On the fragmentation of self-gravitating discs

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Signed: ..........................

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Abstract

I have carried out three-dimensional numerical simulations of self-gravitating discs to determine under what circumstances they fragment to form bound clumps that may grow into giant planets. Through radiation hydrodynamical simulations using a Smoothed Particle Hydrodynamics code, I find that the disc opacity plays a vital role in determining whether a disc fragments. Specifically, opacities that are smaller than interstellar Rosseland mean values promote fragmentation (even at small radii, $R < 25 \text{AU}$) since low opacities allow a disc to cool quickly. This may occur if a disc has a low metallicity or if grain growth has occurred. Given that the standard core accretion model is less likely to form planets in a low metallicity environment, I predict that gravitational instability is the dominant planet formation mechanism in a low metallicity environment. In addition, I find that the presence of stellar irradiation generally acts to inhibit fragmentation (since the discs can only cool to the temperature defined by stellar irradiation). However, fragmentation may occur if the irradiation is sufficiently weak that it allows the disc to attain a low Toomre stability parameter.

With specific reference to the HR 8799 planetary system, I find that it is only possible for fragments to form in the radial range where the HR 8799 planets are located ($R \approx 24 - 68 \text{AU}$) if the disc is massive. In such a high mass regime, mass transport occurs in the disc causing the surface mass density to alter. Therefore, fragmentation is not only affected by the disc temperature and cooling, but also by any restructuring due to the gravitational torques. The high mass discs also pose a problem for the formation of this system because the protoplanets accrete from the disc and end up with masses greater than those inferred from observation and thus, the growth of planets would need to be inhibited. In addition, I find that further subsequent fragmentation at small radii also takes place.

By way of analytical arguments in combination with hydrodynamical simulations using a parameterised cooling method, I explore the fragmentation criteria which in the past, has placed emphasis on the cooling timescale in units of the orbital timescale, $\beta$. I find that at a given radius the surface mass density (i.e. disc mass and profile) and star mass also play a crucial role in determining whether a disc fragments or not as well as where in the disc fragments form. I find that for shallow surface mass density profiles ($p < 2$, where $\Sigma \propto R^{-p}$), fragments form in the outer regions of the disc. However for steep surface mass density profiles ($p \gtrsim 2$), fragments form in the inner regions of a disc. In addition, I find that the critical value of the cooling timescale in units of the orbital timescale, $\beta_{\text{crit}}$, found in previous simulations is only applicable to certain disc surface mass density profiles and for particular disc radii and is not a general rule for all discs. I obtain an empirical fragmentation criteria between the cooling timescale in units of the orbital timescale, $\beta$, the surface mass density, the star mass and the radius. Finally, I carry out crucial resolution testing by performing the highest resolution disc simulations to date. My results cast some serious doubts on previous conclusions concerning fragmentation of self-gravitating discs.
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Declaration

This thesis contains work that has been published with Professor Matthew R. Bate.

The results of Chapter 3 have been published in EXOPLANETS AND DISKS: THEIR FORMATION AND DIVERSITY: Proceedings of the International Conference. AIP Conference Proceedings, Volume 1158, pp. 139-140 (2009).

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