

# Molecular line transfer calculations in star-forming regions

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Signed: .....

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## Abstract

This thesis describes the development, benchmarking and application of a non-LTE, co-moving frame Monte Carlo molecular line radiative transfer module for TORUS. Careful attention has been paid to the convergence, acceleration and optimisation of the code.

I present the results of the application of the code to various benchmarking scenarios, including a collapsing cloud, a circumstellar disc and a very optically thick cloud of interstellar water. Benchmarking is an essential step in verifying the accuracy and efficiency of the code which is vital if it is to be used to analyse real data. In all cases, the code was able to accurately reproduce either the expected analytical solution or (in the absence of such a solution) was able to produce results commensurate with the results of other codes.

In order to facilitate the motivating radiative transfer calculations of a star-forming cluster simulated using smoothed particle hydrodynamics (SPH) performed in this thesis, it was first necessary to devise and test an algorithm that efficiently maps an irregular distribution of smoothed particle hydrodynamics (SPH) particles onto a regular adaptive mesh. Whilst the algorithm was designed with this in mind it has also been used to study the effects of radiative feedback in circumstellar discs as well create a synthetic survey of a simulated galaxy.

Bate et al.'s particle representation was resampled onto an adaptive mesh to enable me to use TORUS to obtain non-LTE level populations of multiple molecular species throughout the cluster and create velocity-resolved datacubes by calculating the emergent intensity using raytracing. I compared line profiles of cores traced by  $\text{N}_2\text{H}^+(1-0)$  to probes of low density gas ( $^{13}\text{CO}$  and  $\text{C}^{18}\text{O}(1-0)$ ) surrounding the cores along the line-of-sight. The relative differences of the line-centre velocities were found to be small compared to the velocity dispersion, matching recent observations. The conclusion is that one cannot reject competitive accretion as a viable theory of star formation based on observed velocity profiles.

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## Declaration

Much of the work described in this thesis has been published in scientific journals. Specifically, significant portions of chapters 2, 3, 4 and 5 have been accepted for publication in the Monthly Notices of the Royal Astronomical Society (MNRAS), arXiv e-print: 1005.1648 (doi:10.1111/j.1365-2966.2010.16982.x).

Chapter 4 also contains the results of work performed by Acreman, Harries and Rundle which is published in MNRAS, 2010, volume 403, pp. 1143 – 1155 as well as work performed using parts of the TORUS radiative transfer code written by me by Acreman, Douglas, Dobbs and Brunt (10.1111/j.1365-2966.2010.16858.x) and Douglas, Acreman, Dobbs and Brunt (doi:10.1111/j.1365-2966.2010.16906.x). The work was facilitated by the SPH to AMR algorithm written by me and described in the same chapter.

Both are available online at the MNRAS Early View website.