High Frequency and Large Dimension Volatility

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature: .................................................................
I am heartily thankful to my supervisor, Richard D. F. Harris, whose encouragement, supervision and support from the preliminary to the concluding level enabled me to develop an understanding of the subject. This thesis would not have been possible without his help and guidance.

I am also indebted to my wife, Li Na, and my parents who supported me in any respect during the completion of this project.
Abstract

Three main issues are explored in this thesis—volatility measurement, volatility spillover and large-dimension covariance matrices. For the first question of volatility measurement, this thesis compares two newly-proposed, high-frequency volatility measurement models, namely realized volatility and realized range-based volatility. It does so in the aim of trying to use empirical results to assess whether one volatility model is better than the other. The realized volatility model and realized range-based volatility model are compared based on three markets, five forecast models, two data frequencies and two volatility proxies, making sixty scenarios in total. Seven different loss functions are also used for the evaluation tests. This necessarily ensures that the empirical results are highly robust. After making some simple adjustments to the original realized range-based volatility, this thesis concludes that it is clear that the scaled realized range-based volatility model outperforms the realized volatility model.

For the second research question on volatility spillover, realized range-based volatility and realized volatility models are employed to study the volatility spillover among the S&P 500 index markets, with the aim of finding out empirically whether volatility spillover exists between the markets. Volatility spillover is divided into the two categories of statistically significant volatility spillover and economically significant volatility spillover. Economically significant spillover is defined as spillover that can help forecast the volatility of another market, and is therefore a more powerful measurement than statistically significant spillover. The findings show that, in reality, the existence of volatility spillover depends on the choice of model, choice of volatility proxy and value of parameters used.
The third and final research question in this thesis involves the comparison of various large-dimension multivariate models. The main contribution made by this specific study is threefold. First, a number of good performance multivariate volatility models are introduced by adjusting some commonly used models. Second, different models and various choices of parameters for these models are tested based on 26 currency pairs. Third, the evaluation criteria adopted possess much more practical implications than those used in most other papers on this subject area.
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## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ABDL</td>
<td>Andersen, Bollerslev, Diebold and Labys</td>
</tr>
<tr>
<td>AMEX</td>
<td>American Stock Exchange</td>
</tr>
<tr>
<td>AR</td>
<td>Autoregressive</td>
</tr>
<tr>
<td>ARCH</td>
<td>Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>CAPM</td>
<td>Capital Asset Pricing Model</td>
</tr>
<tr>
<td>CCC</td>
<td>Constant Conditional Correlation</td>
</tr>
<tr>
<td>CME</td>
<td>Chicago Mercantile Exchange</td>
</tr>
<tr>
<td>DCC</td>
<td>Dynamic Conditional Correlation</td>
</tr>
<tr>
<td>DM</td>
<td>Diebold and Mariano</td>
</tr>
<tr>
<td>EST</td>
<td>Eastern Standard Time</td>
</tr>
<tr>
<td>ETF</td>
<td>Exchange-Traded Fund</td>
</tr>
<tr>
<td>EWMA</td>
<td>Exponentially Moving Average</td>
</tr>
<tr>
<td>FX</td>
<td>Foreign Exchange</td>
</tr>
<tr>
<td>GARCH</td>
<td>Generalized Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>PACF</td>
<td>Partial Autocorrelation Function</td>
</tr>
<tr>
<td>IGARCH</td>
<td>Integrated GARCH</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>National Association of Securities Dealers Automated Quotations</td>
</tr>
<tr>
<td>NYSE</td>
<td>New York Stock Exchange</td>
</tr>
<tr>
<td>RRV</td>
<td>Realized Range based Volatility</td>
</tr>
<tr>
<td>RV</td>
<td>Realized Volatility</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>Standard &amp; Poor’s</td>
</tr>
<tr>
<td>SPDR</td>
<td>Standard &amp; Poor's Depository Receipts</td>
</tr>
<tr>
<td>SV</td>
<td>Stochastic Volatility</td>
</tr>
<tr>
<td>VaR</td>
<td>Value-at-Risk</td>
</tr>
<tr>
<td>VAR</td>
<td>Vector Autoregressive</td>
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