generally to FRs, but not to SVRs.

Secondly, the short-run dynamics show that banks tend not to pass any instantaneous changes onto SVRs, whereas they do for DVRs, and that the readjustment process is stronger in the latter case. On average however, it takes longer for rate changes to pass fully onto DVRs.

Finally, there is no strong statistical evidence that banks alter their rate-setting behaviour depending on whether retail mortgage rates are above or below their equilibrium with wholesale rates. It does however, take considerably longer for banks to re-adjust rates during above equilibrium regimes for SVRs, which again is indicative of imperfect competition in this market.

As for the post-crisis effect, there is strong evidence that both long-run and short-run relationships have been significantly altered. Since the crisis, for eight of the estimated relationships I find strong evidence in the long-run of both a significant jump in equilibrium spreads, and a fall in pass-through, whilst in the short-run there is a considerable weakening of the process that re-adjusts retail rates back towards their equilibrium with the money market.

For future work, given the complex chain of events since August 2007, the use of an *exogenous* time-varying error correction term, that depends on stress in the financial system may be appropriate. Different measures of financial stress could be tested against each other, to find which of them are the most successful at capturing the weakening of the error correction process. The candidates for such a variable need not be restricted to individual series. (Illing & Liu 2006) for example, develop a way of compiling an index in Canada that captures stress in the financial system using a range of financial variables.

Chapter 3

Lending Levels in EU-15 Countries

3.1 Introduction

The predominance of Fixed-Rate (FR) compared to Variable-Rate (VR) mortgage lending differs considerably across Europe. It has been observed that[1] the UK's mortgage system is predominately variable-rate, whilst FR mortgages appear to persist in the Eurozone. Understanding the nature and extent of this diversity, is an important policy issue, because in the event of the UK joining EMU it represents a potential source of asymmetry in the transmission of monetary policy. This chapter presents evidence that, despite obvious differences in institutional histories, differences in the macroeconomic environments of EU-15 countries could be important in explaining this diversity.

Previous research[2] points to an explanation based on non-rational behaviour, focusing on information problems faced by UK mortgage consumers. The simulations presented in this chapter however, show that macroeconomic environments with relatively volatile inflation and relatively stable real interest rates favour the adoption of VR mortgages, and that the history of these variables in the UK is consistent with a more rational explanation.

[1] See, for example (Britton & Whitley 1997)
[2] See (Miles 2004)

The chapter is broken down into three sections. Section 3.2 develops a theory of FR versus VR mortgage choice based on two distinct types of risk. FR contracts expose borrowers and lenders to the risk that unexpected changes in inflation and real interest rates alter the real discounted net present value of the contract, a concept defined in (Campbell & Cocco 2003) as '*wealth*' risk. Although the real discounted net present value of VR contracts is more stable, borrowing constraints may bind more severely when high interest rates coincide with low incomes, forcing immediate painful reductions in consumption. The risk that borrowers experience these liquidity problems is greater under a VR contract, and defined as '*income*' risk.

Having derived equations that allow these risks to be quantified, section 3.3 uses calibrated parameters to generate a baseline simulation measuring the size of wealth and income risk. By adjusting parameters and re-running the simulations, the elasticities of wealth and income risk with respect to macroeconomic parameters is measured, leading to a final statement of what kinds of environment favour VR mortgages.

Section 3.4 presents a summary of how relevant different macroeconomic environments are as an explanatory factor of different levels of FR vs VR mortgage lending across the EU-15. The log of an index of VR lending is regressed onto a range of variables including the means and standard deviations of inflation and real interest rates throughout the 80s and 90s, and proxies for credit market liberalisation (which should reduce income risk), and credit constraints (which should increase income risk). Section 3.5 concludes the findings.

3.2 Measuring Wealth and Income Risk

3.2.1 Wealth Risk

To illustrate the concept, Table 3.2.1 below shows a simple example of wealth risk. The periods shown here are quarterly, where inflation is expected (p^e) to remain stable throughout the year at an annual rate of 2%, and real interest rates (r) are stable at 3%. In reality, inflation turns out to be higher than expected, rising steadily to an annual rate of 8%. If someone took a fixed-rate mortgage out at the beginning of the year, then ignoring tiny second order effects, the competitive nominal rate would be simply 5% per annum, or ignoring compounding, 1.25% per quarter. Imagine, for simplicity that the contract involves borrowing £100,000, which is paid back at the end of the year. This means the nominal cash flow ($nmcf$) evolves as shown in the 5^{th} column. When making a decision at $t = 0$ however, it is not nominal cash flows, but real discounted cash flows that are relevant, so in column 6 there are the real discount factors (rdf) based on the expected course of inflation and real interest rates. Using market rates in this way assumes that the borrower has the same access to capital markets as the lender.

From the borrower's perspective, the real discounted cash flow ($rdmp$) sums to -£100,000, confirming that the fixed-rate is indeed competitive. Moving across to what actually happens, the agreement of a fixed-rate leaves the nominal cash flow unchanged, but the appropriate real discount rates shift downwards. Assessing the borrower's decision at time $t = 0$, they unexpectedly gained almost £3000 of real discounted wealth at the expense of the lender. Similarly, had inflation unexpectedly fallen they would have lost to the benefit of the lender.

Table 3.2.1: Wealth Risk

			Fixed-Rate Contract							
			Expected				Actual			
t	p^e	p	fr	nmcf	rdf	rdmp	fr	nmcf	rdf	rdmp
1	0.50%	0.50%	1.25%	1,250	0.988	1,235	1.25%	1,250	0.988	1,235
2	0.50%	1.00%	1.25%	1,250	0.975	1,219	1.25%	1,250	0.971	1,213
3	0.50%	1.50%	1.25%	1,250	0.963	1,204	1.25%	1,250	0.949	1,187
4	0.50%	2.00%	1.25%	101,250	0.952	96,342	1.25%	101,250	0.924	93,545
					Total:	100,000				97,180
					NPV?	-£100,000				-£97,180

			Variable-Rate Contract							
t	p^e	p	Expected				Actual			
			vr	nmcf	rdf	rdmp	vr	nmcf	rdf	rdmp
1	0.50%	0.50%	1.25%	1,250	0.988	1,235	1.25%	1,250	0.988	1,235
2	0.50%	1.00%	1.25%	1,250	0.975	1,219	1.75%	1,750	0.971	1,699
3	0.50%	1.50%	1.25%	1,250	0.963	1,204	2.25%	2,250	0.949	2,136
4	0.50%	2.00%	1.25%	101,250	0.952	96,342	2.75%	102,750	0.924	94,931
					Total:	100,000				100,000
					NPV?	-£100,000				-£100,000

In the same Table, the example of a VR contract is given below. This shows that as the nominal mortgage rate automatically increases in response to the rising inflation, this compensates evenly for the falling discount factor. This ensures that the net present value of the contract remains at exactly -£100,000 throughout the year, and illustrates the wealth stability of the VR contract.

The same principle applies to real interest rates – if they unexpectedly rise, then again the borrower on the FR gains unexpectedly at the expense of the lender, but with the VR contract wealth is stable.

Under the following assumptions, the variable-rate contract offers pure stability of real discounted wealth (for both the borrower and the lender):

1. There is no one-period uncertainty over inflation and real interest rates

2. Lenders are competitive, and if they require risk premia, and premia for administration/transactions costs, these premia are stable and constant over time

3. The inflation and discount rates faced by the borrower are the same as the lender, which is more likely if they have access to the same financial capital markets. This essentially is equivalent to assuming that borrowers can adjust an asset/debt portfolio of their own, and so do not face any credit constraints.

4. Lenders' expectations are the same as borrowers' expectations, or equivalently, they have the same information set.

3.2.2 Derivation of Wealth Risk Equation

Define r as the real interest rate, π the inflation rate, and for convenience define the variable $x = r + \pi + r\pi$. Subscripts refer to time (where F is the final period of the contract), and superscript e is the expectations operator.

The competitive fixed-rate, ignoring premia is the solution to:

$$\frac{FR_0}{(1+x_1^e)} + \frac{FR_0}{(1+x_1^e)(1+x_2^e)} + ... + \frac{FR_0+1}{(1+x_1^e)(1+x_2^e)...(1+x_F^e)} = 1$$

$$FR_0 = \left[\frac{(1+x_1^e)(1+x_2^e)...(1+x_F^e)-1}{(1+x_2^e)(1+x_3^e)...(1+x_F^e)+...+(1+x_F^e)+1}\right] \tag{3.2.1}$$

The accumulated unexpected gain in real discounted net present value to the borrower, evaluated at time $t = 0$ (where ND is the nominal debt value of the loan) is:

$$\begin{aligned}&\left(\tfrac{FR_0}{(1+x_1^e)} - \tfrac{FR_0}{(1+x_1)}\right)ND + \left(\tfrac{FR_0}{(1+x_1^e)(1+x_2^e)} - \tfrac{FR_0}{(1+x_1)(1+x_2)}\right)ND + ...\\ &+\left(\tfrac{FR_0+1}{(1+x_1^e)(1+x_2^e)...(1+x_F^e)} - \tfrac{FR_0+1}{(1+x_1)(1+x_2)...(1+x_F)}\right)ND\end{aligned}$$

Dividing by ND and rearranging gives:

$$\begin{aligned}&FR_0\left(\begin{array}{c}\frac{1}{(1+x_1^e)} + \frac{1}{(1+x_1^e)(1+x_2^e)} + ... + \frac{1}{(1+x_1^e)(1+x_2^e)...(1+x_F^e)}\\ -\frac{1}{(1+x_1)} - \frac{1}{(1+x_1)(1+x_2)} - ... - \frac{1}{(1+x_1)(1+x_2)...(1+x_F)}\end{array}\right)\\ &+\frac{1}{(1+x_1^e)(1+x_2^e)...(1+x_F^e)} - \frac{1}{(1+x_1)(1+x_2)...(1+x_F)}\end{aligned}$$

By construction, the expected terms multiplied by FR_0 sum to 1, so this leaves:

$$1 - \left[\begin{array}{l} FR_0 \left(\frac{1}{(1+x_1)} + \frac{1}{(1+x_1)(1+x_2)} + ... + \frac{1}{(1+x_1)(1+x_2)...(1+x_F)} \right) \\ + \frac{1}{(1+x_1)(1+x_2)...(1+x_F)} \end{array} \right] \tag{3.2.2}$$

The measure of wealth risk used is the standard deviation of this quantity when the competitive fixed-rate is equal to its mean:

$$\rho_W = SD \left[1 - \left[\begin{array}{l} (\mu + \bar{r}) \left(\frac{1}{(1+x_1)} + \frac{1}{(1+x_1)(1+x_2)} + ... + \frac{1}{(1+x_1)(1+x_2)...(1+x_F)} \right) \\ + \frac{1}{(1+x_1)(1+x_2)...(1+x_F)} \end{array} \right] \right] \tag{3.2.3}$$

3.2.3 Income Risk

To the extent that the previous assumptions are true, the FR contract is riskier than the VR contract. The lender may be unwilling to take this risk on, and so may transfer this cost to the borrower by charging a risk premium. Either way, the VR contract is superior, so why is there a market for FR mortgages? Without exploring issues related to fundamental uncertainty, a plausible explanation is that credit constraints may expose borrowers to the danger that high interest payments coincide with periods of low income, forcing immediate, painful reductions in consumption.

The same example shown in Table 3.2.2 includes the assumption that the borrower begins with a real income of £5000 per quarter. Real discounted labour income is calculated by discounting with each period inflation (p), and multiplying by real labour income growth (l). In this simple example these are equal (6% per annum), so they offset each other, and the stream of real discounted labour income (rdl) is fixed at £5000. A borrower facing credit constraints will experience problems when their real discounted mortgage payments ($rdmp$) are high **as a proportion** of their real discounted labour income (their debt servicing burden, dsb, is high). In the same example, this ratio for the FR contract steadily falls from 25% to 23%, whilst for the VR contract this rises sharply to 51%.

Table 3.2.2: Income Risk

			Fixed-Rate Contract											
			Expected						Actual					
t	r	l	rate	nmcf	rdf	rdmp	rdl	dsb	rate	nmcf	rdf	rdmp	rdl	dsb
1	0.75%	0.75%	1.25%	1,250	0.988	1,235	5,000	25%	1.25%	1,250	0.988	1,235	5,000	25%
2	0.75%	0.75%	1.25%	1,250	0.975	1,219	5,000	24%	1.25%	1,250	0.971	1,213	5,000	24%
3	0.75%	0.75%	1.25%	1,250	0.963	1,204	5,000	24%	1.25%	1,250	0.949	1,187	5,000	24%
4	0.75%	0.75%	1.25%	1,250	0.952	1,189	5,000	24%	1.25%	1,250	0.924	1,155	5,000	23%
			Variable-Rate Contract											
			Expected						Actual					
			rate	nmcf	rdf	rdmp	rdl	dsb	rate	nmcf	rdf	rdmp	rdl	dsb
			1.25%	1,250	0.988	1,235	5,000	25%	1.25%	1,250	0.988	1,235	5,000	25%
			1.25%	1,250	0.975	1,219	5,000	24%	1.75%	1,750	0.971	1,699	5,000	34%
			1.25%	1,250	0.963	1,204	5,000	24%	2.25%	2,250	0.949	2,136	5,000	43%
			1.25%	1,250	0.952	1,189	5,000	24%	2.75%	2,750	0.924	2,541	5,000	51%

Unlike wealth risk, income risk is not symmetrical between borrower and lender, indeed it is unlikely to affect lenders at all, and should only affect borrowers who face liquidity constraints. (Benito & Mumtaz 2006) have estimated that the proportion of UK households facing such constraints is between 20% and 40%. These households display excess sensitivity of consumption in the sense of not smoothing their consumption to the full extent implied by the life-cycle model. They also find that young, unmarried, non-white, degree-educated households without liquid assets, and with negative home equity are more likely to be liquidity constrained. To the extent that these groups are also more likely to be mortgage borrowers, the proportion of borrowers affected by income risk may be higher than this estimate.

3.2.4 Derivation of Income Risk Equation

If l_t is the real labour income growth rate, α is the loan:income ratio, and mr_t the mortgage rate, then for a mortgage contract initiated in period 0, the debt-servicing burden in time t, evaluated at time $t = 0$ is:

$$dsb_t = \frac{\text{present value of real discounted mortgage payment}}{\text{present value of real discounted income}} = \frac{\left(\frac{mr_t}{(1+x_1)(1+x_2)...(1+x_t)} * ND\right)}{\left(\frac{(1+l_1)(1+l_2)...(1+l_t)}{(1+r)_1(1+r)_2...(1+r)_t} * \frac{ND}{\alpha}\right)}$$

For the VR contract, H periods after the contract was agreed this will be:

$$\rho_{IV} = SD\left[\alpha\left(\frac{x_F}{(1+x_1)(1+x_2)...(1+x_H)}\right)\left(\frac{(1+r_1)(1+r_2)...(1+r_H)}{(1+l_1)(1+l_2)...(1+l_H)}\right)\right]$$

And for the FR contract, the risk when the fixed-rate is at its mean is:

$$\rho_{IF} = SD\left[\alpha\left(\frac{(\mu+\bar{r})}{(1+x_1)(1+x_2)...(1+x_H)}\right)\left(\frac{(1+r_1)(1+r_2)...(1+r_H)}{(1+l_1)(1+l_2)...(1+l_H)}\right)\right]$$

The measure of income risk associated with choosing a VR contract instead of a FR contract is then measured as:

$$\rho_I = \rho_{IV} - \rho_{IF}$$

The following section uses this equation, with the previous wealth risk equation (ρ_W) in simulations, to analyse the effect of varying macroeconomic parameters.

3.3 Theoretical Trade-Offs from Risk Simulations

Having established a simple theory of FR and VR mortgage risk, their size depends on the level of inflation, real interest rate, and labour income uncertainty. This chapter follows (Campbell & Cocco 2003) by assuming the following processes, without decomposing real labour income growth into temporary and permanent income shocks:

$$\begin{array}{ll} \pi_t = \mu(1-\phi) + \phi\pi_{t-1} + \varepsilon_t, & \varepsilon_t \sim N\left[0, \sigma_\varepsilon^2\right] \\ r_t = \bar{r} + \psi_t, & \psi_t \sim N\left[0, \sigma_\psi^2\right] \\ l_t = \bar{l} + \delta_t, & \delta_t \sim N\left[0, \sigma_\delta^2\right] \end{array}$$

In some of their analysis (Campbell & Cocco 2003) allowed for interest rate and inflation shocks to be correlated, so the simulations here allow for cross-correlation of all three error terms. The relative attractiveness of the two contracts is considered over a given time horizon. A fixed-rate contract length of four years is used as a medium term financial decision horizon in this analysis.

To investigate risk trade-offs, the baseline parameters were varied, one at a time, across a sensible range, from 0.4% to 3.6% in all cases except cross-correlations of error terms which were varied between -0.8 and +0.8. The curve extrapolation toolbox of MATLAB was then used to derive smooth functioned curves describing the risk trade-offs between parameters and wealth and income risk.[3] A cubic polynomial equation was used for all parameters except inflation persistence, which required a Gaussian fit[4] due to the asymptotic increase in risk as the parameter approaches one. The size of wealth risk as a percentage of income risk, which can be considered as (an unweighted) measure of the 'VR conduciveness' of a particular macroeconomic environment, is plotted on the following graphs.

Figure 3.3.1 shows that 'VR conduciveness' is positively related to the standard deviation of inflation shocks (sigmap), because as this parameter is increased, the unpredictability of wealth associated with the FR contract (wealth risk) increases at a faster rate than the unpredictability of payment shocks mainly associated with the VR contract (income risk). The standard deviation of real interest rate shocks (sigmar) on the other hand, is a negative function of VR conduciveness, because as this parameter increases, wealth risk increases at a slower rate than income risk.

Figure 3.3.2 shows that VR conduciveness is a decreasing function of both mean inflation and mean real interest rates, but increases with mean real income growth. This is because as average levels of inflation and real interest rates increase, wealth risk falls and income risk rises, whereas high income growth reduces income risk and leaves wealth risk unaffected. Figure 3.3.2 also shows that the mean real interest rate parameter is more 'VR conducive elastic' than mean inflation, because the curve is

[3] Graphs illustrating all the fitted wealth and income risk curves are provided in Appendix B.

[4] Using the equation: $a_1 * e^{-((x-b_1)/c_1)^2} + a_2 * e^{-((x-b_2)/c_2)^2}$

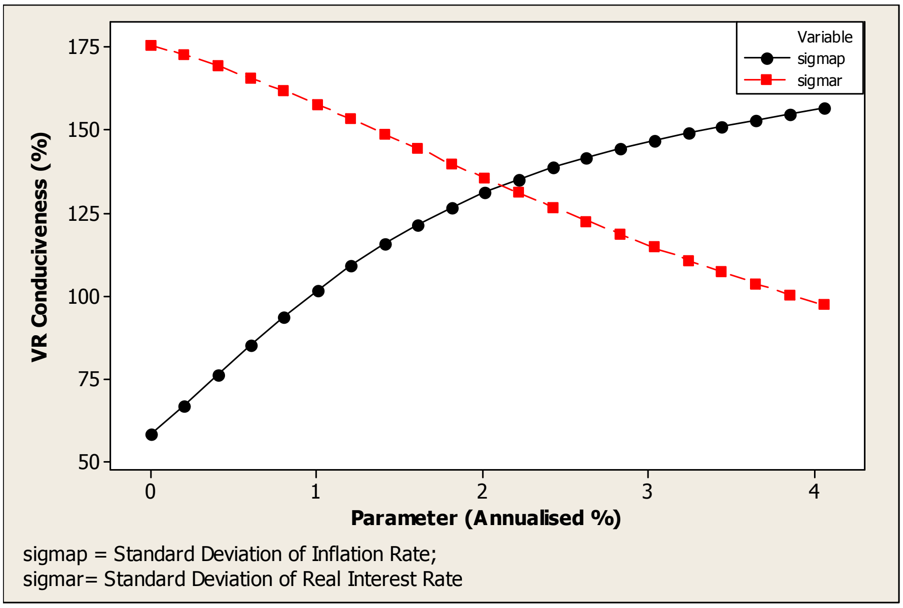


Figure 3.3.1: Volatility Parameters and VR Conduciveness

steeper across the whole range of parameter values.

For low levels of inflation persistence, figure 3.3.3 shows that VR conduciveness is relatively unaffected by this parameter. Above 0.4 however, VR conduciveness increases faster, reaching a peak at 0.65, before falling at an increasing rate towards 1. This is because wealth risk increases at a steadily increasing rate, whilst income risk increases at a slower rate between 0 and 0.7, before rising exponentially.

Changes in VR conduciveness brought about by changes in cross-correlations of the error terms were negligible, except for the correlation between inflation and real interest rate shocks. As this parameter increases, wealth and income risk both increase, but wealth risk rises at a slightly faster rate, resulting in a slow fall in VR conduciveness.

The annualised parameters (ignoring small compounding differences) used in the baseline simulations are shown in Table 3.3.1, together with the point elasticities of wealth and income risk at the baseline. The more positive (negative) these values, the

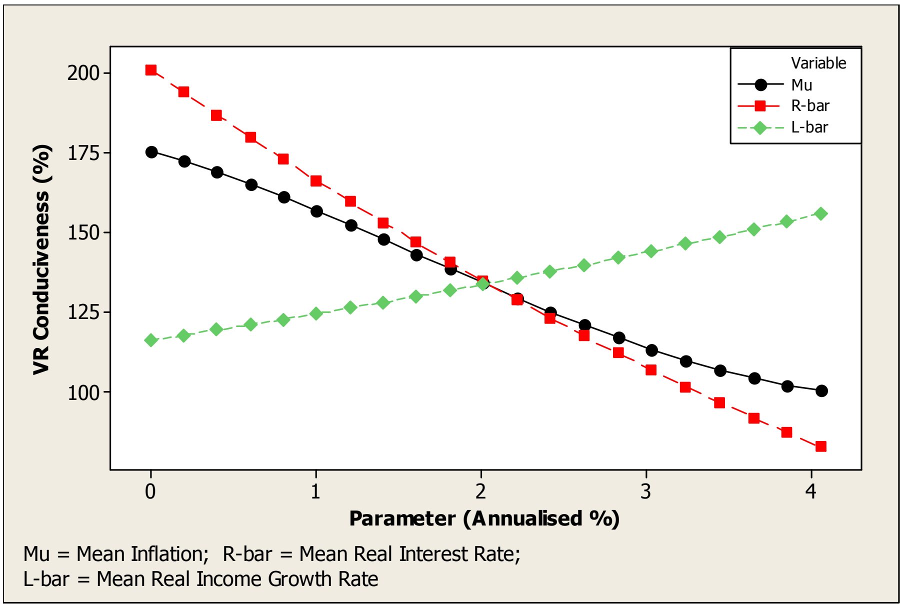


Figure 3.3.2: Mean Parameters and VR Conduciveness

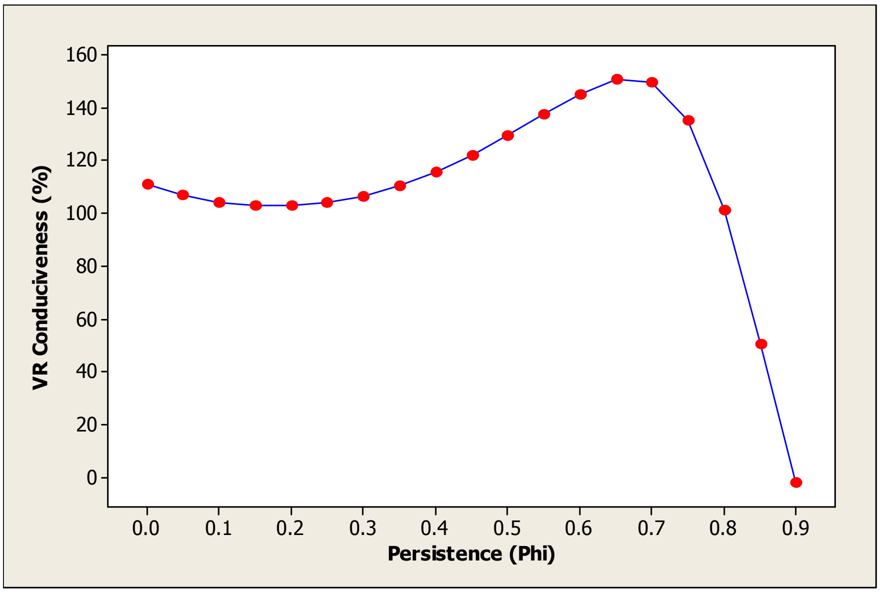


Figure 3.3.3: Inflation Persistence and VR Conduciveness

larger is the percentage increase (decrease) in risk given a 1% increase in the value of the parameter. The last column shows wealth risk elasticity less income risk elasticity, a measure of the 'VR conduciveness' of the parameter, in particular, the percentage increase in wealth risk associated with the FR contract over and above the percentage increase in income risk associated with the VR contract, given a 1% increase in the parameter.

Table 3.3.1: Annualised Parameters

Parameters	Baseline	Wealth η_w	Income η_I	VR Conduciveness Elasticity
Inflation				
μ	2%	-0.11	0.09	-0.19
ϕ	0.5	0.6	0.26	0.34
σ_ε	2%	0.79	0.6	0.18
Real Interest				
r	2%	-0.08	0.17	-0.25
σ_ψ	2%	0.21	0.4	-0.19
Real Income				
I	2%	0	-0.08	0.08
σ_δ	2%	0	0	0
Correlations				
$\sigma_{\delta\psi}$	0	0.43	0.46	-0.03
Time Horizon	4 years			

It should be noted that although sticky inflation appears as the most VR conducive parameter in the Table, this is only the case in the region close to the baseline value of 0.5. For values between 0 and 0.2, and between 0.65 and 1, it was shown earlier that VR conduciveness is a decreasing function of ϕ.

In general, in decreasing order of magnitude, Table 3.3.1 shows that the following macroeconomic conditions favour the development of VR mortgages, *ceteris paribus*:

- Volatile/persistent inflation
- Low mean real interest rate

- Stable real interest rate
- Low mean inflation rate
- High mean real income growth
- Low correlation between inflation and real interest rate

3.4 Explaining Levels of VR/FR Mortgage Lending Across EU-15

3.4.1 VR/FR Mortgage Prevalence Across EU-15

Having established what kind of macroeconomic environment might be expected to favour the development of VR mortgage contracts, this section looks at how lending actually differs across the EU-15. The following table shows an index of how predominant VR mortgages are, constructed from data presented in a paper on EU-15 financial market asymmetries[5]. The UK is not alone with its predominately variable-rate mortgage system: there is more lending at variable-rates in Luxembourg and Sweden, and VR mortgage contracts appear to prevail in Austria, Portugal and Finland. France and Denmark, on the other hand, appear to be at the opposite extreme, with almost all lending at fixed-rates, and in Denmark's case usually for time periods extending to the full 20 or 25 year term.

A detailed discussion of the different regulatory and financial histories is also given in (Maclennan et al. 1998), with no systematic patterns emerging. Denmark, Germany, Sweden, Holland, Austria and Finland are lumped together as 'mortgage bank systems' which raise wholesale funds by selling bonds to institutional investors with no significant local branch networks. The UK, Ireland, France and Spain, on the other hand, are dominated by deposit-taking systems where retail savings institutions

[5] (Maclennan et al. 1998)

Luxembourg	97
Sweden	93
UK	80
Portugal	67
Finland	60
Austria	67
Ireland	57
Spain	53
Germany	53
Greece	47
Italy	27
Holland	28
Belgium	25
France	13
Denmark	7

Table 3.4.1: Variable-Rate Mortgage Prevalence Index

transform the savings of millions of households into long-term mortgages. Credit market liberalisation is considered most complete in UK, Spain, Finland and Sweden.

Whilst institutional histories are likely to be important factors, the relative levels of VR and FR lending may also be explained by macro-economic histories. It is important to be aware that there is a ‘chicken and egg’ style conundrum here, because deciphering causation direction is required, in the sense that macro-environments are themselves likely to be a function of institutional histories. Here it is simply proposed that the two are inter-twined to the extent that it is possible to think of them as the same phenomenon.

The following section 3.4.2 tests this hypothesis by regressing the log of the above index onto the means and standard deviations of inflation and interest rates throughout the 80s and 90s, with credit constraint proxies as additional explanatory variables. Significance of these variables, and signs that are consistent with the relationships presented in the previous section can be thought of as evidence supporting this hypothesis.

3.4.2 Explaining Levels of VR/FR Mortgage Lending Across EU-15

Table 3.4.2 shows the variables considered in a regression of the log of the VR index, *Ln(VI)*. These include the means and standard deviations of inflation and nominal interest rates, based on quarterly data from 1980 (Q1) to 1998 (Q2) of the CPI and Long term government bond rates, accessed via Datastream. To the extent that credit-constraints exist in an economy, income risk should be higher, and hence vr lending should be lower, so two variables are included to proxy credit-constraint. There is extensive literature on the measurement of credit constraints, amongst which (Jappelli & Pagano 1989) have presented detailed evidence that internationally, debt to gdp ratios are a good proxy for the extent of credit constraints. Given that credit market liberalisation can be thought of as an opposing measure, a 'liberalised credit market' dummy variable for UK, Spain, Finland and Sweden is also included.

Table 3.4.2: Regression Variables

	Ln(VI)	NomiSD	InflSD	NomiMean	Libdummy	CreditCons	MeanInfl
Denmark	-1.15	4.50%	0.90%	10.80%	0	65	1.00%
France	-0.89	3.20%	1.00%	9.70%	0	21	1.10%
Belgium	-0.6	2.60%	0.70%	9.10%	0	22	0.80%
Italy	-0.57	4.20%	1.20%	12.50%	0	7	1.80%
Holland	-0.55	1.70%	0.60%	7.70%	0	60	0.60%
Greece	-0.33	2.10%	2.30%	19.20%	0	6	3.60%
Germany	-0.28	1.50%	0.60%	7.10%	0	51	0.70%
Spain	-0.28	3.50%	1.10%	12.30%	1	22	1.70%
Ireland	-0.24	3.60%	1.50%	10.70%	0	27	1.30%
Finland	-0.22	1.90%	1.00%	7.40%	1	30	1.10%
Austria	-0.17	1.60%	0.80%	7.50%	0	32	0.80%
Portugal	-0.17	4.90%	2.10%	14.40%	0	26	2.80%
UK	-0.1	2.30%	1.10%	9.80%	1	57	1.30%
Sweden	-0.03	2.30%	1.10%	10.70%	1	51	1.30%
Lux	-0.01	1.60%	0.80%	7.80%	0	22	0.80%

The variables in Table 3.4.2 are arranged in order of significance (size of t-statistic) from most significant to the least. A regression on the standard deviation of nominal interest rates and inflation, with the liberalised credit market dummy yields:

$$\ln(VI) = -0.33 - 0.3(NomiSD) + 0.9(InflSD) + 0.1(LibDummy) + e$$

p-values: (0.02) (0.00) (0.28) (0.00)

$\overline{R}^2 = 0.72$

The variables in this regression yield signs that are consistent with the theory and simulations presented in the previous sections: volatile (stable) interest rates have a negative (positive) effect on the VR index; volatile inflation has a positive effect; and credit market liberalisation (reducing income risk) has a positive effect

Although the volatility of inflation and interest rates (*NomiSD* and *InflSD*) are the only variables significant at the 5% level, An F-test on the coefficient for *LibDummy* restricted to zero was insignificant. Further regressions without either of these two variables increases AIC, lowers $\overline{R}^2$, and does not change the sign of any of the other variables.

Figures 3.4.1 and 3.4.2 show scatter plots of the VR Index against inflation volatility (*InflSD*), and nominal interest rate volatilty (*NomiSD*) respectively, with a fitted OLS regression line. Their respective upwards and downwards slopes confirm the idea that inflation volatility is a positive function, and interest rate volatility a negative function, of VR lending prevalence, but that individually these variables are not very good at explaining the variation observed across EU-15 countries.

3.5 Conclusion

The simulations show that economies more conducive to variable-rate mortgages are those with relatively volatile, persistent, and low inflation; low and stable real interest rates; high real income growth; and low correlation between inflation and real interest rate shocks. An index of variable-rate prevalence has been constructed, and consistent with the theoretical risk simulations, it is positively and negatively correlated with

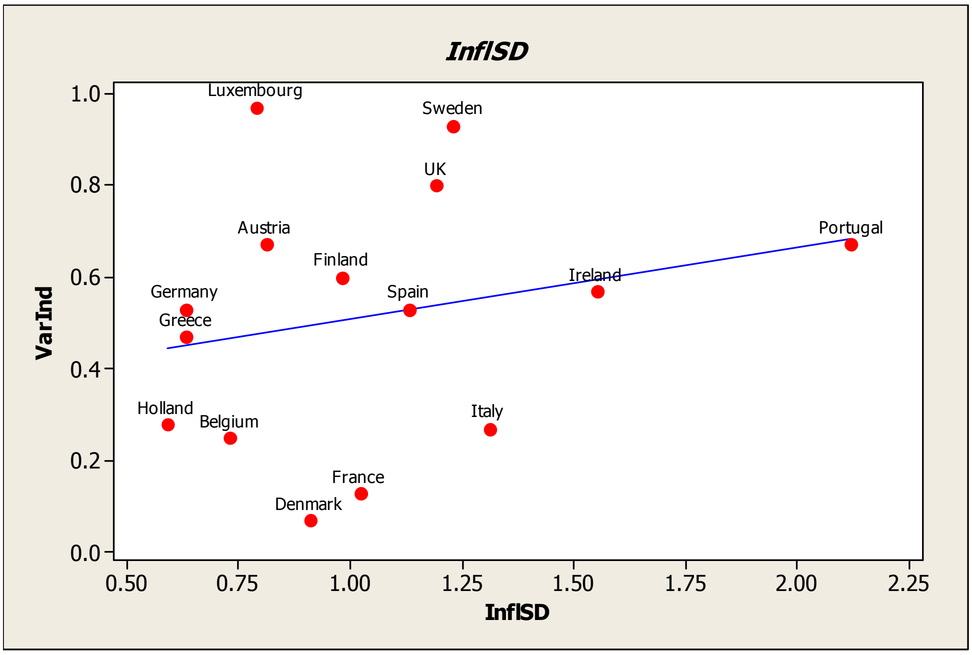


Figure 3.4.1: Scatter Plot of VR Prevalence and Inflation Volatility

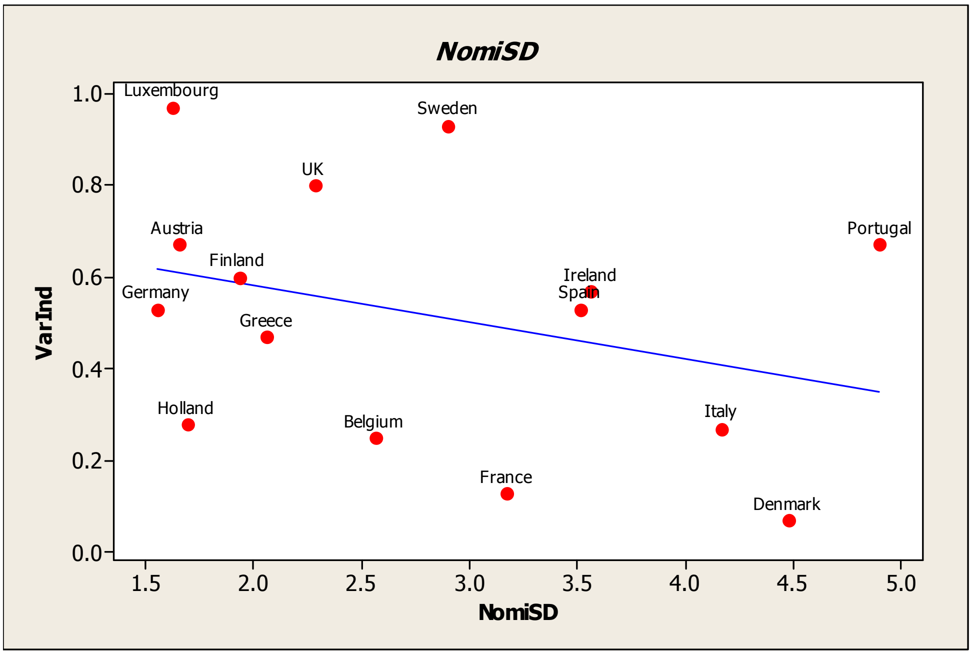


Figure 3.4.2: Scatter Plot of VR Prevalence and Nominal Interest Rate Volatility

inflation and interest rate volatility respectively. A four variable model including a dummy for credit market liberalisation and mean interest rates is also consistent with the theoretical trade-offs. This regression over-predicts the extent of variable-rate lending in France, Finland and Denmark, under-predicts Luxembourg, Spain and Italy, whilst fairly accurately predicting VR mortgage prevalence in Austria, Portugal, Ireland, Germany, Belgium, Holland, Sweden, Greece and UK. Other factors that could be relevant, but which are not analysed in this chapter include fiscal and other types of incentive, and cultural habit-formations.

The regression analysis of the EU-15 presented in this chapter suggests that the UK's relatively volatile inflation combined with its relatively stable interest rates from 1980 to 1998, may help to explain why the general level of fixed-rate (variable-rate) mortgage lending is relatively low (high) compared to other EU-15 countries, in a way that is consistent with associated mortgage risks.

Chapter 4

Optimal Monetary Policy

4.1 Introduction

This chapter investigates the social benefits of fixed versus variable rate debt contracts, by looking at the transmission of productivity shocks under fully optimal (Ramsey) monetary policy. We integrate a quantity-optimising banking sector that lends under either fixed or variable rates within a model with borrowing constrained households, providing a framework that can be used to investigate the relationships between the structure of debt contracts, and the monetary policy transmission mechanism. In particular we study the propagation of a productivity shock in the non-durable sector under Ramsey monetary policy. The introduction of overlapping debt contracts tempers the effect of the financial multiplier. Although the implied steady state allocation of resources unambiguously reduces the deterministic component of social welfare, an appropriate design of debt contracts, their length and the interest rate composition, can reduce volatility of the key economic variables, so that the financial sector can play a stabilising role in the economy. We demonstrate that an intermediate ratio of fixed and variable rate debt contracts is socially optimal.

Our economy is populated by households who would like to borrow under collateral of durable goods and by households who are willing to lend. In a similar way to (Iacoviello 2005), (Kiyotaki & Moore 1997), and (Campbell & Hercowitz 2005), we introduce heteregeneity by assuming some households are relatively impatient. The distinction between two fundamentally different groups of households is a useful device that naturally gives rise to borrowing constraints and debt. The optimising behaviour of households naturally implies that the impatient households become borrowers and the patient households become lenders.

We allow households to increase borrowing if they provide enough collateral.[1] We require a good that can be used as collateral and employ a two good – two factor model, similar to the one developed in (Monacelli 2007), (Calza et al. 2009), and (Monacelli 2009). Durable goods can capture houses and other long term purchases such as cars, whose associated debt contracts are also often specified in nominal variable or fixed rate terms. We assume that both borrowers and savers work, although all firms belong only to savers. In this respect our model is similar to the one developed in (Calza et al. 2009) except that we provide a detailed modelling of financial markets.

We introduce the profit-maximising, price-taking financial sector model of (Graham & Wright 2007), which has the attractive feature of capturing the simultaneous setting of fixed and variable rates in a tractable, yet forward-looking way.[2] Banks borrow from patient households and lend to impatient households under either fixed or variable rates and all debt contracts are in nominal terms, and overlapping Calvo-type contracts ((Calvo 1983)). Every time the contract is re-written, the profit-maximising bank decides on the amount of lending, but sets the fixed rate optimally as a price-competitive firm[3]. The share of contracts with fixed rates is exogenous, however we

[1] The use of a 'financial accelerator' as an explanation for business cycle amplification dates back at least as far as (Bernanke, Gertler & Gilchrist 1999). This term is also used in (Iacoviello 2005), which includes impatient entrepreneurs tied to collateral based constraints, and in (Aoki, Proudman & Vlieghe 2004) in the context of households with collateral based on house value. In contrast to our Monacelli framework however, (Aoki et al. 2004) use rule-of-thumb consumers (who always consume their current income), and there is no analysis of optimal monetary policy.

[2] See also (Gerali, Neri, Sessa & Signoretti 2008) and (Andres & Arce 2008) for similar profit-maximising, price-taking banking sectors with unlimited lending.

[3] See also (Rubio 2009) where there are no overlapping debt contracts but actuarially fair fixed interest rates on debt contracts. Although (Calza et al. 2009) define the fixed rate in a non-optimising

investigate the welfare consequences of the different shares for a central planner. In contrast to (Graham & Wright 2007), we do not assume a fixed size of collateral for every household, but use durable goods as collateral for impatient households. In essence, this paper combines the endogenous collateral constraint of the (Calza et al. 2009) model, with the banking sector that optimally sets debt contracts at fixed and variable rates as in (Graham & Wright 2007).

The paper is organised as follows: In the next Section we set up a two good – two sector model with a financial sector. Section 4.3 defines optimal policy. We discuss calibration in Section 4.5, and demonstrate how to find the steady state of the dynamic model in Section 4.4. Section 4.6 presents results and Section 4.7 concludes.

4.2 The Model

The model builds on (Monacelli 2007) and (Graham & Wright 2007). The economy consists of two types of households, patient and impatient and two sectors – producing durable and non durable goods respectively – each populated by a large number of monopolisticly competitive firms and by a perfectly competitive final goods producer. The relative impatience of one group of households results in their preference for current consumption at the expense of future consumption (their marginal utility of current consumption exceeds the marginal utility of saving). They choose to borrow to maximise their utility and the patient households choose to lend. The presence of household debt reflects equilibrium intertemporal trading between the two types of agents, with the savers acting as standard consumption-smoothers. The borrowing of impatient households is constrained by the amount of collateral of durable goods that they can optimally accumulate. In what follows we will refer to impatient households as 'borrowers' or 'constrained households' and use subscript c to denote relevant

way, they focus on a different research question.

variables. The patient household are ‘savers’ or ‘unconstrained households’ and we use subscript u to denote relevant variables. Subscripts d and nd denote variables for sectors that produce durables and non-durables correspondingly.

The financial sector in this economy is represented by financial intermediaries which compete in quantity but not in price. Each household’s debt contract is renegotiated (they ‘move house’) with constant probability, at which point banks have the opportunity to maximise profit and choose the amount of lending. At the same time they either apply the Central Bank’s rate to variable rate contracts, or apply the fixed rate as determined by a no arbitrage condition.

4.2.1 Preferences

The period utility function for each household of type $j \in \{u, c\}$ is identical:

$$u_{jt} = \frac{X_{jt}^{1-\sigma}}{1-\sigma} - \varkappa \frac{N_{jt}^{1+\phi}}{1+\phi} \tag{4.2.1}$$

where X_{jt} and N_{jt} denote composite consumption, and hours of labour, $\frac{1}{\sigma}$ and $\frac{1}{\phi}$ are the elasticities of intertemporal substitution of consumption and labour, and $\varkappa > 0$ is a scale parameter capturing the relative disutility of working compared to consuming.[4]

For all households X_t is a CES consumption aggregator of the form:

$$X_{ut} \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{ut}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D_{ut}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{4.2.2}$$

$$X_{ct} \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{ct}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D_{ct}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{4.2.3}$$

where C_t is a (Dixit & Stiglitz 1977) (henceforth D-S) aggregator of differentiated non-durable goods, D_t denotes consumption of durable goods, $\alpha > 0$ is the share of durable goods in the composite consumption index, and $\eta > 0$ is the elasticity of substitution between non-durable and durable consumption.

[4] Notice that we follow the now conventional approach of building a monetary model in which households do not derive any utility from holding money, see (McCallum & Nelson 2000) and (Woodford 2003), Ch3.

The differentiated goods are produced by monopolistically competitive intermediate goods firms, indexed by $z \in (0,1)$. Perfectly competitive final goods firms then aggregate varieties into a single consumption good, so that standard D-S aggregator for non-durable goods is:

$$C_t = \left[\int_0^1 c_t\left(z\right)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution between intermediate goods. The corresponding non-durable goods price index is given by:

$$P_{nd,t} \equiv \left[\int_0^1 p_t\left(z\right)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}$$

and the composite price index is determined as:

$$P_t \equiv \left[(1-\alpha) P_{nd,t}{}^{1-\eta} + \alpha P_{d,t}{}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

where $P_{d,t}$ is the rental price of durable goods. We assume that households are indifferent between buying new or existing stocks of durable goods, which, if we pursue an analogy with housing stock, means that once a constant rate of depreciation is allowed for, there is no qualitative consumption difference between the service flows from old and new houses. Demand for each individual variety of non-durable good is given by:

$$c_t\left(z\right) = \left(\frac{p_t\left(z\right)}{P_{nd,t}} \right)^{-\epsilon} C_t$$

where $p_t\left(z\right)$ is the price of variety z.

The stock of durables is updated according to:

$$D_{t+1} = \left(1-\delta\right) D_t + d_t$$

where d_t denotes expenditure on durable goods, and δ is a constant rate of depreciation.

4.2.2 Households

Savers

The optimisation problem for patient households is standard. Each household maximises their infinite horizon present discounted stream of future utility flows:

$$\max U_{ut} = E_t \sum_{s=t}^{\infty} \beta_u^{s-t} u_{us}(C_{us}, D_{us}, N_{us}) \tag{4.2.4}$$

where the period utility function $u_{us}(.)$ is defined by equation (4.2.1).

The budget constraint for patient consumers is:

$$\tilde{W}_{ut} \leq A_{ut} + W_{ut}N_{ut} + T_{ut} - P_{nd,t}C_{ut} - P_{d,t}\left(D_{ut} - (1-\delta)\, D_{ut-1}\right) + E_t\left(Q_{t,t+1}\tilde{D}_{t+1}\right) \tag{4.2.5}$$

where $\tilde{W}_{ut}$ represents the value of the household's end of period portfolio, A_{ut} is beginning of period financial wealth and government transfers are denoted by T_{ut}. The nominal wage rate from working N_{ut} hours is W_{ut}. Consumption consists of both non-durable and durable goods spending, priced nominally at $P_{nd,t}$ and $P_{d,t}$ respectively. $\tilde{D}_{t+1}$ are dividends that realise by the beginning of the new period. We define the stochastic discount factor $Q_{t,t+1}$ with the property that the price in period t of any portfolio with random value A_{ut+1} in the following period is given by:

$$\tilde{W}_{ut} = E_t\left(Q_{t,t+1}A_{ut+1}\right)$$

We also denote:

$$\frac{1}{1+R_t} = E_t\left(Q_{t,t+1}\right)$$

If the household holds riskless assets then:

$$A_{ut+1} = (1+R_t)\,\tilde{W}_{ut}$$

and the budget constraint becomes:

$$A_{ut+1} = (1+R_t)\left(A_{ut} + W_{ut}N_{ut} + T_{ut} - P_{nd,t}C_{ut} - P_{d,t}\left(D_{ut} - (1-\delta)\, D_{ut-1}\right)\right) + \tilde{D}_{t+1}$$

We also specify a limit on borrowing to prevent ‘Ponzi schemes’:

$$\tilde{W}_{ut} \geq -\sum_{s=t+1}^{\infty} E_{t+1}\left[Q_{t+1,s}\left(W_{us}N_{us}+T_{us}\right)\right]$$

and introduce the relative price of durables as:

$$q_t = \frac{P_{d,t}}{P_{nd,t}}$$

which allows us to rewrite the budget constraint in real terms:

$$\begin{aligned}\frac{A_{ut+1}}{P_{nd,t}} &= (1+R_t)\left(\frac{A_{ut}}{P_{nd,t-1}}\frac{P_{nd,t-1}}{P_{nd,t}} + \frac{W_{ut}}{P_{nd,t}}N_{ut} + \frac{T_{ut}}{P_{nd,t}}\right. \\ &\qquad \left. -C_{ut} - q_t\left(D_{ut} - (1-\delta)\,D_{ut-1}\right)\right) + \frac{\tilde{D}_{t+1}}{P_{nd,t}}\end{aligned}$$

Denote $a_{ut} = \frac{A_{ut+1}}{P_{nd,t}}$, $\Pi_t = \frac{P_{nd,t}}{P_{nd,t-1}}$, $w_{ut} = \frac{W_{ut}}{P_{nd,t}}$, $t_{ut} = \frac{T_{ut}}{P_{nd,t}}$, and $\tilde{d}_t = \frac{\tilde{D}_{t+1}}{P_{nd,t}}$ to obtain the simplified real budget constraint:

$$a_{ut} = (1+R_t)\left(\frac{a_{ut-1}}{\Pi_t} + w_{ut}N_{ut} + t_{ut} - C_{ut} - q_t\left(D_{ut} - (1-\delta)\,D_{ut-1}\right)\right) + \tilde{d}_t, \quad (4.2.6)$$

To derive first order efficiency conditions for the household maximisation problem we then write down the following Lagrangian:

$$\begin{aligned}L &= \sum_{s=t}^{\infty}\beta_u^{s-t}\left(U_{us} + \Lambda_s\left(a_{us} - (1+R_s)\left(\frac{a_{us-1}}{\Pi_s} + w_{us}N_{us} + t_{us}\right.\right.\right. \\ &\qquad \left.\left.\left. -C_{us} - q_s\left(D_{us} - (1-\delta)\,D_{us-1}\right)\right) - \tilde{d}_s\right)\right)\end{aligned}$$

where U_{us} is the objective utility function defined by equation (4.2.4), and Λ_s is the shadow value (Lagrange multiplier) associated with the budget constraint. Defining the partial derivative of the objective function with respect to variable x as $\frac{\delta U}{\delta x} = U_x$ (marginal utility), we can write the system of first order efficiency conditions as (see appendix B.2.1 for a complete derivation):

$$
\begin{aligned}
\Lambda_t &= -\frac{U_{nd,ut}}{(1+R_t)} && (4.2.7)\\
q_t &= \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\eta}} \frac{D_{ut}^{-\frac{1}{\eta}}}{C_{ut}^{-\frac{1}{\eta}}} + \beta_u (1-\delta) E_t \left(\frac{X_{ut+1}^{\frac{1}{\eta}-\sigma} C_{ut+1}^{-\frac{1}{\eta}}}{X_{ut}^{\frac{1}{\eta}-\sigma} C_{ut}^{-\frac{1}{\eta}}} q_{t+1} \right) && (4.2.8)\\
w_{ut} &= \frac{\varkappa N_{ut}^{\phi}}{(1-\alpha)^{\frac{1}{\eta}} X_{ut}^{\frac{1}{\eta}-\sigma} C_{ut}^{-\frac{1}{\eta}}} && (4.2.9)\\
1 &= \beta_u E_t \left(\frac{X_{ut+1}^{\frac{1}{\eta}-\sigma} C_{ut+1}^{-\frac{1}{\eta}}}{X_{ut}^{\frac{1}{\eta}-\sigma} C_{ut}^{-\frac{1}{\eta}}} \frac{(1+R_t)}{\Pi_{t+1}} \right) && (4.2.10)
\end{aligned}
$$

The budget constraint equation (4.2.6) determines the path of assets, and conditions (4.2.7)-(4.2.10) determine the constraint shadow value, consumption of durables, the wage, and consumption of non-durables respectively. Note that with unrestricted access to financial markets, these conditions for patient households are standard. Following (Monacelli 2009), equation (4.2.8) can be rearranged to give:

$$
\frac{U_{d,ut}}{U_{nd,ut}} = q_t - \beta_u (1-\delta) E_t \left[\frac{U_{nd,ut+1}}{U_{nd,ut}} q_{t+1} \right] \tag{4.2.11}
$$

This equation shows that patient households will make choices between durables and non-durables in such a way as to equate their marginal rate of substitution, to the user cost of durables. Taking the analogy with houses, the higher expected capital losses due to relative house price falls; the more patient the rate of time preference, the higher the rate of depreciation, and the higher the marginal rate of substitution between future and present non-durable consumption, then the greater is the opportunity cost associated with buying an extra unit of housing. In turn, this high opportunity cost implies a desire for a relatively high marginal utility from housing services (and hence lower total utility), and therefore less demand for houses. In this case, with perfect financial markets, movements in the user cost are dominated by expected house price movements (see (Erceg & Levin 2006)). Put simply, when house prices are expected to fall, their re-sale value is lower, so their demand falls.

Conditions (4.2.7) and (4.2.10) determine the stochastic discount factor:

$$Q_{t,t+1} = \beta_u \frac{U_{c,ut+1}}{U_{c,ut}} \frac{P_{nd,t}}{P_{nd,t+1}} \tag{4.2.12}$$

In contrast to the Monacelli model, we also allow patient households to work. The standard labour supply condition (4.2.9) equates the real wage (in units of non-durable consumption) to the marginal rate of substitution between work disutility and non-durable consumption utility. We assume perfect labour mobility, so household optimisation determines the wage, whereas profit maximisation by firms will determine the amount of labour employed in section 4.2.3.

Borrowers

The utility maximisation problem for impatient households is the same as for patient households:

$$\max U_{ct} = E_t \sum_{s=t}^{\infty} \beta_c^{s-t} u_{cs}(C_{cs}, D_{cs}, N_{cs})$$

where the period utility function $u_{cs}(.)$ is defined by equation (4.2.1).

In contrast to the inequality given by equation (4.2.5) however, these households face a constraint that, given our assumptions, will be always binding. The relative patience of the patient households ensures that (for small enough deviations from the steady state), they always hold a positive value of real assets, a_{ut}, which in turn is channeled through financial intermediaries to the impatient households. The real budget constraint for an *average* impatient household is:

$$a_{ct} = \left(1 + R_t^D\right) \left(\frac{a_{ct-1}}{\Pi_t} + C_{ct} + q_t \left(D_{ct} - (1-\delta) D_{ct-1}\right) - w_{ct} N_{ct} + t_{ct} \right) \tag{4.2.13}$$

where R_t^D is the interest rate they face on debt, which as we will see in section 4.2.5 on bank behaviour, for a specific time t, need not be the same rate that patient households earn. Similarly, although a_{ct} is the real value of debt sourced from the assets of the patient consumers, its dynamics also depend on the profit-maximising financial sector which channels assets via fixed and variable rate loans, as in G&W. We write the constraint for an average impatient household who faces an average rate on debt, and may or may not renew the debt contract at time t.

An important difference however, is that, as in the Monacelli model, impatient households face restrictions on borrowing which are proportional to the *variable* collateral value of the durables they own. In other words, we switch the financial accelerator on. The real value of collateral is defined as:

$$k_t = (1 - \chi)\, q_t D_{ct} \tag{4.2.14}$$

where $(1 - \chi)$ is the constant proportion of durable value that can be used as collateral. In the case of mortgages on houses, χ would be the downpayment ratio, or the inverse loan to value ratio, and is hence a measure of how much debt is rationed. (Kiyotaki & Moore 1997) provide a careful story for the existence of this type of rationing, with banks who lend to farmers with idiosynchratic technology. The difference between liquidation values in the event of default and the value to the owner of collateral leads to moral hazard, and enforcement costs lead to debt rationing. Here, following G&W and Monacelli, we apply a similar argument: when banks lend more than the proportion $(1 - \chi)$ of collateral, the incentive to default kicks in, the probability of default increases, and expected profits fall. Unresticted entry into the financial sector also means that all profit opportunities are exploited, and so banks never lend less than $(1 - \chi)$.

Following G&W, we also assume that debt contracts are reconsidered infrequently, with a Calvo-type stickiness parameter ρ, that we interpret as the probability that the household moves house, and hence must re-negotiate its debt contract. At this point the financial intermediary will decide on the maximum nominal quantity it is willing to lend, at either a variable or fixed rate. It is assumed that variable rates are driven down to the perfectly competitive central bank rate, whilst the competitive fixed rates are determined by individual bank optimisation, and hence are subject to short term profits and losses. We demonstrate in section 4.2.5 that the evolution of real average lending can be described by:

$$a_{ct} = (1-\rho)\frac{a_{ct-1}}{\Pi_t} + \rho\Upsilon(k_t) \tag{4.2.15}$$

Formula (4.2.15) suggests that real debt at time t remains the same as at $t-1$ with probability $(1-\rho)$ when contracts are not reconsidered, and with probability ρ the amount of borrowing is a specific function $\Upsilon(.)$ of the value of collateral owned at time t. The household also knows that borrowing will be determined by the size of their collateral when they make consumption decisions, which opens up the possibility of using durable good ownership as a means of expanding debt, and hence consumption, which in turn may fuel durable goods price increases, and so on. This feature captures the growing phenomenon of Mortgage Equity Withdrawal in the UK, as discussed in (Aoki et al. 2004), which may have amplified recent cycles in economic activity.

The Lagrangian for constrained households can hence be written as:

$$\begin{aligned} L &= \sum_{s=t}^{\infty}\beta_c^{s-t}\left(U_{cs} + \Theta_s\left(a_{cs} - \left(1+R_s^D\right)\left(\frac{a_{cs-1}}{\Pi_s} + C_{cs} - w_{cs}N_{cs}\right.\right.\right. \\ &\quad + t_{cs} + q_s\left(D_{cs} - (1-\delta)D_{cs-1}\right))) \\ &\quad \left.-\Gamma_s\left(a_{cs} - (1-\rho)\frac{a_{cs-1}}{\Pi_s} - \rho\Upsilon\left((1-\chi)q_sD_{cs}\right)\right)\right) \end{aligned}$$

where we have two Lagrange multipliers: the first, Θ_s, associated with the real budget constraint, and the second, Γ_s, associated with the collateral constraint. The associated first order conditions are (see appendix B.2.1 for full derivation):

$$\Theta_t = \frac{U_{nd,ct}}{(1+R_t^D)} \tag{4.2.16}$$

$$U_{nd,ct}q_t = U_{d,ct} + \beta_c(1-\delta)E_t\left(U_{nd,ct+1}q_{t+1}\right) + U_{nd,ct}\rho\Upsilon_{Dc,t}\Xi_t \tag{4.2.17}$$

$$w_{ct} = -\frac{U_{n,ct}}{U_{nd,ct}} \tag{4.2.18}$$

$$\left(1+R_t^D\right)\Xi_t = 1 - \beta_c E_t\left(\frac{U_{c,ct+1}}{U_{c,ct}}\frac{(1+R_t^D)}{\Pi_{t+1}}\left(1-(1-\rho)\Xi_{t+1}\right)\right) \tag{4.2.19}$$

where the partial derivatives of the objective function with respect to C_{ct}, D_{ct}, and N_{ct} are written as $U_{nd,ct}$, $U_{d,ct}$, and $U_{n,ct}$ respectively, $\Upsilon_{Dc,t}$ is $\frac{\delta\Upsilon(.)}{\delta Dc,t}$, and we have defined the shadow price of the collateral constraint as $\Xi_t = \Gamma_t / U_{nd,ct}$. The budget constraint equation (4.2.13) determines the path of non-durable consumption, while conditions (4.2.16) to (4.2.19) determine the shadow value of the budget constraint, consumption of durables, the wage, and the shadow value of the collateral constraint respectively.

Rearranging condition (4.2.17) in the form of condition (4.2.11) for patient households we get:

$$\frac{U_{d,ct}}{U_{nd,ct}} = q_t - \beta_c (1-\delta) E_t \left[\frac{U_{nd,ct+1}}{U_{nd,ct}} q_{t+1} \right] - \rho \Upsilon_{Dc,t} \Xi_t \tag{4.2.20}$$

which is identical to condition (4.2.11) for patient households, except for the addition of an extra user cost term ($\rho \Upsilon_{Dc,t} \Xi_t$) that captures how much the collateral constraint restricts the ability of borrowers to purchase debt and acquire new durables. When the shadow value $\Xi_t = 0$, the collateral constraint is not binding, and the nature of constrained household demand for non-durables relative to durables coincides with patient household behaviour. (Monacelli 2009) emphasises that, through this new collateral channel, movements in Ξ_t affect constrained borrower behaviour, and can break the normally strong link that exists between asset price movements and durable/non-durable choice. Decreases in Ξ_t indicate a more relaxed collateral constraint, a rise in user cost, and hence a lower demand for durables.

Similarly, in the absence of the collateral constraint, and with the appropriate time preference and borrower interest rates, condition (4.2.19) collapses to the standard patient consumption Euler equation (4.2.10).

In contrast to the Monacelli model, we have G&W nominal debt contracts which are Calvo-sticky. As debt renegotiation costs approach zero (or the moving probability ρ approaches 1), the stickiness of debt contracts disappears, and we have the Monacelli case in which the full magnitude of the (expected future) collateral constraint Ξ_{t+1}

becomes relevant. In this case, condition (4.2.19) is identical to equation (14) in (Monacelli 2009). At the opposite extreme, as ρ approaches 0, debt contracts cease to be renegotiated, the effect of changes in the collateral constraint disappears, and the financial accelerator is switched off. Borrowers would no longer be able to use the collateral of their durable good to expand consumption, but would still react to changes in the rate on debt determined by banks, R_t^D. Our model would then closely resemble G&W, although borrowers would still adjust durable and non-durable consumption in response to changes in their relative price q_t.

In general though, the parameter ρ defines our model as an intermediate case between the full accelerator mechanism of Monacelli ($\rho = 1$), and the G&W case with fixed collateral. There are, however, several differences with the model in (Monacelli 2007). First, because of staggered debt contracts, the marginal utility of relaxing the collateral constraint for *an average household* has a relatively small impact if only ρ -share of households are able to renew debt contracts. The collateral constraint can only be weaker or tighter because of changes in price, not quantity. It also implies that it is difficult for the household to move this constraint. In other words, although with staggered debt contracts the presence of the collateral constraint in the user cost increases the contemporary demand for durables, the effect is smaller with less frequent adjustment. We shall discuss in Section 4.2.5 that the dynamics of $\Upsilon_{Dc,t}$ are also affected by the proportion of fixed and variable rate debt contracts, Ψ.

Second, debt contract arrangements affect the way Ξ_t is determined in equation (4.2.19). The effect is twofold. First, infrequent adjustment ($\rho < 1$) implies the future constraint affects the tightness of the current constraint. As the constraint cannot be moved immediately (but only with probability ρ) then the higher future Ξ_{t+1} implies a higher net marginal benefit of acquiring today a unit of the durable asset which in turn allows, by relaxing the collateral constraint at the margin, to purchase additional current consumption. Second, the proportion of fixed rate and variable rate debt contracts affects the dynamics of R_t^D and so also affects the tightness of the out-of-steady state collateral constraint.

4.2.3 Intermediate Goods Firms

In this section we derive firm behaviour in a standard way, with intermediate-goods firms choosing the quantity of labour to employ and the prices to set goods at before they are sold on to a perfectly competitive final-goods sector. Section 4.2.3 presents these derivations for the non-durable goods sector, followed by the durable sector in section 4.2.3. As before, subscript nd indicates employment and output in the non-durable goods sector, and subscript d indicates the durable goods sector. In the non-durable goods sector we have standard imperfect competition, with D-S differentiated goods and nominal price rigidity. For the durable goods market however, given that house purchases are usually negotiated individually, we use the baseline Monacelli framework by assuming perfectly flexible prices.

We split profit maximisation of intermediate-goods firms by dealing with the two problems separately: first, they choose labour to minimise cost intra-temporally, and second, they choose prices to maximise the present value of future profit inter-temporally. Each sector employs two types of labour: those from constrained households with subscript c, and those who are patient with subscript u (also consistent with our earlier notation). There is perfect labour mobility, so for each type of labour, wages are competitively equalised across all firms. We assume that the patient households own all firms, so their discount factor is used to evaluate the present value of expected profits.

Production of Non-Durable Goods

Firms in the non-durable goods sector choose employment and prices to maximise the discounted present value of current and future profits:

$$\max_{\left\{N_{nd,cs}(i),N_{nd,us}(i),p_s^*(i)\right\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s}\left(y_{nd,s}\left(i\right)p_{nds}\left(i\right) - W_{us}N_{nd,us}\left(i\right) - W_{cs}N_{nd,cs}\left(i\right)\right) \tag{4.2.21}$$

subject to a constant returns to scale Cobb-Douglas production technology:

$$y_{ndt}\left(i\right) = Z_{ndt}N_{nd,ut}\left(i\right)^{\nu} N_{nd,ct}\left(i\right)^{1-\nu} \tag{4.2.22}$$

where Z_{ndt} is an exogenous technology shock, and the parameter ν captures the relative productivity of patient labour compared to constrained labour. In the case of homogeneous productivity $\nu = 0.5$.

Profit maximisation is also subject to the demand constraint:

$$y_{ndt}\left(i\right) = Y_{ndt}\left(\frac{p_{ndt}\left(i\right)}{P_{ndt}}\right)^{-\epsilon} \tag{4.2.23}$$

and Calvo price rigidity:

$$\begin{aligned} p_{nd,t}\left(i\right) &= p^*_{nd,t}\left(i\right) \\ p_{nd,t+1}\left(i\right) &= \begin{cases} p^*_{nd,t+1}\left(i\right), \text{ with prob } 1-\theta \\ p_{nd,t+1}\left(i\right), \text{ with prob } \theta \end{cases} \end{aligned} \tag{4.2.24}$$

where θ is the probability that firms have the opportunity to adjust prices.

Employment Firm i minimises nominal cost:

$$\min_{N_{nd,ct}(i),N_{nd,ut}(i)} \left\{W_{ut}N_{nd,ut}\left(i\right) + W_{ct}N_{nd,ct}\left(i\right)\right\}$$

subject to the production constraint (4.2.22).

We can then write down the Lagrangian:

$$L = W_{ut}N_{ndut}\left(i\right) + W_{ct}N_{ndct}\left(i\right) - P_{nd,t}\xi_t\left(Z_{ndt}N_{ndut}\left(i\right)^{\nu} N_{ndct}\left(i\right)^{1-\nu} - y_{ndt}\left(i\right)\right)$$

where ξ_t is the Lagrange multiplier associated with the production constraint.

The associated first order efficiency conditions associated with choosing optimal quantities of patient and constrained labour are:

$$\begin{aligned} w_{ut} N_{nd,ut}(i) &= \nu \xi_t y_{ndt}(i) && (4.2.25)\\ w_{ct} N_{nd,ct}(i) &= (1-\nu)\xi_t y_{ndt}(i) && (4.2.26) \end{aligned}$$

where the real wage is defined as:

$$w_{jt} = \frac{W_{jt}}{P_{nd,t}}$$

It also follows that ξ_t can be derived as:

$$\xi_t = \frac{1}{Z_{ndt}} \left(\frac{w_{ut}}{\nu}\right)^{\nu} \left(\frac{w_{ct}}{1-\nu}\right)^{1-\nu} \tag{4.2.27}$$

and substituting ξ_t into conditions (4.2.26) and (4.2.25) we obtain:

$$\begin{aligned} N_{nd,ut}(i) &= \frac{1}{Z_{ndt}} y_{ndt}(i) \frac{w_{ut}^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{w_{ct}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}\\ N_{nd,ct}(i) &= \frac{1}{Z_{ndt}} y_{ndt}(i) \frac{w_{ut}^{\nu}}{\nu^{\nu}} \frac{w_{ct}^{(-\nu)}}{(1-\nu)^{(-\nu)}} \end{aligned}$$

Substitute real wages from conditions (4.2.9) and (4.2.18) to obtain:

$$\begin{aligned} N_{nd,ut}(i) &= \frac{y_{ndt}(i)}{Z_{ndt}} \frac{N_{us}^{\phi(\nu-1)} C_{ut}^{\frac{1}{\eta}(\nu-1)} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)(\nu-1)}}{\nu^{(\nu-1)}} \frac{N_{cs}^{\phi(1-\nu)} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}}{(1-\nu)^{(1-\nu)}}\\ N_{nd,ct}(i) &= \frac{y_{ndt}(i)}{Z_{ndt}} \frac{N_{us}^{\phi\nu} C_{ut}^{\frac{1}{\eta}\nu} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)\nu}}{\nu^{\nu}} \frac{N_{cs}^{\phi(-\nu)} C_{ct}^{\frac{1}{\eta}(-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(-\nu)}}{(1-\nu)^{(-\nu)}} \end{aligned}$$

Next we define price dispersion $\Delta_t = \int \left(\frac{p_{nd,t}(i)}{P_{nd,t}}\right)^{-\epsilon} di$, to allow aggregation of employment:

$$\begin{aligned} N_{nd,ut} &= \frac{1}{Z_{ndt}} \left(\frac{N_{cs}^{\phi} C_{ct}^{\frac{1}{\eta}} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{us}^{\phi} C_{ut}^{\frac{1}{\eta}} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)}}\right)^{(1-\nu)} \frac{\nu^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} Y_{pt}\Delta_{pt}\\ N_{nd,ct} &= \frac{1}{Z_{ndt}} \left(\frac{N_{us}^{\phi} C_{ut}^{\frac{1}{\eta}} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{cs}^{\phi} C_{ct}^{\frac{1}{\eta}} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)}}\right)^{\nu} \frac{(1-\nu)^{\nu}}{\nu^{\nu}} Y_{pt}\Delta_{pt} \end{aligned}$$

and the marginal cost formula can be determined by substituting wages (equation (4.2.9)) into equation (4.2.27) :

$$mc_t = \frac{1}{Z_{ndt}} \frac{\varkappa N_{ut}^{\phi\nu} C_{ut}^{\frac{1}{\eta}\nu} X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu} N_{ct}^{\phi(1-\nu)} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}}{(1-\alpha)^{\frac{1}{\eta}} \nu^{\nu} (1-\nu)^{(1-\nu)}} \tag{4.2.28}$$

Price setting The setting of prices is standard, closely following derivations in (Woodford 2003), Ch.2. Prices are determined by Calvo-type contracts, with a fixed probability $1-\theta$ that they will be fixed each period, and probability θ that firms will have the opportunity to re-set their prices. Firms will then choose prices to maximise the following expected profit function (4.2.21) which can now be written as:

$$\max_{\{p_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{nds}(i) \, p_{nds}(i) - y_{nds}(i) MC_s \right) \tag{4.2.29}$$

where $MC_s = \xi_t P_{nd,t}$ is nominal marginal cost. Note that wages here do not depend on index i, since labour of each type is assumed to be perfectly mobile between sectors, so wages for patient and constrained labour are equalised across all firms. We then come to the familiar problem of maximising (4.2.29) subject to the constraints (4.2.24) and (4.2.23)

At time s we only need consider the maximisation problem for the proportion θ^{s-t} of firms that have the opportunity to set their prices at time t. Optimal price setting can therefore be re-stated as (and substituting demand from equation (4.2.23)):

$$\max_{\{p_t^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{nds} Y_{nds} \left(\left(\frac{p_{ndt}^*(i)}{P_{nds}} \right)^{1-\epsilon} - \left(\frac{p_{ndt}^*(i)}{P_{nds}} \right)^{-\epsilon} \frac{MC_s}{P_{nds}} \right)$$

The associated first order condition is:

$$E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{nds} y_{nds}(i) \left(\frac{p_{ndt}^*(i)}{P_{nds}} - \mu mc_s \right) = 0 \tag{4.2.30}$$

where the steady state mark-up is $\mu = \frac{\epsilon}{(\epsilon-1)}$ and we have defined real marginal cost as $mc_s = \frac{MC_s}{P_{cs}}$.

Since we assume that patient households own all firms, we substitute their stochastic discount factor (equation (4.2.12)), and rearrange condition (4.2.30) (see appendix (B.2.2)) to obtain our price-setting system for the non-durable goods sector:

$$\frac{1-\theta\Pi_t^{\epsilon-1}}{(1-\theta)} = \left(\frac{G_t}{F_t}\right)^{1-\epsilon} \tag{4.2.31}$$

$$G_t = \mu U_{nd,ut} Y_{ndt} mc_t + \theta\beta E\left[{}_t\Pi_{t+1}^{\epsilon} G_{t+1}\right] \tag{4.2.32}$$

$$F_t = U_{nd,ut} Y_{ndt} + \theta\beta E_t\left[\Pi_{t+1}^{\epsilon-1} F_{t+1}\right] \tag{4.2.33}$$

This system has the form of a New-Keynesian Phillips curve in the sense that current marginal cost and inflation are linked to future inflation.

Because of staggered price contracts, the aggregate price in non-durable sector evolves as:

$$P_{nd,t} = \left[(1-\theta)\left(p^*_{nd,t}\right)^{1-\varepsilon} + \theta P_{nd,t-1}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

So the price dispersion $\Delta_t = \int \left(\frac{p_{nd,t}(i)}{P_{nd,t}}\right)^{-\epsilon} di$ obeys (see (Woodford 2003)):

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_t^{\epsilon}\Delta_{t-1} \tag{4.2.34}$$

Production of Durable Goods

In this section we derive the employment and price-setting behaviour for the durable sector. Each period new stocks of durable goods are produced. The relative stickiness in either sector is important, as discussed in (Monacelli 2009). Durable prices, especially for houses, tend to be subject to individual bargaining processes so we follow the baseline framework of (Monacelli 2007) with perfectly flexible durable prices. We use subscript 'd' for employment and output in this sector.

The profit maximisation problem takes the same form as in (4.2.21), and the same production technology:

$$y_{dt}(i) = Z_{dt} N_{dut}(i)^{\nu} N_{dct}(i)^{1-v} \tag{4.2.35}$$

where Z_{dt} is an exogenous technology shock in the durable goods sector, and the parameter ν captures the relative productivity of patient labour compared to constrained labour, and which we assume is the same as in the non-durable goods sector.

Employment The nominal cost minimisation problem is:

$$\min_{N_{dut}(i),N_{dct}(i)} W_{ut}N_{dut}(i) + W_{ct}N_{dct}(i)$$

subject to the production constraint (4.2.35).

Write down the Lagrangian:

$$L = W_{ut}N_{dut}(i) + W_{ct}N_{dct}(i) - P_{ct}\zeta_t \left(Z_{dt}N_{dut}(i)^{\nu} N_{dct}(i)^{1-\nu} - y_{dt}(i)\right)$$

and the first order conditions are:

$$w_{ut}N_{dut}(i) = \nu\zeta_t y_{dt}(i) \tag{4.2.36}$$

$$w_{ct}N_{dct}(i) = (1-\nu)\zeta_t y_{dt}(i) \tag{4.2.37}$$

It also follows that marginal cost can be derived as:

$$\zeta_t = \frac{1}{Z_{dt}}\left(\frac{w_{ut}}{\nu}\right)^{\nu}\left(\frac{w_{ct}}{1-\nu}\right)^{1-\nu} \tag{4.2.38}$$

and we substitute (4.2.38) into (4.2.36) and (4.2.37):

$$N_{dut}(i) = \frac{1}{Z_{dt}}y_{dt}(i)\frac{w_{ut}^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{w_{ct}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \tag{4.2.39}$$

$$N_{dct}(i) = \frac{1}{Z_{dt}}y_{dt}(i)\frac{w_{ut}^{\nu}}{\nu^{\nu}}\frac{w_{ct}^{(-\nu)}}{(1-\nu)^{(-\nu)}} \tag{4.2.40}$$

Then substitute real wages from (4.2.9) and (4.2.18) to obtain:

$$N_{dut}(i) = \frac{1}{Z_{dt}}y_{dt}(i)\frac{N_{us}^{\phi(\nu-1)}C_{ut}^{\frac{1}{\eta}(\nu-1)}X_{us}^{\left(\sigma-\frac{1}{\eta}\right)(\nu-1)}}{\nu^{(\nu-1)}}\frac{N_{cs}^{\phi(1-\nu)}C_{ct}^{\frac{1}{\eta}(1-\nu)}X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}}{(1-\nu)^{(1-\nu)}}$$

$$N_{dct}(i) = \frac{1}{Z_{dt}}y_{dt}(i)\frac{N_{us}^{\phi\nu}C_{ut}^{\frac{1}{\eta}\nu}X_{us}^{\left(\sigma-\frac{1}{\eta}\right)\nu}}{\nu^{\nu}}\frac{N_{cs}^{\phi(-\nu)}C_{ct}^{\frac{1}{\eta}(-\nu)}X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(-\nu)}}{(1-\nu)^{(-\nu)}}$$

And finally, with flexible prices, output in each firm is the same, and aggregation is straightforward:

$$\begin{aligned}
N_{dut} &= \frac{1}{Z_{dt}}\left(\frac{N_{ct}^{\phi}C_{ct}^{\frac{1}{\eta}}X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{ut}^{\phi}C_{ut}^{\frac{1}{\eta}}X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)}}\right)^{(1-\nu)}\frac{\nu^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_{dt}\\
N_{dct} &= \frac{1}{Z_{dt}}\frac{(1-\nu)^{\nu}}{\nu^{\nu}}\left(\frac{N_{ut}^{\phi}C_{ut}^{\frac{1}{\eta}}X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{ct}^{\phi}C_{ct}^{\frac{1}{\eta}}X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)}}\right)^{\nu}Y_{dt}
\end{aligned}$$

Relative Price Setting In the absence of the Calvo price rigidity constraint (4.2.24), the setting of durable prices relative to non-durables is more straight forward (see appendix (B.2.2). Firms choose prices to maximise expected profit:

$$\max_{\{p_s^*(i)\}_{s=t}^{\infty}} E_t\sum_{s=t}^{\infty}Q_{t,s}\left(y_{ds}\left(i\right)p_{ds}\left(i\right)-y_{ds}(i)MC_{ds}\right)$$

where $MC_{ds}=\xi_t P_{nd,t}$ is nominal marginal cost, subject to the demand constraint:

$$y_{dt}\left(i\right)=Y_{dt}\left(\frac{p_{dt}\left(i\right)}{P_{dt}}\right)^{-\epsilon}$$

The first order condition then is simply:

$$\frac{MC_{dt}}{P_{dt}}=\frac{\epsilon-1}{\epsilon}=\mu$$

from where the price of durables relative to non-durables (q_t) is determined by:

$$q_t=\frac{1}{Z_{dt}}\frac{\varkappa\mu N_{ct}^{\phi(1-\nu)}C_{ct}^{\frac{1}{\eta}(1-\nu)}X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}N_{ut}^{\phi\nu}C_{ut}^{\frac{1}{\eta}\nu}X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu}}{(1-\alpha)^{\frac{1}{\eta}}\nu^{\nu}\left(1-\nu\right)^{(1-\nu)}} \tag{4.2.41}$$

4.2.4 Market Clearing Conditions

Total output is equal to the sum of consumption:

$$\begin{aligned}
Y_{ndt} &= C_{ct}+C_{ut}\\
Y_{dt} &= D_{ut}-(1-\delta)\,D_{ut-1}+D_{ct}-(1-\delta)\,D_{ct-1}\\
Y_t &= Y_{ndt}+Y_{dt}
\end{aligned}$$

Similarly, total hours worked adds up to:

$$\begin{aligned} N_{ct} &= N_{ndct} + N_{dct} \\ N_{ut} &= N_{ndut} + N_{dut} \\ N_t &= N_{ut} + N_{ct} \end{aligned}$$

The net saving in the closed economy is zero:

$$A_{ut} - A_{ct} = 0$$

and we divide by the same price index, so the same condition holds in real terms:

$$a_{ut} - a_{ct} = 0$$

4.2.5 Financial Intermediaries

In this section we incorporate the (Graham & Wright 2007) model of financial sector behaviour into our framework. Nominal debt contracts are typically renegotiated between households and financial intermediaries infrequently. (Miles 2004) for example, provides strong evidence that UK households infrequently re-mortgage. Decisions to renegotiate may come from either party: in the case of mortgages, households may change jobs and move house. On the other hand, mortgage lenders in the UK often provide contracts which are discounted or fixed for an agreed period, after which the nature of the contract changes. Conditions for reassessment of collateral at future dates may also be agreed. Whether the household or the bank trigger re-financing, both situations are captured by simply assuming that all contracts are reconsidered with probability ρ at any given period. In the (Graham & Wright 2007) model, regardless of the type of contract – fixed or variable rate – with probability ρ, the bank decides on the quantity of funds, Z_t to lend to a borrower who provides *nominal* collateral K. When the amount of debt is adjusted, the fixed rate is also changed optimally, whereas variable rates are always set equal to the central bank rate.

In this section we derive the optimal amount of debt, Z_t, that banks will want to lend the impatient borrower, based on the value of their collateral, K_t. In the G&W model this is fixed, but we base it here on the value of owned durable goods, as described by equation (4.2.14). Next, we describe implications for the evolution of aggregate debt, and finally we derive the optimal fixed rate and the implied aggregate average rate payable on impatient household debt. Any impatient household, when resetting the contract, takes the proportion Ψ of the new debt under a variable rate. This share can also be interpreted as a fixed proportion of households who are always tied to variable rate contracts. We assume Ψ is exogenous and time-invariant for simplicity and tractability, but we also investigate implications of different choices for social welfare.[5]

Optimal Debt Offers and Aggregate Debt Evolution

At time t any loan contract is reconsidered with probability ρ. When the financial intermediary adjusts the contract, it changes the nominal value of debt it issues, issuing a new quantity, Z_{t+1}. Its optimisation problem will be to maximise the present discounted value of profit that will flow from this contract, so that at time t, it will discount future periods s only for future scenarios in which the contract is not readjusted, so using $(1-\rho)^{s-t}$. In addition, since patient households own the financial intermediaries, their stochastic discount factor ($Q_{t,s}$ derived in equation (B.2.6)) is also used to discount the future flow of profit, so the problem is written as:

$$\max_{Z_{t+1}} W_t = \sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)_s^{s-t} \Phi_s \tag{4.2.42}$$

where Φ_s is a flow objective.

It is assumed that patient households provide funds to financial intermediaries at

[5] Many (see for example (Maclennan et al. 1998)) have suggested that the different contract structures observed between countries have evolved arbitrarily, in the sense that they are more related to legal and institutional factors than economic.

the central bank rate, so the flow objective of profit can be written as:

$$\Phi_s = (R_s^z - R_s) Z_{t+1} - \Omega_s$$

where

$$\Omega_s = \frac{\varpi}{2} \left(\frac{Z_{t+1} - K_s}{K_s} \right)^2 K_s$$

is the quadratic adjustment cost associated with deviating from the nominal value of the collateral, K_s, and we denote R_s^z the rate that is offered on a new contract,

$$R_t^z = (1 - \Psi) R_t + \Psi R_t^{zF} \tag{4.2.43}$$

Lending more than the value of the collateral is risky, see e.g. (Kiyotaki & Moore 1997), while lending less than the collateral value does not extract the maximum profit. So, the deviation from K_s is costly and parameter ϖ measures how large these quadratic costs are.

The first order condition for the financial intermediary's optimisation problem (4.2.42) implies the following dynamic system for real debt (see appendix (B.2.3):

$$\begin{aligned} \frac{Z_{t+1}}{P_{ndt}} &= \frac{K_t}{P_{ndt}} \frac{M_t}{B_t} \\ M_t &= \beta_u (1 - \rho) \Pi_{t+1}^{-1} \left(M_{t+1} + U_{nd,ut+1} \left(R_{t+1}^z - R_{t+1} + \varpi \right) \right) \\ B_t &= \beta_u (1 - \rho) \Pi_{t+1}^{-1} \left(L_{t+1} + \varpi U_{nd,ut+1} K_t K_{t+1}^{-1} \right) \end{aligned} \tag{4.2.44}$$

This system defines the dynamics of the profit maximising level of real debt that is issued by individual financial intermediaries $\left(\frac{Z_{t+1}}{P_{ndt}}\right)$, as a function of inflation (Π_{t+1}), collateral, K_t, the markup of the average rate on debt over the central bank rate ($R_{t+1}^z - R_{t+1}$), and the marginal utility of consumption of non-durables by patient households ($U_{nd,ut+1}$). Real debt evolution is affected by the discount rate of patient households β_u, the stickiness of debt contracts ρ, and the parameter ϖ (which captures the relative size of quadratic costs associated with deviating from the target level of debt).

Appendix (B.2.3) shows how this behaviour translates into the dynamics of aggregate debt. Since constrained households always take all debt offered to them, constrained debt a_{ct} as defined in equation (4.2.13) follows the same path, and we have our final real debt evolution equation:

$$a_{ct} = (1-\rho)\frac{a_{ct-1}}{\Pi_{ct}} + \rho\frac{M_t}{B_t}k_t \tag{4.2.45}$$

Rates on Debt

Each financial intermediary adjusts the quantity of loan it offers with probability ρ. At the same time, we assume that the proportion of fixed and variable rates offered is always the same as in the whole population. We can think of each individual borrower as holding a portfolio contract containing fixed and variable rates, with the proportion determined by exogeneous factors related to legal structure and convention. Later we will be able to show what proportion will be optimal for welfare.

We assume that competition in prices between financial intermediaries forces variable rates to adjust with the central bank rate each period. Fixed rates are fixed until each new renegotiation, at which point the new fixed rates are reconsidered simultaneously with the quantity of debt. The interest rate on fixed rate contracts is then determined by a no arbitrage condition where the financial intermediary is indifferent between lending at fixed or variable rates:

$$\sum_{s=t+1}^{\infty} Q_{t,s}R_t^{zF}(1-\rho)^{t-s}Z_{t+1} = \sum_{s=t+1}^{\infty} Q_{t,s}(1-\rho)^{t-s}R_sZ_{t+1}$$

Appendix (B.2.3) shows how this condition can be used to derive the dynamic system determining the new fixed rate R_t^{zF}:

$$V_t = \beta_u(1-\rho)\Pi_{t+1}^{-1}(V_{t+1} + U_{c,ut+1}R_{t+1}) \tag{4.2.46}$$

$$\mathcal{U}_t = \beta_u(1-\rho)\Pi_{t+1}^{-1}(\mathcal{U}_{t+1} + U_{c,ut+1}) \tag{4.2.47}$$

$$R_t^{zF} = \frac{V_t}{\mathcal{U}_t} \tag{4.2.48}$$

The average fixed rate then evolves as a simple weighted average of the ρ proportion of financial intermediaries who have had the chance to re-set the contract at time t, and the other $(1-\rho)$ who have not:

$$R_t^F = \rho R_t^{zF} + (1-\rho) R_{t-1}^F \tag{4.2.49}$$

So the average interest rate paid by constrained borrowers is:

$$R_t^D = (1-\Psi) R_t + \Psi R_t^F \tag{4.2.50}$$

4.2.6 Private Sector Equilibrium

A private sector rational expectations equilibrium consists of a plan for allocating the sequence for $\{X_{ut}, X_{ct}, N_{ndt}, N_{dt}, D_{ut}, D_{ct}, C_{ut}, C_{ct}, a_{ct}, \Pi_t, q_t, \Xi_t\}$ given the policy $\{R_t\}$, the exogenous productivity shock process $\{Z_{ndt}\}$ and appropriate initial conditions. The system describing the private sector equilibrium can be simplified to leave only dynamic equations and important definitions, as summarised in Table 4.2.1.

4.3 Monetary Policy

We apply Ramsey optimal policy. Assuming feasible pre-committment, a plan that maximises household welfare is delivered, subject to the economy's resource constraints, and consistent with private sector equilibrium. Here we have heterogeneous households with different time preference rates, and collateral constraints which can be relaxed by purchases and price rises of durable goods. The impatience of the borrowers in our model makes it impossible to jointly 'satisfy' both types of household across time. We assume that the policymaking authority puts a relative weight on patient and impatient household *intra-period* welfare, which in our baseline model is the same as the labour share of these households. Specifically, the following welfare

1 (eqn (4.2.2))	$X_{ut}=\left((1-\alpha)^{\frac{1}{\eta}}(C_{ut})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(D_{ut})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$
2 (eqn (4.2.3))	$X_{ct}=\left((1-\alpha)^{\frac{1}{\eta}}(C_{ct})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(D_{ct})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$
3 (eqn (4.2.8))	$X_{ut}^{\frac{1}{\eta}-\sigma}C_{ut}^{-\frac{1}{\eta}}q_t=\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\eta}}X_{ut}^{\frac{1}{\eta}-\sigma}D_{ut}^{-\frac{1}{\eta}}+\beta_u(1-\delta)E_t\left(X_{ut+1}^{\frac{1}{\eta}-\sigma}C_{ut+1}^{-\frac{1}{\eta}}q_{t+1}\right)$
4 (eqn (4.2.10))	$1=\beta_u E_t\left(\frac{X_{ut+1}^{\frac{1}{\eta}-\sigma}C_{ut+1}^{-\frac{1}{\eta}}}{X_{ut}^{\frac{1}{\eta}-\sigma}C_{ut}^{-\frac{1}{\eta}}}\frac{(1+R_t)}{\Pi_{t+1}}\right)$
5 (eqn (4.2.13))	$a_{ct}=\left(1+R_t^D\right)\left(\begin{array}{c}\frac{a_{ct-1}}{\Pi_t}+\nu\left(C_{ct}+q_t\left(D_{ct}-(1-\delta)D_{ct-1}\right)\right)\\-(1-\nu)\left(C_{ut}+q_t\left(D_{ut}-(1-\delta)D_{ut-1}\right)\right)\end{array}\right)$
6 (eqn (4.2.17))	$q_t\left(1-(1-\chi)\rho\frac{\mathcal{M}_t}{\mathcal{B}_t}\Xi_t\right)=\left(\frac{\alpha}{(1-\alpha)}\frac{C_{ct}}{D_{ct}}\right)^{\frac{1}{\eta}}$ $+\beta_c(1-\delta)E_t\left(\frac{X_{ct+1}^{\frac{1}{\eta}-\sigma}C_{ct+1}^{-\frac{1}{\eta}}}{X_{ct}^{\frac{1}{\eta}-\sigma}C_{ct}^{-\frac{1}{\eta}}}q_{t+1}\right)$
7 (eqn (4.2.19))	$\left(1+R_t^D\right)\Xi_t=1-\beta_c E_t\left(\frac{\left(1+R_t^D\right)}{\Pi_{t+1}}\frac{X_{ct+1}^{\frac{1}{\eta}-\sigma}C_{ct+1}^{-\frac{1}{\eta}}}{X_{ct}^{\frac{1}{\eta}-\sigma}C_{ct}^{-\frac{1}{\eta}}}\left(1-(1-\rho)\Xi_{t+1}\right)\right)$
8 (eqn (4.2.32))	$G_t=\frac{1}{Z_{ndt}}\frac{\mu\varkappa}{\nu^{\nu}(1-\nu)^{(1-\nu)}}C_{ut}^{\frac{\nu-1}{\eta}}X_{ut}^{\frac{(\sigma\eta-1)(\nu-1)}{\eta}}C_{ct}^{\frac{1}{\eta}(1-\nu)}X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}$ $\times\left(\frac{(C_{ct}+C_{ut})}{Z_{ndt}}\Delta_{pt}+\frac{D_{ut}-(1-\delta)D_{ut-1}+D_{ct}-(1-\delta)D_{ct-1}}{Z_{dt}}\right)^{\phi}(C_{ct}+C_{ut})$ $+\theta\beta\Pi_{ct+1}^{\epsilon}G_{t+1}$
9 (eqn (4.2.33))	$F_t=(1-\alpha)^{\frac{1}{\eta}}X_{ut}^{\frac{1}{\eta}-\sigma}C_{ut}^{-\frac{1}{\eta}}(C_{ct}+C_{ut})+\theta\beta\Pi_{ct+1}^{\epsilon-1}F_{t+1}$
10 (eqn (4.2.31))	$1-\theta\Pi_t^{\epsilon-1}=(1-\theta)\left(\frac{G_t}{F_t}\right)^{1-\epsilon}$
11 (eqn (4.2.34))	$\Delta_t=(1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}+\theta\Pi_t^{\epsilon}\Delta_{t-1}$
12 (eqn (4.2.41))	$q_t=\frac{1}{Z_{dt}}\frac{\varkappa\mu}{\nu^{\nu}(1-\nu)^{(1-\nu)}(1-\alpha)^{\frac{1}{\eta}}}C_{ct}^{\frac{1}{\eta}(1-\nu)}X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}C_{ut}^{\frac{1}{\eta}\nu}X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu}$ $\times\left(\frac{(C_{ct}+C_{ut})}{Z_{ndt}}\Delta_{pt}+\frac{D_{ut}-(1-\delta)D_{ut-1}+D_{ct}-(1-\delta)D_{ct-1}}{Z_{dt}}\right)^{\phi}$
13 (eqn (4.2.45))	$a_{ct}=(1-\rho)\frac{a_{ct-1}}{\Pi_{ct}}+\rho\frac{M_t}{B_t}(1-\chi)q_tD_{ct}$
14 (eqn (4.2.44))	$(1+R_t)M_t=(1-\rho)E_t\left(M_{t+1}+\varpi+R_{t+1}^z-R_{t+1}\right)$
15 (eqn (4.2.44))	$(1+R_t)B_t=(1-\rho)E_t\left(\frac{(B_{t+1}+\varpi)}{\Pi_{t+1}}\frac{q_tD_{ct}}{q_{t+1}D_{ct+1}}\right)$
16(eqn (4.2.46))	$(1+R_t)V_t=(1-\rho)E_t(V_{t+1}+R_{t+1})$
17 (eqn (4.2.47))	$(1+R_t)\mathcal{U}_t=(1-\rho)E_t(\mathcal{U}_{t+1}+1)$
18 (eqn (4.2.48))	$R_t^{zF}=\frac{V_t}{\mathcal{U}_t}$
19 (eqn (4.2.43))	$R_t^z=(1-\Psi)R_t+\Psi R_t^{zF}$
20 (eqn (4.2.49))	$R_t^F=\rho R_t^{zF}+(1-\rho)R_{t-1}^F$
21 (eqn (4.2.50))	$R_t^D=(1-\Psi)R_t+\Psi R_t^F$

Table 4.2.1: Summary of dynamic system

objective is minimised:

$$\min_{\{R_t\}_{t\geq 0}} \mathcal{W} = \frac{1}{2}\sum_{t=0}^{\infty} \left(\beta_u^{\nu}\beta_c^{1-\nu}\right)^t \left(\nu\left(\frac{X_u^{1-\sigma}}{1-\sigma} - \varkappa\frac{N_u^{1+\phi}}{1+\phi}\right) + (1-\nu)\left(\frac{X_c^{1-\sigma}}{1-\sigma} - \varkappa\frac{N_c^{1+\phi}}{1+\phi}\right)\right) \tag{4.3.1}$$

subject to constraints summarised in Table 4.2.1, (4.2.2), (4.2.3), (4.2.8)), (4.2.10), (4.2.13), (4.2.17), (4.2.19), (4.2.32), (4.2.33), (4.2.31), (4.2.34), (4.2.41), (4.2.45), (4.2.44), (4.2.44), (4.2.46), (4.2.47), (4.2.48), (4.2.43), (4.2.49), (4.2.50) and (4.2.50).

Note that unless the policymaker discounts intra-period welfare of both types using the same discount factor, consumption of those households whose welfare is more heaviliy discounted rises unboundedly, since the relative share of them diminishes in the policymaker's objective. Our setup guarantees a unique, well defined steady state under Ramsey policy (see (Becker 1980) and (Becker & Foias 1987)).

4.4 Steady State

Table (4.4) shows the recursive system to compute the steady state, following from the system in Table 4.2.1 and given the steady state level of inflation Π. All interest rates in the steady state are equal to R, and optimal policy then determines the steady state rate of inflation.

Equations 1 and 12 from the table can be combined to obtain:

$$\Xi = \frac{\beta_u - \beta_c}{\Pi - \beta_c(1-\rho)}$$

which shows that the borrowing constraint Ξ does not bind without heterogeneity of time preference, as in (Monacelli 2007). The reverse is also true: only a steady state in which there is a borrowing constraint can be consistent with our economy with households with different discount rates.

Combining equations 4 and 12 from the steady state system gives:

$$\frac{C_c}{D_c} = \frac{(1-\alpha)}{\alpha}\left((1-\beta_c(1-\delta)) - \Xi\frac{(1+R)\rho(1-\chi)(\beta_u-\beta_c)}{(\Pi-\beta_c(1+R))}\frac{(\Pi^2-\beta_u(1-\rho))}{(\Pi-\beta_u(1-\rho))}\right)^{\eta} q^{\eta}$$

which shows that the steady state relative demand for durables is increasing in Ξ, consistent with the idea that households will want to hold more durables, the higher is the shadow value of the collateral constraint.

Equations 13-15 from the Table show that in the steady state, constrained households will be more highly leveraged, defined by a high ratio of debt to durable goods owned, the higher is the loan to income ratio $(1-\chi)$:

$$\frac{a_c}{D_c} = (1-\chi)\, q \frac{\Pi\rho(\Pi^2-\beta_u(1-\rho))}{(\Pi-(1-\rho))(\Pi-\beta_u(1-\rho))}$$

The presence of the banking sector with overlapping debt contracts affects steady state allocations of consumption. In our analogue of Monacelli's model $\rho = 1$. Relative to that case, with $\rho < 1$, the collateral constraint binds less in the steady state, and X_u, X_c, and N_c go down, while N_u goes up.

4.5 Calibration

We take the frequency of the model as quarterly, and so most of the parameters we take from (Monacelli 2007) and (Monacelli 2009). The individual discount factor for patient households is set to 0.99, consistent with an annual steady state real rate of interest of 4%, whilst impatient households' discount rate is 0.98. Parameters of household utility are set as in Monacelli, with the elasticities of intertemporal substitution of consumption and labour equal to one. We do allow some substitutability between non-durable and durable goods however, by setting $\eta = 1.5$. In the non-durable goods sector, the stickiness of price contracts is determined by the Calvo parameter θ, which we set to 0.75, implying an average contract length of one year. The elasticity

1	$R = \frac{\Pi}{\beta_u} - 1$
2	$\Delta = \frac{(1-\theta)}{\left(1-\theta\Pi^{\epsilon}\right)} \left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}$
3	$q = \frac{(1-\theta\beta_u\Pi^{\varepsilon})}{(1-\theta\beta_u\Pi^{\varepsilon-1})} \left(\frac{(1-\theta)}{(1-\theta\Pi^{\epsilon-1})}\right)^{\frac{1}{\epsilon-1}}$
4	$\frac{C_c}{D_c} = \frac{(1-\alpha)}{\alpha} \left((1-\beta_c(1-\delta)) - \frac{\rho(1-\chi)(\beta_u-\beta_c)}{(\Pi-\beta_c(1-\rho))} \frac{\left(\Pi^2-\beta_u(1-\rho)\right)}{(\Pi-\beta_u(1-\rho))}\right)^{\eta} q^{\eta}$
5	$\frac{C_u}{D_u} = \frac{(1-\alpha)}{\alpha} \left(1-\beta_u(1-\delta)\right)^{\eta} q^{\eta}$
6	$\frac{X_u}{D_u} = \left((1-\alpha)^{\frac{1}{\eta}} (\frac{C_u}{D_u})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$
7	$\frac{X_c}{D_c} = \left[(1-\alpha)^{\frac{1}{\eta}} (\frac{C_c}{D_c})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$
8	$\frac{D_u}{D_c} = \frac{\nu\left(\frac{(1-\alpha)}{\alpha}\left((1-\beta_c(1-\delta))-\frac{\rho(1-\chi)(\beta_u-\beta_c)}{(\Pi-\beta_c(1-\rho))}\frac{\left(\Pi^2-\beta_u(1-\rho)\right)}{(\Pi-\beta_u(1-\rho))}\right)^{\eta} q^{\eta-1}+\delta\right)+\frac{(1-\beta_u)\rho\left(\Pi^2-\beta_u(1-\rho)\right)(1-\chi)}{(\Pi-\beta_u(1-\rho))(\Pi-(1-\rho))}}{(1-\nu)\left(\frac{(1-\alpha)}{\alpha}(1-\beta_u(1-\delta))^{\eta} q^{\eta-1}+\delta\right)}$
9	$D_c = \left(\begin{array}{c} \frac{\nu^{\nu}(1-\nu)^{(1-\nu)}(1-\alpha)^{\frac{1}{\eta}}}{\mu\varkappa} \frac{(1-\theta\beta_u\Pi^{\varepsilon})}{(1-\theta\beta_u\Pi^{\varepsilon-1})} \left(\frac{(1-\theta)}{1-\theta\Pi^{\epsilon-1}}\right)^{\frac{1}{\epsilon-1}} \left(\frac{C_u}{D_u}\right)^{-\frac{\nu}{\eta}} \left(\frac{X_u}{D_u}\right)^{-\nu\left(\sigma-\frac{1}{\eta}\right)} \\ \times \left(\frac{C_c}{D_c}\right)^{-\frac{(1-\nu)}{\eta}} \left(\frac{X_c}{D_c}\right)^{-\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} \left(\Delta\frac{C_c}{D_c}+\delta+\left(\Delta\frac{C_u}{D_u}+\delta\right)\frac{D_u}{D_c}\right)^{-\phi} \left(\frac{D_u}{D_c}\right)^{-\nu\sigma} \end{array}\right)^{\frac{1}{\sigma+\phi}}$
10	$G = \frac{\mu\varkappa C_u^{\frac{\nu-1}{\eta}} X_u^{\frac{(\sigma\eta-1)(\nu-1)}{\eta}} C_c^{\frac{1}{\eta}(1-\nu)} X_c^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} ((C_c+C_u)\Delta+\delta D_u+\delta D_c)^{\phi}(C_c+C_u)}{\nu^{\nu}(1-\nu)^{(1-\nu)}(1-\theta\beta_u\Pi^{\varepsilon})}$
11	$F = \frac{(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}} (C_c+C_u)}{\left(1-\theta\beta_u\Pi_c^{\varepsilon-1}\right)}$
12	$\Xi = \frac{(\Pi-\beta_c(1+R))}{(1+R)(\Pi-\beta_c(1-\rho))}$
13	$M = \frac{\beta_u(1-\rho)(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}} \varpi}{(\Pi-\beta_u(1-\rho))}$
14	$B = \frac{\beta_u(1-\rho)\varpi(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}}}{(\Pi^2-\beta_u(1-\rho))}$
15	$a_c = \rho\frac{M\Pi(1-\chi)qD_c}{B(\Pi-(1-\rho))}$
16	$U = \frac{\beta_u(1-\rho)(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}}}{(\Pi-\beta_u(1-\rho))}$
17	$V = UR$

of substitution between different assortments of non-durable goods is 6, which works out as a mark-up of around 20%.

The parameter χ determines the down payment ratio, which we set to 0.25 (following Monacelli). The deprecation rate δ for durables is set at 0.01 per quarter, which represents approximately a 4% deprecation in housing per year.

We set the parameter $\nu = 0.5$ so that firms demand 50% of labour from patient agents and 50% of labour from the constrained households. We set α to 0.16, which means that the consumption of durables constitutes 16% of overall consumption, and non-durables constitutes 84%.

The interpretation of parameter ϖ is difficult, since it determines how large a financial institution's costs are when deviating from the target level of debt. We assumed quadratic adjustment costs, as in (Graham & Wright 2007) for tractability, but the costs associated with endogenous default risk and missed profit opportunities relative to the size of the loan are hard to interpret, and indeed may not be symmetrical. We chose a value of 3 in our baseline calibration and focus our sensitivity analysis on ρ, the probability of readjustment, to see how the renegotiability of debt contracts affects results. For policy objectives, we calibrate $a = \nu$, so the social planner uses the weights of the different types of household based on their labour share in the economy.

We study the propagation mechanism of an iid productivity shock in the non-durable sector. We set the standard deviation of the shock to 0.1, implying a standard deviation of inflation of around 0.5%, consistent with data in the UK.

4.6 Results

4.6.1 Variable Rates

We study the propagation of a productivity shock in the non-durable goods sector by introducing an iid exogenous process to $Z_{nd,t}$. Examining the economy's responses to a simple one period shock then allows us to focus attention on the nature of the transmission mechanism, and how it differs between economies with predominantly variable rate contracts, and those with mainly fixed.

We first examine the effect of the presence of overlapping debt contracts. In a baseline case, we plot impulse responses to non-durable (ND) productivity shocks in an economy where the banking sector plays a very simple role, borrowing from patient households and lending to impatient households under the collateral of durable goods. This setup is very similar to the one in (Monacelli 2007) and corresponds to the case $\rho = 1$ and $\Psi = 0$. All contracts are reset every period, borrowing is determined by the value of collateral, and all rates are variable, so we denote it as the 'no banking sector' case, as shown in Figure 4.7.1. We compare results to those in our economy where we assume that 95% of borrowers are on variable rate contracts ($\Psi = 0.05$), and that each contract is reconsidered every 2.5 years on average ($\rho = 0.1$). Although variable rates adjust automatically with the central bank rate, the stickiness of debt contracts restricts financial institutions' decisions on the quantity of debt to issue, which also depends on the amount of collateral of durable goods owned by constrained households. Although households are able to use collateral to expand consumption, they cannot smooth consumption as much as they would like.

Following a positive ND productivity shock real marginal cost in the ND sector falls, and the maximisation of profit by monopolistically competitive firms implies a fall in price and thus in ND inflation. The improvement in productivity allows firms to employ less workers but still produce the same level of output (output will be even higher because of increased demand). For those workers that remain in the ND sector, wages go up, and because of perfect labour mobility between the two sectors, this wage increase spills over into the durable sector. Here, there is no increase in productivity, so employment falls in the durable sector as well. The rise in nominal wages in the durable sector increases firms' costs of production, which in turn causes the nominal price of durable goods to increase. This effect, combined with the fall in the price of non-durables, pushes the relative price of durables (q) up.

The response of monetary authorities to this productivity shock is to initially *increase* nominal interest rates, although by a very small amount (the increase is much more pronounced in the model with 'no banking sector'). Monetary policy only slightly changes the interest rate in the first moment, which together with expectations of high inflation in the future (when the productivity shock will disappear), allows the real rate to be negative in the first period.

The negative real interest rate is inversely related to the change in the marginal utility of consumption for unconstrained households, since a lower real interest rate requires marginal utility to fall in the future. The initial increase in the consumption of non-durable goods, therefore, is explained by the increase in real wages.

With durables relatively expensive, constrained households are then able to expand debt levels to finance purchases. They also experience an initial increase in their real wage, allowing an increase in consumption of both goods, but mainly non-durable as they are relatively cheap. Also, with the stickiness of debt contracts, compared to the Monacelli model, there are only limited possibilities to use durable goods as collateral. In this sense, the financial accelerator mechanism that is fully operational in the Monacelli model, and switched off in the G&W model, is 'partly switched on' here. It is particularly apparent from Figure 4.7.1 that in the model with 'no banking sector' the constrained households try to accumulate far more durable goods than in the model with overlapping debt contracts.

The initial rise in the interest rate in the model with 'no banking sector' essentially prevents high borrowing. This is not needed in the model with overlapping debt contracts as debt is relatively small (although more persistent), because of the partly switched-on financial accelerator.

In the second period following the shock, the initial gain in productivity is lost, and marginal cost in the non-durable sector recovers to near previous levels. An increase in marginal cost then leads to an increase in the price of the non-durable goods, and real wages go down. The second period nominal interest rate level is well below the steady state, in order to drive the real rate down even further. The low nominal interest rate and high inflation rate at this stage makes borrowing relatively cheap, so the amount of debt stays relatively high. The financial accelerator mechanism becomes more apparent, as constrained households buy more durable goods, which can be used as collateral for more borrowing. As patient consumers do not have the same collateral constraint, they choose to reduce their consumption of the overpriced durables significantly, but continue consuming non-durable goods.

This optimal monetary policy response ensures that constrained agents do not face high deviations in their consumption path: the initial drop in interest rates keeps lending and therefore the demand for durables high. The relatively high price of the durable goods allows borrowers to secure against them, and non-durable consumption for the constrained agent remains above the steady state.

Moving onto later periods, as the nominal interest rate returns closer to the steady state, real interest rates also begin to increase. Combined with the trend back towards the steady states for non-durable and durable consumption, this causes a slow reduction in levels of debt accrued by the constrained households. Nominal interest rates substantially overshoot the steady state level after several periods in order to stabilise inflation. As the future interest rate is expected to rise, the rate on fixed rate contracts rises immediately. However, this has practically no effect on the economy, since the majority of constrained households are on variable rates.

4.6.2 Fixed Rates

We now change the proportion of households that are on fixed rate contracts, to see how responses differ from predominately variable rate economies. Figures 4.7.2-4.7.5 demonstrate impulse responses of key variables, with the proportion of constrained households on fixed rate contracts at 5%, 35%, 65% and 95%. In general, the transmission mechanism of the non-durable productivity shock is similar to the one discussed above, except for the timing and strength of responses: with higher Ψ, the response is more sluggish and less pronounced.

The first thing to notice in Figure 4.7.2 is that the amplitude of the drop in policy rate becomes considerably smaller as we move to a fixed rate economy. In anticipation of inflation-fighting hikes in the central bank rate, fixed rates rise early, since they are linked directly to the expected path of short rates. As a result, the average rate that constrained households pay on their debt is considerably higher for the fixed rate economy, and will remain high for a long time. With higher Ψ the average rate R^D increases following R^{zF} and it determines the flow of interest payments and the tightness of the budget constraint. The rate also determines the tilt of consumption towards future consumption. Figure 4.7.3 demonstrates a substantial delay in the adjustment of consumption.

With higher Ψ the average interest rate R^D increases in initial periods and so does the tightness of the collateral constraint. A tighter collateral constraint leads to less consumption of durable goods by the borrowers, whilst the lenders reallocate their consumption towards durables, as their relative price does not rise as much as for small Ψ.

4.6.3 Frequency of Resetting

For an intermediate $\Psi = 0.5$, we demonstrate how longer debt contracts moderate the financial accelerator mechanism even further. Figure 4.7.6 demonstrates that longer debt contracts imply smoother adjustment of durable goods stock accumulated by the borrowers; reduce the level of debt; and require higher interest rates in future periods. In this way, the effect of longer contracts is the same as the effect of a higher proportion of fixed rates. However, unlike Ψ in this model the value of ρ affects steady state values as discussed above, so the welfare implications of changes in ρ may not be unambiguous.

4.6.4 Welfare

The system under optimal control (under Ramsey policy) can be linearised.[6] We are able to compute the second-order approximation of social welfare *along the optimal solution* (see Appendix B.1). As the steady state in this model does not depend on Ψ, we only look at the measure of welfare that is based on the measure of volatility of variables. We present these volatilities, as well as the welfare measure in Figure 4.7.7 (for any variable x_t we plot $E_0 \sum_{t=0}^{\infty} \omega^t \left(x_t x_t'\right))$, based on the iid ND productivity shocks.

Figure 4.7.7 demonstrates that the minimum volatility (and thus the maximum of the *stochastic component* of social welfare) is achieved in an intermediate point, for Ψ close to 0.5. Indeed, if Ψ rises then adjustment becomes slower. This immediately implies higher welfare losses with higher Ψ. However, higher Ψ also moderates the amplitude of immediate adjustment, which carries most of the weight in the discounted flow of losses, so losses may also fall with Ψ. The minimum of losses is achieved when

[6] We solve the model using Dynare's toolkit for Ramsey policy. The output produced by Dynare is the linearised model under the optimal policy. The optimal policy itself is given as a linear rule that includes feedback on Lagrange multipliers.

the first effect outweighs the second. For our calibration of the model the higher ρ (longer debt contracts) amplifies the first effect more than the second, and the loss rises with ρ. The optimal Ψ in this model negatively depends on ρ.

The deterministic component of welfare, however, falls with longer debt contracts. As the financial multiplier is tempered, the households are not able to smooth consumption as they would like.

4.7 Conclusion

The aim of this chapter was to integrate a quantity-optimising banking sector that lends under either fixed or variable rates, with a model with borrowing-constrained households. We have provided a framework that can be used to investigate the relationships between the structure of debt contracts and monetary policy. In particular we focused our study on the propagation of a productivity shock in the non-durable sector under Ramsey monetary policy, and have showed that the introduction of overlapping debt contracts tempers the effect of the financial multiplier. Although the implied steady state allocation of resources unambiguously reduces the deterministic component of social welfare, an appropriate design of debt contracts, their length and the interest rate composition, can reduce volatility of the key economic variables, and banks can hence play a stabilising role in the economy. In particular, we demonstrate that an intermediate ratio of fixed and variable rate debt contracts is socially optimal.

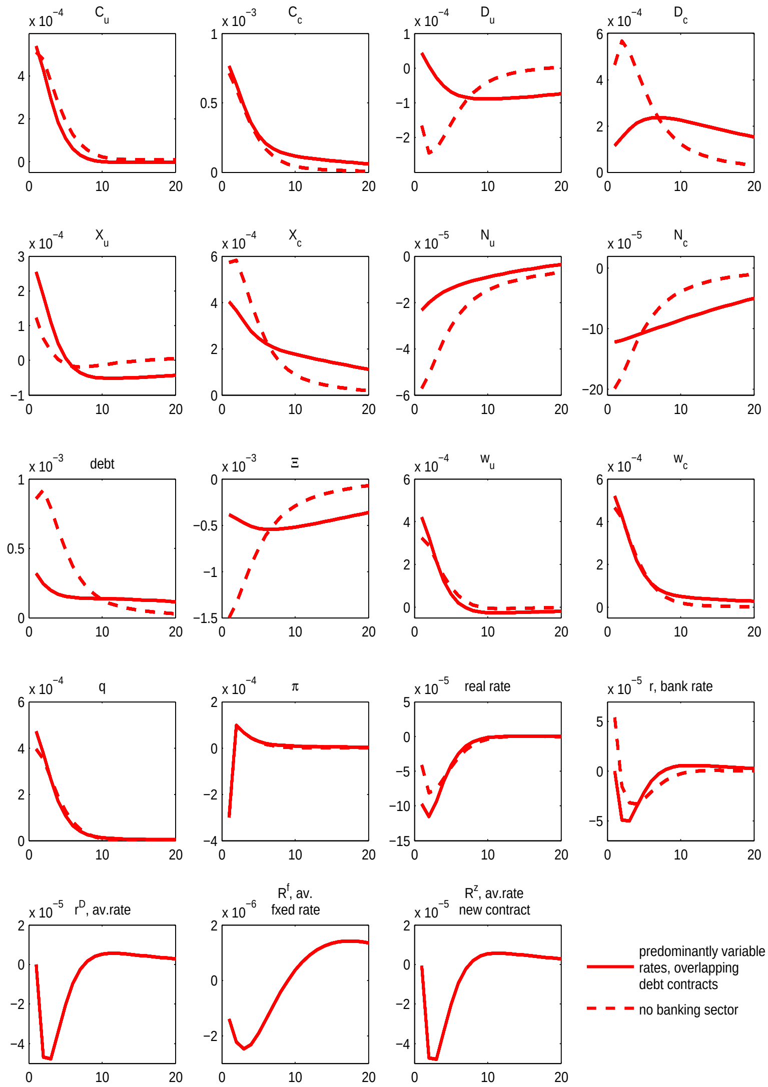


Figure 4.7.1: Impulse responses to a productivity shock in ND sector with s.d. 0.005. In the model with banking sector and overlapping debt contracts 95% of impatient households are on variable rate contracts ($\Psi = 0.05$).

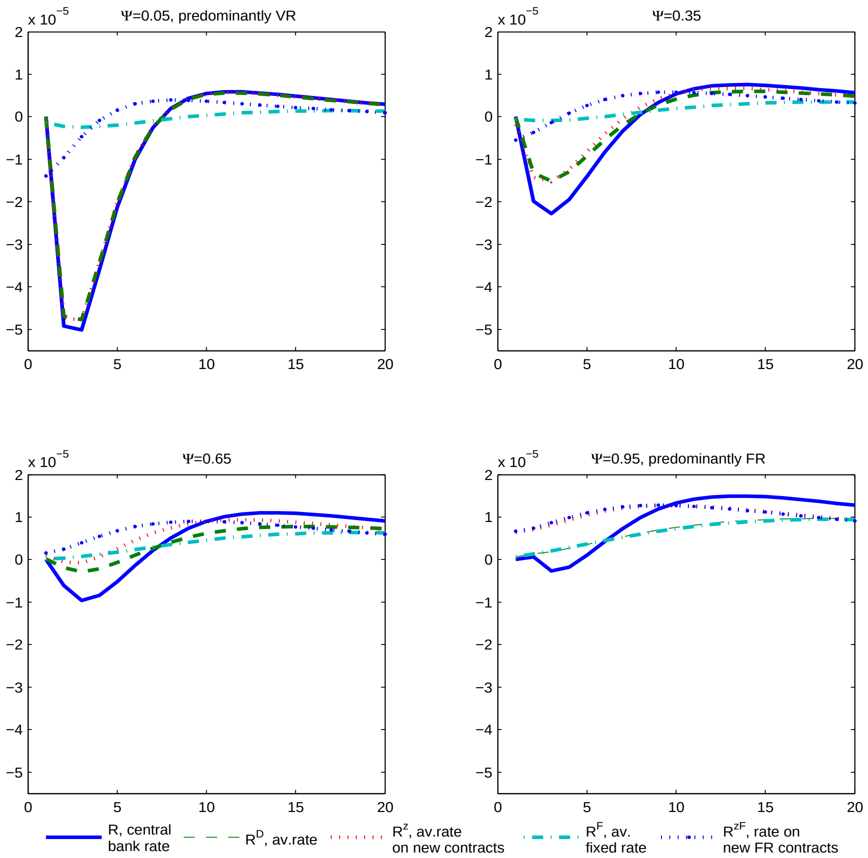


Figure 4.7.2: Impulse responses of interest rates to a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

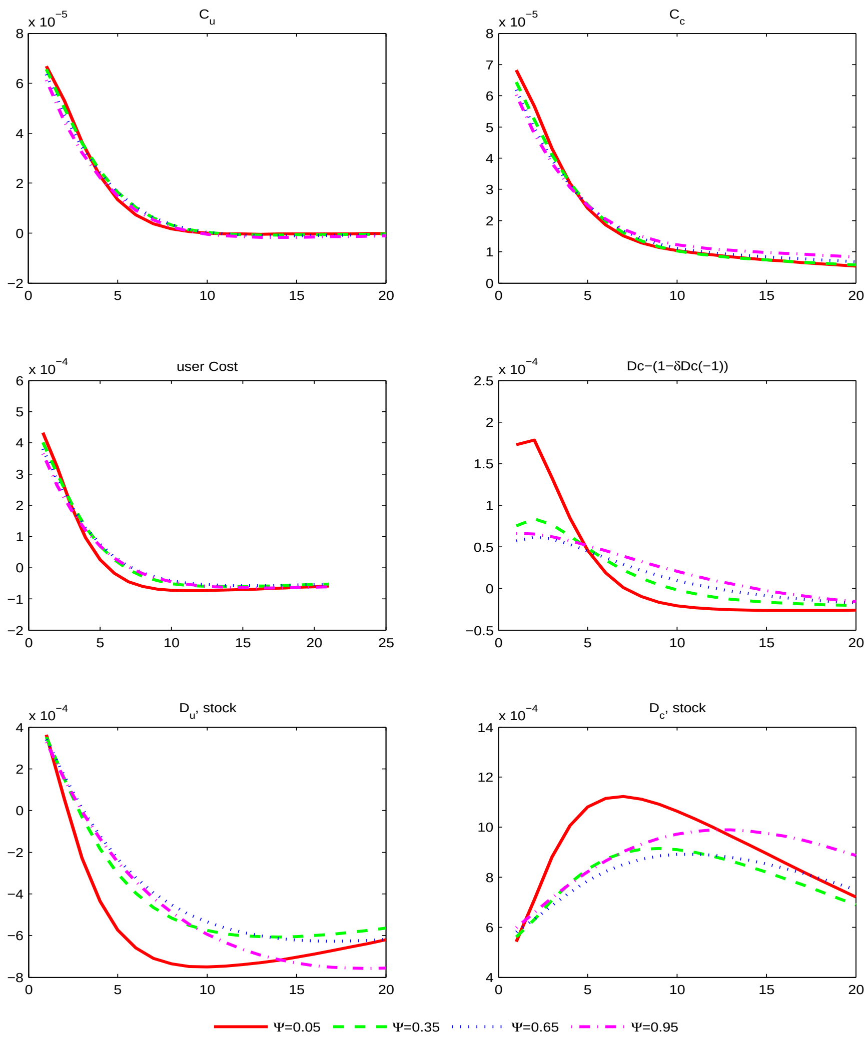


Figure 4.7.3: Impulse responses of consumption to a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

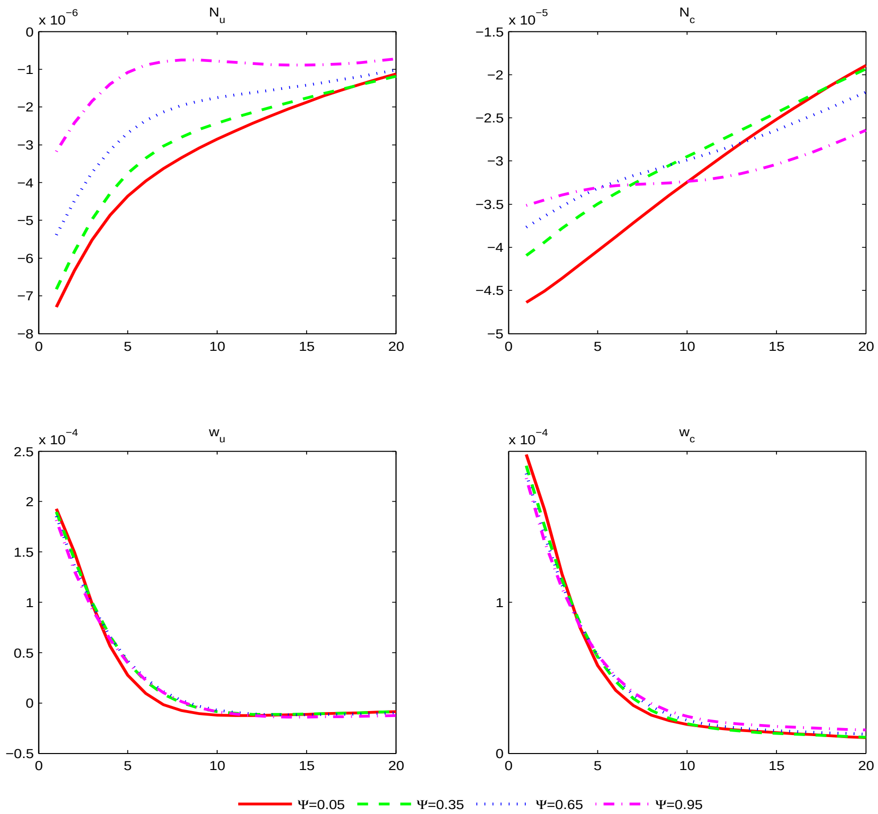


Figure 4.7.4: Impulse responses of labour supply and real wages to a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

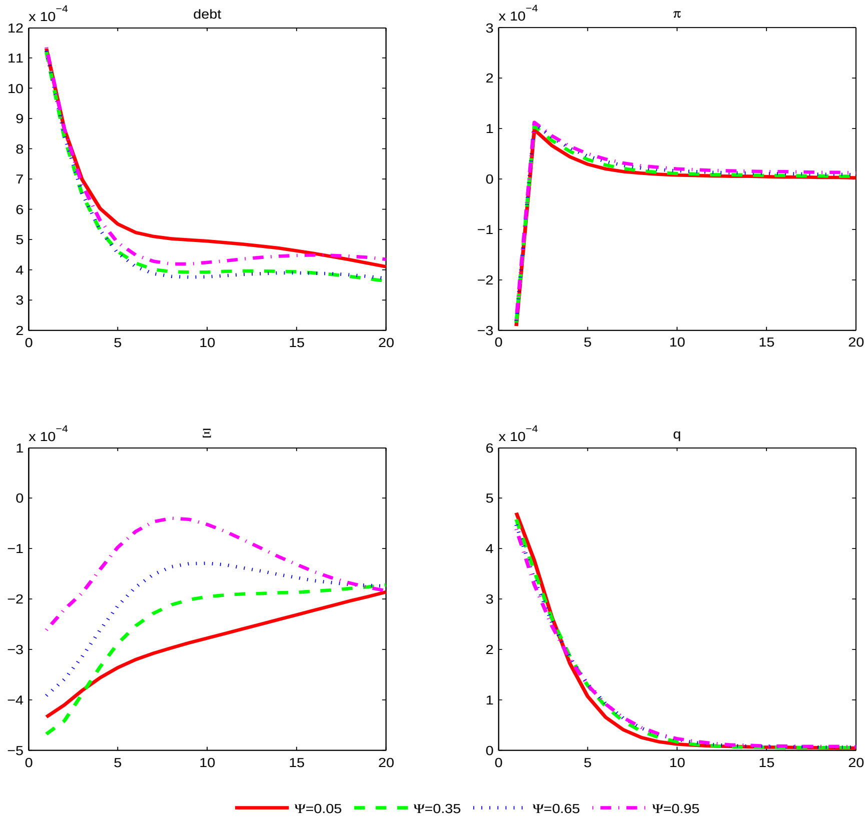


Figure 4.7.5: Impulse responses of debt, inflation, relative price, and the strength of collateral constraint to a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

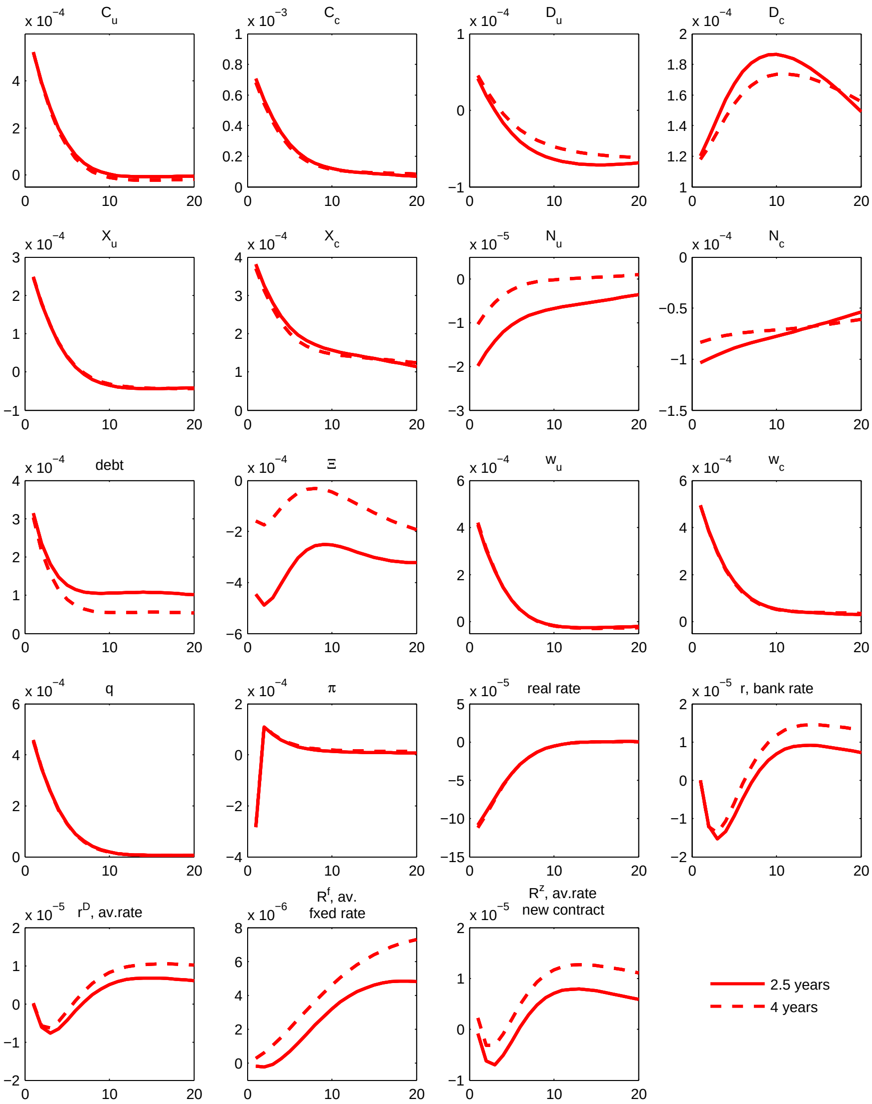


Figure 4.7.6: Impulse responses to an iid ND productivity shock with s.d. 0.005 for different average lengths of debt contracts.

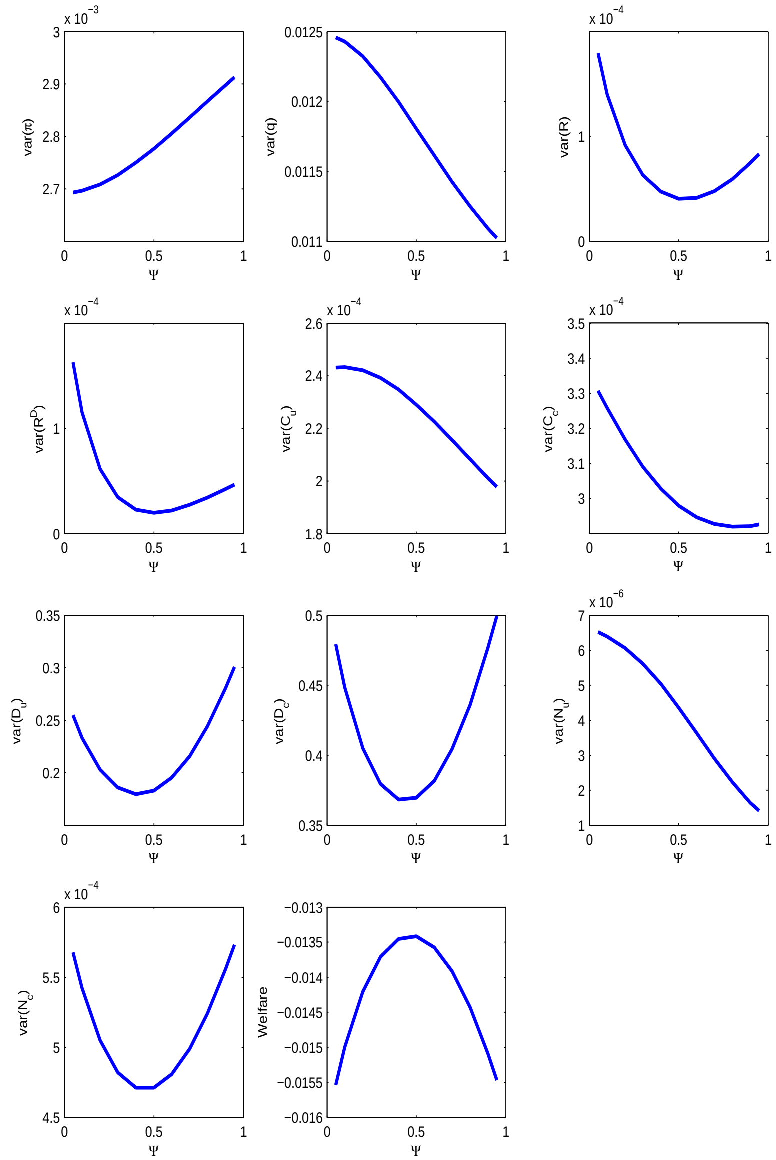


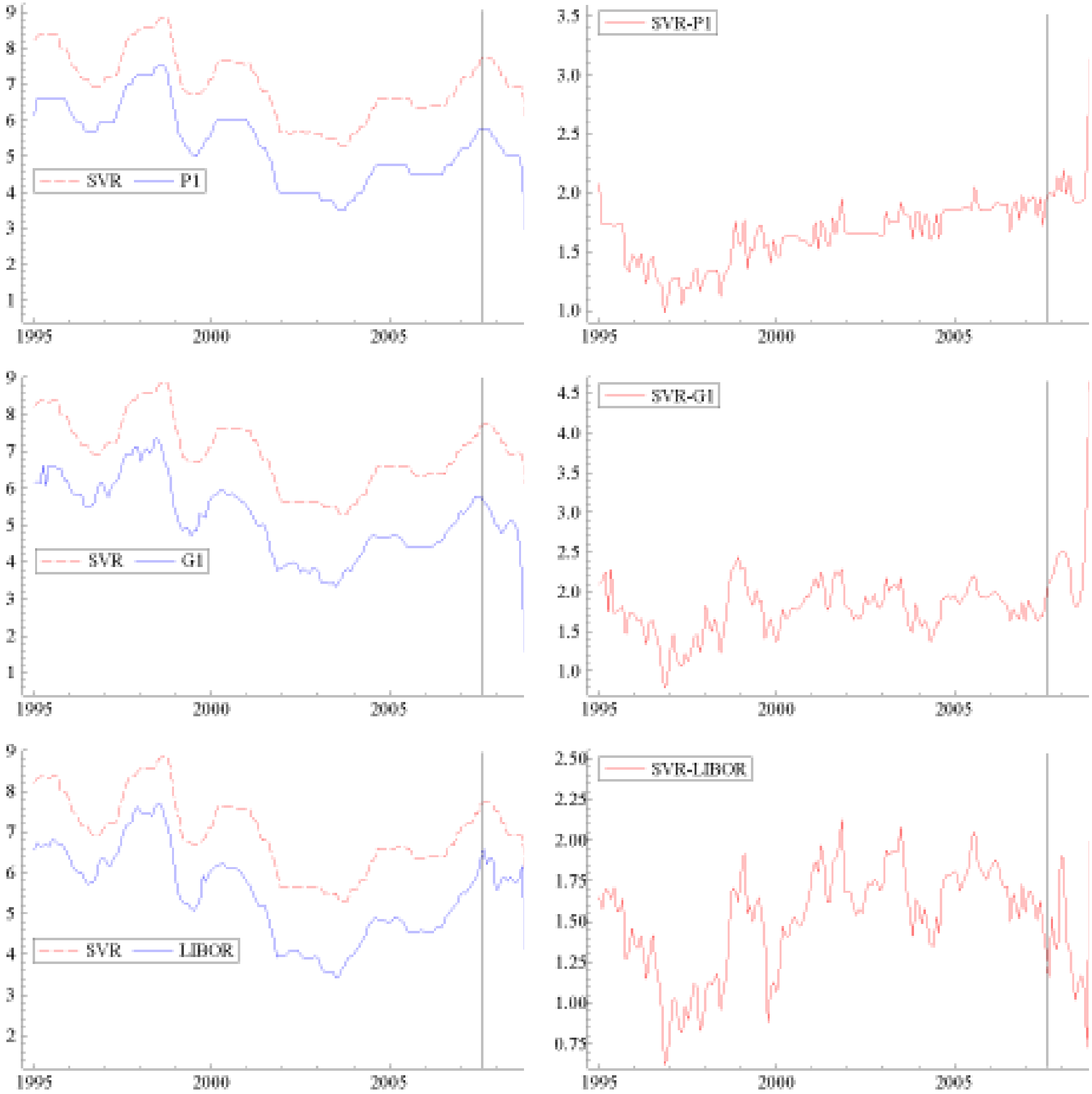
Figure 4.7.7: Variability of key variables as a function of Ψ. Productivity shock in ND sector with s.d. 0.1.

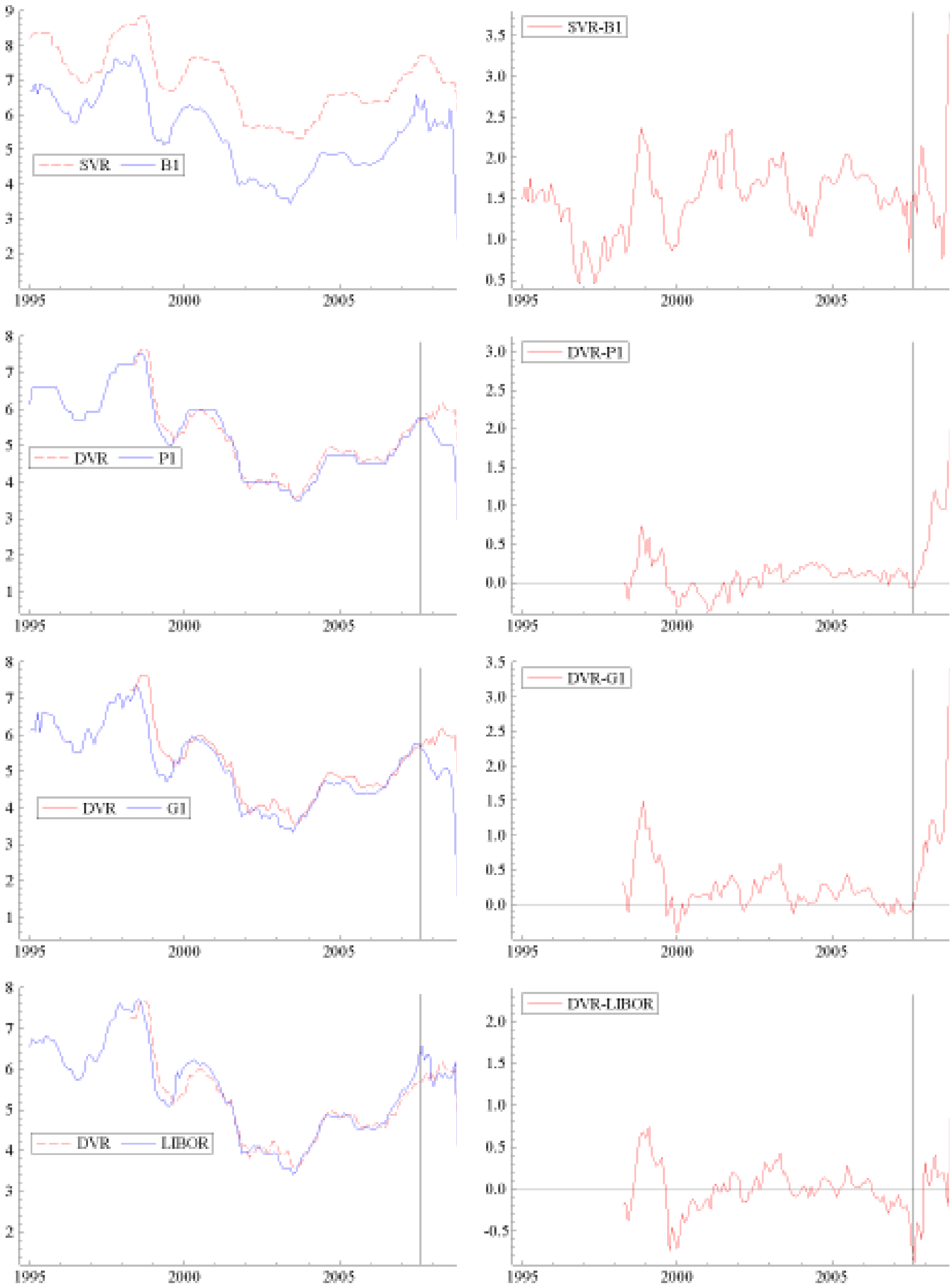
Appendix A

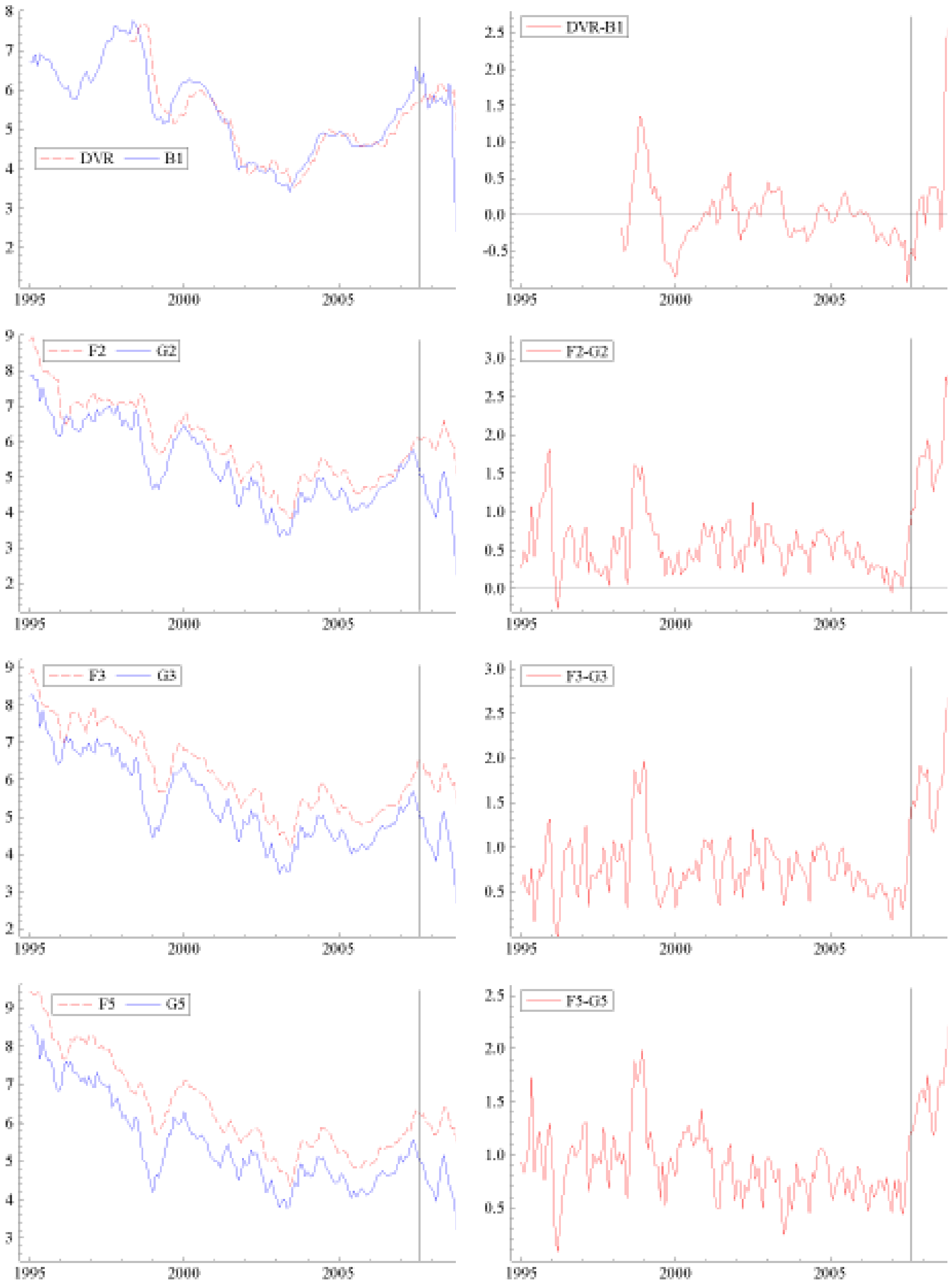
Monetary Transmission to UK Retail Rates

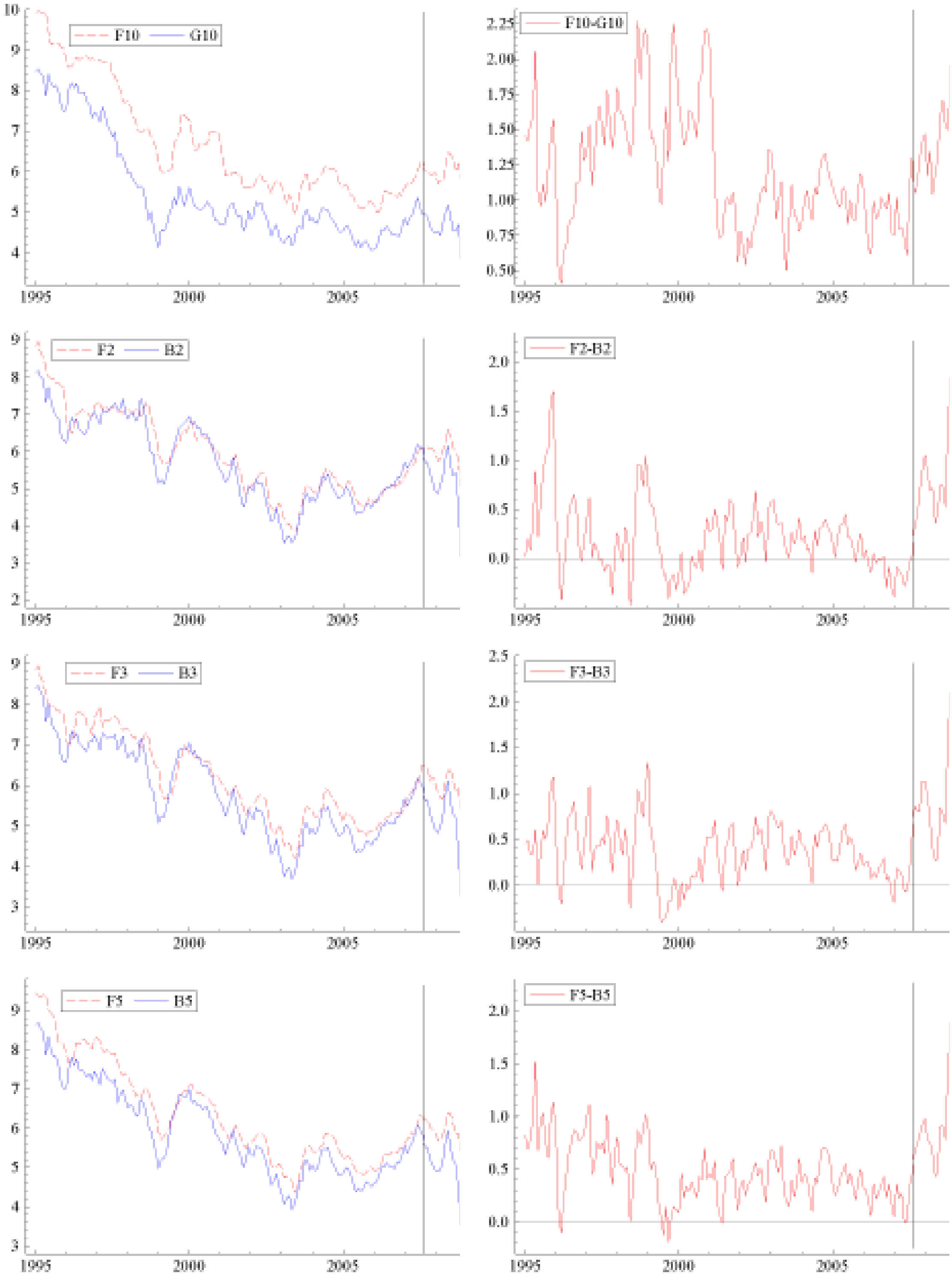
A.1 Interest Rate Graphs

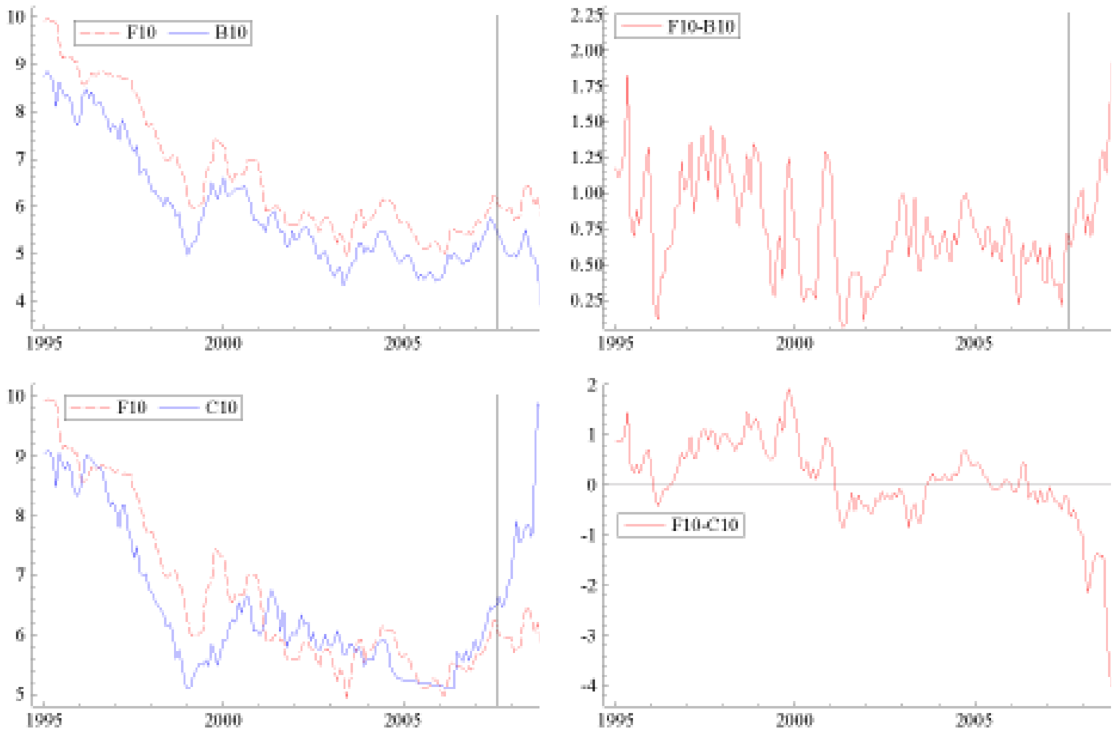
This appendix presents graphs of the 17 pairwise modelled relationships. For all graphs, the x-axis is time during the sample period from January 1995 to May 2009. For each pair, the graph on the left shows the interest rate levels throughout the sample period, and the graph on the right shows their difference (the interest rate spread). The line indicates the crisis period, August 2007. For a summary of variable abbreviations, see Table 2.2.1.


SVR
P1
SVR-P1
SVR
G1
SVR-G1
SVR
LIBOR
SVR-LIBOR
1995
2000
2005


SVR
B1
SVR-B1
DVR
P1
DVR-P1
DVR
G1
DVR-G1
DVR
LIBOR
DVR-LIBOR
1995
2000
2005


DVR
B1
DVR-B1
F2
G2
F2-G2
F3
G3
F3-G3
F5
G5
F5-G5
1995
2000
2005


F10
G10
F10-G10
F2
B2
F2-B2
F3
B3
F3-B3
F5
B5
F5-B5



A.2 Unit Root Tests

This appendix presents four tests, two of which assume the null hypothesis of a stationary, I(0) process (the KPSS and Lo's RS tests), and two assume the null of a unit-root, I(1) process (the ADF and P-P tests). The (unknown) assumptions about the true data-generating process vary according to test, and so ultimately does the (unknown) trade-off between low-power (under-rejection) and size-distortion (over-rejection), so the results should be treated only as indicative. Table A.2.1 shows that all KPSS tests for stationarity are firmly rejected, whilst 10 of the 17 Lo's RS tests are not rejected. None of the tests for non-stationarity are rejected. For the pre-crisis period, there is insufficient evidence to reject any of the tests, other than for the fixed mortgage rate de-trended data.

Table A.2.1: Stationarity Tests (Whole Sample)

Rate	Ho: I(0) (stationary)		Ho: I(1) (non-stationary)	
	KPSS[1]	Lo's RS[2]	ADF[3]	P-P[4]
SVR	0.23 (<0.01)	1.48 (<0.2)	-2.6 (<0.9)	-1.7 (<0.9)
DVR	0.39 (<0.01)	1.75 (<0.05)	-2.5 (<0.9)	-2.0 (<0.9)
LIBOR	0.24 (<0.01)	1.50 (<0.2)	-2.0 (<0.9)	-1.5 (<0.9)
G1	0.15 (<0.05)	1.32 (<0.4)	-2.5 (<0.9)	-1.5 (<0.9)
P1	0.16 (<0.05)	1.24 (<0.5)	-2.6 (<0.9)	-1.4 (<0.9)
B1	0.20 (<0.025)	1.32 (<0.4)	-1.8 (<0.9)	-1.5 (<0.9)
F2	0.44 (<0.01)	1.95 (<0.025)	-2.9 (<0.9)	-2.6 (<0.9)
F3	0.45 (<0.01)	2.06 (<0.025)	-2.7 (<0.9)	-2.6 (<0.9)
F5	0.52 (<0.01)	2.19 (<0.005)	-2.7 (<0.9)	-2.4 (<0.9)
F10	0.58 (<0.01)	2.33 (<0.005)	-2.2 (<0.9)	-2.1 (<0.9)
G2	0.18 (<0.025)	1.31 (<0.4)	-3.1 (<0.1)	-2.3 (<0.9)
G3	0.23 (<0.01)	1.49 (<0.2)	-3.4 (<0.1)	-2.7 (<0.9)
G5	0.35 (<0.01)	1.90 (<0.025)	-2.9 (<0.9)	-2.8 (<0.9)
G10	0.52 (<0.01)	2.27 (<0.005)	-2.3 (<0.9)	-2.3 (<0.9)
B2	0.21 (<0.025)	1.43 (<0.3)	-3.0 (<0.9)	-2.3 (<0.9)
B3	0.22 (<0.01)	1.44 (<0.3)	-3.3 (<0.1)	-2.7 (<0.9)
B5	0.29 (<0.01)	1.68 (<0.1)	-3.2 (<0.1)	-2.9 (<0.9)
B10	0.43 (<0.01)	2.19 (<0.005)	-2.5 (<0.9)	-2.5 (<0.9)
C10	0.49 (<0.01)	2.27 (<0.005)	-0.8 (<0.975)	-0.9 (<0.975)

1. The Kwiatkowski et al. (1992) test of I(0), against I(1)

2.Lo's (1991) R/S test for I(0) against I(d), for d > 0 or d < 0.

3. The augmented Dickey-Fuller test of I(1) against I(0). The number of lags is chosen to optimize the Schwarz information criterion over the range 0 to [T^1/3].

4. Phillips-Perron (1988) test of I(1) against I(0).

Table A.2.2: Stationarity Tests (Pre-Crisis)

Rate	Ho: I(0) (stationary)		Ho: I(1) (non-stationary)	
	KPSS[1]	Lo's RS[2]	ADF[3]	P-P[4]
SVR	0.02 (<1)	0.28 (<1)	-2.1 (<0.9)	-1.7 (<0.9)
DVR	0.43 (<0.01)	1.78 (<0.05)	-1.5 (<0.9)	-1.4 (<0.9)
LIBOR	0.06 (<1)	0.43 (<1)	-1.9 (<0.9)	-1.5 (<0.9)
G1	0.06 (<1)	0.41 (<1)	-2.0 (<0.9)	-1.5 (<0.9)
P1	0.06 (<1)	0.42 (<1)	-1.9 (<0.9)	-1.4 (<0.9)
B1	0.06 (<1)	0.43 (<1)	-1.7 (<0.9)	-1.4 (<0.9)
F2	0.32 (<0.01)	1.63 (<0.1)	-2.0 (<0.9)	-2.0 (<0.9)
F3	0.33 (<0.01)	1.73 (<0.1)	-2.3 (<0.9)	-1.9 (<0.9)
F5	0.38 (<0.01)	1.85 (<0.05)	-1.6 (<0.9)	-1.5 (<0.9)
F10	0.46 (<0.01)	2.13 (<0.005)	-1.6 (<0.9)	-1.7 (<0.9)
G2	0.07 (<1)	0.42 (<1)	-2.4 (<0.9)	-2.3 (<0.9)
G3	0.08 (<1)	0.41 (<1)	-2.5 (<0.9)	-2.5 (<0.9)
G5	0.08 (<1)	0.41 (<1)	-2.6 (<0.1)	-2.6 (<0.1)
G10	0.09 (<1)	0.47 (<1)	-2.4 (<0.9)	-2.4 (<0.9)
B2	0.06 (<1)	0.40 (<1)	-2.3 (<0.9)	-2.3 (<0.9)
B3	0.06 (<1)	0.40 (<1)	-2.4 (<0.9)	-2.4 (<0.9)
B5	0.07 (<1)	0.39 (<1)	-2.4 (<0.9)	-2.4 (<0.9)
B10	0.08 (<1)	0.39 (<1)	-2.3 (<0.9)	-2.3 (<0.9)
C10	0.06 (<1)	0.36 (<1)	-2.3 (<0.9)	-2.3 (<0.9)

1. The Kwiatkowski et al. (1992) test of I(0), against I(1)
2. Lo's (1991) R/S test for I(0) against I(d), for d > 0 or d < 0.
3. The augmented Dickey-Fuller test of I(1) against I(0). The number of lags is chosen to optimize the Schwarz information criterion over the range 0 to [T^1/3].
4. Phillips-Perron (1988) test of I(1) against I(0).

A.3 Cointegration Tests

This appendix presents the Eigenvalue and Trace tests for cointegration between the 17 analysed relationships, assuming both the null hypothesis of no cointegration (rank=0), and full cointegration (rank=1). Across the whole sample, the null hypothesis of cointegration is never rejected, and for the discounted mortgage rate relationships the null of no cointegration is firmly rejected. For the pre-sample period, the null of no cointegration is rejected for all except the standard variable-rate relationships, the relationship between 3-year fixed- and 3-year interbank-rates, and 10-year fixed- and corporate bond rates. The null of full cointegration is rejected only for 3 of the discounted variable-rate relationships.

Table A.3.1: Cointegration Tests (Whole Sample)

Relationship	UVAR Lag Selection[1]	Max Eigenvalue Test		Trace Test	
		Ho: r=0	Ho: r=1	Ho: r=0	Ho: r=1
F2-G2	2	10.4 (<0.5)	1.6 (<1)	11.9 (<0.5)	1.6 (<1)
F3-G3	3	9.5 (<0.5)	3.8 (<0.5)	13.3 (<0.5)	3.8 (<0.5)
F5-G5	3	8.6 (<0.5)	1.8 (<1)	10.4 (<1)	1.8 (<1)
F10-G10	3	13.1 (<0.2)	5.4 (<0.5)	18.5 (<0.1)	5.4 (<0.5)
F2-B2	2	11.7 (<0.2)	2.8 (<1)	14.4 (<0.5)	2.8 (<1)
F3-B3	3	10.1 (<0.5)	5.0 (<0.5)	15.1 (<0.5)	5.0 (<0.5)
F5-B5	3	9.4 (<0.5)	3.7 (<0.5)	13.1 (<0.5)	3.7 (<0.5)
F10-B10	2	13.9 (<0.1)	4.5 (<0.5)	18.3 (<0.1)	4.5 (<0.5)
F10-C10	3	10.6 (<0.5)	1.0 (<1)	11.6 (<0.5)	1.0 (<1)
SVR-LIBOR	3	15.4 (<0.1)	3.4 (<0.5)	18.8 (<0.1)	3.4 (<0.5)
SVR-G1	4	10.4 (<0.5)	1.2 (<1)	11.6 (<0.5)	1.2 (<1)
SVR-B1	4	14.3 (<0.1)	3.4 (<1)	17.7 (<0.2)	3.4 (<1)
SVR-P1	2	10.2 (<0.5)	2.2 (<1)	12.4 (<0.5)	2.2 (<1)
DVR-LIBOR	11	359.3 (<0.01)	4.3 (<0.5)	363.6 (<0.01)	4.3 (<0.5)
DVR-G1	7	203.2 (<0.01)	3.4 (<0.5)	206.6 (<0.01)	3.4 (<0.5)
DVR-B1	9	331.1 (<0.01)	3.4 (<1)	334.5 (<0.01)	3.4 (<1)
DVR-P1	3	167.0 (<0.01)	7.6 (<0.1)	174.5 (<0.01)	7.6 (<0.1)

1. Selected with the Schwarz Information Criterion.

Table A.3.2: Cointegration Tests (Whole Sample)

Relationship	UVAR Lag Selection[1]	Max Eigenvalue Test		Trace Test	
		Ho: r=0	Ho: r=1	Ho: r=0	Ho: r=1
F2-G2	1	45.6 (<0.01)	5.8 (<0.5)	51.4 (<0.01)	5.8 (<0.5)
F3-G3	2	52.2 (<0.01)	7.4 (<0.2)	59.6 (<0.01)	7.4 (<0.2)
F5-G5	3	14.2 (<0.1)	7.5 (<0.2)	21.7 (<0.05)	7.5 (<0.2)
F10-G10	2	16.8 (<0.05)	6.2 (<0.2)	23.1 (<0.025)	6.2 (<0.2)
F2-B2	3	17.2 (<0.05)	6.0 (<0.2)	23.2 (<0.025)	6.0 (<0.2)
F3-B3	3	15.1 (<0.1)	5.9 (<0.5)	20.9 (<0.05)	5.9 (<0.5)
F5-B5	3	19.3 (<0.025)	7.5 (<0.1)	26.8 (<0.01)	7.5 (<0.1)
F10-B10	2	21.9 (<0.01)	6.9 (<0.2)	28.8 (<0.01)	6.9 (<0.2)
F10-C10	2	11.7 (<0.2)	5.8 (<0.5)	17.4 (<0.2)	5.8 (<0.5)
SVR-LIBOR	4	14.2 (<0.1)	2.2 (<1)	16.4 (<0.2)	2.2 (<1)
SVR-G1	4	13.1 (<0.2)	3.2 (<1)	16.3 (<0.2)	3.2 (<1)
SVR-B1	5	10.3 (<0.5)	2.6 (<1)	12.9 (<0.5)	2.6 (<1)
SVR-P1	3	11.8 (<0.2)	2.2 (<1)	14.0 (<0.5)	2.2 (<1)
DVR-LIBOR	3	334.7 (<0.01)	10.6 (<0.05)	345.3 (<0.01)	10.6 (<0.05)
DVR-G1	3	308.5 (<0.01)	11.4 (<0.025)	319.9 (<0.01)	11.4 (<0.025)
DVR-B1	4	338.9 (<0.01)	2.8 (<1)	341.7 (<0.01)	2.8 (<1)
DVR-P1	3	316.3 (<0.01)	10.2 (<0.05)	326.5 (<0.01)	10.2 (<0.05)

1. Selected with the Schwarz Information Criterion.

A.4 Lag and Lead Selection

I follow the suggestion of (Choi & Kurozumi 2008) to use information criteria, and specifically their simplified Cp criterion (see (Mallows 2000)) which minimises bias in simulations. This criterion is given by:

$$C_p = \frac{1}{\widehat{\sigma}_{\varepsilon}^2} \sum_{t=l+2}^{T-u} \widehat{e}_{t,u}^2 + (p+1)(l+u+2) - T, \tag{A.4.1}$$

where $\widehat{\sigma}_{\varepsilon}^2 = \frac{1}{T-u_{\max}-l_{\max}} \sum_{t=l_{\max}+2}^{T-u_{\max}} \widehat{e}_{t,u_{\max}}^2$ and subscripts l and u refer to the number of lags and leads respectively. $l_{\max}$ and $u_{\max}$ are their upper limits, chosen with the suggested function of the sample size, $T^{\frac{1}{4}} \approx 4$. Table A.4.1 shows, for all 17 estimated relationships, the selected P-L model with the chosen number of lags and leads in parentheses (for example, a PL(2,1) model contains 2 lags and 1 lead), together with those chosen with BIC and AIC criteria for the sensitivity analysis. Table A.4.2 shows the estimated parameters for the cases where minimising BIC and AIC resulted in different lags and leads.

Table A.4.1: Cointegration Tests (Whole Sample)

	PL(L,U) Model Minimised with respect to:				PL(L,U) Model Minimised with respect to:		
Relationship	Cp	BIC	AIC	Relationship	Cp	BIC	AIC
Standard Variable Rates				**Fixed Rates**			
SVR-P1	(4,4)	(1,0)	(4,4)	F2-G2	(2,1)	(1,0)	(1,0)
SVR-G1	(4,4)	(3,2)	(4,4)	F3-G3	(4,3)	(2,1)	(2,1)
SVR-LIBOR	(4,4)	(2,1)	(4,4)	F5-G5	(4,4)	(2,1)	(2,1)
SVR-B1	(4,4)	(2,1)	(4,4)	F10-G10	(2,1)	(2,1)	(2,1)
Discounted Variable Rates							
DVR-P1	(4,4)	(1,0)	(1,0)	F2-B2	(2,1)	(1,0)	(2,1)
DVR-G1	(4,4)	(1,0)	(1,1)	F3-B3	(2,1)	(2,1)	(2,1)
DVR-LIBOR	(4,3)	(1,0)	(4,3)	F5-B5	(4,4)	(1,0)	(2,1)
DVR-B1	(4,4)	(1,0)	(4,4)	F10-B10	(2,2)	(1,0)	(2,2)
				F10-C10	(2,1)	(2,1)	(2,1)

Table A.4.2: Cointegration Tests (Whole Sample)

Relationship	Pre-Crisis[1]		Post-Crisis Effect		R^2	DW	Ho: Complete Pass Through[2]	
	Constant	Slope	Constant	Slope			Pre-Crisis	Post-Crisis
Standard Variable Rates								
SVR-P1 PL(1,0)	2.67[c]	0.80[c]	+0.26[c]	-0.04[c]	1.00	2.06	0.000	0.000
	(11.5)	(18.6)	(4.6)	(-3.9)				
SVR-G1, PL(3,2)	2.34[c]	0.88[c]	+0.26[b]	-0.04[a]	0.99	2.06	0.015	0.001
	(9.3)	(18.5)	(3.0)	(-2.2)				
SVR-LIBOR, PL(2,1)	2.30[c]	0.85[c]	+0.05	-0.01	0.99	2.06	0.008	0.002
	(7.5)	(15.4)	(p=0.645)	(p=0.404)				
SVR-B1, PL(2,1)	2.31[c]	0.85[c]	-0.05	+0.01	0.99	2.2	0.000	0.000
	(9.7)	(19.8)	(p=0.524)	(p=0.656)				
Discounted Variable Rates								
DVR-P1, PL(1,0)	0.23	0.97[c]	+0.71[c]	-0.10[c]	0.99	2.07	0.523	0.007
	(p=0.403)	(18.0)	(4.2)	(-3.6)				
DVR-G1, PL(1,0)	-0.11	1.06[c]	+0.74[c]	-0.11[c]	0.99	1.97	0.237	0.971
	(p=0.663)	(20.7)	(5.3)	(-4.2)				
DVR-G1, PL(1,1)	-0.02	1.04[c]	+0.72[c]	-0.11[c]	0.99	1.98	0.399	0.961
	(p=0.930)	(21.1)	(5.1)	(-4.2)				
DVR-LIBOR, PL(1,0)	0.18	0.96[c]	+0.63[c]	-0.11[c]	0.99	1.93	0.336	0.000
	(p=0.409)	(22.4)	(4.9)	(-4.6)				
DVR-B1, PL(1,0)	0.08	0.97[c]	+0.54[c]	-0.09[c]	0.99	1.88	0.564	0.008
	(p=0.754)	(20.6)	(4.6)	(-4.0)				
Fixed Rates								
F2-G2, PL(1,0)	0.63[b]	0.98[c]	0.63	-0.08[b]	0.98	1.94	0.547	0.008
	(3.0)	(25.2)	(4.4)	(-2.8)				
F3-G3, PL(2,1)	1.00[c]	0.95[c]	+0.53[c]	-0.07[a]	0.98	1.98	0.039	0.000
	(4.9)	(25.4)	(3.5)	(-2.2)				
F5-G5, PL(2,1)	0.64[a]	1.04[c]	+0.50[c]	-0.08[a]	0.99	2.02	0.428	0.359
	(2.5)	(22.4)	(3.3)	(-2.6)				
F2-B2, PL(1,0)	0.29	0.98[c]	+0.48[b]	-0.07[a]	0.98	1.99	0.567	0.023
	(p=0.240)	(23.2)	(3.4)	(-2.5)				
F5-B5, PL(1,0)	0.02	1.07[c]	+0.72[c]	-0.12[c]	0.99	1.94	0.014	0.115
	(p=0.881)	(40.0)	(5.2)	(-4.5)				
F5-B5, PL(2,1)	0.04	1.06[c]	+0.65[c]	-0.11[c]	0.99	2.02	0.062	0.261
	(p=0.860)	(30.8)	(4.6)	(-4.0)				
F10-B10, PL(1,0)	0.02	1.07[c]	+0.72[c]	-0.12[c]	0.99	1.94	0.208	0.265
	(p=0.881)	(39.5)	(5.2)	(-4.5)				

1. Pre-crisis includes monthly observations from Jan 1995 up to and including July 2007, post-crisis from August 2007 to May 2009
2. P-values for a Chi-Squared Test with Slope Coefficient restricted =1 (small value indicates evidence for incomplete pass throug
Significance levels of 5%, 1%, and 0.1% are indicated by superscript a, b, and c respectively, numbers in brackets indicate t-ratios except for insignificant coefficients, for which associated p-values are given instead.

A.5 Mean Adjustment Lag Calculation

This appendix shows how Mean Adjustment Lags (MALs) are calculated following (Hendry 1995), for a short run equation that is initially specified in error correction form. The general form for our $SR(x, y)$ model can be written as:

$$\Delta r_t = (\beta_0 + \beta_{00} D)\Delta i_t + \delta\varepsilon_{t-1} + E(L) + F(L) + v_t, \tag{A.5.1}$$

where constants in the disequilibrium term (which do not affect the system's dynamics) are not included; the error term (v_t) is assumed to be a stationary *iid* process (so can be ignored henceforth); and I define:

$$\delta = ((1 - D)(\delta_1\lambda + \delta_2(1 - \lambda)) + D\delta_3) \tag{A.5.2}$$

$$\varepsilon_t = r_t - \alpha_1 i_t - \alpha_3 D i_t \tag{A.5.3}$$

$$E(L) = \sum_{i=1}^{x} \beta_i \Delta i_{t-i} \tag{A.5.4}$$

$$F(L) = \sum_{j=1}^{y} \gamma_j \Delta r_{t-j}. \tag{A.5.5}$$

$E(L)$ and $F(L)$ are lag functions that depend on the (assumed finite) number of lags of the differenced exogenous (retail rate, r_t) and endogenous (wholesale rate, i_t) variables respectively.

From now on I use the standard lag notation, where $x_{t-i} = x_t[L^i]$

If we express the general form in levels with r_t on the LHS and i_t on the RHS:

$$r_t [A(L)] = i_t [B(L)], \tag{A.5.6}$$

then following the algebra on p.215 of (Hendry 1995), the MAL formula is:

$$MAL = \frac{B'(L)}{B(L)} - \frac{A'(L)}{A(L)} \quad \text{for } L = 1, \tag{A.5.7}$$

where prime denotes the first derivative of the lag function with respect to L. Then equation A.5.1 written in the levels form of equation A.5.6 is:

$$\begin{aligned} r_t - r_{t-1} - \delta r_{t-1} - F(L) &= (\beta_0 + \beta_{00} D) i_t - (\beta_0 + \beta_{00} D) i_{t-1} \\ &\quad -\delta \left(\alpha_1 i_{t-1} + \alpha_3 D i_{t-1} \right) + E(L) \\ r_t \left[1 - L^1 (1-\delta) - F(L) \right] &= i_t \left[\begin{array}{c} (\beta_0 + \beta_{00} D) - (\beta_0 + \beta_{00} D) L^1 \\ -\delta L^1 \left(\alpha_1 + \alpha_3 D \right) + E(L) \end{array} \right]. \end{aligned}$$

We therefore have:

$$\begin{aligned} A(L) &= 1 - L^1 (1-\delta) - F(L) \\ A(L)' &= -(1-\delta) - F'(L) \\ B(L) &= \left[(\beta_0 + \beta_{00} D) - (\beta_0 + \beta_{00} D) L^1 - \delta L^1 \left(\alpha_1 + \alpha_3 D \right) + E(L) \right] \\ B'(L) &= -(\beta_0 + \beta_{00} D) - \delta \left(\alpha_1 + \alpha_3 D \right) + E'(L), \end{aligned}$$

and so using formula A.5.7 we have:

$$MAL = \frac{(\beta_0 + \beta_{00} D) + \delta \left(\alpha_1 + \alpha_3 D \right) - E'(1)}{\delta \left(\alpha_1 + \alpha_3 D \right) - E(1)} - \frac{(\delta - 1) - F'(1)}{\delta - F(1)}. \qquad \text{(A.5.8)}$$

The functions $E(L)$ and $F(L)$ have the same form, which can be simplified for any similar differenced lag structure, say $G(L)$, in the following way:

$$\begin{aligned} G(L) &= \sum_{i=1}^{n} \phi_i \Delta x_{t-i} \\ G(L) &= \phi_1 \Delta x_{t-1} + \phi_2 \Delta x_{t-2} ... + \phi_n \Delta x_{t-n} \\ G(L) &= \phi_1 (x_{t-1} - x_{t-2}) ... + \phi_n \Delta (x_{t-n} - x_{t-(n+1)}) \\ G(L) &= x_t \left[\phi_1 \left(L^1 - L^2 \right) ... + \phi_n \left(L^n - L^{n+1} \right) \right], \end{aligned}$$

so for $L = 1$, all terms disappear and we simply have:

$$G(1) = 0. \tag{A.5.9}$$

Taking the first derivative of $G(L)$ with respect to L we have:

$$G^{'}(L) = x_t \left[\phi_1 (1-2L) + \phi_2 \left(2L - 3L^2\right) + ... + \phi_n \left(nL^{n-1} - (n+1)L^n\right)\right],$$

which for $L = 1$ is:

$$\begin{aligned} G^{'}(1) &= x_t \left[\phi_1 (1-2) + \phi_2 (2-3) + ... + \phi_n (n-(n+1))\right] \\ G^{'}(1) &= x_t \left[-\phi_1 - \phi_2 - ... - \phi_n\right] \end{aligned} \tag{A.5.10}$$

Substituting these results A.5.9 and A.5.10 into formula A.5.8 we have:

$$MAL = 1 + \frac{(\beta_0 + \beta_{00}D) + \beta_1 + ... + \beta_x}{\delta\,(\alpha_1 + \alpha_3 D)} + \frac{(1-\delta)}{\delta} - \frac{\gamma_1 + ... + \gamma_y}{\delta}. \tag{A.5.11}$$

For the pre/post-crisis analysis, where $\delta = (\delta_0 + \delta_{00}D)$, we can calculate the MAL for two cases:

Case A: Pre-Crisis Regime ($D = 0$), hence $\delta_a = \delta_0$

Case B: Post-Crisis Regime ($D = 1$), hence $\delta_b = \delta_0 + \delta_{00}$

$$MAL^a = 1 + \frac{\beta_0 + \beta_1 + ... + \beta_x}{\delta_a \alpha_1} + \frac{(1-\delta_a)}{\delta_a} - \frac{\gamma_1 + ... + \gamma_y}{\delta_a} \tag{A.5.12}$$

$$MAL^b = 1 + \frac{(\beta_0 + \beta_{00}D) + \beta_1 + ... + \beta_x}{\delta_b\,(\alpha_1 + \alpha_3 D)} + \frac{(1-\delta_b)}{\delta_b} - \frac{\gamma_1 + ... + \gamma_y}{\delta_b} \tag{A.5.13}$$

For the asymmetry analysis we have $\delta = ((1-D)(\delta_1\lambda + \delta_2(1-\lambda)) + D\delta_3)$. There are two different cases for δ, depending on the particular equilibrium correction regime. For δ, as defined in equation A.5.2, these are:

Case C: Pre-Crisis, Above Equilibrium Regime $(D = 0, \lambda = 1)$, hence $\delta = \delta_1$

Case D: Pre-Crisis, Below Equilibrium Regime $(D = 0, \lambda = 0)$, hence $\delta = \delta_2$

Using notation where MAL^+ and MAL^- denote Mean Adjustment Lags for Case C and D respectively, we have our MAL formulae:

$$MAL^+ = 1 + \frac{\beta_0 + \beta_1 + ... + \beta_x}{\delta\alpha_1} + \frac{(1-\delta_1)}{\delta_1} - \frac{\gamma_1 + ... + \gamma_y}{\delta_1} \tag{A.5.14}$$
$$MAL^- = 1 + \frac{\beta_0 + \beta_1 + ... + \beta_x}{\delta_2\alpha_1} + \frac{(1-\delta_2)}{\delta_2} - \frac{\gamma_1 + ... + \gamma_y}{\delta_2}. \tag{A.5.15}$$

Taking the example of a $SR(2,0)$ model, the mean adjustment lags are:

$$\begin{aligned} MAL^+ &= 1 + \frac{\beta_0 + \beta_1 + \beta_2}{\delta\alpha_1} + \frac{(1-\delta_1)}{\delta_1} \\ MAL^- &= 1 + \frac{\beta_0 + \beta_1 + \beta_2}{\delta_2\alpha_1} + \frac{(1-\delta_2)}{\delta_2}. \end{aligned}$$

A.6 A Forward-Looking Model of Retail Rate-Setting

This appendix outlines the conceptual model of (Mizen et al. 2004), which provides an adjustment cost rationale for asymmetric adjustment of retail rates to changes in policy rates. Retail rates also depend on *forecasts* of policy rates, highlighting the potential for a relationship between current retail rates and *future* wholesale rates, and hence the potential importance of using the P-L estimator to allow for this.

First of all, it is suggested that Banks are able to adjust their retail rates every even period, whilst being able to make an additional adjustment in the following odd period, but only at a fixed cost c. The monetary authority makes a rate decision every even period, after which the bank sets its retail rates for the current and following period. The loss function for the bank is quadratic in the deviation of the retail rate from its desired level, so the optimal rate at time 0 is:

$$r_0^* = \frac{i_0 + E_0(i_1)}{2}. \tag{A.6.1}$$

Following this, unexpected shocks to the policy rate can cause the desired optimal retail rate to change (and in the even time period 1 where it costs the bank to make such an adjustment). Say that base rates can be forecasted up to a white noise error:

$$i_1 = E(i_1) + \varepsilon, \tag{A.6.2}$$

then at even time period 1 the bank would like to re-set to:

$$r_1^* = \frac{i_1 + E(i_2)}{2}.$$

There are two options for the bank at this point. Either it will be worth making this adjustment to r_1^*, if the gain from avoiding the quadratic loss outweighs the fixed marginal cost of adjustment, or not, in which case it will leave its rate unchanged at r_0^*. That is, adjustment is worth it when this condition is fulfilled:

$$E\left[(r_0^* - i_1)^2 - (r_1^* - i_1)^2 + (r_0^* - i_2)^2 - (r_1^* - i_2)^2\right] > c$$

This condition can then be manipulated to give:

$$\begin{aligned} 2(r_0^* - r_1^*)^2 &> c \\ [(r_0^* - r_1^*) - E\Delta i]^2 &> c \end{aligned}$$

The second term is the expected change in the base rate, $E\Delta i = \left(\frac{E_1 i_2 - i_1}{2}\right)$.This means that the bank will *not* adjust iff the difference between its previously set retail rate and the current base rate lies in a region around the expected change in the base rate

$$\left(\frac{E_1 i_2 - i_1}{2}\right) - \sqrt{\frac{c}{2}} < r_0^* - i_1 < \left(\frac{E_1 i_2 - i_1}{2}\right) + \sqrt{\frac{c}{2}}.$$

Appendix B

Optimal Monetary Policy

B.1 Welfare Loss

Suppose we know the evolution of the economy under control, to a linear approximation, is:

$$(Y_{t+1} - Y) = M (Y_t - Y) + B\varepsilon_{t+1}$$

where Y_t is a state variable and the equation is written in terms of deviations from steady state.[1]

Denote:

$$Z_t = (Y_t - Y)$$

then

$$Z_{t+1} = MZ_t + B\varepsilon_{t+1} \tag{B.1.1}$$

In order to derive $E_0 \sum_{t=0}^{\infty} \omega^t Z_t$ we take expectations and sum:

$$E_0 \sum_{t=0}^{\infty} \omega^t Z_{t+1} = ME_0 \sum_{t=0}^{\infty} \omega^t Z_t + BE_0 \sum_{t=0}^{\infty} \omega^t \varepsilon_{t+1}$$

We assume

$$E_0 \varepsilon_{t+1} Z_t' = 0 \tag{B.1.2}$$

so that:

$$\frac{1}{\omega} E_0 \sum_{t=0}^{\infty} \omega^t Z_t = ME_0 \sum_{t=0}^{\infty} \omega^t Z_t + \frac{Z_0}{\omega}$$

[1] This is the default version of output in Dynare 4.04.

where $V = E_0 \sum_{t=0}^{\infty} \omega^t Z_t Z_t'$ and Σ is the variance-covariance matrix of shocks. This yields:

$$E_0 \sum_{t=0}^{\infty} \omega^t Z_t = (I - \omega M)^{-1} Z_0$$

Then we need to derive $V = E_0 \sum_{t=0}^{\infty} \omega^t Z_t Z_t'$. From (B.1.1) and (B.1.2), if we ignore all terms of the third order and higher and assume:

$$\sum_{t=0}^{\infty} \omega^t Z_{t+1} Z_{t+1}' = M \left(\sum_{t=0}^{\infty} \omega^t Z_t Z_t' \right) M' + B \left(\sum_{t=0}^{\infty} \omega^t \varepsilon_{t+1} \varepsilon_{t+1}' \right) B'$$

Therefore:

$$\text{vec}\,(V) = (I - \omega M \otimes M))^{-1} \text{vec}(\frac{\omega}{1-\omega} B \Sigma B' + Z_0 Z_0')$$

Finally, for any arbitrary variable x_t :

$$x_t = N Z_t$$

it follows that:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \omega^t (x_t x_t') &= N \left(E_0 \sum_{t=0}^{\infty} \omega^t (Z_t Z_t') \right) N' = NVN' \\ E_0 \sum_{t=0}^{\infty} \omega^t x_t &= N E_0 \sum_{t=0}^{\infty} \omega^t Z_t = N (I - \omega M)^{-1} Z_0 \end{aligned}$$

Note that for any scalar function f of many variables:

$$f(X_t) = f(X) + \nabla f(X)' (X_t - X) + \frac{1}{2} (X_t - X)' Df(X) (X_t - X) + ...$$

We can write:

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \omega^t f(X_t) \\ = & \sum_{t=0}^{\infty} \omega^t E_0 \left(f(X) + \nabla f(X)' (X_t - X) + \frac{1}{2} (X_t - X)' Df(X) (X_t - X) + ... \right) \\ = & \frac{f(X)}{1-\omega} + \frac{\nabla f(X)' E_0 \sum_{t=0}^{\infty} \omega^t x_t}{1-\omega} + \frac{1}{2} tr \left(Df(X) E_0 \sum_{t=0}^{\infty} \omega^t x_t x_t' \right) + ... \\ = & \frac{f(X)}{1-\omega} + \frac{\nabla f(X)' N (I - \omega M)^{-1} Z_0}{1-\omega} + \frac{1}{2} tr (Df(X) NVN') + ... \end{aligned}$$

B.2 Model Derivations

B.2.1 Households

Savers

Patient households maximise their objective function:

$$\max U_{ut} = E_t \sum_{s=t}^{\infty} \beta_u^{s-t} u_{us}(C_{us}, D_{us}, N_{us})$$

subject to the real budget constraint:

$$a_{ut} = (1 + R_t) \left(\frac{a_{ut-1}}{\Pi_t} + w_{ut} N_{ut} + t_{ut} - C_{ut} - q_t \left(D_{ut} - (1 - \delta) D_{ut-1} \right) \right) + \tilde{d}_t$$

The associated Lagrangian is:

$$L = \sum_{s=t}^{\infty} \beta_u^{s-t} \left(U_{us} + \Lambda_s \left(a_{us} - (1 + R_s) \begin{pmatrix} \frac{a_{us-1}}{\Pi_s} + w_{us} N_{us} + t_{us} \\ -C_{us} - q_s \left(D_{us} - (1 - \delta) D_{us-1} \right) \end{pmatrix} - \tilde{d}_s \right) \right)$$

For the partial derivative of the objective utility function U_{us} with respect to a choice variable x_s, we use notation $\frac{\delta U_s}{\delta x_s} = U_x$. FOCs are then:

$$\begin{aligned}
\frac{\partial L}{\partial C_{us}} &= \beta_u^{s-t} \left(U_{nd} + \Lambda_s \left(1 + R_s \right) \right) = 0 && \text{(B.2.1)} \\
0 &= \frac{\partial L}{\partial D_{us}} = \beta_u^{s-t} \left(U_d + \Lambda_s \left(1 + R_s \right) q_s \right) \\
&\quad - \beta_u^{s-t+1} \Lambda_{s+1} \left(1 + R_{s+1} \right) q_{s+1} \left(1 - \delta \right) && \text{(B.2.2)} \\
\frac{\partial L}{\partial N_{us}} &= \beta_u^{s-t} \left(U_n - \Lambda_s \left(1 + R_s \right) w_{us} \right) = 0 && \text{(B.2.3)} \\
\frac{\partial L}{\partial a_{us}} &= \beta_u^{s-t} \Lambda_s - \beta_u^{s-t+1} \Lambda_{s+1} \frac{(1 + R_{s+1})}{\Pi_{s+1}} = 0 && \text{(B.2.4)} \\
\frac{\partial L}{\partial \Lambda_s} &= a_{us} - (1 + R_s) \begin{pmatrix} \frac{a_{us-1}}{\Pi_{c,s}} + w_{us} N_{us} + t_{us} \\ -C_{us} - q_s \left(D_{us} - (1 - \delta) D_{us-1} \right) \end{pmatrix} - \tilde{d}_s = 0 && \text{(B.2.5)}
\end{aligned}$$

where the objective function partial derivatives are:

$$
\begin{aligned}
U_{nd} &= U_x \frac{\partial X}{\partial C_{us}} = \frac{(1-\alpha)^{\frac{1}{\eta}} X_{us}^{1-\sigma} C_{us}^{-\frac{1}{\eta}}}{\left((1-\alpha)^{\frac{1}{\eta}} (C_{us})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (D_{us})^{\frac{\eta-1}{\eta}}\right)} \\
&= \frac{(1-\alpha)^{\frac{1}{\eta}} X_{us}^{1-\sigma} C_{us}^{-\frac{1}{\eta}}}{X_{us}^{\frac{\eta-1}{\eta}}} = (1-\alpha)^{\frac{1}{\eta}} X_{us}^{\frac{1}{\eta}-\sigma} C_{us}^{-\frac{1}{\eta}} \\
U_d &= U_x \frac{\partial X}{\partial D_{us}} = \frac{\alpha^{\frac{1}{\eta}} X_{us}^{1-\sigma} D_{us}^{-\frac{1}{\eta}}}{\left((1-\alpha)^{\frac{1}{\eta}} (C_{us})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (D_{us})^{\frac{\eta-1}{\eta}}\right)} = \alpha^{\frac{1}{\eta}} X_{us}^{\frac{1}{\eta}-\sigma} D_{us}^{-\frac{1}{\eta}} \\
U_n &= -\varkappa N_{us}^{\phi}
\end{aligned}
$$

We can define the stochastic discount factor by combining FOCs (B.2.1) and (B.2.4):

$$Q_{t,t+1} = \beta_u \frac{U_{nd,ut+1}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,t+1}} \tag{B.2.6}$$

which implies that from period t iterating forward to period s we have:

$$Q_{t,s} = \beta_u^{s-t} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} \tag{B.2.7}$$

Finally, after rearranging, we obtain system (4.2.7)-(4.2.10).

Borrowers

Constrained households maximise their objective function:

$$\max U_{ct} = E_t \sum_{s=t}^{\infty} \beta_c^{s-t} u_{cs}(C_{cs}, D_{cs}, N_{cs})$$

subject to the real budget constraint:

$$a_{ct} = \left(1 + R_t^D\right) \left(\frac{a_{ct-1}}{\Pi_t} + C_{ct} + q_t \left(D_{ct} - (1-\delta) D_{ct-1}\right) - w_{ct} N_{ct} + t_{ct}\right)$$

and the collateral constraint:

$$a_{ct} = (1-\rho) \frac{a_{ct-1}}{\Pi_t} + \rho \Upsilon (k_t)$$

The associated Lagrangian is:

$$L = \sum_{s=t}^{\infty} \beta_c^{s-t} \left(\begin{array}{c} U_{cs} + \Theta_s \left(a_{cs} - \left(1 + R_s^D\right) \left(\begin{array}{c} \frac{a_{cs-1}}{\Pi_s} + C_{cs} - w_{cs} N_{cs} + t_{cs} \\ + q_s \left(D_{cs} - (1-\delta) D_{cs-1}\right) \end{array} \right) \right) \\ -\Gamma_s \left(a_{cs} - (1-\rho) \frac{a_{cs-1}}{\Pi_s} - \rho \Upsilon \left((1-\chi) q_s D_{cs} \right) \right) \end{array} \right)$$

We define $\Xi_s = \Gamma_s / U_{nd,cs}$ and derive FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial C_{cs}} &= \beta_c^{s-t} [U_{nd} - \Theta_s \left(1 + R_s^D\right)] = 0 && \text{(B.2.8)} \\
0 &= \frac{\partial L}{\partial D_{cs}} = \beta_c^{s-t} [U_d - \Theta_s \left(1 + R_s^D\right) q_s + \Xi_s \rho \Upsilon_{Dc,s}] && \text{(B.2.9)} \\
& \quad + \beta_c^{s-t+1} \Theta_{s+1} \left(1 + R_{s+1}^D\right) q_{s+1} (1-\delta) && \text{(B.2.10)} \\
\frac{\partial L}{\partial N_{cs}} &= \beta_c^{s-t} \left(U_n + \Theta_s \left(1 + R_s^D\right) w_{cs} \right) = 0 && \text{(B.2.11)} \\
\frac{\partial L}{\partial \Theta_s} &= a_{cs} - \left(1 + R_s^D\right) \left(\begin{array}{c} \frac{a_{cs-1}}{\Pi_s} + C_{cs} + q_s \left(D_{cs} - (1-\delta) D_{cs-1}\right) \\ - w_{cs} N_{cs} - t_{cs} \end{array} \right) = 0 && \text{(B.2.12)} \\
\frac{\partial L}{\partial \Xi_s} &= \left(a_{cs} - (1-\rho) \frac{a_{cs-1}}{\Pi_s} - \rho \Upsilon \left[(1-\chi) q_s D_{cs} \right] \right) = 0 && \text{(B.2.13)} \\
\frac{\partial L}{\partial a_{cs}} &= \beta_c^{s-t} \left(\Theta_s - \Xi_s\right) + \beta_c^{s-t+1} \left(\begin{array}{c} -\Theta_{s+1} \frac{\left(1 + R_{s+1}^D\right)}{\Pi_{s+1}} \\ + \Xi_{s+1} \frac{(1-\rho)}{\Pi_{s+1}} \end{array} \right) = 0 && \text{(B.2.14)}
\end{aligned}$$

where the objective function partial derivatives are:

$$\begin{aligned}
U_{nd} &= U_x \frac{\partial X}{\partial C_{cs}} = \frac{(1-\alpha)^{\frac{1}{\eta}} X_{ct}^{1-\sigma} C_{ct}^{-\frac{1}{\eta}}}{\left((1-\alpha)^{\frac{1}{\eta}} \left(C_{ct}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(D_{ct}\right)^{\frac{\eta-1}{\eta}} \right)} \\
&= \frac{(1-\alpha)^{\frac{1}{\eta}} X_{ct}^{1-\sigma} C_{ct}^{-\frac{1}{\eta}}}{X_{ct}^{\frac{\eta-1}{\eta}}} = (1-\alpha)^{\frac{1}{\eta}} X_{ct}^{\frac{1}{\eta}-\sigma} C_{ct}^{-\frac{1}{\eta}} \\
U_d &= U_x \frac{\partial X}{\partial D_{cs}} = \frac{\alpha^{\frac{1}{\eta}} X_{cs}^{1-\sigma} D_{cs}^{-\frac{1}{\eta}}}{\left((1-\alpha)^{\frac{1}{\eta}} \left(C_{cs}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(D_{cs}\right)^{\frac{\eta-1}{\eta}} \right)} = \alpha^{\frac{1}{\eta}} X_{cs}^{\frac{1}{\eta}-\sigma} D_{cs}^{-\frac{1}{\eta}} \\
U_n &= -\varkappa N_{cs}^{\phi}
\end{aligned}$$

Finally, after rearranging, we obtain system (4.2.16)-(4.2.19).

B.2.2 Price Setting

Non-Durable Prices

The FOC (4.2.30):

$$E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{nds} y_{nds}(i) \left(\frac{p^*_{ndt}(i)}{P_{nds}} - \mu mc_s \right) = 0$$

is rearranged as:

$$\left(\frac{p^*_{ndt}(i)}{P_{ndt}} \right) = \frac{E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} Y_{nds} \mu mc_s \left(\frac{P_{ndt}}{P_{nds}} \right)^{-(\epsilon+1)}}{E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} Y_{nds} \left(\frac{P_{ndt}}{P_{nds}} \right)^{-\epsilon}} \tag{B.2.15}$$

We substitute the patient household discount factor from equation (4.2.12) and define the numerator in (B.2.15) as:

$$\begin{aligned} G_t &= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} \mu U_{nd}(C_{us}, D_{us}, N_{us}) Y_{nds} mc_s \left(\frac{P_{nds}}{P_{ndt}} \right)^{\epsilon} \\ &= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} g_s \left(\frac{P_{nds}}{P_{ndt}} \right)^{\epsilon} \end{aligned}$$

and the denominator as:

$$\begin{aligned} F_t &= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} U_{nd}(C_{us}, D_{us}, N_{us}) Y_{nds} \left(\frac{P_{nds}}{P_{ndt}} \right)^{\epsilon-1} \\ &= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} f_s \left(\frac{P_{nds}}{P_{ndt}} \right)^{\epsilon-1} \end{aligned}$$

where:

$$\begin{aligned} g_s &= \mu U_{nd} Y_{nds} mc_s \\ &= \frac{1}{Z_{nds}} \frac{\varkappa \mu N_{us}^{\phi\nu} C_{us}^{\frac{\nu-1}{\eta}} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)(\nu-1)} N_{cs}^{\phi(1-\nu)} C_{cs}^{\frac{1}{\eta}(1-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} Y_{nds}}{\nu^{\nu}(1-\nu)^{(1-\nu)}} \\ f_s &= U_{nd}(C_{us}, D_{us}, N_{us}) Y_{nds} = (1-\alpha)^{\frac{1}{\eta}} X_{us}^{\frac{1}{\eta}-\sigma} C_{us}^{-\frac{1}{\eta}} Y_{nds} \end{aligned}$$

It follows that:

$$\begin{aligned} G_t &= g_t + \theta\beta E_t \Pi_{t+1}^{\epsilon} G_{t+1} \\ F_t &= f_t + \theta\beta E_t \Pi_{t+1}^{\epsilon-1} F_{t+1} \end{aligned}$$

The price level in the non-durable sector is determined as:

$$P_{ndt} = \left[(1-\theta)\left(p^*_{ndt}\right)^{1-\epsilon} + \theta P^{1-\epsilon}_{ndt-1}\right]^{\frac{1}{1-\epsilon}}$$

From where:

$$\left(\frac{p^*_{ndt}}{P_{ndt}}\right)^{1-\epsilon} = \frac{\Pi_t^{1-\epsilon}-\theta}{(1-\theta)\Pi_t^{1-\epsilon}} = \frac{1-\theta\Pi_t^{\epsilon-1}}{(1-\theta)} = \left(\frac{p^*_{ct}}{P_{ct}}\right)^{1-\epsilon} = \left(\frac{G_t}{F_t}\right)^{1-\epsilon}$$

And finally we obtain our price-setting equation (New Keynesian Phillips curve) for the non-durable goods sector:

$$\frac{1-\theta\Pi_t^{\epsilon-1}}{(1-\theta)} = \left(\frac{G_t}{F_t}\right)^{1-\epsilon}$$

where

$$\begin{aligned} G_t &= \mu U_{nd,ut} Y_{ndt} mc_t + \theta\beta E_t\left[\Pi^{\epsilon}_{t+1} G_{t+1}\right] \\ F_t &= U_{nd,ut} Y_{ndt} + \theta\beta E_t\left[\Pi^{\epsilon-1}_{t+1} F_{t+1}\right] \end{aligned}$$

Durable Relative Prices

Durable goods firms choose prices to maximise expected their profit function:

$$\max_{\{p^*_s(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s}\left(y_{ds}(i)\, p_{ds}(i) - W_{us} N_{dus}(i) - W_{cs} N_{dcs}(i)\right)$$

Hours worked by patient and constrained households from equations (4.2.39) and (4.2.40) can be written as:

$$\begin{aligned} N_{dut}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{\left(\frac{W_{ut}}{P_{ct}}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(\frac{W_{ct}}{P_{ct}}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} = \frac{1}{Z_{dt}} y_{dt}(i) \frac{W_{ut}^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{W_{ct}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\ N_{dct}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{w^{\nu}_{ut}}{\nu^{\nu}} \frac{w^{(-\nu)}_{ct}}{(1-\nu)^{(-\nu)}} = \frac{1}{Z_{dt}} y_{dt}(i) \frac{W^{\nu}_{ut}}{\nu^{\nu}} \frac{W^{(-\nu)}_{ct}}{(1-\nu)^{(-\nu)}} \end{aligned}$$

Substitute into the profit function to obtain:

$$\begin{aligned}
& E_t \sum_{s=t}^{\infty} Q_{t,s} \begin{pmatrix} y_{ds}(i)\, p_{ds}(i) - W_{us} \frac{1}{Z_{dt}} y_{ds}(i) \frac{W_{us}^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{W_{cs}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\ -W_{cs} \frac{1}{Z_{dt}} y_{ds}(i) \frac{W_{us}^{\nu}}{\nu^{\nu}} \frac{W_{cs}^{(-\nu)}}{(1-\nu)^{(-\nu)}} \end{pmatrix} \\
= \; & E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{ds}(i)\, p_{ds}(i) - \frac{1}{Z_{dt}} y_{ds}(i) \frac{W_{us}^{\nu}}{\nu^{\nu}} \frac{W_{cs}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \right) \\
= \; & E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{ds}(i)\, p_{ds}(i) - y_{ds}(i) MC_{ds} \right)
\end{aligned}$$

where $MC_{ds} = \zeta_t P_{dt}$. Note that wages here do not depend on index i, since labour of each type is assumed to be perfectly mobile and so wages for particular household type are equalised across all firms. So we come to the familiar formulation:

$$\max_{\{p_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{ds}(i)\, p_{ds}(i) - y_{ds}(i) MC_{ds} \right)$$

subject to:

$$y_{dt}(i) = Y_{dt} \left(\frac{p_{dt}(i)}{P_{dt}} \right)^{-\epsilon}$$

The problem for the optimal price setting at time t can equivalently be written as:

$$\max_{\{p_{ds}(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{ds}(i)\, p_{ds}(i) - y_{ds}(i) MC_{ds} \right)$$

Substitute demand:

$$\max_{\{p_{ds}(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{ds} Y_{ds} \left(\left(\frac{p_{ds}(i)}{P_{ds}} \right)^{1-\epsilon} - \left(\frac{p_{ds}(i)}{P_{ds}} \right)^{-\epsilon} \frac{MC_{ds}}{P_{ds}} \right)$$

So the FOC is:

$$\begin{aligned}
\frac{\partial}{\partial p_s(i)} Q_{t,s} P_{ds} Y_{ds} \left(\left(\frac{p_{ds}(i)}{P_{ds}} \right)^{1-\epsilon} - \left(\frac{p_{ds}(i)}{P_{ds}} \right)^{-\epsilon} mc_{ds} \right) &= 0 \\
Q_{t,s} Y_{ds} \left((1-\epsilon) \left(\frac{p_{ds}(i)}{P_{ds}} \right)^{-\epsilon} + \epsilon \left(\frac{p_{ds}(i)}{P_{ds}} \right)^{-\epsilon-1} mc_{ds} \right) &= 0
\end{aligned}$$

$$\frac{p_{dt}(i)}{P_{ds}} - \mu mc_{ds} = 1 - \mu mc_{ds} = 0$$

where $\mu = -\frac{\epsilon}{(1-\epsilon)}$ and $mc_{ds} = \frac{MC_{ds}}{P_{ds}}$.

Real marginal cost will always be set equal to the constant $\frac{\epsilon-1}{\epsilon}$, and switching to time subscript t we have:

$$\begin{aligned}
\frac{MC_{dt}}{P_{dt}} &= \frac{1}{Z_{dt}}\frac{1}{\nu^{\nu}\left(1-\nu\right)^{(1-\nu)}}\frac{W_{ct}^{(1-\nu)}W_{ut}^{\nu}}{P_{dt}}=\frac{\epsilon-1}{\epsilon} \\
\frac{1}{Z_{dt}}\frac{W_{ct}^{(1-\nu)}W_{ut}^{\nu}}{P_{dt}} &= \frac{\epsilon-1}{\epsilon}\nu^{\nu}\left(1-\nu\right)^{(1-\nu)} \\
\frac{1}{Z_{dt}}\left(\frac{W_{ct}}{P_{ndt}}\right)^{(1-\nu)}\left(\frac{W_{ut}}{P_{ndt}}\right)^{\nu}\frac{P_{ndt}}{P_{dt}} &= \frac{\epsilon-1}{\epsilon}\nu^{\nu}\left(1-\nu\right)^{(1-\nu)}
\end{aligned}$$

Substituting wages from the household FOCs (4.2.9) and (4.2.18) we then obtain:

$$\begin{aligned}
&\frac{1}{Z_{dt}}\left(\frac{\varkappa}{\left(1-\alpha\right)^{\frac{1}{\eta}}}N_{ct}^{\phi}C_{ct}^{\frac{1}{\eta}}X_{ct}^{\sigma-\frac{1}{\eta}}\right)^{(1-\nu)}\left(\frac{\varkappa}{\left(1-\alpha\right)^{\frac{1}{\eta}}}N_{ut}^{\phi}C_{ut}^{\frac{1}{\eta}}X_{ut}^{\sigma-\frac{1}{\eta}}\right)^{\nu} \\
= \; & q_t\frac{\epsilon-1}{\epsilon}\nu^{\nu}\left(1-\nu\right)^{(1-\nu)} \\
&\frac{1}{Z_{dt}}\frac{\varkappa}{\left(1-\alpha\right)^{\frac{1}{\eta}}}N_{ct}^{\phi(1-\nu)}C_{ct}^{\frac{1}{\eta}(1-\nu)}X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}N_{ut}^{\phi\nu}C_{ut}^{\frac{1}{\eta}\nu}X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu} \\
= \; & q_t\frac{\epsilon-1}{\epsilon}\nu^{\nu}\left(1-\nu\right)^{(1-\nu)}
\end{aligned}$$

B.2.3 Financial Intermediaries

Optimal Debt Offers

The present discounted value of profits associated with lending amount Z_{t+1} is:

$$W_t=\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(\left(R_s^z-R_s\right)Z_{t+1}-\frac{\varpi}{2}\left(\frac{Z_{t+1}-K_s}{K_s}\right)^2K_s\right)$$

So the FOC for choosing Z_{t+1} to maximise profit is:

$$\frac{\partial W_t}{\partial Z_{t+1}}=\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(\left(R_s^z-R_s\right)-\varpi\left(\frac{Z_{t+1}-K_s}{K_s}\right)\right)=0$$

$$\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(R_s^z-R_s\right)=\varpi\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(\frac{Z_{t+1}-K_s}{K_s}\right) \tag{B.2.16}$$

$$\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(R_s^z-R_s+\varpi\right)=\varpi Z_{t+1}\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}K_s^{-1} \tag{B.2.17}$$

We define the variables (replacing $Q_{t,s}$ with the patient stochastic discount factor of equation (B.2.7):

$$M_t = \sum_{s=t+1}^{\infty} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} \left(\beta_u(1-\rho)\right)^{s-t} \left(R_s^z - R_s + \varpi\right)$$

$$B_t = \varpi \sum_{s=t+1}^{\infty} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} \left(\beta_u(1-\rho)\right)^{s-t} \frac{K_t}{K_s}$$

so equation (B.2.17) becomes:

$$\frac{Z_{t+1}}{K_t} = \frac{M_t}{B_t} \tag{B.2.18}$$

and since variables with subscript t are constant across s, and defining a new variable $L_t = \frac{B_t}{K_t}$, we can write:

$$M_t = \sum_{s=t+1}^{\infty} U_{nd,us} \frac{P_{nd,t}}{P_{nd,s}} \left(\beta_u(1-\rho)\right)^{s-t} \left(R_s^z - R_s + \varpi\right)$$

$$L_t = \varpi \sum_{s=t+1}^{\infty} U_{nd,us} \frac{P_{nd,t}}{P_{nd,s}} \left(\beta_u(1-\rho)\right)^{s-t} K_s^{-1}$$

Now, deal with M_t first. Define $m_s = U_{nd,us}\left(R_s^z - R_s + \varpi\right)$, rearrange, and use the fact that for $s = t$, $M_t = m_t$. Finally we define the variable $\tilde{M}_t = M_t + m_t$, so we have:

$$M_t = \sum_{s=t+1}^{\infty} m_s \left(\frac{P_{nd,s}}{P_{nd,t}}\right)^{-1} \left(\beta_u(1-\rho)\right)^{s-t}$$

$$M_t = -m_t + \sum_{s=t}^{\infty} m_s \left(\frac{P_{nd,s}}{P_{nd,t}}\right)^{-1} \left(\beta_u(1-\rho)\right)^{s-t}$$

$$\tilde{M}_t = M_t + m_t = \sum_{s=t}^{\infty} m_s \left(\frac{P_{nd,s}}{P_{nd,t}}\right)^{-1} \left(\beta_u(1-\rho)\right)^{s-t} \tag{B.2.19}$$

Equation (B.2.19) is a direct analogue of the Calvo price setting formula derived in (Woodford 2003), with appropriate timing.

It then follows that the relationship between $\tilde{M}_t$ and $\tilde{M}_{t+1}$ is defined by:

$$\tilde{M}_t = m_t + \beta_u (1-\rho) \Pi_{t+1}^{-1} \tilde{M}_{t+1}$$

so replacing our definitions $\tilde{M}_t = M_t + m_t$ and $m_t = U_{nd,ut}\left(R_t^z - R_t + \varpi\right)$ we have:

$$M_t = \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \left(M_{t+1} + U_{nd,ut+1}\left(R_{t+1}^z - R_{t+1} + \varpi\right)\right) \tag{B.2.20}$$

We can follow the same procedure for L_t to derive:

$$L_t = \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \left(L_{t+1} + \varpi U_{nd,ut+1} K_{t+1}^{-1}\right)$$

so that given our definition $L_t = \frac{B_t}{K_t}$, B_t is:

$$B_t = \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \left(L_{t+1} + \varpi U_{nd,ut+1} K_t K_{t+1}^{-1}\right) \tag{B.2.21}$$

and we can rearrange the equation (B.2.18) for Z_{t+1}:

$$\frac{Z_{t+1}}{P_{ndt}} = \frac{K_t}{P_{ndt}} \frac{M_t}{B_t} \tag{B.2.22}$$

Equations (B.2.20), (B.2.21) and (B.2.22) define the dynamics of the profit maximising level of real debt that is issued by individual financial intermediaries $\left(\frac{Z_{t+1}}{P_{ndt}}\right)$, as a function of inflation $\left(\Pi_{t+1}^{-1}\right)$, collateral, $\left(\frac{K_t}{K_{t+1}}\right)$, the markup of the average rate on debt over the central bank rate $(R_{t+1}^z - R_{t+1})$, and the marginal utility of consumption of non-durables by patient households $(U_{nd,ut+1})$. These equations also show how real debt evolution is affected by the discount rate of patient households β_u, the stickiness of debt contracts ρ, and the parameter ϖ (which captures the relative size of quadratic costs associated with deviating from the optimal level of debt).

Evolution of Aggregate Debt

Aggregate nominal debt $\mathcal{Z}$ at time t is the sum of all debt contracts written prior to and at time t:

$$\mathcal{Z}_t = \rho \sum_{s=-\infty}^{t} \left(1-\rho\right)^{t-s} Z_s = \rho\left(Z_t + \left(1-\rho\right) Z_{t-1} + \left(1-\rho\right)^2 Z_{t-2} + ...\right)$$

It follows (since $|1-\rho|<1$ we have the sum of a geometric series that converges and is summable) at $t+1$:

$$
\begin{aligned}
\mathcal{Z}_{t+1} &= \rho \sum_{s=-\infty}^{t+1} (1-\rho)^{t+1-s} Z_s = \rho \left(Z_{t+1} + (1-\rho) Z_t + (1-\rho)^2 Z_{t-1} + ... \right) \\
&= \rho Z_{t+1} + \rho (1-\rho) Z_t + \rho (1-\rho)^2 Z_{t-1} + ... \\
&= \rho Z_{t+1} + (1-\rho) \rho \left(Z_t + (1-\rho) Z_{t-1} + ... \right) \\
&= \rho Z_{t+1} + (1-\rho) \mathcal{Z}_t
\end{aligned}
$$

so:

$$
\mathcal{Z}_t = \frac{1}{(1-\rho)} \mathcal{Z}_{t+1} - \frac{\rho}{(1-\rho)} Z_{t+1}
$$

Divide by the non-durable good price index, and substitute the individual quantity of debt issued, Z_{t+1}, as determined by equation (B.2.22) (we define real collateral with lower case k_t):

$$
\begin{aligned}
\frac{\mathcal{Z}_t}{P_{ndt}} &= \frac{1}{(1-\rho)} \frac{\mathcal{Z}_{t+1}}{P_{ndt}} - \frac{\rho}{(1-\rho)} \frac{Z_{t+1}}{P_{ndt}} \\
\frac{\mathcal{Z}_t}{P_{ndt-1}} \frac{P_{ndt-1}}{P_{ndt}} &= \frac{1}{(1-\rho)} \frac{\mathcal{Z}_{t+1}}{P_{ndt}} - \frac{\rho}{(1-\rho)} \frac{M_t}{B_t} k_t
\end{aligned} \tag{B.2.23}
$$

Since constrained households always take all debt offered to them by financial intermediaries, the constrained debt a_{ct} defined in equation (4.2.13) follows the dynamics of $\frac{\mathcal{Z}_{t+1}}{P_{nd,t}}$. We can therefore re-write equation (B.2.23) as:

$$
\begin{aligned}
a_{ct-1} &= \frac{\Pi_t}{(1-\rho)} \left(a_{ct} - \rho \frac{M_t}{B_t} k_t \right) \\
a_{ct} &= (1-\rho) \frac{a_{ct-1}}{\Pi_{ct}} + \rho \frac{M_t}{B_t} k_t
\end{aligned}
$$

Rates on Debt

Its optimisation problem will be to maximise the present discounted value of profit that will flow from this contract, so that at time t, it will discount future periods s only for future scenarios in which the contract is not readjusted, so using $(1-\rho)^{s-t}$.

In addition, since patient households own the financial intermediaries, their stochastic discount factor ($Q_{t,s}$ derived in equation (B.2.6)) is also used to discount the future flow of profit, so the problem is written as:

The optimal rate set on new fixed rate contracts, R_t^{zF}, is determined by a no arbitrage condition that equates the present value of future expected revenues from the variable and fixed rate contracts. As for the determination of the optimal quantity of debt, at time t the financial intermediary will discount future periods s only for future scenarios in which the contract is not readjusted, so using $(1-\rho)^{s-t}$. As before, the patient households' stochastic discount factor ($Q_{t,s}$ derived in equation (B.2.6)) is also used to discount the future flow of profit, so the problem is written as:

$$\sum_{s=t+1}^{\infty} Q_{t,s} R_t^{zF} (1-\rho)^{t-s} Z_{t+1} = \sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{t-s} R_s Z_{t+1}$$

$$R_t^{zF} = \frac{\sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{t-s} R_s}{\sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{t-s}} \tag{B.2.24}$$

We define the variables (replacing $Q_{t,s}$ with the patient stochastic discount factor of equation (B.2.7)):

$$\begin{aligned} V_t &= \sum_{s=t+1}^{\infty} \beta_u^{s-t} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} (1-\rho)^{t-s} R_s \\ \mathcal{U}_t &= \sum_{s=t+1}^{\infty} \beta_u^{s-t} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} (1-\rho)^{s-t} \\ R_t^{zF} &= \frac{V_t}{\mathcal{U}_t} \end{aligned} \tag{B.2.25}$$

and since variables with subscript t are constant across s we can write:

$$\begin{aligned} V_t &= \sum_{s=t+1}^{\infty} (\beta_u (1-\rho))^{s-t} U_{nd,us} R_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \\ \mathcal{U}_t &= \sum_{s=t+1}^{\infty} (\beta_u (1-\rho))^{s-t} U_{nd,us} \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \end{aligned}$$

As for the determination of debt, we organise V_t and $\mathcal{U}_t$ into the same form as for Calvo pricing in (Woodford 2003). Dealing with V_t first we define $v_s = U_{nd,us}R_s$, then use the fact that for $s = t$, $V_t = v_t$, and define $\tilde{V}_t = V_t + v_t$ so we have:

$$\begin{aligned}
V_t &= \sum_{s=t+1}^{\infty} \left(\beta_u \left(1-\rho\right)\right)^{s-t} v_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \\
V_t &= -v_t + \sum_{s=t}^{\infty} \left(\beta_u \left(1-\rho\right)\right)^{s-t} v_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \\
\tilde{V}_t &= V_t + v_t = \sum_{s=t}^{\infty} \left(\beta_u \left(1-\rho\right)\right)^{s-t} v_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \qquad \text{(B.2.26)}
\end{aligned}$$

It then follows that the relationship between $\tilde{V}_t$ and $\tilde{V}_{t+1}$ is defined by:

$$\tilde{V}_t = v_s + \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \tilde{V}_{t+1}$$

so replacing our definitions $\tilde{V}_t = V_t + v_t$ and $v_t = U_{nd,ut}R_t$ we have:

$$V_t = \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \left(V_{t+1} + U_{nd,ut+1}R_{t+1}\right) \qquad \text{(B.2.27)}$$

We can follow the same procedure for $\mathcal{U}_t$ to derive:

$$\mathcal{U}_t = \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \left(\mathcal{U}_{t+1} + U_{nd,ut+1}\right) \qquad \text{(B.2.28)}$$

Equation (B.2.25) combined with (B.2.27) and (B.2.28) describe the dynamics of the new fixed rates R_t^{zF} set each period by those financial intermediaries who have the opportunity to re-set the contract.

B.2.4 Profits of Firms and Financial intermediaries

Aggregate intra-period nominal profit in non-durable goods sector is

$$\begin{aligned}
\Pi_{ndt} &= P_{ndt}Y_{ndt} - W_{ut}N_{nd,ut} - W_{ct}N_{nd,ct} \\
&= P_{ndt}Y_{ndt} - P_{ndt}w_{ut}N_{ndut} - P_{ndt}w_{ct}N_{ndct}
\end{aligned}$$

And in the durable goods sector it is

$$\begin{aligned}\Pi_{dt} &= P_{ndt}Y_{dt} - W_{ut}N_{dut} - W_{ct}N_{dct} \\ &= P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}N_{dut} - P_{ndt}w_{ct}N_{dct}\end{aligned}$$

Total nominal profit is

$$\begin{aligned}\tilde{\Pi}_t &= \Pi_{pt} + \Pi_{dt} = P_{ndt}Y_{pt} - P_{ndt}w_{ut}N_{put} - P_{ndt}w_{ct}N_{pct} \\ & +P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}N_{dut} - P_{ndt}w_{ct}N_{dct} \\ &= P_{ndt}Y_{pt} + P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}\left(N_{put} + N_{dut}\right) - P_{ndt}w_{ct}\left(N_{pct} + N_{dct}\right) \\ &= P_{ndt}Y_{pt} + P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}N_{ut} - P_{ndt}w_{ct}N_{ct} \qquad \text{(B.2.29)}\end{aligned}$$

We assume that the profit is 100% taxed by the government and redistributed according to the following rule:

$$\begin{aligned}t_{ct} &= (1-x)\frac{\tilde{\Pi}_t}{P_{ndt}} \\ t_{ut} &= x\frac{\tilde{\Pi}_t}{P_{ndt}}\end{aligned}$$

where $x = \nu$ in the simplest case, but we can also look at more general case, so I leave it with x for now. We can keep at as a parameter for now. From (B.2.29) it follows that

$$P_{ndt}Y_{pt} + P_{ndt}q_tY_{dt} - W_{ut}N_{ut} - W_{ct}N_{ct} = \tilde{\Pi}_t = P_{ndt}t_{ct} + P_{ndt}t_{ut} \qquad \text{(B.2.30)}$$

We can substitute it in budget constraint and will do it later.

$$\begin{aligned}t_{ct} &= (1-x)\left(Y_{pt} + q_tY_{dt} - w_{ut}N_{ut} - w_{ct}N_{ct}\right) \\ t_{ut} &= x\left(Y_{pt} + q_tY_{dt} - w_{ut}N_{ut} - w_{ct}N_{ct}\right)\end{aligned}$$

The two budget constraints:

$$a_{ut} = (1+R_t)\left(\frac{a_{ut-1}}{\Pi_{c,t}} + w_{ut}N_{ut} + t_{ut} - C_{ut} - q_t\left(D_{ut} - (1-\delta)D_{ut-1}\right)\right) + \tilde{d}_t, \qquad \text{(B.2.31)}$$

$$a_{ct} = \left(1 + R_t^D\right)\left(\frac{a_{ct-1}}{\Pi_{c,t}} + C_{ct} + q_t\left(D_{ct} - (1-\delta)\, D_{ct-1}\right) - w_{ct}N_{ct} - t_{ct}\right)$$

imply that aggregate real profits of financial intermediaries that is distributed as dividends:

$$\tilde{d}_t = \left(R_t^D - R_t\right)\left(\frac{a_{ct-1}}{\Pi_{c,t}} + C_{ct} + q_t\left(D_{ct} - (1-\delta)\, D_{ct-1}\right) - w_{ct}N_{ct} - t_{ct}\right)$$

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