Coupling the Planetary Boundary Layer to the Large Scale Dynamics of the Atmosphere: The Impact of Vertical Discretisation

Submitted by

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Dan Holdaway

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Abstract

Accurate coupling between the resolved scale dynamics and sub-grid scale physics is essential for accurate modelling of the atmosphere. Previous emphasis has been towards the temporal aspects of this so called physics-dynamics coupling problem, with little attention towards the spatial aspects. When designing a model for numerical weather prediction there is a choice for how to vertically arrange the required variables, namely the Lorenz and Charney-Phillips grids, and there is ongoing debate as to which is the optimal. The Charney-Phillips grid is considered good for capturing the large scale dynamics and wave propagation whereas the Lorenz grid is more suitable for conservation. However the Lorenz grid supports a computational mode. In the first half of this thesis it is argued that the Lorenz grid is preferred for modelling the stably stratified boundary layer. This presents the question: which grid will produce most accurate results when coupling the large scale dynamics to the stably stratified planetary boundary layer? The second half of this thesis addresses this question.

The normal mode analysis approach, as used in previous work of a similar nature, is employed. This is an attractive methodology since it allows one to pin down exactly why a particular configuration performs well. In order to apply this method a one dimensional column model is set up, where horizontally wavelike solutions with a given wavenumber are assumed. Applying this method encounters issues when the problem is non normal, as it will be when including boundary layer terms. It is shown that when addressing the coupled problem the lack of orthogonality between eigenvectors can cause mode analysis to break down. Dynamical modes could still be interpreted and compared using the eigenvectors but boundary layer modes could not. It is argued that one can recover some of the usefulness of the methodology by examining singular vectors and singular values; these retain the appropriate physical interpretation and allow for valid comparison due to orthogonality between singular vectors.

Despite the problems in using the desirable methodology some interesting results have been gained. It is shown that the Lorenz grid is favoured when the
boundary layer is considered on its own; it captures the structures of the steady states and transient singular vectors more accurately than the Charney-Phillips grid. For the coupled boundary layer and dynamics the Charney-Phillips grid is found to be most accurate in terms of capturing the steady state. Dispersion properties of dynamical modes in the coupled problem depend on the choice of horizontal wavenumber. For smaller horizontal wavenumber there is little to distinguish between Lorenz and Charney-Phillips grids, both the frequency and structure of dynamical modes is captured accurately. Dynamical mode structures are found to be harder to interpret when using larger horizontal wavenumbers; for those that are examined the Charney-Phillips grid produces the most sensible and accurate results. It is found that boundary layer modes in the coupled problem cannot be concisely compared between the Lorenz and Charney-Phillips grids due to the issues that arise with the methodology. The Lorenz grid computational mode is found to be suppressed by the boundary layer, but only in the boundary layer region.
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