

# High Frequency and Large Dimension Volatility

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## Abstract

Three main issues are explored in this thesis—volatility measurement, volatility spillover and large-dimension covariance matrices. For the first question of volatility measurement, this thesis compares two newly-proposed, high-frequency volatility measurement models, namely realized volatility and realized range-based volatility. It does so in the aim of trying to use empirical results to assess whether one volatility model is better than the other. The realized volatility model and realized range-based volatility model are compared based on three markets, five forecast models, two data frequencies and two volatility proxies, making sixty scenarios in total. Seven different loss functions are also used for the evaluation tests. This necessarily ensures that the empirical results are highly robust. After making some simple adjustments to the original realized range-based volatility, this thesis concludes that it is clear that the scaled realized range-based volatility model outperforms the realized volatility model.

For the second research question on volatility spillover, realized range-based volatility and realized volatility models are employed to study the volatility spillover among the S&P 500 index markets, with the aim of finding out empirically whether volatility spillover exists between the markets. Volatility spillover is divided into the two categories of statistically significant volatility spillover and economically significant volatility spillover. Economically significant spillover is defined as spillover that can help forecast the volatility of another market, and is therefore a more powerful measurement than statistically significant spillover. The findings show that, in reality, the existence of volatility spillover depends on the choice of model, choice of volatility proxy and value of parameters used.

The third and final research question in this thesis involves the comparison of various large-dimension multivariate models. The main contribution made by this specific study is threefold. First, a number of good performance multivariate volatility models are introduced by adjusting some commonly used models. Second, different models and various choices of parameters for these models are tested based on 26 currency pairs. Third, the evaluation criteria adopted possess much more practical implications than those used in most other papers on this subject area.

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## Abbreviations

ABDL	Andersen, Bollerslev, Diebold and Labys
AMEX	American Stock Exchange
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroskedasticity
CAPM	Capital Asset Pricing Model
CCC	Constant Conditional Correlation
CME	Chicago Mercantile Exchange
DCC	Dynamic Conditional Correlation
DM	Diebold and Mariano
EST	Eastern Standard Time
ETF	Exchange-Traded Fund
EWMA	Exponentially Moving Average
FX	Foreign Exchange
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
PACF	Partial Autocorrelation Function
IGARCH	Integrated GARCH
NASDAQ	National Association of Securities Dealers Automated Quotations
NYSE	New York Stock Exchange
RRV	Realized Range based Volatility
RV	Realized Volatility
S&P	Standard & Poor's
SPDR	Standard & Poor's Depository Receipts
SV	Stochastic Volatility
VaR	Value-at-Risk
VAR	Vector Autoregressive