

Milli-kelvin Thermodynamic and Transport Measurements of Low Dimensional Systems in High Magnetic Fields

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.....Martin J Smith

Abstract

This thesis presents an investigation into aspects of the integer quantum Hall effect, specifically the near-dissipationless state of the longitudinal resistivity ρ_{xx} between Landau levels, and the associated broadening of the levels. Eddy currents induced by a time varying magnetic field B are considered in chapter 4. The temperature dependences of the eddy currents were measured over the range 100 mK to 1600 mK. The peak current at filling factor $\nu = 2$ was shown to saturate at $\gtrsim 800$ mK, more robust than previously observed, but was reduced by elevating the temperature to 1600 mK. The saturated regime is associated with a breakdown of the quantum Hall effect, and in this case, the most likely candidate for the saturation is an electron heating effect.

Sweep-rate dependences were characterised for a range of filling factors and temperatures, and even for the lowest sweep rates, never entered a linear regime. Induced currents $\nu = 1, 2$ and 4 all saturated at the same critical value at 100 mK, but $\nu = 4$ was shown to reduce with slower sweep rates, consistent with the prediction that the ρ_{xx} minima is not as small as for lower Landau levels. Induced current decays were measured to be similar to previous work, a fast initial decay attributed to breakdown of the QHE followed by a much longer slow decay. The eddy decay of $\nu = 1$ at low temperature, in the slow decay regime, is among the most persistent reported. It was shown that the assumptions of previous work had not evaluated the mutual inductance of the eddy current in the presence of the magnet sufficiently. By fitting a suitable function to the IV characteristic of $\nu = 1$ the shape of the induced current was modeled. The model agreed with the data, producing a similar shape and a very long time constant for the slow decay.

In chapter 5 the hysteresis in the magnetoresistance of a quantum point contact was investigated, through a simultaneous transport and magnetometry measurement. Induced currents corresponding to filling factors up to $\nu = 8$ were measured. Three corresponding features were measured in the magnetoresistance of a QPC, one more than previously seen. The temperature dependence was measured simultaneously, and for Landau level filling factor $\nu = 1$, the general shape of the curves was the same. The sweep rate IV characteristics of the the two experiments were similar. Sweeping the magnetic field B to a fixed field position and waiting, demonstrated that both phenomena decay with time, a fast decay of seconds and a slow decay taking more than 10,000 seconds. An attempt was made to affect the induced eddy current by switching the QPC gate on/off. Experiments on a fast timescale, 10 ms, resolved structure in the induced currents that has previously been attributed to the noisy breakdown of the quantum Hall effect. By performing a simultaneous measurement, individual breakdown events were seen and correlated.

After investigating the zero-resistance state in chapter 4 and chapter 5 with induced currents, exactly how the zero-resistance state varied between Landau levels was the topic of chapter 6. A method was presented for the fabrication of a novel device, to measure the magnetisation and the heat capacity of a 2DES at the same time. AuGe thin film resistors were grown in only 10 bilayers, reducing the heat capacity per unit area by approximately an order of magnitude on previous workers. The AuGe thermometers were shown to be ‘tunable’, i.e. the temperature dependence was dictated by the annealing conditions after growth, so thermometers with different gold concentrations due to growth conditions, could be tuned to have similar temperature dependences. Low temperature thermometers with small heat capacities were repeatably produced, and thermometer D5 is presented in this thesis with a sensitivity of $S = 0.58$.

At an elevated refrigerator temperature of nearly 300 mK, heat pulses of ~ 26 nJ were resolved on a device which had a 100% front processing success rate, but was not etched from the back. It was shown that a device to measure the broadening of the low temperature, high magnetic field 2DES density of states is possible.

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