Conditional Betas, Higher Comoments and the Cross-Section of Expected Stock Returns

by

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

To my parents

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Abstract

This thesis examines the performance of different models of conditional betas and higher comoments in the context of the cross-section of expected stock returns, both in-sample and out-of-sample.

I first examine the performance of different conditional market beta models by using monthly returns of the Fama-French 25 portfolios formed by the quintiles of size and book-to-market ratio in Chapter 3. This is a cross-sectional test of the conditional CAPM. The models examined include simple OLS regressions, the macroeconomic variables model, the state-space model, the multivariate GARCH model and the realized beta model. The results show that the state-space model performs best in-sample with significant betas and insignificant intercepts. For the out-of-sample performance, however, none of the models examined can explain returns of the 25 portfolios.

Next, I examine the recently proposed realized beta model, which is based on the realized volatility literature, by using individual stocks listed in the US market in Chapter 4. I extend the realized market beta model to betas of multi-factor asset pricing models. Models tested are the CAPM, the Fama-French three-factor model and a four-factor model including the three Fama-French factors and a momentum factor. Realized betas of different models are used in the cross-section regressions along with firm-level variables such as size, book-to-market ratio and past returns. The in-sample results show that market beta is significant and additional betas of multi-factor models can reduce although not eliminate the effects of firm-level variables. The out-of-sample results show that no betas are significant. The results are robust across different markets such as NYSE, AMEX and NASDAQ.

In Chapter 5, I test if realized coskewness and cokurtosis can help explain the cross-section of stock returns. I add coskewness and cokurtosis to the factor pricing models tested in Chapter 4. The results show that the coefficients of coskewness and cokurtosis have the correct sign as predicted by the higher-moment CAPM theory but only cokurtosis is significant. Cokurtosis is significant not only in-sample but also out-of-sample, suggesting

cokurtosis is an important risk. However, the effects of firm-level variables remain significant after higher moments are included, indicating a rejection of higher-moment asset pricing models. The results are also robust across different markets such as NYSE, AMEX and NASDAQ.

The overall results of this thesis indicate a rejection of the conditional asset pricing models. Models of systematic risks, i.e. betas and higher comoments, cannot explain the cross-section of expected stock returns.

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Abbreviations

AMEX	American Stock Exchange
APT	Arbitrage Pricing Theory
AR	Auto Regression
ARCH	Auto Regressive Conditional Heteroskedasticity
ARMA	Auto Regression Moving Average
BM	Book-to-Market Ratio
CAPM	Capital Asset Pricing Model
CAY	Consumption-to-Wealth Ratio
CML	Capital Market Line
CRSP	The Centre for Research in Security Prices
D/E	Debt-to-Equity Ratio
DCC	Dynamic Conditional Correlation
FF3F	Fama-French Three-Factor Model
GARCH	Generalized Auto Regressive Conditional Heteroskedasticity
GDP	Gross Domestic Product
GLS	Generalized Least Squares
GMM	Generalized Methods of Moments
ICAPM	Intertemporal Capital Asset Pricing Model
MA	Moving Average
MCMC	Markov Chain Monte Carlo
NASDAQ	National Association of Securities Dealers Automated Quotations
NYSE	New York Stock Exchange
OLS	Ordinary Least Squares
P/E	Price-to-Earnings Ratio
WLS	Weighted Least Squares

Chapter 1

Introduction

1.1 Background

One of the central problems of finance is understanding the cross-section of asset returns. In academic research, it is at the centre of both theoretical and empirical studies of asset pricing models. In theoretical studies, a successful asset pricing model must be able to explain the cross-sectional patterns of returns of different assets. In empirical studies, the cross-section of returns has been explored in order to test theories and find interesting patterns for further theoretical research and practical use. In practice, investors also need to know what drives the different performances among assets when they make investments.

Although the first stock market was established more than 400 years ago (in Amsterdam in 1602), the first theory of the cross-section of returns was proposed only in the 1960s, the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). In the CAPM, cross-sectional differences between returns are decided only by differences of systematic risk, called market beta. This conclusion is easy to understand because only systematic risk will be compensated and idiosyncratic risk will be diversified away in a well-diversified portfolio.

Early empirical tests of the CAPM focus on the relationship between returns and market beta. Researchers generally reject the model but find a positive coefficient of market beta in cross-sectional regressions (e.g. Black et al., 1972; Fama and MacBeth, 1973), indicating market beta is a priced risk although it alone cannot fully explain the cross-section of stock returns. Since the late 1970s, researchers have found other firm-level fundamental variables are also related to the cross-section of stock returns, e.g. price-to-earnings ratio (Basu, 1977), size (Banz, 1981) and leverage (Bhandari, 1988). These findings indicate a rejection of the CAPM.

In 1992, Fama and French, in an influential study (Fama and French, 1992), comprehensively examine the cross-sectional relationship between stock returns, market beta and firm-level variables. They show that market beta is not priced but firm-level variables are significantly related to returns. Among firm-level variables, the combination of size and book-to-market ratio (BM) can drive out the explanatory abilities of other variables. Furthermore, Fama and French (1993, 1996) show that portfolios formed by size and BM are particularly challenging for the CAPM. They propose a new model with three factors related to the market, size and BM and show that the three-factor model can explain the returns of portfolios formed by different firm-level variables, except momentum portfolios. The studies of Fama and French have stimulated a rapidly expanding literature on the cross-section of stock returns.

Another important finding is the momentum effect of Jegadeesh and Titman (1993). They find that past returns within twelve months are positively correlated with future returns: past winners continue to be winners and past losers continue to be losers. The return differences between winners and losers cannot be explained by the CAPM or the Fama-French three-factor model. The momentum portfolios, along with the Fama-French size/BM portfolios, are among the most serious challenges to the CAPM.

Huge academic efforts have been devoted to explain the effects of size, BM and momentum. Within the asset pricing framework, there are three explanations: the conditional CAPM, multi-factor models and the higher-moment CAPM. At this point, I will briefly introduce the three categories, with details to follow in the next four chapters.

The CAPM of Sharpe and Lintner is an unconditional model where market beta is assumed to be constant. Hansen and Singleton (1982) prove that the conditional CAPM may hold even if the unconditional CAPM fails. In the conditional CAPM, conditional market beta is time-varying. Therefore, many researchers have tried to develop models for time-varying market beta. Widely used models in the literature include the popular rolling window estimation, the macroeconomic variables model (e.g. Shanken, 1990; Ferson and Harvey, 1999), the state-space model (e.g. Faff et al., 2000; Jostva and Philipov, 2004), the multivariate GARCH model (e.g. Braun et al., 1994; Bali, 2008) and the recently proposed realized beta model (Anderson et al., 2005, 2006).

The market return is the single risk factor generating returns of individual assets in the CAPM. Some researchers attribute the failure of the CAPM to the fact that the market return alone is not enough to explain asset returns so that other risk factors should be included. Theoretical frameworks include the intertemporal CAPM (ICAPM) of Merton (1973) and the arbitrage pricing theory (APT) of Ross (1974). However, the theory of multi-factor models does not give factors explicitly so that researchers must find them from empirical studies. Early studies use macroeconomic variables as factors (e.g. Chen et al., 1986). Recently, due to the success of explaining the cross-section of stock returns, models based on empirical findings of firm-level variables have become popular such as the Fama-French three-factor model (Fama and French, 1996) and Carhart's four-factor model (Carhart, 1997). Subsequent studies put multi-factor models into a conditional framework so that multi-factor betas are also time-varying (e.g. Ferson and Harvey, 1999; Wang, 2003).

The CAPM is developed based on the mean-variance analysis of Markowitz (1952) where investors are assumed to care only about the mean and variance of returns. If returns do not follow an elliptical distribution and investors care about higher moments, such as skewness and kurtosis, then higher moments will be priced. This intuition has led to the development of the higher-moment CAPM. Kraus and Litzenberger (1976) propose a three-moment CAPM where coskewness is added into the traditional CAPM.

Fang and Lai (1997) extend this model to the four-moment CAPM. More recently, the conditional higher-moment CAPM has achieved some success in explaining the cross-section of stock returns. Harvey and Siddique (1999) and Smith (2007) find conditional coskewness is important; Dittmar (2002) proposes a conditional four-moment CAPM and finds that it cannot be rejected by using industry portfolios.

In modern finance, the cross-section of asset returns remains at the centre of finance research. New patterns have been found and existing patterns have been refined[1]; new models have been proposed and tested. In practice, practitioners, such as portfolio managers, pay close attention to academic findings in the cross-section of stock returns so that they can gain more guidelines in their investments. Overall, the cross-section of stock returns is one of the central problems in both academia and practice.

1.2 Motivation

The existing literature on testing conditional asset pricing models mainly focuses on their in-sample performance. For example, in tests of the conditional CAPM, Lettau and Ludvigson (2001) propose using the consumption-to-wealth ratio (CAY) to model market beta and find this variable is useful in explaining size/value portfolios in-sample; Jostova and Philipov (2004) propose the stochastic beta model, which is actually a state-space model estimated by the Markov chain Monte Carlo (MCMC) method, and test its in-sample cross-sectional performance by using individual stocks; Bali (2008) uses a bivariate GARCH model for the conditional beta and this study is also in-sample.

In the context of multi-factor models and the higher-moment CAPM, many studies also focus on in-sample performance. The original three-factor model of Fama and French (1996) is proposed and tested in an unconditional form. Subsequent tests of its conditional version focus on its in-sample performance such as He et al. (1996). For the

[1] See chapter 2 for details.

higher-moment CAPM, due to the difficulties of modelling coskewness and cokurtosis, most studies also focus on in-sample performance such as Kim (1987), Ditmmar (2002) and Smith (2007).

However, a true test of conditional asset pricing models should be an out-of-sample test. In the theoretical setting of conditional asset pricing models, investors only use information available when they make investment decisions. Using the full sample to estimate models inevitably utilises information beyond investors' information set and therefore can lead to some bias, such as the over-conditioning bias studied by Boguth et al. (2008). In practice, it could be misleading if investors make decisions based on the in-sample performance of a model because a model's out-of-sample performance may be substantially different from its in-sample performance.

Based on the considerations above, I examine whether different conditional models can explain the cross-section of stock returns not only in-sample but also out-of-sample. In out-of-sample tests, I use information only available at time period t to estimate the model and then use estimated parameters to forecast betas or higher comoments of time period $t+1$. The one-step-ahead forecasted betas or higher comoments are used in the cross-sectional regressions. In this way, I can test if a conditional model can truly explain the cross-section of stock returns out-of-sample. This is more relevant to conditional asset pricing models both in academic theory and in practice.

1.3 Contributions

The first contribution of the thesis is the examination of the out-of-sample performance of different conditional asset pricing models. As discussed in the last section, out-of-sample tests of conditional asset pricing models are more important than in-sample tests but many studies focus only on in-sample tests.

Some existing studies do use out-of-sample cross-sectional tests of conditional models but they mainly restrict their techniques to the rolling window OLS regressions. For example, the study by Avramov and Chordia (2006) uses a 36-month rolling window to estimate their models. Researchers who propose more advanced techniques usually only examine in-sample performance such as the studies cited in the last section (e.g. Jostova and Philipov, 2004; Bali, 2008).

The second contribution of the thesis is the examination of the performance of the recently proposed realized beta model in the cross-section of stock returns by using both portfolios and individual stocks listed in the US market, both in-sample and out-of-sample. Realized beta (Andersen et al., 2005, 2006) is based on the recent literature of realized volatility (Andersen et al., 2003; Barndorff-Nielson and Shephard, 2004). Andersen et al. (2005) study the time series properties of realized market beta of the Fama-French 25 portfolios and Morana (2009) tests the in-sample cross-sectional relationship between realized multi-factor betas and returns of those 25 portfolios. Andersen et al. (2006) study the properties of realized market beta of the 30 stocks in the Dow Jones Industrial Index (DJIA). However, no studies have examined the out-of-sample relationship between realized betas and returns. In this thesis, I first examine the out-of-sample relationship between forecasted realized market beta and returns of the Fama-French 25 portfolios. Then, I examine both in-sample and out-of-sample relationships between realized betas and individual stock returns by using all the stocks listed in the US market (NYSE, AMEX and NASDAQ). I also extend the realized single-factor market beta to multi-factor betas. This is the first study to examine comprehensively the relationships between realized betas and returns by using such a large universe of stocks.

The third contribution of the thesis is to extend the realized beta model to the measurement of higher comoments. Based on the realized beta model, I use high frequency returns to compute low frequency coskewness and cokurtosis and then examine the relationship between returns and higher comoments both in-sample and out-of-sample. The method of computing higher comoments is simpler than many techniques used in current literature (e.g. Harvey and Siddique, 1999). The test assets are also stocks listed in the US market.

1.4 Empirical results

The overall results show that some conditional beta models, such as the state-space model for the Fama-French 25 portfolios and the realized beta model for individual stocks, can explain part of the effects of size, value and momentum in-sample but none of the models examined can explain those effects out-of-sample.

In Chapter 3, I examine different conditional market beta models of the conditional CAPM by using monthly returns of the Fama-French 25 portfolios. The models include simple OLS regression, the macroeconomic variables model, the state-space model, the multivariate GARCH model and the realized beta model. The monthly portfolio returns are regressed on each of those betas in the cross-section. In-sample, the state-space model performs very well in the sense of a significant beta, an insignificant alpha and a high value of R-squared. Out-of-sample, however, none of the models examined can generate a significantly priced conditional beta. The results are robust across different subsamples and estimation intervals.

In Chapter 4, the recently proposed realized beta model is tested using individual stocks in the US market. I use daily returns within each month to estimate betas of different factor pricing models. The models considered are the CAPM, the Fama-French three-factor model and a four-factor model, which is the Fama-French three-factor model augmented by a momentum factor. Betas used in the cross-sectional regressions are contemporaneously measured betas, in-sample forecasted betas and out-of-sample forecasted betas. For contemporaneously measured betas, betas of the market, size factor (SMB) and momentum factor (WML) are significant while beta of the value factor (HML) is insignificant; the inclusion of betas of additional factors besides the market return does reduce the effects of size, BM and momentum although it does not eliminate those effects. For in-sample forecasted betas, betas of the market and WML remain significant but betas of SMB and HML are not. For out-of-sample forecasted betas, no betas are significantly priced. My results show that in-sample and out-of-sample forecasted betas can have very different performance. Testing a model only based on its in-sample performance may lead to the wrong conclusion. For example, Bali et al. (2009) find a significantly positive risk premium of in-sample forecasted realized market beta. My results, in contrast, show that out-of-sample forecasted realized market beta has a negative coefficient.

In Chapter 5, I extend the method of realized beta to estimate coskewness and cokurtosis. I add coskewness and cokurtosis into the cross-section regressions to examine if they can help explain the cross-section of stock returns. Similar to Chapter 4, I use contemporaneously measured, in-sample forecasted and out-of-sample forecasted coskewness and cokurtosis. The coefficients of both contemporaneously measured coskewness and cokurtosis have the correct signs but only cokurtosis is significant. Cokurtosis is an important risk because it is significant both in-sample and out-of-sample which is consistent with existing evidence of leptokurtosis of stock returns. Coskewness, however, is insignificant both in-sample and out-of-sample.

1.5 Conclusion

The unconditional CAPM cannot explain the effects of firm-level variables on the cross-section of stock returns. Academic efforts have been devoted to explain the failure of the unconditional CAPM. The explanations within the asset pricing framework include the conditional CAPM, multi-factor models and the higher-moment CAPM. In modern finance, multi-factor models and the higher-moment CAPM are also put in a conditional framework. Therefore, tests of those models focus on their conditional performance. Specifically, the question is whether conditional betas and higher comoments can explain the cross-section of stock returns.

This thesis examines different techniques of conditional betas and higher comoments. The main focus is on the comparison of in-sample and out-of-sample performance of those techniques. In in-sample analysis, the state-space model is the best model for the Fama-French 25 portfolios and realized betas and higher comoments are significant for individual stocks except beta of HML and coskewness. Out-of-sample, however, none of the models examined can generate significant betas or higher comoments. The only exception is that out-of-sample forecasted cokurtosis is significant. The results of this thesis indicate a rejection of conditional asset pricing models.

The results of the thesis cast some doubt on testing conditional asset pricing models based on their in-sample performance, which is the focus of many previous studies. This thesis shows that the results of out-of-sample forecasted betas can be substantially different from in-sample estimated betas. Betas significant in-sample will not necessarily be significant out-of-sample because the success of models' in-sample performance may be subject to over conditioning and over fitting bias. Furthermore, it may be misleading if we make judgements based on a model's in-sample performance. Therefore, it is important to test a model not only based on its in-sample performance but also out-of-sample performance.

1.6 Organization of this thesis

The remainder of this thesis is organized as follows. In Chapter 2, I review comprehensively the literature on the cross-section of expected returns within an asset pricing framework. Other explanations such as behavioural finance are also mentioned briefly to give readers a complete picture of this area.

In Chapter 3, I test different conditional market beta models by using the Fama-French 25 size/value portfolios. The state-space model performs best in-sample with insignificant intercepts and significant market beta, which is consistent with existing literature. None of the models, however, can explain the cross-section of returns of the 25 portfolios out-of-sample. The results indicate a rejection of the conditional CAPM.

Chapter 4 tests the conditional CAPM and multi-factor models using the recently proposed realized beta model. Test assets are individual stocks listed in the US market. The results show that betas of the market, size factor and momentum factor are significant in-sample but insignificant out-of-sample. Furthermore, betas cannot fully explain the effects of firm-level variables.

Chapter 5 adds realized coskewness and cokurtosis to the factor pricing models tested in Chapter 4 to test whether higher comoments are priced and can help explain the cross-section of stock returns. The results show that cokurtosis is a significant risk both in-sample and out-of-sample, which is consistent with the evidence on the leptokurtosis of returns' distributions. Coskewness, however, is insignificant. Adding coskewness and cokurtosis to factor pricing models cannot help explain the cross-section of stock returns.

The last chapter, Chapter 6, makes conclusions of the thesis and gives future research directions.

Chapter 2

Literature Review

In this chapter, I review the literature on the cross-section of stock returns within the asset pricing framework. Understanding the cross-section of stock returns has long been the centre of finance in both academia and practice, i.e. why different stocks have different expected returns. The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), which is based on the mean-variance framework of Markowitz (1952, 1959), states that the cross-section of returns is only decided by differences in stocks' systematic risk, market beta. Early empirical tests of the CAPM by Black et al. (1972) and Fama and MacBeth (1973) find there is a positive relationship between market beta and returns but this relationship is too flat. Since the 1980s, however, researchers have found that the cross-section of stock returns is related to firm-level variables such as P/E (Basu, 1977), size (Banz, 1981), and book-to-market ratio (BM) (Fama and French, 1992). Fama and French (1992) comprehensively study the relationship between returns, market beta and firm-level variables. They show that market beta is not priced but firm-level variables such as size and BM are significant. This is among the most serious challenges of the CAPM. Subsequent research has been focused on the explanation of those anomalies. Within the asset pricing framework, there are three major approaches. The first is the conditional CAPM which focuses on the time-varying property of market beta. The second is the multi-factor model such as Fama and French (1993) which uses other factors to explain the cross-section of stock returns. The third is the higher moment CAPM which adds coskewness and cokurtosis into the CAPM (e.g. Kraus and Litzenberger, 1976).

Based on the brief introduction above, the following review will start with the mean-variance analysis and the CAPM because the CAPM is the first asset pricing model and is still used as a benchmark model in both academia and practice. Furthermore, all the anomalies are actually abnormal returns under the CAPM. Then, I will give a brief review of the abnormal returns associated with different firm-level variables with focus on size, BM and past returns. A lot of research has been devoted to explaining those abnormal returns. I focus on the explanation within the asset pricing framework under the assumption of rational investors. Specifically, I give a detailed review of the conditional CAPM, multi-factor models and the higher-moment CAPM. Of course, there are other explanations such as irrational investors within the behavioural finance framework and the effects of market microstructure. However, the focus of this thesis is on asset pricing in a rational expectations framework and so I will only give a brief review of the other explanations. Asset pricing models can also be expressed in discount factor form (e.g. Cochrane, 2001) but the techniques for conditional betas cannot enter the discount factor easily, so I will only mention the discount factor models when necessary.

2.1 Mean-variance analysis and the CAPM

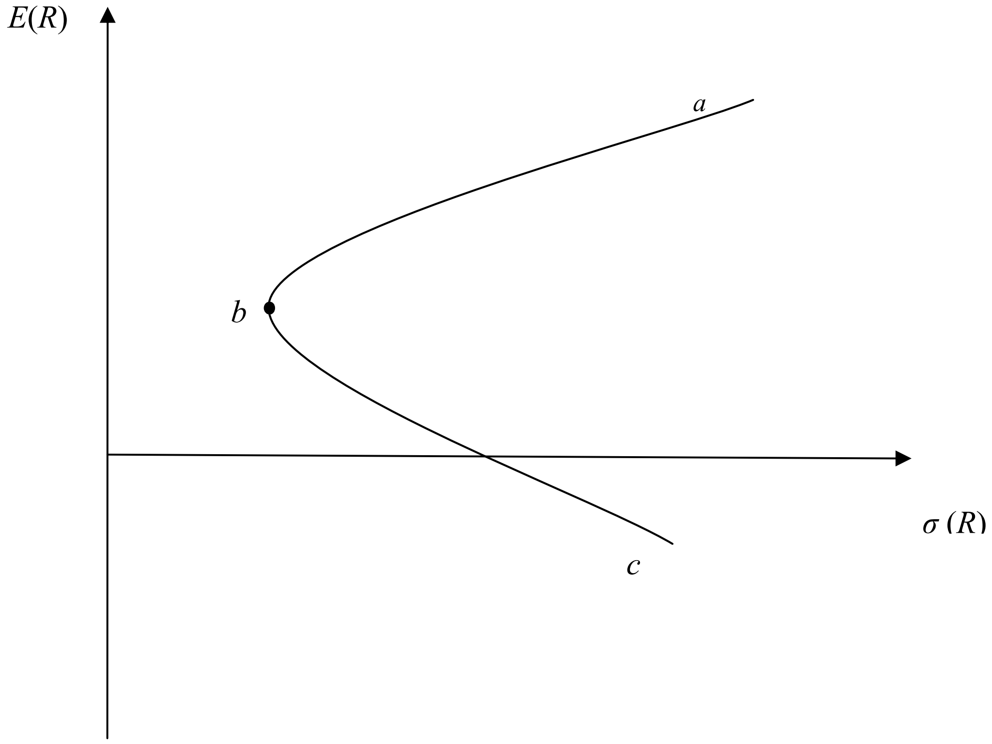
2.1.1 Mean-variance analysis

There has been a long history of dealing with risk in financial markets. The first stock market can be tracked back to 1602 when shares of the East India Company began trading in Amsterdam (Perold, 2004). However, the theoretical foundation of decision making under uncertainty was developed only from the 1940s. Von Neumann and Morgenstern (1944) develop the utility function of payoff and uncertainty and formally state the trade-off between risk and return.

Markowitz (1952, 1959) puts the risk-return trade-off in a portfolio framework and formally uses variance as a measure of risk. The assumptions of Markowitz's model include that investors are risk averse and only care about the mean and variance of portfolios for one period. Under those assumptions, investors will choose mean-variance efficient portfolios which minimize the variance for a given expected return or maximize expected return for a given variance. Figure 2.1 graphs the possible

Figure 2.1 Mean-Variance Analysis: Flexible and Efficient Set

The figure plots the flexible and efficient set of the mean and variance analysis of Markowitz. *E(R)* is the expected return and $\sigma(R)$ is the standard deviation of returns. Point *b* denotes the global minimum variance portfolios (GMVP). The curve *abc* and the area within are the flexible set but only the curve above *b* is efficient in the sense that lower standard deviations for a given return or higher returns for a given standard deviation. The curve above b is called efficient set or efficient frontier.



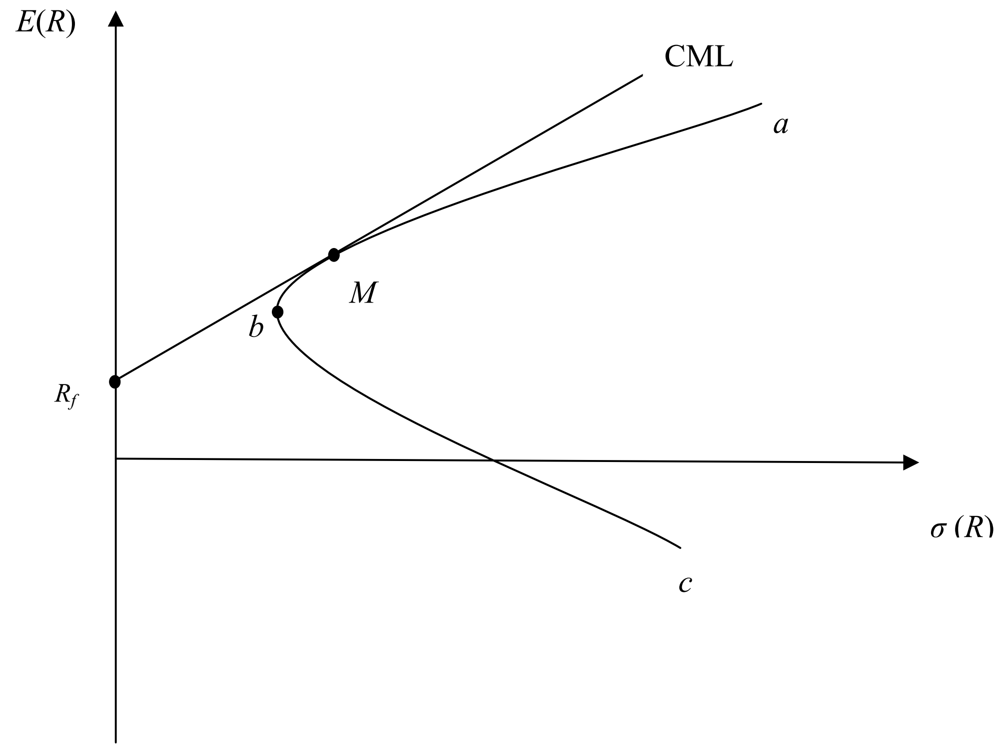
investment opportunities for investors. The curve *abc* and the area within it are all investment opportunities but only the curve above point *b*, which is the global minimum variance portfolios (GMVP), is the efficient set, also called the efficient frontier. Therefore, investors will only choose portfolios on the curve from *b* to *a* based on their utility functions.

2.1.2 The Sharpe-Lintner CAPM

Built on the mean-variance analysis of Markovitz, Sharpe (1964) and Lintner (1965) propose the CAPM by adding additional assumptions. First, investors have identical expectations of the distribution of asset returns. The second is that all investors can borrow and lend any amount at the same risk free rate. The last one is that the market is in equilibrium and is complete with no frictions. Under those additional assumptions,

Figure 2.2 The Capital Market Line (CML)

The figure plots the capital market line of Sharpe (1964) and Lintner (1965). *E(R)* is the expected return and $\sigma(R)$ is the standard deviation of returns. R_f is the risk free rate. Point *b* denotes the global minimum variance portfolios (GMVP) and the curve *abc* and the area within are the flexible set when there are only risky assets. *M* denotes the market portfolio, which is the tangent point from R_f. The straight line from R_f and *M* is the new efficient frontier when there is a risk free asset.



investors will choose the same risky assets and therefore these risky assets form the market portfolio. The efficient frontier now becomes the tangent line of the efficient frontier with only risky assets from the risk free rate, which is called the capital market line (CML). The graph of CML is in Figure 2.2. The efficient frontier is the tangent line crossing R_f and *M,* which is the market portfolio. Different investors will invest different weights in the risk free asset and the market portfolio but they will hold the same risky asset portfolio, i.e. the market. The CAPM also implies that the market portfolio is efficient. For individual assets, the CAPM implies the following relationship,

$$E\left(R_i\right) = R_f + \beta_i\left(E(R_M) - R_f\right), i = 1,..., N \qquad (2.1)$$

where β_i is called market beta and defined as

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}. \qquad (2.2)$$

According to the CAPM, asset returns are decided only by their market beta, the systematic risk. Therefore, the cross-sectional differences of asset returns are only attributed to the cross-sectional differences of market beta:

$$E(r_i) = \lambda \beta_i \qquad (2.3)$$

where r_i is the excess return of asset *i*. The risk premium, λ, should be positive, so assets with high betas should have higher returns than those with low betas.

Black (1972) relaxes the assumption that there is a risk free asset. He proves that the market portfolio is still efficient if unrestricted short sales are allowed. But the risk free rate in equation (2.1) is replaced by the return of a zero-beta portfolio. Of course, the assumption of unrestricted short sales is unrealistic but the market portfolio is no longer efficient without this assumption.

2.1.3 The conditional CAPM of Hansen and Richard (1987)

The Sharpe-Lintner CAPM states the relationship between unconditional expected return and beta. Using unconditional expectations omits conditioning information used by investors when they make decisions. Information accumulates over time and investors update their expectations when new information arrives, which in turn will result in new portfolio choices. Therefore, asset pricing models should incorporate the conditional expectations of investors.

Hansen and Richard (1987) study the conditional portfolio choice problem of investors. They solved both the unconditional and conditional mean-variance optimization problems. The unconditional optimization is to minimize the unconditional portfolio variance for given unconditional expected returns,

$$\text{Min}_w w'\Sigma w \quad s.t. \quad w'E = \mu;\, w'\iota = 1, \tag{2.4}$$

where w is a vector of weights of individual assets, ι is a vector of 1, E and $\sum$ are mean and variance/covariance matrix of returns, respectively. The conditional optimization is to minimize the conditional portfolio variance for given conditional expected returns,

$$\text{Min}_w w'\Sigma_t w \quad s.t. \quad w'E_t = \mu;\, w'\iota = 1 \tag{2.5}$$

where E_t and $\sum_t$ are the conditional mean and variance/covariance matrix of returns, respectively. Hansen and Richard find that the solution of the conditional optimization is different from the unconditional optimization.[2]

Furthermore, they prove that a portfolio on the conditional frontier may not be on the unconditional frontier. Therefore, the CAPM may hold conditionally even if it fails unconditionally. The conditional CAPM is

$$E_t(R_{i,t+1}) = r_f + \beta_{i,t}\left(E_t(R_{M,t+1}) - r_f\right), \tag{2.6}$$

where

$$\beta_{i,t} = \frac{Cov(r_{i,r+1}, r_{m,t+1} \mid I_t)}{Var(r_{m,t+1} \mid I_t)}. \tag{2.7}$$

The conditional CAPM states the relationship between conditional expected returns and

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[2] See Lemma 3.3 and 3.4 in Hansen and Richard (1987).

beta, which is the conditional counterpart of the Sharpe-Lintner CAPM.

2.2 Empirical tests of the CAPM

In this subsection, I focus on the cross-sectional tests of the CAPM which test the two implications from equation (2.3): the first is that market beta can fully explain the cross-section of asset returns and no other variables have marginal explanatory abilities and the second is that market beta has a positive risk premium. Of course, there are other approaches of testing the CAPM such as testing the efficiency of the market portfolio and time series tests of zero intercepts. My thesis focuses on the cross-section of stock returns, betas and firm-level variables so I mainly review the literature of cross-sectional tests and mention other approaches only when necessary.

Before summarizing the empirical results, it is necessary to highlight the difficulties inherent in empirical tests of the CAPM. The first is the well-known critique of Roll (1977) that the CAPM is untestable because the true market portfolio is unobservable. In empirical tests, researchers often use an index of a broad market such as the CRSP index of the US market as a proxy for the true market portfolio. Second, market beta is also unobservable. Therefore, only estimates of the true beta are used in the cross-section regressions, which cause the error-in-variables problem and can distort the estimate of the market risk premium. To overcome this problem, one can adjust the errors in estimated betas directly (e.g. Kim, 1996). A more common approach is to sort stocks into portfolios according to their betas or other variables such as size and BM because a diversified portfolio's beta can be estimated more accurately than individual assets' beta. However, sorting stocks into portfolios suffers from the well-known data snooping bias (Lo and MacKinlay, 1990). Therefore, more recent tests also attempt to use individual stocks as test assets and try to mitigate the error-in-variables problem at the same time (e.g. Brennan et al., 1998).

2.2.1 Early empirical tests of the CAPM

Early empirical tests of the CAPM focus on the following cross-sectional regressions,

$$\overline{R}_i = \alpha + \lambda \hat{\beta}_i + \varepsilon_i \tag{2.8}$$

where $\overline{R}_i$ is the sample average return and $\hat{\beta}_i$ is estimated beta of asset *i,* which is typically an OLS estimated slope of asset *i*'s returns on the market return. If the CAPM holds, α should be equal to the risk free rate and λ should be equal to the market excess return. The results of early empirical tests (e.g. Douglas, 1968; Black et al., 1972; Miller and Scholes, 1972; Blume and Friend, 1973) reject the CAPM although some find there is a positive risk premium on market beta. α is found consistently greater than the returns of the U.S. Treasury bill, which is used as a proxy of the risk free rate, and the risk premium λ is too small: less than the average excess returns of a portfolio of US common stocks.

The residuals of regression (2.4) are generally correlated due to the common sources of variation such as factors related to the whole economy and the industry and have heteroskedasticity due to firm-specific effects. It is well-known that correlation and heteroskedasticity cause an inconsistent estimate of standard errors and that OLS estimator is not generally efficient. A natural way to deal with the correlated residuals is generalized OLS (GLS). Shanken (1985) proposes this method and later proves that the GLS estimator is efficient (Shanken, 1992). GLS needs to estimate the full variance/covariance matrix of the residuals and therefore may not perform well in finite samples. In econometrics, weighted OLS (WLS) is used to deal with this problem. Researchers often ignore the covariances between residuals and use a diagonal matrix containing only variances on the diagonal. Litzenberger and Ramaswamy (1979) use this method in testing the CAPM.

GLS and WLS are asymptotically efficient but perhaps biased in finite sample (Shanken and Zhou, 2007). Instead of dealing with the residual variances/covariances directly, Fama and MacBeth (1973) propose a method for dealing with this problem. This method runs cross-sectional regressions period by period,

$$R_{i,t} = \alpha_t + \lambda_t \hat{\beta}_{i,t} + \varepsilon_{i,t} \,. \tag{2.9}$$

α and γ have a subscript of t in equation (2.5) because they are estimated each period, which is different from equation (2.4). After running regressions for each period, we get a series of estimated parameters. Then the time series means of the estimates are used as final estimates and the usual statistical inferences about sample means can be used, i.e.

$$\overline{\theta} = \frac{\sum_{t=1}^{T} \hat{\theta}_t}{T} \tag{2.10}$$

where $\hat{\theta}_t = (\hat{\alpha}_t, \hat{\lambda}_t)'$ is the estimated parameter vector of equation (2.5). The standard error of $\overline{\theta}$ is the usual standard error of the sample mean: standard deviations divided by the square root of T. According to Fama and MacBeth, the period-to-period variation in the coefficients can fully capture the effects of residual correlation on the standard error estimation. Another advantage of this approach is that it can easily deal with conditional betas and other time-varying variables such as firm size and BM which are not easily incorporated into equation (2.4). Therefore, this approach has now become standard in the literature. The empirical results in Fama and MacBeth (1973) also reject the CAPM with similar findings: the intercept is too high and the slope is too low.

Non-regression based approaches of the cross-sectional test are also proposed. Gibbons (1982) is the first to propose the maximum likelihood (ML) method to test the CAPM. Shanken (1992) and Shanken and Zhou (2007) solve the ML function explicitly. More recently, Cochrane (2001) proposed the use of GMM which can easily accommodate correlation and heteroskedasticiy of the residuals. However, the two methods cannot easily deal with time-varying betas and other variables and therefore are not commonly used.

The time series implications of the CAPM were first noted by Jensen (1968) who points out that the intercept of regressions of individual asset excess returns on the market excess returns should be zero if the CAPM holds,

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t} \,. \tag{2.11}$$

where $r_{i,t}$ and $r_{m,t}$ are excess returns of asset i and market, respectively; α_i is the time series intercept of asset i, known as Jensen's alpha. Early time series empirical tests also reject the CAPM: high beta assets have negative alphas and low beta assets have positive alphas (e.g. Black et al., 1972; Blume and Friend, 1973; Stambough, 1982).

2.2.2 Recent tests: the CAPM and the cross-section of expected returns

Since the late 1970s and early 1980s, tests of the CAPM have shifted to see whether variables other than market beta have effects on the cross-section of stock returns. The CAPM states that only market beta can explain the cross-section of expected returns. Therefore, if other variables are found that have effects on the cross-section of stock returns and these effects cannot be fully explained by market beta, then the CAPM is rejected.

Many researchers have found that accounting fundamentals have an effect on the cross-section of stocks returns. Basu (1977) finds the effect of price-earnings ratios (P/E): low P/E stocks have higher returns than high P/E stocks. Banz (1981) finds a well-known size (defined as price times shares outstanding) effect: small stocks outperform large stocks in average returns. Bhandari (1988) documents a leverage effect:

stocks with high debt-to-equity ratios (D/E) have returns too high to be explained by their market beta. Finally, Stattman (1980) and Rosenberg et al. (1985) find the value effect: stocks with high BM have higher returns than stocks with low BM and this effect cannot be explained by market betas.

Past returns have also been found to have predictive abilities for future returns. DeBondt and Thaler (1985, 1987) find the long-term reversal effect that over horizons of three to five years stock returns are negatively auto-correlated, i.e. stocks with low past long-term returns have higher returns than stocks with high past long-term returns. More recently, Jegadeesh and Titman (1993) find a short-term momentum effect that over horizons of three to twelve months stock returns are positively auto-correlated: i.e. stocks with high short-term past returns (winners) continue to outperform stocks with low short-term past returns (losers).

In their influential paper, Fama and French (1992) provide strong evidence on the empirical failure of the CAPM. They find that size, P/E, D/E, BM and long-term returns all have explanatory abilities of the cross-section of stock returns after market beta is included. They show that the combination of size and BM can drive out the explanatory abilities of other firm-level variables. Furthermore, they find that market beta is not related to the cross-section of returns. In their subsequent papers, Fama and French (1993, 1996) use time series tests and also firmly reject the CAPM. They propose a three-factor model to explain these effects except the momentum effect, which has now become a benchmark model in finance. This model will be explained later in this chapter. Based on their findings of the importance of size and BM effect, Fama and French (1993, 1996) form 25 portfolios based on the quintiles of size and BM, which are among the most serious challenges to the CAPM.

There is also international evidence of these effects. Chan et al. (1991) find a strong BM effect in the Japanese stock market and Capaul et al. (1993) report a similar effect in the

European market. Fama and French (1998) examine twelve non-US markets and find that price ratios which affect the US market have similar effects in those twelve markets.

Variables related to market frictions also have been found as predictors of future returns. The intuition is that investors require higher returns for greater frictions. Empirical work focuses on the impact of liquidity risk on returns. Amihud and Mendelson (1986) find that the bid-ask spread is positively related to returns. Subsequent studies have suggested using other variables to measure liquidity risk. For example, Amihud (2002) uses the ratio of absolute return to trading volume. Brennan and Subrahmanyam (1996) suggest using the relation between price changes and order flows. Brennan et al. (1998) use share turnover and find it is negatively correlated with stock returns.

The CAPM states that only systematic risk affects returns because idiosyncratic risk can be diversified away by forming portfolios. Therefore, idiosyncratic risk should not affect returns if the CAPM holds. However, recent studies have found that idiosyncratic risk has some relationship with returns. Lehman (1990) and Fu (2009) find that idiosyncratic risk is positively priced but Ang et al. (2006) find a negative relationship between idiosyncratic risk and returns. The different results are due to the different techniques used to estimate idiosyncratic risk.

2.3 Explanations: what causes the failure of the CAPM?

The failure of the CAPM to explain the cross-section of expected returns has made researchers think about the reasons causing its failure. In the asset pricing framework, there are three explanations. The first is the conditional CAPM which explains the failure of the Sharpe-Lintner CAPM due to its static property. In the conditional CAPM, both beta and the market premium are time-varying, which are typically assumed constant, at least within a short window, in the traditional CAPM tests. The second is the multi-factor model. This approach states that the single factor, the market excess return, in the CAPM is not enough to capture all the risks and therefore additional factors are needed. Recently, multi-factor models have also been put into a conditional framework. The last approach is to relax the mean-variance assumptions. This is the higher-moment CAPM which relates investors' preferences to skewness and kurtosis in addition to mean and variance. The higher-moment CAPM is also examined in its conditional version in modern finance.

2.3.1 The conditional CAPM

The Sharpe-Lintner version of the CAPM assumes that investors make investments only for one period. Therefore, market beta and the market risk premium are constant. Hansen and Richard (1987) relax this assumption and assume that investors optimize their investments period by period over multi-period horizons. At the start of each period, investors optimize their portfolios based on the information available, which leads to a conditional optimization problem. The conditional CAPM states that in each period conditional returns are decided by conditional market beta,

$$E(r_{i,t+1} \mid I_t) = \beta_{i,t} E(r_{m,t+1} \mid I_t) \tag{2.12}$$

where I_t is the information set available to investors at the end of period *t*, $E(\bullet \mid I_t)$ is the conditional expectation based on I_t, and

$$\beta_{i,t} = \frac{Cov(r_{i,r+1}, r_{m,t+1} \mid I_t)}{Var(r_{m,t+1} \mid I_t)} \tag{2.13}$$

is conditional market beta. A crucial difference between the conditional and unconditional CAPM is that market beta is time-varying in the conditional CAPM but constant in the unconditional CAPM. Therefore, modelling time-varying market beta plays a central role in tests of the conditional CAPM.

Actually, researchers had used time-varying market beta before the proposal of the conditional CAPM. For example, Fama and MacBeth (1973) use a 60-month rolling window to estimate market beta and this method is still widely used now. However, it is after the proposal of the conditional CAPM that great efforts were devoted to the modelling of conditional market beta. More sophisticated techniques have been applied to model market beta since the late 1980s.

The first approach is to use a function of macroeconomic variables. Shanken (1990) models market beta as a linear function of the interest rate and its volatility and uses regression to estimate the coefficients and conditional market beta. This method is also widely used today. Lettau and Ludvigson (2001) use the consumption-to-wealth ratio (CAY) to model conditional market beta and find that their model can explain the Fama-French 25 portfolio returns very well. The extension to multi-factor models has been given by Ferson and Harvey (1999) and Avramov and Chordia (2006). Ferson and Schadt (1996) apply this method in fund performance measurement and Ferson and Siegel (2001) discuss the portfolio optimization problem under conditioning variables. Other researchers put the conditional CAPM into a GMM framework such as Harvey (1989, 1991). The advantage of GMM is that it does not need the usual assumptions of OLS and it can also model the conditional market return as functions of conditioning variables. The extension of GMM to multi-factor models has been given by He et al. (1996). Early studies use linear functions to model conditional market beta. Recently, Wang (2003) uses non-parametric techniques to estimate conditional market beta and multi-factor betas and finds that betas are nonlinearly related to conditioning variables. His results show that non-parametric betas perform much better than unconditional betas.

Starting from the late 1980s, the auto-regressive conditional heteroskedasticiy (ARCH) model of Engle (1982) and the generalized ARCH (GARCH) model of Bollerslev(1986) have been applied to modelling market beta. Bollerslev et al. (1988) use a multivariate GARCH model to estimate conditional market beta. Braun et al. (1995) use a bivariate exponential GARCH (EGARCH) model and find that conditional market beta is very persistent. More recently, Bali (2008) has used a bivariate GARCH model to estimate conditional market beta and finds that this model can explain the returns of the Fama-French 25 portfolios sorted by size and BM. However, the curse of dimension limits the use of the multivariate GARCH model because the number of parameters becomes overwhelming when the number of assets is increased. A strategy used in econometrics to overcome this problem is to model conditional correlations. Bollerslev (1990) assumes constant correlations between assets. Engle (2002) proposes a dynamic conditional correlation GARCH (DCC-GARCH) model in which conditional correlations are modelled like conditional variances. In this model, the assumption of all correlations having the same dynamics and the two-step estimation method significantly reduce the estimation difficulties of high-dimensional multivariate GARCH models.

The third method to model time-varying betas is the state-space model. In the state-space model, the equation from the CAPM is treated as an observation equation or measurement equation,

$$r_{i,t} = \beta_{i,t} r_{m,t} + \varepsilon_{i,t} \,. \tag{2.14}$$

The intercept is usually omitted for simplicity of estimation. Conditional market beta is treated as an underlying unobservable process. In empirical studies, the most commonly used processes include a stationary AR(1) process,

$$\beta_{i,t} = (1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{i,t-1} + u_{i,t} \tag{2.15}$$

and a random walk process with unit root,

$$\beta_{i,t} = \beta_{i,t-1} + u_{i,t} \,. \tag{2.16}$$

The state-space model can be estimated by either the Kalman filter or the Markov chain Monte Carlo (MCMC) method. Recently, the state-space model has achieved some success in explaining the cross-section of stock returns. Jostova and Philipov (2004) use the model with an AR(1) market beta to explain the cross-section of stock returns. They find that the intercept and other firm-level variables are insignificant and market beta is highly significant by using individual stocks. Ang and Chen (2007) use a similar model to explain the value portfolios from 1926 to 2002 and find the intercepts are insignificant. Both models above are estimated by the MCMC method. Adrian and Franzoni (2009) use the Kalman filter method to estimate an AR(1) model and find that market beta can explain the returns of the Fama-French 25 portfolios.

The comparison of different techniques mentioned above has been done by many researchers. The general findings are that the state-space model outperforms other models. For example, in cross-section regressions, Jostova and Philipov (2004) use individual stocks while Marti (2004) uses industry portfolios. In time series tests, Faff et al. (2000), Mergner and Bulla (2008) and Choudhry and Wu (2009) also find that the state-space model is preferred.

Recently, the availability of high frequency data has allowed researchers to estimate variance/covariance matrix more accurately by using intra-period data. The idea of using intra-period data to estimate the variance was first proposed by Merton (1980). He proves that variance can be estimated more accurately as the frequency increases. In his paper, he uses daily returns to estimate monthly variance. This method was subsequently used by French et al. (1987). Nelson and Foster (1994) prove theoretically that estimated volatility can be arbitrarily accurate as the frequency goes infinitely high and give the optimal weights of the intra-period data. In the literature on beta estimation, Scholes and Williams (1977) use daily data to estimate conditional beta under non-synchronous trading conditions. More recently, this idea has been formalized within the theory of quadratic variation such as Andersen et al. (2001a, 2003) and Barndorff-Nielsen and Shephard (2004). The estimated variance from intra-period is called realized volatility in the literature. Andersen et al. (2005, 2006) and Barndorff-Nielsen and Shephard (2004) apply the technique of realized volatility to model realized beta. The advantage of this method is its simplicity because realized volatilities are just sums of the squared intra-period returns. The disadvantage is that it requires intra-period data which limits the history of available data. For example, intra-day data for US equities is only available from 1993, which is too short for tests of asset pricing models. Therefore, daily frequency is perhaps the highest frequency for asset pricing tests. Some researchers have already used daily data to test the conditional CAPM. Morana (2009) uses daily returns to compute monthly realized betas and cross-section tests to test the CAPM and multi-factor models. By using the Fama-French 25 portfolios, he finds a negative coefficient of realized market beta. In time series tests, Lewellen and Nagel (2006) use a short-window regression method and also reject the conditional CAPM.

All the methods discussed above try to model conditional beta directly. Jagnnathan and Wang (1996), however, propose a different approach. They derive the unconditional implications of the conditional CAPM, which is a two-beta model. The first beta is the usual market beta while the second one is beta with respect to the market risk premium. They use the default premium as a proxy of the market premium because the market premium cannot be observed. They also include labour income as an additional risk factor. In their empirical results, they find that this model can explain the cross-section of returns of 100 portfolios formed by the deciles of size and BM and both additional betas are significantly priced.

More recently, Campbell and Vuolteenaho (2004) have also proposed a two-beta model.

They decompose the unexpected market return into news of future cash flows and discount rates. The news of future discount rates is estimated by a vector autoregressive (VAR) system while the news of future cash flow is backed out by the realized market return and the estimated discount rate news from the VAR system. Correspondingly, market beta is decomposed into a cash flow beta and a discount rate beta. They argue that the cash flow beta has a higher risk premium than the discount rate beta because cash flow changes are permanent but changes of discount rates can be offset by changes in future investment opportunities. Therefore, they call the cash flow beta "bad beta" and the discount rate beta "good beta". Their empirical results show that small and value stocks have higher cash flow betas which means those stocks are indeed riskier than large and growth stocks.

2.3.2 Multi-factor models

This subsection gives a brief review of the development of various multi-factor models. The details of the models used in the empirical study will be given in Chapter 4.

In the CAPM, the market excess return is the only risk factor. In the 1970s, two theories of multi-factor models were proposed. Merton (1973) proposes the intertemporal CAPM (ICAPM) which states that variables correlated with the future investment opportunity set also affect asset returns besides the market return. Ross (1976) proposes the arbitrage pricing theory (APT) which states that asset returns are decided by multiple factors. Neither of the two models gives guidelines for the factors. Therefore, additional factors can only be motivated by empirical studies.

Early studies use macroeconomic variables as additional factors (e.g. Chen et al., 1986) or factors extracted from principle component analysis (e.g. Connor and Korajczyk, 1988). Later studies have shifted to factors related to abnormal returns. Fama and French (1993, 1996) propose a three-factor model based on their empirical findings (Fama and French, 1992). Besides the market excess return, they include two additional factors (SMB and HML) which correspond to the size and value effects, respectively. The details of the definitions of the two additional factors will be given in Chapter 4. Fama and French (1996) use time series regressions to test their model and find that it can explain the effects of firm-level variables such as size and BM but not the momentum effect of Jegadeesh and Titman (1993). Due to its success in explaining the cross-section of expected returns, this model has now been widely used in many areas such as fund performance (e.g. Carhart, 1997) and cost of capital estimation (e.g. Fama and French, 1997).

The Fama-French three-factor model is purely motivated by empirical findings. Therefore it is interesting to understand the economic sources behind the two additional factors. Fama and French (1993) argue that the two additional factors proxy for an underlying distress factor. Vasslou (2003) relates SMB and HML to news of future GDP growth and Petkova (2006) relates the two factors to the innovations in predictive variables.

Although the Fama-French three-factor model performs very well in time series tests, some researchers reject it in cross-sectional tests. Daniel and Titman (1997) find that it is size and BM instead of betas of the additional factors that decide the cross-sectional differences of expected returns and therefore the model is rejected. Brennan et al. (1998) find that the size and BM effects are reduced under the Fama-French model but remain significant. In tests of the conditional version of this model, both He et al. (1996) and Ferson and Harvey (1999) reject it while Wang (2003) and Avramov and Chordia (2006) find some support for this model.

Other factors have also been proposed based on the variables which affect asset returns. Motivated by the momentum effect of Jegadeesh and Titman (1993) and the Fama-French model's inability in explaining this effect, Carhart (1997) proposes a four-factor model which includes the three factors of the Fama-French model and a momentum factor (WML or UMD). Avramov and Chordia (2006) also test the four-factor model and find that past returns remain significant after the momentum factor is included. Based on the findings of the effects of liquidity risk on the cross-section of stock returns, Pastor and Stambough (2003) propose a liquidity factor, which is based on the sensitivities of stocks to liquidity. Ang et al. (2006) propose the use of market volatility as an additional factor, which is shown to help explain the cross-section of stock returns. Adrian and Rosenberg (2008) use the two-component GARCH model of Engle and Lee (1999) to model market volatility as long-run and short-run components and use the two components as additional factors of the market return. They show that their model can explain the Fama-French 25 portfolios very well.

2.3.3 The higher-moment CAPM

The CAPM is based on the mean-variance analysis proposed by Markovitz (1952), which assumes that investors' utility functions only have two inputs, i.e. the mean and variance of portfolio returns. However, if asset returns do not have an elliptical distribution and investors also care about higher moments of return distributions, then it is natural to extend the CAPM to include those higher moments such as skewness and kurtosis. The intuition behind the preference of skewness and kurtosis is straightforward. Skewness measures the asymmetry of distributions. Suppose there are two assets with the same mean and variance of returns but one has positive skewness while the other is negatively skewed, a rational investor would prefer the former to the latter because there is greater probability of better than average realized returns. Therefore, assets with greater skewness would have greater demand and lower returns, and vice versa. Similarly, kurtosis measures the tails of distributions which are extreme values. Risk-averse investors will prefer lower kurtosis to avoid extreme movement of their asset values. Therefore, assets with lower kurtosis would have lower returns and vice versa.

In finance theory, Rubinstein (1973) has proved that investors will care about all moments of returns if the assumption of elliptical distribution is relaxed and investors do not have quadratic utility functions. The first model incorporating higher moments is proposed by Kraus and Litzenberger (1976) which indicates asset returns are linearly related to their beta and coskewness. Fang and Lai (1997) extend this model to include cokurtosis. Early empirical tests of the higher-moment CAPM give mixed results. The empirical study of Kraus and Litzenberger (1976) finds support for their model. But Friend and Westerfield (1980) show that the three-moment CAPM is generally rejected by more detailed tests such as using different subsamples and predictive measures of coskewness. Lim (1989) uses GMM to test the three-moment CAPM and find that coskewness is priced in most periods. Hwang and Satchell (1999) examine the emerging markets and find that the higher-moment CAPM can explain emerging market returns better than the conventional CAPM.

More recently, conditional versions of the higher-moment CAPM have attracted much interest from researchers. Harvey and Siddique (1999) find substantial evidence of time-varying skewness in industry and size/value portfolios and coskewness is significantly priced. Dittmar (2002) puts the higher moments in a stochastic discount factor framework and finds that the four-moment CAPM cannot be rejected. Adesi et al. (2004) find that coskewness can explain the size effect. Chung et al. (2006) find that the Fama-French factors become insignificant when up to the tenth moment are included. Smith (2007) models both beta and coskewness as a linear function of information variables and find that his model cannot be rejected. Ang et al. (2006) use cross-sectional regressions and find that coskewness has a negative coefficient while cokurtosis has a positive one, both of which are significant and consistent with predictions of theory. All of the above studies use US data. Using UK data, however, Hung et al. (2004) find that higher moments are not priced.

2.3.4 Other explanations

In this subsection, I briefly mention some other explanations for the failure of the

CAPM or the effects of some firm-level variables on the cross-section of stock returns.

Breeden (1979) proposes the consumption CAPM (CCAPM) which relates expected returns to covariances of returns with aggregate consumption. Early empirical tests only find weak support for this model (e.g. Breeden et al., 1989). Recently, researchers have used different measures of consumption risk and find some support for the CCAPM. Lettau and Ludvigson (2001) find the consumption-to-wealth ratio improves the performance of the CCAPM (their model can also be explained as a conditional CAPM). Parker and Julliard (2005) find that the long-run cumulative consumption growth is useful.

Labour income has also been found to explain the cross-section of expected returns. Jagnnathan and Wang (1996) find that including beta with labour income as an additional risk with their two betas can explain the 100 size/value portfolio returns very well. Dittmar (2002) also includes labour income in his four-moment conditional CAPM. Santos and Veronesi (2006) show that the labour-income-to-consumption ratio is useful in explaining expected returns.

Lo and MacKinglay (1990) show that data-snooping can reject the CAPM more frequently. If we group stocks based on their fundamental variables such as size and BM, then the CAPM can be rejected more frequently by using grouped portfolio returns. Conrad et al. (2003) show that data-snooping can account for 50% of the in-sample relationship between firm-level variables and returns. Furthermore, researchers usually double sort stocks based on two variables such as size and BM, which will result in more bias.

Data problems can also cause the effects of firm-level variables. Kothari et al. (1995) show that survivorship bias can cause the effect of BM on the cross-section of stock returns. However, researchers also find the BM effect using data sets corrected for survivorship bias (see Fama and French, 2004). Knez and Ready (1997) find that the size effect disappears after 1% extreme observations are excluded from the sample.

Finally, in an influential paper, Lakonishok et al. (1994) show that irrational investors can cause the value effect. They argue that the value effect is due to investors extrapolating past growth too far in the future (see the survey of Subrahmanyam (2009) for further references).

2.4 Conclusion

The CAPM of Sharpe (1964) and Lintner (1965) is a fundamental asset pricing model. It states that the cross-sectional differences of expected asset returns are only decided by systematic risk or market beta. Empirical studies, however, generally reject the CAPM. Alternative explanations of the failure of the CAPM have been proposed. There are three explanations under the rational expectations framework: the conditional CAPM, the multi-factor models and the higher-moment CAPM. In modern finance, the multi-factor models and the higher-moment CAPM are also put into a conditional framework.

A key problem in testing conditional asset pricing models is the difficulty of modelling betas and higher comoments. Ghysels (1998) shows that the conditional CAPM performs even worse than the unconditional CAPM if the conditional market beta is not modelled properly. This critique also applies to the test of conditional multi-factor models and the conditional higher-moment CAPM. In empirical studies, different techniques can give contradictory conclusions. For example, Fama and French (1993, 1996) show that the CAPM cannot explain the value effect but Ang and Chen (1996) use a state-space model for market beta and find the conditional CAPM cannot be rejected.

Overall, existing literature generally agrees that unconditional models, including the CAPM, multi-factor models and the higher-moment CAPM, tend to be rejected. The debate is on the conditional versions of those models. Different techniques will give different results. Therefore, no conclusion has been reached so far. I expect that new techniques will be proposed to modelling systematic risks and the debate will continue to be the focus of many studies.

Chapter 3

Conditional Market Beta and the Cross-Section of Stock Returns

3.1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has long been the basic model to describe the relationships of expected returns and risk. In the CAPM, the market portfolio is efficient and the cross-sectional differences in returns are determined by differences in market beta only. This has two implications for the cross-section of asset returns: first, market beta should have a positive coefficient and the intercept should be zero (using excess returns and assuming there is a risk free asset); second, there are no other variables having any marginal explanatory abilities, i.e. any other variables will not be related with returns in the cross-section.

Early empirical tests focused on the first implication: market beta has a positive coefficient and the intercept is zero. The results generally rejected the CAPM by finding a significant intercept and a too flat coefficient on market beta (e.g. Black et al., 1972; Fama and MacBeth, 1973). However, the coefficient on market beta is still positive as predicted by the CAPM. Therefore, market beta seems a priced risk although it alone cannot fully explain the cross-section of stock returns. Subsequent empirical work has shifted to test the second implication by finding other variables related to returns. Many studies have documented that cross-sectional differences of returns are also related to firm-level variables such as size (Banz, 1981), book-to-market ratio (Stattman, 1980; Rosenberg et al., 1985) and past returns (Jegadeesh and Titman, 1993).

In their influential paper, Fama and French (1992) give a comprehensive study of the relationship between stock returns, market beta and firm-level variables. Different from early tests, they find a negative coefficient of market beta in cross-sectional tests and strongly reject the CAPM. They also show that the combination of size and book-to-market ratio drives out the explanatory abilities of other variables. One explanation of this is that the market return alone is not enough to explain stock returns. Therefore, multi-factor asset pricing models should be used. Fama and French (1993,

1996) in their subsequent papers propose a three-factor model to explain cross-sectional differences of stock returns. Their model can explain the impact of firm specific characteristics on stock returns except the momentum effect. Other factors to account for different firm-specific characteristics have also been proposed such as the four-factor model of Carhart (1997).

Another explanation is that the failure of the CAPM is due to its unconditional static nature. The CAPM is derived from a one-period setting in which market beta is assumed to be constant. However, both theoretical and empirical work has shown market beta is time-varying. Theoretically, Hansen and Richard (1987) show that a conditional version of the CAPM could hold even if the unconditional CAPM fails. Empirical work has found some support of the time-varying property of market beta and the conditional CAPM (e.g. Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001; Ang and Chen, 2006). Due to its theoretical soundness and empirical success, the conditional CAPM is becoming popular in academia. The debate of CAPM has shifted from its unconditional version to its conditional version.

In the conditional CAPM, both market beta and the market risk premium are time-varying. Therefore, it is important to model both of them appropriately. In the cross-sectional test of the CAPM, the market risk premium is estimated each period as the coefficient of market beta while market beta is modelled using different time series techniques. Hence, time-varying market beta plays a key role in the cross-sectional test of the conditional CAPM. If a wrong model of market beta is used then a wrong market risk premium and intercept will be estimated, which in turn will lead to a wrong conclusion. Ghysels (1998) shows the conditional CAPM performs even worse than the unconditional CAPM if conditional market beta is modelled inappropriately. Unfortunately, the theory of the conditional CAPM does not give how to model time-varying market beta. Therefore, researchers have to develop empirical models mainly from the statistical or econometric view.

Different conditional market beta models have been proposed in order to accommodate the time-varying properties of market beta. For example, Shanken (1990) proposes to use macroeconomic economic variables to model market beta. This method has been widely used in the literature, such as by He et al. (1996) and Ferson and Harvey (1999). Existing studies generally reject the conditional CAPM but Lettau and Ludvigson (2001) use the consumption-to-wealth ratio (CAY) as an information variable and find support for the conditional CAPM. More recently, the state-space model has achieved success. For example, Jostova and Philipov (2005) use estimated market beta in cross-section tests by individual stocks and find market beta is significant and drives out the significance of other variables such as size and BM; Ang and Chen (2007) put their model in a time series context to examine the value effect by using a long sample from 1926 to 2001 and find the conditional CAPM cannot be rejected; Bali (2008) uses a bivariate GARCH model to estimate market beta and find it is significant by using industry and size/value portfolios.

Some studies compared the performance of different models in the context of explaining stock returns. In the cross-section comparison, Jostova and Philipov (2004) use individual stocks while Marti (2005) uses industry portfolios and both of the two studies find the state-space model is the best model among those examined such as the rolling window regression model and different multivariate GARCH models.[3]

All the studies cited above use estimated betas from the whole sample in the cross-sectional regressions. Their out-of-sample forecast performance in explaining the impact of firm-level size and book-to-market ratio on the cross-sectional differences of expected stock returns has rarely been examined. A problem of using in-sample estimated beta is over-conditioning bias: using information beyond the information set of the investors when they make investment decisions (Boguth et. al., 2008). The conditional CAPM is an ex-ante model but in-sample estimated market beta is an ex-post measure so that it is inappropriate for a true test of the conditional CAPM. This problem can be solved by using the out-of-sample forecast of conditional market beta in the cross-sectional regression. Therefore, it is necessary to examine whether out-of-sample forecasted market beta from those different models can explain the cross-section of stock returns.

Based on this motivation, in this chapter, I examine whether the different models of conditional market beta can explain the cross-section of stock returns not only in-sample but also out-of-sample. The returns used are the Fama-French 25 portfolios sorted by firm size and book-to-market ratio (Fama and French, 1993, 1996), which are

[3] Some other studies focus on the time series test, e.g. Faff et al. (2000), Mergner and Bulla (2008) and Choudhry and Wu (2009) and also find the state-space models perform the best.

among the most serious challenges of the CAPM. The first model is the unconditional OLS coefficient in a regression of individual portfolio's return on the market return using the whole sample. The second model is also OLS but using data only available until the end of each period, which is the simplest way to model time-varying market beta. The third model is to model market beta as a linear function of predetermined information variables, e.g. Shanken (1990) and Ferson and Harvey (1999). The fourth model is a state-space model in which market beta is the time-varying parameter on the market return. The structure of the time-varying parameter is modelled by two forms: (1) a mean-reversion first order autoregressive (AR(1)) process with constant mean and (2) a random walk process because the AR(1) coefficient in the AR process is very close to one in some cases. Each of the two forms is estimated by two methods: the Kalman filter and the MCMC method. Broadly speaking, all the approaches of conditional market beta in the literature fall into two categories. One is to model market beta directly. All the models described above fall into this category. The other category is to model the covariance of individual returns and the market return and the conditional variance of the market return because market beta is the ratio of the two. The next two models are in this category. The fifth model is the dynamic conditional correlation multivariate generalized autoregressive conditional heteroskedasticiy (DCC-GARCH) model of Engle (2002). This model can be used in a high dimensional system with a small number of parameters, and it is fast to estimate by the two-step method. Finally, the sixth model is the realized beta model of Andersen et al. (2005, 2006) which is built on the recent literature of realized volatility (e.g. Andersen et al., 2003; Barndoff-Nielsen and Shephard, 2004).

I first use the whole sample to estimate the different models and the resulting conditional market beta, then examine the cross-sectional relationship between the portfolio returns and conditional market beta. Consistent with the existing literature, I find that market beta from the state-space model does explain the cross-sectional differences of returns very well: the time series average of the cross-sectional regression coefficients on market beta is significantly positive and that of the intercepts is insignificantly different from zero.

Next I examine whether out-of-sample forecasts of different conditional market beta models can explain the cross-sectional differences of stock returns. I use information only available until the end of each period to estimate the models and generate one-step ahead forecasted market beta for the next period. Forecasted market beta is used in the cross-sectional regressions. The results show that out-of-sample forecasted market beta performs much worse than in-sample estimated market beta: none of conditional beta models can generate significantly positive average of the cross-sectional regression coefficient on conditional beta, which is consistent with evidence in the literature that beta has less predictability than the variance and covariance (Andersen et al., 2006).

The rest of the chapter is organized as follows. In section 3.2, I describe the models of conditional market beta and the Fama-MacBeth cross-sectional regression method. Section 3.3 describes the data used in the empirical work. Section 3.4 presents the empirical results. Section 3.5 concludes.

3.2. Empirical framework

3.2.1 The unconditional CAPM and constant beta

The CAPM of Sharpe (1964) and Lintner (1965) is expressed in the form of unconditional expectations of returns,

$$E(r_i) = \beta_i E(r_m), \qquad (3.1)$$

where r_i is the excess simple return of portfolio *i*, r_m is market excess simple return, $\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$ is the systematic risk. The econometric model of the unconditional CAPM is a linear regression of individual portfolio's returns on the market return:

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t} \qquad (3.2)$$

where $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$ is assumed to have zero mean, constant variance σ_i^2 and to be independently and identically distributed (IID). The coefficient on the market return, β_i, is assumed to be constant and estimated by OLS. The estimated β_i is used in the cross-sectional regression.

3.2.2 The conditional CAPM and models of time-varying beta

The conditional CAPM is set up in a dynamic environment and expressed in the form of conditional expectations of returns,

$$E(r_{i,t+1} \mid I_t) = \beta_{i,t} E(r_{m,t+1} \mid I_t) \tag{3.3}$$

where I_t is the information set available to investors at the end of period *t*, $E(\bullet \mid I_t)$ is the conditional expectation based on I_t, and

$$\beta_{i,t} = \frac{Cov(r_{i,r+1}, r_{m,t+1} \mid I_t)}{Var(r_{m,t+1} \mid I_t)} \tag{3.4}$$

is time-varying systematic risk hence has a subscript of *t,* which means market beta in each time period is different from each other. Efforts in tests of the conditional CAPM have been focused on how to model time-varying market beta. There are two categories of modelling strategies: one is based on equation (3.2) but replace the constant coefficient on the market return with a time-varying coefficient; the other one focuses on the two components of conditional market beta: conditional variance of the market return and conditional covariance of individual portfolio's returns with the market return. The remainder of this subsection will describe different models of conditional market beta, each of which falls into one of the above two categories. I will start with models focusing on conditional market beta directly and then describe models focusing on conditional variance and covariance.

3.2.2.1 The short-window regression

This method only uses a short window of data to estimate market beta. There are two ways to choose the window length: one is expanding the sample by one observation each time and the other is the popular rolling-window estimation. This method has been used by many researchers, e.g. Fama and MacBeth (1973). It assumes market beta to be constant within a short time interval, which is typically 60 months and uses equation (3.2) in each interval to estimate market beta. This is the simplest method to model time-varying market beta but its shortcomings are obvious: if market beta is time-varying each month, this method will be inappropriate.

In this chapter, I follow the common approach of the literature by using 60-month rolling window and an expanding sample regression to estimate time-varying market beta of each month. For the in-sample estimation, data of month *t*-59 through month *t* are used in the regression for each month to count in the effect of current month. For the out-of-sample forecast, an expanding window is used to be consistent with other models and the results of rolling window of month *t*-60 though month *t*-1 are reported in Appendix 3.

3.2.2.2 The macroeconomic variables model

In the macroeconomic variables model, market beta is a linear function of predetermined macroeconomic variables, e.g. Shanken (1990) and Ferson and Harvey (1999). The econometric model can be expressed as

$$r_{i,t} = \alpha_i + \beta_{i,t} r_{m,t} + \varepsilon_{i,t} \tag{3.5}$$

where

$$\beta_{i,t} = \delta_{0i} + \delta_{1i}^{'} Z_{t-1}\,, \tag{3.6}$$

Z_{t-1} is a *k*-dimensional vector of the lagged macroeconomic variables. δ_{0i} is a constant coefficient and δ_{1i} is a *k*-vector of coefficients on the macroeconomic variables. OLS can be applied to

$$r_{i,t} = \alpha_i + (\delta_{0i} + \delta_{1i}^{'} Z_{t-1}) r_{m,t} + \varepsilon_{i,t} \tag{3.7}$$

to estimate δ_{0i} and δ_{1i}. If the estimated values of δ_{1i} are different from zero, then market beta is time-varying. Hence a test of constant market beta can be carried out by testing whether δ_{1i} is jointly equal to zero. This model links market beta to macroeconomic variables, which is a property of systematic risk supported by many studies (e.g. Ferson and Harvey, 1999). However, there is no theoretical guidance of what variables are in the information set of investors of time *t* and the linear function may be misspecified.[4] In empirical implementation, the information variables are chosen according to previous empirical studies and the linear functional form of market beta as equation (3.6) is chosen for its simplicity.

For in-sample estimation, I use the whole sample to estimate δ_{0i} and δ_{1i} and use the estimated parameters to calculate market beta of each period *t* according to equation (3.6). For the out-of-sample forecast, I use data available only until the end of period *t* to estimate δ_{0i} and δ_{1i} and use the estimates with macroeconomic variable of period *t* to forecast market beta for period *t*+1 according to equation (3.6).

3.2.2.3 The state-space model

Both short-window regression and macroeconomic variable models are based on linear regressions. A more advanced approach to model the time-varying coefficient is the state-space model. When it is applied to beta modelling, equation (3.5) can be treated as the measurement equation. In order to simplify the estimation of the model, I omit the intercept in the equation. This way also gets better results than the way that the intercept is included.[5] So the measurement equation is

$$r_{i,t} = \beta_{i,t} r_{m,t} + \varepsilon_{i,t}. \tag{3.8}$$

In the state-space model, market beta is treated as an unobservable latent variable and is needed to be modelled explicitly. So the next step is to specify the process of market beta, which is called the transition equation. Following the popular method of the literature, I use two processes to model market beta. The first is an AR(1) process,

$$\beta_{i,t} = (1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{i,t-1} + u_{i,t}, \tag{3.9}$$

where ϕ_{i0} is the long-run mean of market beta, ϕ_{i1} measures the persistence, $\varepsilon_{i,t}$ and $u_{i,t}$ are Gaussian white noise processes and uncorrelated with each other. The AR(1) model assumes market beta is a stable mean-reversion process which is

[4] See Cochrane (2001) for the critics of the first point and Wang (2003) for the evidence of nonlinear relationship of beta and information variables.

[5] Previous studies such as Jostova and Philipov (2004) and Adrian and Franzoni (2009) also exclude the intercept.

documented in the theory of market beta (Gomes et al., 2003) and empirical studies (e.g. Chang and Weiss, 1991). Some empirical studies (e.g. Braun, Nelson and Sunier, 1995) find the estimated value of the AR(1) coefficient in equation (3.9) is very close to 1 which suggests market beta may have a unit root. Therefore, I also consider a random walk process of beta,[6] i.e.

$$\beta_{i,t} = \beta_{i,t-1} + u_{i,t}. \tag{3.10}$$

The advantage of the state-space model is that it allows to model market beta directly by using time series techniques. The disadvantage is that the estimation of the parameters and market beta is much more difficult than regression-based models. A popular way is to use the Kalman filter and maximum likelihood method (see Harvey, 1989; Durbin and Koopmans, 2001). The Kalman filter has been used by many researchers in beta modelling (e.g. Faff et al., 2000). I use filtered beta as in-sample estimated beta because there is no counterpart of smoothed beta in the MCMC method described below. The out-of-sample forecast can be easily achieved according to equations (3.9) and (3.10), respectively. The details of the Kalman filter are given in Appendix 3A.

Recently, advances in computing facilities and computational methods have increased the application of the Bayesian method in econometrics. One application is the MCMC method. It has been used in the estimation of the state-space model of market beta by some researchers recently (e.g. Jostova and Philipov, 2004; Ang and Chen, 2007). Jostova and Philipov (2004) use their Bayesian beta in the cross-sectional regressions with individual stock returns and find a significantly positive coefficient of market beta and insignificant coefficients of the intercept and other firm-specific variables.However, they use in-sample estimated market beta only and do not examine the out-of-sample forecast performance of their Bayesian beta in the cross-sectional regression. In this chapter, I apply the estimation method of Jostova and Philipov (2004) to estimate the models of (3.9) and (3.10). The details of the prior and posterior distributions of the parameters and estimation are in Appendix 3B. I not only estimate market beta in-sample but also forecast market beta out-of-sample.

The empirical results show that the two methods give similar results. But the Kalman

[6] The studies of Marti(2005), Mergner and Bulla (2008) and Choudhry and Wu (2009) also consider the random walk model.

filter method is much faster than the MCMC method.

3.2.2.4 The DCC-GARCH model

All the models above focus on market beta directly. Alternatively, the DCC-GARCH model and the realized beta model described in the next subsection model the conditional variance and covariance. The GARCH class models have long been popular in the volatility literature since the development of Engle's (1982) ARCH model and Bollerslev's (1986) GARCH model. The multivariate GARCH model has been used to model conditional market beta by many researchers such as Bollerslev et al. (1988) and Bali (2008). There is a key problem of the multivariate GARCH model: it is highly parameterized. When the dimension of the equations increases, the number of parameters becomes overwhelming. Hence, most applications of the multivariate GARCH model are low dimensional problems such as the bivariate GARCH model. In finance, however, high-dimensional problems are not uncommon. One method to overcome the dimension problem is to model the conditional correlation. Bollerslev (1990) assumes a constant correlation between any two series. By assuming constant correlations, one can just estimate the conditional variance processes therefore the number of parameters is reduced significantly. Obviously, the drawback of this model is the assumption of the constant correlations, which is hardly to be true. Engle (2002) proposes the DCC-GARCH model which uses a GARCH type model for the conditional correlation to generate a time-varying conditional correlation and maintain a small number of parameters at the same time. So the DCC-GARCH model is an ideal tool to deal with high dimensional multivariate volatility problems. In this chapter, I apply this newly proposed DCC-GARCH(1,1) model to the return series of individual portfolios and market to get conditional market beta.

It is easier to describe the DCC-GARCH model in vector/matrix form. Suppose there is a multivariate return series $\{r_t\}$ under consideration, where $r_t = (r_{1,t},...,r_{N,t})'$ is an N-vector of returns, the DCC-GARCH(1,1) model is:

$$r_t = \mu_t + a_t ,$$

$$a_t \sim N(0,\Sigma_t) ,$$

$$\Sigma_t \equiv \left(\sigma_{ij,t}\right)_{N\times N} = D_t \rho_t D_t ,$$

where $\mu_t = E\left(r_r \mid I_{t-1}\right)$ is the conditional mean of r_t, $D_t = \text{diag}\left\{\sqrt{\sigma_{ii,t}}\right\}$ is the $N \times N$ diagonal matrix of time-varying standard deviations with $\sqrt{\sigma_{ii,t}}$ on the *i*th diagonal, and ρ_t is the time-varying conditional correlation matrix. The conditional variance of each series is modelled by a univariate GARCH(1,1) model,

$$\sigma_{ii,t}^2 = \omega_i + \kappa_i a_{i,t-1}^2 + \lambda_i \sigma_{ii,t-1}^2 , \tag{3.11}$$

for *i*=1,…,*N*. The model for the conditional correlation is

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 e_{t-1} e_{t-1}^{'} + \theta_2 Q_{t-1} \quad , \tag{3.12}$$

$$\rho_t = J_t^{-1} Q_t J_t^{-1} , \tag{3.13}$$

where $Q_t \equiv \left(q_{ij,t}\right)_{N \times N}$, $e_t = D_t^{-1} a_t$ is the standardized residuals, $\bar{Q}$ is the unconditional covariance matrix of e_t, ρ_t is the correlation matrix of e_t and $J_t = \text{diag}\left\{\sqrt{q_{ii,t}}\right\}$ is the $N \times N$ diagonal matrix with $\sqrt{q_{ii,t}}$ on its *i*th diagonal. There are some restrictions on the parameters to guarantee Σ_t to be positive definite for all *t* (Engle and Sheppard, 2001). The usual restrictions on univariate GARCH(1,1) parameters apply to ω_i, κ_i, and λ_i such as $\omega_i > 0$, $\kappa_i \geq 0$, $\lambda_i \geq 0$ and $0 < \kappa_i + \lambda_i < 1$ for all *i*. For the DCC equation (3.12), θ_1 and θ_2 are non-negative scalar parameters satisfying $0 < \theta_1 + \theta_2 < 1$ and $\bar{Q}$ is positive definite.

The estimation of the DCC-GARCH model can be done via the two-stage estimation method, which has consistent estimates (Engle, 2002; Engle and Sheppard, 2001). The first stage is to estimate equation (3.11) as a univariate GARCH model for each *i* and get residuals series a_t and variance matrix series Σ_t. Using the estimates from stage one, the second stage is to estimate equation (3.12). The two-stage estimation method significantly reduces the difficulties of estimating a high-dimensional multivariate GARCH model. After the estimation is done, we can get the conditional standard deviations of each series and their correlations. Let $r_{N,t}$ in r_t be the market excess return and $r_{i,t}$ be portfolio i's return for i=1,…,N-1in time t; we can write conditional market beta of portfolio i as

$$\beta_{i,t} = \rho_{iN,t} \frac{\sqrt{\sigma_{ii,t}}}{\sqrt{\sigma_{NN,t}}}, \tag{3.14}$$

where $\rho_{iN,t}$ is the (iN)th element of ρ_t. In-sample estimated beta is calculated from estimated conditional standard deviations and correlations using the whole sample. For out-of-sample forecasts, estimations are done by using data until time t and one-step-ahead forecasts of conditional standard deviations and correlations are carried out according to equations (3.11)-(3.13), then conditional beta in time t+1 is from equation (3.14).

3.2.2.5 The realized beta model

Recently, due to the availability of high-frequency data, the modelling of volatility using intra-period data has drawn significant attention. The use of intra-period data to estimate volatility has been used as early as Merton (1980) and extended by French et al. (1987). The method of French et al. (1987) has been widely used since it was proposed. Nelson and Foster (1994) give theoretical proof that estimated volatility can be arbitrarily accurate as the frequency goes infinitely high. In the literature of beta modelling, Scholes and Williams (1977) use daily data to estimate conditional market beta under non-synchronous trading conditions. Recently, research on realized volatility has been based on the theory of quadratic variation (e.g. Andersen et al., 2003; Barndorff-Nielsen and Shephard, 2004).

Suppose that the logarithmic $N \times 1$ vector price process, $p_t \equiv (p_{1,t},...,p_{N,t})$, follows a multivariate continuous-time stochastic volatility diffusion,

$$dp_t = \mu_t dt + \Sigma_t dW_t, \tag{3.15}$$

where W_t is a standard N-dimensional Brownian motion, μ_t is the instantaneous drift, Σ_t is the $N \times N$ matrix of instantaneous variance and covariance. Then, the conditional distribution of the continuously compounded h-period return, $r_{t,t+h} = p_{t+h} - p_t$, based on the sample path realization of μ_t and Σ_t, is

$$r_{t+h,h} \mid \sigma\{\mu_{t+\tau}, \Sigma_{t+\tau}\}_{\tau=0}^{h} \sim N\left(\int_0^h \mu_{t+\tau} d\tau, \int_0^h \Sigma_{t+\tau} d\tau\right), \tag{3.16}$$

where $\sigma\{\mu_{t+\tau}, \Sigma_{t+\tau}\}_{\tau=0}^{h}$ is the σ-field generated by the sample path of $\mu_{t+\tau}$ and $\Sigma_{t+\tau}$ for $0 \le \tau \le h$. Therefore, the integrated diffusion matrix $\int_0^h \Sigma_{t+\tau} d\tau$ provides a natural measure of the true underlying h-period volatility.

By the theory of quadratic variation, we can estimate the integrated volatility using intra-period data,

$$\sum_{j=1}^{[h/\Delta]} r_{t+j\cdot\Delta} r'_{t+j\cdot\Delta} \to \int_0^h \Sigma_{t+\tau} d\tau \tag{3.17}$$

as the sampling frequency of returns increases, or $\Delta \to 0$. This estimate is called realized volatility in the literature. As analysis in (3.2.2.4), I let the N'th element of p_t be the market price and the other elements of p_t to be the individual portfolios' prices, so that $\Sigma_{NN,t}$ is the integrated market variance of month t, $\sigma^2_{M,t}$, and $\Sigma_{iN,t}$ is the integrated covariance of portfolio i with market, $\sigma_{iM,t}$. The corresponding estimate of $\sigma^2_{M,t}$ is called realized market variance,

$$\hat{\sigma}^2_{M,t,t+h} = \sum_{j=1}^{[h/\Delta]} r^2_{N,t+j\cdot\Delta}, \tag{3.18}$$

and the corresponding estimate of $\sigma_{iM,t}$ is called realized covariance of individual portfolio i with market,

$$\hat{\sigma}_{iM,t,t+h} = \sum_{j=1}^{[h/\Delta]} r_{i,t+j\cdot\Delta} r_{N,t+j\cdot\Delta}. \tag{3.19}$$

Then the associated realized beta is defined as

$$\hat{\beta}_{i,t,t+h} = \frac{\hat{\sigma}_{iM,t,t+h}}{\hat{\sigma}^2_{M,t,t+h}}, \tag{3.20}$$

which converges to the underlying integrated beta:

$$\hat{\beta}_{i,t,t+h} \rightarrow \beta_{i,t,t+h} = \frac{\sigma_{iM,t,t+h}}{\sigma^2_{M,t,t+h}}. \tag{3.21}$$

Hence, realized beta is a measure of underlying *h*-period beta.

Obviously, the sampling frequency within each period, Δ, plays a key role in estimation of realized beta. Ideally, we want to use as high frequency as we can because the estimate will be more and more accurate as the frequency increases. However, market microstructure limits the choice of the frequency (Andersen et al., 2001). More important in asset pricing model tests, a long history of data should be used to test the relationship between market beta and returns but the ultra-high frequency intra-day data is only available from 1993. Due to this constraint, I use daily data which is available from 1963 to construct monthly realized variance and covariance. Therefore, $h = 1$ and Δ is roughly equal to 1/22 because there are slightly different numbers of trading days each month.

Due to the practical limit in sampling frequency, the intra-period data is sampled discretely instead of continuously, so there will be estimation errors of the realized variance and covariance. To reduce the estimation error, I treat realized beta as an observation of underlying integrated beta instead of using realized beta directly. A natural way to do this is the state-space model. In the observation equation, realized beta is an unbiased measure of integrated beta,

$$\hat{\beta}_{i,t} = \beta_{i,t} + \varepsilon_{i,t}, \tag{3.22}$$

where $\varepsilon_{i,t}$ is Gaussian white noise and mutually independent for any $j \neq i$ and the variance of $\varepsilon_{i,t}$ is computed according to equation (34) of Barndorff-Nielsen and

Sheppard (2004). Then integrated beta is modelled by an AR(1) process,

$$\beta_{i,t} = (1-\alpha_{1i})\alpha_{0i} + \alpha_{1i}\beta_{i,t-1} + u_{i,t}\,, \tag{3.23}$$

where α_{0i} corresponds to the mean of beta and α_{1i} is the autoregressive coefficient, $u_{i,t}$ is Gaussian white noise, mutually independent for any $j \neq i$ and independent with $\varepsilon_{i,t}$. By the Kalman filter, the model can be estimated and underlying integrated beta can be extracted. For in-sample estimated market beta, I use filtered beta from the Kalman filter in the cross-sectional regression; both smoothed beta and raw realized beta have similar results. For out-of-sample forecasted market beta, I estimate the model in each month *t* and generate forecasts according to equation (3.23) for month *t*+1. Forecasted beta is used in the cross-sectional regression of return in period *t*+1.

3.2.3 The Fama-MacBeth approach

The Fama-MacBeth (1973) approach has been widely used in the empirical work of finance to test the cross-sectional relationship of one variable with others. It includes two steps. In the first step, suppose there are *T* periods of data, a cross-sectional regression of excess returns on conditional beta is run for each period *t*=1,…,*T*,

$$r_{i,t} = \alpha_t + \gamma_t \hat{\beta}_{i,t} + \varepsilon_{i,t}\,, \tag{3.24}$$

where α_t and γ_t are parameters, $\varepsilon_{i,t}$ is a cross-section Gaussian white noise, $\hat{\beta}_{i,t}$ is in-sample estimated market beta or out-of-sample forecasted market beta from those different models described in section (3.2.2). α_t is the abnormal return and γ_t is the market risk premium of period *t*. After the first step, we will have two series of estimated parameters, $\{\hat{\alpha}_t\}$ and $\{\hat{\gamma}_t\}$ for *t*=1,…,*T* .

If conditional market beta can explain the cross-section of stock returns, the average of $\{\hat{\alpha}_t\}$ should be zero and that of $\{\hat{\gamma}_t\}$ should be positive because the conditional CAPM indicates that the cross-sectional differences of stock returns are solely attributed to the differences of conditional market beta. The second step is to test whether the two hypotheses are true. The usual *t*-test can be used,

$$t = \frac{\bar{X}}{\sigma_{\bar{X}}}, \tag{3.25}$$

where X is the series under consideration, either $\{\hat{\alpha}_t\}$ or $\{\hat{\gamma}_t\}$, $\bar{X}$ is the sample mean and $\sigma_{\bar{X}}$ is the standard error of $\bar{X}$,

$$\sigma_{\bar{X}} = \sqrt{\frac{\sum_{t=1}^{T}(X - \bar{X})^2}{T(T-1)}}. \tag{3.26}$$

The distribution of t is student-*t* with (T-1) degrees of freedom. The null hypothesis is $\bar{\alpha}_t$ =0 and $\bar{\gamma}_t \leq 0$, respectively. The inference can be made in the usual fashion.

3.3. Data

3.3.1 Market and individual portfolio returns

The proxy for the market is the value-weighted index of all stocks listed in NYSE, AMEX and NASDAQ from the Center for Research in Security Prices (CRSP). The risk free rate is the one-month Treasury bill rate. The individual portfolios are the Fama and French (1993) 25 portfolios sorted by stocks' size and book-to-market ratios (BM). To construct the portfolios, Fama and French (1993) rank all NYSE stocks in the CRSP based on the quintiles of size in June of each year. Then, they assign stocks in NYSE, AMEX and NASDAQ to each of the quintiles according to their size. They also sort all stocks in NYSE, AMEX and NASDAQ by their BM in December of the previous year and assign stocks to each BM quintile. Then the intersections of the five quintiles of ME and BM generate 25 portfolios (5×5). The value-weighted returns are used. Following common practice in the literature, I denote each portfolio by its quintiles of size and BM, for example, the portfolio of stocks in size quintile 1 and BM quintile 1 is denoted by S1B1, and similarly the portfolio of stocks in size quintile 5 and BM quintile 5 is denoted by S5B5. Monthly returns are available from July 1926 to December 2007 but daily returns are available only from July 1963.[7]

[7] The data is downloaded from Ken French's online data library.

The portfolio S5B5 (large value stocks) has missing values from July 1930 to June 1931. I replace the missing values with the average returns on the other four portfolios within the same size quintile (S5B1, S5B2, S5B3 and S5B4) as in Bali (2008).

3.3.2 Macroeconomic variables

The macroeconomic variables used in equation (3.7) include dividend yield of S&P500 which is defined as the sum of previous twelve months' dividends divided by the price of current month, one-month Treasury bill rate, monthly change of logarithm industry production, default spread defined as Moody's Baa rated corporate bonds' yield minus Aaa rated corporate bonds' yield, and term spread defined as Moody's Aaa rated corporate bonds' yield minus one month Treasury bill rate[8]. Those variables are chosen according to previous studies of conditional market beta (e.g. He et al., 1996; Ferson and Harvey, 1999).

3.3.3 Summary statistics

Table 3.1 presents summary statistics of data. Panel A gives means and standard deviations of the Fama-French 25 portfolios' returns from July 1926 to December 2007. The well-known size effect and value effect are obvious. Returns monotonically decrease with size (along the columns) except the growth portfolios (the column of B1), and increase with BM (along the rows). Previous empirical work has documented that this pattern cannot be explained by cross-sectional differences of market beta (Fama and French, 1992, 1993). Therefore, it is among the most serious challenges of the CAPM to explain the cross-section of returns of those 25 portfolios. The lower panel of Panel A gives the unconditional OLS regression results of the 25 portfolios' returns on the market return. Like the raw return, the intercept, α, generally decreases with size except the growth portfolios (B1 and B2) and increases with BM, and is significant in many cases especially for the small and value portfolios, which suggests the unconditional CAPM cannot explain the returns of these portfolios very well. The estimated market beta generally decreases with size and value portfolios (B5) have higher market beta than growth portfolios (B1) except in the smallest portfolios (S1). This means that the well-known negative relationship between market beta and returns might not be so prominent in the whole sample.

[8] The usual definition of term spread is a long term Treasury bond's yield minus a short-term Treasury bill's yield but both data are unavailable through the whole sample. I replace long-term Treasury bond's yield by Moody's Aaa rated corporate bonds' yield and short-term Treasury bill's yield by one month Treasury bill as Adrian and Franzoni (2009).

Table 3.1 Summary Statistics of Data

Panal A and panel B present summary statistics of monthly portfolio returns from July 1926 and July 1963 to December 2007, respectively. The returns are simple nominal return in excess of one-month Treasury bill rate, measured in percent. S1 through S5 stand for quintiles of size (from small to large), while B1 through B5 stand for quintiles of book-to-market ratio (from low to high). α and β are parameters from the unconditional regression using whole sample,

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t}.$$

Newey-West heteroskedasticity and autocorrelation robust standard errors are in parenthesis. Panel B gives summary statistics of monthly observations of macroeconomic variables. DIV is dividend yield of S&P500, defined as the sum of previous twelve months' dividend divided by the price of current month, RF is the one-month Treasury bill rate, IP is the monthly change of logarithm industry production, DEF is the default spread defined as Moody's Baa rated corporate bonds' yield minus Aaa rated corporate bonds' yield, and TERM is the term spread defined as Moody's Aaa rated corporate bonds' yield minus one month Treasury bill rate.

Panel A: The Fama and French 25 Size/BM Portfolios 1926:07-2007:12										
	B1	B2	B3	B4	B5	B1	B2	B3	B4	B5
	Means					Standard Deviations				
S1	0.46	0.82	1.03	1.20	1.41	12.40	10.70	9.30	8.70	9.61
S2	0.57	0.95	1.04	1.10	1.22	8.02	7.91	7.33	7.61	8.71
S3	0.68	0.88	0.98	1.00	1.12	7.68	6.57	6.74	6.80	8.67
S4	0.68	0.75	0.87	0.96	1.08	6.23	6.26	6.33	7.02	9.01
S5	0.60	0.62	0.68	0.74	0.94	5.49	5.24	5.72	6.92	7.60
	α					β				
S1	-0.60	-0.14	0.13	0.35	0.51	1.65	1.48	1.40	1.31	1.39
	(0.23)	(0.17)	(0.16)	(0.14)	(0.17)	(0.14)	(0.12)	(0.07)	(0.08)	(0.09)
S2	-0.24	0.13	0.27	0.31	0.34	1.25	1.28	1.19	1.23	1.36
	(0.13)	(0.11)	(0.10)	(0.11)	(0.14)	(0.06)	(0.06)	(0.06)	(0.06)	(0.07)
S3	-0.15	0.15	0.23	0.27	0.22	1.28	1.13	1.15	1.13	1.40
	(0.10)	(0.08)	(0.08)	(0.09)	(0.13)	(0.04)	(0.03)	(0.04)	(0.04)	(0.08)
S4	-0.01	0.04	0.17	0.21	0.15	1.07	1.10	1.09	1.18	1.45
	(0.07)	(0.07)	(0.07)	(0.09)	(0.14)	(0.03)	(0.03)	(0.04)	(0.06)	(0.09)
S5	-0.02	0.02	0.05	0.01	0.20	0.97	0.92	0.98	1.14	1.16
	(0.05)	(0.05)	(0.07)	(0.10)	(0.14)	(0.02)	(0.02)	(0.04)	(0.07)	(0.05)
Panel B: The Fama and French 25 Size/BM Portfolios 1963:07-2007:12										
	Means					Standard Deviations				
S1	0.22	0.80	0.83	1.03	1.14	8.09	6.93	5.94	5.56	5.87
S2	0.40	0.67	0.91	0.95	1.01	7.34	5.95	5.30	5.11	5.67
S3	0.43	0.73	0.74	0.85	1.01	6.72	5.38	4.85	4.67	5.31
S4	0.53	0.53	0.73	0.85	0.87	5.95	5.07	4.79	4.63	5.23
S5	0.42	0.50	0.48	0.59	0.61	4.69	4.46	4.23	4.17	4.74
	α					β				
S1	-0.47	0.21	0.31	0.55	0.65	1.47	1.25	1.09	1.01	1.04
	(0.22)	(0.19)	(0.16)	(0.16)	(0.17)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
S2	-0.29	0.12	0.42	0.48	0.51	1.45	1.18	1.04	0.99	1.06
	(0.16)	(0.13)	(0.12)	(0.12)	(0.15)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)
S3	-0.21	0.20	0.28	0.42	0.54	1.37	1.12	0.98	0.92	1.00
	(0.13)	(0.10)	(0.11)	(0.11)	(0.15)	(0.03)	(0.03)	(0.04)	(0.04)	(0.05)
S4	-0.06	0.02	0.26	0.41	0.40	1.26	1.08	0.99	0.92	1.00
	(0.10)	(0.09)	(0.10)	(0.11)	(0.14)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)
S5	-0.06	0.05	0.07	0.21	0.22	1.01	0.95	0.85	0.79	0.83
	(0.08)	(0.07)	(0.09)	(0.10)	(0.13)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)

Table 3.1 *(Continued)*

Panel C: Macroecnomic Variables						
Variable	Mean	Std.dev.	Cross Correlations			
DIV	0.47	0.21				
RF	0.31	0.25	-0.24			
IP	0.28	1.85	-0.09	-0.07		
DEF	1.12	0.71	0.61	-0.07	-0.11	
TERM	2.30	1.37	0.05	-0.33	0.03	0.40

Panel B gives the summary statistics of the Fama-French portfolios' returns for the subsample from July 1963 to December 2007 because this subsample is used to test the performance of out-of-sample forecasted market beta. The means and standard deviations of the subsample are lower than those of the whole sample but with similar patterns: returns decrease with size (except growth portfolios B1) and increase with BM. Unconditional alpha is generally greater than the whole sample with a few exceptions while unconditional market beta is lower except the growth portfolios (column B1). Unconditional market beta generally decreases with BM in the second subsample, which indicates a negative relationship between market beta and returns. Therefore, the size/value effects are more prominent in the second subsample and it is more challenging for conditional market beta to explain the cross-section of stock returns in this subsample.

Panel C gives summary statistics of the macroeconomic variables. Cross correlations of the five series are small, all within -0.40 and 0.40 except the correlation between dividend yield and default premium (0.61), therefore there will be little problem of multicolinearity in the regression of equation (3.7).

3.4 Empirical results

3.4.1 Estimation of conditional market beta models

This section gives the whole sample estimation results of models of conditional market beta described in section 3.2.2, i.e. the macroeconomic variables model, the state-space model, the DCC-GARCH (1,1) model and the realized beta model.

3.4.1.1 The macroeconomic variables model

Table 3.2 reports the estimation results of coefficients of conditional beta equation of the macroeconomic variables model. From the results, we can see that the constant is the most important item of market beta and is similar to unconditional beta. Each macroeconomic variable has significant coefficients in some of the 25 portfolios except the TERM variable, which is insignificant in all of the 25 portfolios. The most significant variable is the one month Treasury bill rate, RF, which is significant in 18 portfolios. The Wald test statistics for the coefficients of the macroeconomic variables equal to zero are all significant at 5% level except the portfolio S5B2, which shows evidence of time-varying properties of conditional market beta.

3.4.1.2 The state-space model

Table 3.3 reports the estimation results of parameters of conditional beta in the state-space model of equation (3.8) and (3.9), using either the Kalman filter method or the MCMC method. Both methods generate very similar results but the Kaman filter is much faster than the MCMC method. The parameter ϕ_{i0} corresponds to the unconditional mean of conditional beta *i*, which is similar to the pattern of unconditional beta. The AR(1) coefficient, ϕ_{i1}, is significant in 21 portfolios and its value ranges from -0.16 to 0.96, which shows substantial evidence of time-varying property and cross-sectional differences. 13 portfolios have estimated ϕ_{i1} below 0.5 which has two implications: the first is that mean-reversion is important for beta, the second is that conditional beta is not as persistent as conditional volatility, the AR(1) coefficient in conditional volatility models is very close to 1. Therefore conditional market beta has lower predictability compared to conditional volatility. The lower predictability makes it difficult to forecast conditional market beta out-of-sample.

Table 3.2 Estimation Results of the Macroeconomic Variables Model

This table reports the coefficients of conditional beta in the macroeconomic variables model using the whole sample from July 1926 to Dec 2007,

$$r_{i,t} = \alpha_i + \beta_{i,t} r_{m,t} + \varepsilon_{i,t},$$
$$\beta_{i,t} = \delta_{0i} + \delta_{1i} Z_{t-1}.$$

The first column denotes the Fama-French 25 portfolios, S1 through S5 stand for size quintiles and B1 through B5 stand for BM quintiles. Columns 2-7 are the coefficients of conditional beta: a constant and macroeconomic variables including: DIV is dividend yield of S&P500 which is defined as the sum of previous twelve months' dividend divided by the price of current month, RF is the one-month Treasury bill rate, IP is the monthly change of logarithm industry production, DEF is the default spread defined as Moody's Baa rated corporate bonds' yield minus Aaa rated corporate bonds' yield, and TERM is the term spread defined as Moody's Aaa rated corporate bonds' yield minus one month treasury bill rate. The last column is the Wald test statistics for all the coefficients of the macroeconomic variables are equal to zero. The asterisk * denotes significance level of 5%.

Portfolio	Constant	DIV	RF	IP	DEF	TERM	Wald
S1B1	1.13*	-0.51	0.17	0.13*	0.08	0.03	12.23*
	(0.45)	(1.00)	(0.31)	(0.05)	(0.25)	(0.06)	
S1B2	1.45*	-0.73	-0.49*	0.08	0.47	-0.10	12.43*
	(0.39)	(0.83)	(0.25)	(0.07)	(0.28)	(0.08)	
S1B3	1.63*	-0.59	-0.90*	0.05*	0.28*	-0.06	38.11*
	(0.21)	(0.39)	(0.21)	(0.02)	(0.11)	(0.04)	
S1B4	1.34*	-0.51	-0.71*	0.09	0.37*	-0.08	34.92*
	(0.14)	(0.39)	(0.17)	(0.05)	(0.19)	(0.05)	
S1B5	1.43*	-0.13	-0.88*	0.09	0.26	-0.08	29.87*
	(0.20)	(0.42)	(0.25)	(0.05)	(0.18)	(0.05)	
S2B1	1.40*	-0.56*	0.14	0.05*	0.10	0.01	67.62*
	(0.12)	(0.19)	(0.16)	(0.02)	(0.07)	(0.03)	
S2B2	1.18*	-0.52	-0.18	0.06	0.29*	-0.03	13.42*
	(0.13)	(0.33)	(0.13)	(0.03)	(0.14)	(0.04)	
S2B3	1.15*	-0.48	-0.32*	0.05	0.28	-0.04	16.55*
	(0.10)	(0.34)	(0.12)	(0.03)	(0.15)	(0.04)	
S2B4	1.27*	-0.33	-0.59*	0.04	0.23	-0.05	42.49*
	(0.10)	(0.30)	(0.12)	(0.03)	(0.13)	(0.04)	
S2B5	1.24*	0.27	-0.68*	0.04	0.08	-0.02	51.16*
	(0.14)	(0.29)	(0.16)	(0.03)	(0.11)	(0.04)	

Table 3.2 (*continued*)

S3B1	1.17*	-0.61*	0.26*	0.04*	0.27*	-0.02	25.66*
	(0.09)	(0.23)	(0.10)	(0.02)	(0.09)	(0.03)	
S3B2	1.26*	-0.39*	-0.18*	0.02	0.12*	-0.02	12.83*
	(0.07)	(0.15)	(0.08)	(0.01)	(0.06)	(0.02)	
S3B3	1.16*	-0.30	-0.38*	0.01	0.18*	-0.03	36.96*
	(0.10)	(0.20)	(0.10)	(0.01)	(0.07)	(0.03)	
S3B4	1.06*	0.04	-0.45*	0.04*	0.09	-0.01	77.38*
	(0.07)	(0.19)	(0.10)	(0.01)	(0.08)	(0.03)	
S3B5	1.12*	0.49*	-0.74*	0.03*	0.05	0.01	209.06*
	(0.10)	(0.18)	(0.15)	(0.01)	(0.06)	(0.02)	
S4B1	1.17*	-0.23*	0.31*	0.00	-0.01	0.00	122.28*
	(0.06)	(0.09)	(0.06)	0.00	(0.02)	(0.01)	
S4B2	0.95*	-0.07	0.08	0.00	0.05	0.02	12.66*
	(0.07)	(0.18)	(0.07)	(0.01)	(0.06)	(0.02)	
S4B3	0.94*	0.09	-0.15	0.02*	0.04	0.02	20.79*
	(0.10)	(0.18)	(0.08)	(0.01)	(0.04)	(0.02)	
S4B4	0.90*	0.39*	-0.42*	0.02*	0.02	0.02	93.86*
	(0.09)	(0.16)	(0.09)	(0.01)	(0.05)	(0.02)	
S4B5	1.22*	0.23	-0.82*	0.01	0.15	-0.02	235.60*
	(0.10)	(0.20)	(0.14)	(0.02)	(0.09)	(0.03)	
S5B1	1.03*	0.00	0.05	-0.01*	-0.03	0.00	49.97*
	(0.04)	(0.05)	(0.06)	0.00	(0.02)	(0.01)	
S5B2	0.89*	0.08	0.12*	0.00	-0.04	0.01	9.80
	(0.04)	(0.08)	(0.05)	(0.01)	(0.03)	(0.01)	
S5B3	0.72*	0.09	0.00	0.01	0.08*	0.01	107.09*
	(0.05)	(0.08)	(0.07)	(0.01)	(0.03)	(0.01)	
S5B4	0.87*	0.28*	-0.47*	0.00	0.10*	-0.01	337.36*
	(0.08)	(0.12)	(0.09)	(0.01)	(0.04)	(0.02)	
S5B5	1.52*	-0.36	-1.15*	0.03*	0.11	-0.02	60.69*
	(0.17)	(0.28)	(0.17)	(0.01)	(0.07)	(0.03)	

Table 3.3 Estimation Results of the State-Space Model

This table reports the coefficients of conditional beta in the state-space model using whole sample from July 1926 to Dec 2007,

$$r_{i,t} = \beta_{i,t} r_{m,t} + \varepsilon_{i,t},$$
$$\beta_{i,t} = (1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{i,t-1} + u_{i,t}.$$

The first column denotes the Fama-French 25 portfolios, S1 through S5 stand for size quintiles and B1 through B5 stand for BM quintiles. Columns 2-5 are the coefficients of conditional beta. KF stands for the estimation via the Kalman filter method and MCMC stands for the Markov chain Monte Carlo method. The standard errors are in parenthesis. The asterisk * denotes significance level of 5%.

Portfolio	ϕ_{i0}		ϕ_{i1}	
	KF	MCMC	KF	MCMC
S1B1	1.54*	1.53*	-0.16	-0.16
	(0.06)	(0.06)	(0.09)	(0.09)
S1B2	1.35*	1.35*	-0.16*	-0.16*
	(0.04)	(0.05)	(0.07)	(0.07)
S1B3	1.27*	1.26*	0.11	0.13
	(0.04)	(0.04)	(0.11)	(0.10)
S1B4	1.16*	1.16*	0.24*	0.25*
	(0.04)	(0.04)	(0.09)	(0.08)
S1B5	1.28*	1.27*	0.18*	0.19*
	(0.05)	(0.04)	(0.09)	(0.09)
S2B1	1.32*	1.32*	0.40*	0.39*
	(0.04)	(0.04)	(0.14)	(0.15)
S2B2	1.21*	1.21*	0.18	0.17
	(0.03)	(0.03)	(0.10)	(0.09)
S2B3	1.11*	1.11*	0.34*	0.35*
	(0.03)	(0.03)	(0.08)	(0.08)
S2B4	1.14*	1.13*	0.50*	0.50*
	(0.03)	(0.03)	(0.09)	(0.09)
S2B5	1.27*	1.27*	0.22*	0.22*
	(0.04)	(0.04)	(0.10)	(0.09)
S3B1	1.24*	1.23*	0.49*	0.48*
	(0.03)	(0.03)	(0.11)	(0.11)
S3B2	1.12*	1.12*	0.31	0.36*
	(0.02)	(0.02)	(0.16)	(0.16)
S3B3	1.08*	1.08*	0.84*	0.83*
	(0.03)	(0.04)	(0.05)	(0.06)
S3B4	1.07*	1.07*	0.50*	0.51*
	(0.02)	(0.03)	(0.10)	(0.11)
S3B5	1.22*	1.22*	0.86*	0.85*
	(0.06)	(0.06)	(0.04)	(0.04)

Table 3.3 (*continued*)

Portfolio	ϕ_{i0}		ϕ_{i1}	
	KF	**MCMC**	**KF**	**MCMC**
S4B1	1.15*	1.15*	0.96*	0.96*
	(0.05)	(0.26)	(0.01)	(0.02)
S4B2	1.06*	1.06*	0.79*	0.77*
	(0.02)	(0.02)	(0.08)	(0.06)
S4B3	1.05*	1.05*	0.59*	0.56*
	(0.02)	(0.02)	(0.10)	(0.10)
S4B4	1.09*	1.08*	0.80*	0.78*
	(0.04)	(0.04)	(0.05)	(0.05)
S4B5	1.23*	1.23*	0.82*	0.81*
	(0.06)	(0.06)	(0.06)	(0.05)
S5B1	0.99*	0.98*	0.03	0.06
	(0.01)	(0.01)	(0.23)	(0.21)
S5B2	0.94*	0.95*	0.82*	0.65*
	(0.02)	(0.02)	(0.09)	(0.14)
S5B3	0.89*	0.89*	0.48*	0.48*
	(0.02)	(0.02)	(0.10)	(0.11)
S5B4	0.97*	0.97*	0.94*	0.93*
	(0.06)	(0.06)	(0.02)	(0.03)
S5B5	1.11*	1.11*	0.80*	0.80*
	(0.06)	(0.06)	(0.06)	(0.06)

3.4.1.3 The DCC-GARCH(1,1) model

Table 3.4 reports the estimation results of the DCC-GARCH (1,1) model. Panel A includes the coefficients of the univariate GARCH(1,1) model of the returns of the 25 portfolios and the market. Consistent with the GARCH literature, the sum of the ARCH coefficient(κ_i) and GARCH coefficient (λ_i) is very close to one：greater than 0.9 for all equations, which means volatility is very persistent. The estimated coefficients of the conditional correlation equation are in Panel B. Again, the results are similar to the existing empirical work (e.g. Engle, 2002). The conditional correlation is also very persistent with θ_2 equal to 0.942.

3.4.1.4 The realized beta model

The estimation results of the realized beta model are reported in Table 3.5. The parameters α_{oi}, which correspond to the unconditional mean of integrated beta, are increasing with size and decreasing with book-to-market ratio. The AR(1) parameters α_{1i} are all greater than 0.75 indicating integrated beta is a very persistent process. This is different from the AR(1) coefficients of the state-space model in Table 3.4, which are lower. This is due to the different modelling techniques. Therefore, different techniques can give very different conditional market beta.

3.4.2 A comparison of the different models

Table 3.6 reports the sample means and standard deviations of different market beta generated by the different models along with unconditional beta. The means of conditional beta are similar to corresponding unconditional beta but the standard deviations are very high. Among conditional beta, beta from the state-space model has higher standard deviations than other beta, and the random walk model has the highest standard deviation, which makes sense because a random walk model is not mean reversion. Therefore, the time-varying property of conditional beta is substantial. Finally, the means of most realized beta are lower than other beta, which is because realized beta is only available from July 1963 and most betas have a decreasing trend.

Table 3.4 Estimation Results of the DCC-GARCH model

This table reports the estimation results of the conditional variance and correlation equations of the DCC-GARCH (1,1) model,

$$r_t = \mu_t + a_t, \;\; a_t \sim N(0, \Sigma_t),$$

$$\Sigma_t \equiv \left(\sigma_{ij,t}\right)_{N \times N} = D_t \rho_t D_t,$$

$$\sigma_{ii,t}^2 = \omega_i + \kappa_i a_{i,t-1}^2 + \lambda_i \sigma_{ii,t-1}^2,$$

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 e_{t-1} e_{t-1}' + \theta_2 Q_{t-1} \quad ,$$

$$\rho_t = J_t^{-1} Q_t J_t^{-1},$$

where each return series is assumed to be an AR(1) process, $D_t = \text{diag}\left\{\sqrt{\sigma_{ii,t}}\right\}$, $Q_t = \left(q_{ij,t}\right)_{N \times N}$, $e_t = D_t^{-1} a_t$ and $J_t = \text{diag}\left\{\sqrt{q_{ii,t}}\right\}$. The sample is monthly returns of the Fama-French 25 portfolios and the value-weighted CRSP stock index from July 1926 through December 2007. S1 through S5 stand for size quintiles and B1 through B5 stand for BM quintiles. Mkt is the value-weighted CRSP stock index. The standard errors are in parenthesis. The asterisk * denotes significance level of 5%.

Panel A: Univariate GARCH Equation			
Portfolio	ω_i	κ_i	λ_i
S1B1	1.928*	0.115*	0.873*
	(0.464)	(0.013)	(0.016)
S1B2	1.365*	0.129*	0.868*
	(0.275)	(0.010)	(0.011)
S1B3	0.640*	0.099*	0.894*
	(0.187)	(0.016)	(0.015)
S1B4	0.889*	0.107*	0.880*
	(0.224)	(0.015)	(0.017)
S1B5	0.707*	0.098*	0.897*
	(0.131)	(0.013)	(0.011)
S2B1	1.158*	0.114*	0.873*
	(0.399)	(0.017)	(0.018)
S2B2	1.203*	0.129*	0.852*
	(0.407)	(0.018)	(0.021)
S2B3	0.709*	0.117*	0.872*
	(0.236)	(0.017)	(0.018)
S2B4	1.056*	0.109*	0.867*
	(0.293)	(0.017)	(0.020)
S2B5	1.043*	0.115*	0.871*
	(0.279)	(0.018)	(0.018)

Table 3.4 (*continued*)

S3B1	1.360*	0.171*	0.814*
	(0.428)	(0.021)	(0.020)
S3B2	0.915*	0.120*	0.861*
	(0.286)	(0.020)	(0.020)
S3B3	1.033*	0.139*	0.838*
	(0.284)	(0.023)	(0.023)
S3B4	1.262*	0.117*	0.849*
	(0.326)	(0.022)	(0.026)
S3B5	1.254*	0.130*	0.850*
	(0.283)	(0.018)	(0.019)
S4B1	0.841*	0.130*	0.855*
	(0.291)	(0.021)	(0.021)
S4B2	0.786*	0.131*	0.852*
	(0.239)	(0.019)	(0.018)
S4B3	0.962*	0.129*	0.847*
	(0.250)	(0.017)	(0.016)
S4B4	0.823*	0.114*	0.866*
	(0.246)	(0.019)	(0.021)
S4B5	0.963*	0.128*	0.858*
	(0.311)	(0.018)	(0.019)
S5B1	0.675*	0.120*	0.861*
	(0.220)	(0.017)	(0.016)
S5B2	0.455*	0.126*	0.862*
	(0.168)	(0.017)	(0.014)
S5B3	0.587*	0.119*	0.863*
	(0.181)	(0.019)	(0.018)
S5B4	0.677*	0.147*	0.836*
	(0.230)	(0.024)	(0.024)
S5B5	0.914*	0.116*	0.865*
	(0.312)	(0.018)	(0.022)
MKT	0.611*	0.127*	0.856*
	(0.223)	(0.023)	(0.021)
Panel B: Conditional Correlation Equation			
	θ_1	θ_2	
	0.023*	0.942*	
	(0.001)	(0.005)	

Table 3.5 Estimation Results of the Realized Beta Model

This table reports the estimation of the parameters of the following model:

$$\hat{\beta}_{i,t} = \beta_{i,t} + \varepsilon_{i,t},$$

$$\beta_{i,t} = (1 - \alpha_{1i})\alpha_{0i} + \alpha_{1i}\beta_{i,t-1} + u_{i,t}.$$

The sample is monthly returns of the Fama-French 25 portfolios from July 1926 through December 2007. S1 through S5 stand for size quintiles and B1 through B5 stand for BM quintiles. The standard errors are in parenthesis. The asterisk * denotes significance level of 5%.

	α_{0i}					α_{1i}				
	B1	B2	B3	B4	B5	B1	B2	B3	B4	B5
S1	0.98*	0.85*	0.74*	0.67*	0.67*	0.85*	0.90*	0.92*	0.93*	0.92*
	(0.06)	(0.07)	(0.07)	(0.07)	(0.06)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
S2	1.13*	0.90*	0.82*	0.75*	0.85*	0.76*	0.89*	0.94*	0.94*	0.93*
	(0.04)	(0.05)	(0.08)	(0.07)	(0.08)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
S3	1.12*	0.88*	0.79*	0.77*	0.86*	0.79*	0.86*	0.91*	0.93*	0.91*
	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)
S4	1.11*	0.89*	0.84*	0.85*	0.93*	0.86*	0.84*	0.86*	0.93*	0.91*
	(0.03)	(0.02)	(0.03)	(0.05)	(0.06)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)
S5	1.14*	1.01*	0.94*	0.91*	0.97*	0.85*	0.89*	0.88*	0.89*	0.91*
	(0.02)	(0.03)	(0.03)	(0.03)	(0.05)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)

Table 3.6 Comparison of the Unconditional and Conditional Betas

The table reports the summary statistics of the unconditional and conditional beta of the Fama-French 25 portfolios. The sample is monthly returns of the Fama-French 25 portfolios from July 1926 through December 2007 except the realized beta model, which uses monthly realized beta series from July 1963 through December 2007. S1 through S5 stand for size quintiles and B1 through B5 stand for BM quintiles. OLS stands for the OLS estimation of unconditional beta. Rolling is the 60-month rolling window regression estimation. Macro is beta from the macroeconomic variables. KF_AR is the state-space model with AR(1) state equation and KF_RW is state-space model with random walk state equation, both of which are estimated by the Kalman filter. MC_AR and MC_RW are state-space models with AR(1) state equation and random walk state equation respectively and estimated using the MCMC method. DCC is the DCC-GARCH(1,1) model. Realized is the realized beta model. Standard deviations of conditional beta are in parenthesis.

Portfolio	OLS	Rolling	Macro	KF_AR	KF_RW	MC_AR	MC_RW	DCC	Realized
S1B1	1.65	1.50	1.52	1.54	1.50	1.53	1.49	1.55	0.98
		(0.32)	(0.29)	(0.48)	(0.38)	(0.48)	(0.56)	(0.40)	(0.34)
S1B2	1.48	1.33	1.27	1.35	1.32	1.35	1.32	1.36	0.84
		(0.27)	(0.27)	(0.51)	(0.70)	(0.51)	(0.51)	(0.31)	(0.32)
S1B3	1.40	1.22	1.27	1.27	1.23	1.26	1.23	1.23	0.73
		(0.36)	(0.25)	(0.46)	(0.55)	(0.46)	(0.48)	(0.37)	(0.30)
S1B4	1.31	1.14	1.12	1.17	1.15	1.16	1.15	1.16	0.66
		(0.31)	(0.27)	(0.38)	(0.56)	(0.38)	(0.46)	(0.30)	(0.30)
S1B5	1.39	1.22	1.22	1.28	1.27	1.27	1.26	1.27	0.66
		(0.38)	(0.29)	(0.45)	(0.65)	(0.45)	(0.54)	(0.42)	(0.28)
S2B1	1.25	1.33	1.33	1.32	1.32	1.32	1.32	1.32	1.13
		(0.25)	(0.15)	(0.25)	(0.39)	(0.25)	(0.32)	(0.25)	(0.27)
S2B2	1.28	1.19	1.16	1.21	1.20	1.21	1.19	1.19	0.90
		(0.20)	(0.17)	(0.26)	(0.39)	(0.26)	(0.31)	(0.20)	(0.26)
S2B3	1.19	1.10	1.06	1.11	1.12	1.11	1.11	1.11	0.80
		(0.22)	(0.17)	(0.26)	(0.38)	(0.27)	(0.32)	(0.22)	(0.30)
S2B4	1.23	1.12	1.09	1.14	1.14	1.13	1.13	1.12	0.74
		(0.24)	(0.18)	(0.24)	(0.38)	(0.26)	(0.32)	(0.22)	(0.28)
S2B5	1.36	1.24	1.21	1.27	1.28	1.27	1.26	1.25	0.85
		(0.30)	(0.22)	(0.35)	(0.45)	(0.35)	(0.38)	(0.30)	(0.31)

Table 3.6 (*continued*)

Portfolio	OLS	Rolling	Macro	KF_AR	KF_RW	MC_AR	MC_RW	DCC	Realized
S3B1	1.28	1.26	1.22	1.24	1.23	1.23	1.22	1.24	1.12
		(0.20)	(0.17)	(0.19)	(0.32)	(0.21)	(0.26)	(0.23)	(0.21)
S3B2	1.13	1.11	1.12	1.12	1.12	1.12	1.12	1.11	0.88
		(0.14)	(0.08)	(0.14)	(0.21)	(0.14)	(0.18)	(0.14)	(0.17)
S3B3	1.15	1.06	1.04	1.08	1.07	1.07	1.08	1.06	0.78
		(0.18)	(0.12)	(0.18)	(0.26)	(0.20)	(0.21)	(0.16)	(0.20)
S3B4	1.13	1.06	1.03	1.07	1.09	1.07	1.07	1.06	0.77
		(0.20)	(0.15)	(0.21)	(0.29)	(0.22)	(0.25)	(0.17)	(0.22)
S3B5	1.40	1.21	1.21	1.23	1.23	1.22	1.22	1.21	0.86
		(0.33)	(0.26)	(0.32)	(0.43)	(0.36)	(0.37)	(0.29)	(0.26)
S4B1	1.07	1.14	1.15	1.14	1.14	1.14	1.14	1.14	1.10
		(0.15)	(0.10)	(0.14)	(0.18)	(0.16)	(0.16)	(0.17)	(0.15)
S4B2	1.10	1.07	1.05	1.06	1.06	1.06	1.07	1.07	0.89
		(0.12)	(0.05)	(0.12)	(0.17)	(0.13)	(0.15)	(0.13)	(0.12)
S4B3	1.09	1.04	1.02	1.05	1.05	1.05	1.04	1.04	0.84
		(0.13)	(0.08)	(0.14)	(0.21)	(0.15)	(0.18)	(0.12)	(0.14)
S4B4	1.17	1.06	1.04	1.09	1.09	1.09	1.08	1.06	0.85
		(0.20)	(0.17)	(0.22)	(0.32)	(0.25)	(0.28)	(0.19)	(0.20)
S4B5	1.44	1.23	1.20	1.24	1.24	1.23	1.23	1.22	0.93
		(0.35)	(0.26)	(0.31)	(0.41)	(0.35)	(0.37)	(0.31)	(0.27)
S5B1	0.97	1.00	1.00	0.99	1.00	0.98	1.00	0.99	1.15
		(0.07)	(0.03)	(0.06)	(0.09)	(0.05)	(0.07)	(0.09)	(0.12)
S5B2	0.92	0.94	0.95	0.94	0.95	0.94	0.95	0.94	1.01
		(0.08)	(0.03)	(0.09)	(0.15)	(0.10)	(0.11)	(0.10)	(0.13)
S5B3	0.98	0.89	0.87	0.90	0.89	0.89	0.89	0.90	0.94
		(0.12)	(0.07)	(0.14)	(0.15)	(0.14)	(0.13)	(0.12)	(0.15)
S5B4	1.13	0.97	0.94	0.98	0.98	0.97	0.97	0.96	0.91
		(0.24)	(0.18)	(0.21)	(0.27)	(0.23)	(0.24)	(0.20)	(0.16)
S5B5	1.16	1.07	1.09	1.11	1.11	1.10	1.11	1.09	0.98
		(0.30)	(0.28)	(0.32)	(0.43)	(0.36)	(0.38)	(0.28)	(0.22)

Figure 3.1 plots the different conditional betas of portfolios S1B1 and S1B5. Betas from the state-space models estimated by the MCMC method are not graphed because they are very similar to those estimated from the Kalman filter method. From the graph, it is clear that conditional betas are different from each other. Beta from rolling window estimation is smoother than other conditional betas; it makes sense that each time only one new observation is added and the oldest observation is discarded. The AR(1) state-space model generates the most varying beta, which corresponds to its mean reverting property, while the random walk state-space model generates a much smoother beta series due to the unit root. The macroeconomic variables model and the DCC-GARCH model generate beta varying between the rolling window model and the AR(1) state-space model, which is due to the fact that information variables used in the macroeconomic variable model are very persistent and the conditional correlation of the DCC-GARCH(1,1) model is also very persistent, respectively. Most of the time, realized beta is the lowest among conditional betas for portfolios S1B1 and S1B5.

Due to the unobservable property of beta, it is hard to say which model is better only from the estimation results. So it is only able to see if any models are better within a specific application. The next subsection examines if these conditional market beta models can explain the cross-section of stock returns, which is among the most serious challenges of the CAPM.

Figure 3.1 Plots of Betas

This figure plots conditional betas of portfolios S1B1 and S1B5. OLS is unconditional beta. Rolling is the 60-month rolling window regression. Macro is the macroeconomic variable model. KF_AR and KF_RW are state-space model with AR(1) and random walk state equations, respectively, estimated by Kalman filter. DCC is the DCC-GARCH(1,1) model. Realized is the realized beta model.

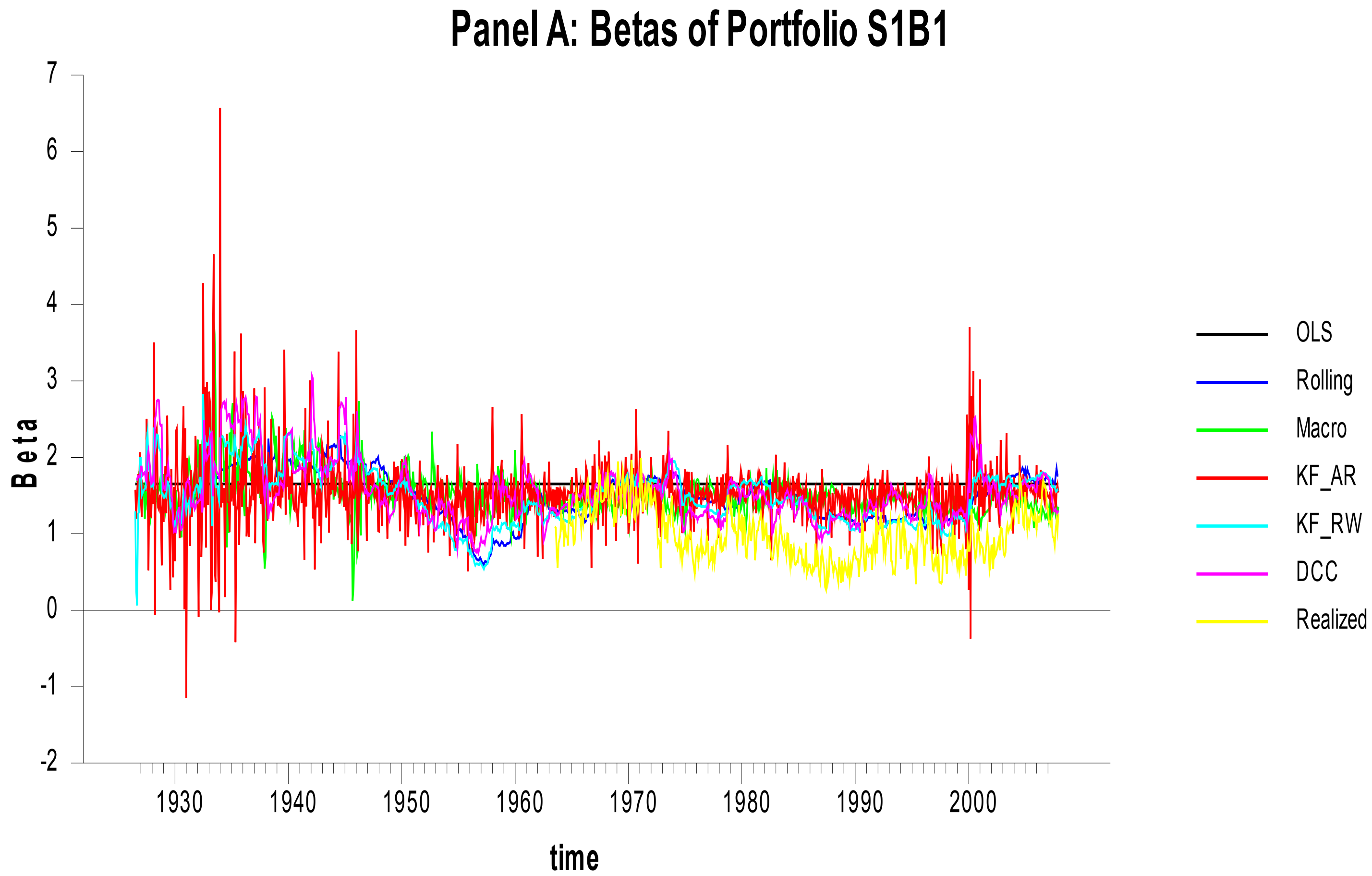
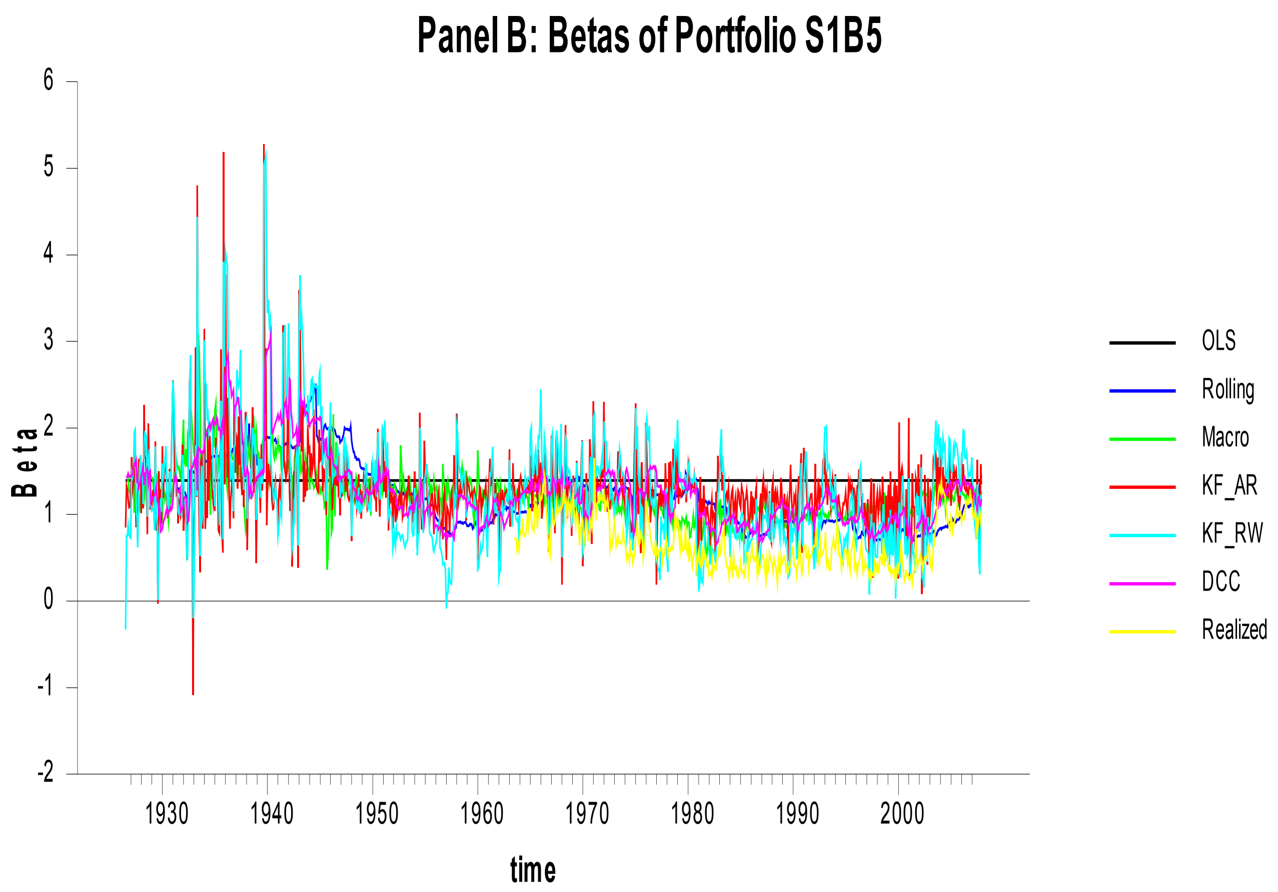


Figure 3.1 (*continued*)



3.4.3 Cross-sectional regression results

3.4.3.1 In-sample estimated market beta

This subsection reports Fama-French cross-sectional regression results using in-sample estimated beta. The rolling window regression is estimated every 60 months including current month, i.e. for each month *t*, data of month *t*-59 through month *t* are used, then estimated beta is used in the cross-sectional regression with returns of month *t*. The inclusion of returns of month *t* is to count in the effect of the current month. All the other models are estimated using the whole sample. For the state-space models estimated by the Kalman filter, beta used in the cross-sectional regressions is filtered beta, which is comparable with beta estimated by the MCMC method. The realized beta used in the cross-sectional regression is also filtered betas from the Kalman filter and both smoothed and raw realized beta give similar results.

Panel A of Table 3.7 reports the time series statistics of the cross-sectional regressions of the whole sample. The state-space model with a random walk beta estimated by either the Kalman filter or the MCMC method produces the best results: small and insignificant constant and positive and significant risk premium of market beta. The estimated risk premium of market beta is 0.70% from the Kalman filter and 0.67% from the MCMC, both of which are close to the sample mean of the monthly market excess return (0.65%). The R^2 (37% for Kalman filter and 35% for MCMC) is much greater than that of unconditional beta model (21%). The mean-reversion beta model has the highest R^2 (53% for Kalman filter and 50% for MCMC) and a significant positive risk premium, but alpha of the mean-reversion beta model is higher than the random walk beta model and significantly different from zero. This result is consistent with the existing literature (e.g. Faff et al., 2000; Marti, 2005; Mergner and Bulla, 2008). All other conditional beta models have significant alphas and fail to generate significant risk premiums of market beta. The R^2 of these conditional beta models is only slightly greater than that of unconditional beta, which suggests no improvement.

To check the robustness of the above results, I divide the whole sample into two subsamples, the first subsample is from July 1926 to June 1963 and the second is from July 1963 to December 2007. The results of the two subsamples are reported in panel B and panel C of Table 3.7, respectively. Consistent with the results of the whole sample,

Table 3.7 Fama-MacBeth Cross-Sectional Regression Results of in-sample Estimated Beta

This table reports the time series means and standard errors of sample mean (in parenthesis) of the estimated parameters of the regressions:

$$r_{i,t} = \alpha_t + \gamma_t \hat{\beta}_{i,t} + \varepsilon_{i,t}\,.$$

using the in-sample estimated betas. R^2 is the average of the time series R-squared of each cross-sectional regression. The data is the returns of Fama-French 25 portfolios and the in-sample estimated beta is from July 1926 through December 2007 except that realized beta is only available from July 1963. OLS is unconditional beta. Rolling is the 60-month rolling window regression. Macro is the macroeconomic variable model. KF_AR and KF_RW are the state-space model with AR(1) and random walk state equations, respectively, estimated by the Kalman filter while MC_AR and MC_RW are the same models but estimated by the MCMC method. DCC is the DCC-GARCH(1,1) model. Realized is the realized beta model. The asterisk * denotes significance level of 5%.

	OLS	Rolling	Macro	KF_AR	KF_RW	MC_AR	MC_RW	DCC	Realized
Panel A: Full Sample from July 1926 through December 2007									
α	0.41	0.70*	0.87*	-0.62*	-0.07	-0.51*	-0.02	0.83*	
	(0.33)	(0.23)	(0.26)	(0.16)	(0.10)	(0.15)	(0.13)	(0.23)	
γ	0.40	0.16	-0.09	1.21*	0.70*	1.10*	0.67*	0.02	
	(0.36)	(0.31)	(0.31)	(0.27)	(0.22)	(0.26)	(0.24)	(0.27)	
R^2	0.21	0.24	0.22	0.53	0.37	0.50	0.35	0.23	
Panel B: Subsample from July 1926 through June 1963									
α	0.57	0.52	0.76*	-1.08*	-0.16	-0.95*	-0.33	0.96*	
	(0.59)	(0.40)	(0.37)	(0.26)	(0.15)	(0.24)	(0.21)	(0.36)	
γ	0.44	0.59	0.16	1.75*	0.90*	1.61*	1.07*	0.08	
	(0.69)	(0.56)	(0.47)	(0.46)	(0.38)	(0.44)	(0.41)	(0.44)	
R^2	0.22	0.22	0.20	0.58	0.37	0.55	0.36	0.20	
Panel C: Subsample from July 1963 through December 2007									
α	0.27	0.83*	0.97*	-0.24	0.01	-0.13	0.24	0.72*	0.93*
	(0.35)	(0.28)	(0.36)	(0.20)	(0.14)	(0.18)	(0.17)	(0.28)	(0.22)
γ	0.36	-0.14	-0.29	0.77*	0.54*	0.68*	0.33	-0.03	-0.28
	(0.34)	(0.34)	(0.40)	(0.32)	(0.26)	(0.31)	(0.28)	(0.33)	(0.25)
R^2	0.20	0.26	0.24	0.50	0.36	0.47	0.34	0.25	0.22

the state-space model with random walk beta gives the best results. Beta estimated by the Kalman filter method generates insignificant alphas and significantly positive risk premiums in both subsamples. The risk premium of beta is 0.90% for the first subsample and 0.54% for the second one; both are close to the corresponding sample means of the market excess return (0.85% and 0.47%, respectively). But the risk premium of the MCMC estimated beta is no longer significant in the second subsample. The R^2 of the random walk beta model is around 35%. The mean-reversion beta model has the highest R^2 (around 50%) and significantly positive risk premiums in both subsamples and a significant alpha in the first subsample and an insignificant one in the second subsample, which suggests the mean reversion beta model succeeds only in the second subsample but fails in the first subsample. All other conditional beta models have significant alphas (except the rolling window model in the first subsample) and insignificant risk premiums. Noticeably, the risk premiums estimated using different beta models are lower in the second subsample than the first subsample and all the risk premiums are negative in the second subsample except those of the state-space model.

Overall, the results suggest that the state-space model with a random walk beta generates the best results in-sample among the different conditional beta models. The state-space model with a mean-reversion beta works well only in the second subsample. The Kalman filter method gives better results than the MCMC method. The next step is to examine if any models have out-of-sample forecast ability to explain the cross-section of stock returns, which is the main purpose of this chapter.

3.4.3.2 Out-of-sample forecasted market beta

The out-of-sample forecast starts from July 1963, which is the most serious challenging period, by using the first subsample from July 1926 to June 1963 as the initial sample except the realized beta model. The forecast is done by the expanding sample method, i.e. for each month t, all the data available from July 1926, the beginning of sample, until month t-1 are used. I also tried a rolling window method and got similar results but the algorithms of the state-space models and DCC-GARCH(1,1) model by using the expanding sample method converge better than those of the rolling window method[9]. So I report the results from the expanding sample method in the main text and attach the results of the rolling window method in Appendix 3C. For the realized beta model, the forecast starts from July 1968 by using the expanding sample method; the results of rolling window method are also in Appendix 3C.

Panel A of Table 3.8 reports the time series statistics of Fama-French cross-sectional regression parameters. The results are very different from those by using in-sample estimated beta: no conditional beta models can generate a significantly positive coefficient of beta. Coefficients of beta are still positive for the state-space models with either random-walk beta or AR(1) beta but much lower than those of in-sample estimated beta and are no longer significantly different from zero. Expanding sample OLS also has a positive but insignificant coefficient of beta (0.40). All other models, i.e. the macroeconomic model, the DCC-GARCH (1,1) model and the realized beta model,

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[9] The algorithms of the expanding sample method converge in all cases but some cannot converge in the rolling window method.

Table 3.8 Fama-MacBeth Cross-sectional Regression Results of out-of-sample Forecasted Beta from Expanding Sample Method

This table reports the time series means and standard errors of sample mean (in parenthesis) of the estimated parameters of the regressions:

$$r_{i,t} = \alpha_t + \gamma_t \hat{\beta}_{i,t} + \varepsilon_{i,t} \cdot$$

using the out-of-sample forecasted betas from the expanding sample method. R^2 is the average of the time series R-square of each cross-sectional regression. The data is the returns of Fama-French 25 portfolios and the out-of-sample forecasted betas from July 1963 through December 2007 except that the realized betas are available from July 1968. OLS is the expanding sample regression. Macro is the macroeconomic variable model. KF_AR and KF_RW are state-space model with AR(1) and random walk state equations, respectively, estimated by Kalman filter while MC_AR and MC_RW are the same models but estimated by the MCMC method. DCC is the DCC-GARCH(1,1) model. Realized is the realized beta model. The asterisk * denotes significance level of 5%.

	OLS	Macro	KF_AR	KF_RW	MC_AR	MC_RW	DCC	Realized
Panel A: Full sample from July 1963 to December 2007								
α	0.19	1.11*	0.54	0.31	0.52	0.31	0.73*	1.26*
	(0.32)	(0.35)	(0.31)	(0.21)	(0.31)	(0.21)	(0.29)	(0.27)
γ	0.40	-0.40	0.12	0.31	0.15	0.32	-0.05	-0.72*
	(0.27)	(0.35)	(0.33)	(0.20)	(0.33)	(0.20)	(0.33)	(0.26)
R^2	0.18	0.18	0.23	0.20	0.23	0.20	0.23	0.18
Panel B: Subsample from July 1963 to December 1985								
	OLS	Macro	KF_AR	KF_RW	MC_AR	MC_RW	DCC	Realized
α	-0.51	0.95*	-0.15	0.27	-0.17	0.20	0.62	1.49*
	(0.36)	(0.33)	(0.40)	(0.28)	(0.40)	(0.29)	(0.40)	(0.43)
γ	0.90*	-0.31	0.65	0.24	0.67	0.31	-0.07	-1.17*
	(0.32)	(0.31)	(0.44)	(0.28)	(0.44)	(0.28)	(0.47)	(0.42)
R^2	0.19	0.15	0.24	0.23	0.24	0.23	0.25	0.18
Panel C: Subsample from January 1986 to December 2007								
	OLS	Macro	KF_AR	KF_RW	MC_AR	MC_RW	DCC	Realized
α	0.91	1.28*	1.26*	0.35	1.22*	0.41	0.83*	1.07*
	(0.53)	(0.61)	(0.47)	(0.31)	(0.48)	(0.31)	(0.42)	(0.34)
γ	-0.11	-0.49	-0.42	0.38	-0.39	0.33	-0.02	-0.37
	(0.45)	(0.64)	(0.49)	(0.28)	(0.49)	(0.27)	(0.45)	(0.33)
R^2	0.17	0.20	0.22	0.17	0.22	0.17	0.21	0.18

have a negative coefficient of beta. The R^2 of each model is around 20% suggesting little power of explanation of the cross-section of the 25 portfolio returns.

I further divide the whole sample into two subsamples with roughly the same length and the first subsample is from July 1963 to December 1985. The results of the first subsample are very similar to those of the whole sample: the state-space models have a positive but insignificant coefficient on beta and the macroeconomic model, the DCC-GARCH (1,1) model and the realized beta model have a negative coefficient of beta. The exception is the expanding sample OLS method, which generates a significantly positive coefficient on beta and insignificant intercept. The results of the second sample are reported in panel C. Only the state-space model with random walk beta has a positive coefficient on beta but insignificantly different from zero; all other models have a negative coefficient on beta. The results suggest that the difficulties of the conditional CAPM in explaining the cross-section of stock returns are in the more recent period.

Therefore, my results show that no conditional beta models can explain the cross-section of stock returns out-of-sample in the period from July 1963 to December 2007, which is a suggestion of the failure of these models.

3.5 Conclusion

Previous empirical studies of conditional market beta and the cross-section of stock returns focus on in-sample estimated market beta (e.g. Jostova and Philipov, 2004; Bali, 2008). However, the conditional CAPM is an ex-ante model with information available only at the time when investors make decisions but in-sample estimated market beta uses information beyond that. Therefore, this chapter examines if any conditional market beta models can explain the cross-section of stock returns not only in-sample but also out-of-sample. The models examined include: unconditional beta, the short window regression method, the macroeconomic variables model, the state-space model with a mean-reversion beta and a random walk beta estimated by either the Kalman filter or the MCMC method, a DCC-GARCH(1,1) model and a realized beta model.

In-sample, the state-space model does a good job. The random walk beta model generates an insignificant intercept and a significantly positive coefficient of beta in the whole sample and both subsamples while the mean-reversion beta model is successful only in the second subsample. All the other models fail to explain the cross-section of stock returns in either the whole sample or any of the two subsamples. The results are consistent with the existing literature (e.g. Jostova and Philipov, 2004; Marti, 2005).

For out-of-sample forecasted market beta, no models examined can generate significantly positive coefficients on beta and therefore fail to explain the cross-section of returns. The results are similar in both subsamples with only one exception that the expanding OLS method has an insignificant intercept and a significantly positive coefficient of beta in the first subsample.

Overall, out-of-sample forecasted betas cannot explain the cross-section of stock returns although in-sample estimated beta of the state-space model does a good job. The results suggest a rejection of the conditional CAPM if we use out-of-sample tests.

Appendix 3

***A.* The Kalman filter**

This section gives the details of the Kalman filter of the state-space model,

$$r_{i,t} = \beta_{i,t} r_{m,t} + \varepsilon_{i,t} \tag{3.8}$$

$$\beta_{i,t} = (1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{i,t-1} + u_{i,t}\,. \tag{3.9}$$

where $\varepsilon_{i,t} \sim N(0,\sigma^2_{\varepsilon_i})$ and $u_{i,t} \sim N(0,\sigma^2_{u_i})$ are white noise series and uncorrelated with each other. Equation (3.8) is the measurement equation and equation (3.9') is the state equation. In order to derive the Kalman filter, we need the following theorem (see Tsay, 2005):

Theorem A4.1 Suppose that x, y and z are three random vectors such that their joint distributions are multivariate normal. In addition, assume the covariance matrix of *x, y* and *z* are non-singular and *y* and *z* are uncorrelated. Then

(1) $E(x \mid y) = \mu_x + \Sigma_{xy}\Sigma_{yy}^{-1}(y-\mu_y)$,

(2) $Var(x \mid y) = \Sigma_{xx} - \Sigma_{xx}\Sigma_{yy}^{-1}\Sigma_{yx}$,

(3) $E(x \mid y,z) = E(x \mid y) + \Sigma_{xz|y}\Sigma_{zz|y}^{-1}(z-\mu_z)$,

(4) $Var(x \mid y,z) = Var(x \mid y) - \Sigma_{xz|y}\Sigma_{zz|y}^{-1}\Sigma_{zx|y}$,

where $\mu_w = E(w)$ and $\Sigma_{mw|v} = Cov(m,w \mid v)$.

Denote the conditional mean and variance of $\beta_{i,s}$ for given information set of period *j* by $\beta_{i,s|j}$ and $\Sigma_{i,s|j}$, respectively. From equation (3.9), we have

$$\beta_{i,t|t-1} = E[(1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{t-1} + u_{i,t} \mid I_{t-1}] = (1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{t-1|t-1}\,, \tag{A3.1}$$

$$\Sigma_{t|t-1} = Var[(1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{t-1} + u_{i,t} \mid I_{t-1}] = \phi_{i1}^2\Sigma_{t-1|t-1} + \sigma_{u_i}^2, \tag{A3.2}$$

$$y_{t|t-1} = E(y_t \mid I_{t-1}) = \beta_{i,t|t-1} r_{m,t}, \tag{A3.3}$$

$$\begin{aligned} v_t &= y_t - y_{t|t-1} \\ &= y_t - \beta_{i,t|t-1} r_{m,t} \\ &= \beta_{i,t} r_{m,t} + \varepsilon_{i,t} - \beta_{i,t|t-1} r_{m,t} = (\beta_{i,t} - \beta_{i,t|t-1}) r_{m,t} + \varepsilon_{i,t}, \end{aligned} \tag{A3.4}$$

$$V_t = Var(v_t) = Var[(\beta_{i,t} - \beta_{i,t|t-1}) r_{m,t} + \varepsilon_{i,t}] = \Sigma_{t|t-1} r_{m,t}^2 + \sigma_{\varepsilon_i}^2, \tag{A3.5}$$

When we observe a new data at time t, then we can update the expectation and variance of $\beta_{i,t}$ according to theorem A4.1 because $I_t = \{I_{t-1}, y_t) = \{I_{t-1}, v_t\}$. We have the following formulas,

$$\begin{aligned} \beta_{i,t|t} &= E(\beta_{i,t} \mid I_t) = E(\beta_{i,t} \mid I_{t-1}, v_t) \\ &= E(\beta_t \mid I_{t-1}) + Cov(\beta_{i,t}, v_t \mid I_{t-1}) Var(v_t)^{-1} v_t \\ &= \beta_{i,t|t-1} + C_t V_t^{-1} v_t, \end{aligned} \tag{A3.6}$$

where

$$\begin{aligned} C_t &= Cov(\beta_{i,t}, v_t) = Cov[\beta_{i,t}, (\beta_{i,t} - \beta_{i,t|t-1}) r_{m,t} + \varepsilon_{i,t} \mid I_{t-1}] \\ &= Cov[\beta_{i,t}, (\beta_{i,t} - \beta_{i,t|t-1}) r_{m,t} \mid I_{t-1}] \\ &= \Sigma_{t|t-1} r_{m,t}. \end{aligned} \tag{A3.7}$$

For the updated variance of $\beta_{i,t}$, we have

$$\begin{aligned} \Sigma_{t|t} &= Var(\beta_{i,t} \mid I_t) \\ &= Var(\beta_{i,t} \mid I_{t-1}) - Cov(\beta_{i,t}, v_t \mid I_{t-1}) V_t^{-1} Cov(\beta_{i,t}, v_t \mid I_{t-1})' \\ &= \Sigma_{t|t-1} - C_t V_t^{-1} C_t'. \end{aligned} \tag{A3.8}$$

Now we get the Kalman filter, which is a recursive updating of $\beta_{i,t}$ for initial values of

$\beta_{i,0|0}$ and $\Sigma_{i,0|0}$,

$$\begin{aligned}
&\beta_{i,t|t-1} = (1-\phi_{i0})\phi_{i1} + \phi_{i1}\beta_{t-1|t-1}, \\
&\Sigma_{t|t-1} = \phi_{i1}^2\Sigma_{t-1|t-1} + \sigma_{u_i}^2, \\
&\beta_{i,t|t} = \beta_{i,t|t-1} + \Sigma_{t|t-1}r_{m,t}(\Sigma_{t|t-1}r_{m,t}^2 + \sigma_{\varepsilon_i}^2)^{-1}(y_t - \beta_{i,t|t-1}r_{m,t}), \\
&\Sigma_{t|t} = \Sigma_{t|t-1} - (\Sigma_{t|t-1}r_{m,t})^2(\Sigma_{t|t-1}r_{m,t}^2 + \sigma_{\varepsilon_i}^2)^{-1}.
\end{aligned} \tag{A3.9}$$

The initial values $\beta_{i,0|0}$ and $\Sigma_{i,0|0}$ can be set equal to the unconditional mean and variance of $\beta_{i,t}$ respectively or according to previous studies, for example it is a common practice to set $\beta_{i,0|0}$ equal to 1.

The parameters $\{\phi_{i0,}\phi_{i1,}\sigma_{\varepsilon_i}, \sigma_{u_i}\}$ can be estimated by maximum likelihood method. The logarithm of the likelihood function is

$$\ln L = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\ln(V_t) + \frac{v_t^2}{V_t}\right), \tag{A3.10}$$

where $v_t = y_t - \beta_{i,t|t-1}r_{m,t}$, and $V_t = \Sigma_{t|t-1}r_{m,t}^2 + \sigma_{\varepsilon_i}^2$. The likelihood function can be evaluated recursively by Kalman filter in equation (3.9) until it converges. In the empirical work, the initial values of $\beta_{i,0|0}$ and $\Sigma_{i,0|0}$ are set equal to the unconditional mean and variance of $\beta_{i,t}$.

B. The MCMC Estimation

In this Appendix, I give prior and posterior distributions of parameters of the state-space model, which are used in the MCMC method. For details of the derivation of those distributions, please refer to Jostova and Philipov (2005).

The model is

$$r_{i,t} = \beta_{i,t}r_{m,t} + \varepsilon_{i,t}, \tag{3.8}$$

$$\beta_{i,t} = (1-\phi_{i1})\phi_{i0} + \phi_{i1}\beta_{i,t-1} + u_{i,t}, \tag{3.9}$$

for t=1,…T, where $\varepsilon_{i,t} \sim N(0,\sigma_{\varepsilon_i}^2)$ and $u_{i,t} \sim N(0,\sigma_{u_i}^2)$ are independent Gaussian white noise processes. Time-varying beta is treated as parameters. Let $\beta_i = (\beta_1,...,\beta_T)$ be the vector of time-varying beta then the parameter vector for each portfolio i is $\theta_i = (\phi_{i0},\phi_{i1},\sigma_{\varepsilon_i}^2,\sigma_{u_i}^2,\beta_i)'$.

The priors for parameters are assumed as follows,

$$p(\sigma_{\varepsilon_i}^2) = IG(a_1,b_1) \propto (\sigma_{\varepsilon_i}^2)^{-(a_1+1)} \exp(-\frac{b_1}{\sigma_{\varepsilon_i}^2}) \tag{A3.11}$$

$$p(\phi_{i0}) = N(\mu_0,\sigma_0^2) \propto \frac{1}{\sigma_0}\exp(-\frac{(\phi_{i0}-\mu_0)^2}{2\sigma_0^2}) \tag{A3.12}$$

$$p(\phi_{i1}) = truncated_{(-1,1)}N(\mu_1,\sigma_1^2) \propto \frac{1}{\sigma_1}\exp(-\frac{(\phi_{i1}-\mu_1)^2}{2\sigma_1^2}) \tag{A3.13}$$

$$p(\sigma_{u_i}^2) = IG(a_2,b_2) \propto (\sigma_{u_i}^2)^{-(a_2+1)} \exp(-\frac{b_2}{\sigma_{u_i}^2}) \tag{A3.14}$$

where $truncated_{(-1,1)}N()$ is the truncated normal distribution with lower bound equal to -1 and upper bound equal to 1.

In the empirical work, the following values are used,

$$a_i = 0.001, b_i = 0.001, i = 1,2$$

$$\mu_0 = 1.0, \sigma_0^2 = 100$$
$$\mu_1 = 0.5, \sigma_1^2 = 100 .$$

The conditional posteriors used in the Gibbs sampler are as follows. For each time t=1,…,T-1,

$$p(\beta_{i,t} \mid rest) \propto N(\frac{\sigma_{u_i}^2 r_{m,t} r_{i,t} + \sigma_{\varepsilon_i}^2(\phi_{i0}(1-\phi_{i1})^2 + \phi_{i1}(\beta_{i,t-1}+\beta_{i,t+1}))}{\sigma_{\varepsilon_i}^2(1+\phi_{i1})^2 + \sigma_{u_i}^2 r_{m,t}^2}, \frac{\sigma_{u_i}^2\sigma_{\varepsilon_i}^2}{\sigma_{\varepsilon_i}^2(1+\phi_{i1})^2 + \sigma_{u_i}^2 r_{m,t}^2}). \tag{A3.15}$$

where *rest* stands for all the other parameters in the parameter vector. For time t=1, the initial condition of $\beta_{i,t-1} = \beta_{i,0}$ is set to be 1.0.

For t=T, the conditional posterior is

$$p(\beta_{i,T} \mid rest) \propto N(\frac{\sigma_{u_i}^2 r_{m,T} r_{i,T} + \sigma_{\varepsilon_i}^2 (\phi_{i0} + \phi_{i1}(\beta_{i,T-1} - \phi_{i0}))}{\sigma_{\varepsilon_i}^2 + \sigma_{u_i}^2 r_{m,T}^2}, \frac{\sigma_{u_i}^2 \sigma_{\varepsilon_i}^2}{\sigma_{\varepsilon_i}^2 + \sigma_{u_i}^2 r_{m,t}^2}). \tag{A3.16}$$

The conditional posteriors for other parameters are:

$$p(\phi_{i0} \mid rest) \propto N(\frac{(1-\phi_{i1})\sigma_0^2 \sum_{t=1}^{T} (\beta_{i,t} - \beta_{i,t-1}) + \mu_{i,0}\sigma_{u_i}^2}{T(1-\phi_{i1})^2 \sigma_{\varepsilon_i}^2 + \sigma_{u_i}^2}, \frac{\sigma_{u_i}^2 \sigma_{\varepsilon_i}^2}{T(1-\phi_{i1})^2 \sigma_{\varepsilon_i}^2 + \sigma_{u_i}^2}) \tag{A3.17}$$

$$p(\phi_{i1} \mid rest) \propto truncated_{(-1,1)} N(\frac{\sigma_1^2 \sum_{t=1}^{T} (\beta_{i,t} - \phi_{i0})(\beta_{i,t-1} - \phi_{i0}) + \mu_{i,1}\sigma_{u_i}^2}{\sigma_1^2 \sum_{t=1}^{T} (\beta_{i,t-1} - \phi_{i0})^2 + \sigma_{u_i}^2}, \frac{\sigma_{u_i}^2 \sigma_{\varepsilon_i}^2}{\sigma_1^2 \sum_{t=1}^{T} (\beta_{i,t-1} - \phi_{i0})^2 + \sigma_{u_i}^2}), \tag{A3.18}$$

$$p(\sigma_{u_i}^2) \propto Inv \sim \chi^2 (T + 2a_2, \frac{\sum_{t=1}^{T} (\beta_{i,t} - \phi_{i0} - \phi_{i1}(\beta_{i,t-1} - \phi_{i0}))^2 + 2b_2}{T + 2a_2}), \tag{A3.19}$$

$$p(\sigma_{\varepsilon_i}^2) \propto Inv \sim \chi^2 (T + 2a_1, \frac{\sum_{t=1}^{T} (r_{i,t} - \beta_{i,t} r_{m,t})^2 + 2b_1}{T + 2a_1}). \tag{A3.20}$$

For the random walk beta model, ϕ_{i0} and ϕ_{i1} are restricted to be zero and one, respectively.

C. Results of the Rolling Window Estimation

The cross-sectional regression results of out-of-sample forecasted beta using rolling window method are reported in Table A3.1. The window length is 60 months for rolling window OLS, the macroeconomic variables model and the realized beta model, which is the standard length used in the literature; it is 444 months, the length of the initial sample from July 1926 to June 1963, used to generate the first forecast for other models. The reason is that a longer data series makes the algorithms converge better.

For the whole sample, similar to the results in Table 3.8, no models have a significantly positive coefficient on market beta. Market beta from the state-space model with AR(1) beta and random walk beta estimated by the MCMC method have a positive but insignificant coefficient and all other models have a negative coefficient. All intercepts are significantly different from zero except the MC_AR model. The R^2s are all around 20%.

The results of the two subsamples are very similar to those of the whole sample. None of the coefficients on market beta is significantly positive and most intercepts are significantly different from zero with R^2s all around 20%.

Table A3.1 Fama-MacBeth Cross-sectional Regression Results of out-of-sample Forecasted Beta from the Rolling Window Method

This table reports the time series means and standard errors of sample mean (in parenthesis) of the estimated parameters of the regressions:

$$r_{i,t} = \alpha_t + \gamma_t \hat{\beta}_{i,t} + \varepsilon_{i,t} \cdot$$

using the out-of-sample forecasted betas from the rolling window method. R^2 is the average of the time series R-square of each cross-sectional regression. The sample is the returns of Fama-French 25 portfolios and the out-of-sample forecasted betas from July 1963 through December 2007 except that the realized betas are available from July 1968. OLS is the rolling window regression. Macro is the macroeconomic variable model. KF_AR and KF_RW are state-space model with AR(1) and random walk state equations, respectively, estimated by Kalman filter while MC_AR and MC_RW are the same models but are estimated by the MCMC method. DCC is the DCC-GARCH (1,1) model. Realized is the realized beta model. The asterisk * denotes significance level of 5%.

	OLS	Macro	KF_AR	KF_RW	MC_AR	MC_RW	DCC	Realized
Panel A: Full sample from July 1963 to December 2007								
α	0.82*	0.78*	0.70*	1.08*	0.32	0.47*	0.98*	1.28*
	(0.28)	(0.21)	(0.31)	(0.25)	(0.23)	(0.22)	(0.29)	(0.27)
γ	-0.13	-0.08	-0.03	-0.36	0.32	0.17	-0.22	-0.74*
	(0.33)	(0.17)	(0.35)	(0.24)	(0.26)	(0.23)	(0.33)	(0.25)
R^2	0.24	0.19	0.23	0.13	0.20	0.21	0.23	0.19
Panel B: Subsample from July 1963 to December 1985								
	OLS	**Macro**	**KF_AR**	**KF_RW**	**MC_AR**	**MC_RW**	**DCC**	**Realized**
α	0.29	0.70*	-0.03	1.16*	0.04	0.38	0.70	1.68*
	(0.39)	(0.29)	(0.38)	(0.32)	(0.33)	(0.31)	(0.39)	(0.43)
γ	0.33	-0.12	0.55	-0.49	0.50	0.16	-0.09	-1.37*
	(0.46)	(0.22)	(0.45)	(0.31)	(0.41)	(0.31)	(0.45)	(0.41)
R^2	0.28	0.21	0.25	0.15	0.22	0.24	0.24	0.19
Panel C: Subsample from January 1986 to December 2007								
	OLS	**Macro**	**KF_AR**	**KF_RW**	**MC_AR**	**MC_RW**	**DCC**	**Realized**
α	1.36*	0.85*	1.44*	1.00*	0.61	0.56	1.27*	0.96*
	(0.42)	(0.31)	(0.48)	(0.37)	(0.32)	(0.33)	(0.44)	(0.34)
γ	-0.61	-0.04	-0.63	-0.23	0.14	0.18	-0.36	-0.25
	(0.49)	(0.27)	(0.54)	(0.37)	(0.34)	(0.34)	(0.48)	(0.31)
R^2	0.21	0.17	0.22	0.11	0.18	0.17	0.21	0.18

Chapter 4

Realized Betas and the Cross-Section of Stock Returns

4.1. Introduction

In Chapter 3, I focused on whether the conditional CAPM can explain the cross-section of stock returns by using the Fama-French 25 size/book-to-market ratio (BM) portfolios. As explained in previous chapters, while the conditional CAPM is an effort to overcome the shortcomings of the unconditional CAPM, another approach is the multi-factor explanation: the CAPM with only one factor, the market return, omits some other important factors. Different multi-factor models on the line of Merton's (1973) intertemporal CAPM (ICAPM) or Ross's (1976) arbitrage pricing theory (APT) have been proposed according to the empirical findings of abnormal returns associated with firm-level characteristics such as size, BM and past returns. In a series of papers, Fama and French (1992, 1993 and 1996) propose a three-factor model with a market factor, a factor corresponding to size (SMB) and a factor corresponding to BM (HML). Fama and French (1996) show that their three-factor model can explain the cross-section of stock returns associated with size, BM, price-to-earnings ratio and leverage, but not momentum. Carhart (1997) adds a momentum factor into Fama-French's three-factor model to account for the momentum effect of Jegadeesh and Titman (1993).[10]

Empirical results of tests of these multi-factor models are mixed. Fama and French (1996) test the unconditional version of their model by running time series regressions and find insignificant Jensen's alphas of different portfolio sorting methods except the momentum portfolios. However, subsequent tests of the conditional version of the Fama-French three-factor model by He et al. (1996) and Ferson and Harvey (1999) reject this model. Wang (2003) uses a non-parametric method and find some support for Fama-French's model. In the cross-sectional test, Brennan et al. (1998) and Avramov and Chordia (2006) reject Fama-French's three-factor model and a four-factor model which is Fama-French's three-factor model augmented by a momentum factor by using

[10] Other factors are also proposed such as the liquidity factor of Pastor and Stambaugh (2003) and the volatility factors of Ang et al. (2006) and Adrian and Rosenberg (2008).

abnormal returns of individual stocks.

As in the cross-sectional test of the conditional CAPM, the modelling of multi-factor betas plays a key role in the cross-sectional test of multi-factor models. In a multi-factor model, the returns of individual assets are determined by their betas or factor loadings of different factors.[11] Different methods of modelling betas can generate different results. For example, Ferson and Harvey (1999) reject Fama-French's model by using a linear function of macroeconomic variables to model betas but Wang (2003) finds some support for the conditional Fama-French model by using a nonparametric method. Relying on macroeconomic variables has two drawbacks: the first is the choice of variables because researchers cannot observe the information set of investors; the second is the functional form as different specifications will give different results. The argument of Ghysels (1998) about the conditional CAPM also applies to conditional multi-factor models: if we cannot model conditional multi-factor betas correctly, then conditional models are more likely to be rejected.

Recently, some researchers have used daily data to test the conditional CAPM and multi-factor models due to its availability (e.g. Lewellen and Nagel, 2006; Bali et al., 2009; Morona, 2009). The use of daily returns to construct monthly or quarterly betas can overcome the drawbacks of relying on the information variables because it is a nonparametric method and does not need to specify any external variables. Multi-factor betas are computed from the variance/covariance matrix of individual assets' returns and factor returns and it is known from Merton (1980) that the variance/covariance matrix can be estimated accurately when the data frequency goes to infinitely high. Recently, built on the realized volatility literature, realized beta has attracted much interest (e.g. Bollerslev and Zhang, 2003; Anderson et al., 2005, 2006; Hooper et al., 2008; Morona, 2009). Bollerslev and Zhang (2003) show that realized market beta generates more accurate forecast of returns than the traditional rolling-window approach using monthly returns. Andersen et al. (2005, 2006) and Hooper et al. (2008) study the time series properties of realized market beta. They find that realized market beta has short memory properties in contrast with the long memory properties of the realized variance and covariance. Morona (2009) tests different pricing models in cross-section regressions and finds support for the model of Jagnnathan and Wang (1996) by using

[11] In the literature, the name of the coefficients of the factors is called betas, factor loadings or factor sensitivities. In this and the following chapters, I use betas in order to make it consistent with Chapter 3.

realized multi-factor betas.

Another issue in asset pricing tests is the choice of testing assets. In Chapter 3, I use the Fama-French 25 portfolios sorted by size and BM as many previous studies.[12] The advantage of using portfolios is that the noises of individual stocks can be averaged out in a portfolio and therefore estimated beta of portfolios is more precise than individual stocks (e.g. Blume, 1970; Friend and Blume, 1970; Black et al., 1972). Since beta is estimated more precisely, the errors-in-variables problem in the cross-section regressions of returns on beta is diminished. However, using portfolios also causes other problems. The first is the well-known data-snooping bias in portfolio-based asset pricing tests (Lo and MacKinlay, 1990). If we sort stocks into portfolios according to firm-level characteristics such as size or BM, tests will be biased to reject the null hypothesis more frequently than the usual significance level. Another problem is loss of information, pointed out by Litzenberger and Ramaswamy (1979). Using individual stocks in asset pricing tests can overcome these shortcomings as long as beta of individual stocks can be estimated precisely.[13]

Based on the considerations above, in this chapter I examine if realized betas of different factor pricing models, computed from daily returns, can explain the cross-section of stock returns using individual stocks both in-sample and out-of-sample. On the one hand, using daily data to estimate monthly and quarterly betas can give more precise estimates; on the other hand, using individual stocks in tests can avoid data-snooping and loss of information problems caused by using portfolios. Factor models considered are the CAPM, the Fama-French (1993, 1996) three-factor model and a four-factor model including Fama-French's three factors and a momentum factor. Data used in cross-section tests is monthly and quarterly returns of all common stocks listed in NYSE, AMEX and NASDAQ from July 1963 to December 2007. Monthly and quarterly realized betas are computed from daily returns of individual stocks and different factors.

The results show that contemporaneous market beta of the CAPM does have a

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[12] Some examples include He et al. (1996), Ferson and Harvey (1999), Lettau and Ludvigson (2001), Wang (2003), Bali (2008) and Adrian and Franzoni (2009).

[13] Examples of using individual stocks in asset pricing include Fama and French (1992), Brennan et al. (1998) and Avramov and Chordia (2006).

significantly positive coefficient in cross-section regressions, which contrasts with Fama and French's (1992) results where a negative coefficient of beta is found and is similar to the results of Ang et al. (2006). In-sample forecasted market beta also has a significantly positive coefficient which is consistent with findings of Bali et al. (2008) who group stocks into portfolios according to their in-sample forecasted market beta and find high-beta portfolios have higher returns than low-beta portfolios. Out-of-sample forecasted market beta, however, has a negative coefficient, which suggests that market beta is difficult to forecast. The lagged size, BM and past returns are all significant in all cases suggesting market beta alone cannot explain the cross-section of stock returns.

Moving to the Fama-French three-factor model, contemporaneous market beta is still significant and additional betas of SMB and HML reduce although do not eliminate the effects of size and BM but the coefficient on HML beta is insignificant. For in-sample forecasted betas, only market beta is significant and the other two betas are not. None of out-of-sample forecasted betas has a significantly positive coefficient. The results are consistent with Daniel and Titman (1997) who show that it is firm-level size and BM instead of betas of SMB and HML factors that have an impact on the cross-section of stock returns. Therefore, betas on SMB and HML, especially forecasted betas, cannot drive out the significance of lagged size and BM in the cross-section of stock returns.

The results of the four-factor model are similar to the Fama-French model. Contemporaneous momentum beta reduces but does not eliminate the effect of past returns. For in-sample forecasted betas, different from betas of SMB and HML, momentum beta is still significant but it no longer reduces the effect of past returns. For out-of-sample forecasted betas, no betas are significantly priced, similar to the results of the CAPM and the Fama-French model.

The remainder of this chapter is organized as follows. In section 4.2, I describe factor pricing models and the construction of realized multi-factor betas. Section 4.3 defines the different factors used in the empirical work. Section 4.4 describes the data. Empirical results are reported in section 4.5. Section 4.6 gives conclusions.

4.2. Factor pricing models and realized betas

4.2.1 Factor pricing models and measurement of realized betas

In a conditional *K*-factor pricing model, returns are generated by the *K* factors,

$$R_{i,t} = E_{t-1}(R_{i,t}) + \sum_{k=1}^{K} \beta_{i,k,t} \tilde{f}_{k,t} + \varepsilon_{i,t} \tag{4.1}$$

where $R_{i,t}$ is the return on asset *i* at time *t*, E_{t-1} is the conditional expectation based on information available only at time *t*-1, $\tilde{f}_{k,t}$ is the innovation of the return on factor *k* with respect to the information available at time *t*-1, $\beta_{i,k,t}$ is conditional beta of factor *k* of asset *i* at time *t* based on information available at time *t*-1. Assuming there is a risk-free asset, $E_{t-1}(R_{i,t})$ is modelled by

$$E_{t-1}(R_{i,t}) = R_{f,t} + \sum_{k=1}^{K} \lambda_{k,t} \beta_{i,k,t} \tag{4.2}$$

where $R_{f,t}$ is the risk-free rate and $\lambda_{k,t}$ is the risk premium of factor *k* at time *t*. If a factor is a portfolio's return, which is the case in this chapter, we have $\lambda_{k,t} = E_{t-1}(f_{k,t})$, where $f_{k,t}$ is the return of factor *k* at time *t*. Then the return generating process of equation (4.1) can be written as

$$R_{i,t} = R_{f,t} + \sum_{k=1}^{K} \beta_{i,k,t} f_{k,t} + \varepsilon_{i,t} \,. \tag{4.3}$$

The empirical work usually deals with the excess return, so the risk-free rate is subtracted from each side,

$$r_{i,t} = \sum_{k=1}^{K} \beta_{i,k,t} f_{k,t} + \varepsilon_{i,t} \tag{4.4}$$

Where $r_{i,t} = R_{i,t} - R_{f,t}$ is the excess return of asset *i* at time *t*. Let $\beta_{i,t} = (\beta_{i,1,t}, ..., \beta_{i,K,t})'$

be the vector of conditional multi-factor betas, then conditional betas are given by

$$\beta_{i,t} = Cov_{t-1}(f_t, f_t)^{-1} \times Cov_{t-1}(f_t, r_{i,t}) \tag{4.5}$$

Similar to the issue of modelling market beta in the conditional CAPM, it is also critical to model multi-factor betas in tests of multi-factor models. It will be a natural extension of techniques used in conditional market beta modelling to a multivariate case. However, the curse of dimension makes some techniques difficult to implement in high-dimensional problems such as the GARCH model and the MCMC method. Therefore, the most popular way of modelling multi-factor betas is the rolling window regression method and the macroeconomic variables model (see chapter 3 for examples) and the other methods in market beta modelling in Chapter 3 are rarely used.

Notice that the beta vector in equation (4.5) is computed from the variance matrix of the factors and the covariance matrix of the factors and individual stocks; the idea behind realized market beta in Chapter 3 is also applicable in multi-factor betas. From Merton (1980), it is known that the conditional variance/covariance can be estimated arbitrarily well through the use of intra-period data. For example, the monthly variance of the factors can be estimated well by using daily data or even intra-day data. The empirical implementation of this method focuses on the estimation of the conditional market volatility. For example, French et al. (1987) and Ghysels et al. (2005) use daily market return to estimate conditional monthly market volatility. Nelson and Fostor (1996) give the theoretical background of this estimator.

More recently, the literature of using intra-period data has moved to realized volatility, which is built on the quadratic variation theory (Andersen et al., 2001a, 2001b, 2003; Barndoff-Nielsen and Shephard, 2004). Realized beta framework described in Chapter 3 is directly applicable here. For convenience of reference, I replicate the analysis of realized beta in Chapter 3 here but in a multivariate expression. Following Andersen et al. (2005, 2006) and Barndoff-Nielsen and Shephard (2004), suppose the $N \times 1$ vector logarithm price process, p_t, follows a multivariate continuous-time stochastic volatility diffusion,

$$dp_t = \mu_t d_t + \Sigma_t dW_t \tag{4.6}$$

where W_t is a standard N-dimensional Brownian motion, Σ_t is a stationary diffusion process and independent of the W_t process. Then, the conditional distribution of the continuously compounded h-period return, $r_{t,t+h} = p_{t+h} - p_t$, based on the sample path realization of μ_t and Σ_t, is

$$r_{t+h,h} \mid \sigma\{\mu_{t+\tau}, \Sigma_{t+\tau}\}_{\tau=0}^{h} \sim N\left(\int_0^h \mu_{t+\tau} d\tau, \int_0^h \Sigma_{t+\tau} d\tau\right), \tag{4.7}$$

where $\sigma\{\mu_{t+\tau}, \Sigma_{t+\tau}\}_{\tau=0}^{h}$ is the σ-field generated by the sample path of $\mu_{t+\tau}$ and $\Sigma_{t+\tau}$ for $0 \le \tau \le h$. Therefore, the integrated diffusion matrix $\int_0^h \Sigma_{t+\tau} d\tau$ provides a natural measure of the true underlying h-period volatility.

By the theory of quadratic variation, we can estimate the integrated volatility using the intra-period data,

$$\sum_{j=1}^{[h/\Delta]} r_{t+j\cdot\Delta} r'_{t+j\cdot\Delta} \rightarrow \int_0^h \Sigma_{t+\tau} d\tau \tag{4.8}$$

as the sampling frequency of returns increases, or $\Delta \rightarrow 0$. This estimate is called realized volatility in the literature. Then we have

$$[f,f]_{t+h} = \sum_{j=1}^{[h/\Delta]} f_{t+j\cdot\Delta} f'_{t+j\cdot\Delta} \rightarrow \int_0^h Cov(f_{t+\tau}, f_{t+\tau}) d\tau \tag{4.9}$$

and

$$[f,r_i]_{t+h} = \sum_{j=1}^{[h/\Delta]} f_{t+j\cdot\Delta} r'_{i,t+j\cdot\Delta} \rightarrow \int_0^h Cov(f_{t+\tau}, r_{i,t+\tau}) d\tau\,. \tag{4.10}$$

Realized beta vector of asset i is constructed by

$$\widehat{\beta}_{i,t+h} = ([f,f]_{t+h})^{-1} [f,r_i]_{t+h}\,. \tag{4.11}$$

Then $\widehat{\beta}$ converge to underlying integrated beta vector and also conditional beta vector of period t+h by Merton (1980).

Notice that realized multi-factor betas are equal to the coefficients of an OLS regression without intercept using data within each period, i.e.

$$r_{i,t+j\bullet\Delta} = \sum_{k=1}^{K} \beta_{i,k,t+1} f_{k,t+j\bullet\Delta} + \varepsilon_{i,t+j\bullet\Delta}, \quad j = 1,\ldots\frac{h}{\Delta} \tag{4.12}$$

where ε is a Gaussian white noise. For example, if daily returns are used to compute monthly betas, then h=1 and Δ is roughly equal to 1/22. Therefore, monthly realized betas are simply the coefficients of OLS regression of daily returns on daily factor returns within each month. The difference is the asymptotic theories behind the quadratic variation theory and ordinary regressions.

The advantages of using realized betas are obvious. First, it does not assume any information variables and the functional form of betas with these information variables. Second, it employs high-frequency data which may include richer information than low frequency data, which are the monthly returns typically used in the literature. Third, it is easy to compute even in the multi-factor model. Other techniques are too complex to be implemented in a multivariate case such as Ang and Chen's (2007) MCMC method and the multivariate GARCH model used by Braun et al. (1995).

Similar to realized market beta in Chapter 3, the choice of frequency, Δ, plays an important role in the estimation of realized multi-factor betas. On the one hand, we want the frequency as high as possible; on the other hand, data availability and market microstructure limits the frequency of data. Based on this, I use daily data to construct monthly realized betas as in Chapter 3 and quarterly betas in addition. It is well-known that non-synchronous trading can make betas estimated from high-frequency data biased. Scholes and Williams (1977) propose a measurement robust to the non-synchronous trading properties of the stock market by adding leads and lags of market returns. This measurement has similar results as using only contemporaneous market returns. Therefore, the main analysis is based on realized betas computed only by contemporaneous market returns, and the results of Scholes and William's betas are reported in the Appendix.[14]

Recently, many researchers have used daily data to construct monthly betas. Ang et al. (2006) use daily returns from an overlapping 12-month window to construct their downside beta and find downside beta is a significant risk. Bali et al. (2009) use daily data within each month to construct realized market beta and sort stocks according to their in-sample forecasted beta. They find that high-beta portfolios have higher returns than low-beta portfolios even after controlling size, BM and other variables. Morana (2009) uses daily returns of Fama-French 25 size/BM portfolios to construct monthly realized betas of different factor pricing models and finds support for Jagannathan and Wang's (1996) model.

However, no researchers have examined the cross-sectional relationship of realized betas and returns by using individual stocks' returns. Furthermore, whether out-of-sample forecasted realized betas can explain the cross-section of stocks returns has never been examined. The main contribution of this chapter is to examine the abilities of realized betas associated with different factors in explaining the cross-section of stock returns both in-sample and out-of-sample.

4.2.2 Modelling realized betas

Realized betas are an ex-post measure of underlying integrated betas. However, conditional asset pricing models are ex-ante models. Therefore, we need to use forecasted realized betas in tests of conditional asset pricing models.

The forecast of realized betas is based on information only available at the forecast time,

$$\beta_{i,k,t} = f(X_{t-1}), \tag{4.13}$$

where $\beta_{i,k,t}$ is asset i's beta of factor k at time t, $f(.)$ is any function used to generate forecast and X_{t-1} is information variables available in time t-1 including lags of realized betas.

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[14] Andersen et al. (2006) and Morana (2009) also only use contemporaneous market and factor returns.

In this chapter, I model realized betas as a simple AR(1) process as Bollerslev and Zhang (2003) and Bali et al. (2008) and add lagged macroeconomic variables in the AR(1) model because many previous studies have documented that betas are related to macroeconomic variables[15],

$$\beta_{i,k,t} = \alpha_0 + \alpha_1 \beta_{i,k,t-1} + \sum_{j=1}^{J} \alpha_{X,j} X_{j,t-1} + \varepsilon_{i,k,t}. \qquad (4.14)$$

Robustness checks are given by including more lags of realized betas such as AR(2) and AR(3) models, a moving average item such as ARMA(1,1) model. All these models have very similar results, which are reported in the Appendix.[16]

4.2.3 Cross-sectional test and the Fama-MacBeth method

If a factor asset pricing model can explain assets' returns, then betas of the factors are the only determinant of the cross-section of returns, i.e.

$$r_{i,t} = \sum_{k=1}^{K} \lambda_{k,t} \beta_{i,k,t} \ . \qquad (4.15)$$

The implication of this equation is that the intercept of a cross-sectional regression of returns on betas should be insignificantly different from zero and any other variables do not have explanatory abilities. Therefore, empirical tests of linear factor models focus on the following regression,

$$r_{i,t} = \alpha_t + \sum_{k=1}^{K} \lambda_{k,t} \beta_{i,k,t} + \sum_{j=1}^{J} c_{j,t} Z_{i,j,t-1} + \varepsilon_t \,, \qquad (4.16)$$

where $Z_{j,t-1}$ is the *j*th firm-level variable of time t-1 such as size, BM and past returns.

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[15] An incomplete list includes: Shanken (1990), Ferson and Harvey(1993), Ferson and Korjczyk (1995), Ferson and Schadt (1996) and Ferson and Harvey (1999).

[16] I do not use the Kalman filter in Chapter 3 for two reasons: first it performs poorly for individual stocks, many stocks cannot converge; second, it is too slow to make out-of-sample forecasts for each of the stock, for example, it takes three days for a dual core laptop to run an in-sample Kalman filter for each stock, therefore, making it infeasible for out-of-sample forecasts.

If an asset pricing model holds, α_t and $c_{j,t}$ should be zero and $\lambda_{k,t}$ should be significantly different from zero. The usual focus is on the $c_{j,t}$ because Berk (1995) and Jaganathan and Wang (1998) show that $c_{j,t}$ will be significantly different from zero if an asset pricing model is misspecified. For forecasted betas,

$$r_{i,t} = \alpha_t + \sum_{k=1}^{K} \lambda_{k,t} \beta_{i,k,t|t-1} + \sum_{j=1}^{J} c_{j,t} Z_{i,j,t-1} + \varepsilon_t \tag{4.17}$$

where $\beta_{i,k,t|t-1}$ is forecasted realized betas based on forecasting models.

The popular method of cross-section tests of asset pricing model is the Fama and MacBeth (1973) method which is based on the test of time series average of the estimated parameters of each time *t*. Let $\theta_t = (\alpha_t, \lambda_{1,t}, \ldots, \lambda_{K,t}, c_{1,t}, \ldots, c_{J,t})'$ be the parameter vector, then the standard Fama and MacBeth (1973) method is

$$\bar{\hat{\theta}}_i = \frac{1}{T} \sum_{t=1}^{T} \hat{\theta}_{i,t} \tag{4.18}$$

where $\hat{\theta}_{i,t}$ is OLS estimate of $\theta_{i,t}$, the *i*th element of θ_t, of each time *t*. The usual *t*-test of sample mean can be applied to $\bar{\hat{\theta}}_i$.

4.3. Asset pricing models and factors

The models examined in this chapter include: (i) the CAPM, (ii) the Fama and French (1993) three-factor model, and (iii) a four-factor model including Fama-French's three factors and a momentum factor.

4.3.1 The CAPM

The CAPM of Sharpe (1964) and Lintner (1965) is the simplest asset pricing model with market excess return as the only factor. However, the market is unobservable and therefore only proxy of market can be used. The common practice in empirical finance is to use an index broadly including many assets. In this chapter, I use the value-weighted CRSP index of all stocks listed in NYSE, AMEX and NASDAQ, which is the most popular market proxy of empirical studies. Realized market beta of the CAPM is computed in the same way as Chapter 3.

4.3.2 The Fama-French three-factor model

Fama and French (1993) propose a three-factor model to explain returns of portfolios sorted by size, BM and other variables. They show that this model can explain those portfolios' returns very well except the momentum portfolios. This model has now become the benchmark model in many applications such as asset pricing, abnormal return analysis, fund performance measurement and capital budgeting.

The three factors used are a market factor, an SMB factor corresponding to size and an HML factor corresponding to BM. The market factor is the usual choice of the market proxy: a broad index including many assets. In practice, the CRSP value-weighted index of all stocks listed in NYSE, AMEX and NASDAQ is used.

The SMB and HML factors are constructed as follows. First, at the end of June of each year *t*, all stocks listed in NYSE, AMEX and NASDAQ with a valid measure of size and BM are assigned into two size portfolios and three BM portfolios. The size measure is the market equity of June of year *t* while the BM measure is the book equity of fiscal year *t*-1 divided by market equity of December of year *t*-1. The breakpoint for size portfolios are the median of the NYSE stocks market equity. Stocks with market equity smaller than the median are assigned to the small portfolio and other stocks are assigned to the big portfolio. Meanwhile, the breakpoints for BM portfolios are the 30^{th} and the 70^{th} percentiles of the NYSE stocks' BM. The growth portfolio includes all stocks with BM lower than the 30^{th} percentile, the neutral portfolio includes all stocks with BM between the 30^{th} and 70^{th} percentiles, and all other stocks are included in the value portfolio. Second, six portfolios are formed according to the intersections of the size and BM portfolios, i.e. small growth, small neutral, small value, big growth, big neutral and big value portfolios. Then the value-weighted returns of these six portfolios are computed for July of year *t* to June of year *t*+1. Third, the average of returns across the small, big, growth and value portfolios are computed,

$$small = \frac{1}{3}(small\,growth + small\,neutral + small\,value)$$

$$big = \frac{1}{3}(big\,growth + big\,neutral + big\,value)$$

$$value = \frac{1}{2}(samll\,value + big\,value)$$

$$growth = \frac{1}{2}(samll\,growth + big\,growth)\,.$$

Finally, the SMB factor is defined as small minus big and the HML factor is defined as value minus growth,

$$SMB = small - big$$

$$HML = value - growth\,.$$

These two factors, combined with the market excess return, are the Fama-French three-factor model.

Although the three-factor model has achieved great success in empirical studies and become a benchmark model in many applications, its theoretical judgements are still unclear. The SMB and HML factors are purely from empirical studies without any theoretical background. Fama and French (1996) argue that their model is a three-factor equilibrium model of Merton's (1973) intertemporal CAPM (ICAPM) or Ross's (1976) arbitrage pricing theory (APT). The two factors SMB and HML mimic combinations of two underlying risk factors or state variables related to future investment opportunity sets. They suggest that the SMB and HML factors are related to a distress factor. Vasslou (2003) relates SMB and HML to news of future GDP growth and Petkova (2006) relates the two factors to innovations in predictive variables. Understanding the economic meaning of the two factors remains an interesting problem in finance. If the SMB and HML factors are truly equilibrium pricing factors, then betas on the two factors should have explanatory abilities of cross-section of stock returns. This chapter will examine whether this is true.

4.3.3 Fama-French model augmented by a momentum factor: a four-factor model

Jegadeesh and Titman (1993) show that past returns can predict future returns. Portfolios of past winners have higher returns than portfolios of past losers. This effect is called momentum in the literature. Fama and French (1996) show that their three-factor model is able to explain most abnormal returns associated with the CAPM but unable to capture the momentum effect. Motivated by this, Carhart (1997) proposes a four-factor model, which includes the Fama-French three factors and a momentum factor. His momentum factor is the return of an equal-weighted portfolio of past winners minus the return of an equal-weighted portfolio of past losers. In this chapter, the momentum factor used is the return of WML, which is similar to Carhart's momentum factor and will be described below.

First, similar to the construction of SMB and HML factors, in the end of June of each year, all stocks listed in NYSE, AMEX and NASDAQ with a valid size measure of June are assigned into two size portfolios: small and big. The breakpoint is the median of the NYSE stocks market equity. For each month *t*, all stocks with a valid price of month *t*-13 and a good return of month *t*-2 are assigned into three momentum portfolios, low, medium and high, according to prior returns from month *t*-12 to month *t*-2 (2-12). The breakpoints are the 30th and 70th percentile of NYSE stocks' prior returns (2-12). Second, in each month *t*, six portfolios are formed according to the intersection of the size and momentum portfolios, i.e. small low, small medium, small high, big low, big medium and big high. Value-weighted returns of each portfolio are computed for each month *t*. Third, the average across the low and high portfolios' returns are computed,

$$high = \frac{1}{2}(small\,high + big\,high),$$

$$low = \frac{1}{2}(small\,low + big\,low)\,.$$

Finally, the WML factor is defined as the difference of high and low,

$$WML = high - low\,.$$

Similar to the SMB and HML factors, WML is also purely from empirical findings without theoretical background. Because the purpose of proposing the WML factor is to explain the momentum effect, beta on WML should add explanatory abilities of the model if it is an equilibrium asset pricing model. This chapter will test this question as well.

4.3.4 Summary

The most general form of the models considered in this chapter is the four-factor model which can be written as

$$r_{i,t} = \alpha_i + \beta_{i,t} Mkt_t + s_{i,t} SMB_t + h_{i,t} HML_t + m_{i,t} WML_t + \varepsilon_{i,t}, \qquad (4.19)$$

Where $r_{i,t}$ is the excess return of asset *i* at time *t*, α_i should be zero if the model holds. The CAPM has only one factor *Mkt*, and the Fama-French three-factor model has three factors: *Mkt, SMB* and *HML*.

Then the cross-sectional tests are based on returns and realized betas,

$$r_{i,t} = \alpha_t + \lambda_{1,t}\hat{\beta}_{i,t} + \lambda_{2,t}\hat{s}_{i,t} + \lambda_{3,t}\hat{h}_{i,t} + \lambda_{4,t}\hat{m}_{i,t} + \sum_{j=1}^{J} c_{j,t} Z_{i,j,t-1} + u_t. \qquad (4.20)$$

The CAPM only has one risk: market beta. The Fama-French model has three betas: $\beta, s,$ and *h*. The four-factor model has all four betas. The Fama-MacBeth method is based on the time series average of the estimated parameters. If a model holds, the averages of α_t and $c_{j,t}$ should be insignificantly different from zero and those of λ s should be significantly different from zero.

4.4. Data

The data used are monthly and daily returns of all common stocks listed in NYSE, AMEX and NASDAQ from the Centre for Research in Security Prices (CRSP). The sample is from July 1963 through December 2007. NASDAQ stocks enter the sample only from January 1972. Following common practice in the literature, financial companies are excluded. Only stocks with at least 24 available realized betas are included for the purpose of modelling realized betas. This screening process yields a total of 12,622 stocks, of which 3,580 are listed in NYSE and AMEX and 9,042 are listed in NASDAQ.

The cross-sectional regression is mainly based on monthly returns and monthly realized betas which are computed from daily returns. Quarterly regressions are used as a robustness check. The fundamental data include size, BM and past returns, which are defined as follows:

SIZE: The natural logarithm of market value of previous month. Robustness checks are done by using size of the second to the last month, as Brennan et al. (1998) and Avramov and Chordia (2006), and give similar results;

BM: For July of each year *t* to June of year *t*+1, BM is logarithm of ratio of the book equity of fiscal year *t*-1 to market equity of December of year *t*-1. The book equity is defined as the sum of the book value of equity and deferred tax from Compustat;

RET2_12: For each month *t*, RET2_12 is the natural logarithm of cumulative returns from month *t*-12 to month *t*-2. Robustness checks are done by using three different measures of past returns: RET2_3, RET4_6 and RET7-12 which are defined as logarithm of cumulative returns from month *t*-3 to *t*-2, *t*-4 to *t*-6 and *t*-7 to *t*-12, respectively. All the three measures have similar results.

All the above three variables are subtracted by their cross-sectional means of each month when included in the cross-sectional regression.

To be included in monthly cross-sectional regressions, each stock must have a valid measure of size, BM and RET2_12. Companies with negative book value are excluded, and in each month, BM values less than 0.005 fractile or greater than 0.995 fractile are set equal to 0.005 and 0.995 fractile values, respectively. To overcome the survivorship problem, the first two years of Compustat data of each company is dropped as in Fama and French (1992) and Kothari et al. (1995). For realized betas, only months with at least 12 daily returns are used. Robustness checks are given with months with at least 20 daily returns and the results are similar. When forecasting realized betas in-sample, I only use stocks with at least 24 realized betas. For the out-of-sample forecast of realized betas, a 60-month rolling window is used, and only stocks with at least 24 realized betas in the past 60 months are used. Then only stocks with valid realized betas are used in the cross-sectional regression. The data screening process is outlined in Figure 4.1.

There are 522 months of cross-section regressions for the whole sample and stocks from NYSE and AMEX, and 408 months of cross-section regressions for stocks from NASDAQ. The average of number of stocks in each month is 2,723 for the whole sample, 1,333 for the sample of NYSE and AMEX stocks, and 1,779 for the sample of NASDAQ stocks.

Table 4.1 presents the summary statistics of stock excess returns and firm-level variables. Panel A reports the results for all stocks while panel B and panel C report results for NYSE and AMEX stocks only and NASDAQ stocks only, respectively. I report the time series averages of the cross-sectional means, medians and standard deviations of different variables. The mean and median of the excess return of all stocks are 0.90% and -0.30%, respectively. NYSE and AMEX stocks have lower mean return (0.81%) but higher median (0.02%) than NASDAQ stocks (mean 1.17% and median -0.65%). The mean of SIZE variable is 4.31 for all stocks. NYSE and AMEX stocks have greater size (5.14) than NASDAQ stocks (3.58). The mean BM is -0.47 for the whole sample and -0.38 and -0.56 for the two subsamples, respectively. The variable RET2_12 has a mean of 0.16 for the whole sample. NYSE and AMEX stocks have a slightly lower RET2_12 (0.15) than NASDAQ stocks (0.18).

The macroeconomic variables used in forecasting realized betas include dividend yield of S&P500 which is defined as the sum of the previous 12 months' dividends divided by the price of the current month, the one-month Treasury bill rate, the difference between three-month treasury rate and one-month treasury rate, default spread defined as Moody's Baa rated corporate bonds' yield minus Aaa rated corporate bonds' yield, and term spread defined as one-year treasury yield minus one-month Treasury bill rate. Those variables are chosen according to previous studies of conditional beta (e.g. He et al., 1996; Ferson and Harvey, 1999).

4.5. Empirical results

In this section, I report the cross-sectional regression results using realized betas. First, contemporaneously measured realized betas are used. In each month *t*, returns are regressed on realized betas of month *t* and firm-level variables. Then, the results of in-sample forecasted betas are reported. The third subsection reports the results of out-of-sample forecasted betas.

Figure 4.1 Data Screening Process

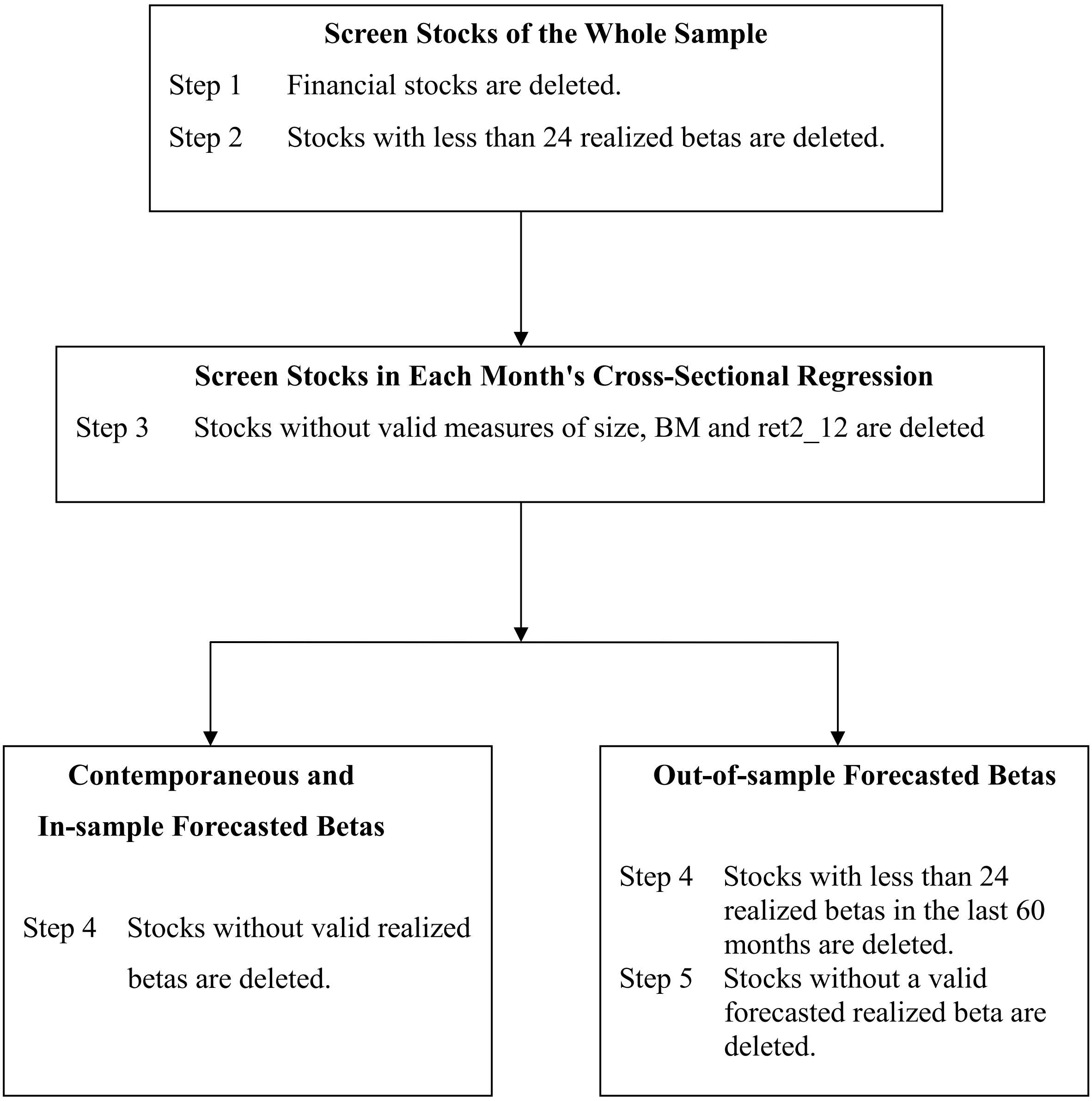

Table 4.1 Summary Statistics

This table presents the time series averages of the cross-sectional means, medians and standard deviations of different variables. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. RET is the excess return of stocks. SIZE is logarithm of market equity (price times shares outstanding). BM is the logarithm of book-to-market ratio except that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively.. RET2_12 is the cumulative returns of months *t*-12 to *t*-2 for each month *t*.

	Mean	Median	Std.
Panel A: NYSE, AMEX and NASDAQ Stocks			
RET(%)	0.90	-0.30	14.75
SIZE	4.37	4.21	1.95
BM	-0.47	-0.38	0.83
RET2_12	0.16	0.06	0.59
Panel B: NYSE and AMEX Stocks Only			
RET(%)	0.81	0.02	11.66
SIZE	5.14	5.12	1.96
BM	-0.38	-0.31	0.74
RET2_12	0.15	0.09	0.46
Panel C: NASDAQ stocks only			
RET(%)	1.17	-0.65	18.43
SIZE	3.58	3.54	1.57
BM	-0.56	-0.46	0.93
RET2_12	0.18	0.04	0.73

4.5.1 Contemporaneously measured realized betas

Table 4.2 summarizes the Fama-MacBeth regression results of contemporaneously measured realized betas from different asset pricing models. Panel A reports the results of stocks listed in NYSE, AMEX and NASDAQ. The second column is the results of the CAPM with market beta as the only systematic risk. The average of the constant in the cross-sectional regression, alpha, is 0.32 and is only marginally significant. Market beta has a significantly positive coefficient of 0.95, which is different from the results of Fama and French (1992) and Shanken and Zhou (2008) who find a negative coefficient of market beta using unconditional market beta. The estimated market risk premium is greater than the ex-post average of market return (0.65%). My results show the importance of the measurement of market beta used in cross-section tests of the CAPM. However, coefficients of the three firm-level variables are highly significant with the usual sign in the existing literature, i.e. negative coefficient on size and positive coefficient on BM and RET2_12, which suggests market beta alone cannot explain the cross-section of stock returns. Finally, the average adjusted R^2 is only 9.3% which further suggests the poor ability of market beta to explain the cross-section of returns. Overall, the results suggest that market beta is a significant risk but it alone cannot capture the effects of size, value and past returns.

Turning to the Fama-French three-factor model in column 3, alpha is negative with much smaller absolute value than that of the CAPM and remains insignificantly different from zero, which suggests additional betas explain more of the cross-section of returns. The average of coefficient on market beta is 0.893, similar to that of the CAPM, and is significantly positive. Beta on SMB, *s*, has an average coefficient of 0.393, significantly different from zero and greater than the average of SMB (0.233). But beta on HML, *h*, has an insignificant average coefficient (much less than the average of HML, 0.425), which contradicts the proposal of HML in Fama and French (1992, 1993), who show that BM combined with size can drive out other variables' abilities of explaining the cross-section of stock returns and the value effect is the most serious challenge of the CAPM. The size and value effects are attenuated by additional betas, the average coefficient on SIZE is reduced in absolute value to -0.217 from -0.305 and that on BM is reduced to 0.211 from 0.330, but both estimates are still significantly different from zero, suggesting the two additional betas do capture some, but not all, of the size and value effects. The results are consistent with Brennan et al. (1998). The momentum effect, however, still persists with an average coefficient of 0.565 and is significant, indicating the Fama-French model does a poor job in explaining the momentum effect. Finally, the average adjusted R^2 is increased dramatically to 27%.

Column 4 of panel A in Table 4.2 reports the results of the four-factor model. The constant coefficient is 0.052 and insignificant. The estimated coefficients on β, *s* and *h* are similar to those of the Fama-French three-factor model with β and *s* having a significant coefficient and *h* having an insignificant one. The new beta on WML, *m*, has a significant coefficient of 0.59 which is less than the average of WML (0.84%). The coefficient on SIZE (-0.222) is similar to the three-factor model and that on BM is reduced further to 0.173. More important, the momentum effect is attenuated: the coefficient on RET2_12 is reduced to 0.300 from 0.565 in the three-factor model. However, all the coefficients on SIZE, BM and RET2_12 remain significantly different from zero. The adjusted R^2 is increased further to 34%. The results indicate that the four-factor model can explain the momentum and value effects better but still cannot capture all the effects of SIZE, BM and RET2_12.

Table 4.2 Fama-MacBeth Regression Results with Contemporaneous Realized Betas

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of contemporaneous monthly realized betas. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, s, h and m are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month t-12 to month t-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.320	-0.046	0.052
	(0.167)	(0.101)	(0.090)
β	0.954	0.893	0.896
	(0.149)	(0.152)	(0.155)
s		0.393	0.373
		(0.115)	(0.119)
h		0.124	0.124
		(0.106)	(0.108)
m			0.590
			(0.165)
SIZE	-0.305	-0.217	-0.222
	(0.049)	(0.033)	(0.029)
BM	0.330	0.211	0.173
	(0.056)	(0.034)	(0.031)
RET2_12	0.691	0.565	0.300
	(0.120)	(0.083)	(0.059)
$\bar{R}^2$(%)	9.324	27.003	33.995
Panel B: NYSE and AMEX stocks only			
α	0.196	-0.157	-0.074
	(0.141)	(0.084)	(0.075)
β	0.884	0.854	0.850
	(0.149)	(0.154)	(0.158)
s		0.307	0.295
		(0.112)	(0.115)
h		0.154	0.153
		(0.106)	(0.108)
m			0.566
			(0.157)
SIZE	-0.203	-0.095	-0.106
	(0.044)	(0.025)	(0.023)
BM	0.269	0.151	0.129
	(0.052)	(0.032)	(0.029)
RET2_12	0.900	0.740	0.418
	(0.100)	(0.095)	(0.067)
$\bar{R}^2$(%)	10.448	28.637	35.851

Table 4.2 (*continued*)

Panel C: NASDAQ Stocks Only			
α	0.654	0.180	0.286
	(0.231)	(0.139)	(0.124)
β	1.159	1.049	1.058
	(0.192)	(0.186)	(0.189)
s		0.431	0.406
		(0.139)	(0.142)
h		0.109	0.110
		(0.132)	(0.133)
m			0.596
			(0.203)
SIZE	-0.560	-0.508	-0.499
	(0.070)	(0.057)	(0.050)
BM	0.336	0.225	0.174
	(0.069)	(0.049)	(0.045)
RET2_12	0.468	0.378	0.205
	(0.105)	(0.076)	(0.059)
$\bar{R}^2$(%)	7.430	25.116	32.162

I then divide the whole sample into two subsamples to check whether the results are robust across different markets. The first subsample includes stocks listed in NYSE and AMEX only and the second includes stocks listed in NASDAQ only. Panel B of Table 4.2 reports the results of using the first subsample. The results are similar to the whole sample but with some different patterns. The constants are smaller than the whole sample and become negative in the three-factor and four-factor models, suggesting stocks in NYSE and AMEX have smaller risk-adjusted returns on average. The coefficients on β, s and m are all significant, while those on h are insignificant. The effects of size and value are less significant while the momentum effect is more significant in the first sample. The adjusted R^2 are slightly higher than the whole sample.

Panel C of Table 4.2 reports the results of the second subsample. The constant, α, is higher than the whole sample and the first sample. The CAPM and the four-factor model have a significant positive constant but the Fama-French three-factor model has an insignificant one. The results are consistent with Avramov and Chordia (2006) who include a dummy variable of NASDAQ stocks in the cross-section regression and find a positive coefficient. The coefficients on β, s and m are all greater than the first subsample, indicating higher risk premiums of NASDAQ stocks required by investors. Beta on HML, h, remains insignificant. SIZE and BM have greater coefficients than the first subsample while RET2_12 has a lower coefficient, which is consistent with Brennan et al. (1998) and Avramov and Chordia (2006). Finally, the adjusted R^2 are lower than the first subsample.

Overall, the results show that market beta remains a significant risk even when other betas and firm-level variables are included in the regression and contemporaneous multi-betas can help explain some, but not all, of the size, value and momentum effects. The next step is to examine whether forecasted betas have any relationships with returns. The next two subsections deal with this problem.

4.5.2 In-sample forecasted realized betas

The results of the cross-sectional regressions using in-sample forecasted realized betas are reported in Table 4.3. Panel A summarizes the results of the whole sample. The estimation results of the CAPM, reported in the second column, are similar to those of using contemporaneously measured market beta (column 2 of panel A of Table 4.2). The constant is much lower (0.189) and remains insignificant. The coefficients of β and the firm-level variables are all similar to those reported in column 2 of panel A of Table 4.2. However, the adjusted R^2 is much lower (5.16%) than that of contemporaneously measured beta (9.32%), suggesting forecasted beta loses some explanatory abilities. This is reasonable because forecasted betas have some errors imbedded in the forecast. Overall, the results indicate in-sample forecasted beta is still a significant risk although not enough to capture the effects of firm-level variables. This finding is consistent with Bali et al. (2008) who group stocks into portfolios based on in-sample forecasted monthly realized betas and find high-beta portfolios have higher returns than low-beta portfolios.

The third column reports the results of the Fama-French three-factor model. The constant and the coefficient of market beta are similar to those of the CAPM but slightly lower. Betas on SMB and HML, however, have negative coefficients. The coefficients on SIZE, BM and RET2_12 are all similar to those under the CAPM in the second column suggesting additional betas, s and h, lose their explanatory abilities.

Table 4.3 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas from an AR(1) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.189	0.152	0.307
	(0.161)	(0.156)	(0.176)
β	1.008	0.943	0.837
	(0.196)	(0.138)	(0.114)
s		-0.028	-0.065
		(0.085)	(0.074)
h		-0.134	-0.118
		(0.081)	(0.063)
m			0.688
			(0.068)
SIZE	-0.297	-0.288	-0.297
	(0.056)	(0.047)	(0.047)
BM	0.364	0.360	0.362
	(0.054)	(0.051)	(0.056)
RET2_12	0.595	0.543	0.506
	(0.124)	(0.121)	(0.126)
$\bar{R}^2$(%)	5.159	6.162	6.126
Panel B: NYSE and AMEX Stocks Only			
α	0.291	0.233	0.342
	(0.149)	(0.142)	(0.156)
β	0.641	0.782	0.743
	(0.187)	(0.156)	(0.132)
s		-0.195	-0.255
		(0.086)	(0.077)
h		-0.093	-0.115
		(0.089)	(0.069)
m			0.709
			(0.076)
SIZE	-0.160	-0.182	-0.198
	(0.049)	(0.042)	(0.042)
BM	0.282	0.274	0.290
	(0.054)	(0.049)	(0.052)
RET2_12	0.763	0.705	0.641
	(0.140)	(0.136)	(0.140)
$\bar{R}^2$(%)	5.796	6.928	7.003

Table 4.3 (*continued*)

Panel C: NASDAQ stocks only			
α	0.200	0.033	0.299
	(0.213)	(0.203)	(0.234)
β	1.620	1.257	1.004
	(0.244)	(0.153)	(0.114)
s		0.214	0.225
		(0.104)	(0.092)
h		-0.137	-0.109
		(0.098)	(0.072)
m			0.719
			(0.079)
SIZE	-0.638	-0.595	-0.589
	(0.082)	(0.074)	(0.071)
BM	0.399	0.387	0.362
	(0.064)	(0.065)	(0.071)
RET2_12	0.426	0.391	0.345
	(0.110)	(0.107)	(0.109)
$\bar{R}^2$(%)	3.496	4.633	4.668

Furthermore, the adjusted R^2 (6.16%) is only slightly higher than that of the CAPM and much lower than that from contemporaneously measured realized betas. The results show that additional betas of the Fama-French model are not priced and cannot explain the impacts of the firm-level variables, a rejection of the model.

Turning to the four-factor model, column four shows that the results are similar to those of the Fama-French model with beta on WML, m, having a significant positive coefficient. Market beta, β, is still significantly positive (0.837) given the existence of all other betas and all the firm-level variables, which suggests β is a robust systematic risk. The coefficient on m is significantly positive indicating it is also a priced systematic risk. Different from the results from contemporaneously measured betas, the coefficient on RET2_12 remains the same as that of the CAPM and the Fama-French model, suggesting that forecasted beta on the momentum factor fails to capture the impact of past returns. Finally, the adjusted R^2 is even lower than that of the Fama-French model. The results show that the additional beta on the momentum factor helps only a little, if any, in explaining the cross-section of stock returns, although it has a significant positive premium.

Panel B reports the results of using stocks only listed in NYSE and AMEX, and panel C reports the results of NASDAQ stocks. The pattern of panel B compared to panel A is similar to that in Table 4.2. The overall results of the two subsamples are similar to those from the whole sample. Market beta consistently has a significantly positive coefficient across the two subsamples and different models with smaller coefficients in the first subsample than the second. The coefficients on both *s* and *h* are negative in the first subsample (panel B) but *s* is significantly positive in the second subsample (panel C). Beta on WML, *m*, has a significantly positive coefficient in both subsamples. All the three firm-level variables are highly significant. The adjusted R^2 of the three-factor and four-factor models is similar and only slightly higher than that of the CAPM. The difference from Table 4.2 is that constants of Panel B are greater than those of panel C.

Overall, the results show that market beta is the most important systematic risk among betas of different factors. In-sample forecasted betas of SMB and HML lose their explanatory abilities. In-sample forecasted beta of WML is still significantly priced but helps little, if any, in explaining the cross-section of stock returns. Finally, the adjusted R^2 indicates multi-factor models improve only a little, if any, over the CAPM.

4.5.3 Out-of-sample forecasted betas

In this subsection, I examine whether out-of-sample forecasted betas are significantly priced and can explain the cross-section of stock returns. Out-of-sample forecasted betas are of more interest than contemporaneous and in-sample forecasted betas both theoretically and practically. Theoretically, conditional asset pricing models assume investors only use information available when they make investment decisions but both contemporaneous and in-sample forecasted betas use information beyond the information set available to investors in each period. Practically, only out-of-sample forecasted betas are relevant and available for investors to make their investments.

The Fama-MacBeth regression results of out-of-sample forecasted betas are in Table 4.4. The results are very different from those of contemporaneous betas (Table 4.2) and in-sample forecasted betas (Table 4.3). Panel A reports the results using the whole sample. The second column is the results from the CAPM. This time the constant, α, has a value of 0.943, much greater than that of in-sample forecasted betas (0.189) and highly significant. Market beta now has an insignificant and negative coefficient of -0.094. All three firm-level variables have significant coefficients and the usual sign. The adjusted R^2 is only 3.62%. The results show that forecasted realized market beta cannot explain the cross-section of stock returns.

Table 4.4 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of out-of-sample forecasted monthly realized betas from an AR(1) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market factor. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.943	0.884	0.862
	(0.234)	(0.263)	(0.270)
β	-0.094	-0.017	0.002
	(0.087)	(0.030)	(0.020)
s		-0.016	-0.008
		(0.014)	(0.011)
h		-0.007	0.005
		(0.013)	(0.010)
m			0.010
			(0.013)
SIZE	-0.172	-0.177	-0.176
	(0.052)	(0.050)	(0.050)
BM	0.304	0.320	0.326
	(0.067)	(0.074)	(0.074)
RET2_12	0.558	0.571	0.569
	(0.137)	(0.142)	(0.144)
$\bar{R}^2$(%)	3.622	3.212	3.187
Panel B: NYSE and AMEX Stocks Only			
α	0.811	0.773	0.759
	(0.207)	(0.237)	(0.244)
β	-0.084	-0.034	-0.026
	(0.097)	(0.034)	(0.025)
s		-0.021	-0.009
		(0.018)	(0.015)
h		0.013	0.017
		(0.015)	(0.012)
m			0.002
			(0.015)
SIZE	-0.084	-0.086	-0.084
	(0.047)	(0.045)	(0.045)
BM	0.253	0.265	0.271
	(0.061)	(0.065)	(0.065)
RET2_12	0.724	0.732	0.728
	(0.154)	(0.160)	(0.161)
$\bar{R}^2$(%)	4.471	3.967	3.954

Table 4.4 (*continued*)

Panel C: NASDAQ stocks only			
α	1.254	1.204	1.215
	(0.280)	(0.312)	(0.317)
β	0.002	0.036	0.035
	(0.098)	(0.035)	(0.021)
s		0.020	0.013
		(0.018)	(0.014)
h		-0.038	-0.013
		(0.017)	(0.013)
m			0.024
			(0.016)
SIZE	-0.396	-0.403	-0.403
	(0.071)	(0.066)	(0.066)
BM	0.229	0.236	0.236
	(0.077)	(0.087)	(0.087)
RET2_12	0.492	0.510	0.510
	(0.105)	(0.114)	(0.114)
$\bar{R}^2$(%)	2.386	2.052	2.047

The third and last columns report the results from the Fama-French three-factor model and the four-factor model, respectively. The results are similar to the CAPM. The constant is very large and significant while none of betas has a significantly positive coefficient. All the three firm-level variables are highly significant. The adjusted R^2 is only slightly above 3%. The results show that additional betas from multi-factor models cannot add any explanatory abilities to the CAPM. Panels B and C report the results of the subsample of NYSE and AMEX stocks and the subsample of NASDAQ stocks, respectively. The overall results are similar to the whole sample. None of betas is significantly priced, suggesting that out-of-sample forecasted betas cannot explain the cross-section of stock returns.

The results show that out-of-sample forecasted betas have very different results than in-sample forecasted betas. For example, using in-sample forecasted betas, Bali et al. (2008) find a significant positive premium associated with market beta. However, out-of-sample forecasted market beta has an insignificant and negative coefficient from my results. Therefore, using in-sample forecasted betas in asset pricing tests potentially suffers from over-conditioning bias because information beyond investors' information set is used to estimate conditional betas. In practice, it can be misleading if investors use in-sample forecasted betas to make investment decisions.

My results are consistent with Daniel and Titman (1997) who show that it is firm characteristics rather than betas that decide the cross-section of stock returns. They show that portfolios formed by betas of the Fama-French three-factor model do not have return patterns once firm characteristics are controlled. Therefore, betas of the Fama-French factors cannot explain the cross-section of stock returns. My results show that additional betas of the Fama-French model, i.e. betas on SMB and HML, do not have significantly positive coefficients both in-sample and out-of-sample and market beta is only significant in-sample. On the other hand, firm-level variables are always significant no matter what betas and what models are used.

Obviously, one reason for the failure of forecasted betas is the simple linear models used because contemporaneous betas do capture quite a portion of the cross-section of returns. Therefore, more sophisticated and non-linear models will perhaps get better results. This will be examined in the future.

4.6. Conclusion

This chapter examines the cross-sectional relationships between stock returns, betas (factor loadings) of different models and firm-level variables using individual stocks listed in NYSE, AMEX and NASDAQ. Betas used are three kinds of realized betas: contemporaneously measured, in-sample forecasted and out-of-sample forecasted. Asset pricing models examined are the CAPM, the Fama-French three-factor model and a four-factor model which is the Fama-French three-factor model augmented by a momentum factor. Firm-level variables are size, BM and past returns.

The results show that contemporaneous realized betas can explain quite a portion of the cross-section of stock returns and multi-factor models do outperform the CAPM. Betas associated with factors corresponding to different firm-level variables effects have significantly positive coefficients (except beta on HML) and do attenuate although not eliminate the effects of size, BM and momentum. This is consistent with findings of Brennan et al. (1998). The results from in-sample forecasted betas, however, indicate that market beta is the most significant risk and betas on SMB and HML are no longer significant while beta on WML still has a significantly positive coefficient. In-sample forecasted betas of multi-factor models cannot help explain the effects of size, book-to-market ratio and momentum any more. Turning to the results of out-of-sample forecasted betas, none of betas has significantly positive coefficient and the constant and all firm-level variables are highly significant.

The results show that betas cannot explain the cross-section of stock returns from an ex-ante view although they can explain part of the effects of firm-level variables ex-post. Out-of-sample forecasted betas of multi-factor models cannot help explain the cross-section of stock returns and do not have significantly positive coefficients. The results of this chapter indicate a rejection of the three models examined from an ex-ante view.

The results of this chapter indicate that ex-post realized betas do capture part of the effects of firm-level variables and have a relative high adjusted R^2. This suggests that it is important to model realized betas carefully. The techniques used in this chapter are all linear models; therefore, a natural extension will be using more sophisticated non-linear models to forecast realized betas, which will be the research direction in the future.

Appendix 4: Robustness checks

A. Alternative measures of size and past returns

Brennan et al. (1998) and Avramov and Chordia (2006) use the following measures of size and past returns:

SIZE: the market capitalisation of the second to the last month;

RET2_3: the logarithm of cumulative return from month *t*-3 to month *t*-2;

RET4_6: the logarithm of cumulative return from month *t*-4 to month *t*-6;

RET7_12: the logarithm of cumulative return from month *t*-7 to month *t*-12.

Table A4.1 reports the results of using the above size and past return measures. Panel A summarizes the results of the whole sample. The results are similar to Table 4.2. The constant is insignificant. *β, s* and *m* are significantly positive but *h* is not. All firm level variables' coefficients are highly significant and have the expected sign: negative size effect and positive value and momentum effect. Panel B gives the results of NYSE and AMEX stocks and panel C are the results of NASDAQ stocks. Both panels have similar results to corresponding panels of Table 4.2 except that the RET2_3 has a negative coefficient in panel C. The results are consistent with Avramov and Chordia (2006).

Table A4.1 Fama-MacBeth Regression Results with Alternative Measures of SIZE and Past Returns
This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of contemporaneous monthly realized betas. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market factor. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the two-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_3, RET4_6, RET7_12 are the cumulative returns from month *t*-3 to month *t*-2, from month *t*-6 to month *t*-4 and from month *t*-12 to month *t*-7, respectively. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.328	-0.047	-0.066
	(0.170)	(0.103)	(0.077)
β	0.942	0.891	0.843
	(0.146)	(0.150)	(0.157)
s		0.396	0.299
		(0.114)	(0.114)
h		0.126	0.151
		(0.105)	(0.107)
m			0.569
			(0.155)
SIZE	-0.269	-0.191	-0.092
	(0.046)	(0.031)	(0.022)
BM	0.355	0.229	0.148
	(0.053)	(0.032)	(0.029)
RET2_3	0.517	0.401	0.423
	(0.231)	(0.171)	(0.166)
RET4_6	0.679	0.582	0.465
	(0.192)	(0.134)	(0.126)
RET7_12	0.668	0.529	0.426
	(0.120)	(0.087)	(0.084)
$\bar{R}^2$(%)	9.642	27.150	36.027

Table A4.1 (*continued*)

Panel B: NYSE and AMEX Stocks Only			
α	0.212	-0.149	0.052
	(0.144)	(0.085)	(0.092)
β	0.866	0.845	0.894
	(0.147)	(0.153)	(0.154)
s		0.312	0.376
		(0.111)	(0.117)
h		0.155	0.124
		(0.105)	(0.107)
m			0.601
			(0.163)
SIZE	-0.182	-0.084	-0.197
	(0.042)	(0.024)	(0.028)
BM	0.286	0.166	0.193
	(0.051)	(0.032)	(0.030)
RET2_3	0.968	0.834	0.117
	(0.254)	(0.195)	(0.139)
RET4_6	0.949	0.894	0.243
	(0.227)	(0.161)	(0.103)
RET7_12	0.878	0.748	0.267
	(0.139)	(0.108)	(0.069)
$\bar{R}^2$(%)	10.922	28.883	34.113
Panel C: NASDAQ Stocks Only			
α	0.668	0.191	0.298
	(0.235)	(0.143)	(0.126)
β	1.136	1.036	1.045
	(0.189)	(0.185)	(0.188)
s		0.430	0.405
		(0.138)	(0.141)
h		0.119	0.119
		(0.131)	(0.132)
m			0.612
			(0.200)
SIZE	-0.457	-0.427	-0.426
	(0.063)	(0.053)	(0.048)
BM	0.408	0.279	0.225
	(0.062)	(0.045)	(0.042)
RET2_3	-0.309	-0.216	-0.317
	(0.244)	(0.176)	(0.149)
RET4_6	0.388	0.355	0.138
	(0.181)	(0.143)	(0.116)
RET7_12	0.500	0.346	0.163
	(0.117)	(0.085)	(0.072)
$\bar{R}^2$(%)	7.646	25.207	32.253

$_{i,k,l}$

B. Alternative measures of realized betas

*B.*1 Scholes and Williams (1977)

Scholes and Williams (1977) propose a beta estimate to correct the bias caused by non-synchronous trading in the market. They include the lead and lag betas as well as contemporaneous beta in the final beta estimate. Specifically, three regressions of individual returns on lead, lag and contemporaneous market returns are run separately,

$$r_{i,t} = \alpha_{i,l} + \beta_{i,l} r_{m,t-l} + \varepsilon_{i,t-l}, \quad l = -1,0,1. \tag{A4.1}$$

Then the final estimate of beta is computed as

$$\hat{\beta}_i = \frac{\sum_{l=-1}^{1} \hat{\beta}_{i,l}}{1 + 2\rho_m}, \tag{A4.2}$$

where $\hat{\beta}_{i,l}$ is OLS estimate from equation (A4.1) and ρ_m is the first order autocorrelation of the market return. This measure is easily applied to the multivariate case as in Bollerslev and Zhang (2003). Three separate regressions are run by using lead, lag and contemporaneous factor returns,

$$r_{i,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k,l} f_{k,t-l} + \varepsilon_{i,l}, \; l = -1,0,1. \tag{A4.3}$$

Then beta of factor *k* is computed as

$$\hat{\beta}_{i,k} = \frac{\sum_{l=-1}^{1} \hat{\beta}_{i,k,l}}{1 + 2\rho_k}, \tag{A4.4}$$

where $\hat{\beta}_{i,k,l}$ is OLS estimate from equation (A4.3) and ρ_k is the first order autocorrelation of the *k*th factor.

Realized betas of different pricing models are computed according to equation (A4.3) and (A4.4) with the intercept in equation (A4.3) excluded, using daily returns within each month. Table A4.2 reports the results of the cross-sectional regression using realized betas of Scholes and Williams (1977). Compared with Table 4.2, constants are greater and significantly positive in all three models and β, s and m have a lower coefficient but remain significantly positive. This is because betas of Scholes and Williams (1977) are more volatile than unadjusted betas (see Sercu et al., 2008). Therefore, the slope of the regression is flatter and the intercept is greater. The coefficient of SIZE is similar to Table 4.2 but the coefficients of BM and RET2_12 are reduced (except the coefficient of RET2_12 of the CAPM) and BM is no longer significant in Fama-French model and the four-factor model. Finally, the adjusted R^2 is greater than that in Table 4.2.

Results of the two subsamples are reported in panel B and panel C, respectively. The pattern is similar to that in Table 4.2. The exception is that BM is generally insignificant under multifactor models (only significant in the Fama-French model in panel B) and RET2_12 is only significant in the subsample of NYSE and AMEX stocks under the four-factor model.

Overall, the results show that betas of Scholes and Williams (1977) do a better job than unadjusted betas in capturing the effects of size, BM and past returns. But this should be explained with caution because constants in the cross-sectional regressions are greater and highly significant and the coefficients on betas are flatter.

Table A4.2 Fama-MacBeth Regression Results of Scholes and Williams (1977) Betas

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of contemporaneous monthly realized betas adjusted by the method of Scholes and Williams (1977). Only stocks have at least 12 available daily returns are used. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.383	0.246	0.378
	(0.143)	(0.071)	(0.063)
β	0.645	0.498	0.358
	(0.133)	(0.114)	(0.113)
s		0.171	0.199
		(0.079)	(0.080)
h		0.114	0.139
		(0.084)	(0.084)
m			0.379
			(0.131)
SIZE	-0.297	-0.223	-0.233
	(0.044)	(0.025)	(0.020)
BM	0.264	0.068	0.012
	(0.051)	(0.025)	(0.022)
RET2_12	0.728	0.397	0.119
	(0.111)	(0.066)	(0.042)
$\bar{R}^2$(%)	15.791	45.382	55.671
Panel B: NYSE and AMEX Stocks Only			
α	0.236	0.079	0.187
	(0.118)	(0.059)	(0.054)
β	0.597	0.494	0.349
	(0.131)	(0.114)	(0.114)
s		0.129	0.156
		(0.078)	(0.081)
h		0.130	0.158
		(0.084)	(0.083)
m			0.370
			(0.126)
SIZE	-0.186	-0.106	-0.122
	(0.039)	(0.018)	(0.015)
BM	0.222	0.059	0.018
	(0.049)	(0.026)	(0.022)
RET2_12	0.897	0.521	0.177
	(0.124)	(0.076)	(0.048)
$\bar{R}^2$(%)	16.666	46.877	57.511

Table A4.2 (*continued*)

Panel C: NASDAQ Stocks Only			
α	0.743	0.525	0.656
	(0.207)	(0.101)	(0.088)
β	0.757	0.551	0.417
	(0.165)	(0.137)	(0.134)
s		0.198	0.244
		(0.092)	(0.093)
h		0.116	0.139
		(0.104)	(0.104)
m			0.352
			(0.159)
SIZE	-0.560	-0.489	-0.488
	(0.063)	(0.046)	(0.038)
BM	0.239	0.026	-0.039
	(0.062)	(0.037)	(0.033)
RET2_12	0.474	0.257	0.060
	(0.098)	(0.062)	(0.046)
$\bar{R}^2(\%)$	14.096	43.605	53.989

B.2 More days within each month

Realized betas can be a more accurate measure of underlying integrated betas with more observations within each interval. In this subsection, I examine whether the results are affected by using only months with at least 20 available daily returns. Table A4.3 reports the results, which are very similar to those of Table 4.2. But this cannot be treated as that using more intra-period observation has no effect on the estimates because both 20 and 12 are relatively small numbers of intra-period observations compared with ultra high-frequency intraday data. The use of high frequency data needs to be examined in the future.

Table A4.3 Fama-MacBeth Regression Results of Months Having at Least 20 Daily Returns

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of contemporaneous monthly realized betas. Only stocks have at least 20 available daily returns are used. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.336	-0.058	0.045
	(0.183)	(0.112)	(0.100)
β	1.046	0.960	0.956
	(0.159)	(0.162)	(0.167)
s		0.452	0.431
		(0.126)	(0.129)
h		0.054	0.054
		(0.111)	(0.112)
m			0.533
			(0.177)
SIZE	-0.311	-0.216	-0.222
	(0.054)	(0.036)	(0.031)
BM	0.286	0.198	0.161
	(0.059)	(0.036)	(0.033)
RET2_12	0.678	0.573	0.310
	(0.132)	(0.092)	(0.066)
$\bar{R}^2$(%)	9.184	26.114	33.106
Panel B: NYSE and AMEX Stocks Only			
α	0.189	-0.170	-0.083
	(0.154)	(0.093)	(0.083)
β	0.968	0.911	0.899
	(0.160)	(0.166)	(0.170)
s		0.365	0.355
		(0.121)	(0.125)
h		0.081	0.083
		(0.112)	(0.113)
m			0.494
			(0.168)
SIZE	-0.208	-0.092	-0.103
	(0.048)	(0.027)	(0.025)
BM	0.256	0.153	0.129
	(0.055)	(0.034)	(0.031)
RET2_12	0.842	0.756	0.427
	(0.147)	(0.105)	(0.074)
$\bar{R}^2$(%)	10.385	27.765	34.992

Table A4.3 (*continued*)

Panel C: NASDAQ Stocks Only			
α	0.697	0.173	0.284
	(0.250)	(0.151)	(0.135)
β	1.256	1.120	1.120
	(0.205)	(0.198)	(0.203)
s		0.504	0.477
		(0.150)	(0.153)
h		0.035	0.035
		(0.136)	(0.137)
m			0.510
			(0.216)
SIZE	-0.548	-0.504	-0.494
	(0.075)	(0.061)	(0.054)
BM	0.283	0.205	0.160
	(0.073)	(0.052)	(0.048)
RET2_12	0.456	0.389	0.218
	(0.115)	(0.083)	(0.064)
$\bar{R}^2$(%)	7.389	24.603	31.662

C. Quarterly realized betas and returns.

In this Appendix, I use quarterly intervals. Realized betas are computed by using daily returns within each quarter. Then cross-sectional regressions of quarterly returns of individual stocks on quarterly realized betas are run for each quarter. The time series means of the estimates and standard errors of the means are reported.

Table A4.4 reports the results of quarterly regressions. The second column of panel A is the results of the CAPM. The estimates are roughly three times those in Table 4.2, which is due to the quarterly compounding of monthly returns. The statistical inference is the same. The constant is positive but insignificant while beta is significantly positive. All three firm-level variables are highly significant. The adjusted R^2 is 8.91%, similar to that from monthly regressions. The third column of panel A reports the results of the Fama-French model. The intercept is still insignificant. β and s are both significantly positive and have magnitudes of roughly three times of those in Table 4.2. The coefficient on h is still insignificant and similar to that in Table 4.2, which suggests it is not a priced risk. The coefficients on the three firm-level variables are reduced

Table A4.4 Fama-MacBeth Regression Results of Quarterly Intervals

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of contemporaneous quarterly realized betas. Only stocks have at least 20 available daily returns are used. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.692	-0.135	-0.015
	(0.594)	(0.404)	(0.385)
β	3.048	2.716	2.786
	(0.599)	(0.538)	(0.529)
s		1.081	1.027
		(0.358)	(0.359)
h		0.157	0.132
		(0.391)	(0.390)
m			2.199
			(0.518)
SIZE	-0.787	-0.553	-0.570
	(0.194)	(0.135)	(0.129)
BM	1.133	0.972	0.930
	(0.186)	(0.121)	(0.117)
RET2_12	1.632	1.318	0.755
	(0.391)	(0.315)	(0.255)
$\bar{R}^2$(%)	8.914	17.363	20.753
Panel B: NYSE and AMEX Stocks Only			
α	0.484	-0.294	-0.211
	(0.505)	(0.344)	(0.325)
β	2.595	2.472	2.574
	(0.559)	(0.538)	(0.531)
s		0.819	0.779
		(0.348)	(0.346)
h		0.375	0.321
		(0.350)	(0.349)
m			2.187
			(0.484)
SIZE	-0.473	-0.240	-0.265
	(0.168)	(0.105)	(0.103)
BM	0.954	0.713	0.689
	(0.176)	(0.116)	(0.111)
RET2_12	2.050	1.789	1.043
	(0.447)	(0.360)	(0.287)
$\bar{R}^2$(%)	10.041	18.979	22.671

Table A4.4 (*continued*)

Panel C: NASDAQ stocks only			
α	1.498	0.241	0.347
	(0.816)	(0.532)	(0.512)
β	3.855	3.347	3.440
	(0.773)	(0.658)	(0.645)
s		1.195	1.125
		(0.441)	(0.441)
h		0.054	0.038
		(0.519)	(0.517)
m			2.221
			(0.639)
SIZE	-1.479	-1.335	-1.339
	(0.282)	(0.245)	(0.235)
BM	1.296	1.174	1.087
	(0.217)	(0.170)	(0.166)
RET2_12	1.256	1.066	0.739
	(0.354)	(0.277)	(0.226)
$\bar{R}^2$(%)	6.416	14.613	17.656

compared to column two but remain highly significant. The R^2 is increased to 17.36% from the CAPM but much smaller than that from monthly regressions in Table 4.2 (27.00%). The last column is the results of the four-factor model. The estimates are similar to the third column except the additional coefficient on *m* is significant and the coefficient of RET2_12 is reduced to 0.755 from 1.318. The R^2 is increased to 20.75% but again much smaller than the monthly regressions in Table 4.2 (34.00%). Panel B and panel C report the results of the two subsamples. The patterns in the results are similar to those of Table 4.2.

Overall, the results of quarterly regressions are consistent with those of monthly regressions but quarterly realized betas explain less of the cross-section of stock returns than monthly realized betas as seen from the smaller adjusted R^2.

D. Alternative models of realized betas

In this Appendix, I examine whether the results of forecasted betas are affected by forecasting models of realized betas. In the main text, I use a simple AR(1) model with lagged macroeconomic variables for realized betas. Here, I consider some more sophisticated models:

AR(2): Hooper et al. (2008) report that an AR(2) model performs best among different autoregressive models in forecasting realized quarterly betas of 40 stocks listed in FTSE 100.

AR(3): Hooper et al. (2008) also consider an AR(3) model and show that the AR(3) model performs best in some cases.

ARMA(1,1): Chang and Weiss (1991) show that estimated betas should follow an ARMA(1,1) model. Therefore, I also consider an ARMA(1,1) model for realized betas.

All the models above also include lagged macroeconomic variables as the AR(1) model in the main text. In Appendix *D*.1, I compare the results of in-sample forecasted betas from different models and the comparison of out-of-forecasted betas is reported in Appendix *D*.2. The results of all the alternative models are very similar. Therefore, I mainly compare the results of AR(2) with those of AR(1) in Table 4.3 and state the differences of other models when necessary.

D.1 In-sample forecasted betas

Table A4.5 reports the results of in-sample forecasted betas from the AR(2) model. Comparing with Table 4.3, we can see that the results of the two tables are very similar. Panel A is the results of using the full sample. The constant is insignificant for all the three models. The coefficients on market beta of the AR(2) model are smaller than those in Table 4.3 but remain significantly positive. The coefficients on *s* become positive but remain insignificantly different from zero. *h* still has negative coefficients as in Table 4.3. Momentum beta, *m*, remains significantly positive although becomes smaller than that in Table 4.3. The magnitudes of the coefficients on the three firm-level variables are also similar to those in Table 4.3. The adjusted R^2 is also similar to Table 4.3 for all three models. Overall, the results show that the AR(2) model has slightly, if any, improvement over the AR(1) model. Panel B reports the results of the subsample of NYSE and AMEX stocks only. The estimated coefficients are generally smaller than those in panel B of Table 4.3 except the constant of the CAPM while the adjusted R^2 is slightly higher. Panel C is the results of the subsample of NASDAQ stocks only. The overall results are similar to the panel C in Table 4.3. Overall, the AR(2) model performs slightly better than the AR(1) model but the statistical inference are unchanged except that the coefficients on *s* become significantly positive for NASDAQ stocks.

Tables A4.6 and A4.7 report the results of models AR(3) and ARMA(1,1), respectively. The results are very similar to those of the AR(2) model. The exception is that the coefficients of *s* in ARMA(1,1) are positive and significant for the full sample (panel A of Table A4.7). The significance of *s* is driven by NASDAQ stocks because it is insignificant in the first subsample (panel B of Table A4.7).

Table A4.5 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas from an AR(2) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas from an AR(2) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.226	0.146	0.320
	(0.164)	(0.161)	(0.181)
β	0.963	0.851	0.723
	(0.186)	(0.127)	(0.103)
s		0.067	0.027
		(0.081)	(0.069)
h		-0.083	-0.064
		(0.074)	(0.056)
m			0.606
			(0.064)
SIZE	-0.293	-0.271	-0.278
	(0.055)	(0.046)	(0.046)
BM	0.362	0.357	0.354
	(0.055)	(0.052)	(0.058)
RET2_12	0.597	0.554	0.517
	(0.124)	(0.122)	(0.127)
$\bar{R}^2$(%)	5.143	6.231	6.166
Panel B: NYSE and AMEX Stocks Only			
α	0.306	0.219	0.332
	(0.151)	(0.145)	(0.160)
β	0.626	0.703	0.659
	(0.180)	(0.144)	(0.121)
s		-0.090	-0.141
		(0.081)	(0.071)
h		-0.026	-0.069
		(0.083)	(0.065)
m			0.621
			(0.072)
SIZE	-0.159	-0.162	-0.176
	(0.049)	(0.041)	(0.042)
BM	0.280	0.267	0.283
	(0.054)	(0.050)	(0.053)
RET2_12	0.763	0.711	0.653
	(0.140)	(0.137)	(0.141)
$\bar{R}^2$(%)	5.792	6.979	7.051

Table A4.5 (*continued*)

Panel C: NASDAQ Stocks Only			
α	0.271	0.087	0.366
	(0.218)	(0.211)	(0.242)
β	1.516	1.093	0.846
	(0.230)	(0.138)	(0.101)
s		0.300	0.284
		(0.098)	(0.085)
h		-0.089	-0.052
		(0.088)	(0.064)
m			0.640
			(0.073)
SIZE	-0.624	-0.576	-0.569
	(0.080)	(0.072)	(0.069)
BM	0.392	0.377	0.349
	(0.065)	(0.066)	(0.072)
RET2_12	0.434	0.405	0.363
	(0.110)	(0.108)	(0.111)
$\bar{R}^2$(%)	3.484	4.793	4.816

Table A4.6 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas from an AR(3) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas from an AR(3) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, s, h and m are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month t-12 to month t-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.243	0.155	0.313
	(0.166)	(0.163)	(0.184)
β	0.943	0.797	0.683
	(0.181)	(0.120)	(0.096)
s		0.116	0.071
		(0.077)	(0.065)
h		-0.075	-0.045
		(0.070)	(0.052)
m			0.552
			(0.061)
SIZE	-0.292	-0.262	-0.269
	(0.055)	(0.046)	(0.046)
BM	0.362	0.356	0.352
	(0.055)	(0.053)	(0.059)
RET2_12	0.594	0.547	0.519
	(0.124)	(0.122)	(0.128)
$\bar{R}^2$(%)	5.147	6.308	6.242
Panel B: NYSE and AMEX Stocks Only			
α	0.304	0.210	0.327
	(0.151)	(0.147)	(0.163)
β	0.634	0.688	0.628
	(0.176)	(0.138)	(0.111)
s		-0.057	-0.097
		(0.076)	(0.067)
h		-0.024	-0.057
		(0.079)	(0.060)
m			0.572
			(0.068)
SIZE	-0.161	-0.158	-0.168
	(0.048)	(0.041)	(0.041)
BM	0.281	0.266	0.280
	(0.054)	(0.051)	(0.054)
RET2_12	0.761	0.711	0.657
	(0.140)	(0.137)	(0.141)
$\bar{R}^2$(%)	5.809	7.004	7.068

Table A4.6 (*continued*)

Panel C: NASDAQ Stocks Only			
α	0.306	0.125	0.383
	(0.221)	(0.215)	(0.246)
β	1.460	1.018	0.796
	(0.223)	(0.130)	(0.096)
s		0.327	0.301
		(0.093)	(0.080)
h		-0.074	-0.035
		(0.083)	(0.061)
m			0.583
			(0.071)
SIZE	-0.615	-0.566	-0.559
	(0.080)	(0.071)	(0.069)
BM	0.389	0.375	0.347
	(0.066)	(0.067)	(0.073)
RET2_12	0.432	0.411	0.372
	(0.111)	(0.108)	(0.112)
$\bar{R}^2$(%)	3.517	4.961	5.013

Table A4.7 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas from an ARMA(1,1) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas from an ARMA(1) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.271	0.113	0.298
	(0.167)	(0.164)	(0.186)
β	0.928	0.744	0.641
	(0.178)	(0.113)	(0.092)
s		0.245	0.165
		(0.080)	(0.065)
h		-0.040	-0.023
		(0.065)	(0.050)
m			0.628
			(0.062)
SIZE	-0.295	-0.252	-0.261
	(0.054)	(0.045)	(0.045)
BM	0.361	0.354	0.340
	(0.055)	(0.053)	(0.058)
RET2_12	0.589	0.519	0.486
	(0.124)	(0.122)	(0.128)
$\bar{R}^2$(%)	5.253	6.775	6.862
Panel B: NYSE and AMEX Stocks Only			
α	0.352	0.173	0.290
	(0.150)	(0.147)	(0.163)
β	0.590	0.659	0.595
	(0.175)	(0.125)	(0.105)
s		0.041	0.024
		(0.078)	(0.064)
h		-0.020	-0.053
		(0.073)	(0.058)
m			0.626
			(0.069)
SIZE	-0.162	-0.147	-0.153
	(0.048)	(0.040)	(0.040)
BM	0.278	0.266	0.282
	(0.054)	(0.050)	(0.053)
RET2_12	0.756	0.671	0.603
	(0.140)	(0.137)	(0.141)
$\bar{R}^2$(%)	5.914	7.297	7.386

Table A4.7 (*continued*)

Panel C: NASDAQ Stocks Only			
α	0.316	0.073	0.396
	(0.224)	(0.217)	(0.249)
β	1.459	0.951	0.754
	(0.216)	(0.119)	(0.094)
s		0.470	0.356
		(0.097)	(0.081)
h		-0.016	0.010
		(0.077)	(0.058)
m			0.649
			(0.072)
SIZE	-0.613	-0.560	-0.555
	(0.078)	(0.069)	(0.068)
BM	0.390	0.369	0.320
	(0.066)	(0.067)	(0.073)
RET2_12	0.423	0.387	0.349
	(0.111)	(0.108)	(0.112)
$\overline{R}^2(\%)$	3.635	5.667	5.995

D.2 Out-of-sample forecasted betas

Table A4.8 reports the results of out-of-sample forecasted betas from the AR(2) model. The results are very similar to those of Table 4.4. None of betas has a significantly positive coefficient while the constant and the three firm-level variables are highly significant. Tables A4.9 and A4.10 report the results of models AR(3) and ARMA(1,1), respectively. The results are very similar to those of the AR(2) model.

Table A4.8 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas from an AR(2) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas from an AR(2) model. The forecast is done by a 60-month rolling window. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month t-12 to month t-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.929	0.868	0.856
	(0.239)	(0.266)	(0.271)
β	-0.079	-0.009	0.006
	(0.076)	(0.025)	(0.016)
s		-0.009	-0.005
		(0.012)	(0.010)
h		-0.004	0.001
		(0.011)	(0.008)
m			0.003
			(0.011)
SIZE	-0.174	-0.175	-0.175
	(0.052)	(0.050)	(0.050)
BM	0.307	0.323	0.326
	(0.068)	(0.074)	(0.074)
RET2_12	0.562	0.566	0.569
	(0.138)	(0.143)	(0.144)
$\bar{R}^2$(%)	3.559	3.180	3.159
Panel B: NYSE and AMEX Stocks Only			
α	0.785	0.752	0.744
	(0.212)	(0.240)	(0.245)
β	-0.047	-0.014	-0.014
	(0.088)	(0.029)	(0.021)
s		-0.011	-0.001
		(0.015)	(0.013)
h		0.008	0.010
		(0.013)	(0.010)
m			-0.004
			(0.013)
SIZE	-0.085	-0.085	-0.083
	(0.046)	(0.045)	(0.045)
BM	0.261	0.267	0.273
	(0.062)	(0.065)	(0.066)
RET2_12	0.728	0.728	0.725
	(0.154)	(0.160)	(0.161)
$\bar{R}^2$(%)	4.433	3.925	3.937

Table A4.8 (*continued*)

Panel C: NASDAQ Stocks Only			
α	1.256	1.203	1.217
	(0.285)	(0.315)	(0.319)
β	-0.004	0.028	0.035
	(0.085)	(0.027)	(0.018)
s		0.020	0.010
		(0.015)	(0.013)
h		-0.022	-0.014
		(0.015)	(0.011)
m			0.023
			(0.013)
SIZE	-0.395	-0.404	-0.405
	(0.070)	(0.066)	(0.066)
BM	0.227	0.234	0.231
	(0.079)	(0.087)	(0.087)
RET2_12	0.497	0.510	0.516
	(0.106)	(0.114)	(0.114)
$\bar{R}^2(\%)$	2.338	2.056	2.047

Table A4.9 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas from an AR(3) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas from an AR(3) model. The forecast is done by a 60-month rolling window. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.928	0.861	0.856
	(0.243)	(0.267)	(0.272)
β	-0.081	-0.002	0.010
	(0.068)	(0.021)	(0.015)
s		-0.010	-0.012
		(0.010)	(0.009)
h		0.001	0.000
		(0.010)	(0.007)
m			0.010
			(0.010)
SIZE	-0.172	-0.174	-0.174
	(0.052)	(0.050)	(0.050)
BM	0.307	0.323	0.325
	(0.069)	(0.074)	(0.075)
RET2_12	0.557	0.575	0.562
	(0.139)	(0.143)	(0.144)
$\bar{R}^2$(%)	3.520	3.184	3.176
Panel B: NYSE and AMEX Stocks Only			
α	0.774	0.751	0.750
	(0.215)	(0.242)	(0.246)
β	-0.037	-0.016	-0.013
	(0.081)	(0.025)	(0.018)
s		-0.010	-0.009
		(0.014)	(0.012)
h		0.012	0.006
		(0.011)	(0.009)
m			-0.002
			(0.011)
SIZE	-0.085	-0.085	-0.085
	(0.046)	(0.045)	(0.045)
BM	0.265	0.267	0.265
	(0.062)	(0.065)	(0.066)
RET2_12	0.729	0.732	0.723
	(0.155)	(0.160)	(0.161)
$\bar{R}^2$(%)	4.378	3.934	3.933

Table A4.9 *(continued)*

Panel C: NASDAQ Stocks Only			
α	1.269	1.198	1.215
	(0.290)	(0.316)	(0.319)
β	-0.027	0.042	0.039
	(0.075)	(0.023)	(0.016)
s		0.009	0.000
		(0.013)	(0.011)
h		-0.018	-0.012
		(0.013)	(0.009)
m			0.027
			(0.012)
SIZE	-0.393	-0.403	-0.401
	(0.070)	(0.067)	(0.066)
BM	0.225	0.235	0.235
	(0.080)	(0.087)	(0.087)
RET2_12	0.487	0.523	0.505
	(0.108)	(0.114)	(0.114)
$\bar{R}^2(\%)$	2.318	2.072	2.066

Table A4.10 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas from an ARMA(1,1) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas from an ARMA(1,1) model. The forecast is done by a 60-month rolling window. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. CAPM has only market beta. FF3F is the Fama-French three-factor model with market, SMB and HML as factors. 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

	CAPM	FF3F	4F
Panel A: NYSE, AMEX and NASDAQ Stocks			
α	0.943	0.877	0.841
	(0.243)	(0.269)	(0.272)
β	-0.109	-0.013	0.008
	(0.067)	(0.020)	(0.014)
s		-0.016	0.000
		(0.010)	(0.008)
h		0.002	0.002
		(0.010)	(0.007)
m			0.003
			(0.009)
SIZE	-0.166	-0.174	-0.172
	(0.052)	(0.050)	(0.050)
BM	0.308	0.321	0.310
	(0.068)	(0.075)	(0.075)
RET2_12	0.561	0.554	0.563
	(0.139)	(0.144)	(0.144)
$\bar{R}^2$(%)	3.510	3.164	3.155
Panel B: NYSE and AMEX Stocks Only			
α	0.802	0.762	0.730
	(0.216)	(0.243)	(0.246)
β	-0.085	-0.019	0.000
	(0.077)	(0.022)	(0.017)
s		-0.016	-0.010
		(0.013)	(0.011)
h		0.010	0.010
		(0.011)	(0.009)
m			-0.004
			(0.010)
SIZE	-0.080	-0.084	-0.083
	(0.046)	(0.045)	(0.045)
BM	0.258	0.264	0.260
	(0.062)	(0.066)	(0.066)
RET2_12	0.718	0.707	0.730
	(0.155)	(0.162)	(0.161)
$\bar{R}^2$(%)	4.329	3.913	3.917

Table A4.10 (*continued*)

Panel C: NASDAQ Stocks Only			
α	1.279	1.222	1.210
	(0.288)	(0.317)	(0.319)
β	-0.049	0.016	0.030
	(0.075)	(0.023)	(0.015)
s		0.005	0.013
		(0.013)	(0.010)
h		-0.014	-0.010
		(0.012)	(0.009)
m			0.007
			(0.012)
SIZE	-0.387	-0.401	-0.395
	(0.070)	(0.067)	(0.067)
BM	0.226	0.240	0.219
	(0.079)	(0.087)	(0.088)
RET2_12	0.508	0.511	0.511
	(0.107)	(0.115)	(0.117)
$\bar{R}^2$(%)	2.305	2.039	2.028

Chapter 5

Can Higher Comoments Help Explain the Cross-Section of Stock Returns?

5.1 Introduction

The CAPM of Sharpe (1964) and Lintner (1965) assumes that investors only care about the mean and variance of asset returns which are the first two moments of distributions. However, if returns have non-elliptical distributions and investors do not have quadratic utility functions then they will care about all moments of returns (Rubinstein, 1973). Therefore, the failure of the CAPM is also perhaps due to omission of higher moments such as skewness and kurtosis. The intuition of preference for higher moments is straightforward. Risk averse investors will prefer positive to negative skewness because of a higher probability of greater than average outcomes. For kurtosis, risk averse investors will prefer lower to higher kurtosis to avoid extreme events. Therefore, returns should be negatively related to skewness and positively related to kurtosis.

Academic efforts of incorporating higher (co)moments in asset pricing models started in the 1970s. The first model of a higher-moment CAPM is proposed by Kraus and Litzenberger (1976), which is a three-moment CAPM including coskewness of individual assets with the market. In this model, the investors' utility function is expanded by a third order Taylor series. In this way, coskewness of an asset can enter the model naturally. The results of empirical tests of this model are mixed. Kraus and Litzenberger (1976) find some support for including coskewness but subsequent tests of Friend and Westerfield (1980) reject the model by using more data and subsample analysis. Lim (1989) uses GMM to test the model and finds coskewness is priced. Fang and Lai (1997) extend the model of Kraus and Litzenberger (1976) to incorporate cokurtosis of individual assets with the market.

More recently, the higher moment CAPM has been put into its conditional version. Harvey and Siddique (2000) give a comprehensive study of coskewness in a conditional framework. They develop a conditional three-moment CAPM in a stochastic discount factor form and use different measures of coskewness. They find that adding coskewness can increase the explanatory abilities of the CAPM and the Fama-French model significantly. Furthermore, they also find that the effects of size, book-to-market ratio and momentum are related to coskewness. For example, in the size decile portfolios, large portfolios have smaller returns and larger coskewness than small portfolios, i.e. mean returns and coskewness are negatively correlated, consistent with the theoretical prediction.

This study stimulated new interest in the higher-moment CAPM. Dittmar (2002) extends the model of Harvey and Siddique (2000) to the four-moment CAPM by a third-order Taylor series expansion and includes labour income in the market portfolio as suggested by Jannathan and Wang (1996). His empirical results support this model and reject both the traditional CAPM and the Fama-French three-factor model. Smith (2007) tests a conditional version of Kraus and Litzenberger's model and finds this model cannot be rejected but the conditional CAPM and Fama-French model are rejected. Furthermore, Chung et al. (2006) find that the Fama and French factors SMB and HML are no longer significant in explaining the cross-section of stock returns when 3-10 comoments of stock returns are included. They conclude that the factors SMB and HML may be related to higher moments of return distributions.

Although the higher-moment CAPM achieves success in empirical tests, the difficulties of modelling time-varying higher comoments limit its practical use. For example, the autoregressive conditional skewness approach of Harvey and Siddique (1999) is too complicated and is rarely used by any other researchers; the model of Smith (2007) is too parametric and is also of little use in practice. Therefore, the most popular method is the rolling window sample coskewness and cokurtosis estimation due to its simplicity. This method, however, may not give accurate estimates because it only uses past sample returns. Harvey and Siddique (2000) compare the performance of constant coskewness and rolling-window estimated coskewness and find constant coskewness model performs better, which is also consistent with the argument of Ghysels (1998) about the conditional CAPM. This drawback makes them assume constant coskewness in their study. Hence, it is important to model the conditional higher comoments appropriately in the cross-section test and practical use of the higher-moment CAPM.

In this chapter, I use daily returns within each month to estimate monthly higher comoments and then use these estimates in the cross-sectional regression. The purpose is to examine whether higher comoments can help explain the cross-section of stock returns in addition to betas of the market and other factors. The use of daily returns to estimate monthly higher comoments is in the line of recent literature on realized volatility (e.g. Andersen et al., 2003). The advantage of using daily returns is that it can make more accurate estimates than using monthly returns and is easy to compute and therefore can overcome the difficulties of estimating higher comoments encountered by previous studies. Recent empirical studies have already started employing high frequency data to model higher (co)moments. For example, Beine et al. (2004) use intra-day data to model realized skewness of exchange rates and Ang et al. (2006) use daily data to estimate coskewness and cokurtosis of stock returns.

I add coskewness and cokurtosis into different factor models to examine if they are significant in pricing stock returns. The factor models considered are the same as in Chapter 4: the CAPM, the Fama-French three-factor model and a four-factor model including the Fama-French three factors and a momentum factor. I use betas of different factors and higher comoments from contemporaneous measurement, in-sample forecasts and out-of-sample forecasts in the cross-sectional regressions, respectively. The results show that cokurtosis is a significantly priced risk both in-sample and out-of-sample. It is the only significant risk priced out-of-sample, indicating investors do care about the leptokurtosis of stock returns. Coskewness, however, is insignificantly priced both in-sample and out-of-sample.

The rest of this chapter is organized as follows. Section 5.2 introduces the higher-moment CAPM. The estimation of realized higher comoments is in section 5.3. Section 5.4 describes the data. The empirical results are reported in section 5.5. Section 5.6 makes conclusions.

5.2 The higher-moment CAPM

Kraus and Litzenberger (1976) expand the investors' utility function by a third-order Taylor series to incorporate skewness into the asset pricing model. In this section, I follow their method to incorporate kurtosis (see also Hwang and Satchell, 1999). It is assumed that investors invest an amount of W_t and have a utility function of wealth at period t+1, W_{t+1}, in each period t. To incorporate skewness and kurtosis of returns, I expand the function U at the point of expected wealth $E_t(W_{t+1})$ by a fourth-order Taylor series,

$$\begin{aligned} U(W_{t+1}) &= U(\bar{W}_{t+1}) + U^{'}(\bar{W}_{t+1})(W_{t+1}-\bar{W}_{t+1}) + \frac{1}{2}U^{''}(\bar{W}_{t+1})(W_{t+1}-\bar{W}_{t+1})^2 \\ &+ \frac{1}{3!}U^{'''}(\bar{W}_{t+1})(W_{t+1}-\bar{W}_{t+1})^3 + \frac{1}{4!}U^{''''}(\bar{W}_{t+1})(W_{t+1}-\bar{W}_{t+1})^4 + o(W_{t+1}^4) \end{aligned} \quad (5.1)$$

where $\bar{W}_{t+1} \equiv E_t\left(W_{t+1}\right)$ for the simplicity of denotations and $o(W_{t+1}^4)$ is in higher order than W_{t+1}^4. After taking an expectation of both sides of equation (5.1), we have

$$E_t(U(W_{t+1})) = U(\bar{W}_{t+1}) + \frac{1}{2}U^{''}(\bar{W}_{t+1})\sigma_{w,t}^2 + \frac{1}{3!}U^{'''}(\bar{W}_{t+1})\gamma_{w,t}^3 + \frac{1}{4!}U^{''''}(\bar{W}_{t+1})\theta_{w,t}^4 + o(W_{t+1}^4) \quad (5.2)$$

where

$$\begin{aligned} W_{t+1} &= w_f R_f + \sum_{i=1}^{N} w_i R_{i,t+1} \\ \sigma_{w,t} &= \sum_{i=1}^{N} w_i \beta_{ip} \sigma_{p,t} \\ \gamma_{w,t} &= \sum_{i=1}^{N} w_i \gamma_{ip} \gamma_{p,t} \\ \theta_{w,t} &= \sum_{i=1}^{N} w_i \theta_{ip,t} \theta_{p,t}, \end{aligned}$$

are the future value, conditional variance, skewness and kurtosis of wealth, respectively, and

$$\begin{aligned} & w_f + \sum_{i=1}^{N} w_i = 1, \\ & \beta_{ip,t} = \frac{E_t[(R_{i,t+1}-E_t(R_{i,t+1}))(R_{p,t+1}-E_t(R_{p,t+1}))]}{E_t[(R_{p,t+1}-E_t(R_{p,t+1}))^2]} = \frac{Cov_t(R_{i,t+1}, R_{p,t+1})}{Var_t(R_{p,t+1})}, \\ & \gamma_{ip,t} = \frac{E[(R_{i,t+1}-E_t(R_{i,t+1}))(R_{p,t+1}-E_t(R_{p,t+1}))^2]}{E[(R_{p,t+1}-E_t(R_{p,t+1}))^3]}, \\ & \theta_{ip,t} = \frac{E[(R_{i,t+1}-E_t(R_{i,t+1}))(R_{p,t+1}-E_t(R_{p,t+1}))^3]}{E[(R_{p,t+1}-E_t(R_{p,t+1}))^4]}. \end{aligned} \quad (5.3)$$

In this way, the investor's expected utility is a function of mean, variance, skewness and kurtosis

$$E_t(U) = f(\overline{W}_{t+1,} \sigma_{w,t}, \gamma_{w,t}, \theta_{w,t}) . \tag{5.4}$$

For the preference of the investors about the different moments, the usual assumptions of positive marginal utility of wealth and risk aversion imply

$$f_{\overline{W}_{t+1}} > 0 \text{ and } f_{\sigma_{w,t}} < 0 \tag{5.5}$$

where f_x is the partial derivative of f with respect to x. For skewness and kurtosis, we need to know the properties of higher order derivatives of the utility function. Kraus and Litzenberger (1976) prove $U''' > 0$ under the condition of non-decreasing absolute risk aversion because

$$\begin{aligned} &\frac{d(-\frac{U''}{U'})}{dW} \leq 0 \Rightarrow \frac{-U'U''' + (U'')^2}{(U')^2} \leq 0 \\ &\Rightarrow U''' \geq \frac{(U'')^2}{U'} > 0. \end{aligned} \tag{5.6}$$

Scott and Horvath (1980) prove that $U'''' < 0$ under assumption of investors being strictly consistent in preference direction[17] and positive third order derivative. So we have

$$f_{\gamma_{w,t}} > 0 \text{ and } f_{\theta_{w,t}} < 0 . \tag{5.7}$$

Equations (5.5) and (5.7) mean that investors prefer higher mean and skewness and

[17] Strictly consistent in preference direction is

$$U^n(W) > 0 \; for \; all \; W,$$
$$U^n(W) = 0 \; for \; all \; W, or$$
$$U^n(W) < 0 \; for \; all \; W.$$

An investor who is not strictly consistent has utility function either

$$U^n(W) \geq 0 for \; all \; W, or$$
$$U^n(W) \leq 0 for \; all \; W,$$

where U^n is the n-th order derivative of U.

lower variance and kurtosis.

Each investor maximizes his utility function $f(E(R_p),\sigma_p,\gamma_p,\theta_p)$ subject to the constraint $w_f + \sum_{i=1}^{N} w_i = 1$. The solution of this maximization problem is

$$E(R_i) - R_f = b_1\beta_{ip} + b_2\gamma_{ip} + b_3\theta_{ip} \qquad (5.8)$$

where

$$\begin{aligned} b_1 &= -\frac{f_{\sigma_p}}{f_{E(R_p)}}\sigma_p > 0, \\ b_2 &= -\frac{f_{\gamma_p}}{f_{E(R_p)}}\gamma_p \text{ which has an opposite sign as } \gamma_p, \\ b_2 &= -\frac{f_{\theta_p}}{f_{E(R_p)}}\theta_p > 0. \end{aligned} \qquad (5.9)$$

The derivation is given in Appendix A.2. In order to arrive at the higher-moment CAPM, we need to assume that all investors have the same belief of asset return distributions. Under this assumption, Cass and Stiglitz (1970) prove that a necessary and sufficient condition for investors to hold the same risky portfolio is that each investor's risk tolerance is a linear function of wealth with the same cautiousness for all investors, i.e. $-U_i^{'}/U_i^{"} = a_i + bW_i$. In this case, all investors will hold the same risky portfolio which is the market portfolio now. Therefore, equation (5.8) is changed to

$$E(R_i) - R_f = \lambda_1\beta_{im} + \lambda_2\gamma_{im} + \lambda_3\theta_{im} \qquad (5.10)$$

where β_{im} is the usual market beta in the conventional CAPM, γ_{im} and θ_{im} are coskewness and cokurtosis, respectively; λ_i, i=1,2,3 are the risk premiums and have similar expressions of b_i in equation (5.8) except replacing all the moments of portfolio p with corresponding moments of the market return.

Equation (5.10) is the four-moment CAPM. In the conventional CAPM, only market beta is included. In a three-moment CAPM, market beta and coskewness are included.

5.3 The estimation of higher comoments

Although the intuition and theoretical derivation of the higher-moment CAPM are both straightforward, the empirical estimation of higher (co)moments is difficult. This difficulty can be seen from the fact that there is little literature of techniques of estimating time-varying higher moments compared with the vast literature of volatility modelling. Harvey and Siddique (1999) propose an autoregressive conditional skewness model on the line of the GARCH model of volatility, but their model is too complicated and therefore of little practical use. Smith (2007) models different conditional (co)moments by linear functions of lagged macroeconomic variables. This method has so many parameters that it has the potential for over fitting the data. Due to the difficulty of using advanced techniques, the most popular method in modelling higher comoments is using the sample counterparts from previous data, such as sample coskewness and cokurtosis computed from past five years data (Kraus and Litzenberger, 1976; Chung et al., 2006).

In this chapter, I use daily returns within each month to estimate monthly coskewness and cokurtosis of individual stocks on the line of the recent literature of realized volatility (Andersen et al., 2003; Barndorff-Nielsen and Shephard, 2004). This method utilizes the rich information content in high frequency data and is easy to use. More important, it is more accurate than using monthly returns. Recent work of Ang et al. (2006) also uses daily data to compute coskewness and cokurtosis.

In the empirical test of the higher-moment CAPM, coskewness is defined as Ang et al. (2006),

$$\text{coskew}_{i,t} = \frac{E_t[(r_{i,t+1} - \mu_{i,t})(r_{m,t+1} - \mu_{m,t})^2]}{\sqrt{\text{var}_t(r_{i,t+1})}\,\text{var}_t(r_{m,t+1})}, \tag{5.11}$$

This definition is also similar to Harvey and Siddique (2000). It has two advantages over equation (5.3). First, this coskewness measure should always have a negative coefficient in the cross-sectional test while λ_2 in equation (5.10) has an opposite sign of γ_m, market skewness, which is very difficult to test in empirical work because γ_m is very difficult to estimate from an ex-ante view. Second, the denominator of equation (5.11) only has the second moment of returns, the variance, which can be estimated more accurately than the third moment in the denominator of equation (5.3). Cokurtosis is defined in a similar way,

$$\text{cokurt}_{i,t} = \frac{E_t[(r_{i,t+1} - \mu_{i,t})(r_{m,t+1} - \mu_{m,t})^3]}{\sqrt{\text{var}_t(r_{i,t+1})}\,\text{var}_t(r_{m,t+1})^{3/2}}. \tag{5.12}$$

The empirical estimates of coksewness and cokurtosis are the sample counterparts of equation (5.11) and (5.12) of daily returns within each month,

$$\widehat{\cos\text{kew}}_{i,t} = \frac{\frac{1}{J_t}\sum_{j=1}^{J_t}(r_{i,j} - \overline{r}_{i,t})(r_{m,j} - \overline{r}_{m,t})^2}{\sqrt{\frac{1}{J_t}\sum_{j=1}^{J}(r_{i,j} - \overline{r}_{i,t})^2}\,(\frac{1}{J_t}\sum_{j=1}^{J}(r_{m,j} - \overline{r}_{m,t})^2)} \tag{5.13}$$

and

$$\widehat{\text{co}\,\text{kurt}}_{i,t} = \frac{\frac{1}{J_t}\sum_{j=1}^{J_t}(r_{i,j} - \overline{r}_{i,t})(r_{m,j} - \overline{r}_{m,t})^3}{\sqrt{\frac{1}{J_t}\sum_{j=1}^{J}(r_{i,j} - \overline{r}_{i,t})^2}\,(\frac{1}{J_t}\sum_{j=1}^{J}(r_{m,j} - \overline{r}_{m,t})^2)^{3/2}}, \tag{5.14}$$

where J_t is the number of trading days within month *t*, $r_{i,j}$ and $r_{m,j}$ are the returns of asset *i* and market of the *j*th trading day of month *t*, respectively, and $\overline{r}_{i,t}$ and $\overline{r}_{m,t}$ are sample means of daily returns of asset *i* and market of month *t*, respectively. [18]

The estimated coskewness and cokurtosis are ex-post measures. To test if they can

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[18] The reason to use sample estimates instead of the method of estimating realized betas in Chapter 4 is that there is no theory of the convergence of realized higher moments. Recent theory on realized power variation (Barndorff-Niesen and Sheppard, 2003) relates $\sum_i |r_i|^n$ to $\int \sigma^r(s)ds$. Clearly, $\sum_i r_i^4$ does not converge to kurtosis. Furthermore, this theory does not apply to skewness because it's the power of raw returns instead of the absolute returns. Therefore, using daily returns to compute monthly higher comoments is an explorative method.

explain the cross-section of returns from an ex-ante view I use forecasted coskewness and cokurtosis. Similar to Chapter 3, forecasts are based on an AR(1) model with lagged macroeconomic variables and robustness checks are done by using other models in Appendix B.

In the empirical test, I add higher comoments to betas of different factors to see if they can help explain the cross-section of stock returns. Factor pricing models are those considered in Chapter 4, i.e. the CAPM, the Fama-French three-factor model and a four-factor model with Fama-French three factors and a momentum factor. Betas of different factors are estimated the same as in Chapter 4. Then the cross-sectional regressions are run on betas, higher comoments and firm-level variables. The most general form can be written as,

$$r_{i,t} = \alpha_t + \lambda_{1,t}\hat{\beta}_{i,t} + \lambda_{2,t}\hat{s}_{i,t} + \lambda_{3,t}\hat{h}_{i,t} + \lambda_{4,t}\hat{m}_{i,t} + \lambda_{5,t}\widehat{\text{coskew}}_{i,t} + \lambda_{6,t}\widehat{\text{cokurt}}_{i,t} + \sum_{j=1}^{J} c_{j,t} Z_{i,j,t-1} + u_t \qquad (5.15)$$

where $\hat{\beta}_{i,t}$, $\hat{s}_{i,t}$, $\hat{h}_{i,t}$ and $\hat{m}_{i,t}$ are defined the same as in Chapter 4. The CAPM does not have $\hat{s}_{i,t}$, $\hat{h}_{i,t}$ and $\hat{m}_{i,t}$, the Fama-French model does not have $\hat{m}_{i,t}$ and the four-factor model has all of the items in equation (5.15).

5.4 Empirical results

The data used is the same as in Chapter 4 and is screened in the same way.

5.4.1 Contemporaneous betas and higher comoments

Table 5.1 reports the cross-sectional regression results of contemporaneous market beta and higher comoments. I first regress returns on each comoment separately to test if comoment is significantly priced. The reason is that the multicolinearity between estimated comoments can distort the results if they are pooled together. For example, the correlation between market beta and cokurtosis is 0.68. Panel A reports the results of the whole sample. The first column is the results using only realized market beta in the cross-sectional regressions. As indicated by the CAPM, realized beta has a significantly positive coefficient of 0.844. The adjusted R^2 is 13.816% which is relatively high for a single explanatory variable. The second and third columns report the results of coskewness and cokurtosis, respectively. Both higher comoments have the right sign, negative for coskewness (-0.427) and positive for cokurtosis (0.542); but only cokurtosis's coefficient is significant. The adjusted R^2 is only 1.925% for coskewness and much higher for cokurtosis (5.391%). The last column reports the results of pooling all three comoments together with a constant and firm-level variables. The effect of multicolinearity is obvious. The coefficient on beta is increased dramatically to 1.587 and cokurtosis now has a negative coefficient (-1.815). Coskewness now has a significantly negative coefficient of -0.675. All three firm-level variables have significant coefficients with the right sign.

To check the robustness of the results, I divide the whole sample into two subsamples. The first subsample contains only stocks listed in NYSE and AMEX while the second contains only stocks in NASDAQ. Panel B and panel C report the results of the two subsamples, respectively. The overall results are similar to panel. The magnitudes of most coefficients are larger in the second subsample than the first subsample but the adjusted R^2 are lower in the second subsample.

Similar to realized betas in Chapter 4, realized higher comoments are ex-post measures. To test the models from an ex-ante view, we need to use forecasted comoments. The next two subsections deal with this problem by using in-sample and out-of-sample forecasted higher comoments, respectively.

Table 5.1 Fama-MacBeth Regression Results with Contemporaneous Realized Betas and Higher Comoments

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of contemporaneous monthly realized betas and higher comoments. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. β is realized market beta. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative return from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks				
α				0.593
				(0.204)
β	0.844			1.587
	(0.208)			(0.247)
coskew		-0.427		-0.675
		(0.296)		(0.217)
cokurt			0.542	-1.815
			(0.168)	(0.259)
SIZE				-0.142
				(0.043)
BM				0.345
				(0.052)
RET2_12				0.680
				(0.111)
$\bar{R}^2$(%)	13.816	1.925	5.391	12.956

Panel B: NYSE and AMEX stocks only				
α				0.463
				(0.178)
β	0.818			1.513
	(0.202)			(0.243)
coskew		-0.107		-0.663
		(0.298)		(0.191)
cokurt			0.564	-1.515
			(0.156)	(0.219)
SIZE				-0.045
				(0.038)
BM				0.294
				(0.049)
RET2_12				0.857
				(0.125)
$\bar{R}^2$(%)	15.729	2.559	6.832	14.217

Table 5.1 (*continued*)

Panel C: NASDAQ stocks only				
α				0.913
				(0.264)
β	1.072			1.944
	(0.264)			(0.326)
coskew		-0.484		-0.570
		(0.353)		(0.313)
cokurt			0.579	-2.375
			(0.212)	(0.372)
SIZE				-0.425
				(0.065)
BM				0.340
				(0.065)
RET2_12				0.446
				(0.099)
$\overline{R}^2$(%)	10.472	1.220	3.354	10.849

5.4.2 In-sample forecasted betas and higher comoments

Table 5.2 reports the cross-sectional regression results of in-sample forecasted betas and higher comoments. Panel A gives the results of the whole sample. Column 2 is the results of the four-moment CAPM. Market beta has a significantly positive coefficient of 0.928, similar to that in Table 4.3, suggesting beta is a robust risk. Coskewness has the wrong sign and insignificantly different from zero. Cokurtosis' coefficient is 0.644 and significantly positive. The coefficients on the three firm-level variables are all similar to those in Table 4.3. The adjusted R^2 is 5.238%, only slightly greater than the CAPM. The results suggest that cokurtosis is a significantly priced risk but higher comoments cannot help explain the cross-section of stock returns. The third column is the results of the Fama-French model with higher comoments. The coefficients on β, s and h are all similar to the Fama-French model without higher comoments in Table 4.3. β has a significantly positive coefficient but s and h have negative and insignificant coefficients. Coskewness still has the wrong sign as in column 2 and

Table 5.2 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas and Higher Comoments

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas and higher comoments from an AR(1) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks			
	CAPM	FF3F	4F
α	-0.156	-0.250	-0.126
	(0.166)	(0.154)	(0.168)
β	0.928	0.863	0.731
	(0.197)	(0.137)	(0.108)
s		-0.017	-0.042
		(0.085)	(0.074)
h		-0.119	-0.102
		(0.081)	(0.062)
m			0.679
			(0.068)
coskew	0.038	0.157	0.187
	(0.267)	(0.264)	(0.269)
cokurt	0.644	0.719	0.777
	(0.092)	(0.089)	(0.099)
SIZE	-0.347	-0.346	-0.354
	(0.057)	(0.049)	(0.049)
BM	0.343	0.340	0.345
	(0.054)	(0.051)	(0.057)
RET2_12	0.612	0.566	0.515
	(0.123)	(0.120)	(0.124)
$\bar{R}^2$(%)	5.238	6.227	6.211

Table 5.2 (*continued*)

Panel B: NYSE and AMEX stocks only			
α	-0.006	-0.097	-0.013
	(0.160)	(0.147)	(0.154)
β	0.575	0.684	0.632
	(0.194)	(0.154)	(0.127)
s		-0.181	-0.238
		(0.086)	(0.077)
h		-0.081	-0.094
		(0.088)	(0.067)
m			0.709
			(0.075)
coskew	0.051	0.175	0.198
	(0.315)	(0.312)	(0.315)
cokurt	0.512	0.567	0.614
	(0.101)	(0.094)	(0.103)
SIZE	-0.197	-0.222	-0.243
	(0.049)	(0.043)	(0.044)
BM	0.265	0.258	0.272
	(0.054)	(0.049)	(0.052)
RET2_12	0.778	0.722	0.647
	(0.139)	(0.136)	(0.138)
$\bar{R}^2(\%)$	5.930	7.027	7.110
Panel C: NASDAQ stocks only			
α	-0.090	-0.321	-0.075
	(0.215)	(0.201)	(0.225)
β	1.579	1.228	0.972
	(0.243)	(0.153)	(0.110)
s		0.228	0.221
		(0.104)	(0.090)
h		-0.112	-0.094
		(0.098)	(0.071)
m			0.710
			(0.080)
coskew	0.076	0.176	0.174
	(0.347)	(0.338)	(0.343)
cokurt	0.586	0.670	0.697
	(0.113)	(0.116)	(0.121)
SIZE	-0.674	-0.644	-0.628
	(0.084)	(0.076)	(0.072)
BM	0.389	0.379	0.359
	(0.064)	(0.064)	(0.071)
RET2_12	0.437	0.412	0.354
	(0.109)	(0.107)	(0.111)
$\bar{R}^2(\%)$	3.541	4.684	4.750

cokurtosis still has a significantly positive coefficient (0.719). The firm-level variables all have significant coefficients with the expected signs. The last column presents the results of the four-factor model with higher comoments. The coefficients on *β, s, h* and *m* are also similar to those of the four-factor model without higher comoments. *β* and *m* have significantly positive coefficients while *s* and *h* have negative coefficients. Coskewness and cokurtosis have similar coefficients to columns 2 and 3. Firm-level variables all have significant coefficients with the expected signs but the coefficient on RET2_12 is reduced to 0.515.

Panel B and panel C report the results of the first and second subsamples, respectively. Statistical inferences are generally the same as the whole sample: *β* and cokurtosis have significantly positive coefficients and *h* and coskewness are insignificant. The exception is that *s* is significant in NASDAQ stocks (panel C). The magnitudes of the parameters in panel B are generally smaller than panel C. Firm-level variables are all significant with the expected signs with panel C having larger magnitudes (except RET2_12). The patterns are similar to Table 4.3 in chapter 4. The firm-level variables effects of size and BM are more prominent in NASDAQ stocks, the second subsample.

5.4.3 Out-of-sample forecasted betas and higher comoments

This subsection examines if out-of-sample forecasted higher comoments can help explain the cross-section of stock returns. The one-step ahead out-of-sample forecasts are generated by 60-month rolling window estimation. The results are reported in Table 5.3. Panel A is the results of using the whole sample. The second column reports the results of the four-moment CAPM. Comparing with Table 5.2, the coefficient on beta becomes negative and insignificant. The coefficient on coskewness still has the wrong sign and remains insignificant. Cokurtosis, however, remains significantly positive. The third column reports the results of the Fama-French model augmented by higher comoments. The coefficient on beta is still negative. Both of *s* and *h* are negative and insignificant. Coskewness also has the wrong sign and is insignificant. Cokurtosis, however, remains significantly positive although the coefficient is reduced to 0.169 from 0.382. The last column reports the results of the four-factor model augmented by higher comoments. Among the systematic risks, four betas and two higher comoments, only cokurtosis is significantly positive at 10% level. The three firm-level variables are highly significant and the R^2 is very low, only around 3.5%, under all of the three models.

Table 5.3 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas and Higher Comoments

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of out-of-sample forecasted monthly realized betas and higher comoments from an AR(1) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month t-12 to month t-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks			
	CAPM	FF3F	4F
α	0.814	0.780	0.772
	(0.236)	(0.239)	(0.242)
β	-0.244	-0.028	-0.001
	(0.099)	(0.025)	(0.017)
s		-0.012	-0.002
		(0.014)	(0.011)
h		-0.006	0.003
		(0.012)	(0.010)
m			0.013
			(0.012)
coskew	0.093	0.090	0.099
	(0.117)	(0.123)	(0.123)
cokurt	0.382	0.169	0.144
	(0.064)	(0.077)	(0.080)
SIZE	-0.227	-0.215	-0.212
	(0.051)	(0.054)	(0.054)
BM	0.295	0.319	0.324
	(0.067)	(0.071)	(0.071)
RET2_12	0.595	0.594	0.591
	(0.137)	(0.141)	(0.141)
$\bar{R}^2$(%)	3.755	3.453	3.457

Table 5.3 (*continued*)

Panel B: NYSE and AMEX stocks only			
α	0.700	0.684	0.680
	(0.209)	(0.214)	(0.216)
β	-0.227	-0.042	-0.026
	(0.113)	(0.030)	(0.022)
s		-0.017	-0.004
		(0.017)	(0.015)
h		0.014	0.016
		(0.014)	(0.011)
m			0.001
			(0.015)
coskew	0.168	0.168	0.173
	(0.127)	(0.129)	(0.129)
cokurt	0.295	0.111	0.090
	(0.068)	(0.074)	(0.077)
SIZE	-0.128	-0.114	-0.111
	(0.046)	(0.048)	(0.048)
BM	0.239	0.263	0.268
	(0.061)	(0.064)	(0.064)
RET2_12	0.738	0.738	0.734
	(0.153)	(0.157)	(0.157)
$\overline{R}^2$(%)	4.638	4.259	4.274
Panel C: NASDAQ stocks only			
α	1.156	1.101	1.111
	(0.280)	(0.285)	(0.288)
β	-0.160	0.019	0.021
	(0.111)	(0.033)	(0.022)
s		0.025	0.019
		(0.018)	(0.015)
h		-0.035	-0.014
		(0.016)	(0.013)
m			0.025
			(0.016)
coskew	0.030	0.016	0.023
	(0.174)	(0.177)	(0.177)
cokurt	0.440	0.251	0.247
	(0.087)	(0.107)	(0.109)
SIZE	-0.451	-0.449	-0.448
	(0.070)	(0.072)	(0.072)
BM	0.231	0.246	0.246
	(0.077)	(0.082)	(0.082)
RET2_12	0.538	0.543	0.544
	(0.106)	(0.111)	(0.111)
$\overline{R}^2$(%)	2.487	2.249	2.257

Panel B and panel C report the results from the two subsamples, respectively. The overall results are similar to the whole sample. The exception is cokurtosis. In panel B, cokurtosis is significantly positive only in the higher-moment CAPM (column 2) but is insignificant under the Fama-French model and the four-factor model. In panel C, cokurtosis is significantly positive under all of the three models. The results suggest that kurtosis risk is more prominent in the NASDAQ market.

Overall, the results show that cokurtosis is a robust systematic risk under different models and markets and is more prominent in the NASDAQ market. However, out-of-sample forecasted betas and coskewness are insignificant. Furthermore, higher comoments cannot help explain the effects of size, BM and past returns.

5.5 Conclusion

If investors have a preference over higher moments than mean and variance and asset returns do not have an elliptical distribution, those higher moments should be priced in the cross-section according to the higher-moment CAPM. This chapter examines whether realized higher comoments are significantly priced systematic risks and whether they can help explain the effects of firm-level variables.

I use daily returns within each month to estimate realized coskewness and cokurtosis and then use time series techniques to model them. The results show that realized cokurtosis is significantly priced both in-sample and out-of-sample but coskewness is insignificant and has the wrong sign in cases of in-sample and out-of-sample forecasts. The firm-level variables, i.e. size, book-to-market ratio and past returns, remain highly significant with expected signs after higher comoments added in the cross-sectional regressions. This suggests that the higher comoments cannot help explain the effects of firm-level variables.

Contemporaneous cokurtosis is highly significant but the coefficient of out-of-sample forecasted cokurtosis is reduced significantly. Therefore, similar to the modelling of betas in Chapter 4, possible improvement may be achieved by using more sophisticated nonlinear models, which will be done in the future.

Cokurtosis is an important risk although it cannot help explain the effects of firm-level variables. The implications are important for portfolio optimization and risk management. Traditional portfolio and risk management typically focus on variance but the results of this chapter show it is not enough. To obtain a complete view of a portfolio's risks, it is necessary to include kurtosis in the risk management of portfolios. This is useful implications for practical portfolio management.

Appendix 5

***A*. Derivation of the equation 5.8 and 5.9**

***A*.1 Proof of $\sum_{i=1}^{N} w_i\beta_{ip}$, $\sum_{i=1}^{N} w_i\gamma_{ip}$ and $\sum_{i=1}^{N} w_i\theta_{ip}$ are equal to 1**

$$
\begin{aligned}
\sum_{i=1}^{N} w_i\beta_{ip} &= \sum_{i=1}^{N} w_i \frac{E[(R_i - E(R_i))(R_p - E(R_p))]}{E[(R_p - E(R_p))^2]} \\
&= \frac{E[\sum w_i(R_i - E(w_iR_i))(R_p - E(R_p))]}{E[(R_p - E(R_p))^2]} \\
&= \frac{E[(R_p - E(R_p))(R_p - E(R_p))]}{E[(R_p - E(R_p))^2]} \\
&= 1.
\end{aligned}
\qquad \text{(A5.1)}
$$

The proof of $\sum_{i=1}^{N} w_i\gamma_{ip}$ and $\sum_{i=1}^{N} w_i\theta_{ip}$ equal to 1 is similar.

***A*.2 The Derivation of equation (5.8) and (5.9)**

The maximization problem is

$$\text{Max} \quad f(E(R_p), \sigma_p, \gamma_p, \theta_p) \quad \text{subject to the constraint } w_f + \sum_{i=1}^{N} w_i = 1.$$

Then the Lagrangian function is

$$L = f(E(R_p), \sigma_p, \gamma_p, \theta_p) - \lambda(w_f + \sum_{i=1}^{N} w_i - 1). \quad \text{(A5.2)}$$

The first-order condition with respect to w_f and w_i is

$$\begin{aligned} &\frac{\partial f}{\partial w_f} = f_{E(R_p)}(1+R_f) - \lambda = 0, \\ &\frac{\partial f}{\partial w_i} = f_{E(R_p)}(1+R_i) + f_{\sigma_p}\beta_{ip}\sigma_p + f_{\gamma_p}\gamma_{ip}\gamma_p + f_{\theta_p}\theta_{ip}\theta_p - \lambda = 0. \end{aligned} \quad \text{(A5.3)}$$

Combine the first order conditions above to get equation (5.8) and (5.9)

B. Alternative models of realized betas and higher comoments

In this Appendix, I examine whether the results are robust to different models of realized betas and higher comoments. Similar to chapter 4, I add more lags of realized betas or higher-comoments and an additional moving average item. Specifically, the models considered are AR(2), AR(3) and ARMA(1,1) models. Each model also includes lagged macroeconomic variables like the AR(1) model in the main text.

B.1 In-sample forecasted betas and higher comoments

Table A5.1 reports the results of in-sample forecasted betas and higher comoments from an AR(2) model with lagged macroeconomic variables. Comparing with Table 5.2 in the main text, the results are very similar. In panel A, results of the whole sample, market beta and cokurtosis have significantly positive coefficients under all of the three models. Betas of the Fama-French factors SMB and HML, *s* and *h*, are insignificant while beta of the momentum factor has a significantly positive coefficient. Coskewness has the wrong sign and is insignificant. Panel B and panel C report the results of the two subsamples, respectively. Overall, the statistical inferences are similar between the two subsamples. Market beta and cokurtosis are highly significant in both subsamples but NASDAQ stocks have higher risk premiums. The exception is beta of SMB, *s,* which is significantly priced in NASDAQ market. This makes sense because NASDAQ stocks are smaller than NYSE and AMEX stocks on average. The results of the AR(3) are reported in table A5.2 which are very similar to the AR(2) model.

Table A5.1 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas and Higher Comoments from an AR(2) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas and higher comoments from an AR(2) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and Panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. The *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks

	CAPM	FF3F	4F
α	-0.096	-0.220	-0.082
	(0.168)	(0.158)	(0.173)
β	0.894	0.790	0.641
	(0.188)	(0.126)	(0.097)
s		0.077	0.050
		(0.080)	(0.070)
h		-0.075	-0.054
		(0.073)	(0.054)
m			0.599
			(0.063)
coskew	0.024	0.132	0.129
	(0.247)	(0.244)	(0.248)
cokurt	0.590	0.643	0.702
	(0.086)	(0.083)	(0.094)
SIZE	-0.340	-0.324	-0.330
	(0.056)	(0.048)	(0.049)
BM	0.342	0.340	0.339
	(0.055)	(0.052)	(0.058)
RET2_12	0.613	0.571	0.521
	(0.123)	(0.122)	(0.125)
$\bar{R}^2$(%)	5.222	6.289	6.256

Table A5.1 (*continued*)

Panel B: NYSE and AMEX Stocks Only			
α	0.050	-0.061	0.022
	(0.162)	(0.149)	(0.157)
β	0.581	0.633	0.577
	(0.189)	(0.142)	(0.116)
s		-0.084	-0.128
		(0.079)	(0.072)
h		-0.017	-0.053
		(0.081)	(0.063)
m			0.624
			(0.071)
coskew	0.189	0.221	0.209
	(0.290)	(0.290)	(0.293)
cokurt	0.438	0.472	0.524
	(0.093)	(0.086)	(0.096)
SIZE	-0.193	-0.197	-0.215
	(0.049)	(0.042)	(0.043)
BM	0.265	0.254	0.270
	(0.054)	(0.050)	(0.053)
RET2_12	0.776	0.721	0.654
	(0.139)	(0.137)	(0.139)
$\bar{R}^2$(%)	5.922	7.063	7.167

Panel C: NASDAQ Stocks Only			
α	-0.012	-0.253	0.002
	(0.220)	(0.210)	(0.234)
β	1.483	1.073	0.822
	(0.229)	(0.139)	(0.098)
s		0.314	0.280
		(0.098)	(0.083)
h		-0.072	-0.040
		(0.088)	(0.063)
m			0.631
			(0.074)
coskew	0.001	0.171	0.156
	(0.317)	(0.308)	(0.313)
cokurt	0.554	0.632	0.671
	(0.111)	(0.114)	(0.118)
SIZE	-0.659	-0.625	-0.607
	(0.082)	(0.074)	(0.071)
BM	0.382	0.371	0.346
	(0.066)	(0.066)	(0.072)
RET2_12	0.442	0.424	0.369
	(0.110)	(0.109)	(0.113)
$\bar{R}^2$(%)	3.534	4.845	4.892

Table A5.2 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas and Higher Comoments from an AR(3) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas and higher comoments from an AR(3) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks			
	CAPM	FF3F	4F
α	-0.023	-0.151	-0.026
	(0.170)	(0.161)	(0.176)
β	0.893	0.767	0.626
	(0.183)	(0.119)	(0.091)
s		0.121	0.089
		(0.077)	(0.065)
h		-0.067	-0.033
		(0.069)	(0.051)
m			0.550
			(0.061)
coskew	-0.131	-0.091	-0.075
	(0.232)	(0.230)	(0.234)
cokurt	0.477	0.515	0.575
	(0.080)	(0.077)	(0.087)
SIZE	-0.329	-0.305	-0.312
	(0.055)	(0.047)	(0.048)
BM	0.346	0.343	0.338
	(0.055)	(0.053)	(0.059)
RET2_12	0.611	0.566	0.527
	(0.124)	(0.122)	(0.125)
$\bar{R}^2$(%)	5.225	6.373	6.352

Table A5.2 (*continued*)

Panel B: NYSE and AMEX Stocks Only			
α	0.090	-0.021	0.061
	(0.161)	(0.149)	(0.159)
β	0.602	0.646	0.566
	(0.183)	(0.136)	(0.106)
s		-0.052	-0.085
		(0.075)	(0.067)
h		-0.019	-0.040
		(0.078)	(0.059)
m			0.575
			(0.068)
coskew	-0.022	-0.036	-0.053
	(0.269)	(0.267)	(0.270)
cokurt	0.348	0.363	0.425
	(0.086)	(0.080)	(0.090)
SIZE	-0.187	-0.184	-0.200
	(0.048)	(0.041)	(0.043)
BM	0.269	0.259	0.267
	(0.054)	(0.050)	(0.054)
RET2_12	0.777	0.727	0.663
	(0.140)	(0.137)	(0.139)
$\bar{R}^2$(%)	5.938	7.092	7.177
Panel C: NASDAQ Stocks Only			
α	0.067	-0.180	0.071
	(0.223)	(0.214)	(0.238)
β	1.431	1.008	0.783
	(0.223)	(0.130)	(0.094)
s		0.341	0.293
		(0.094)	(0.079)
h		-0.056	-0.025
		(0.083)	(0.060)
m			0.579
			(0.072)
coskew	-0.071	0.008	0.026
	(0.303)	(0.299)	(0.306)
cokurt	0.467	0.548	0.574
	(0.103)	(0.104)	(0.108)
SIZE	-0.645	-0.611	-0.595
	(0.081)	(0.073)	(0.070)
BM	0.380	0.368	0.342
	(0.066)	(0.067)	(0.074)
RET2_12	0.439	0.427	0.377
	(0.111)	(0.109)	(0.113)
$\bar{R}^2$(%)	3.567	5.022	5.107

Table A5.3 Fama-MacBeth Regression Results with in-sample Forecasted Realized Betas and Higher Comoments from an ARMA(1,1) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of in-sample forecasted monthly realized betas and higher comoments from an ARMA(1,1) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of the book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks			
	CAPM	FF3F	4F
α	0.016	-0.128	0.027
	(0.167)	(0.157)	(0.174)
β	0.871	0.706	0.592
	(0.178)	(0.111)	(0.088)
s		0.241	0.165
		(0.080)	(0.065)
h		-0.045	-0.028
		(0.064)	(0.049)
m			0.616
			(0.062)
coskew	-0.264	-0.160	-0.213
	(0.184)	(0.185)	(0.185)
cokurt	0.485	0.455	0.500
	(0.069)	(0.067)	(0.082)
SIZE	-0.334	-0.289	-0.302
	(0.055)	(0.047)	(0.047)
BM	0.343	0.340	0.326
	(0.054)	(0.053)	(0.058)
RET2_12	0.600	0.526	0.493
	(0.124)	(0.121)	(0.125)
$\bar{R}^2$(%)	5.316	6.808	6.931

Table A5.3 (*continued*)

Panel B: NYSE and AMEX Stocks Only			
α	0.112	-0.049	0.051
	(0.156)	(0.146)	(0.154)
β	0.523	0.598	0.536
	(0.176)	(0.123)	(0.101)
s		0.045	0.032
		(0.079)	(0.064)
h		-0.010	-0.054
		(0.071)	(0.057)
m			0.620
			(0.069)
coskew	-0.303	-0.256	-0.229
	(0.226)	(0.230)	(0.233)
cokurt	0.425	0.388	0.415
	(0.076)	(0.076)	(0.087)
SIZE	-0.192	-0.174	-0.183
	(0.049)	(0.041)	(0.042)
BM	0.260	0.251	0.268
	(0.054)	(0.050)	(0.053)
RET2_12	0.770	0.681	0.614
	(0.139)	(0.137)	(0.139)
$\bar{R}^2(\%)$	6.010	7.383	7.520

Panel C: NASDAQ Stocks Only			
α	0.111	-0.143	0.147
	(0.222)	(0.210)	(0.238)
β	1.442	0.935	0.739
	(0.217)	(0.119)	(0.094)
s		0.465	0.353
		(0.096)	(0.081)
h		-0.021	0.004
		(0.077)	(0.058)
m			0.635
			(0.072)
coskew	-0.149	-0.060	-0.194
	(0.229)	(0.229)	(0.229)
cokurt	0.421	0.444	0.474
	(0.089)	(0.082)	(0.092)
SIZE	-0.647	-0.598	-0.592
	(0.079)	(0.071)	(0.069)
BM	0.381	0.361	0.313
	(0.065)	(0.067)	(0.072)
RET2_12	0.427	0.393	0.356
	(0.111)	(0.108)	(0.111)
$\bar{R}^2(\%)$	3.677	5.654	5.970

Table A5.3 reports the results of the ARMA(1,1) model. The coefficients of market beta and cokurtosis are similar to AR models but beta on SMB, *s*, now becomes significantly positive. If we further examine the results of the two subsamples (panel B and panel C), we can see that the significance of *s* is due to the NASDAQ stocks, similar to the results of the AR models because *s* is only significant in the NASDAQ stocks, the second subsample. Coskewness has the right sign this time but remains insignificant under all the models within different samples.

Overall, the results of the in-sample forecasted betas and higher comoments show that market beta and cokurtosis are significantly priced risks and beta on SMB, *s*, is only significant in NASDAQ stocks. Coskewness is not significant under different models and within different samples.

***B*.2 Out-of-sample forecasted betas and higher comoments**

Table A5.4 reports the results of out-of-sample forecasted betas from an AR(2) model. The results are similar to the Table 5.3. In panel A, market beta has an insignificant coefficient in all of the three models. Coskewness has the wrong sign (positive) and insignificant. Cokurtosis is significantly positive under the higher-moment CAPM (column 2) but insignificant under multi-factor models (columns 3 and 4). The Fama-French factors SMB and HML and the momentum factor WML are all insignificant. Panel B and panel C report the results of the two subsamples. It can be seen that cokurtosis effect is more prominent in the NASDAQ stocks. In panel B, cokurtosis is only significant under the CAPM, similar to the whole sample. But in panel C, it is significant at 10% level under the multi-factor models. Table A5.5 reports the results of the AR(3) model, which are similar to the AR(2) model but cokurtosis is no longer significant under multi-factor models.

The results of the ARMA(1,1) model are reported in Table A5.6. In panel A, coskewness has negative coefficient this time but remains insignificant but cokurtosis is highly significant. Comparing the results in panel B and panel C, it can be seen that cokurtosis is significant in panel C only. Therefore, cokurtosis risk is mainly significant in NASDAQ stocks. Other estimation results are similar to AR models.

Table A5.4 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas and Higher Comoments from an AR(2) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of out-of-sample forecasted monthly realized betas and higher comoments from an AR(2) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks			
	CAPM	FF3F	4F
α	0.841	0.802	0.804
	(0.239)	(0.244)	(0.247)
β	-0.159	-0.011	0.005
	(0.082)	(0.021)	(0.014)
s		-0.005	-0.001
		(0.012)	(0.010)
h		-0.003	0.000
		(0.010)	(0.008)
m			0.006
			(0.010)
coskew	0.092	0.086	0.092
	(0.101)	(0.106)	(0.106)
cokurt	0.239	0.103	0.083
	(0.053)	(0.068)	(0.070)
SIZE	-0.211	-0.202	-0.200
	(0.052)	(0.053)	(0.053)
BM	0.303	0.322	0.323
	(0.068)	(0.071)	(0.072)
RET2_12	0.593	0.588	0.588
	(0.138)	(0.141)	(0.142)
$\bar{R}^2$(%)	3.673	3.403	3.406

Table A5.4 *(continued)*

Panel B: NYSE and AMEX Stocks Only			
α	0.715	0.696	0.695
	(0.212)	(0.219)	(0.221)
β	-0.105	-0.017	-0.010
	(0.099)	(0.026)	(0.019)
s		-0.005	0.003
		(0.015)	(0.013)
h		0.009	0.009
		(0.012)	(0.010)
m			(0.003)
			(0.013)
coskew	0.200	0.205	0.192
	(0.110)	(0.111)	(0.112)
cokurt	0.151	0.065	0.052
	(0.058)	(0.065)	(0.067)
SIZE	-0.115	-0.107	-0.104
	(0.046)	(0.047)	(0.048)
BM	0.250	0.263	0.268
	(0.061)	(0.064)	(0.064)
RET2_12	0.740	0.737	0.734
	(0.154)	(0.158)	(0.158)
$\bar{R}^2$(%)	4.578	4.188	4.225
Panel C: NASDAQ Stocks Only			
α	1.192	1.135	1.149
	(0.284)	(0.290)	(0.293)
β	-0.094	0.021	0.027
	(0.091)	(0.027)	(0.019)
s		0.022	0.016
		(0.015)	(0.013)
h		-0.020	-0.015
		(0.014)	(0.011)
m			0.024
			(0.013)
coskew	0.023	0.001	0.029
	(0.150)	(0.153)	(0.152)
cokurt	0.279	0.160	0.153
	(0.072)	(0.091)	(0.092)
SIZE	-0.433	-0.436	-0.436
	(0.070)	(0.071)	(0.071)
BM	0.233	0.244	0.240
	(0.078)	(0.083)	(0.083)
RET2_12	0.536	0.543	0.545
	(0.106)	(0.112)	(0.112)
$\bar{R}^2$(%)	2.416	2.225	2.226

Table A5.5 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas and Higher Comoments from an AR(3) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of out-of-sample forecasted monthly realized betas and higher comoments from an AR(3) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with the exception that book-to-market ratios greater than the 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks

	CAPM	FF3F	4F
α	0.853	0.818	0.822
	(0.242)	(0.249)	(0.251)
β	-0.131	-0.003	0.011
	(0.072)	(0.019)	(0.013)
s		-0.007	-0.009
		(0.010)	(0.009)
h		0.001	-0.003
		(0.009)	(0.007)
m			0.013
			(0.010)
coskew	0.065	0.075	0.043
	(0.092)	(0.097)	(0.097)
cokurt	0.171	0.065	0.051
	(0.046)	(0.059)	(0.060)
SIZE	-0.200	-0.192	-0.191
	(0.051)	(0.052)	(0.053)
BM	0.304	0.322	0.322
	(0.068)	(0.072)	(0.072)
RET2_12	0.586	0.595	0.579
	(0.139)	(0.141)	(0.142)
$\bar{R}^2$(%)	3.634	3.391	3.408

Table A5.5 *(continued)*

Panel B: NYSE and AMEX Stocks Only			
α	0.714	0.697	0.703
	(0.215)	(0.222)	(0.224)
β	-0.067	-0.013	-0.008
	(0.089)	(0.024)	(0.017)
s		-0.006	-0.007
		(0.014)	(0.012)
h		0.009	0.005
		(0.011)	(0.009)
m			(0.003)
			(0.011)
coskew	0.144	0.152	0.164
	(0.099)	(0.101)	(0.102)
cokurt	0.109	0.059	0.054
	(0.052)	(0.059)	(0.060)
SIZE	-0.108	-0.105	-0.105
	(0.046)	(0.047)	(0.047)
BM	0.252	0.261	0.258
	(0.062)	(0.064)	(0.064)
RET2_12	0.735	0.740	0.730
	(0.155)	(0.158)	(0.158)
$\bar{R}^2$(%)	4.505	4.178	4.196

Panel C: NASDAQ Stocks Only			
α	1.210	1.155	1.170
	(0.288)	(0.295)	(0.297)
β	-0.073	0.035	0.033
	(0.081)	(0.023)	(0.016)
s		0.011	0.004
		(0.013)	(0.011)
h		-0.015	-0.016
		(0.012)	(0.009)
m			0.031
			(0.012)
coskew	0.018	0.033	-0.037
	(0.137)	(0.141)	(0.141)
cokurt	0.188	0.099	0.091
	(0.063)	(0.078)	(0.079)
SIZE	-0.418	-0.420	-0.419
	(0.070)	(0.070)	(0.070)
BM	0.235	0.245	0.244
	(0.079)	(0.083)	(0.083)
RET2_12	0.528	0.553	0.532
	(0.108)	(0.112)	(0.112)
$\bar{R}^2$(%)	2.402	2.213	2.223

Table A5.6 Fama-MacBeth Regression Results with out-of-sample Forecasted Realized Betas and Higher Comoments from an ARMA(1,1) model

This table reports the time series average of individual stock cross-sectional OLS regression coefficient estimates of out-of-sample forecasted monthly realized betas and higher comoments from an ARMA(1,1) model. Panel A presents the results of all stocks listed in NYSE, AMEX and NASDAQ. Panel B uses stocks only in NYSE and AMEX and panel C uses stocks only in NASDAQ. The sample is from July 1963 to December 2007. Besides coskewness and cokurtosis, the CAPM has only market beta, FF3F is the Fama-French three-factor model with market, SMB and HML as factors and 4F is the four-factor model with the Fama-French three factors and a momentum factor WML. β, *s, h* and *m* are realized betas of the market, SMB, HML and WML, respectively. *coskew* and *cokurt* are monthly coskewness and cokurtosis, respectively. SIZE is the logarithm of the one-month lagged market capitalization. BM is the logarithm of book-to-market ratio with exception that book-to-market ratios greater than 0.995 fractile and less than 0.005 fractile are set equal to 0.995 fractile and 0.005 fractile, respectively. RET2_12 is the cumulative returns from month *t*-12 to month *t*-2. $\bar{R}^2$ is the time series average of adjusted R^2. Standard errors of sample mean are in parenthesis.

Panel A: NYSE, AMEX and NASDAQ Stocks			
	CAPM	FF3F	4F
α	0.852	0.809	0.782
	(0.240)	(0.250)	(0.252)
β	-0.164	-0.017	0.010
	(0.068)	(0.018)	(0.012)
s		-0.015	0.000
		(0.010)	(0.008)
h		0.003	0.001
		(0.009)	(0.007)
m			0.003
			(0.009)
coskew	-0.005	-0.025	-0.033
	(0.087)	(0.093)	(0.093)
cokurt	0.194	0.097	0.074
	(0.041)	(0.058)	(0.059)
SIZE	-0.204	-0.206	-0.201
	(0.052)	(0.053)	(0.053)
BM	0.304	0.316	0.307
	(0.068)	(0.073)	(0.073)
RET2_12	0.587	0.569	0.576
	(0.138)	(0.141)	(0.141)
$\bar{R}^2$(%)	3.611	3.382	3.384

Table A5.6 *(continued)*

Panel B: NYSE and AMEX Stocks Only			
α	0.732	0.700	0.675
	(0.215)	(0.225)	(0.227)
β	-0.129	-0.019	0.003
	(0.081)	(0.020)	(0.016)
s		-0.013	-0.009
		(0.013)	(0.011)
h		0.009	0.008
		(0.011)	(0.009)
m			(0.004)
			(0.011)
coskew	0.035	0.027	0.020
	(0.094)	(0.100)	(0.100)
cokurt	0.114	0.047	0.033
	(0.044)	(0.052)	(0.054)
SIZE	-0.105	-0.104	-0.103
	(0.046)	(0.047)	(0.047)
BM	0.254	0.263	0.256
	(0.062)	(0.065)	(0.065)
RET2_12	0.725	0.714	0.732
	(0.154)	(0.158)	(0.158)
$\overline{R}^2$(%)	4.443	4.142	4.165
Panel C: NASDAQ Stocks Only			
α	1.193	1.141	1.124
	(0.285)	(0.296)	(0.297)
β	-0.120	0.005	0.029
	(0.074)	(0.022)	(0.015)
s		0.008	0.015
		(0.013)	(0.010)
h		-0.009	-0.009
		(0.012)	(0.009)
m			0.009
			(0.012)
coskew	-0.057	-0.064	-0.079
	(0.136)	(0.140)	(0.141)
cokurt	0.279	0.189	0.173
	(0.057)	(0.081)	(0.080)
SIZE	-0.436	-0.447	-0.435
	(0.070)	(0.070)	(0.070)
BM	0.228	0.241	0.229
	(0.079)	(0.084)	(0.085)
RET2_12	0.543	0.533	0.532
	(0.108)	(0.112)	(0.116)
$\overline{R}^2$(%)	2.362	2.206	2.189

Chapter 6

Conclusion and Future Research

6.1 Conclusion

This thesis tests whether conditional asset pricing models can explain the cross-section of expected stock returns both in-sample and out-of-sample. The results show that all conditional asset pricing models are rejected in out-of-sample tests although some modes can explain the cross-section of returns in-sample, which indicates an out-of-sample failure of conditional models.

I first test the conditional CAPM in Chapter 3 by using the Fama-French 25 size/value portfolios, which are among the most serious challenges of the CAPM. Conditional market beta should fully explain the cross-section of these 25 portfolios' returns if the conditional CAPM holds. Specifically, there should be an insignificant intercept and a significantly positive coefficient on market beta in cross-sectional regressions. Models of conditional market beta examined include the short window regression model, the macroeconomic variables model, the state-space model, the DCC-GARCH model and the realized beta model. The results show that the state-space model with random walk market beta can explain the cross-section of returns very well in-sample and all other models are rejected. In the out-of-sample test, however, all models are rejected. The reason is that conditional market beta is difficult to forecast out-of-sample.

In Chapter 4, I test different conditional asset pricing models by using the recently proposed realized beta model to estimate conditional betas. Test assets are individual stocks listed in the US market. Realized betas and firm-level variables such as size, BM and past returns are included in cross-sectional regressions. If a conditional asset pricing model holds, betas should have significantly positive coefficients and intercept and firm-level variables should be insignificantly different from zero. I use daily returns within each month to compute monthly realized betas and then model them by time series techniques. Asset pricing models examined include: the CAPM, the Fama-French three-factor model and a four-factor model including Fama-French three factors plus a momentum factor. The results show that contemporaneous market beta has a significantly positive coefficient but firm-level variables are all significant, which indicates that market beta is a priced risk but cannot fully explain the effects of firm-level variables. Contemporaneous multi-factor betas of the Fama-French model and the four-factor model, which are also significant except beta of the value factor (HML), can reduce but not eliminate the effects of firm-level variables. For in-sample forecasted betas, market beta and beta of the momentum factor (WML) still have a significantly positive coefficient but additional betas of the Fama-French model become insignificant. Additional betas in multi-factor models, the Fama-French model and the four-factor model, cannot reduce the effects of firm-level variable any more. For out-of-sample forecasted betas, no betas have a significantly positive coefficient and firm-level variables are highly significant, which indicates a strong rejection of conditional asset pricing models.

In Chapter 5, I test whether higher comoments, coskewness and cokurtosis, are significant systematic risks and can help explain the cross-section of expected returns. I use daily returns within each month to compute monthly realized coskewness and cokurtosis and then add them to different factor pricing models tested in Chapter 4. The results show that cokurtosis is a significant risk both in-sample and out-of-sample. Cokurtosis is the only out-of-sample significant systematic risk among all betas and higher comoments examined in this thesis. Coskewness, however, is insignificant both in-sample and out-of-sample. Firm-level variables remain highly significant even after coskewness and cokurtosis are added to factor pricing models. The results show that cokurtosis is an important risk but it cannot help explain the cross-section of expected stock returns.

The overall results indicate a rejection of different conditional asset pricing models, i.e. the conditional CAPM, the conditional Fama-French model, the conditional four-factor model and the conditional higher-moment CAPM. The reason is that conditional systematic risks, betas and higher comoments, are difficult to estimate and forecast. Simple techniques such as regression based methods are not enough to describe the dynamics of systematic risks. More advanced techniques such as the state-space model and multivariate GARCH model are difficult to estimate, especially for high-dimensional problems, and may have over fitting problem, i.e. good in-sample performance but poor out-of-sample performance. Realized betas and higher comoments are easy to compute but require high-frequency data, which are difficult to obtain especially for illiquid small stocks. The difficulties of estimating and modelling systematic risks make it difficult for conditional models to success in explaining the cross-section of expected returns. The next subsection will give some suggestions for future research with emphasis on overcoming difficulties of modelling systematic risks.

6.2 Future Research

The results of this thesis show that some ex-post estimated or in-sample forecasted betas and cokurtosis are significant and can explain some or all of the effects of firm-level variables such as the state-space model in Chapter 3, realized betas in Chapter 4 and realized cokurtosis in Chapter 5. The difficulty is the out-of-sample forecast of betas and higher comoments. The Forecasting models used in this thesis are mostly linear models. Therefore, more advanced models may provide better results if they can generate better forecasts.

The linear models used in this thesis assume that betas and higher comoments have the same dynamics during different market conditions. Recent literature, however, have found that asset returns have asymmetric comovement during bear market and normal market. For example, Ang and Chen (2002) find that US stocks are more correlated with

US market index during downside market moves, especially for extreme downside moves; Longin and Solnik (2001) find statistically significant asymmetric correlation of international equity returns.

Some researchers also propose theoretical models which relate betas to market risk premium which is high during bear market (e.g. Santos and Veronesi, 2004). Zhang (2005) shows that value stocks are riskier than growth stocks in bad times. Subsequent empirical work by Petkova and Zhang (2005) find that value stocks' beta is higher than growth stocks' when market risk premium is high.

A popular model to deal with asymmetry is the regime switching model (Hamilton, 1989). Some studies have examined the regime switching model in the context of asset allocation during bear and normal markets. Ang and Bekaert (2002, 2004) show that international diversification is still valuable in bear market when the correlation between the US and international markets are high; Guidolin and Timmermann (2008) add regime switching, coskewness and cokurtosis in the standard international CAPM.

Therefore, econometric models of systematic risks should incorporate the asymmetry properties of returns' comovement. It is interesting to see if the forecast of betas and higher comoments can be improved by applying the regime switching model to systematic risks in different market regimes. The regime switching model may describe the dynamics of systematic risks better if systematic risks have different dynamics in different regimes. Related research includes the asymmetric beta of Gu (2005) in the state-space model. Gu uses regime switching techniques to model market beta and betas of the Fama-French model. He divides market into two regimes: up market when the market return is positive and down market when the market return is negative. His results, however, still reject the conditional CAPM and the Fama-French model. The limit of his approach is that he assumes betas are constant within each regime. A natural extension is to model beta as a dynamic process. This approach is also related to the downside risk of Ang et al. (2006) who model the downside risk as market beta when the market return is below a threshold such as zero or the risk free rate. The authors find that the downside risk is significant in the cross-section of stock returns.

In this thesis, daily data are used to estimate monthly realized betas and higher comoments due to data limitation. It is very interesting to see whether intraday data can give better results with improved accuracy of estimated realized betas and higher comoments. Empirical studies of using intraday data to estimate realized betas include Bollerslev and Zhang (2003) and Anderson et al. (2005, 2006), who find that using intraday data can improve the accuracy of estimated realized betas.

One problem of using intraday data is non-synchronous trading of high frequency data, which can cause biased estimates. I use the method of Scholes and Williams (1977) in Chapter 4 to overcome the non-synchronous trading problem but this method is developed under daily frequency and assumes there is only one day lag of information between active stocks and inactive stocks. Therefore, it may not be suitable for intraday data. An interesting approach to deal with this problem is the realized kernel approach of Barndorff-Nielsen et al. (2008, 2009, 2010), which uses non-parametric kernel method to intraday data. They show that this approach can improve the accuracy of realized volatility estimated from noisy and non-synchronous data. It is very interesting to apply this method to realized beta estimation.

After we get more accurate realized betas, the next step is to develop time series models to make forecasts. The forecasting models of realized betas and higher comoments are linear ARMA models, which may be too simple to forecast realized betas and higher comoments. The asymmetry of returns' comovement can also cause realized betas to have different dynamics under different market conditions. Therefore, it is interesting to test if stocks' returns have asymmetric realized betas and if realized betas have different dynamics under different market conditions. If the asymmetry is significant, more advanced non-linear models such as regime switching models or non-parametric models may improve the forecast of realized betas and higher comoments. It is especially interesting to develop more advanced models for realized betas estimated from high frequency intraday data.

Estimating and forecasting betas and higher comoments are among the most challenging work for conditional asset pricing models. Only estimated betas and higher comoments are used in empirical studies and therefore poor estimates can lead to wrong conclusions. For practitioners, poorly estimated betas and higher comoments can lead to wrong evaluation of portfolios' systematic risks, which may further result in wrong investment decisions. Overall, the importance of betas and higher comoments as systematic risks makes it at the centre of finance research. New models will be proposed continuously and it is interesting to see whether these models can improve the estimation and forecast of systematic risks.

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