

**Optoelectronic properties of carbon-based nanostructures:  
Steering electrons in graphene by electromagnetic fields**

Submitted by Richard Rudolph Hartmann,  
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## Abstract

Graphene has recently become the focus of enormous attention for experimentalists and theorists alike mainly due to its unique electronic properties. However, the limited way in which one can control these properties is a major obstacle for device applications. The unifying theme of this thesis is to propose and thoroughly justify ways to control the electronic properties of graphene and carbon nanotubes by light or static electric and magnetic fields and to harness these properties for optoelectronic applications.

A linearly polarized excitation is shown to create a strongly anisotropic distribution of photoexcited carriers in graphene, where the momenta of photoexcited carriers are aligned preferentially normal to the polarization plane. This effect offers an experimental tool to generate highly directional photoexcited carriers which could assist in the investigation of "direction-dependent phenomena" in graphene-based nanostructures. The depolarization of hot photoluminescence is used to study relaxation processes in graphene, both free standing and grown on silicon carbide. This analysis is extended to include the effect of a magnetic field, thereby allowing one to obtain the momentum relaxation times of hot electrons. The analysis of momentum alignment in the high frequency regime shows that a linearly polarized excitation allows the spatial separation of carriers belonging to different valleys.

Quasi-metallic carbon nanotubes are considered for terahertz applications. They are shown to emit terahertz radiation when a potential difference is applied across their ends and their spontaneous emission spectra have a universal frequency and bias voltage dependence. It is shown that the same intrinsic curvature which opens the gap in the quasi-metallic carbon nanotube energy spectrum also allows optical transitions in the terahertz range. The exciton binding energy in narrow-gap carbon nanotubes is calculated and found to scale with the band gap and vanishes as the gap decreases, even in the case of strong electron-hole attraction. Therefore, excitonic effects should not dominate in narrow-gap nanotubes.

Contrary to widespread belief, it is shown that full confinement is possible for zero-energy states in pristine graphene. The exact analytical solutions for the zero-energy modes confined within a smooth one-dimensional potential  $V = \alpha/\cosh(\beta x)$  are presented. This potential provides a good fit for the potential profiles of top-gated graphene structures. It is shown that there is a threshold value of the characteristic potential strength  $\alpha/\beta$  for which the first mode appears, in striking contrast to the non-relativistic case. A relationship between the characteristic strength and the number of modes within the potential is found. An experimental setup is proposed for the observation of these modes. The proposed geometry could be utilized in future graphene-based devices with high on/off current ratios.

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