Hedging and Value at Risk

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Abstract

In this paper, we show that although minimum-variance hedging unambiguously reduces the standard deviation of portfolio returns, it tends to increase portfolio kurtosis and consequently the effectiveness of hedging in terms of a more general measure of risk such as VaR is uncertain. We compare the reduction in standard deviation with the reduction in 99% VaR for thirteen cross-hedged currency portfolios using both in-sample and out-of-sample approaches. We find that minimum-variance hedging reduces standard deviation considerably more than it reduces VaR. Indeed, for some portfolios, the out-of-sample reduction in VaR is negligible. As an alternative, we propose a minimum-VaR hedging strategy that minimises the historical simulation VaR of the hedge portfolio. Minimum-VaR hedge ratios are found to be significantly lower than minimum-variance hedge ratios. The minimum-VaR hedging strategy offers a significant improvement over the minimum-variance hedging strategy in terms of VaR. Moreover, in many cases, it actually yields a larger out-of-sample reduction in standard deviation also.

Keywords: Hedging; Value at risk; Skewness; Kurtosis; Historical simulation.

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1. Introduction

The theory of hedging is now well established and is commonly used by practitioners, either to offset the risk of a position in the cash or spot market by taking a position in the derivatives market, or as part of an overall investment strategy such as in a hedge fund. In both cases, the primary objective is to estimate the size of the short position that must be held, as a proportion of the long position, that maximises the agent's expected utility, which is defined over the risk and expected return of the hedged portfolio. This is the problem of estimating the optimal hedge ratio, or OHR.

In the mean-variance framework, risk is measured by the variance, (or, equivalently, standard deviation) of the hedged portfolio. Varying the hedge ratio traces out a feasible set for the hedged portfolio in expected return–standard deviation space. The OHR is the hedge ratio that equates the agent's marginal rate of substitution between the expected return and the standard deviation of the hedged portfolio with the slope of this feasible set (see Cecchetti, Cumby and Figlewski, 1988). Very commonly, however, optimal hedging is taken to mean minimum-variance hedging.[1] One justification for this is that if the futures price follows a martingale (i.e. it is an unbiased estimator of the future spot price), the expected futures return is zero and so the OHR is simply the hedge ratio that minimises the variance of the hedged portfolio (see, for example, Benninga, Eldor and Zilcha, 1983).

The principles of minimum-variance hedging are grounded in portfolio theory, resting on the result that a weighted portfolio of two assets will have a variance lower than the weighted average variance of the two individual assets, as long as the two assets are not perfectly positively correlated. Hedging involves taking two negatively correlated positions (a long position in one asset and a short position in another positively correlated asset) and so typically the standard deviation of the hedged portfolio can be substantially reduced. Indeed, in the case of futures hedging, the only reason why the risk of the hedged portfolio cannot be entirely eliminated is because of

[1] The minimum-variance hedge ratio is sometimes known as the pure hedging component of the OHR since it ignored the speculative component related to the expected return of the hedge portfolio (see, for example, Anderson and Danthine, 1981).

a mismatch between the maturity of the long and short positions (a delta hedge) or in the underlying asset (a cross hedge).

The use of standard deviation as a measure of risk – and hence the use of minimum-variance hedging as a method of reducing portfolio risk – is justified by assuming either that investors have quadratic utility or that asset returns are drawn from an elliptical distribution, that is to say, the skewness and kurtosis (and indeed the higher moments) of a portfolio are the same as those of the assets that it comprises.[2] Under the first assumption, portfolio returns may be skewed or leptokurtic but this does not affect investor utility because their utility functions are defined solely over expected return and the standard deviation. Under the second assumption, investors may care about portfolio skewness and kurtosis but the hedge portfolio has the same distribution as the underlying assets and so investors base their hedging decision solely on expected return and standard deviation.

In contrast with theory, however, short horizon financial asset returns are characterised by very significant leptokurtosis and, to a lesser degree, skewness.[3, 4] Additionally, evidence suggests that the skewness and kurtosis of returns are not preserved when assets are formed into portfolios, implying that returns are not drawn from an elliptical distribution (see, for example, Simkowitz and Beedles, 1978; Aggarwal et al, 1989; Tang and Choi, 1998). Moreover, Scott and Horvath (1980)

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[2] A portfolio of assets drawn from an elliptical distribution will have the same elliptical distribution as the original assets, except for the mean and standard deviation. The normal distribution is one such elliptical distribution commonly employed in finance, but others include the Student-t and uniform distributions. Notably, the lognormal distribution is not elliptical (see, for example, Ingersoll, 1987).

[3] Leptokurtosis and skewness are difficult to identify separately since standard test statistics for skewness and kurtosis are only valid under the null hypothesis of normality. In particular, the distribution of the skewness statistic is severely distorted when the data is leptokurtic and so a significant skewness statistic may simply be a manifestation of leptokurtosis. Peiro (1999) investigates the skewness of financial asset returns using non-parametric tests and finds that skewness is much less important than conventional test statistics would imply.

[4] The leptokurtosis of returns is partly explained by time-varying volatility: if returns were drawn each day from a normal distribution with a variance that changes over time, the unconditional distribution of returns would be leptokurtic (see Engle, 1982). However, even allowing for time-varying volatility, short horizon financial asset returns appear to be highly leptokurtic (see, for example, Baillie and DeGennaro, 1990; Bollerslev, Chou and Kroner, 1992).

show that in general, investors should have preferences for odd moments of the return distribution and preferences against even moments (see also Arditti, 1967; Kane, 1982). When returns are not elliptically distributed and investors do not have quadratic utility, then standard deviation is no longer an appropriate measure of risk since it fails to capture all of the characteristics of portfolio returns that investors consider to be important. A number of authors have proposed asset pricing models that explicitly allow for investors' preferences over the higher moments of returns (see, for example, Kraus and Litzenberger, 1976; Fang and Lai, 1997; Dittmar, 2002). These studies typically find that the co-skewness and co-kurtosis of asset returns (which measure the contribution that an asset makes to the skewness and kurtosis of a portfolio) are priced by the market, suggesting that investors do indeed have preferences over portfolio skewness and kurtosis.

While hedging unambiguously reduces the standard deviation of portfolio returns, its effect on skewness and kurtosis is uncertain. A number of papers have shown empirically that portfolios formed from skewed or leptokurtic assets are generally less skewed or leptokurtic than the original assets (see, for example, Simkowitz and Beedles, 1978; Aggarwal et al, 1989). In this paper, we show that in contrast, hedging, which involves forming portfolios from negatively correlated assets, has little effect on skewness but invariably leads to a substantial *increase* in portfolio kurtosis. The implication of this is that while hedging unambiguously serves to reduce risk in terms of portfolio standard deviation, the impact on risk more generally is uncertain.

A natural framework in which to consider the higher moments of the return distribution of hedged portfolios is Value at Risk (VaR), which is defined as the maximum loss on a portfolio over a certain period of time that can be expected with a certain probability. When returns are normally distributed and the mean return is assumed to be zero (as is commonly the case in practice when dealing with short horizon returns), the VaR of a portfolio is simply a constant multiple of the standard deviation of the portfolio. However, when the return distribution is non-normal, the VaR of a portfolio is determined not just by the standard deviation of returns but also by their skewness and kurtosis (and indeed by their higher moments as well). Clearly VaR is not a perfect measure of risk, since it completely ignores the expected size of a loss in the event that the VaR of a portfolio is exceeded.[5] Despite its shortcomings, however, VaR is now perhaps the most widely used risk measure amongst practitioners, largely because of its adoption by the Basle Committee on Banking Regulation for the assessment of the risk of the trading books of investment banks and its use in setting investment banks' capital requirements (see Jorion, 2000). More recently, VaR has gained a much wider usage and is now used by fund managers for asset allocation and performance evaluation and by non-financial corporations to summarise the risk of cash-flow shortfalls (see, for example, Dowd, 1998).

Increasingly, VaR is also being used as an input to portfolio theory. For example, Alexander and Baptista (2002) propose a mean-VaR framework that is analogous to the traditional mean-variance framework, but in which portfolios are located in mean-VaR space instead of mean-standard deviation space. Under the assumption that portfolio returns are normally distributed, they prove a number of results including concavity of the feasible set and the existence of a unique market portfolio. A number of authors (Huisman, Koedik and Pownall, 1999; Campbell, Huisman and Koedik, 2001; Alexander and Baptista, 2004) have suggested combining the two frameworks by solving the portfolio optimisation problem in the mean-variance framework but with an additional constraint on the maximum portfolio VaR. In the context of hedging, Duarte (1998) proposes a scenario-based VaR approach to compute the optimal hedging strategy for derivatives portfolios.

In this paper, we consider two very simple, but nevertheless important questions. The first is whether the substantial reductions in portfolio variance that can be gained from hedging offer correspondingly large reductions in portfolio risk, when risk is measured by a more general metric such as VaR. We answer this question empirically using data on thirteen cross-hedged currency portfolios. Specifically, we construct minimum-variance hedge portfolios and compare the risk reduction in terms of standard deviation with the risk reduction in terms of VaR. Ex ante, it is not obvious what to expect: hedging reduces portfolio standard deviation but simultaneously

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[5] In this sense, VaR is not a *coherent* measure of risk (see, for example, Artzner et al, 1999). This is a problem addressed by Conditional VaR (CVaR). CVaR, also known as the expected shortfall, is the expected loss on a portfolio, conditional on the loss being greater than VaR (see, for example, Tasche, 2002).

increases kurtosis, and so the overall effect on the quantile of the hedge portfolio return distribution is uncertain. We investigate the effectiveness of the minimum-variance hedging strategy by considering both in-sample and out-of-sample performance. In terms of in-sample performance, we find that minimum-variance hedging reduces VaR by considerably less than it reduces standard deviation. For example, for a long position in the Singapore Dollar hedged with a short position in the US Dollar, minimum-variance hedging reduces standard deviation by 33% but reduces VaR by only 15%. In terms of out-of-sample performance, the differences can be substantial, with minimum-variance hedging giving, in some cases, a negligible reduction in VaR.

The second question is whether a greater reduction in portfolio VaR can be obtained by explicitly minimising VaR rather than standard deviation. While only risk-neutral investors would actually seek to minimise VaR (as opposed to maximising their utility, which could be assumed to be some non-linear function of VaR and expected return), this analysis is in the spirit of the existing literature on hedging, which is currently focussed on variance minimisation rather than utility maximisation. In addressing this question, we employ a historical simulation approach. Specifically, we estimate the hedge ratio that minimises the (negative of the) appropriate quantile of the empirical distribution of hedged portfolio returns. Again, we consider both a static in-sample approach and a dynamic out-of-sample approach. We find that minimum-VaR hedge ratios are typically considerably smaller than minimum-variance hedge ratios, suggesting that smaller short positions are required to minimise VaR than to minimise variance. In terms of hedging performance, minimum-VaR hedging leads to a hedge portfolio that has VaR that is, on average, of the order of fifteen percent lower than the VaR of the minimum-variance hedge portfolio. Moreover, minimum-VaR portfolios have a standard deviation that is similar to that of minimum-variance portfolios. Indeed, in some cases, minimum-VaR hedging actually provides a *larger* out-of-sample reduction in standard deviation compared with minimum-variance hedging.

The outline of this paper is as follows. In Section 2 we discuss the theoretical distribution of hedge portfolio returns. Section 3 reports evidence on the empirical distribution of hedge portfolio returns and quantifies the risk reduction of hedging in terms of both standard deviation and VaR for the minimum-variance hedging strategy. Section 4 presents the results for the minimum-VaR hedging strategy. Section 5 concludes.

2. The Distribution of Hedge Portfolio Returns

In this section, we summarise the principles of minimum-variance hedging and derive expressions for the standard deviation, skewness and kurtosis of the minimum-variance hedge portfolio. Our aim in this section is to show that while minimum-variance hedging necessarily reduces the standard deviation of portfolio returns, its effect on skewness and kurtosis – and hence on portfolio VaR – is ambiguous.

Suppose that there are two assets, Asset 1 and Asset 2, with per-period returns r_1 and r_2, and that a short position in Asset 2 is used to hedge the risk exposure of a long position in Asset 1. We assume that the mean return for both assets is zero.[6] The hedge portfolio, given by a long position in Asset 1 and a fraction, *h*, of a short position in Asset 2, has a return equal to

$$r_p = r_1 - hr_2 \tag{1}$$

The variance of the hedge portfolio return is given by

$$\begin{aligned}\sigma_p^2 &= \mathrm{var}(r_1 - hr_2)\\ &= \sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2\end{aligned} \tag{2}$$

where σ_1^2 is the variance of r_1, σ_2^2 is the variance of r_2 and $\rho_{1,2}$ is the correlation between r_1 and r_2. The skewness coefficient of the hedge portfolio is given by

[6] This is a common assumption when dealing with daily financial asset returns, but could easily be relaxed to allow for a non-zero mean

$$
\begin{aligned}
s_p &= \frac{E[r_p^3]}{\sigma_p^3} \\
&= \frac{s_1\sigma_1^3 - 3hs_a\sigma_1^2\sigma_2 + 3h^2 s_b\sigma_1\sigma_2^2 - h^3 s_2\sigma_2^3}{(\sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2)^{3/2}}
\end{aligned}
\tag{3}
$$

where the skewness and co-skewness coefficients of the two assets are defined by

$$
s_1 = \frac{E[r_1^3]}{\sigma_1^3},\ s_2 = \frac{E[r_2^3]}{\sigma_2^3},\ s_a = \frac{E[r_1^2 r_2]}{\sigma_1^2\sigma_2},\ s_b = \frac{E[r_1 r_2^2]}{\sigma_1\sigma_2^2} \tag{4}
$$

The kurtosis coefficient of the hedge portfolio is given by

$$
\begin{aligned}
k_p &= \frac{E[r_p^4]}{\sigma_p^4} \\
&= \frac{k_1\sigma_1^4 - 4hk_a\sigma_1^3\sigma_2 + 6h^2 k_b\sigma_1^2\sigma_2^2 - 4h^3 k_c\sigma_1\sigma_2^3 + h^4 k_2\sigma_2^4}{(\sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2)^2}
\end{aligned}
\tag{5}
$$

where the kurtosis and co-kurtosis coefficients of the two assets are defined by

$$
k_1 = \frac{E[r_1^4]}{\sigma_1^4},\ k_2 = \frac{E[r_2^4]}{\sigma_2^4},\ k_a = \frac{E[r_1^3 r_2]}{\sigma_1^3\sigma_2},\ k_b = \frac{E[r_1^2 r_2^2]}{\sigma_1^2\sigma_2^2},\ k_c = \frac{E[r_1 r_2^3]}{\sigma_1\sigma_2^3} \tag{6}
$$

We now derive the standard deviation, skewness and kurtosis of the minimum-variance hedge portfolio. The minimum-variance hedge ratio is the value of *h* that minimises (2), which, by differentiating (2) and setting equal to zero, yields

$$
h = \rho_{1,2}\frac{\sigma_1}{\sigma_2} \tag{7}
$$

Substituting (7) into (2), the standard deviation of the minimum-variance hedge portfolio is given by

$$\sigma_p = \sigma_1 (1-\rho_{1,2}^2)^{1/2} \tag{8}$$

The correlation coefficient, $\rho_{1,2}$, is bounded by plus one and minus one, and so minimum-variance hedging will always reduce the standard deviation of the hedge portfolio relative to that of Asset 1, except in the extreme case that the returns of Asset 1 and Asset 2 are uncorrelated. In the event that the returns are perfectly (positively or negatively) correlated, the hedge is perfect and the standard deviation of the hedge portfolio is zero. Thus the standard deviation of the hedge portfolio can never by higher than that of Asset 1.

Substituting (7) into (3) and (5), the skewness and kurtosis coefficients of the minimum-variance hedge portfolio are given by

$$s_p = \frac{s_1 - 3\rho_{1,2} s_a + 3\rho_{1,2}{}^2 s_b - \rho_{1,2}{}^3 s_2}{(1-\rho_{1,2}^2)^{3/2}} \tag{9}$$

$$k_p = \frac{k_1 - 4\rho_{1,2} k_a + 6\rho_{1,2}^2 k_b - 4\rho_{1,2}^3 k_c + \rho_{1,2}^4 k_2}{(1-\rho_{1,2}^2)^2} \tag{10}$$

For uncorrelated assets (i.e. those with $\rho_{1,2} = 0$), the skewness and kurtosis coefficients are simply equal to the skewness and kurtosis coefficients of Asset 1, namely s_1 and k_1. When $\rho_{1,2} = \pm 1$, the skewness and kurtosis coefficients are not defined but L'Hopital's rule can be used to show that as $\rho_{1,2} \to \pm 1$, $k_p \to k_2$ and $s_p \to s_2$, i.e. the skewness and kurtosis coefficients of the hedge portfolio converge to those of Asset 2. However, when $0 < |\rho_{1,2}| < 1$, s_p and k_p are not bounded by s_1 and k_1 (or even by s_2 and k_2). To see that this is the case, suppose for simplicity that $s_1 = s_2 = s$ and $k_1 = k_2 = k$. Differentiating (9) and (10) with respect to $\rho_{1,2}$ and evaluating for $\rho_{1,2} = 0$ (where $s_p = s$ and $k_p = k$) yields $-3s_a$ and $-4k_a$, respectively. For s and k to be the maxima of s_p and k_p, these derivatives would have to be zero, and hence we would have to have $s_a = k_a = 0$, which would be the case, for example, if r_1 and r_2 were stochastically independent, or were drawn from an elliptical distribution. More generally, s_a and k_a will not be zero, and so neither will be the derivatives, in which case, s and k cannot be the maxima of s_p and k_p, respectively. Thus, while minimum-variance hedging will unambiguously reduce the standard deviation of the hedge portfolio relative to that of Asset 1, the skewness and kurtosis coefficients of the minimum-variance hedge portfolio can be larger in magnitude than those of both Asset 1 and Asset 2 depending on the values of the co-skewness and co-kurtosis coefficients. Hence the effect of minimum-variance hedging on a more general measure of risk that incorporates investors' preferences over skewness and kurtosis will be uncertain.

One approach to measuring risk that implicitly addresses the issue of higher moments of portfolio returns is Value at Risk (VaR). VaR, which has its origins in the 'safety-first' criterion of Roy (1952), is defined as the largest loss on a portfolio that can be expected with a particular probability over a certain horizon. When the mean return is zero, the VaR of a portfolio can be written as

$$VaR_p = -\sigma_p q_p^\alpha (s_p, k_p) \qquad (11)$$

where q_p^α is the α percent quantile of the standardised distribution (i.e. zero mean and unit variance) of hedged portfolio returns and α is equal to one minus the VaR confidence level. This quantile is generally a function of the skewness and kurtosis coefficients of hedged portfolio returns, s_p and k_p. When returns are normally distributed, $s_p = 0$ and $k_p = 3$ and the VaR of a portfolio is simply a constant multiple of the standard deviation of portfolio returns. At high VaR confidence levels (those typically used in practice), reducing skewness or increasing kurtosis will increase the VaR of a portfolio for a given standard deviation.[7] Consequently portfolios with low standard deviation can potentially have high VaR if they are highly leptokurtic or significantly negatively skewed. In the following two sections,

[7] Note that even if hedging reduces the absolute value of skewness through diversification, this will increase VaR when the unhedged asset is positively skewed.

we investigate, empirically, how the skewness and kurtosis of portfolio returns is affected by hedging and the consequences for portfolio risk as measured by VaR.

3. Data and Sample Selection

In the empirical analysis, we consider the risk reduction for cross-hedged currency portfolios. Specifically, we assume that a GBP investor has a long position in a foreign currency and hedges the risk exposure of this position using a short position in another currency.[8] While it might have been natural to consider hedging the long currency position with a short position in the futures market, there are several advantages to considering cross-hedging instead. The first is that it is easier to obtain reliable data. Spot and futures markets generally close at different times of the day and so the use of closing spot and futures prices will introduce a spurious variability in the hedged portfolio daily return due to the non-synchronous nature of the data. In contrast, it is straightforward to obtain prices for a large number of currencies recorded at the same time of day. Secondly, at any one time, there is typically a range of different futures contracts available with different maturity dates and very different liquidity levels. The use of cross-hedging avoids the need to arbitrarily choose among these contracts. Thirdly, cross-hedging avoids having to arbitrarily choose when to rollover the futures contract, either on the basis of volume traded or the date. In the empirical analysis, we use daily returns provided by Reuters for ten developed market currencies (AUD, CAD, EUR, JPY, NZD, NOK, SGD, SEK, CHF and USD) measured against the GBP for the period 03/01/1994 to 09/07/2004 (a total of 2745 observations), which is the longest common sample available.[9] The rates that we use are the mid-rates recorded each day at 4.00pm in London. From the quoted mid-rates, we calculate continuously compounded daily returns.

Panel A of Table 1 gives summary statistics for the ten currency return series, including the mean, standard deviation and skewness and excess kurtosis coefficients.

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[8] We use the standard ISO currency abbreviations: AUD is Australian Dollar, CAD is Canadian Dollar, EUR is Euro, GBP is British Pound, JPY is Japanese Yen, NZD is New Zealand Dollar, NOK is Norwegian Kroner, SGD is Singapore Dollar, SEK is Swedish Kroner, CHF is Swiss Franc and USD is US Dollar.

[9] The EUR exchange rate before its inception on 01.01.1999 is a synthetic rate computed by Reuters using the entry weights of the EUR countries.

For all ten series, the mean daily return is very close to zero. The standard deviations of the return series lie between 0.45% for EUR and 0.74% for JPY. All the series are highly leptokurtic with excess kurtosis coefficients ranging from 1.16 for EUR to 6.18 for NOK. The evidence of skewness is much less pronounced. For five of the ten series, the skewness coefficient is negative and, in most cases, it is small in magnitude. Using the Bera-Jarque statistic, which tests the joint null hypothesis of zero skewness and zero excess kurtosis, all ten series are shown to be highly non-normal.

Panel B of Table 1 reports the correlation matrix for the ten currency return series. The correlations range from 0.105 for CHF/GBP and CAD/GBP to 0.869 for CHF/GBP and EUR/GBP. For the empirical analysis, we choose the thirteen pairs of currencies for which the correlation is 0.50 or greater. While this is to make the analysis more manageable, it is consistent with the idea that investors would generally choose to hedge using only highly correlated currencies. These thirteen currency pairs are highlighted in the table. For each currency pair, we consider the portfolio that comprises a long position in the relatively minor currency, hedged with a short position in the relatively major currency. As a check, we also conducted the analysis with the relatively minor currency hedged using the relatively major currency. This led to exactly the same qualitative conclusions.[10]

[Table 1]

4. Empirical Results: Minimum-variance Hedging

In this section, we consider the effect of minimum-variance hedging on the risk of each of the thirteen hedged currency portfolios. Specifically, we estimate the minimum-variance hedge ratio using (7) and use this to construct the minimum-variance hedge portfolio. We then consider the effect of minimum-variance hedging, firstly on the standard deviation of the hedged portfolio, and secondly on the VaR of the hedged portfolio. We consider two cases. In the first, we measure the in-sample performance of the hedged portfolio constructed with a static hedging strategy that

[10] These results are available from the authors.

uses the entire sample to compute the hedge ratio. In the second, we consider the out-of-sample performance of the hedged portfolio constructed with a dynamic hedging strategy that uses a rolling window to estimate the hedge ratio.

4.1 In-Sample Results

Table 2 reports the estimated hedge ratio, the standard deviation, the skewness and excess kurtosis coefficients and the 99% one-day VaR, for both the unhedged currency and the hedged portfolio, using the static hedging strategy. We also report the percentage reduction in standard deviation and VaR relative to the unhedged currency. The last line gives the average value of each measure across all thirteen portfolios.

[Table 2]

In all cases, owing to the relatively high correlation between each pair of currencies, minimum-variance hedging yields a substantial reduction in portfolio standard deviation, ranging from a 13.97% reduction for the SGD/AUD portfolio compared to the unhedged currency, to a 50.51% reduction for the CHF/EUR. Thus minimum-variance hedging yields large reductions in risk when risk is measured by standard deviation. However, for all but two of the thirteen portfolios, minimum-variance hedging increases the kurtosis coefficient relative to the hedged currency and in some cases the increase is substantial, such as for the SGD/USD, NOK/EUR and NOK/CHF. Only for the SGD/JPY and SGD/AUD is the kurtosis reduced relative to the hedged currency, and then only slightly. Minimum-variance hedging increases the average kurtosis coefficient across the thirteen portfolios from 2.84 to 6.54. For skewness, the picture is less clear. There is evidence of diversification effects for many of the portfolios, but in others, the skewness coefficient becomes larger in absolute value. For four of the thirteen series, the skewness coefficient becomes more negative. The average skewness coefficient goes from -0.03 to 0.07. These results suggest that minimum-variance hedging substantially reduces standard deviation, has little effect on skewness, but substantially increases portfolio kurtosis. This is consistent with the discussion in the previous section.

We now consider the effect of minimum-variance hedging on the VaR of the hedge portfolio. The reduction in VaR ranges from 11.14% for the AUD/CAD to 49.97% for CHF/EUR. For ten of the thirteen portfolios, the reduction in VaR is less than the reduction in standard deviation and in some cases the differences are considerable. For example, for the SGD/USD, minimum-variance hedging reduces standard deviation by 32.98% but reduces VaR by only 15.49%. For SGD/JPY, NOK/EUR and NOK/CHF, VaR is reduced by more than standard deviation. For these three portfolios, hedging increases positive skewness, which in the case of NOK/EUR and NOK/CHF outweighs the increase in kurtosis. The average reduction in standard deviation across all thirteen portfolios is -27.08%, while the average reduction in VaR is -24.11%.

4.2 Out-of-Sample Results

The static hedging strategy described above cannot be implemented in practice since it requires the ex post optimal hedge ratio. We therefore also consider a dynamic hedging strategy in which the minimum-variance hedge ratio is estimated using a rolling window and used to construct the following day's hedge portfolio. We then consider the standard deviation and VaR of the resulting series of hedge portfolio returns. We use a window length of 1000 observations (approximately four years of data) in order to estimate the hedge ratio, although we also tried window lengths of 250 and 500 but the results are qualitatively similar.[11] The sample used for the evaluation of the dynamic hedging strategy is 04/11/1997 to 09/07/2004 (1745 observations). Table 3 reports the same measures as Table 2 but for the dynamic hedging strategy.

[Table 3]

Comparing Table 3 with Table 2, the dynamic minimum-variance hedging strategy yields a similarly large reduction in standard deviation. Again, however, hedging increases kurtosis in most cases, and for almost half of the portfolios, also increases negative skewness. For four of the thirteen portfolios, minimum-variance hedging

[11] The results for the 250 and 500 window lengths are available from the authors.

reduces VaR by more than it reduces standard deviation, but for the remaining nine portfolios, the reduction in VaR is generally much lower, and in some cases, negligible. For example, for the SGD/USD portfolio, the dynamic minimum-variance hedging strategy reduces standard deviation by 27.95% but reduces VaR by only 3.61%. Minimum-variance hedging seems to yield particularly poor results in terms of VaR also for the CAD/USD, AUD/CAD, SGD/CAD and SGD/AUD portfolios. Using the dynamic minimum-variance hedging strategy, the average reduction in standard deviation is 26.41%, while the average reduction in VaR is only 21.28%.

Thus, minimum-variance hedging, while yielding substantial reductions in standard deviation, simultaneously increases portfolio kurtosis, which in many cases serves to generate much lower reductions in VaR. Based on the in-sample results, minimum-variance hedging reduces standard deviation on average by about 27%, while it reduces VaR by about 24%. Based on the out-of-sample results, the difference is more substantial (a 26% reduction in standard deviation versus a 21% reduction in VaR). Looking at the individual portfolios shows that there is considerable heterogeneity in the effectiveness of the minimum-variance hedging strategy in terms of VaR. In some cases, while the reduction in standard deviation is substantial, the reduction in VaR is negligible. For example, for the SGD/USD portfolio, minimum-variance hedging reduces standard deviation by 30%, while it reduces VaR by only 4%.

5. Empirical Results: Minimum Value at Risk Hedging

In this section, we consider the effect of minimum-VaR hedging, as opposed to minimum-variance hedging, on the risk of each of the thirteen hedged currency portfolios. We consider three questions. The first is whether for a particular portfolio, the composition of the minimum-VaR hedge portfolio is very different from the minimum-variance hedge portfolio. The second is whether, measured by VaR, the risk of the minimum-VaR hedge portfolio is significantly lower than the risk of the minimum-variance hedge portfolio. The third is whether, measured by standard deviation, the risk of the minimum-VaR portfolio is significantly higher than the risk of the minimum-variance portfolio. In order to estimate the minimum-VaR hedge ratio, we employ the historical simulation approach (see, for example, Jorion, 2000). Specifically, we choose an arbitrary hedge ratio, calculate hedge portfolio returns and use the historical simulation approach to estimate the VaR of the resulting hedge portfolio. We then use a numerical optimisation procedure in order to estimate the value of the hedge ratio that minimises the hedge portfolio VaR. This is the minimum-VaR hedge ratio. While the historical simulation VaR of the hedge portfolio is not generally a globally convex function of the hedge ratio, a grid search approach quickly leads to a global minimum. Again, we provide results both for a static hedging strategy that considers the in-sample performance of the hedged portfolio and a dynamic hedging strategy that considers the out-of-sample performance of the hedged portfolio.

5.1 In-Sample Results

Table 4 reports the estimated hedge ratio, the standard deviation, the skewness and kurtosis coefficients and the 99% one-day VaR, for both the unhedged currency and the minimum-VaR hedged portfolio, using the static hedging strategy. We also report the percentage reduction in the standard deviation and VaR relative to the currency being hedged.

[Table 4]

Comparing Table 4 with Table 2, it can be seen that the estimated minimum-VaR hedge ratios are lower than the corresponding minimum-variance hedge ratios for all but two of the portfolios, suggesting that minimising VaR typically involves taking a smaller position in the futures market. The average minimum-VaR hedge ratio across all thirteen portfolios is 0.61 compared to the average minimum-variance hedge ratio of 0.71. The kurtosis coefficient is lower for the minimum-VaR portfolio than for the minimum-variance portfolio for nine of the thirteen portfolios, which is consistent with the idea that minimum-variance hedging attaches no importance to kurtosis and hence leads to portfolio VaR that is not minimised. The VaR of the minimum-VaR hedge portfolio is in every case lower than that of the minimum-variance hedge portfolio, which is to be expected from the in-sample results. In some cases, the differences are not particularly large, but they do nevertheless represent an improvement. For several of the assets, the differences are more substantial. For example, for SGD/USD, minimum-VaR hedging leads to a 19.06% reduction in VaR, while minimum-variance hedging leads to a reduction of only 15.49%. The average reduction in VaR is 25.93% compared with 24.11% for minimum-variance hedging. Clearly then, when risk is measured by VaR rather than standard deviation, some improvement can be made over minimum-variance hedging by focussing explicitly on the minimisation of VaR. In terms of standard deviation, minimum-VaR hedging performs almost as well as minimum-variance hedging with a reduction in standard deviation of 25.98%, compared with a reduction of 27.08% for minimum-variance hedging.

5.2 Out-of-Sample Results

Table 5 reports the estimated hedge ratio, the standard deviation, the skewness and kurtosis coefficients and the 99% one-day VaR, for both the unhedged currency and the minimum-VaR hedged portfolio, using the dynamic hedging strategy. Again, we also report the percentage reduction in the standard deviation and VaR relative to the currency being hedged.

[Table 5]

Comparing Table 5 with Table 3, it can be seen that the dynamic minimum-VaR hedging strategy leads to considerably greater reductions in VaR compared with the dynamic minimum-variance hedging strategy. In several cases, the differences are very substantial. For example, for the SGD/USD, minimum standard deviation hedging yielded a reduction in VaR of only 3.61%, while minimum-VaR hedging yields a reduction of 19.42%. The differences are also substantial for the NZD/AUD, CAD/USD, AUD/CAD, SGD/CAD and SGD/AUD portfolios. For four portfolios (CHF/EUR, SGD/JPY, SEK/CHF and NOK/CHF), minimum-VaR hedging reduces VaR by less than minimum-variance hedging, although for three of these, the difference is not large. The average reduction in VaR is 24.68% compared with 21.28% for minimum-variance hedging. Surprisingly, in many cases, minimum-VaR hedging actually leads to a greater reduction in standard deviation than does minimum-variance hedging. Indeed, the average reduction in standard deviation is 28.63% compared with 26.41% for minimum-variance hedging. This suggests that minimum-VaR hedging can reduce risk more than minimum-variance hedging, not only in terms of VaR, but also in terms of standard deviation.

5. Conclusion

Minimum-variance hedging is commonly used by practitioners, either to offset the risk of a position in the cash or spot market by taking a position in the derivatives market, or as part of an overall investment strategy such as in a hedge fund. While minimum-variance hedging unambiguously reduces the standard deviation of portfolio returns, its effect on skewness and kurtosis, and hence on more general measures of risk such as VaR, is ambiguous. In this paper, we examine the performance of minimum-variance hedging in a VaR framework. We find that minimum-variance hedging, while reducing portfolio standard deviation, substantially increases portfolio kurtosis. Consequently, the reduction in VaR from minimum-variance hedging is considerably lower than the reduction in standard deviation. Indeed in some cases, the out-of-sample reduction in VaR is negligible. The finding that minimum-variance hedging significantly increases portfolio kurtosis is consistent with, and indeed helps to explain the fact that the returns of hedge funds, many of which take a combination of long and short positions either in the same market or across different markets, tend to be highly leptokurtic (see, for example, Kat and Lu, 2002). This finding therefore has implications for both hedge fund managers and their investors.

We also investigate the extent to which minimum-VaR hedging strategies are able to improve upon minimum-variance hedging strategies in the VaR framework, using a historical simulation approach. We find that minimum-VaR hedge ratios are typically considerably smaller than minimum-variance hedge ratios, suggesting that smaller short positions are typically required to minimise VaR than to minimise variance. In terms of hedging performance, the VaR of the minimum-VaR portfolio is of the order of fifteen percent lower than the VaR of the minimum-variance hedge portfolio. Moreover, minimum-VaR portfolios have a standard deviation that is typically only marginally higher than the standard deviation of minimum-variance portfolios. Indeed in some cases, minimum-VaR hedging actually provides a *larger* out-of-sample reduction in standard deviation compared with minimum-variance hedging.

Interesting avenues for future research would include the investigation of the consequences of hedging using derivative instruments, such as futures, and the investigation of hedging in other asset markets, such as equities and bonds. Also of interest would be the hedging of derivative portfolios in the VaR framework. The return distribution of underlying assets such as currencies, equities and bonds are leptokurtic but approximately symmetric, and so the impact of hedging is mainly on the kurtosis of the hedged portfolio, as shown above. In contrast, non-linear portfolios, such as those that contain options, have return distributions that are often heavily skewed, and so hedging may have interesting consequences for these portfolios in terms of skewness as well as kurtosis. Finally, in this paper, we have used the historical simulation approach to analyse the consequences of hedging in the VaR framework. It would be natural to also investigate alternative parametric approaches to the estimation of VaR and perhaps the derivation of an analytical minimum-VaR hedge ratio.

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Table1: Summary Statistics and Correlations

Panel A: Summary Statistics

	Mean	SD	Skewness	Kurtosis	Bera-Jarque
AUD	0.01%	0.69%	0.04	2.05	480.78
CAD	0.01%	0.56%	0.13	1.45	247.46
EUR	0.00%	0.45%	-0.19	1.16	170.56
JPY	0.01%	0.74%	-0.26	3.59	1505.44
NZD	0.00%	0.69%	0.09	2.55	746.12
NOK	0.00%	0.52%	0.25	6.19	4404.48
SGD	0.01%	0.54%	-0.21	3.30	1266.13
SEK	0.00%	0.54%	-0.02	1.27	185.57
CHF	0.00%	0.54%	-0.20	1.43	250.41
USD	0.01%	0.47%	0.05	1.64	308.25

Panel B: Correlation Matrix

	AUD	CAD	EUR	JPY	NZD	NOK	SGD	SEK	CHF	USD
AUD	1.00	0.54	0.24	0.29	0.78	0.27	0.51	0.33	0.13	0.45
CAD		1.00	0.20	0.31	0.47	0.27	0.62	0.34	0.11	0.77
EUR			1.00	0.33	0.27	0.75	0.27	0.67	0.87	0.21
JPY				1.00	0.32	0.32	0.571	0.29	0.36	0.38
NZD					1.00	0.30	0.49	0.32	0.19	0.41
NOK						1.00	0.30	0.66	0.66	0.27
SGD							1.00	0.32	0.20	0.74
SEK								1.00	0.55	0.30
CHF									1.00	0.12
USD										1.00

Notes: Panel A reports the mean, standard deviation, skewness coefficient, excess kurtosis coefficient and Bera-Jarque statistic for daily continuously compounded returns. The sample period is 03/01/1994 to 09/07/2004. The Bera-Jarque statistic has a chi-squared distribution with two degrees of freedom under the null hypothesis that returns are normally distributed. Panel B reports the correlation matrix for the ten return series. The highlighted pairs of currencies are those that are used in the empirical analysis.

Table 2: In-Sample Effectiveness of Minimum-Variance Hedge Portfolio

	Hedge ratio	SD (unhedged)	SD (hedged)	Skewness (unhedged)	Skewness (hedged)	Kurtosis (unhedged)	Kurtosis (hedged)	99% VaR (unhedged)	99% VaR (hedged)	Reduction in SD	Reduction in VaR
NZD/AUD	0.77	0.69%	0.43%	0.09	0.19	2.55	2.62	1.67%	1.13%	-37.00%	-32.60%
SEK/NOK	0.69	0.54%	0.41%	-0.02	-0.01	1.27	2.15	1.30%	1.02%	-24.98%	-21.54%
CHF/EUR	1.03	0.54%	0.27%	-0.20	-0.56	1.43	4.43	1.52%	0.76%	-50.51%	-49.97%
CAD/USD	0.92	0.56%	0.36%	0.13	0.02	1.45	1.99	1.33%	0.97%	-36.23%	-27.23%
AUD/CAD	0.66	0.69%	0.58%	0.04	0.08	2.05	2.24	1.65%	1.47%	-15.98%	-11.14%
SEK/EUR	0.81	0.54%	0.40%	-0.02	0.19	1.27	2.24	1.30%	1.05%	-26.11%	-19.43%
SGD/JPY	0.42	0.54%	0.44%	-0.21	0.17	3.30	3.06	1.42%	1.10%	-17.91%	-22.28%
NOK/EUR	0.86	0.52%	0.34%	0.25	0.88	6.19	21.21	1.31%	0.82%	-33.57%	-37.58%
SEK/CHF	0.55	0.54%	0.45%	-0.02	0.35	1.27	2.62	1.30%	1.09%	-16.22%	-15.95%
NOK/CHF	0.64	0.52%	0.39%	0.25	1.03	6.19	16.07	1.31%	0.92%	-24.96%	-30.00%
SGD/CAD	0.59	0.54%	0.42%	-0.21	-0.46	3.30	6.93	1.42%	1.17%	-21.59%	-17.23%
SGD/AUD	0.40	0.54%	0.46%	-0.21	-0.13	3.30	2.73	1.42%	1.23%	-13.97%	-13.00%
SGD/USD	0.85	0.54%	0.36%	-0.21	-0.89	3.30	16.74	1.42%	1.20%	-32.98%	-15.49%
Average	0.71	0.56%	0.41%	-0.03	0.07	2.84	6.54	1.41%	1.07%	-27.08%	-24.11%

Notes: The table reports the minimum-variance hedge ratio and the standard deviation, skewness coefficient, excess kurtosis coefficient and one-day 99% VaR for the minimum-variance hedge portfolio. The table also reports the percentage reduction in standard deviation and VaR of the minimum-variance hedge portfolio, relative to the standard deviation and VaR of the long asset. The sample period is 03/01/94 to 09/07/04.

Table 3: Out-of-Sample Effectiveness of Minimum-Variance Hedge Portfolio

	Hedge ratio	SD (unhedged)	SD (hedged)	Skewness (unhedged)	Skewness (hedged)	Kurtosis (unhedged)	Kurtosis (hedged)	99% VaR (unhedged)	99% VaR (hedged)	Reduction in SD	Reduction in VaR
NZD/AUD	0.78	0.76%	0.46%	0.11	0.12	2.24	2.73	1.74%	1.19%	-38.78%	-31.76%
SEK/NOK	0.72	0.55%	0.40%	-0.07	-0.19	1.55	2.95	1.29%	1.01%	-27.14%	-21.62%
CHF/EUR	1.03	0.51%	0.24%	-0.15	-0.56	1.26	6.49	1.43%	0.65%	-53.92%	-54.72%
CAD/USD	0.98	0.58%	0.40%	0.20	0.01	1.06	1.33	1.34%	1.00%	-31.01%	-24.94%
AUD/CAD	0.68	0.72%	0.62%	0.02	0.00	2.24	2.02	1.69%	1.56%	-13.51%	-8.08%
SEK/EUR	0.81	0.55%	0.38%	-0.07	0.16	1.55	3.15	1.29%	1.03%	-29.64%	-20.27%
SGD/JPY	0.42	0.58%	0.46%	-0.22	0.18	3.14	2.99	1.45%	1.09%	-20.25%	-24.85%
NOK/EUR	0.86	0.54%	0.39%	0.40	0.99	7.39	18.53	1.22%	0.86%	-28.55%	-29.48%
SEK/CHF	0.60	0.55%	0.44%	-0.07	0.43	1.55	3.82	1.29%	1.06%	-20.20%	-18.08%
NOK/CHF	0.65	0.54%	0.42%	0.40	1.24	7.39	16.59	1.22%	0.93%	-21.72%	-23.84%
SGD/CAD	0.63	0.58%	0.47%	-0.22	-0.52	3.14	6.48	1.45%	1.34%	-18.64%	-7.23%
SGD/AUD	0.43	0.58%	0.51%	-0.22	-0.06	3.14	2.04	1.45%	1.33%	-12.04%	-8.13%
SGD/USD	0.87	0.58%	0.41%	-0.22	-0.90	3.14	13.99	1.45%	1.40%	-27.95%	-3.61%
Average	0.73	0.59%	0.43%	-0.01	0.07	2.98	6.39	1.41%	1.11%	-26.41%	-21.28%

Notes: The table reports the minimum-variance hedge ratio and the standard deviation, skewness coefficient, excess kurtosis coefficient and one-day 99% VaR for the minimum-variance hedge portfolio using a rolling window of 1000 observations to calculate the hedge ratio. The table also reports the percentage reduction in standard deviation and VaR of the minimum-variance hedge portfolio, relative to the standard deviation and VaR of the long asset. The sample period is 04/11/97 to 09/07/04.

Table 4: In-Sample Effectiveness of Minimum-VaR Hedge Portfolio

	Hedge ratio	SD (unhedged)	SD (hedged)	Skewness (unhedged)	Skewness (hedged)	Kurtosis (unhedged)	Kurtosis (hedged)	99% VaR (unhedged)	99% VaR (hedged)	Reduction in SD	Reduction in VaR
NZD/AUD	0.72	0.69%	0.43%	0.09	0.19	2.55	2.77	1.67%	1.11%	-36.79%	-33.79%
SEK/NOK	0.53	0.54%	0.41%	-0.02	0.05	1.27	1.67	1.30%	0.99%	-23.37%	-23.46%
CHF/EUR	1.07	0.54%	0.27%	-0.20	-0.56	1.43	4.33	1.52%	0.76%	-50.41%	-50.34%
CAD/USD	0.87	0.56%	0.36%	0.13	0.04	1.45	1.90	1.33%	0.95%	-36.07%	-28.41%
AUD/CAD	0.54	0.69%	0.58%	0.04	0.07	2.05	2.30	1.65%	1.43%	-15.38%	-13.70%
SEK/EUR	0.66	0.54%	0.41%	-0.02	0.15	1.27	2.14	1.30%	1.01%	-25.14%	-22.39%
SGD/JPY	0.31	0.54%	0.45%	-0.21	0.02	3.30	3.15	1.42%	1.06%	-16.50%	-24.97%
NOK/EUR	0.80	0.52%	0.35%	0.25	0.88	6.19	21.50	1.31%	0.82%	-33.39%	-37.73%
SEK/CHF	0.48	0.54%	0.46%	-0.02	0.29	1.27	2.40	1.30%	1.08%	-15.91%	-17.09%
NOK/CHF	0.64	0.52%	0.39%	0.25	1.03	6.19	16.07	1.31%	0.92%	-24.96%	-30.01%
SGD/CAD	0.40	0.54%	0.44%	-0.21	-0.40	3.30	6.19	1.42%	1.12%	-18.99%	-20.84%
SGD/AUD	0.44	0.54%	0.46%	-0.21	-0.12	3.30	2.60	1.42%	1.20%	-13.81%	-15.31%
SGD/USD	0.52	0.54%	0.39%	-0.21	-0.65	3.30	11.01	1.42%	1.15%	-27.01%	-19.06%
Average	0.61	0.56%	0.42%	-0.03	0.08	2.84	6.00	1.41%	1.05%	-25.98%	-25.93%

Notes: The table reports the minimum-VaR hedge ratio and the standard deviation, skewness coefficient, excess kurtosis coefficient and one-day 99% VaR for the minimum-VaR hedge portfolio. The table also reports the percentage reduction in standard deviation and VaR of the minimum-VaR hedge portfolio, relative to the standard deviation and VaR of the long asset. The sample period is 03/01/94 to 09/07/04.

Table 5: Out-of-Sample Effectiveness of Minimum-VaR Hedge Portfolio

	Hedge ratio	SD (unhedged)	SD (hedged)	Skewness (unhedged)	Skewness (hedged)	Kurtosis (unhedged)	Kurtosis (hedged)	99% VaR (unhedged)	99% VaR (hedged)	Reduction in SD	Reduction in VaR
NZD/AUD	0.71	0.76%	0.44%	0.11	0.18	2.24	2.84	1.74%	1.11%	-41.72%	-36.02%
SEK/NOK	0.59	0.55%	0.41%	-0.07	0.05	1.55	1.75	1.29%	1.00%	-24.69%	-22.73%
CHF/EUR	1.03	0.51%	0.27%	-0.15	-0.48	1.26	4.23	1.43%	0.76%	-48.29%	-47.21%
CAD/USD	0.95	0.58%	0.37%	0.20	0.10	1.06	1.72	1.34%	0.92%	-36.19%	-31.33%
AUD/CAD	0.60	0.72%	0.59%	0.02	0.08	2.24	2.50	1.69%	1.46%	-18.00%	-13.95%
SEK/EUR	0.63	0.55%	0.41%	-0.07	0.08	1.55	2.30	1.29%	1.01%	-25.63%	-21.61%
SGD/JPY	0.47	0.58%	0.45%	-0.22	0.11	3.14	3.43	1.45%	1.11%	-21.16%	-23.16%
NOK/EUR	0.87	0.54%	0.35%	0.40	0.90	7.39	22.05	1.22%	0.84%	-36.08%	-31.04%
SEK/CHF	0.52	0.55%	0.46%	-0.07	0.29	1.55	2.46	1.29%	1.08%	-16.51%	-16.93%
NOK/CHF	0.66	0.54%	0.39%	0.40	0.98	7.39	15.25	1.22%	0.94%	-27.60%	-22.80%
SGD/CAD	0.51	0.58%	0.43%	-0.22	-0.45	3.14	6.52	1.45%	1.20%	-25.64%	-17.32%
SGD/AUD	0.38	0.58%	0.48%	-0.22	-0.10	3.14	2.63	1.45%	1.20%	-16.70%	-17.38%
SGD/USD	0.63	0.58%	0.38%	-0.22	-0.75	3.14	12.66	1.45%	1.17%	-33.95%	-19.42%
Average	0.66	0.59%	0.42%	-0.01	0.08	2.98	6.18	1.41%	1.06%	-28.63%	-24.68%

Notes: The table reports the minimum-VaR hedge ratio and the standard deviation, skewness coefficient, excess kurtosis coefficient and one-day 99% VaR for the minimum-VaR hedge portfolio using a rolling window of 1000 observations to calculate the hedge ratio. The table also reports the percentage reduction in standard deviation and VaR of the minimum-variance hedge portfolio, relative to the standard deviation and VaR of the long asset. The sample period is 04/11/97 to 09/07/04.