Information and Ambiguity: Contrarian and Herd Behaviour in Financial Markets[¶]

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Abstract

The paper studies the impact of informational ambiguity, encapsulated in neo-additive capacities, on history-dependent price behaviour in the standard microstructure model of sequential trading in financial markets. Differences in ambiguity attitudes between market makers and traders are shown to generate contrarian and herding behaviour. The existence of a bid-ask spread can reduce such history-dependent behaviour. We also consider bubbles, market expectations and the bid-ask spread in the long-run.

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1 Introduction

Herding in financial markets has been explored extensively in the literature in recent years[1]. The majority of the existing literature has focused on rational herding which can be catalogued as: informational herding, reputation-based and compensation-based herding. In this paper we confine our attention to informational herding in financial markets; and as well as demonstrating its occurrence we also show that in our framework contrarian behaviour can also arise.

The market structure of our theoretical framework, which we summarise in Section 2, follows that developed by Bikhchandani, Hirshleifer and Welch (1992), based on the work of Glosten and Milgrom (1985), which itself is a derivative of that of Copeland and Galai (1983); and this type of framework has featured in one form or another in much of the subsequent literature, including the influential paper by Avery and Zemsky (1998). We modify that paradigm by introducing the notion of uncertainty or ambiguity, but where uncertainty is captured by non-additive beliefs (Schmeidler (1989)). In effect, either or both of the main players in the market (informed traders and the market maker) are assumed to be uncertain about the probabilities (as in Knight (1921)) of the value of the asset that is being traded.

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[1] Hirshleifer and Teoh (2003) provide an overview of the recent theoretical and empirical research on this topic.

The model of uncertainty we use is Choquet expected utility (henceforth CEU) which is due to Schmeidler (1989). In this model, individuals' beliefs are represented as capacities (non-additive beliefs). This model is able to explain evidence that individuals do not assign subjective probabilities to events, for instance Ellsberg's (1961) paradox.

Experiments on decision-making under risk (i.e. with known probabilities) have shown that individuals tend to overweight both high and low probability events. This can be explained by insensitivity of perception in the middle of the range. For instance. the change from a probability of 0.55 to 0.60 is not perceived as great as the change from 0 to 0.05. It has been argued that the decision weights assigned to events are an inverse S-shaped function over the "given" probabilities of occurrence of those events (see, Gonzalez and Wu (1999), Abdellaoui (2000), and Bleichrodt and Pinto (2000)). That function can be approximated by this simple piece wise linear function,

$$\phi(p) = \begin{cases} 1 & for \quad p = 1 \\ \lambda + (1 - \lambda - \gamma) \cdot p & for \quad 0 < p < 1 \\ 0 & for \quad p = 0 \end{cases} \tag{1}$$

where $\phi(p)$ denotes a subjective of the probability distribution p.

If probabilities are not known, then we have a problem of decision-making under ambiguity. In this case a similar phenomena has been found (see, Kilka and Weber (2001)). Individuals do not assign subjective probabilities to events. Instead they overweight both highly likely and highly unlikely events. (In this case the likelihood of events is subjective.) Chateauneuf, Eichberger and Grant (2006) have axoimatised decision-making under ambiguity. They show that preferences may be represented as a Choquet integral with respect to a neo-additive capacity. A neo-additive capacity is analogous to the piecewise linear distortion of probability in equation (1) but applies to uncertainty rather than risk.

We shall make the assumption that is dominant in the literature that a given agent is risk-neutral, so that the function which maps outcomes into utility is linear. Together with the assumption of CEU preferences with beliefs represented as neo-additive capacities, this implies that preferences may be represented as a weighted average of the expected value of utility, the maximum value of utility and the minimum value of utility. This is expressed as,

$$\lambda Max\,(w) + \gamma Min\,(w) + (1-\gamma-\lambda)\,E_{\pi}w, \tag{2}$$

where $Max\,(w)$ (resp. $Min(w)$) is the maximum (resp. minimum) value of, say, trading an asset, in our model, and $E_{\pi}w$ is the expected value of trading an asset given the set of relevant probabilities, π. The neo in the capacity epitomises the fact it is a *n*on-*e*xtreme-*o*utcome additive capacity.

Knight (1921) maintained that agents differ in their attitudes to ambiguity. The majority of people are ambiguity-averse, behaving more cautiously when probabilities are undefined, while a significant minority of individuals appear to be the opposite, being ambiguity-loving (see the experimental evidence in Camerer and Weber (1992)). In CEU, agents are ambiguity-averse if they put more weight on "bad" outcomes than do EU maximisers, while they are ambiguity-loving if they put more weight on "good" outcomes. We define ambiguity-averse behaviour as *pessimism* (*optimism*) when they place more weight on the possibly low (high) value of an asset.

As our focus is the acquisition of new information, specifically a private signal about the value of the asset received by informed traders, and its impact on prior (public or market) beliefs about that value, it is necessary to formulate a process by which those beliefs are up-dated conditional on the new information. In respect of non-additive beliefs, Bayes' rule is still well-defined. However, Gilboa and Schmeidler (1993) show that the application of Bayes' rule to non-additive beliefs corresponds to 'optimistic updating' in the sense that new information gained is always regarded as good news. It seems, therefore, to conflict with the assumption that players are ambiguity-averse. In fact, there have been a number of proposals for updating CEU preferences (see, for instance, Gilboa and Schmeidler (1993), Kelsey (1995)[2] and Eichberger, Grant and Kelsey (2003)), but no consensus has emerged as to the appropriate updating rule for non-additive beliefs. Rather, it has been agreed that different updating rules are appropriate for different circumstances.[3] The rules, nevertheless, all coincide with Bayesian updating when beliefs are additive. Our chosen up-dating rule for neo-additive capacities is the one proposed by Eichberger, Grant and Kelsey (2003), which they label a Generalized Bayesian Updating rule, since this is consistent with our assumption that agents can be either ambiguity-averse or ambiguity-loving.

Using the neo-additive capacity to capture uncertainty about information, either on behalf of the market maker and/or of the informed trader, we are able to demonstrate that herd and contrarian behaviour can occur. That behaviour arises from rational choices; and so contrasts with the claims of others (such as, Shleifer and Summers (1990 and Kirman (1993)). Indeed, there is a range of market (public) expectations of the value of the asset being traded over which such behaviour can occur. Furthermore, herd and contrarian behaviour arise under the same set of information about the alternative values that the asset can take and the arrival of private signals of informed traders about the probabilities of those alternative values. Therefore, we do not require different informational frameworks to see the possibility of either kind of behaviour, as do, for example, Avery and Zemsky (1998); neither do we need such elaborate frameworks. This we shall see, and comment upon, as we proceed. Our approach introduces uncertainty about the reliability of the informational structure in a way that we could contend encapsulates any uncertainty in one encompassing mechanism; a mechanism that is given credibility by the experimental evidence on the psychology of choice in the absence of (complete) knowledge.

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[2] In this literature, the 'Dempster-Shafer rule' is discussed to update non-additive beliefs, which is shown to be a pessimistic updating rule.

[3] Eichberger and Kelsey (1996) conclude that there is no 'correct' updating rule for non-additive beliefs and argue that the updating rule should depend on the application.

The remainder of the paper is organized as follows. In Section 2 we outline the Glosten-Milgrom market micro structure that we use and the informational properties that we consider. In section 3, we note some basic principles of ambiguity theory, and present the neo-additive capacity and its updating. Section 4 provides (conventional) definitions of herding/contrarian behaviour. In Section 5 we demonstrate the possibility of contrarian and herd behaviour in our framework and present two propositions and corollaries. In Section 6 we provide some (partly heuristic) analysis of bubbles and the dynamics of price behaviour. This is followed in Section 7 by some consideration of the bid-ask spread and the convergence of the price to the market's expectation over time. Some concluding observations are offered in Section 8.

2 Market micro structure

Market Mechanism. The market is for a risky asset exchanged for money among market makers (henceforth, whenever MM appears in the main text, or M as a subscript in equations or as m in Figures, these are all shorthand for market maker(s)) and two types of traders, informed and uninformed, which latter are usually referred to in the literature as liquidity or idiosyncratic or noise traders. The true value of the asset is $w \in \{0, 1\}$, information about which arrives slowly in the market. Trading occurs sequentially and one trader (who can be of either type, of course) is randomly selected in each period. There is an infinite sequence of traders indexed by $t = 0, 1, 2, ...$ MMs set the trading prices at the beginning of each trading period. At any given period t, the selected trader can buy or sell a unit of an asset or not trade, and then must exit the market. We note that we adopt the notation w for the value of the asset rather than what has perhaps become the standard symbol of V, to avoid any confusion later with the capacities that we introduce to replace probabilities, to which we have attached the standard symbol (v).

Traders. Informed traders receive a private signal concerning the value of the asset in addition to the publicly available information (see below) on the past history of trades. If, as in the mainstream literature, they experience no ambiguity about the value of the asset consequent upon receiving their signal, then informed traders base their trading decision on their expected utility of the asset, which equates with its expected value since informed traders are assumed to be risk-neutral: the utility function being a linear function of the value of the asset. In a world of ambiguity, informed traders base their assessment on their CEU (therefore, the Choquet expected value) of the asset. Idiosyncratic traders trade for different reasons, regardless of the price and any other information. Their decisions or pay-offs are not our concern here. In the following, we use traders as a shorthand for informed traders and focus on their behaviour.

Table 1

Signal probabilities

	$P(x=h\|w)$	$P(x=l\|w)$
$w=1$	p	$1-p$
$w=0$	$1-p$	p

The private signals (x) received by informed traders at any point in time concerning the value of the asset, are identical to those proposed by Bikhchandani, Hirshleifer and Welch (1992), being two in number: $x \in \{h, l\}$. These indicate that the value of the asset is "high" (h) or "low" (l). Their probabilities are given in Table 1, and it is assumed that $p > 1/2$. Intuitively, signal l is more likely when $w = 0$ and it can be interpreted as a "Bearish" signal. Similarly, $x = h$ can be interpreted as a "Bullish" signal. We have these expected values of the asset: $E\left[w|x=l\right] < E\left[w\right] < E\left[w|x=h\right]$. These signals have been labelled "value uncertainty" by Avery and Zemsky (1998);

which they label "monotonic signals", to which we refer later. Those signals, effectively, can be seen as defining signals that cannot produce herd or contrarian behaviour given the assumed objective of the informed trader. Avery and Zemsky (1998) introduced a further layer of uncertainty, what they call "event uncertainty" following the finance literature where this was first introduced by Easley and O'Hara (1987) (see, also Easley and O'Hara (1992)). They combine the two types of uncertainty to produce non-monotonic signals and thereby herding behaviour. They also introduce the notion of "composition uncertainty" which when combined with value uncertainty can lead to contrarian behaviour. Again, we only note these concepts here, and refer to them later.

Market Makers. As argued in Chari and Kehoe (2003), including market makers is a convenient way of modelling trade between informed and uninformed traders. We assume, in accord with the literature upon which we are drawing, that there is finite number of risk-neutral market makers under Bertrand competition. Market makers make money from the uninformed (idiosyncratic) traders but lose money to the informed traders, thereby making zero expected profits.

Public belief. Traders act sequentially and observe H_t, the history of actions (trades and their type) and prices (bid, spread), up until time t. We define $\pi_1^t = P\left(w = 1 | H_t\right)$ as the public belief at time t, the probability in effect, that the value is high, conditional on the public history H_t. For any given trader's action $a \in \{s, b, n\}$, where s represents selling, b as buying, and n no trading, the public beliefs update according to the Bayes' rule. We say that a trader's action a is informative when it affects the public belief: $P\left(w = 1 | H_t, a\right) \neq \pi_1^t$. Finally we define H_t as a positive (negative) history if $\pi_1^t > \pi_1^0$ ($\pi_1^t < \pi_1^0$).

Private belief. An informed trader at given t receives a private signal concerning the value of the asset in addition to the public information π_1^t. He then updates the probabilities that the value is high (π_1^t) or low ($1 - \pi_1^t$) and evaluates the concomitant expected value (utility) of the asset, should he hold unambiguous beliefs about those outcomes.[4] If he is selected as the trader at time t he then has to compare this expected value with the bid and ask prices of the market maker. The updating of the probabilities is carried out using the standard Bayesian rule on conditional probabilities. However, the informed traders might not take those probabilities as definitive, being uncertain of their reliability; tending to reduce high probabilities (so, as it were, playing down high outcomes) and to increase lower ones (so, as it were, to raise the prominence of unfavourable outcomes) to accommodate their ambiguity about the outcomes. Such informed traders turn the probabilities into *neo-additive capacities*. Accordingly, the informed traders decide on their trading strategy on the basis of the (signal-updated) Choquet Expected Utility (Value) of the asset, CEU, and not on its (signal-updated) Expected Utility, EU.

Market Makers' belief and price-setting. We follow most (but not all of the literature) in assuming that the MMs do not receive private signals about the value of the asset and have only the same information as that in the public domain. However, if they are assumed to know that private or insider information is available to some market participants, as is the normal supposition, and if they are risk-neutral, then they must set a bid-ask spread around π_1^t (see, for example, Glosten and Milgrom (1985)). Nevertheless, the spread is frequently overlooked or ignored in the literature; with authors taking π_1^t, the trader's expected value of the asset, as the *price* of the asset, or taking it to be approximately so. We do not adopt that procedure, and all the scenarios that we discuss will consider bid and ask prices: in this way the possibilities of no trade are highlighted. Those prices will be assumed to be determined in the way specified by Glosten and Milgrom; and their explicit introduction permits us to introduce notion of event uncertainty of Easley and O'Hara (1987, 1992) in the specific way in which they used it, rather than in the way that it was embodied in Avery and Zemsky (1998). To be precise: Easley and O'Hara consider the situation where, before he sets the bid and ask prices prior to the next trading period, the market maker assigns a probability that a specific signal will have been received by the informed trader: this is their α, which is incorporated in the bid and ask prices below. In addition, the MM might be uncertain about the bid and ask prices that he has calculated, in which event he will be set those prices according to their CEU.

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[4] We note that we follow the published the literature and assume that there are no transactions costs. Additionally, we note that papers by Romano (2006) and Cipriani and Guarino (2006) consider aspects of transactions costs in the Glosten-Milgrom model; though not in our framework.

Bid and Ask prices. The ask price is the price at which the market maker will sell the asset to a potential buyer. Following Glosten and Milgrom (1985) it can be supposed that in the absence of ambiguity, given that the market maker is concerned solely with maximising expected profit (or its expected utility), he must set the ask price so that it equals his expected value (utility) of the asset, should the next trader be a purchaser of the asset. In that way he can make up for the fact that he will gain on average from the liquidity traders but will lose out on average to the informed traders; and his expected profit (or utility) from a sale or a purchase will be zero. The market maker's expected value of the asset then must turn out to be what he sold it for, or what he paid for it. If we assume to begin with that the market maker is concerned with the expected value of the asset then he will set the ask price (A^t) equal to his expectation of the asset's value consequent upon a purchase by the next trader. That is:

$$A^t = E[w_t \mid b] = w_l P[w = w_l \mid \Lambda_t] + w_h P[w = w_h \mid b] \ . \tag{3}$$

Here: w_l and w_h are the two, low and high, values of the asset; P denotes probability; and b denotes the decision to buy by the trader t. Given that $w_l = 0$, its component in A^t is otiose here. Of course, $P[w = w_h \mid b]$ is the probability that the value of the asset is high if the market maker's trade in the next period should be a sale.

From Bayes' Rule:

$$P[w = w_h \mid b] = \frac{P[w = 1]P[b \mid w = 1]}{P[w = 1]P[b \mid w = 1] + P[w = 0]P[b \mid w = 0]} \quad . \tag{4}$$

The conditional probabilities in (4) depend obviously (O'Hara (1997)) on these probabilities: (i) that the trader will be informed or uniformed ; (ii) that the informed trader will have received a signal; (iii) of any high (low) signal, p and $(1 - p)$, respectively, should a signal have been received; (iv) that the informed trader will buy given the receipt of either signal; and, (v) that the uninformed trader will buy a unit of the asset at the ask price when the value is high or low.

Assumptions made about the probabilities of the trader being informed vary. One frequent supposition, which we adopt here, is that the market maker knows, somehow, through the previous history of trading, the proportion of traders who are informed (say, ξ); accordingly, that ratio is used as the probability that the next trader will be informed. Under the notion that there is "event uncertainty" in addition to value uncertainty, Easley and O'Hara (1992) introduce probability (ii) above (their α), the probability in the mind of the market marker that as the trading day opens a signal will have been received by informed traders. In that situation, for example, the probability that the next trader will be an informed trader who will buy the asset under a high signal, will equal $\xi\alpha p$. If we make the additional (standard) assumption that the market maker sets the probability that the informed trader will purchase the asset when he has received a high (low) signal at 1 (0), and that the market maker possesses no finer set of information than the market, so that his priors for the value of the asset as trader t comes to the market must be π_1^t, namely, $P[w = w_h = 1 \mid H_t]$, and $P[w = w_l = 0 \mid H_t] = 1 - \pi_1^t$; we can write the ask price as:

$$A^t = \left[\frac{\pi_1^t(\xi\alpha p + (1/3)(1 - \xi))}{\pi_1^t(\xi\alpha p + (1/3)(1 - \xi)) + (1 - \pi_1^t)(1/3)(1 - \xi)}\right] . \tag{5}$$

On similar assumptions, the bid price counterpart to equation (5) for the situation in which the market trader determines his expectation of the asset's value consequent upon his observing a sale by the next trader is:

$$B^t = \left[\frac{\pi_1^t(1/3)(1-\xi))}{\pi_1^t(1/3)(1-\xi)) + (1-\pi_1^t)(\xi(1-\alpha p) + (1/3)(1-\xi))} \right]. \tag{6}$$

It is easily established that the ask (bid) price is concave (convex) in π_1^t and that $A^t > \pi_1^t > B^t$. The market maker's expected profit from a sale or a purchase must be zero by construction. Thus, the arrival of a trader on the market at t of, for example, a purchaser of the asset, will be:

$$E[(A^t - w)] = (A^t - E[w \mid \Lambda_t])P(\Lambda_t) = 0. \tag{7}$$

Further, it is immediate that the ask (bid) price will increase (fall) when the market maker finds that the proportion of informed traders in the market (ξ) is higher than he previously thought. The intuition behind those responses is self-evident, since he is dealing with better-informed group of traders.

The preceding formulations of the ask and bid prices hold in two of the four scenarios that we consider in the next but one section. In the two other situations they have to be adapted for the ambiguity attitudes of the market maker. Those ambiguity attitudes do not alter the zero expected profit, or zero "expected utility" outcome, but they do affect the relationship between the ask/bid prices and the expected value or expected utility of the informed trader; and thereby they affect the part played by adverse selection in determining prices and the bid-ask spread.

3 Modelling Ambiguity and CEU

We now outline single person decisions when there is ambiguity. Ambiguity is modelled by non-additive beliefs and preferences are represented as a Choquet integral with respect to these beliefs, as in Schmeidler (1989). We begin by stating the now well-known general features of capacities and then turn our attention to neo-additive capacities and to CEU based upon them. Throughout we use the following notation:

Notation We consider a finite set of states of nature S. The set of events is taken to be the set of all subsets of S, which we denote by Σ. The set of possible outcomes or consequences is denoted by X. An act is a function from S to X. The set of all acts is denoted by $A(S)$.

3.1 Capacities and the Choquet Integral

A capacity generalises the notion of probability and assigns non-additive weights to events. We use a special case of the Schmeidler model axiomatised by Chateuaneuf, Eichberger and Grant (2006). In this beliefs are represented as neo-additive capacities, which are defined below.

Definition 3.1 *Let* γ, λ *be real numbers such that* $0 \leqslant \gamma \leqslant 1, 0 \leqslant \lambda \leqslant 1-\gamma$, *and let* π *be an additive probability on* S, *define a neo-additive-capacity* ν *by* $\nu(A) = \lambda + (1-\gamma-\lambda)\pi(A), \emptyset \subsetneqq A \subsetneqq S; \nu(\varnothing) = 0; \nu(S) = 1.$

Chateuanеuf, Eichberger and Grant (2006), show that the Choquet expected value of a function $f : S \to \mathbb{R}$ with respect to the neo-additive capacity ν is given by:

$$\mathrm{CEU}(v) = \int f dv = \gamma \cdot \inf_{s\in S}(f) + \lambda \cdot \sup_{s\in S}(f) + (1-\lambda-\gamma)\mathbf{E}_{\pi}(f).$$

The Choquet integral is like an expectation as it is a weighted sum of utilities. The weight assigned to a state depends on how the outcome is "ranked".[5] For a neo-additive capacity, the Choquet integral is a weighted averaged of the highest payoff, the lowest payoff and the expected payoff. The response to ambiguity is partly optimistic represented by the weight given to the best outcome, λ, and partly pessimistic represented by γ. The neo additive capacity models the observation of Kilka and Weber (2001) that individuals tend to overweight highly likely and highly unlikely events in their decision-making. As a consequence intermediate events are under weighted.

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[5] Gilboa (1987), Schmeidler (1989) and Sarin and Wakker (1992) provide axiomatisations for CEU preferences. Wakker (2001) characterises capacities representing ambiguity-averse or pessimistic attitudes of a decision maker.

Neo-additive capacities allow both optimistic and pessimistic attitudes to ambiguity. We regard optimism (pessimism) as prevailing when the individual over-weights the favourable (unfavourable) outcome. Therefore, a neo-additive capacity represents pure optimism if $\gamma = 0$, and pure pessimism if $\lambda = 0$.[6]

Intuitively, CEU with respect to neo-additive capacities describes a situation in which agents accept the events that underline EU with additive probabilities. However, they lack confidence in this prediction. In part they react to this in an optimistic way measured by λ and in part in a pessimistic fashion measured by γ. We can, therefore, interpret the additive part of CEU, $\mathbf{E}_{\pi}\left(f\right)$, as the agent's belief and $(1 - \lambda - \gamma)$ as his(her) degree of confidence in that belief.[7].

3.2 Up-dating Neo-additive Capacities

To apply CEU with neo-additive beliefs to a dynamic process, it is necessary to model how agents update their beliefs upon the arrival of new information; and thence their CEU. Consider first then the up-dating of the neo-additive capacity conditional upon the receipt of new information. We define the capacity for any event, A given the occurrence of any event F, using the Generalised Bayesian Updating rule axiomatised by Eichberger, Grant and Kelsey (2003).

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[6] Chateauneuf, Eichberger and Grant (2006) define the neo-additive capacity as:

$v(E) = \delta(1 - \alpha) + (1 - \delta)\pi(E)$, which is identical with the definition adopted here when: $\delta = \lambda + \gamma; \alpha = \frac{\gamma}{\gamma+\lambda}$. So that $\lambda + \gamma$ can be interpreted as a measure of ambiguity and α as a measure of ambiguity attitude.

[7] The following examples relate CEU to some more familiar decision rules:

1. if $\lambda = 0$, preferences have the maximin form and are extremely pessimistic;
2. if $\gamma = 0$, preferences exhibit the maximal degree of optimism;
3. if $\gamma + \lambda = 1$, preferences coincide with the Hurwicz (1951) criterion.

The latter has been developed and axiomatised by Arrow and Hurwicz (1972).

Definition 3.2 *Let $\nu = \lambda + (1-\gamma-\lambda)\pi$ be a neo-additive capacity and let F be a subset of S. Then if A is a non-empty subset of F we define the updated neo-additive capacity $\nu_F(A)$ by*

$$\nu_F(A) = \begin{cases} \delta_F\lambda + (1-\delta_F(\gamma+\lambda))\,\pi_F(A) & \text{if } A \subsetneqq F \\ 1 & \text{if } A = F, \end{cases}$$

$$\begin{aligned} \text{where } \delta_F &= \frac{1}{(1-\gamma-\lambda)\,\pi(F)+\gamma+\lambda} \\ \text{and } \pi_F(A) &= \begin{cases} \pi(A)/\pi(F) & \text{if } \pi(F) > 0 \\ 0 & \text{if } \pi(F) = 0 \end{cases}. \end{aligned}$$

This rule has the advantage that the updated preferences can again be represented as a Choquet integral with respect to a neo-additive capacity. Thus we remain within the original class of preferences. A second advantage is that if the original preferences exhibit both optimistic and pessimistic responses to ambiguity so, in general, do the updated preferences.[8]

Lemma 3.1 *The Choquet expected utility with respect to a conditional neo-additive capacity is,* $\mathrm{CEU}(f|v_F(E|\pi,\delta)) = [1-\delta_F(\gamma+\lambda)]\,\mathbf{E}_{\pi|F}(f) + \delta_F(\lambda \sup f + \gamma \inf f)$.

The proof is immediate. Note, the Choquet utility (here also expected value) of a random variable with respect to any conditional neo-additive capacity is well defined even if the conditioning event is an ex ante zero probability event, provided $\lambda > 0$ or $\gamma > 0$. More generally, the more unlikely (in terms of the additive 'prior' π) is the event, the less confidence (the lower is $1-\delta_F(\lambda+\gamma)$) the individual has in the 'additive part of the theory' and the more weight (the greater $\delta(F)$ is) he places on 'extreme' outcomes (in proportion to his relative degree of optimism, λ, versus his relative degree of pessimism, γ). A consistent signal ($F = E$) reduces confidence less than does an inconsistent signal ($F \neq E$).

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[8] As shown in Eichberger, Grant and Kelsey (2003) neither the optimistic updating rule nor the Dempster-Shafer updating rule share this property.

4 Herd and Contrarian Behaviour

4.1 Definitions

We adopt essentially the same definition of herding and contrarian behaviour as Avery and Zemsky (1998). However, we cannot state the definitions in exactly the same way because we work with the actual trading prices (bid and ask) of the market trader, rather than the market's expectation of the value of the asset. Formally, we define herd behaviour as:

Definition 4.1 *(i) Given a positive history of trades, $\pi_1^t > \pi_1^0$, so that π has been rising prior to t, a trader with a low private signal $x = l$ will engage in a herd buy at time t, if $E_{T,x=l}^t(w) > E(A^t)$, since the optimum strategy will be to purchase the asset*; *and, (ii) Given a "negative" history of trades, so that prior to t, π has been falling, a trader with a high private signal $x = h$ will engage in a sell herd at time t if $E_{T,x=h}^t(w) < E(B^t)$, since the optimum strategy will be to dispose of the asset.*

Here, for example: $E_{T,x=l}^t(w)$ represents the informed trader's expected value of the asset, or its EU_T or CEU_T, whichever happens to constitute his decision index, at t given the signal $x = l$.

Concomitantly, we define contrarian behaviour as: $E(A^t)$ represents the expected ask price of the market maker which can be itself, or, identically, its EU_{MM} or its CEU_{MM}.

Definition 4.2 *(i) Given a positive history of trades, $\pi_1^t > \pi_1^0$, so that π has been rising prior to t, a trader with a high private signal $x = h$, will engage in contrarian behaviour at time t, if $A^t > E_{T,x=h}^t(w) < B^t$, since the optimum strategy will be to* *dispose of the asset; (ii) Given a "negative" history of trades, so that prior to t,* π *has been falling, a trader with a low private signal* $x = l$, *will engage in contrarian buying at time t, if* $E^t_{T,x=l}(w) > A^t$, *since the optimum strategy will be to purchase the asset.*

In effect, an informed trader will engage in herd behaviour when, for whatever reason he, as it were, overrides what his private signal is indicating about the value of the asset, and trades in line with market sentiment. Such a trader will engage in contrarian behaviour when he discounts his private signal to trade against the market trend.[9] We note that this overriding of the private signal is the definition of an information cascade.

These outcomes need not arise because of ambiguity in the mind of the informed trader, they might arise because of that or ambiguity in the mind of the market maker, or both. That we shall see in the following section. For instance, it might be thought that when traders are "highly" optimistic in their valuation of the asset, they might buy even when their private signal is negative. However, it might be that the market maker suffers from doubts about his set of information and adjusts his ask price (equation 5) and bid price (equation 6) accordingly. Some degree of pessimism on his part will result in those prices being discounted at "highish" values of the public's expectation and to their being upgraded at "lowish" values of that expectation. Consequently, without any ambiguity in the behaviour of the informed trader, for example, receipt of a low signal might still take his expected value of the asset above the "adjusted" ask price; so with highish and rising prices the informed trader might buy the asset and herd.

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[9] Such behaviour has been labelled by Chari and Kehoe (2003) as generating one of "waves of optimism and pessimism" rather than of herd behaviour. However, logically our definitions do, indeed, define herd and contrarian behaviour by a given trader; if it happens that a sequence of informed traders all appear over a period and they have all receive high signals then should the circumstances delineated in (i) of Definition (4.1), this will coincide with a boom in prices and what seems like a wave of optimism. But it is not necessarily the case that the latter is prevalent in the market from the observed trades; and, under (i) of Definition (4.1) there will not be optimism in the market in the usual sense, since low signals have been received by the traders.

4.2 Analysis of Trading with CEU

We now turn to study in detail how differences in the information, beliefs, and concomitant assessments of the asset's value between the MM and informed traders can generate different trading behaviour and prices. It will be recalled that we assume for simplicity that the market maker possesses the same information set as is publicly available (the past history of trading prices, be they ask or bid prices, plus the knowledge that informed traders exist and receive high and low signals with probability p and $1-p$, respectively). Consequently, to reiterate this, the market expectation of the value of the asset at time t (H_t) is the probability that the value of the asset will be 1: π_1^t.

Any informed trader will update that price/market expected value of the asset, consequent upon his receiving any private signal. It follows from Bayes' rule that the updated belief about that value, namely, the probability that $w=1$ (given that we can ignore the other value of $w=0$) conditional on the type of signal is: [10]

$$\pi(w|x)=\pi_x(w)=\begin{cases}\pi\left(w=1|x=1\right)=\frac{p\pi_1^t}{p\pi_1^t+(1-p)\left(1-\pi_1^t\right)}=\pi_{x_1}(w)\\ \pi\left(w=1|x=0\right)=\frac{(1-p)\pi_1^t}{(1-p)\pi_1^t+p\left(1-\pi_1^t\right)}=\pi_{x_0}(w)\,.\end{cases}\qquad(8)$$

If the trader's objective is to maximise the expected utility of the asset, which here will be its expected value, taking full cognisance of his private signal, then that utility or value will be given by $\pi_{x_1}(w)$, for a high signal and $\pi_{x_0}(w)$ for a low signal in equation (8).

However, should the trader's objective be to maximise his Choquet expected value of the asset, then we need to define how he up-dates his neo-additive beliefs consequent upon the receipt of a signal. Thereafter, we need to formulate his $CEU_T(w)$ based on that up-date.

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[10] As noted, when $w=0$, there is no need to figure out $\pi_1(0)$ and $\pi_0(0)$, since they do not appear in any of the expected value (utility) formulations.

Denote by $v_x(w)$ the conditional neo-additive belief of w given x, then:

Definition 4.3 *The conditional belief of* w *given* x *is the function,* $v_x(w|\pi,\lambda,\gamma) = (1-\delta(x)(\lambda+\gamma))\,\pi_x(w) + \delta(x)\,\lambda$

where

$$\delta(x) = [(1-\lambda-\gamma)\pi(x)+\lambda+\gamma]^{-1}. \tag{9}$$

Correspondingly, the informed trader's CEU with respect to the conditional neo-additive capacity $v_x(w|\pi,\lambda,\gamma)$ is:

$$CEU_T(f|v_x(w|\pi,\lambda,\gamma)) = (1-\delta(x)(\gamma+\lambda))\,E_{\pi|x}(f) + \delta(x)(\lambda\sup f + \gamma\inf f), \tag{10}$$

where, $\sup f = 1$ and $\inf f = 0$.

Without the signal x, CEU_T of the asset's value would be:

$$\begin{aligned} CEU_T(f|v(w|\pi,\lambda,\gamma)) &= (1-\lambda-\gamma)\,E_\pi(f) + \lambda\sup f + \gamma\inf f\ ; & (11)\\ CEU_T(f|v(w|\pi,\lambda,\gamma)) &= (1-\lambda-\gamma)\,\pi_1^t + \lambda. & (12)\end{aligned}$$

Suppose that the market maker's behaviour is characterised by his maximising his $CEU_M(w)$ or, equivalently, the Choquet expected value of the asset, when determining A^t and B^t. His $CEU_M(w)$ for the ask and bid prices will follow immediately from the above:

$$CEU_M = (1-\lambda_M-\gamma_M)A^t + \lambda_M, \tag{13}$$

$$CEU_M = (1-\lambda_M-\gamma_M)B^t + \lambda_M. \tag{14}$$

We show how diversity in expectations and perceived uncertainty between MMs and traders can generate different trading behaviour in the market in different conditions.

(1) *With a private signal but no ambiguity in the perceptions of MMs and informed traders.*

The trader's action now depends upon his EU or the expected value of the asset. They are given by the first expression in equation(8) for the "Bullish" signal and for the second expression for the "Bearish" signal. that they are, respectively, concave and convex is immediate.[11] We observe that both of those equations have identical magnitudes at, respectively, $\pi_1^t = 0$ and $\pi_1^t = 1$.

Now, how do $EU_{T,h}(w)$ and $EU_{T,l}(w)$ relate, respectively, to A^t and B^t? We have already seen that A^t is concave, and B^t is convex, with respect to π_1^t. Consider, for example, the relationship between $EU_{T,h}(w)$ and A^t (equation (5)). It is at once apparent that they will only intersect, in fact, coincide, at $\pi_1^t = 0$ and $\pi_1^t = 1$. However, the one can lie above or below the other (in a diagram such as Figure 1) at all intermediate values of π_1^t. This is most readily seen by choosing an intermediate value of $\pi_1^t = 0.5$. Then:

$$A^t \overset{\geq}{\underset{<}{=}} EU_{T_{x=h}}(w)\ as\ : 3\xi\alpha p^2 + [2(1-\xi) - 3\xi\alpha]p + (\xi - 1) \overset{\leq}{\underset{>}{=}} 0. \qquad (15)$$

Since A^t is an increasing function of α (the probability that a signal has been received) and ξ (the market maker's prediction of the proportion of informed traders in the market), then it is obvious that the higher they are the greater is the probability that equation (15) is negative. In other words, it is possible that should the next trader be one who has received a high signal *no trading* will take place. Even for moderate values of, say, ξ, for a highish value α, $p > 0.5$, can be of such a magnitude that equation (15) is negative. In Figure 1 we have assumed that $\alpha = 1$, which follows Glosten-Milgrom (1985)); $\xi = 0.25$; and, $= 0.7$; and so $EU_{T,h}(w) > A^t$; however, should ξ have been set at 0.5, the outcome would have been reversed.

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[11] Thus, for example, we find that:

$$\frac{\partial EU_{T,h}(w)}{\partial \pi_1^t} = \frac{p(1-p)}{z} > 0; \frac{\partial}{\partial \pi_1^t}\left(\frac{\partial EU_{T,h}(w)}{\partial \pi_1^t}\right) = \frac{-2p(1-p)(2p-1)}{z^3} < 0,$$

where:$z \equiv p\pi_1^t + (1-p)(1-\pi_1^t) > 0; p > 1/2$.

We turn to consider the relationship between the bid price (equation 6) and the expected utility of the informed trader under the low signal, the second expression in equation (8). Again, obviously, the two equations cannot intersect and they are equal in value at the limit points, $\pi_1^t = 0$ and $\pi_1^t = 1$. Also again, let $\pi_1^t = 0.5$; whence the informed trader's expected utility becomes $1-p$. In general:

$$B^t \gtreqless EU_{T_{x=l}}(w) \ as: 3\xi\alpha p^2 - (3\xi\alpha + \xi + 2)p + (1 + 2\xi) \lesseqgtr 0. \tag{16}$$

Consider the case of the receipt of a high signal by the informed trader. Let the market maker's parameters be such that equation (5) lies below that of $EU_{T_{x=h}}(w)$ on Figure 1. Then the trader will trade, buying the asset given his favourable indication as to its value. Even supposing that the market's expectations (π_1) over period (trader) 0 to t, have fallen consistently (along the 45° line) in Figure 1, the trader will not ignore his high signal and engage in herd selling; neither will he engage in contrarian behaviour. This is also the outcome if $EU_{T_{x=h}}(w)$ lies below B^t, the situation illustrated in Figure 1.

In effect, when $E\left[w|x=l\right] < B^t < E(w_t) = \pi_1^t < A^t < E\left[w|x=h\right]$, we have a situation where *the private signals are monotonic*, as defined by Avery and Zemsky (1998). Alternatively, this situation can be encapsulated in the condition that: $1 = P(w = E_{market}^{t=0}(w)) > 0$, given effectively by their definition of *event uncertainty* which arises when: $1 > P(w = E_{market}^{t=0}(w)) > 0$. Essentially, an environment where such an outcome can arise is one where there are at least three possible values of the asset, the third lying between the two extremes outcomes of Table 1: so that, say, $w \in \{0, 1/2, 1\}$. It can then prove impossible to deduce $P(w = E_{market}^{t=0}(w))$ and the priors for the three competing outcomes; so knowledge is incomplete, even for the informed trader who is wishing to up-date the market's valuation, consequent upon his receipt of a signal indicating which of the three outcomes is likely to be the "true" value. With monotonic signals the market has complete information about the probability of (its own) expected value of the asset: here, that is, with only two feasible values for the asset, one of which is 0, the market can deduce the probability of the two possible values of the asset, π_1^0, for $w = 1$; and $1-\pi_1^0$, for $w = 0$.

(2) *With a private signal and market maker ambiguity.*

In this situation we are concerned with the CEU of the market maker, whereby the ask and bid prices are given, respectively, by equations (13) and (14); and in those equations A^t and B^t are given by, respectively, equations (5) and (6). The informed trader's actions are still determined by the values of $EU_{T,x}(w)$, the two expressions in equation (8).

Hence, for easy reference, for the high signal:

$$EU_{T,x=h} = \frac{p\pi_1^t}{p\pi_1^t + (1-p)\left(1-\pi_1^t\right)} \tag{17}$$

and the price that the market maker now decides to ask on the basis of his ambiguity-affected initial price,

$$CEU_{MM}(A^t) = (1-\lambda_M-\gamma_M)\left[\frac{\pi_1^t(\xi\alpha p + (1/3)(1-\xi))}{\pi_1^t(\xi\alpha p + (1/3)(1-\xi)) + (1-\pi_1^t)(1/3)(1-\xi)}\right] + \lambda_M. \tag{18}$$

The companion equations for the low signal are:

$$EU_{T,x=l} = \frac{(1-p)\pi_1^t}{(1-p)\pi_1^t + p\left(1-\pi_1^t\right)}, \tag{19}$$

$$CEU_{MM}(B^t) = (1-\lambda_M-\gamma_M)\left[\frac{\pi_1^t(1/3)(1-\xi)}{\pi_1^t(1/3)(1-\xi) + (1-\pi_1^t)(\xi(1-\alpha p) + (1/3)(1-\xi))}\right] + \lambda_M. \tag{20}$$

We can state this Proposition:

Proposition 4.1 *For given γ, λ for market makers, and traders with no ambiguity, there exist π^*, π^{**} such that:*

(a). If $\pi \in [0, \pi^*]$, with a high signal herd selling behaviour occurs with positive probability.

(b). If $\pi \in [\pi^{**}, 1]$, with a low signal herd buying behaviour occurs with positive probability.

Proof: Figure 1 still portrays the equations for $EU_{T,x}(w)$. Now the equation for A^t, becomes $CEU_{MM}(A^t)$, equation (18). That equation is concave and has the values λ_M at $\pi_1^t = 0$ and $1 - \gamma_M$ at $\pi_1^t = 1$. Consequently, it must intersect $EU_{T,x=h}(w)$ at some intermediate value of π_1^t; say, π^* (Figure 2: where, $\alpha = 1; p = 0.7; \xi = 0.5; \lambda_M = 0.2$; and, $\gamma_M = 0.4$). Suppose that there has been a recent history of falling prices (downwards movement along the 45° line) then when they begin to fall beyond π^* (see, Figure 2), the informed trader will ignore his favourable signal and sell with the herd. Assume now that there has been a low signal. $CEU_{MM}(B^t)$, equation (20), will have identical intercepts at $\pi_1^t = 0$ and $\pi_1^t = 1$ as $CEU_{MM}(A^t)$. It is concave and so it must intersect $EU_{T,x=l}(w)$ at some intermediate value of π_1^t, π^{**}. Consequently, in a period of rising price of the asset, when the price exceeds π^{**}, the informed trader will ignore his own low signal and herd buy; because the market maker discounts the upward trend in the market's expectation of the value of the asset. ■

We may add that on Figure 2, if the bid-ask spread is ignored and the market maker is assumed to set his price at π_1^t, when he is uncertain about that price, $CEU_{MM}(A^t)$, will be replaced by equation $CEU_{MM}(\pi_1^t)$, which is obviously (18) with A^t replaced by π_1^t. $CEU_{MM}(\pi_1^t)$ will be the straight line connecting the two intercepts, λ_M and $1-\gamma_M$ in Figure 2. It will be noted that $CEU_{MM}(\pi_1^t)$ increases the range over which both types of herding occur, which is immediate since $A^t > \pi_1^t > B^t$. Therefore, the bid-ask spread can make it more difficult for herding behaviour to arise, but it cannot eliminate it altogether.

It can also be ascertained from Figure 2 that there are price ranges for the high and low signal over which the *informed trader will not trade.* In the case of the high signal this will be for π_1^t in the range $[\pi^*, \overline{\pi}]$. When there is a low signal the range will be $[\widehat{\pi}, \pi^{**}]$.

Proposition 4.1 gives rise to this Corollary:

Corollary 4.1 *A ceteris paribus increase in the optimism of the market maker increases the range of herd selling by the informed trader, and a ceteris paribus increase in his pessimism increases the range over which the informed trader will engage in herd buying.*

Proof: Consider an increase in λ_M when γ_M is unchanged: this raises the intercept of $CEU_M(A^t)$ at $\pi_1^t = 0$, whilst leaving the intercept at $\pi_1^t = 1$ unaffected in Figure 2, but also reducing its (still clearly positive) slope. However, it is only at the value of $\pi_1^t = 1$ that the old and new $CEU_M(A^t)$ can intersect. So the new $CEU_M(A^t)$ must always lie above the old except at that "intersection point" of $\pi_1^t = 1$.[12] It then follows that the intersection of the given $EU_{T,x=h}(w)$ and the new $CEU_M(A^t)$ must be at a higher value of π^*; and the range over which the informed trader will indulge in herd selling ($0 < \pi_1^t < \pi^*$) must increase. When there is a ceteris paribus increase in γ_M this reduces the intercept of $CEU_M(A^t)$ at $\pi_1^t = 1$ in Figure 2 whilst leaving the value of $CEU_M(A^t)$ at $\pi_1^t = 0$ unchanged; and its slope falls. Therefore, it must intersect $EU_{T,x=l}(w)$ at a lower value of π^{**} than previously. That establishes the second element of the corollary. ■

It is evident that Proposition 4.1 and Corollary 4.1 hold when we ignore bid and ask prices and let the market maker's price be set nominally at π_1^t; when under ambiguity this will be represented by $CEU_M(\pi_1^t)$ on Figure 2. The signals, though monotonic in the strict sense of Avery and Zemsky (1998), are not monotonic in the implied sense that seemed to inspire their definition; because their definition is, in effect, a definition of uncertainty that is formulated to exclude action choices that conflict with private information signals. The uncertainty that causes the herd behaviour could be seen as a way of representing the Avery and Zemsky (1998) event uncertainty in a world of value uncertainty; but here envisaged as doubt in the opinion of the market maker about the reliability of some or of all of the several probabilities that underlie the setting of the ask and bid prices, including perhaps especially the market's expectation of the asset's value. As noted under case (4) below, herd behaviour can also arise when, in addition, the informed trader experiences uncertainty about the probabilities that are relevant to his trading decision; the probabilities of the signals and the reliability of the market's expectation of the asset's value.

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[12] Thus, let λ_{M1} and λ_{M2} depict the lower and the higher values of the market maker's degree of optimism, and the concomitant $CEUs$ be CEU_{M2} and CEU_{M1}, then:

$CEU_{M2} - CEU_{M1} = (\lambda_{M2} - \lambda_{M1})(1 - A^t)$.

(3) *With a private signal, informed trader ambiguity and no market maker ambiguity.*

In this case, the choice criterion for the setting of ask and bid prices by the market maker is the maximisation of expected value (utility) under a high and a low signal, respectively: namely, equations (5) and (6), respectively. For the informed trader the choice of action on the market is given by his $CEU(w)$ under the receipt of either signal. For the "Bullish" signal "Bearish" signal, respectively, these are:

$$CEU_{T,x_h}(w) = [1 - \delta(x)(\lambda + \gamma)] \cdot E_{\pi|x_h} + \delta(x)\lambda, \tag{21}$$

$$CEU_{T,x_l}(w) = [1 - \delta(x)(\lambda + \gamma)] \cdot E_{\pi|x_l} + \delta(x)\lambda. \tag{22}$$

In equation (21) and equation (22), the conditional expectations are given, respectively, by equations (17) and (19), and $\delta(x)$ is given by (9). In regard to the latter, we note that it alters as the signal alters for given ambiguity parameters; since it depends upon the "primitive" values of the probability of each signal at the time the neo-additive capacity is being formed. Clearly, there are many ways in which we can model those two probabilities for a given trader t. However, for our purposes we can make the simplifying assumption that $\pi(x = h) = \pi(x = l) = 1/2$.

We can then state this Proposition, which is illustrated in Figure 3:

Proposition 4.2 *For given γ_T, λ_T for traders, and market makers with no ambiguity, there exist π^*, π^{**} such that:*

(a). If $\pi \in [0, \pi^*]$, then contrarian buying behaviour occurs with positive probability.

(b). If $\pi \in [\pi^{**}, 1]$, then contrarian selling behaviour occurs with positive probability.

Proof: Consider (a): by definition such contrarian behaviour has to occur when the informed trader receives a low signal. In order for him to purchase the asset in that event $CEU_{T,x_l}(w) > A^t$. We note that $CEU_{T,x_l}(w)$ is convex[13], with values of $\delta(x)\lambda_T$ at $\pi_1^t = 0$ and $1 - \delta(x)\gamma_T$ at $\pi_1^t = 1$. Since A^t is concave and is zero at $\pi_1^t = 0$ it must be intersected by $CEU_{T,x_l}(w)$ at some value $\pi^*, 0 < \pi^* < 1$. Therefore, for $0 < \pi_1^t < \pi^*$, $CEU_{T,x_l}(w) > A^t (> B^t)$. If recent prices (and hence, π_1^t) have been falling, even when he obtains a low signal the trader will purchase the asset against the trend. Consider (b): such contrarian behaviour requires that despite receiving a high signal, the informed trader sells the asset in a rising market. That is, $CEU_{T,x_h}(w) < B^t$. $CEU_{T,x_h}(w)$ is concave[14], whilst B^t is convex. Accordingly, they must intersect at some $\pi_1^t = \pi^{**}$; such that $(A^t >)\ CEU_{T,x_h}(w) > B^t$, for $1 < \pi_1^t < \pi^{**}$. So, when there is recent history of rising prices, the informed trader decides to sell the asset over that range of prices, despite the receipt of a good signal.

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[13] Thus: $\frac{\partial CEU_{T,xl}(w)}{\partial \pi_1^t} = \frac{z}{q^2}[(1-p)p]; \frac{\partial^2 CEU_{T,xl}(w)}{\partial \pi_1^t \partial \pi_1^t} = -\frac{2z(1-p)p(1-2p)}{q^3}$.

Here: $z = 1 - \delta(x)(\lambda_T + \gamma_T) > 0; q = (1-p)\pi_1^t + p(1-\pi_1^t)$; so that with $p > 0.5$, both derivatives are positive.

[14] Thus: $\frac{\partial CEUh(w)}{\partial \pi_1^t} = \frac{z}{r^2}[(1-p)p]; \frac{\partial^2 CEU_{T,xh}(w)}{\partial \pi_1^t \partial \pi_1^t} = -\frac{2z(1-p)p(2p-1)}{r^3}$.

Here: $z = 1 - \delta(x)(\lambda_T + \gamma_T) > 0; r = p\pi_1^t + (1-p)(1-\pi_1^t)$; so that with $p > 0.5$, the first derivative is positive and the second is negative.

■

Some observations should now be made about this situation and the illustration of it given in Figure 3 (where $\alpha = 1; \xi = 0.5; p = 0.7; \lambda_T = 0.4$;and, $\gamma_T = 0.2)$[15]. Of economic importance, we can note that situations of *no trade* are feasible over a wide range of prices (see also, Dow and Werlang (1992)). Thus, with a high signal and the representation of $CEU_{T,x_h}(w)$ and A^t in Figure 3, the no trade price range for the informed trader is $[\widehat{\pi}, \pi^{**}]$. For the case of $CEU_{T,x_l}(w)$ and B^t the range is $[\pi^*, \overline{\pi}]$. It can readily be determined that the ranges over which contrarian buying and selling can occur will be smaller than would be the case if the sale and purchase prices of the market maker were identical, as is assumed to be the case when π_1^t (given by the 45° line) is taken to be "the" price set by the market maker. That outcome is simply the result of the relationship $A^t > \pi_1^t > B^t$.

Why does contrarian behaviour emerge when there is ambiguity in the mind of the trader but not in the mind of the market maker? If the trader has an unambiguous view of the true value of the asset and, hence has perfect belief/confidence in his up-dated market expectation of that value (as in Figure 2), when, he receives, for example, a high signal his expected value $EU_{T,x_h}(w)$ must exceed $A^t = EU_M$ (excluding the possible situation of no trade) within the unit interval. However, once the trader holds ambiguous opinions he will have less than complete confidence or belief in his up-dated value of the market's expectations of the value of the asset. His assessment of the value of the asset will then be based on its CEU and, for example, given at least some degree of optimism, will exceed the market valuation of the asset (π_1^t) at zero and near zero values of that valuation (as in Figure 3). As π_1^t rises, CEU_T, increases at a slower rate, since irrespective of the absolute/relative values of the trader's two degrees of ambiguity, he downgrades higher probabilities, so that if we were to take π_1^t as the market maker's price of the asset, as it approaches unity, CEU_T must eventually exceed it. The informed trader, despite his high signal, eventually discounts its value and so reduces his expected value of the asset below that of the market. There will be contrarian selling even in a market where π_1 is rising. That divergence between π_1^t and CEU_T must increase proportionally as π_1^t approaches unity. Consequently, despite having received a signal such that his expected valuation of the asset exceeds π_1^t, the concomitant doubt about the consequent up-dated valuation, causes the individual to adjust the value $(CEU_{T,x_h}(w))$ downwards increasingly on a rising market, such that it implies that the best strategy is to sell the asset against the market trend. The trader engages in contrarian selling, doubting the wisdom of slavishly following market sentiment.

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[15] The values of CEU for the two signals are identical here at $\pi_1^t = 0$, and at $\pi_1^t = 1$, of course, because of the assumption that the prior probabilities of the occurrence of either signal are also identical.

Following on from Proposition (4.2), we can state this corollary:

Corollary 4.2 *An increase in the optimism of the informed trader increases the price range over which contrarian buying behaviour occurs; an increase in the pessimism of informed traders increases the price range over which contrarian selling behaviour occurs.*

Proof: This is straightforward. In regard to contrarian buying, our focus is the receipt of a low signal. In that situation we find that a ceteris paribus increase in the degree of optimism increases the value of $CEU_{T,x_l}(w)$ in Figure 3 at both $\pi_1^t = 0$ and $\pi_1^t = 1$ (see the previous footnote). It follows, therefore, that since the slope of $CEU_{T,x_l}(w)$ is always positive, when it embodies an increase in λ_T must intersect A^t at a higher value of π^* than previously. This proves the first part of the corollary. The second is readily proved in similar fashion. When a high signal is received, an increase in γ_T will reduce the level of $CEU_{T,x_h}(w)$ at $\pi_1^t = 0$ and $\pi_1^t = 1$. Again, since the slope of $CEU_{T,x_h}(w)$ is always positive, it follows that in Figure 3 the new $CEU_{T,x_h}(w)$ must intersect B^t at a lower value of π^{**} than it did previously. The price range over which contrarian selling occurs is increased.■

In Avery and Zemsky (1998) it is necessary for them to combine value uncertainty (that is, the type of information about signal probabilities, such now as that given in Table 1) with their third aspect of uncertainty, what they label "compositional uncertainty". The latter is seen as arising when the probability of informed traders of different types is unknown. This will relate to our ξ now applied to a given trader, t, where, of course, t, also identifies the time at which the trader comes to the market; it being possible that though the traders come to the market sequentially, groups of traders might have the same signals, with some groups maybe possessing better or "stronger" information signals than others. Their argument is that such uncertainty will, given value uncertainty, make it difficult for participants to draw inferences from market prices, and so this can lead to what they call "extreme short-run price effects due to herding" (Avery and Zemsky (1998), p. 735)).

(4) *With a private signal and symmetric ambiguity for both market makers and informed traders.*

There is no necessity to consider this case. It is at once plain from our preceding analyses that when market maker ambiguity is incorporated alongside informed trader ambiguity that this can generate *herd behaviour.* In those circumstances we are able to prove that Proposition(4.1) holds.

5 Heuristics on Herd behaviour, Price dynamics and Bubbles

The possible consequences of herd behaviour are, of course, excess volatility (Shiller (2001)) and, especially, price bubbles. Regarding market information efficiency, conclusions in the literature are mixed[16]. In our framework we will see that herd buying

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[16] For example, Vives (1996) suggested informational inefficiency and implied price distortions in the long-run. However AZ paper suggested the short-run mispricing assets but long-run efficient markets, which our results agree with.

(resp. selling) caused by sufficient optimism (resp. pessimism) can lead to extreme price effects, probably to volatility, maybe to bubbles. This is especially likely to be the case when, for example, the market maker adjusts λ_M and γ_M on the basis of the current price.

Consider, for example, herd buying. From case (2) of the previous section we know informed traders make their investment decision according to the signs of $EU_{T,x_l}(w) - CEU_M(A^t)$ and $EU_{T,x_l}(w) - CEU_M(B^t)$. Herd buying by the informed trader when the market is buoyant occurs when the expected utility (value) of the asset to that trader, even with a receipt of a low signal, exceeds the price at which the market maker is prepared to ask for the asset at a given market's assessment of its value, π_1 at a given next trading date. Such a situation will have occurred because the market maker discounts the value of the market expected value as that value increases beyond some point; so that he becomes more dubious, ambiguous, about what the market will bear in terms of his ask and bid prices, as the market's expectation climbs towards 1. In a sense, we could see this as a process whereby he discounts the market expectation; whilst when the market's expectations are low, the market maker gives greater weight to them than does the market, which takes the probability of an asset valuation of 1 to be π_1^t for time t. Accordingly, the market maker loses out, as it were, further to the informed trader when the expected value of the asset is high and has been rising steadily over recent trades. The informed trader is able to exploit his informational advantage; for even with a low signal further continuing increases in π_1^t increase the informed trader's expected utility (value) at a higher, and at a increasingly greater, rate than they do $CEU_M(B^t)$ and, the more so, $CEU_M(A^t)$. However, the market maker might not lose out to the extent that is implied in the preceding, by adjusting his ambiguity about the price of the asset, as the following suggests.

In this Glosten-Milgrom (1985) market organisation it is difficult to see how the market maker can distinguish whether traders are herding or are trading in response to their own information; because traders arrive sequentially one at a time, do not trade in bulk, and, in any event, might or might not be informed. However, we might conjecture that upon seeing a rising trend in prices, and at the upper end of the possible price range, the market maker might come to believe that more and more traders are informed and are continually receiving a high signal. He will wish to protect himself from this in the normal way by increasing his ask price; but he might adjust that price further consequent upon seeing the price trend, by reducing his pessimism about the high value of the asset (so that γ_M is a negative function of the trend in the past price) and increasing his optimism (so that λ_M is a positive function of the past price trend). In such an eventuality, for example, the value of $CEU_M(A^t)$ will increase at both $\pi_1^t = 0$ and $\pi_1^t = 1$. In that way time he will reduce the amount of the potential for herd buying at each higher value of the price, after the "threshold level" which prompts a reduction in γ_M. However, his adjustment will tend always to lag behind that of the informed trader. The static illustration given in Figure 4 gives some representation of such a dynamic possibility. The graphs labelled $CEUm(A1)$ and $CEUm(B1)$ embody a higher values of λ_M (0.5) and a lower value of γ_M (0.1) than do $CEUm(A)$ and $CEUm(B)$, respectively. Again, the graphs $CEUm(Pi)$ and Pi are, respectively, those of $CEU_M(\pi_1^t)$ and the 45° line.

In fact, it is feasible for the market maker not only to put a halt to herd buying but to curb the boom in prices. If he becomes very optimistic that the true value of the asset is high, he might reduce his degree of pessimism to zero and increase his degree of optimism to a figure that approaches 1. Thus, as an illustration, in Figure 5, we have assumed that $\gamma_M = 0$ and $\lambda_M = 0.8$; the graphs of $EU_{T,x_h}(w)$ and $EU_{T,x_l}(w)$ are as drawn on Figure 4. In these circumstances whatever the signal, informed traders at $t+2$ will sell, engendering a fall in the price of the asset; possibly further causing a downward revision of public expectation of π_1 for the next period's trading. The boom, ceteris paribus, will then be halted.

Consequently, the market maker's actions could lead to price volatility. His (likely) downward revision of ask and bid prices, consequent upon what we must expect would be a concomitant reduction in the public expectation of π_1, would lead, ceteris paribus, to a purchase of the asset should the next trader, $t+2$, receive either a high or a low signal. That outcome might cause the market's expectation of the value of the asset to increase should the sale price in $t+2$ exceed π_1^{t+1}. Then, even if the uncertainty/ambiguity parameters of the market maker remain unaltered, his ask and bid prices for $t+3$ will exceed their values for $t+2$. Thereafter, of course, the permutations as to what might transpire thereafter, given the two possible signals, the possibility that any subsequent trader is informed or a liquidity trader, are too numerous to enable anything other than even more heuristic scenarios to be imagined than our possible portrayal of the emergence of volatility.[17]

6 A note on the long-run: bid-ask spread and convergence of market maker's and the market's expected value of the asset

We have drawn attention to the type of circumstances in which there will be no trade by a given informed trader. In a sense, this causes the market to break down (in regard to trading by informed traders) but the market maker, if he has a stock of the asset, must attempt to re-activate it again. We have offered some heuristic observations on how he might endeavour to do so in course of the discussion of the preceding section. Since there is a positive probability that trade will break down under ambiguity, the ingenious Proposition 3 and proof provided by Glosten and Milgrom (1985) (p. 85-87) that there will be a bound to the bid-ask spread, no longer holds; because it is predicated on the probability that a given type of trade (a purchase or a sale) will occur lies between 0 and 1. However, should it do so then their proposition would hold when the bid and ask prices depend upon the ambiguity parameters of the market maker.

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[17] It is perhaps worthy of mention that our framework could possibly explain house price booms, with estate agents (and valuers) assuming the role of market makers.

Similarly, their Proposition 4 (Glosten and Milgrom (1985), pp. 87-88), that the differences between the ask (bid) price and the market's (public) expected value of the asset converge to zero almost surely as the number of trades increases, so that there is a consensus over the value, is no longer valid in our market environment, given the increased chances that no trades by informed traders will occur under ambiguity. Again, if situations of no trade could be ruled out, the proposition would also hold under our framework of uncertainty. The theorem that leads to the Chebychev Inequality would still be the vehicle through which the convergence almost surely of the ask and bid prices to the market's expectation of the value of the asset could be demonstrated. The presence of ambiguity parameters in the ask and bid prices, for example, under market maker ambiguity, would not affect the convergence; for the resultant expected values of the asset and the expected value are obviously part of the same random distribution, that of π_1 for given t.

7 Concluding Remarks

We have re-examined herding behaviour in a financial market where trade is sequential and prices of assets are endogenously determined. To investigate the effects of ambiguity in financial markets, we modelled agents' beliefs as neo-additive capacities and their preference as CEU. We have demonstrated that herd and contrarian behaviour can be rational for informed traders when ambiguity and ambiguity attitudes condition their trading strategy on the market. Additionally, there is more extensive scope for situations where informed traders can be seen, rationally, not to trade; and the market break down. We offered some heuristic observations on how the herd behaviour could develop into a bubble; which could lead to a crash or to volatility, as a consequence of the reactions of market makers to the uncertain or, ambiguous, information that they can ascertain from the recent trading activity.

The approach that we take in this paper to the modelling of informational uncertainty and attitudes to it is one that has support from the literature on the psychology of decision-making and associated laboratory experiments. It is an approach that does not need the various informational structures that are used in the influential study of Avery and Zemsky (1998). Of itself, it formally only requires the notion which underlies all the literature in this field, that of value uncertainty. The notion that there is ambiguity about relevant informational probabilities, in the minds of either or both the informed trader and the marker maker, can been as a general umbrella for uncertainty concepts such as "value uncertainty" and "compositional uncertainty". In effect, the transformation of probabilities via the neo-additive capacity and the concomitant degrees of ambiguity, can be regarded as embodying the response to those forms of uncertainty.

Figures: The notation that has to be used for the Figures differs slightly from that used in the main text, since Greek letters could not be used in the graphics. However, its equivalent here is almost immediate. Thus: for example: $pi{=}\pi_1^t; pi\hat{} = \widehat{\pi_1^t}$; $pi\tilde{} = \overline{\pi_1^t}$; $CEUt(x = h) = CEU_{T,x=h}; CEUm(A) = CEU_M(A^t)$.

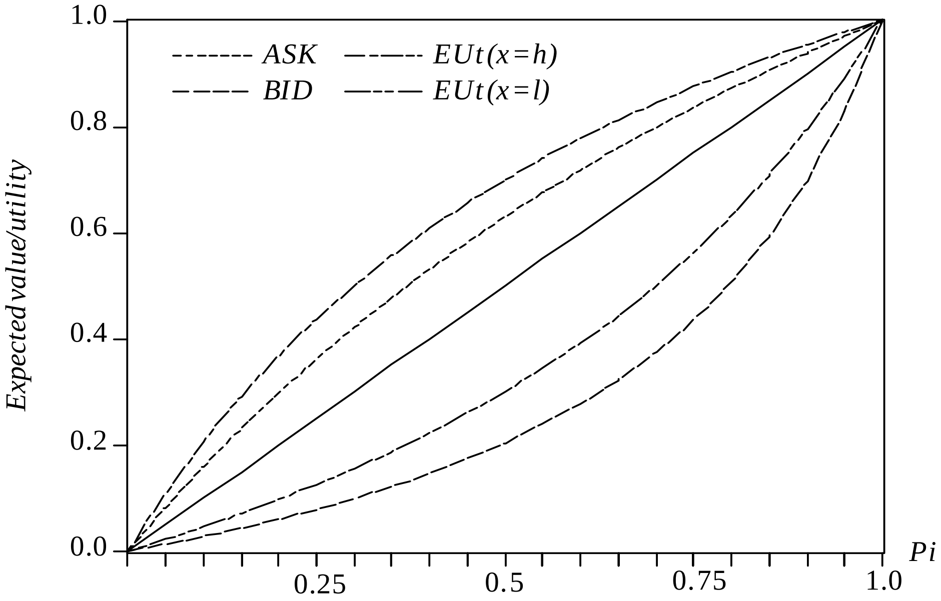

Figure 1: No Ambiguity

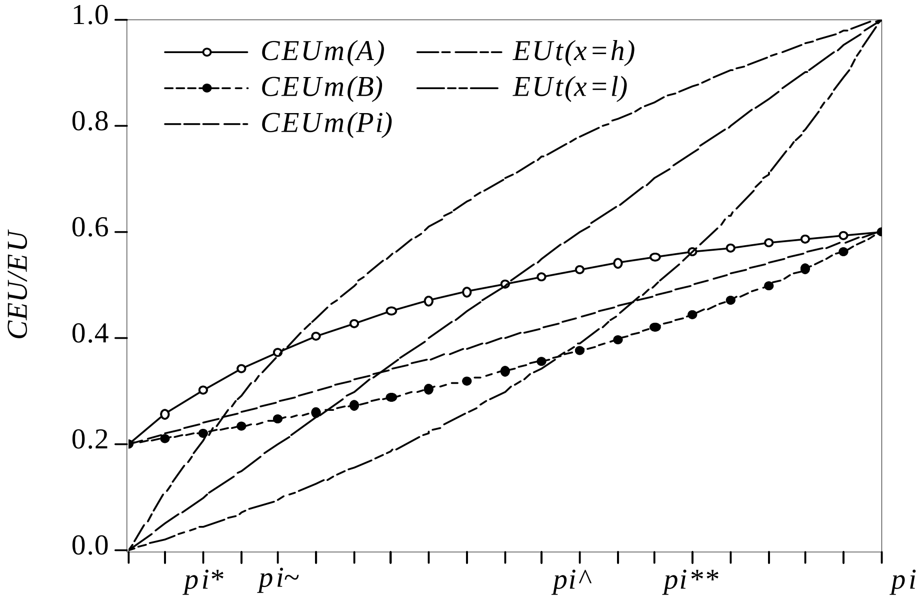

Figure 2: Market Maker Ambiguity

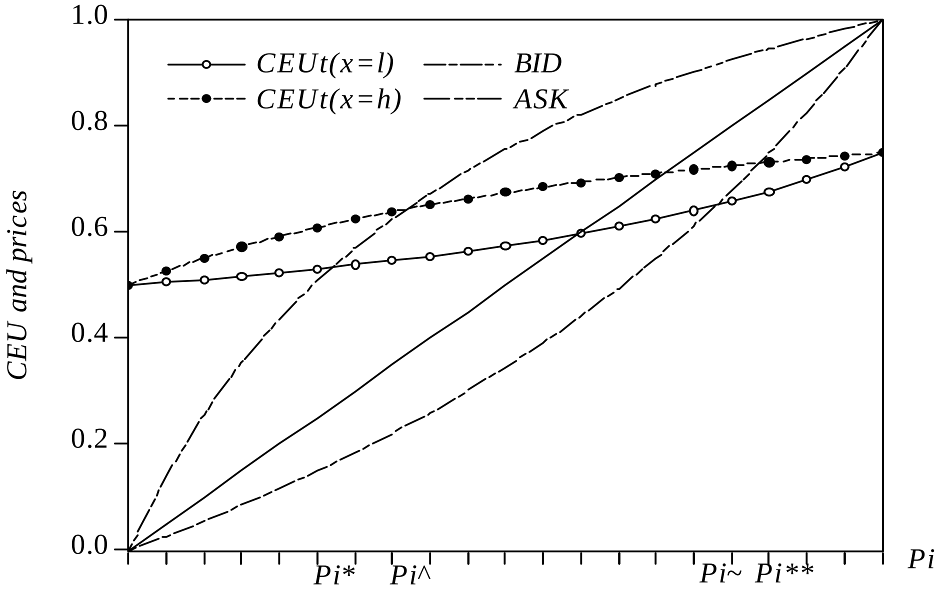

Figure 3: Informed Trader Ambiguity

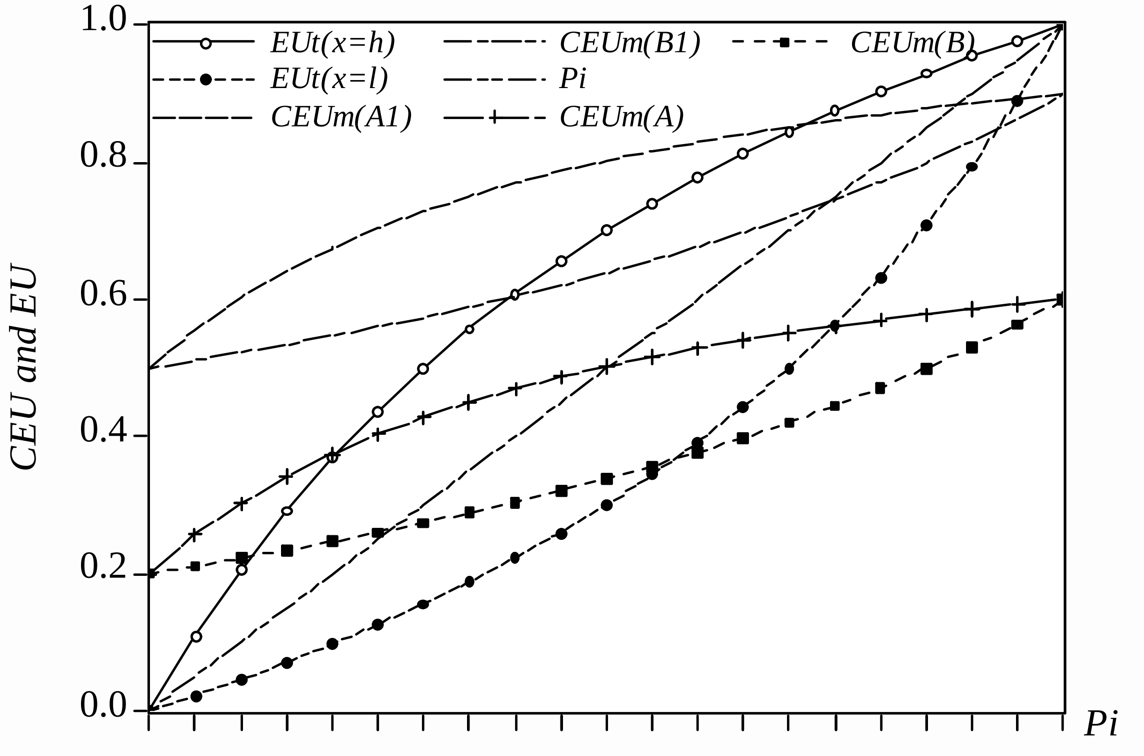

Figure 4: Change in the Market Maker's Ambiguity

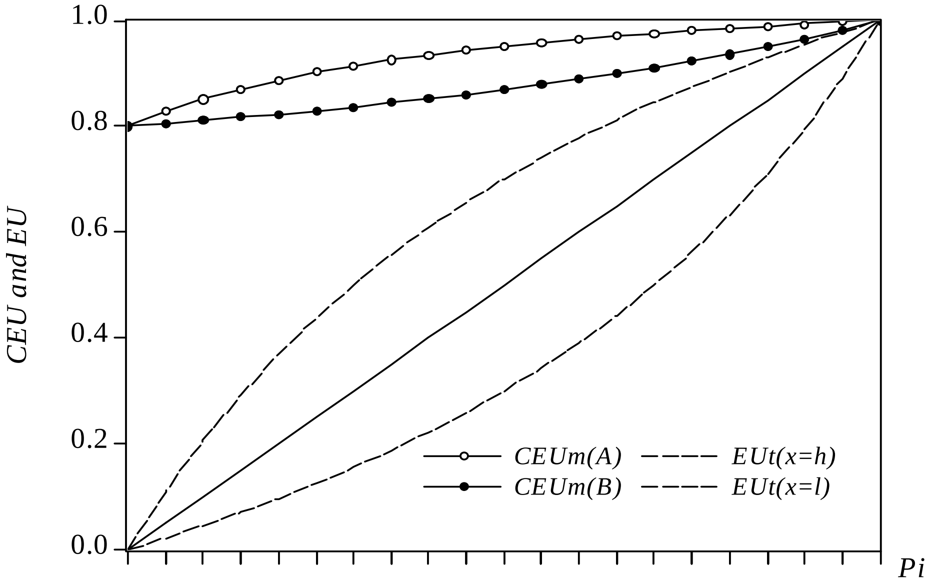

Figure 5: Increase in the Market Maker's Optimism

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