Uncertainty and Financial Fragility

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Abstract

My thesis analyzes various types of uncertainties and their effects on financial fragility in the context of information asymmetries and bank-run models. When various generations of currency crisis are considered, it is observed that the financial system and fragilities associated with it plays a critical role in more recent crisis episodes. Therefore, focusing on the financial system can possibly lead to a better understanding of how and why these crises took place. The analysis presented here aims to provide some new insights about this topic. In the first chapter, I tried to analyze how public borrowing can affect financial fragility when how a private bank finances its lending to the government is private information. I built a simple theoretical model where the government basically borrows from a commercial bank. The objective of the government is to realize borrowing at the lowest possible cost but at the same time it cares about the financial stability. The risk-averse commercial bank, on the other hand, maximizes utility by allocating the financing of its lending among a safe and a risky loan where the amount it uses from the safe source considered to be a measure of financial stability. Moral hazard arises as the amount of safe loan used is not observable to the government. Under the assumption that the risk premium is decreasing in income, I show, when the government is not able push the rate down below a certain level, it can trade a rise in borrowing costs with some financial stability. In other words, although pushing the rate down is good both for borrowing costs and financial stability, under asymmetric information, it may be optimal to design a contract with a reward scheme and accept a higher cost for borrowing for a relatively more reliable financial system. This chapter contributes to the literature by identifying a potential moral hazard problem in the process of public borrowing and displays how it can lead to a higher than optimal level of financial fragility when the economic policy gets obsessed with lowering the borrowing costs. The analysis provided is also interesting as it displays an unusual case where the borrower rather than the lender faces issues resulting from asymmetric information. In the second chapter, a bank-run model used to analyze effects of uncertainty on financial fragility in terms of maturity mismatch. I use
an extended version of the well-known Diamond and Dybvig model by introducing short term borrowing where the future cost of borrowing is unknown. This creates an additional source of maturity mismatch and the demand deposit contracts are now vulnerable to both depositor and lender panics. The key is when the borrowing and investment decisions are made the total cost of borrowing is unknown but the deposit contract can be written contingent on this cost. This creates different consumption paths for patient and impatient agents and they bear different degrees of interest rate risk. The characterization of the contract shows interest risk is mainly borne by early consumers particularly for higher roll over costs. In times of crisis the most liquid funds are the ones that are used first and hence consumers who need urgent liquidity suffers most. The main contribution of this part is that, it combines a bank run model with aggregate uncertainty with short-term borrowing. It also sheds some light on the dynamics of financial problems in developing countries. The last chapter analyzes risk sharing under private banking. Once again a version of Diamond and Dybvig framework is used. Instead of assuming a banking structure where consumers form a union to achieve optimal risk sharing, I consider a private bank that maximizes profits. I analyze the deposit contract under different assumptions about how the bank and the depositors consider the probability of a bank run. The original Diamond and Dybvig model, implicitly assumes the probability of a bank-run is sufficiently small to ensure participation. With a private bank, I allow partial participation and optimizing depositors automatically establish individual rationality. This leads to a supply of deposits (or demand for risk sharing function) which varies along with the payments offered in the contract. Therefore, the bank faces a trade-off between the rates it offer and the amount of deposits it can attract. This basically leads a new set of equilibrium contracts to come out which are not possible under standard risk sharing. Depending on the risk averseness of the consumers these alternative contracts produce different levels of financial fragility. This last chapter contributes to the literature by considering the possible risk sharing contracts under a profit maximizing monopolistic commercial bank. It also briefly discusses how this may affect financial fragility.
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Chapter 1

Introduction

The notion of financial stability became quite popular in the world of economic research in the past decade. Moreover, it is also established as one of the key parameters of how economic policy is conducted. This was mainly due to the nature of the relatively recent crisis episodes which took place after the first half 1990s. The first and second generation models, while being able to explain the dynamics of the former currency crises quite adequately, failed to account fully for the happenings in the more recent episodes. The key factors of the first and second generation views such as fiscal imbalances, output or employment costs of defending the exchange rate pegs were partly or completely nonexistent in these relatively recent occurrences. Instead, a common feature of them was some sort of banking system failure which eventually led to a sudden reversal of international capital flows and hence the abandoning of the exchange rate peg. In almost all cases the currency crash came with a financial breakdown. This suggests a third generation view which assigns a critical role to domestic financial system, its institutional structure and vulnerabilities.

This thesis aims to provide some new insights in financial fragility in the context of information asymmetries and banking system failures. It is mainly inspired by the financial crisis occurred in November 2000 in Turkey, which eventually led to the collapse of the current pegged exchange rate regime and a severe devaluation of the domestic currency (Turkish Lira -TL) in February 2001. The former incident in
November was due to a liquidity shortage in the domestic money market and the structure of the economic policy at that time made it difficult to handle the problem. Although it was eventually solved, the damage on expectations was proven to be permanent and played a critical role in the inevitable devaluation three months later. This also meant the end of the exchange rate based stabilization program launched in January 2000.

Past experience showed, attacks on central bank’s foreign exchange (FX) reserves and a result of severe devaluation are far from being surprising when fixed exchange rate regimes are considered. However, what makes the Turkish case more interesting is the effect of the former liquidity crisis on expectations, management of which is critical in exchange rate based stabilization programs. The risks associated with this regime has already been anticipated in the economic program and therefore it was designed to have three stages. For the first eighteen months the value of the FX basket was announced in advance. This was the first stage where the exchange rate was pre-determined. After that, for the second stage, a temporary transition period was planned where the exchange rate would be left floating in a gradually widening band. In the third and the final stage switching to a fully floating exchange rate regime would be realized. The first ten months of the program was quite successful. Just before November, although there were a couple of red flags such as increasing real exchange rates and widening current account deficit, these were all expected and credibility of the program was still strong. The turning point was the liquidity shortage in November after which a fast deterioration of economic expectations was initiated. Therefore, it is still argued, whether the ultimate collapse of the program in February might have been avoided if the incident in November had not happened. Although there exist different views about if it was a mistake to initiate a risky program like that at the beginning, most of the economists, analysts and policy makers agreed on one particular thing, what happened in November was the beginning of the end.

\[^1\text{For more information on exchange rate based stabilization programs and the risks associated, see (Mendoza & Uribe 1999) and (Calvo & Vegh 1999)}\]
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Given the widely accepted importance of the November incident in the whole process, the fragility of the Turkish financial system has become the main discussion topic of the time. There exists several studies that analyze the crisis and the role of financial fragility. These studies, which are reviewed later in this chapter, are assessments of the crisis in the light of historical developments in Turkish economy, data concerning the financial markets and economic policies of 1980s and 90s. It is widely accepted that, high public sector borrowing requirement (PSBR) contributes to the financial fragility in Turkey. This aspect of the economic structure is reinstated by several studies in explaining the origins of 2000-2001 financial crisis.

In the context of this introduction we provide a summary of currency crisis models next, focusing mainly on the third generation models which place financial fragility at the center of the problem. In the following subsection, we review the recent economic history of Turkey and discuss the public debt and its effects on the financial structure with reference to existing literature.

1.1 Currency Crisis and Financial Stability

The first generation of currency crisis models relies mainly on the fiscal imbalances caused by expansionary policies. Krugman (1979) shows how in a pegged regime the government loses its control over the way of budget deficit being financed. If the government keeps on running deficits, this eventually lead to a reduction in FX reserves. At a certain stage before the reserves are completely exhausted, a sudden speculative attack drains the rest of them leading to the collapse of the currency peg. Krugman (1979) also mentions the speculative attack is costless for the individual investor even when the currency peg remains intact and therefore it is likely to happen when the reserves are running low. This main scenario of the first generation models is also supported by the empirical evidence of 1970s and 1980s. The crises in Mexico (1973-1982), in Argentina (1978-1981) can be explained quite accurately with the features of first generation models. The excessively expansionary pre-crisis...
fundamentals are a common feature of these episodes.

Krugman’s analysis builds on the model by Salant & Henderson (1978) in which they study a stabilization scheme where the government uses a stockpile of a certain resource to stabilize its price. They display how this process ends with a speculative attack in which the agents acquire the remaining stock of the resource. While applying this framework to the preservation of the exchange rate peg by using the reserves, Krugman was not able to provide the exact timing of the collapse in the fixed rate regime. Following his work, Flood & Garber (1991) show how an explicit solution for this timing can be derived in a linear model. Agenor, Bhandari & Flood (1992) provides a survey about how the Krugman-Flood-Garber insight is extended and applied to various crisis episodes. The analysis of Mexican (Blanco & Garber 1986) and Argentinian (Cumby & van Wijnbergen 1989) experiences both identify domestic credit growth as a critical factor behind the speculative attacks.

The main feature of the first generation models is that they are based on some kind of policy failure. The fiscal deficits compounded with fixed exchange rates and the resulting loss of the control over deficit finance is the main factor that leads to the depletion of the reserves and eventually triggers the speculative attack. However the speculative attacks of 1990s put some doubt on this view as in many of them the fiscal disequilibria was not present. Moreover, these new episodes showed the crisis may occur when the central bank still have a considerable amount of reserves. After the first generation view fell out of fashion in the early 1990s, Obstfeld (1994) proposed a second generation view which is mainly founded on the real costs of defending the peg. In other words, the government may choose to abandon the exchange rate peg not because it simply ran out of resources but because it finds it too costly for other objectives such as employment or growth. This new approach also involves a possibility that crisis can be driven by self-fulfilling expectations. The anticipation that the peg cannot be maintained can be enough for the it to collapse. This is because the cost of defending the fixed rate could be well related to the expectations about its sustainability. If many agents doubt the fixed rate system is going to be abandoned, defending it becomes extremely costly and at some point
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The speculative attacks in Europe in 1992-93 sets a typical example of second generation type of crisis. The European Exchange Rate Mechanism was collapsed by a self-fulfilling prophecy where the investors’ doubted about how the others evaluate the sustainability of the parites. Obsfeld’s emphasis of mounting unemployment and recession also fits to the European experience. The self-fulfilling attacks on the currency can also be explained by information cascades [Banerjee (1992) and Bikhchandani, Hirshleifer & Welch (1992)]. The cascades story considers a sequential decision rule where each agent relies on the actual observation of others’ actions when making a decision of her own. An investor may disregard his own information or belief that the peg will be maintained if she observes sufficiently more agents who came before her acted differently. Therefore, the sequential rule leads to herding behavior. Calvo & Mendoza (1996) on the other hand, depart from the sequential decision making framework and consider a global market where the actions are taken simultaneously. They show that as the world capital market grows, due to information frictions, incentives of obtaining country-specific information is reduced. Therefore, small rumors may initiate herding behavior and lead all the investors to move their capital from one country to the other.

The first generation type of crisis were considered to be much more predictable as the indicators of them can be easily observed. However, the self-fulfilling nature of the second generation crisis make them much more difficult to predict. What really causes the negative expectations which eventually derive the economy to an attack equilibrium is not really obvious. Therefore, after the early 1990s the researchers who want to understand the origins of the recent crisis have turned to exploratory empirical models (Flood & Marion 1999). Eichengreen, Rose, Wyplosz, Dumas & Weber (1995) construct a speculative pressure index using a panel of 20 countries. The pressure index is a weighted average of exchange rate, interest rate and reserve changes. The extreme values of this index relative to its mean are then defined as crisis episodes. Their selection method did a good job in catching historical episodes if the crises have large unpredictable components. However, this also means that
they are also less likely to be correlated with variables selected as crisis determinants and thus the empirical methods may not be able to predict these crises very well. While Eichengreen et al. (1995) use data from only industrialized countries, Kaminsky, Lizondo & Reinhart (1998) include developing countries in their pane hoping the variation in data may help increasing the predictive power of their model. Sachs, Tornell, Velasco, Calvo & Cooper (1996) perform a similar analysis using a panel consists of only developing countries. Both studies manage to find some variables that are important indicators of the pressure index.2

Although the self-fulfilling scenario of second generation view remained to be accurate for the most recent episodes of currency crisis, the mechanism that was put forward by Obstfeld (1994) was not able to account fully for the pre-crisis dynamics of these episodes. The second generation view emphasizes the output and unemployment costs of defending the peg and relate these costs to the anticipations of the agents about the future commitment to the fixed rate system. However, weakness in real activity was not a feature of Mexican crisis in 1994 or the Asian crisis in 1997. On the contrary, the Asian countries were growing quite quickly not very long before their financial meltdown. The main features of these recent episodes were troubled financial system and sudden reversals of short-term international capital. In almost all cases including the Asian experience, financial system was collapsed along with the currency. This suggests a new generation of models where the domestic banking sector, its structure and other fragilities play a critical role.

The joint occurrence of banking and currency crisis, often named as "twin" crisis was a common feature of the many recent crisis episodes. In fact, in an open economy, given that the foreign assets and liabilities are parts of domestic bank balance sheets, expecting a connection between banking and currency crisis is quite reasonable. The third generation models are based on this basic scenario. The problems in the domestic banking sector and the behavior of the domestic banks lead to the currency attacks eventually. With the help of the latest technological

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2See also Berg & Pattillo (1999) and Frankel & Rose (1996) for early indicators of currency crisis.
developments, international capital has become extremely mobile recently. This urges the countries who choose fixed exchange rate systems to have a safe and sound domestic financial system. This is because, any trouble in there may easily lead short-term international capital to flow out of the country and hence causing a currency crash.

The main explanations for banking crisis are moral hazard problems, large macro-economic shocks that cause drastic changes in the value of assets or excessive risk taking behavior by domestic banks. The mismatch of maturities of assets and liabilities is one of the most common risks that may easily lead to bank-runs. The structure of the international capital market is also very suitable for this kind of risky behavior. The short term international capital is often cheap and attractive for domestic banks. On the other hand, they may choose to invest in longer term assets. Given the mobile nature of the capital, a panic by foreign creditors may easily lead the bank to fail before the maturity of its assets. The celebrated work of Diamond & Dybvig (1983) shows how an attempt of risk sharing can end up with a bank run because of the maturity mismatch. Chang & Velasco (2001) extended their work for a small open economy and study the foreign creditor panics as well.

There are a few empirical studies that investigate the association between banking and currency crisis. Kaminsky & Reinhart (1999) find banking troubles help predict currency crisis. Eichengreen & Rose (1998) studies the influence of exchange rate volatility and the regime on the probability of banking crisis for a large sample of developing countries. Glick & Hutchison (2000) report twin crisis phenomenon is specific for emerging market economies that are financially liberalized and cannot be generalized for a broader set of countries. They also examine the value of banking crisis in predicting currency crisis and vice versa by using a signal-to noise ratio methodology and study the lagged relationship between them by using probit regressions. Their results show that, in emerging market countries banking crisis is a strong indicator for future currency crisis. On the other hand, currency crisis does not appear to a good signal for future banking problems. Goldstein, Kaminsky & Reinhart (2000) provides empirical tests of early indicators of banking and currency
crisis in emerging markets while studying the interaction between the two.

When it comes to theoretical work, Obstfeld (1994) mentioned there might be a link between banking and currency crisis in the context of the second generation models. He argues weak banking system can cause currency crisis if the government chooses inflation over exchange rate stability. In other words, because of the banking problems defending the peg may become too costly in terms of inflation which eventually causes the fixed rate system to be abandoned. He also mentions the pressure on exchange rate can cause a banking crisis if the government chooses to defend the peg by raising the interest rates. Therefore, he defines the link consistent with the main story of the second generation models. There exists other theoretical studies that consider a causality from banking sector distress to currency crisis. The government bail-outs are quite common in case of banking sector problems. Velasco (1987) and Calvo (1998) argue that if the extra liquidity created by this bail-out is inconsistent with the fixed rate regime it can lead to currency attacks. Miller (1999) states a devaluation might be one of the plausible choices of policy for a government when it is facing a threat of a bank-run. González-Hermosillo (1996) shows that substitution of foreign assets for domestic assets by agents can lead to currency crisis in weak financial systems.

The simultaneous occurrence of banking and currency crisis may be due to a common factor. The reversal of international capital can be one of these factors. When creditors panicked, refused to roll-over and demanded immediate pay back in Asia, there was almost no option left for the government to avoid the crash of both the exchange rate regime and the financial system. Radelet & Sachs (1998) argue Asian crisis was caused by a boom of international lending and then a sudden withdrawal of funds. They place large scale capital inflows into the financial system at the core of the crisis. Chang & Velasco (1998) also defined Asian episode as a crisis of international illiquidity. They analyze financial liberalization, shortening maturity structure of the debt and foreign exchange composition of assets and liabilities to explain how financial crisis turns into currency crisis where the government tries to fix the exchange rate and act as a lender of last resort at the same time. Rodrik &
Velasco (1999) study short term capital flows in the context of the maturity and the cost of external borrowing. They show empirically that short-term debt to reserves ratio is a robust predictor of financial crisis and higher short-term vulnerability causes more severe crisis in case of the reversal of international capital. Goldfajn & Valdes (1997) also display how adverse shocks of productivity or international interest rates magnified by the behavior of foreign investors can trigger sudden outflow of capital.

Another common factor may be financial liberalization. Considering information problems in a poorly developed banking sector, excessive international openness may lead banks to take on risky portfolios with foreign currency liabilities. McKinnon & Pill (1996) emphasize the role of financial fragility in creating the dynamics for twin crisis. They argue financial liberalization coupled with deposit insurance may lead to a lending boom which eventually causes a banking and currency crisis.\(^3\)

Mentioning of deposit insurance by McKinnon & Pill (1996) is also quite important since along with other factors, the excessive leverage and guarantees play a critical role in banking sector weakness and vulnerability. Goldstein (1998) captured these guarantees together with insufficient capitalization, weak supervision and directed lending under the term "crony capitalism". He records this structure as one of the key features of Asian countries before the crisis. Corsetti, Pesenti & Roubini (1999) also talks about guarantees for external liabilities and mentions moral hazard as the main reason for overinvestment and excessive borrowing. Similarly, Krugman (1998) points out an implicit government guarantee on liabilities in Asian countries and too much short-term borrowing by financial intermediaries due to moral hazard. He states this borrowing behavior led to asset price inflation which made the banks' financial condition look better than it was. Once the bubble burst the asset prices fell drastically and the banks became insolvent.

Considering the observations and empirical finding of the existing academic work on currency crisis, Krugman (1998) summarizes the main aspects of crisis preven-

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\(^3\)See also Demirguc-Kunt & Detragiache (1999) for the relation between financial liberalization and financial fragility and the link between banking and currency crisis.
tion under four tactical elements. First is maintaining a strong domestic financial sector. Well capitalized and supervised banks, effective corporate governance and bankruptcy codes and credible means of contract enforcement are listed as important features of such a system. It is also mentioned, the amount of debt that can be sustained depends on how strong is the financial structure. The second element is the choice of exchange rate system. The trade-off between an independent monetary policy and flexibility of the exchange rate regime is emphasized. A sound and stable policy environment where, the vulnerabilities are minimized and fiscal deficits beyond the sustainable financing capacity are avoided, is listed as the third issue. The last but not the least is the reduction of vulnerability against rollover and balance sheet risks. Avoiding policy biases toward short-term capital and having foreign liabilities that can become a claim consistent with the economy’s foreign reserves are pointed out as two crucial strategies.

The Turkey 2000-2001 experience has many similarities with some of the crisis models outlined above. Next section discusses the main characteristics of Turkish economy and tries to identify the main features of the banking and currency crises experienced at that time.

1.2 A Review of Turkish Economic Structure

In this section, we focus on the past three decades of the Turkish economy prior to the crisis period to help us understand the structure that leads to the problems of the financial system better. In 1980s and 1990s, Turkish economy is associated with high and chronic inflation, extremely volatile growth rates and continuously increasing public debt stock. There were two crucial incidents in 1980s that would shape the economic developments in the following decades. First one is the choice of issuing domestic debt as the main method to finance budget deficits starting from 1986. Figure 1.2.1 displays the course of public sector borrowing requirement and domestic debt stock as a share of the gross domestic product. The borrowing need
of the public sector has gone up to 6.8 percent of the GDP on average in 1990s from 3.8 percent in 1980s. The ratio of domestic debt stock to the GDP, on the other hand, was on an upward trend particularly after 1990, exceeding 20 percent by the end of the decade.

![Figure 1.2.1: PSBR and Domestic Debt Stock](source)

More interestingly, the change in the method of finance can clearly be seen in Figure 1.2.2 which shows the shares of treasury bills and government bonds in the total domestic debt stock. The share of total government securities in the domestic debt stock was around 22 percent in 1986. Beyond that time it followed a steep upward trend and in 1998 all domestic debt was financed by issuing bonds and t-bills. The maturity structure of the debt instruments also followed an unfavorable course making it more and more problematic in time to service the domestic debt. The share of t-bills (securities that have a maturity shorter than one year) in domestic debt stock relative to the share of bonds (securities that have a maturity longer than one year) has gone up after 1994. Rolling over this debt became the central issue of the economic policy in 1990s and thereafter.
The second incident was the full liberalization of the capital account in 1989 causing a fast dollarization in the economy in 1990s. Starting from the second half of the 1980s capital mobility increased globally. This led many middle-income countries to liberalize their international accounts by easing or removing the controls over capital flows (Celasun 2002). Turkey followed this trend by liberalizing the international capital flows in 1989. Celasun (2002) explains the motivation for this decision with the current economic conditions. He states the average growth rate which was 6.4 percent in 1984-87 has fallen to 1.6 percent in 1988-89 period, the net capital inflows have dropped below one billion dollars and real wages have become unsustainably low at that time. Therefore, the liberalization was motivated by the aim of raising real wages and public expenditures to initiate a boom in the economy via increased capital inflows. However, Celasun (2002) also mentions the liberalization is often criticized of being premature because it was not backed up by a sound macroeconomic structure. Inflation was still high, FX reserves were inadequate and financial supervision was quite weak. Cizre-Sakallioglu & Yeldan (2000) lists the main goals of financial liberalization to be achieving fiscal and monetary stability and business confidence, promoting stable economic growth, encouraging privatiza-
tion and controlling inflation. However, they argue it brought more instability and a worsened the income distribution. The problems associated with the premature financial liberalization in late 1980s are also stated by other studies such as Ertugrul & Selcuk (2001), Boratav & Yeldan (2006), Yeldan (2001a), Onis (2004) and Rodrik (1990). Onis (2004) summarizes the severe consequences using the following words.

"The decision to liberalize the capital account in an environment of high degree of macroeconomic instability and the absence of an adequate institutional framework to regulate the financial sector rendered the Turkish economy highly dependent on short-term and highly speculative capital flows. Short-term capital inflows magnified the degree of instability in the Turkish economy as political actors used these funds to finance rising budget deficits thereby postponing costly adjustment decisions to the future. It was not surprising in this context that Turkish economy experienced successive financial crises with serious real economy consequences."

The liberalization of the capital account led to a fast dollarization in the economy which played a critical role in today’s economic structure in Turkey. Here we can talk about two key characteristics of the Turkish economy. The first one is the high and chronic inflation in Turkey with the volatile economic growth performance. The dollarization caused a high rate of pass through from exchange rates to prices. The main reason for this is of course the increased openness of the economy after the policy reversal. Ozkan (2005) shows how the degree of openness (trade volume to GDP ratio) made two steep jumps after the trade liberalization in early 1980s and the capital account liberalization at the end of the decade. However, due to real appreciation of TL, the increased trade volume was mainly in the favor of imports during 1990s and in time Turkish economy became highly import oriented. Moreover, a great deal of imports are intermediate goods which makes the prices of imports and hence the exchange rates a major determinant of production costs. This boosts exchange rate pass-through even further making the economy extremely
vulnerable to external shocks. As a result, both inflation and economic growth are highly influenced by the exchange rates. In times of favorable external conditions, if foreign currency is cheap, Turkey experiences relatively lower inflation with higher growth. On the other hand, a negative external shock leads to high inflation and recession at the same time\(^4\). The politicians are tend to blame the external economic conditions in times of poor economic performance, but they are often reluctant to mention their contribution when the economy is performing well. The relatively good growth rates observed in 2002-2008 period in Turkey together with historically low inflation figures are influenced by the strong domestic currency (besides other factors) during the same period. This is also consistent with the points we make about the importance of the international economic conditions in Turkish economy.

Another interesting feature that enhances the vulnerability of Turkish economy to exchange rate shocks is the indexation behavior. The economic agents in Turkey have a strong memory of high inflation and therefore they constantly seek financial instruments that can prevent real losses for their savings. After the full-scale opening up of the capital account, foreign currencies, particularly the US Dollar, became a popular financial instrument. Therefore, even the middle-income class started to follow the exchange rates closely everyday. In time, exchange rate became one of the major factors that shape the expectations of the economic agents and a strong indexation behavior is developed beside the actual cost-side effect of the exchange rate changes. In an empirical study using Johansen’s cointegration analysis, Yavuz (2003) shows that, the long run relation between prices and exchange rates prevails even when the prices of non-tradable goods are considered. This points out an indexation behavior free from the cost side effect which is not relevant for non-tradable goods. Although a rise in exchange rates does not raise the production costs of the non-tradables, via a psychological effect, level of the exchange rate still plays a role in determination of the prices of such goods.

\(^4\)In a study about calculation of output-sacrifice ratio, Cetinkaya & Yavuz (2002) conclude that the sacrifice ratio in Turkey’s disinflation episodes is negligibly small if not negative. This result supports the idea that inflation is counter-cyclical. Because of the supply-side shocks resulting from the changes in exchange rates, disinflation often comes with economic growth leading to a negative sacrifice ratio.
The second structural feature of the Turkish economy that is influenced significantly by the financial liberalization is the structure of the banking sector. This effect is closely related to the other key incident mentioned earlier, the switch from foreign to domestic means of financing the public debt and relying mainly on bond issuing to cover domestic borrowing needs. We study the effects of these two major events on the banking sector in detail in the next section. However, in general, we can say that the environment created by the economic policy perspective of the 1980s made Turkish economy strongly dependent to international capital. Starting from early 1990s, sustainability of the domestic debt stock became the central issue of the economic policy. While the Treasury was relying more and more on domestic borrowing, as a result of the financial liberalization, the domestic banks often found it profitable to borrow internationally and lend to the government. Figure 1.2.3 displays the share of loans extended to private sector and to the government in commercial banks’ total assets. The upward trend in the latter is evident pointing the effect of debt financing on the banking sector balance sheets. Starting from 1998 the share of private credits fell rapidly whereas the share of government securities went up reaching to 20 percent by the end of 1999. This also indicates how difficult it became to manage domestic debt by the end of 1990s. The other problem arises due to the term structure of the respective borrowing and lending rates. Given the unstable nature of the Turkish economy, the foreign capital was mostly short-term rather than being in the form of direct investment. On the other hand, the bank lending to the government was longer term creating a maturity mismatch between the assets and the liabilities of the banking sector. Moreover, banks often borrowed in FX and then lend to the government in TL, carrying an exchange rate risk as well in the process. All in all, the economical environment shaped by the critical policy shifts of 1980s created a new system of debt finance. Instead of borrowing internationally, the government relies on the domestic banking system. However, it is still very much dependant on the foreign capital, availability of which determines the conditions of the domestic borrowing. In the process though, both the exchange rate and interest rate risks are transferred on the domestic banks.
Turkish economy has still been paying for the costs of irresponsible public spending and large budget deficits. But the way it suffers is quite different than what the other countries (especially the South American economies) experienced. This is due to the way of financing the deficits. In most of the South American economies the huge deficits were monetized at some point leading to hyper-inflation episodes. Whereas in Turkey, domestic borrowing strategy prevented Turkey from seeing hyper-inflations but instead it led to economy with high and stable inflation with a fragile banking system. Ozatay & Sak (2002) also mentions this financing mechanism limiting excess money supply avoid much higher than realized inflation rates and they provide data about how consolidated budget is financed between 1995 and 2001. It can clearly be seen, usage of central bank resources was zero starting from 1997, how the government relied on domestic borrowing over other ways of finance and net foreign borrowing was mainly negative (Table 1.2.1).
The 1990s was mainly a transition to a new phase of neo-liberalism for Turkish economy. A managed float system has chosen as an exchange rate regime. Exchange rates, being one of the key determinants of macroeconomic performance, were let to float with the active involvement of the central bank as a major player in the market. In this system, the central bank does not have a transparent exchange rate target, but it intervenes the market whenever it finds necessary. This created uncertainty on the policy side as well.

The economy was hit by a crisis in 1994 which can be considered as the first repercussion of the premature liberalization of the capital account. Early 1990s was mainly a period of overvalued TL and hence despite the negative effects of the Gulf War in 1991, the annual average GDP growth was over five percent in this period (1990-93). However, consistent with the structure explained above, this period was also associated with a rise in public debt, growing short-term international liabilities in the banking sector and a corresponding deterioration of the current account balance (Celasun 2002). Finally, the reversal of short-term international capital in 1994 resulted in a significant devaluation and a six percent fall in national income5.

This crisis was the first real warning for the Turkish economy displaying that the macroeconomic and institutional structure is not strong enough for a fully opened economy. Therefore, the necessity of a set of immediate structural reforms was obvious. The IMF supported economic program launched after the crisis aimed to take serious measures to address this problems. However, these efforts was proven unsuccessful as the recovery from the crisis happened quite quickly thanks to the favorable international conditions. 1995-97 period was a time where the international

\[ \begin{array}{lcccccc}
\text{Table 1.2.1: Financing the public sector deficit (percent of GNP)} \\
\hline
\hline
\text{Net domestic borrowing} & 3.6 & 7.1 & 8.5 & 8.6 & 12.4 & 7.4 & 12.6 \\
\text{Net foreign borrowing} & -1.0 & -0.9 & -1.5 & -1.9 & 0.6 & 2.1 & -2.4 \\
\text{Central Bank advances} & 1.2 & 1.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
\hline
\text{Source: Treasury, Ozatay and Sak (2002)}
\end{array} \]

\[ ^5 \text{For a detailed analysis of 1994 crisis see (Ozatay 2000).} \]
capital flows to emerging market economies made a peak. This was enough to ease
the pressure on the banking sector and led to a GDP growth of 8 percent just a year
after the crisis in 1995. After that, the economic program was abandoned along
with the structural reforms.

However, the international conditions changed significantly after the Asian crisis
and along with the increasing interest rates the pressure on public debt increased.
In addition to these, Turkey was hit by two large earthquakes in 1999. All these
developments made it almost impossible to finance the public debt by the end of
1990s. This is when the a new economic program was launched with the support of
the IMF.

The exchange rate based stabilization program aimed to exploit the dollarized
nature of the economy by pre-announcing the value of a FX basket (consists of one
US dollar and 0.77 Euros) for the next 18 months. As it is explained above the
exchange rate market was often being intervened by the central bank, but this time
it was totally transparent in an attempt to manage the expectations. The program
uses one of the most important economic variables as a nominal anchor, to lower
the inflation and (by reducing uncertainty) ease the pressure on public borrowing.
Obviously, controlling the exchange rates was not enough to create an economically
safer environment that would help sustaining the borrowing at reasonable rates. The
noble feature of this program was it contained much more than a pegged exchange
rate regime. It was supported by tight monetary policy, fiscal discipline and wide
range of structural reforms. Moreover, the program had an exit strategy from the
relatively unstable exchange rate policy. After 18 months, the exchange rate would
be allowed to float in a widening band leading to a fully floating regime eventually.
This was announced at the beginning of the program so that all the agents would
know instability created by the pegged exchange rates would only be for a limited
time period. While these were the strong features of the program, as every pegged
exchange rate regime it leaves the determination of the interest rates to the market.
By pegging the exchange rate the central bank guarantees to meet any demand for
CHAPTER 1. INTRODUCTION

FX on the determined prices and therefore loses its control over the money supply. If it keeps providing liquidity in the domestic money market this may lead to an attack on central bank reserves. This was considered in the program by a limitation put on the net domestic assets of the central bank. This weakness on the domestic money market side was quite important given the fragility of the banking system.

Ten months after the launch of the program, on November 2000, a liquidity crisis took place in the domestic money market. It started with the difficulties faced one mid-size bank and rolled into a full scale crisis in the debt market and the whole domestic banking system. Central bank could not intervene at first due to the limitations introduced by the design of the economic program. All in all, the fragility of the banking system was on display when an incident in the money market turned into a crisis in a short time. Ozatay & Sak (2002) provides a detailed explanation of how this process developed, interested readers can refer to their study. We discuss the fragility in the financial system and the structure of the bank balance sheets in more detail, in the next section.

Although with a one-time violation of the net domestic assets limitation and taking over the bank caused the incident the crisis was handled temporarily and it did not turn into an attack on the FX reserves, it definitely worsened the expectations about the future of the exchange rate based stabilization program. With the help of other triggering factors, the peg collapsed in February 2001 and the exchange rate was allowed to float. The nominal TL value of the US dollar was almost doubled the next day and the program came to an end before seeing the widening band that was supposed to be in practice only a few months later.

Ozatay & Sak (2002) provides an excellent assessment of the crisis by discussing the nature of it in the context of the different generations of crisis identified in the literature. According to their analysis, although first and second generation models have some common features with what was observed in Turkey in 2000 and 2001, they are not able to fully explain the crisis. The existence of high public sector debt and hence borrowing requirement brings to mind a first generation type of crisis.
However, because of three aspects of economic environment Ozatay & Sak (2002) concludes Turkish crisis cannot be explained well by first generation models. First, despite the existence of high public deficits, they were not financed by inflation tax, so a loose monetary policy was not in place prior to the crisis. Secondly, no depletion of the central bank reserves was observed. Finally, no continuous deterioration of economic fundamentals was in place. In fact, in terms of the fundamentals 1999 was much worse than 2000. Therefore, the lack of some key features of first generation type of crisis is evident in the Turkish case.

Next, when they consider second generation type of crisis, they mention it is possible to think due to high public sector debt and fragile banking system, the speculators may have anticipated that the current peg will not be able to survive and therefore a self-fulfilling attack on central bank reserves could have triggered the crisis. However, they underline the fact that a common feature of second generation crisis is an expansion and favorable short-run macroeconomic conditions just after the crisis. This was not the case in Turkey as the economy has contracted by 8.5 percent in 2001. This leads to a possibility of a third generation type of crisis where the root cause of the problem is financial fragility. Ozatay & Sak (2002) argues the main reason for the crisis was the fragile banking system along with a set of triggering factors that make the fragility clear.

Ozkan (2005) also argues the Turkish experience had features relevant to all three generations of crisis. However, in this case the unhealthy condition of fiscal balances did not work through expanding domestic credit but created distortions in the financial system contributing to the overall vulnerability of the economy. She also underlines the key role of the financial system and points out banking sector restructuring and precautions for a sound fiscal balances as policy measures to prevent future crisis. In the next section we discuss the fragility in the financial sector prior to the crisis in line with the events and structural issues that are outlined so far.
1.3 Financial Fragility Prior to the Crisis

So far we mention two types of risks imposed on the banking system mainly due to the policy choices of 1980s and 1990s. In this section we try to explain these risks in depth by providing some data in reference to studies analyzing Turkish financial system. While discussing the effects of financial liberalization and deficit financing by issuing domestic debt, we point out the emergence of a new mechanism where domestic banks make use of the cheap international capital by borrowing in FX and lending it to the government (buying securities) in TL. First of all, such an act inflicts an exchange rate risk on the bank. The bank receives from the return of government securities is in domestic currency, however it will pay its debt in FX. Therefore how much profit it makes depends on the change in the value of domestic currency between the time of investment and the time it pays the debt back. Meanwhile, if the domestic currency depreciates, the bank makes a capital loss because of the exchange rates. Secondly, bank also carries an interest rate risk due to the maturity mismatch between its assets and liabilities. This is because the maturity of international borrowing is shorter compared to the maturity of the government securities. Therefore, the bank has to roll over its short term debt a few times before it can collect the returns of the securities. During the process if the interest on bank borrowing goes up, once again the bank makes a capital loss.

In a typical economy the main mission of the banking system is simply to transmit the savings into investment via collecting deposits and extending loans. In Turkey, particularly after financial liberalization, the share of non-deposit liabilities in total liabilities increased significantly. This can mainly be observed for the share of FX denominated liabilities. Ertugrul & Selcuk (2001) reports the share of FX denominated liabilities which was 25 percent in 1988, rose to 48 percent in 1999. On the assets side, the share of government debt instruments kept increasing and made a peak with 34 percent in 1999. Over time, the banking sector’s main function turns into financing budget deficits. With the changed nature of banking system, banks start to borrow internationally and lend to the government domestically without
strong control mechanisms or risk management strategies. This behavior shortly named as "borrowing abroad - lending home" strategy by Ertugrul & Selcuk (2001), dominated Turkish banking sector starting with the liberalization of the capital account in 1989.

Ertugrul & Selcuk (2001) displays how exchange rate risk increased by the upward trend in FX short positions (FX denominated liabilities - FX denominated assets) of the banking sector prior to both crisis episodes in 1994 and 2000 (Figure 1.3.1). They explained the risk taking behavior of the private banks in the following words.

"A combination of disinflationary efforts and short-term borrowing based deficit financing policies made the banking system more vulnerable against foreign exchange and interest rate risks. Higher interest rate commitment on domestic assets, lower depreciation rate, and an increase in the PSBR built up the foreign exchange reserves of the Central bank but also opened the banking sector to speculative attacks. A more risk taking behavior of the privately owned banks and their large short positions in foreign currency raised the question about the sustainability of the short-term capital inflow based external balance policy."

Ozatay & Sak (2002) points out the tendency of running FX short positions is also related to the inflation memory of the domestic banks. Also being discussed as a common feature for developing countries, adopting to the high inflation environment these banks are unable to borrow long term in their own currency. They also provide banking data to display the exchange rate risk. It is observed that, the FX denominated assets to FX denominated liabilities ratio was declining on the road to the crisis. Moreover, the liquid FX denominated assets to FX denominated liabilities ratio was also decreasing, indicating an additional vulnerability against sudden reversals (Table 1.3.1).
Maturity mismatch and the interest rate risk associated with it is another source of fragility that was pointed out earlier. Ozatay & Sak (2002) presents evidence in this matter as well. Table 1.3.2 provides information about the length of maturities of assets and liabilities in the banks’ balance sheets. The total assets to liabilities ratio for short term items seems quite low in general. For the first nine months of 2000 just before the crisis, it is relatively lower compared to the previous three years. The data also reveals the deposits are also mainly shorter term and their maturity got even shorter in 2000. The share of very short term repos in total liabilities also displayed an increasing trend towards the crisis.
CHAPTER 1. INTRODUCTION

Based on our discussion about the main characteristics of the Turkish financial system so far, we can conclude Turkish economy can easily be derived from a really favorable state to a rather disastrous one. In the context of the multiple equilibria models instead of a "good" and a "bad" equilibrium there is "heaven" and there is "hell". Prior to the crisis period, the economy was in fair condition only months after two historically large earthquakes in 1999. This is possible because of the credibility and confidence imposed by the newly introduced economic program. However, just 11 months later, the turmoil in financial market turned the expectations upside down leading to huge devaluation and a period of recession and high inflation. Due to this fragile structure any period with favorable economic conditions in Turkey can only be experienced in the expense of increased risks. When these risks are realized, the economy faces a crisis period which incurs excessive costs for the majority of the society.

After the crisis with the realization of the risks we outlined above the banks were hit by three main shocks. The first two shocks are related to the maturity mismatch and interest rate risk. First, the sharp increase in the interest rates led to rise in funding costs for the bank who were running short-term debt to finance longer term government securities. Second, the rise in the interest rates also caused a decline in the value of the government securities portfolio of the banks which constituted a significant share in their total assets. On the other hand, the third shock was due to the exchange rate risk. The high amount of short positions held led to enormous capital losses with the severe devaluation. The government launched a banking sector restructuring program only three months after the crisis in May 2001. This meant recognition of banking sector fragility as the root cause of the crisis. Next, we discuss two more points about the financial fragility in the following two subsections which are the contribution of exchange rate stabilization program and the differences among the banks.
1.3.1 Financial Fragility and the Stabilization Program

Our evaluation of the Turkish financial system above reveals that the problems associated with the banking sector have their roots in the policy choices of 1980s. Therefore, it would be extremely harsh to blame the exchange rate based stabilization program that was launched in the end of 1999 for what happened in November 2000 and then in February 2001. However, also consistent with some of the data provided earlier, the program might have contributed to the fragility which already existed in the banking sector. We mentioned the vulnerability created by the loss of control on the money supply in the former section. This is typical for any fixed or pre-determined exchange rate regime. Beside that, it is also argued the program might have contributed to the financial fragility by easing the short-term exchange rate risk.

The program by pre-announcing the path of the exchange rate led to a higher degree of capital inflow to the country. This is also evident theoretically considering the uncovered interest parity condition. Considering there was no improvement in the longer run risks, the capital that came in was mainly short term. It came mainly in the form of foreign borrowing creating more incentives for the domestic banks to run short positions in FX and hence reinforcing the "borrowing abroad - lending home" strategy. Moreover, inflow of international capital also increased the liquidity in the domestic money market leading to a decline in the interest rates. However, given the uncertainties about the sustainability of the exchange rate regime in the longer run, the decline in short term-interest rates was sharper compared to the decline in longer term interest rates. As a result, the yield curve became steeper and the spread between overnight borrowing rates and longer term return on government debt instruments grew larger.

The change in the slope of the yield curve also strengthened the tendency of operating under higher interest risk for the banks. Borrowing short term and lending to government became more attractive and profitable. In general, bank behavior during the stabilization program was basically unchanged compared to the era of
... financial liberalization, however certain features of the program fed the incentives for this sort of behavior and hence boosted the fragility in the financial sector.

Despite all these, it would be unfair to neglect the restructuring planned for the banking sector in the economic program. The program anticipated the problems in the sector and it included strong measures to eliminate the fragility. However, all these measures could be effective in a longer time period and the current structure of the banking system failed to survive in the newly introduced environment by the launch of the program. It is a fact that, in the situation that was reached by the end of 1990s where the sustainability of the domestic debt stock has become almost impossible, there were not too many choices and something more radical must be tried. This stabilization program was chosen. It had a lot of strong features that was compatible with certain characteristics of the Turkish economy. Nevertheless, all the errors made in the last two decades rendered it ultimately unsuccessful.

Some studies evaluating the financial crisis of 2000-2001, point out some flaws in the design and implementation of the program as mistakes of International Monetary Fund and Turkish policy makers. Akyuz & Boratav (2003) argue banking sector reformation should have been done before the launch of the program because making the necessary legal and institutional arrangements was critical before attacking inflation. They argue the restructuring was only attempted after the outbreak of the November turmoil ignoring that worst time to reform is in the middle of a crisis. They also mention the IMF hardened its position to put the most of the blame on the government once it became clear the program was no longer viable and this also did not help to reestablish the market confidence.

Alper (2001) outlines three critical factors that played a role in the outbreak of the financial crisis. First, he underlines the importance of maintaining the stream of good news in such economic programs and argues the government failed to do this because of the slow pace in the outlined agenda of structural reforms. Secondly, he thinks the financial backing up by the IMF was not enough to provide insurance that would completely eliminate the exchange rate risk. Finally, he indicates the
importance of interest rate undershooting in the financial fragility created by the
program and identifies the "no sterilization" rule in the design of monetary policy
as a significant factor in the fall of the interest rates. He argues if some of the
capital inflows were sterilized, the fall in the interest rate would not be so abrupt
and the liquidity crisis could have been avoided. Ozkan (2005) states currency pegs
encourage domestic banks to borrow internationally and hence carry great risks
in the absence of proper supervision and monitoring of financial intermediaries.
In countries whose financial systems are fragile, pegging the exchange rate is a
significantly risky strategy as the collapse of the peg can easily turn into a full-scale
financial crisis.

Finally, Yeldan (2001b) points out the effect of the economic program on moner-
tary policy and therefore, on the financial system using the following words.

"...the IMF-directed Turkish disinflation episode all too clearly spells
the dangers of restricting the monetary policy of an economy to specu-
lative in-and-out-flows of short term foreign capital, which by itself, is
excessively liquid, excessively volatile, and is subject to herd psychology.
The program, by dismantling all the tools of stabilization and monetary
control of the Central Bank, has left the economy defenseless against a
speculative run and a sudden stop. Trapped within the confines of a
pre-announced program of exchange rate devaluation, and of a mone-
tary rule administered effectively by short term arbitrage speculation,
the Turkish Central Bank’s monetary effectiveness was reduced to the
miniscule role of an accounting officer. Under this role, the CB lost all
its power to steer the economy in the advent of a disruptive shock or a
change in the investors’ perceptions leading to a sudden stop."
1.3.2 Heterogeneity of Banking Sector

The features we discussed above are in general valid for the whole banking system in Turkey. The dominance of government debt instruments on the balance sheets, the behavior of running FX short positions and hence bearing interest and exchange rate risks can be considered as the common characteristics of the banks. However, the scale of the risks carried can be different among banks and at different times. In this context, Ozatay & Sak (2002) underline two kinds of dichotomy in the banking system. The first is the distinction between state and private banks. In many occasions, state banks acted differently compared to the private banks. The problem of state banks became clear particularly after the crisis in 1994. Ertugrul & Selcuk (2001) reports that these banks extended concessionary credits to the agricultural sector and small and medium sized enterprises as a part of the government policies a great deal of which were populist efforts to attract political support. After the crisis, when the market interest rate was rising these banks continued credit expansion and the result was the so-called duty losses which were amount to 20 billion US Dollars by the end of 2000. The duty losses made up higher than 10 percent of the GDP and 14 percent of the total assets of the banking system. The state banks were not only a problem for the high budget deficit and PSBR but the behavior their behavior affected the money markets negatively at different times.

The second dichotomy was about the good and bad banks among the private banks. As mentioned earlier the general type of risk taking behavior in terms of exchange rates and maturity mismatch is quite common among all banks. However some of them can take these risks in a more extreme way than the others. The events at the time of the November 2000 financial crisis is a very good example of this. The triggering factor of the crisis is the cut in credit lines of one particular bank, called Demirbank, which was bearing unacceptable level of interest rate risk at the time. What Demirbank did was simply to borrow overnight and buy much longer term government securities. Then it just rolled over the overnight debt until its investment matures. Given the difference we explained earlier between the short
and longer term interest rates, such an act could have been very profitable if the bank had been able to keep borrowing in low rates until the government securities are due.

Ozatay & Sak (2002) also provided some data about the differences between Demirbank and the rest of the sector. Table (1.3.3) displays the information about investment in government debt instruments (GDI) and the short-term repos as a method of finance. First of all, the data shows excessive investment in GDI which was heavily financed by borrowing short term on Demirbank’s account. The share of GDI in total assets was 67.5 percent for Demirbank for the last quarter of 2000 while it was only 25.16 percent for the rest of the sector. Moreover, it is observed that this share followed an increasing trend since the launch of the economic program at the beginning of the year 2000. On the other hand, for the rest of the sector a fall in investment in GDI relative to other assets was recorded throughout the program. Secondly, Demirbank relied heavily on repos for the finance of the investment in GDI. Despite its large portfolio of GDI, repos to GDI plus repos ratio was much higher for Demirbank compared to the rest of the sector. By the end of 2000, this ratio was 70.69 percent for Demirbank whereas it was only 33.04 percent for the others.

Table 1.3.3: Demirbank vs Others

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</thead>
<tbody>
<tr>
<td>GDI / Assets</td>
<td>Demirbank</td>
<td>58.54</td>
<td>57.09</td>
<td>65.12</td>
<td>58.33</td>
<td>54.88</td>
<td>63.53</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>31.72</td>
<td>29.03</td>
<td>34.64</td>
<td>31.45</td>
<td>30.56</td>
<td>28.82</td>
</tr>
<tr>
<td>Repos / GDI + repos</td>
<td>Demirbank</td>
<td>68.30</td>
<td>57.16</td>
<td>69.48</td>
<td>66.40</td>
<td>59.10</td>
<td>64.81</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>60.24</td>
<td>48.51</td>
<td>48.66</td>
<td>37.51</td>
<td>38.06</td>
<td>40.94</td>
</tr>
<tr>
<td>FX / TL Liabilities</td>
<td>Demirbank</td>
<td>43.59</td>
<td>109.89</td>
<td>69.41</td>
<td>88.14</td>
<td>112.63</td>
<td>73.78</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>135.48</td>
<td>155.11</td>
<td>194.23</td>
<td>213.31</td>
<td>213.21</td>
<td>219.72</td>
</tr>
</tbody>
</table>

Source: Central Bank of Turkey, Ozatay and Sak (2002)

Another interesting point is the different behavior in terms of FX liabilities. The rest of the sector held much higher FX liabilities relative to TL liabilities compared to Demirbank. This was because instead of borrowing in FX, Demirbank borrowed domestically in the money market for short-term and kept rolling over its debt on
the ongoing domestic interest rate. The only reason it can borrow for low rates was
the liquidity in the system due to other banks borrowing in FX for reasonable rates
with the abundant foreign capital. That was in fact how the crisis was triggered.
Some of the banks were running FX short positions and choose to lend the excess
liquidity in the domestic money market on a short-term basis. Therefore, at a
first glance these banks did not have a problem of maturity mismatch as they were
borrowing short-term and lending short-term. However, another bank in the system
was borrowing from them and investing in longer term assets and it kept rolling
over this debt. In other words, this bank did not have the liquidity to pay back at
a given time. Anticipating this, the lender banks decided not to lend to this bank
at some point and this led to a liquidity crisis in the domestic money market.

Obviously this does not mean the only reason for the crisis is a single bank
acting too risky and the crisis would not happen if Demirbank did not behave in
this way. It only sets a perfect example how the conditions of the financial system
and the ongoing exchange rate regime can encourage such a behavior for making
high profits. There might be some banks who have a stronger financial position
and who do not need such a speculative strategy for making profits. But as long
as there are others who are willing to take severe risks, the well-being of the whole
financial system will be under threat. Once again this shows the prerequisite of such
an economic program is a sound and strong banking system with careful monitoring
and supervision.

All in all, we can say the increasing spread between short and longer term interest
rates motivated some banks to exploit this difference by bearing high interest rate
risk for a certain time period. The fragility induced by this behavior, given other
risks such as exchange rate risk carried by the lender banks, led to a banking crisis
eventually. In the context of multiple equilibria models, the expectations determine
which equilibrium prevails. Given the structure of the Turkish banking system, this
heterogeneity has pushed the economy towards a bad equilibrium.
Chapter 2

Public Borrowing and Financial Fragility

2.1 Introduction

In this chapter, we try to build a theoretical model with moral hazard to analyze, the conditions and behavior that lead to a fragile financial system in the process of public borrowing. Our analysis contributes to the literature by providing a theoretical foundation to the widely accepted detrimental effects of excessive public borrowing on the Turkish financial system. We believe, our model also helps to provide a better understanding of the economic crisis which can be vital in shaping the economic policy and preventing such incidents in the future. The dominance of public sector in balance sheets of Turkish banks is evident by the space occupied by treasury bonds on the assets side, however, a theoretical model can help to reveal the dynamics of bank behavior influenced by this dominance. Our analysis also contributes to the moral hazard literature by applying it to the process of public borrowing. In a borrowing process, problems arising from asymmetric information such as moral hazard or adverse selection are often issues for the lender. The lender usually has imperfect information about how the borrower uses the funds and he is concerned about if the borrower can pay the loan back. An interesting aspect of our model is that, the borrower (government) is the party who has imperfect
information about how the lender (the bank) finances the loan. The bank can go for risky ways of finance which can create fragility in the system. Different from any private borrower, this may be an issue for the government¹.

In this context, this chapter is organized to have four sections including this introduction. In the second section, we provide a brief summary for the theoretical background of the principal-agent models with asymmetric information. Next section presents our model and results, the last section concludes.

### 2.2 Theoretical Background

The model we build in this chapter is based on the well-known principal-agent model by Laffont & Martimort (2002). In principal agent model one of the parties (the principal) simply delegates a task to the other (the agent) which often causes incentive problems to arise. The delegation of the task is based on a contract, but the contract can only be based on common information. However, most of the time the agent has some private information that cannot be observed by the principal. This unobserved characteristic or action on the agent’s side affects the principal’s objective. Therefore, the principal tries to create incentive mechanisms in the contract to make the agent behave accordingly (or to choose the agent with the right characteristics) to realize its objective optimally.

Earlier economic models were not interested in incentive questions. They were mainly concerned about how the prices are determined in the market assigning the role of profit maximization to the industrial organization. How the organization coordinates the actions of its members to achieve this goal was out of question. Once the economists started to think about how various tasks are carried out inside

¹One can argue that the lender still faces a default risk. However, when the borrower is the government who has the ability to print money, it can be considered much more safer compared to an ordinary borrower. Besides, in countries like Turkey where most of the public debt is domestically owned, refusing to pay would lead to the collapse of the whole financial system, so it is reasonable to think default risk is minimal.
the industrial organization, the flow of information became important. The effect of the flow of information on behavior of the economic agents was evident when choice under uncertainty is introduced by Von Neumann & Morgenstern (1944). The next step was the discovery of possibly different objectives among the different members of the same organization. After that, incentive questions has become the central issue of the economic problems. These two features, different objectives and different information sets are the stepping stones of the incentive theory. The principal-agent framework is a set up where two such agents try to fulfil a certain task together based on a contractual relationship.

Note that the delegation of the task is rather straightforward when there is no imperfect information, i.e. the principal observes everything. The importance of private information in terms of incentive problems is explained by Laffont & Martimort (2002)

"Delegation of a task to an agent who has different objectives than the principal who delegates this task is problematic when information about the agent is imperfect. This problem is the essence of incentive questions. If the agent has a different objective function but no private information, the principal could propose a contract that perfectly controls the agent and induces the latter’s actions to be what he would like to do himself in a world without delegation. Again incentive issues would disappear."

Laffont & Martimort (2002) mention two critical assumptions for the principal-agent model. First, it is exempt from any negotiation issues. It simply assumes the principal makes a take it or leave it offer and hence he has the negotiation power\textsuperscript{2}. He

\textsuperscript{2}The importance of information sets when choosing any particular action in game theory is apparent. Therefore, who plays first and hence who has the negotiation power affects the outcome directly. One of the best examples of this is the Stackelberg’s leader firm (Von Stackelberg 1934). It displays very clearly how one firm can make higher profits compared to the other only by choosing first, although both firms are completely identical in any other way. However, principal-agent framework focused more on how the principal can affect the agents behavior which he cannot observe perfectly by creating incentives. Although, he has all the power in terms of negotiation, he also has other problems resulting from imperfect information.
offers a contract and the agent chooses to accept or reject it due to the individual rationality comparing his utility under the contract with his outside opportunity. The second assumption is the enforceability of the contract. The model assumes the presence an impartial court of law that can impose proper punishment in case one of the parties deviate from the actions they have committed in the contract.

They outline three types of information problems, namely adverse selection, moral hazard and non-verifiability. Adverse selection is a problem of hidden knowledge. The agent has some private information which is often about his own productivity or cost structure in carrying out the task. This information also affects the principal’s objective but he does not possess it and this is what creates the problem. The second one, moral hazard, is a problem of hidden action. This time an action carried out by the agent after the contract is signed cannot be observed by the principal. This action again affects the objective of the principal and hence leads to trouble on the principal's side. The final problem is the case non-verifiability. It occurs when there exists some information shared by the principal and the agent but it cannot be observed by a third party and most importantly by any court of law.

Adverse selection is often referred as the information problem itself. However, more accurately it is a common result of the hidden information problem. When the principal does not know a certain feature of the agent, he offers a contract based on the respective probabilities of facing different types of agents. Under certain conditions, the resulting offer may wipe all the agents with the preferred characteristics off the market, leaving the principal with only 'bad' type of agents. A good example of this is the "lemons" problem introduced by Akerlof (1970). He considers the second-hand cars market. In the context of the principal agent model, we let the principal to be the buyer and the agent to be the seller. The principal faces an agent and makes an offer to buy his car. The seller knows the quality of the car for certain, but for the buyer there is a certain probability that the car is a "lemon". The value of a lemon both to the buyer and the seller is lower compared to a good car. Given the positive probability of ending up with a lemon, the offer
of the principal will be a weighted average of the value of a lemon and the value of a good car. If the probability of a lemon is sufficiently high and its relative value is sufficiently low, the offer of the principal will be too low to attract any seller with a good car and only lemons will remain in the market.

Another case of adverse selection can be seen in insurance market (Rothschild & Stiglitz 1976). When an insurance company (principal) offers say a car insurance contract, it does not know how risky the customers (agent) drive. On the other hand, profits of the company depends on the probability of any claims by the customer so the hidden information affects the principal’s objective. Initially, let us suppose the company only offers full insurance (covers all loss in case of an collusion). The positive probability of risky drivers buying the contract derives up the premium in the contract. However, the safer drivers will be willing to pay relatively lower premium as they already have a smaller chance of having an accident. Therefore, just as in the used car market, in the insurance market an adverse selection problem can arise if the premium offered by the company is too high for the safe drivers.

The novelty of principal-agent framework is that, it makes it possible for the principal to discriminate between safe and risky drivers by offering two different contracts. Although the company cannot possibly know the type of driver he sells the contract, by providing different options it can make the driver reveal his type by choosing a particular contract. Rothschild & Stiglitz (1976) does this by offering incomplete insurance. It simply offers two contracts say one is with full coverage and a higher premium and the other with incomplete coverage with a lower premium. If the principal adjusts these two arguments of the contracts accordingly, he can make the risky driver’s to choose the full coverage and pay a higher premium and the safe drivers to choose incomplete coverage and pay a lower premium. The intuition here is that, risky drivers will be willing to pay more for higher coverage as they have a higher chance to have an accident. There are two things the principal should consider in designing the contracts. First, the contract that is meant for safe drivers should have sufficiently low coverage to prevent the risky drivers to pick that contract as well and second, it should still be attractive enough for the safe drivers so that they
will not choose to buy no insurance at all. The former condition that the principal should meet is called incentive compatibility constraint while the latter one is called the individual rationality constraint. Basically, the principal can derive the utility of the safe drivers down to the level where they are indifferent between accepting the contract or not, however the risky drivers will enjoy a higher utility than they would in case of no insurance. Note that this is due to the private information of being risky. In case of complete information, the principal can drive both types of agents to their reservation utilities by offering two different contracts contingent on their types. Just because of the private information, in order to make them reveal their type, the principal should pay an "information rent" to the risky drivers over their reservation utility. Laffont & Martimort (2002) explain this in the following way.

"...the information gap between the principal and the agent has some fundamental implications for the design of the bilateral contract they sign. In order to reach an efficient use of economic resources, this contract must elicit the agent’s private information. This can only be done by giving up some information rent to the privately-informed agent. Generally this rent is costly for the principal... At the optimal second-best contract the principal trades off his desire to reach allocative efficiency against the costly information rent given up to the agent to induce information revelation."

Monopoly price discrimination is another popular topic where the principal agent framework can be applied. The monopolist (principal) knows that a group of consumers (agent) say higher-income consumers are willing to pay a higher price for a better quality product and others say lower-consumers cannot simply afford the higher price even if the quality is much better. The problem here is that if the monopolist raises the price and the quality of the product to extract more from

\[3\] Once again this is due to the assumption we mentioned earlier that principal holds all the bargaining power by making a take it or leave it offer.
the higher income consumers, he may loose the lower income consumers. The price discrimination idea is about supplying two types of goods with different quality and price and keep selling to both types of consumers. A good example for this is the airline tickets. The company simply tries to sell economy class tickets to lower income consumers while it provides higher price, higher quality first class tickets for the higher income consumers. For examples of monopoly price discrimination see Pigou & Aslanbeigui (2001) and Mussa & Rosen (1978).

The second information problem that was pointed out earlier is moral hazard. This time the information gap between the principal and the agent is about an action the agent takes after the contract is signed. This action can be observed neither by the principal nor by the court of law and hence, it cannot be a part of the contract. However, this action may still be important as it affects the objective of the principal. As the principal cannot write a contract on the unobserved action, the contract can only be made contingent on another performance parameter which is observable and also affected by that action. The problem arises because the performance parameter is a noisy signal of the agent’s action, that is observing the parameter the principal can only deduct the probability distribution of the action and not the action itself. If the relation between the performance parameter and the action is deterministic, there will be no information problem as observing the parameter, the action will be known for certain. In other words, the action will become indirectly observable. Given the stochastic relation between the parameter and the action, the principal tries to offer a transfer schedule contingent on the parameter that would lead the agent to exert the preferable action.

Note that, adverse selection and moral hazard cannot be distinguished with strict borders. For example, the insurance contract example we outline can also be formulated as a problem of moral hazard under adverse selection. The main difference is that under moral hazard, the uncertainty is endogenous rather than exogenous. When we consider adverse selection, the uncertainty is about the type of the driver where he can be risky or safe and it is exogenously given. Under moral hazard, we have only one type of agent but after signing the contract, he can drive
safely or dangerously. The problem occurs as the agent after buying insurance can be tend to drive carelessly as any damage would be paid by the insurance company anyway. The insurance firm cannot observe how the agent drives but it can observe if he has an accident. Therefore, it can design the contract to offer rewards if no accident occurs and/or punishments if the accident happens. As the agent who drives safely will have lower probability of having an accident, if the reward/punishment schedule of the contract is strong enough, it can induce the agent to drive safely. No claims bonus offered by all insurance companies is a good example of this.

A well-known application of moral hazard problem is the efficiency wages theory introduced by Shapiro & Stiglitz (1984). They analyzed a principal-agent relation between the employer and the worker. The employer after signing an employment contract with the agent is concerned about the effort that the worker exerts. In their set-up, the effort of the worker can be observed by monitoring but perfect monitoring is extremely costly and that is the factor that leads to the moral hazard problem. They show that due to the high cost of monitoring, the employer performs only random or incomplete monitoring and make the contract contingent on the results of it. Therefore, monitoring results become a noisy signal of the workers performance. The contract offers a certain wage provided that the worker is not caught shirking and it imposes of a punishment of being fired if he is. If the employer offers a high enough wage, he can increase opportunity cost of shirking and induce a positive level effort on the worker's side. This is the main idea of their model as higher wages may lead to equilibrium unemployment supporting the new-Keynesian views about labor markets. The firm simply pays higher wages than the marginal productivity of the worker because of the information rent it pays as a result of asymmetric information.

Credit markets are often associated with information problems as well. When extending loans to entrepreneurs, banks have to deal with information asymmetries. An adverse selection problem may arise if the existence of risky entrepreneurs derives the loan rates up wiping the safer borrowers of the market. A bank may face a moral hazard problem if it cannot observe how risky investments the borrower takes on.
These are due to the default risk. If the borrower cannot pay the loan back, the bank can only extract a certain amount from him and extraction often involves other costs as well. In the next section, we consider a credit market where the government borrows from the commercial bank. However, this is a special case because it is the borrower who faces the information problem as he is interested in how the lender finances the loan which is private information. The government trades off lowering its borrowing costs against the desire of motivating the bank to chooses safer ways of financing the loans.

2.3 The Model

We start with a simple model where the commercial bank optimally decides how it is going to finance its lending to the government. Given the environment provided by the exchange rate based stabilization program, while modelling the behavior of Turkish banks, we ignore any exchange rate risk and focused on the interest rate risk only. The safe and risky ways of finance can be thought as different alternatives for the bank. If it picks the safe loan to finance the lending, we may suppose it picks a longer maturity loan to avoid maturity mismatch. If it picks the risky loan, it can end up with a much lower cost if everything goes well (presumably, he can roll over the short term debt without any problems until the investment matures) or it can end up with a much higher cost if the bad state occurs (presumably, creditors panic lead to high roll over costs). Note that the model we consider in this chapter is static and it does not have maturities. We try to cover interest rate risk by different levels of financing cost with different probabilities. In the next two chapters, we use a more dynamic framework to analyze maturity mismatch and the weakness resulting from it. After we analyze the commercial bank’s decision process, we proceed by adding the government and information problems to analyze the possible results.
2.3.1 A Simple Model of Public Borrowing

We consider a simple model with a government and a commercial bank. The government needs to borrow a certain amount of funds at the lowest possible cost. The only source of borrowing is the commercial bank and the government promises to pay $R > 1$ dollars back for every dollar it borrows. The commercial bank, on the other hand, finances its lending to the government by borrowing from another source. There are two types of loans available to the commercial bank. The first type of loan is a safe loan where the bank pays $r_f > 1$ dollars back for each dollar it borrows. The second type of loan is a risky one. The bank pays $r$ dollars for each dollar it borrows where $r$ is a discrete random variable which is greater than one with probability one. The risky rate $r$ can only take two values. With probability $p$, $r$ equals to $r_H$ and with probability $(1 - p)$, $r$ equals to $r_L$ where $r_H > r_L > 1$.

For simplicity, we normalize the funds the government has to raise at 1 dollar. Then, the commercial bank lends a dollar to the government and it allocates the finance of this 1 dollar between the safe and the risky loan to maximize its utility. We further assume the commercial bank is risk averse and has utility function $U(\pi)$ with $U' > 0$ and $U'' < 0$ where $\pi$ is the payoff resulting from lending to the government. Let $s \in [0, 1]$ be the proportion of lending financed by the safe asset. The payoffs for the bank in two different states of the risky loan can then be written as

$$\pi^H = R - r_f s - r^H (1 - s)$$

and

$$\pi^L = R - r_f s - r^L (1 - s)$$

The expected utility of the commercial bank is

$$p U(\pi^H) + (1 - p) U(\pi^L)$$

(2.3.1)
The commercial bank simply chooses $s$ to maximize (2.3.1) and the first order condition for an interior solution can be written as

$$p(r^H - r_f)U'(\pi^H) + (1 - p)(r^L - r_f)U'(\pi^L) = 0$$  \hfill (2.3.2)

Equation (2.3.2) simply states the ex-ante marginal utility of $s$ should be equal to zero for optimality. Note that, this is possible if and only if $r^L < r_f < r^H$. We also need stronger conditions to rule out the corner solutions. First, for $s = 0$, the ex-ante marginal utility should be strictly positive.

$$p(r^H - r_f)U'(R - r^H) + (1 - p)(r^L - r_f)U'(R - r^L) > 0$$  \hfill (2.3.3)

Second, for $s = 1$, the ex-ante marginal utility should be strictly negative.

$$p(r^H - r_f)U'(R - r_f) + (1 - p)(r^L - r_f)U'(R - r_f) < 0$$  \hfill (2.3.4)

which implies

$$E(r) < r_f$$  \hfill (2.3.5)

Condition (2.3.5) is intuitive as the risk averse bank demands a positive risk premium to use the risky loan. In other words, the expected cost of the risky loan should be lower than the cost of the safe loan for an interior solution. Otherwise, the risk averse bank simply uses the safe loan only, choosing $s = 1$.

We can now conclude Equation (2.3.2) solves the bank’s maximization problem as long as the second order condition holds.
CHAPTER 2. PUBLIC BORROWING AND FINANCIAL FRAGILITY

\[ p(r^H - r_f)^2U''(\pi^H) + (1 - p)(r^L - r_f)^2U''(\pi^L) < 0 \quad (2.3.6) \]

Note that, given \( U'' < 0 \), Equation (2.3.6) holds, Equation (2.3.1) is strictly concave in \( s \) and the second order condition is satisfied. The solution to the commercial bank’s problem is summarized in Proposition 1.

**Proposition 1** The optimal proportion of the safe loan used by the risk averse commercial bank \( 0 < s^* < 1 \) is characterized by the following three conditions:

\[ p(r^H - r_f)U'((\pi^H)) + (1 - p)(r^L - r_f)U'((\pi^L)) = 0 \]

\[ p(r^H - r_f)U'(R - r^H) + (1 - p)(r^L - r_f)U'(R - r^L) > 0 \]

\[ E(r) < r_f \]

Next, we derive some comparative statics results which will be useful when extending the model in the following sections. First, we look at what happens to \( s^* \) as the return \( R \) changes. To begin, let:

\[ D = p(r^H - r_f)^2U''(\pi^H) + (1 - p)(r^L - r_f)^2U''(\pi^L) \]

and recall that \( D < 0 \) by the concavity of the utility function and also by the second order condition. Without a specific form for the utility function and hence a closed form solution for \( s^* \), we totally differentiate the first order condition Equation (2.3.2) and let the change on all parameters except \( R \) to be zero in order to get the following equation.

\[ \{p(r^H - r_f)U''(\pi^H)\frac{d\pi^H}{dR} + (1 - p)(r^L - r_f)U''(\pi^L)\frac{d\pi^L}{dR}\} \ dR + D \ ds = 0 \]
Given that $\frac{ds^H}{dR} = \frac{ds^L}{dR} = 1$, we have

$$
\frac{ds^*}{d\bar{R}} = -\frac{p(r^H - r_f)U''(\pi^H) + (1 - p)(r^L - r_f)U''(\pi^L)}{D}
$$

(2.3.7)

With $D < 0$, the sign of $\frac{ds^*}{d\bar{R}}$ is determined by the sign of the numerator. The numerator can be rewritten as:

$$
p(r^H - r_f)U'(\pi^H)U''(\pi^H) + (1 - p)(r^L - r_f)U'(\pi^L)U''(\pi^L)
$$

(2.3.8)

The first order condition implies

$$
p(r^H - r_f)U'(\pi^H) = -(1 - p)(r^L - r_f)U'(\pi^L)
$$

Imposing this, Equation (2.3.8) becomes

$$
p(r^H - r_f)U'(\pi^H) \left[ \frac{U''(\pi^H)}{U'(\pi^H)} - \frac{U''(\pi^L)}{U'(\pi^L)} \right]
$$

(2.3.9)

Next, let $\lambda(\pi) = -\frac{U''(\pi)}{U'(\pi)}$ be the Arrow-Pratt measure of absolute risk aversion. Equation (2.3.9) can then be rewritten as:

$$
p(r^H - r_f)U'(\pi^H) \left[ \lambda(\pi^L) - \lambda(\pi^H) \right]
$$

(2.3.10)

Therefore, the sign of (2.3.8) depends on $\frac{d\lambda}{d\pi}$ or in other words how the degree of risk aversion changes with the associated payoff. Consequently, given $\pi^L > \pi^H$, we have:

$$
\frac{ds^*}{d\bar{R}} < 0 \quad \text{if} \quad \frac{d\lambda}{d\pi} < 0 \quad \text{and hence} \quad \lambda(\pi^L) < \lambda(\pi^H)
$$
\[
\frac{ds^*}{dR} > 0 \quad \text{if} \quad \frac{d\lambda}{d\pi} > 0 \quad \text{and hence} \quad \lambda(\pi^L) > \lambda(\pi^H)
\]

\[
\frac{ds^*}{dR} = 0 \quad \text{if} \quad \frac{d\lambda}{d\pi} = 0 \quad \text{and hence} \quad \lambda(\pi^L) = \lambda(\pi^H)
\]

This is also related to the third derivative of the utility function. The derivative of the degree of risk aversion parameter \(\lambda\) with respect to the payoff \(R\) gives

\[
\frac{d\lambda}{d\pi} = -\frac{U'''(\pi)U'(\pi) - (U''(\pi))^2}{(U'(\pi))^2}
\] (2.3.11)

Therefore, given \(U'(\pi) > 0\), \(\frac{d\lambda}{d\pi} > 0\) if \(U'''(\pi) < 0\) which implies marginal utility is concave. If \(U'''(\pi) > 0\), on the other hand, the sign of \(\frac{d\lambda}{d\pi}\) depends on the relative sizes of the terms in numerator. The third derivative of the utility function is often referred as an indicator of "prudence" and associated with the preference towards precautionary savings. A convex marginal utility function and hence positive \(U'''\) causes the consumer to hold precautionary savings against risk. The notion of prudence is introduced by Kimball (1990) and plays an important role in intertemporal consumption decision under uncertainty \(^4\).

Given the precautionary savings motive, assuming \(U''' < 0\) does not seem very realistic. Moreover, most common functional forms used for utility functions in the economic literature has in fact a positive third derivative. The widely used functions such as the natural logarithm or the square root all imply risk aversion and positive prudence. Therefore, within the context we discuss above, the effect of a change in the bank’s payoff on its degree of absolute risk aversion will depend on the relative sizes of first, second and third derivatives. At this point, we consider another utility function, namely the constant relative risk aversion (CRRA) utility function given as

\(^4\)For a detailed explanation of prudence and its relation with risk aversion see Kimball (1990).
where the Arrow-Pratt measure of relative risk aversion is constant and equal to $\gamma$.

$$\gamma = - \frac{U''(\pi)}{U'(\pi) \pi}$$

or equally,

$$\lambda(\pi) = \frac{\gamma}{\pi}$$

and hence,

$$\frac{d\lambda}{d\pi} = - \frac{\gamma}{\pi^2} < 0$$

Therefore, if we assume CRRA preferences, we have $\frac{d\lambda}{dR} < 0$. Decreasing absolute risk aversion is a more plausible assumption and many commonly used utility functions are consistent with this criteria. In the context of our model, this also means the safe loan is an inferior good. A rise in $R$ is a positive income shock for the bank which raises the payoff in both states of the risky rate $r$. By assuming increasing risk aversion, we assume banks tend to take more risk as they become richer. This is why a rise in $R$ increases the amount of risky loan used and decreases the share of the safe loan in the bank’s liabilities. As the usage of the safe loan falls with higher income, we conclude safe loan is an inferior good.

Although it is quite obvious, this result is yet alone important in understanding the bank behavior in Turkey. The need for public borrowing derives the rates high and encourage the banks to take higher risks. This fact is underlined by many economists and according to many of them the only way of stabilizing Turkish
financial system is to lower the interest rates as much as possible. All the banking system and Treasury have a constant fear of not being able to sustain the debt stock. This put extra pressure on expectations and they often are shaped by the path of the public borrowing rates. If we consider the link between public borrowing rates and short term rates, mainly, the flow of unstable short-term capital in the economy determines them. In other words, to be able to borrow on more favorable conditions, the government cannot do anything but welcome all the capital; short-term or not.

All in all, considering the principal agent framework, given the bank’s risk preferences, it is in the government’s best interest to push $R$ down as much as possible. If the government derives utility for higher usage of the safe loan in terms of financial stability, pushing $R$ down will not only help for that but also lowers the borrowing costs easing the pressure on the debt stock. At the first glance, there seems no trade-off for the government. However, changing a few conditions proves the opposite. In the next sub-section, we outline these conditions and try to show how a moral hazard problem may emerge.

### 2.3.2 Moral Hazard

All the examples concerning the information problem, the moral hazard which we mention earlier in this chapter, has an agent who tends to behave against the interests of the principal. An insurance customer is tend to drive risky when he buys insurance or a worker tends to shirk after signing the employment contract. In the contractual relation between the bank and the government, this is not that obvious. The bank just makes a choice between safe and risky loan, there is no obvious reason why it tends to take more risk after it signed the contract. Up to this point, we try to establish how the spread between short and longer term rates affects financial fragility in Turkey. In other words, high public borrowing rates are seemed to be responsible for the vulnerability of the banking system. However, the income effect which is explained also shows that pushing the borrowing rate down, eases the cost pressure and stabilizes the system as well. Below, we extend our simple model by
adding some uncertainty. Then, we show how the inability to push the borrowing
rate down compounded with asymmetric information leads to moral hazard and how
the government can improve its situation by offering a transfer program contingent
on the bank’s contribution to financial stability.

As before, we consider a credit market where a government borrows from a
commercial bank. The objective of the government is to realize borrowing at the
lowest possible cost but at the same time, it cares about the financial stability.
The commercial bank, on the other hand, maximizes utility considering the rate it
receives from lending and the costs incurred in the finance of the lending.

We assume two possible ways of finance are available to the commercial bank.
The first one is a safe loan where the bank pays back \( r_f \) for each unit of borrowing.
The second one is a risky loan where the payback \( r \) is a random variable. The
distribution of \( r \) is discrete and known. With probability \( p \), \( r \) equals to \( r^H \) and with
probability \( (1 - p) \), it equals to \( r^L \) where \( r^H > r^L \). We assume the bank gives \$1 to
the government and it can obtain this money by using either of the available loans.
We further assume the bank is risk averse and it chooses the amount it gets from
the safe loan \( s \in [0, 1] \) to maximize utility.

For financial stability, the government wants to promote the safe loan by offering
a reward scheme in terms of the return on public borrowing. However, there is the
moral hazard as the government cannot observe the amount of safe loan used \( s \).
It can only observe the stock price of the bank \( \sigma \), which can either be high \( \sigma^H \) or
low \( \sigma^L \). The stock price is therefore a discrete random variable whose probability
distribution is conditional on \( s \) for any \( s \in [0, 1] \), \( \Pr(\sigma^H \mid s) = s \). Thus, given \( s \),
there exist four possible states of nature that the bank can be in, each of which
is associated with a different payoff. The probabilities and possible payoffs for the
commercial bank are given below (Figure 2.3.1).

Given the payoff scheme, the objective of the bank is to maximize expected
utility is:
Figure 2.3.1: The Probabilities and Possible Payoffs for the Commercial Bank

\[ spU(\pi^{HH}) + s(1 - p)U(\pi^{HL}) + (1 - s)pU(\pi^{LH}) + (1 - s)(1 - p)U(\pi^{LL}) \]  

(2.3.12)

Then, the first order condition for an interior solution implies ex-ante marginal utility of \( s \) equals to zero.

\[
pU(\pi^{HH}) + spU'(\pi^{HH}) \frac{d\pi^{HH}}{ds} + (1 - p)U(\pi^{HL}) + s(1 - p)U'(\pi^{LH}) \frac{d\pi^{LH}}{ds} - pU(\pi^{LH}) \\
+(1 - s)pU'(\pi^{LH}) \frac{d\pi^{LH}}{ds} - (1 - p)U(\pi^{LL}) + (1 - s)(1 - p)U'(\pi^{LL}) \frac{d\pi^{LL}}{ds} = 0 \]  

(2.3.13)

Rearranging and substituting results;

\[
\frac{d\pi^{HH}}{ds} = \frac{d\pi^{LH}}{ds} = (r^H - r_f) \quad \text{and} \quad \frac{d\pi^{HL}}{ds} = \frac{d\pi^{LL}}{ds} = (r^L - r_f)
\]
We have

\[ p[U(\pi_{HH}) - U(\pi_{LH})] + (1 - p)[U(\pi_{HL}) - U(\pi_{LL})] \]

\[ + p(r^H - r_f)[sU'(\pi_{HH}) + (1 - s)U'(\pi_{LH})] \]

\[ + (1 - p)(r^L - r_f)[sU'(\pi_{HL}) + (1 - s)U'(\pi_{LL})] = 0 \] (2.3.14)

The first two terms in ex-ante marginal utility of \( s \) in Equation (2.3.14) correspond to the extra utility associated with the raised probability of receiving the higher return \( R^H \) rather than \( R^L \), in two different states of the risky borrowing rate \( r^H \) and \( r^L \) respectively. Given that \( R^H > R^L \), we have \( \pi_{HH} > \pi_{LH} \) and \( \pi_{HL} > \pi_{LL} \) which means both terms are positive for all \( s \in [0,1] \). The remaining two terms on the left hand side of Equation (2.3.14) show the trade off between using the safe or the risky loan. Note that, necessary condition for an interior solution is \( r^H > r_f > r^L \). Under this condition, an additional unit of safe loan used decreases the utility when \( r = r^L \), i.e. the last term in Equation (2.3.14) is negative. Therefore, with \((1 - p)\)sufficiently large an interior solution is possible.

Although above condition is necessary, it is not sufficient for an interior solution. We have to rule out the two extreme cases to find an optimal \( s \in (0,1) \). First, if \( s = 0 \), then we have

\[ p[U(R^H - r^H) - U(R^L - r^H)] + (1 - p)[U(R^H - r^L) - U(R^L - r^L)] \]

\[ + p(r^H - r_f)U'(R^L - r^H) + (1 - p)(r^L - r_f)U'(R^L - r^L) > 0 \] (2.3.15)
Secondly, if \( s = 1 \), then we have

\[
[U(R^H - r_f) - U(R^L - r_f)]
\]

\[+ [p(r^H - r_f) + (1 - p)(r^L - r_f)]U'(R^H - r_f) < 0 \tag{2.3.16}
\]

which implies

\[E(r) < r_f - \frac{U(R^H - r_f) - U(R^L - r_f)}{U'(R^H - r_f)} \tag{2.3.17}
\]

The condition in Equation (2.3.17) points out a higher risk premium for the risk averse bank compared to the one in the basic model. The expected cost of the risky loan should sufficiently be lower than the cost of the safe loan for the bank to choose \( s < 1 \). If Equation (2.3.17) does not hold, the bank simply finances all investment using the safe loan.

After establishing ex-ante marginal utility of \( s \) is positive for \( s = 0 \) and negative for \( s = 1 \), we need one more necessary condition to complete the solution to commercial bank’s problem. The second order condition implies ex-ante utility is concave in \( s \).

\[
2p(r^H - r_f)[U'((\pi^{HH}) - U'(\pi^{HL})] + 2(1 - p)(r^L - r_f)[U'(\pi^{HL}) - U'(\pi^{LL})]
\]

\[+ p(r^H - r_f)^2[sU''(\pi^{HH}) + (1 - s)U''(\pi^{HL})]
\]

\[+ (1 - p)(r^L - r_f)^2[sU''(\pi^{HL}) + (1 - s)U''(\pi^{LL})] < 0 \tag{2.3.18}
\]
CHAPTER 2. PUBLIC BORROWING AND FINANCIAL FRAGILITY

Given the concavity of the utility function, last two terms in (2.3.18) are negative for all \( s \in [0, 1] \). Similarly, \( U'' < 0 \) also implies \( U'(\pi^{HH}) < U'(\pi^{LH}) \) and \( U'(\pi^{HL}) < U'(\pi^{LL}) \). Therefore, the first term is also negative as \((r^H - r_f) > 0\). That leaves the second term to be the only problematic one, as it is positive when \((r^L - r_f) < 0\). This term shows the change in extra utility gained by an increase in probability of getting \( R^H \) as \( s \) increases. It is being positive implies that the potential extra utility becomes larger as the bank’s usage of the safe loan gets larger. However, we assume this effect will be surpassed by the negativity of other three terms and hence ex-ante marginal utility of \( s \) is decreasing in \( s \) overall. The concavity of the utility function alone does not guarantee Equation (2.3.12) to be concave in \( s \) and that is the reason we need this extra assumption for the second order condition Equation (2.3.18) to hold.

Next, we analyse the government’s problem. The government wants to offer the optimal contract to maximize its objective function which has two parts. First, it derives utility from a more healthier economy, and second, it aims to minimize the borrowing cost. Therefore, the objective function can be written in the following form.

\[
\max s q^H + (1-s) q^L - s R^H - (1-s) R^L
\]  (2.3.19)

where \( q^H \) and \( q^L \) show the value of a higher stock price to the government. Suppose the probability of a strong financial environment is given by the function \( q(\sigma) \). As \( \sigma \) can only take two values, for notational simplicity, we let \( q(\sigma^H) \) to be equal to \( q^H \) and \( q(\sigma^L) \) to be equal to \( q^L \). We further assume \( q(\sigma) \) is increasing in \( \sigma \) and hence \( q^H > q^L \).

To avoid any negative payoffs, we also assume the following limited liability constraints for the transfers to the commercial bank

\[
R^H \geq r^H
\]  (2.3.20)
Given $s$, the worst case scenario for the commercial bank is a low return $R_L$ and a high rate $r^H$ which corresponds to the lowest possible payoff $\pi^{LH}$. When Equation (2.3.21) holds $\pi^{LH}$ will be non-negative so as all other higher payoffs. In the government’s program, Equation (2.3.14) becomes the incentive compatibility (IC) constraint as $s$ satisfies this condition pays a higher payoff than any other $s$ that can be chosen. Therefore, the government maximizes Equation (2.3.19) subject to Equations (2.3.14), (2.3.20) and (2.3.21).

The limited liability constraints are vital for our analysis because they characterize the inability of reducing $R$ on the government’s side. We mention earlier principal-agent framework is exempt from any negotiation issues and we just assume the principal makes the offer. This is how we take into account this sort of issues. In a country with high public debt where the government is desperate to borrow, it is not possible to push the rates under a certain level. We assume this level to be bad state risky rate just to avoid complication regarding negative payoffs\(^5\).

The first case we analyze is the case of complete information. To find the first best level of $R_L$, $R^H$ and $s$, we assume initially that the government can observe $s$. This means a contract can be written on $s$ and the government by laws and sanctions can penalize the bank heavily to induce any level of $s$. Under complete information the government can simply ban the risky loan to induce $s = 1$ and choose $R_L = R^H = r^H$ making both limited liability constraints binding. The IC constraint does not take into account the perfect info case so the only thing that the bank should be careful about is the bank’s participation constraint. Since the bank is not given any choice other than $s = 1$, it can consider its utility and may choose not to sign the contract anyway. Here, we make another simplifying assumption and

\(^5\)Most of the utility functions are not defined for negative payoffs. However this could be handled by giving the bank an initial income in case of which negative payoffs would cease to be a problem. For the sake of simplicity, we stick with our assumption here.
assume the reservation utility for the bank is simply zero. Therefore, given \( s = 1 \) and \( R_L = R_H = r_H \) participation constraint of the bank holds. These conditions simply lead to \( \pi^{HH} = \pi^{HL} = \pi^{LH} = \pi^{LL} = r_H - r_f \) and the participation constraint is

\[ U(r^H - r_f) \geq 0 \] (2.3.22)

With this condition, we establish that the government is able to induce the highest possible \( s \) with the lowest possible cost given our assumptions under complete information.

If we switch to private information, imposing \( s \) is no longer observable and a contract can only be designed contingent on \( \sigma \). The first implication is that, now the IC constraint must be taken into account.

\[
p[U(\pi^{HH}) - U(\pi^{LH})] + (1 - p)[U(\pi^{HL}) - U(\pi^{LL})] \\
+ p(r^H - r_f)[sU'(\pi^{HH}) + (1 - s)U'(\pi^{LH})] \\
+ (1 - p)(r^L - r_f)[sU'(\pi^{HL}) + (1 - s)U'(\pi^{LL})] = 0 \] (2.3.23)

The Equation (2.3.23) makes sure the bank chooses \( s \) optimally and hence, it will choose no other \( s \) given the transfer schedule. To see what happens if the government chooses not to offer a reward scheme depending on \( \sigma \), we take \( R_L = R_H \). Once two borrowing rates \( R_L \) and \( R_H \) are equal, we have \( \pi^{HH} = \pi^{LH} \) and \( \pi^{HL} = \pi^{LL} \). Hence, the first order condition reduces to

\[
p(r^H - r_f)U'(\pi^{HH}) + (1 - p)(r^L - r_f)U'(\pi^{HL}) = 0 \] (2.3.24)
Now let \( s = \phi(R^L, R^H) \) be the level of \( s \) that solves the IC condition. This condition just reduces to the first order condition of the model which we consider in the former sub-section. Given the income effect analysis we did in there, we will get a decreasing function \( \phi \) in \( R^L = R^H \). In other words, given the objective of the government, there will be no trade off and the government will choose the lowest possible rate \( R^L = R^H = r^H \). The only difference from the complete info case is that this time \( s \) is determined by the IC condition from the function \( \phi \).

This is the best government can do by offering a single independent rate. How good is the outcome depends on how high is \( s \). Although this way, the government certainly imposes the least possible cost, how high the \( s \) can be achieved depends on the parameters. This is where the limited liability constraint becomes vital. If the government can push \( R \) down sufficiently, it can induce even \( s = 1 \). And if this can be done, we end up with the complete info case and the private information does not change anything. Once again this is thanks to the income effect. On the other hand, as we assume in the developing countries, particularly in Turkey, if \( r^H \) is sufficiently high, \( s \) might be small or even zero. Then, there might be room for improvement with reward schedule. Let us look at what happens when the government increases \( R^H \) slightly over \( r^H \). The first two terms in Equation (2.3.23) becomes positive creating an incentive to increase \( s \) for the bank. For the remaining terms, the income effect kicks in again and they will tell the bank to reduce \( s \). However, if the positive effect dominates, we have \( s = \phi(R^H, r^H) \) with \( \phi'(R^H, r^H) > 0 \). This also makes room for improvement by increasing \( R^H \) further.

Substituting the function \( s = \phi(R^H, r^H) \) into the objective of the government we get

\[
\max \phi q^H + (1 - \phi)q^L - \phi R^H - (1 - \phi)r^H
\]  

(2.3.25)

Note that limited liability constraint for \( R^L \) is still binding and it is equal to \( r^H \). Then the first order condition is
\[ [\Delta q - (R^H - r^H)](\phi' - \phi) = 0 \]  \hspace{1cm} (2.3.26)

The term in brackets is the net worth of increasing \( R^H \) over \( R^L = r^H \). That is the gain from increased financial stability, \( \Delta q \) minus the extra cost of the reward scheme. It affects the objective of the government to the extend it increases the optimal \( s \) so it is multiplied by \( \phi' \). The other negative term shows the effect of the increase in probability of paying a higher rate as optimal \( s \) is now increased. Note that, the level of \( s \) that can be induced is between 0 and 1. Therefore, the slope of \( \phi, \phi' \) is positive only between 0 and 1. If \( r^H \) is low enough and optimal \( s \) already equals 1, the slope \( \phi' \) will be zero and \( \phi \) will be one and the first order condition collapses.

### 2.4 Conclusion

We tried to outline the main problems that Turkish banking system have been suffering for about two decades. One of the most important problem is the excessive interest rate risk the banks bear related to public sector borrowing scheme. We set up a model where banks are able to choose the amount of interest rate risk they would like to take given the conditions of public sector borrowing. Our analysis reveal a few points about the factors that cause the fragility based on interest rate risk and how it can be handled.

An issue that has been told about Turkish economy quite often is the incredibly high real interest rates. Our model shows how they can be the root cause of financial instability by encouraging banks to behave risky. Obviously, this also depends on the risk preferences of the banks. However, when the rates are really high and the government is really desperate to raise funds, potential profits for the banks from their business with the government exceeds the opportunities they can find elsewhere such as extending private loans etc.. Compounded with the opportunities of short
term borrowing much cheaper, this type of financial environment leads to a fragile banking structure.

The novelty with our analysis is that it displays how asymmetric information together with inability of lowering the borrowing rates lead to a moral hazard problem in the public sector debt market. This problem may not even exists in developed economies even when the banks finance their government lendings through private information. If the government is able to borrow at lower rates, it can easily avoid any financial vulnerability, but when it is not easy to push the interest rates down, the information problems emerge.

An implication of ignoring this feature in an economy like Turkish economy is the notion of becoming obsessed with lowering the borrowing rates. The sustainability of domestic debt stock is such a big issue that it is normal to be concerned about the borrowing rates. Besides, it is argued that lowering the rates will serve for improving financial stability as well, so the best an economic policy can attempt is to lower these rates as much as possible. This belief is so strong, sometimes it creates problems for the central bank which tries to target inflation and adjust the interest rates accordingly. A decision to increase interest rate in an attempt to fight with inflation, being anticipated as a commitment for the central bank to meet the inflation target, can raise concerns about the debt stock and affect the expectations badly. Our model shows that, lowering the rates really works for financial stability if you can really push them down. If you cannot do that, even when you achieve some improvement by lowering the cost of borrowing to a certain extend and ease the pressure on the debt stock, you can still be stuck in an equilibrium of high interest rates and high financial vulnerability. Therefore, our model displays if the government is unable to lower the rates sufficiently, it can apply a reward scheme to promote usage of safer methods of finance. In other words, it can buy some financial stability in return of an increase in borrowing costs. If the rates stay high anyway, focusing on minimizing borrowing costs may be sub-optimal. This can be considered as the main result of this study. The biggest problem that should be tackled in implementing such a reward scheme is finding an appropriate verifiable
component to be contracted on. For this component, (we use the stock price in our model), to be effective, it should be highly correlated with the bank’s financing decision and also it must be a good measure of financial stability for the government to rely on. Finding such a component might not be easy.

Finally, we can point out the importance of the government’s objective function. If we assume the government does not care at all about financial stability, there would be no moral hazard problem. The government would just push the single rate down until limited liability constraint binds and it would never read an incentive mechanism to promote the safer loan. Of course, this is the extreme case. More plausibly, we can assume the government weighs his concerns about financial stability and borrowing costs differently. In the objective function, we assign no weights, they are sort of equally important. If we assigned weights though, the weight that the government gives to its desire for a strong financial environment would determine how much it would be willing to sacrifice in terms of borrowing costs to promote it. In an economy like Turkey where public debt is an issue but at the same time, financial crisis can occur quite frequently, it is really hard to assign weights for these issues.
Chapter 3

Interest Rate Risk and Financial Fragility

3.1 Introduction

We summarized the characteristics of Turkish financial system in the introduction chapter and explained how Turkish banks are tend to rely on short-term borrowing from international sources to lend to the domestic government. In this chapter, we consider a Diamond and Dybvig (D&D) type model to analyze such behavior. Our model introduces short term borrowing in the basic D&D framework. Moreover, it assumes the future cost of this borrowing is unknown initially. We, then, analyze the characteristics of the risk sharing demand deposit contract in this environment.

When a bank borrows short-term and lends it for longer term, the mismatch of the maturities of its assets and liabilities makes it necessary for the bank to roll-over its debt at least for once. This is what creates an additional fragility besides depositor panics. The future roll-over cost of the bank’s debt is often subject to shocks and other economic developments that cannot be predicted in advance. When the bank gets involved in such an action, it adds uncertainty about the interest rate in its problem. Moreover, unlike the uncertainty of early or late consumption, this
uncertainty is not washed away when the agents come together and share risk. Hellwig (1994) explains this situation in the following way:

"Interest rate risk poses an important problem for banks and other financial intermediaries that take in short-term funds and finance long-term. If the returns on the long-term investment are given and subsequently the market rate of interest turns out to be very high, it is quite possible that at some point refinancing costs may exceed investment returns, or, equivalently, that the market value of the initial investment may fall below the repayment obligation to the initial financiers."

This scenario is consistent with the story we tell in Chapter two about the financial crisis in Turkey in November 2000. There was a bank which borrows mostly overnight and buys much longer term government debt instruments with those funds. Then, it rolls-over its debt on the overnight interbank market on the ongoing short-term interest rate. If at any point in time during this period the interest rate rises significantly, this bank might end up paying more than it is going to gain when the government bonds mature. Moreover, if the lenders panic and refuse to roll-over its debt, it might even fail. This was what exactly happened in Turkey in November 2000.

Hellwig (1994) points out another important feature of the interest rate risk. He says:

"...fluctuations in interest rates affect the economy as a whole, so that interest rate risk is not diversifiable. The interest-induced valuation risks of long-lived real assets can be shifted from one agent to another, or they can be shared between agents, but they cannot be diversified away".

If the interest rate rises in the market, that probably affects the values of all financial assets. During the November financial crisis in Turkey, when a jump in short term rates was observed, this obviously lowers the value of the government
securities that the bank holds at the same time. Therefore, a two-way effect is present. The borrowing cost goes up, besides, the liquidation value of the long-term investment goes down. The interest rate risk is not about putting all the eggs in the same basket. The important thing is the total exposure of the financial system to the interest rate risk and how this risk is allocated between different agents efficiently. Our deposit contract aims to provide insights on this matter.

Our analysis in this chapter is mainly inspired from the Turkish experience. We believe it contributes to the better understanding of the conditions that make Turkish economy and other emerging market economies vulnerable to banking crisis. Another noble feature of this study is that it combines two different extensions made to the basic D&D model. It introduces borrowing and aggregate uncertainty at the same time. We also allow the contract to be contingent on the uncertain borrowing rate. We think it might be interesting to see how a variable rate deposit contract allocates the risk among patient and impatient agents.

We review studies that have D&D model with aggregate uncertainty or borrowing follows this introduction. Then, the model and the results are presented. Final section concludes.

### 3.2 D&D Model, Aggregate Uncertainty and Short-term Borrowing

Chang & Velasco (2001) consider a D&D set up where each consumer has access to an international capital market and is able to borrow a certain amount of goods. The consumers act collectively by forming a bank and they surrender their endowments and their right to borrow to this bank. Then, the bank chooses how much to borrow and invest to maximize the utility of a typical ex-ante consumers. In their model though, the conditions of foreign borrowing is very advantageous. There is no interest on borrowing and it is available in both periods. The only
risk is a creditor panic. On the other hand, they assume early liquidation is costly and hence foreign borrowing becomes a perfect tool to pay the consumption needs of the impatient in the middle period. With this set up, Chang & Velasco (2001) analyze how different commitments by the bank about its foreign and domestic debt (deposits) or its ability to borrow more (openness, financial liberalization) affects the financial fragility at the equilibrium.

We built our model according to their basic set up. However, we explicitly define borrowing to be short term and no interest condition is only valid if you pay back one period later. If the debt is rolled over, i.e. same amount is borrowed again to pay the previous debt, an interest rate applies when it is paid back in the final period. That interest rate, the roll over cost, is stochastic and not known initially. However, the deposit contract can be written contingent on it.

There are other studies that introduce aggregate uncertainty into the basic D&D set up. Allen & Gale (1998) consider a model where the return of the long term project is stochastic. They consider two cases. When the contract can be written contingent on the stochastic return, first best risk sharing can be implemented. In the case of non-contingent contracts though, implementing first best is possible only if liquidation is costless. Their assumption about how the bank services deposits (abandoning the sequential servicing in D&D) makes it possible for them to define optimal bank runs that implement risk sharing even in the case of non-contingent contracts. However, in case of costly short term liquidation, this does not work and there is a role for the government. We also benefit from the first part of Allen & Gale (1998) where they consider contingent contracts. However, having a stochastic return and having a stochastic borrowing rate yields completely different outcomes.

Another study that analyzes interest rate risk by introducing aggregate uncertainty into D&D framework is done by Hellwig (1994). He considers an alternative short-term investment opportunity besides the illiquid but productive long-term project. The risk arises because the yield of investing short term consecutively is stochastic. Under these conditions, the bank allocates resources between short and
long term investment opportunities. He, as well, considers contingent contracts. Although his set up is the closest to ours, once again having short term investments with uncertain yield can lead to different results than having short-term borrowing with uncertain roll-over cost.

There are also other studies that introduce aggregate uncertainty in D&D model. Chari & Jagannathan (1988) and Jacklin & Bhattacharya (1988) introduce uncertainties about the bank run process in an attempt to explain how the runs happen. Also, there exists studies that consider models where the proportion of impatient consumers is random ((Champ, Smith & Williamson 1996), (Chari 1989), (Wallace 1990)).

3.3 The Model

3.3.1 Basic Framework

The benchmark model is inspired from Chang & Velasco (2001) and it is an extended version of the well-known Diamond & Dybvig (1983) model. We consider a closed economy occupied by many individuals who are either consumers or lenders. There exists three time periods (date = 0, 1, 2) and a single good that can be consumed and invested. The model is in real terms, there assumed to be no money and hence we will be speaking of units of consumption and/or investment.

The first group of the agents are consumers and they are born with an endowment of $e$ units of the good on date 0. There are two types of consumers. A proportion of $\lambda < 1$ of the consumers are type 1 (impatient) and they only get utility from consuming on date 1. The other $(1 - \lambda)$ proportion of the consumers, on the other hand, is type 2 (patient) and they only get utility from consuming on date 2. There is a long-term investment project that each consumer can enjoy such that one unit of the good invested in the project on date 0 yields $R > 1$ units on date 2. However, if it is liquidated early on date 1 the project yields only $\varepsilon < 1$. In other words, the
project is productive if and only if it can be held till it matures and otherwise, it can only be liquidated with a net loss. Alternatively, the good can be stored and the storage technology is such that 1 unit of the good stored simply yields 1 unit in the next period.

There is also a domestic capital market where the consumers can borrow goods from the second group of agents, the lenders. It is assumed that lenders are small and they are willing to lend subject to the conditions below as long as they expect the borrower will be able to pay the debt back.

(i) The borrowing is short-term i.e. it has to be paid back one period ahead.

(ii) The interest rate is zero for period 0 loans but there exists positive interest on loans that are taken in period 1. Therefore, for any amount that is borrowed at $T = 0$, say $b$, $b$ is paid back on date 1, however if the debt is rolled over, the borrower should pay $rb$ on date 2 where $r > 1$.

(iii) There exists an exogenously given borrowing ceiling and only a maximum of $B$ units of the good can be borrowed each period.

Although, the long-term project seems only optimal to a patient consumer, the decision process of the consumers is not straightforward because as in Diamond and Dybvig (1983) the consumers are ex-ante identical. In other words, we assume the consumers have no information regarding to their type on date 0 when the investment is actually made. They discover their types on date 1 and therefore, each consumer faces a risk of being impatient with a probability $\lambda$ and force to consume early. Moreover, each consumer’s type is private information. We assume the preferences of the consumers is given by the utility function $U(c)$ and it satisfies the Inada conditions.

Following Chang and Velasco (2001), we assume the consumers form a coalition which is called a "bank" to attain optimal risk sharing. Without aggregate uncertainty, this type of collective behavior is optimal and yields to the social optimum. The bank’s aim is to maximize the expected utility of its representative member. The problem of a representative consumer is to determine an optimal borrowing
and investment strategy for each of its members. Let $b$ show the amount of goods borrowed on date 0 and $p$ show the portion which is paid back on date 1 leaving $(b - p)$ of debt rolled over to be paid on date 2. Let $k$ and $s$ show the amount of goods that are invested to the long-term project and stocked respectively, and let $l$ denote the part of the long term investment that is liquidated early on date 1. In the benchmark model, we assume $r$, the cost of borrowing in period one, is deterministic and known on date 0. Furthermore, for the sake of simplicity, we assume the proportion of patient and impatient agents are one to one ($\lambda = 1/2$) and hence, be ignored during the optimization. The optimization problem that solves the social optimum is as follows:

$$\max U(c_1) + U(c_2)$$

s.t

$$k + s \leq b + e$$

$$c_1 \leq s + \epsilon l - p$$

$$c_2 \leq (s + \epsilon l - p - c_1) + R(k - l) - r(b - p)$$

$$c_2 \geq c_1$$

$$b \leq B$$

$$p \leq \min\{s + \epsilon l, b\}$$

$$l \leq k$$

$$c_1, c_2, k, s, l, b_0, b_1 \geq 0$$

Constraint (3.3.2) shows the investment decision in period 0. The total amount of goods available to the agent is her endowment, $e$, and the initial borrowing, $b_0$, which will be allocated in either the long term investment or stocked. Constraint (3.3.3) is the resource constraint on date 1. The bank can meet the consumption needs of the impatient in three alternative ways: by storing the necessary amount at period 0, by liquidating the long term investment or by net borrowing on date 1. Similarly,
Constraint (3.3.4) is the resource constraint for period 2 which simply states that what is in the bank in period 2 will be allocated between patient agents after the debt from period 1 is honored. Constraint (3.3.6) states that the borrowing cannot exceed the exogenously given borrowing ceiling $B$ and Constraint (3.3.7) ensures payback cannot exceed the initially borrowed amount. Similarly, Constraint (3.3.5) ensures liquidation cannot be higher than the total amount invested in the long term project. Constraint (3.3.5) is the incentive compatibility constraint, that ensures that no patient agent is better off by acting as impatient.\footnote{This constraint is a direct application of the Revelation Principle which states the Bayesian Nash equilibrium of the bank’s problem can be reproduced by an equilibrium where each agent truthfully reveal their types. Therefore, attention can be restricted to the equilibria where no agent has an incentive to misinterpret her own type. For more details of the Revelation Principle, see Myerson (1991).} The last two constraints are quite straightforward. Constraint (3.3.8) ensures that the amount liquidated early cannot exceed the total investment and the non-negativity conditions (3.3.9) are self-explanatory.

\textbf{Proposition 2.1:} When $r$ is known and $r < R$, the solution to optimization problem (3.3.1) is characterized by the following conditions:

(i) $l^* = p^* = 0$

(ii) $b^* = B$

(iii) $c_1^* = s^*$ and $c_2^* = Rk^* - rb$

Then, $s^*$ and $k^*$ are determined by the following equations:

(iv) $U'(s^*) = RU'(Rk^* - rb)$

(v) $k^* + s^* = B + e$

\textbf{Proof of Proposition 2.1:} The Kuhn-Tucker necessary conditions for the maximization problem (3.3.1) implies:

$$U'(c_1) = RU'(c_2)$$

(3.3.10)

The first order conditions for $l$ and $p$ from date 1 are:

$$\varepsilon U'(c_1) - R U'(c_2) - \gamma_3 \leq 0$$

and
\[ r U'(c_2) - U'(c_1) - \gamma_4 \leq 0 \]
where \( \gamma_3 \) and \( \gamma_4 \) are Lagrange multipliers for Constraints (3.3.7) and (3.3.8).

If Equation (3.3.10) is imposed, we have:

\[ \varepsilon U'(c_1) - R U'(c_2) < 0 \text{ given } \varepsilon < 1 \quad \text{and} \quad r U'(c_2) - U'(c_1) < 0 \text{ given } R > 0. \]
This implies \( l^* = p^* = 0 \).

The optimality condition for the choice of \( b \) implies \( U'(c_1) - r U'(c_2) - \gamma_5 \leq 0 \)
where \( \gamma_5 \) is the Lagrange multiplier for the Constraint (3.3.6). If Equation (3.3.10)
is imposed, we have \( U'(c_1) - r U'(c_2) > 0 \) which implies \( b^* = B \). Then, Constraints (3.3.3) and (3.3.4) hold as equalities and imply \( c_1^* = s^* \) and \( c_2^* = Rk^* - rb \). Next, \( s^* \)
and \( k^* \) are determined by the Equation (3.3.10) and the Constraint (3.3.2).

This solution also makes sense economically. Given the availability of the no
cost storage technology, the first implication is that, early liquidation is not optimal,
which follows \( l = 0 \). Secondly, as \( r \) is known, there is no point in borrowing on date 0
and paying it back on date 1 implying \( p = 0 \). Therefore, the optimal way of
meeting the consumption needs of the impatient is storing the necessary amount.\(^3\)

Then Constraint (3.3.3) is binding as there is no point in storing more than the
needs of the impatient.

\[ c_1 = s \] (3.3.11)

Next decision is about the initial borrowing \( b \). This is a binary decision. If
\( R \geq r \), then the bank will use all the borrowing limit since the yield will be higher
than the cost. Otherwise, the bank chooses not to borrow at all and the capital
market would be obsolete. Therefore, we focus on the former case, leading \( b = B \).
Constraint (3.3.4) will also bind as all the goods in the bank are distributed to the
patient consumers in period 2.

\(^3\)It is evident that as period 1 borrowing has a positive cost, borrowing only makes sense if it is
used for investing in the long-term technology. This technology is available at \( T=0 \), therefore,
what is borrowed at \( T=0 \) is rolled over at \( T=1 \) which implies \( b_0 = b_1 \).

\(^3\)This result is different from both Diamond and Dybvig (1983) and Chang and Velasco (2001).
In Diamond and Dybvig (1983) early liquidation is costless and in Chang and Velasco (2001) the
interest rate on loans is zero.
\[ c_2 = Rk - rB \] (3.3.12)

The Constraint (3.3.2) is also binding, as all the goods are either stocked or invested on date 0.

\[ k + s = B + e \] (3.3.13)

The optimal values \( k^* \) and \( s^* \) can then be found as functions of the exogenous variables \( \{e, r, R\} \) using the optimality conditions to smooth out the respective marginal utilities. The solution is completed by recalling \( l^* = p^* = 0 \) and \( b^* = B \).\(^4\) Also note that, incentive compatibility condition (3.3.5) holds with strict inequality at the optimum.\(^5\)

### 3.3.2 Demand Deposits and Bank-runs

The solution given in the previous subsection is the best outcome that the bank can accomplish. One way of achieving that is by using demand deposits as suggested by Diamond and Dybvig (1983). The demand deposit system has the following features. At date 0, the consumers deposit their endowments and give their right to borrow to the bank. The bank then borrows \( B \), stores \( s^* \) of the goods and invest \( k^* \) in the long-term technology. In return, the consumers can either withdraw \( c_1^* \) on date 1 or \( c_2^* \) on date 2. Moreover, we assume that the bank has a sequential service constraint, which requires that the consumers who are tend to withdraw resources are serviced sequentially. In other words, the bank’s payoff to any consumer depends only to her place in the line.

\(^4\)The benchmark model and the solution of our model is very similar to Chang and Velasco (2001). There are only a few differences. First of all, the lending is provided by domestic lenders in a closed economy instead of foreign creditors. Secondly, maturity of the debt is explicitly defined as one period. Finally, the interest rate on period 1 loans is positive instead of zero. These modifications are made to fit with the appropriate extensions that will follow in the coming sections.

\(^5\)Recall \( s^* \) and \( k^* \) are determined by \( U'(c_1) = RU'(c_2) \). Then, with \( R > 1 \), one has \( U'(c_1) > U'(c_2) \) and with \( U'' < 0 \). This implies \( c_2 > c_1 \).
Given the environment, each consumer decides to withdraw or wait after discovering her type at $T=1$. The bank meets the demand from consumers first from its stocked goods and then, by liquidating the long term investment until it runs out of resources. If the requests exceed the total liquidation value of the bank’s assets the bank fails and closes. Therefore, any consumer comes to the bank at $T=1$ can withdraw $c_1$ units if the bank is still open. If the bank survives, it pays its debt at $T=2$ and then, it is dissolved and the goods are distributed among the consumers who did not withdraw at $T=1$.

The demand deposit system can lead to the optimal allocation if each consumer’s withdrawal decision is consistent with her own type. If only impatient agents withdraw in the first period, then the bank does not close and pay $c_2$ to the patient depositors in the second period. This is considered to be the good equilibrium. Note that each consumer’s withdrawal strategy and each lender’s lending strategy are optimal, given the final outcome. However, there is another equilibrium of the game in which a bank run occurs. Suppose now all the consumers go to the bank to reclaim their funds and that the lenders refuse to roll over the debt in period 1. Then the bank will fail if

$$f = c_1 + b^* - s^* - \varepsilon k^* > 0 \quad (3.3.14)$$

That is the case when bank’s short term liabilities exceed its short term assets. Notice that this run scenario is another equilibrium because if Equation (3.3.14) holds it is optimal for patient consumers to withdraw in the first period and it is optimal for lenders not to roll over the debt because the bank will not be able to honour its debt in period 2. Moreover, $f$, the mismatch between bank’s short term assets and liabilities, can be used as an index of financial fragility. In other words, the more positive $f$ is, for a wider range of parameter values, the runs will be possible.

In the next subsection, we extend the model by assuming the short term interest rate on period one loans, $r$, is unknown while the investment decision is made and we will analyze its effects on financial fragility.
### 3.3.3 Uncertainty and Interest Rate Risk

In this section, we assume $r$ is random and on date 0, the bank decides on investment and borrowing before observing it and by knowing only its distribution. Moreover, the distribution of $r$ is such that $r$ may or may not be smaller than $R$, but it is greater than one. To be more clear about the timing of the actions, a timeline is given in Figure 3.3.1.

![Figure 3.3.1: Timeline of the events](image)

In the new setup, while the bank makes the investment and borrowing decisions on date 0, it does not know what will the roll-over cost be for initial debt. On the other hand, bank’s decisions on date 1 is made after $r$ is observed. As seen in the timeline above, bank decides on early liquidation $l$ and payback $p$ after it observes the roll over cost. The consumption allocation between early and late consumers is also made after $r$ is realized. Another way of considering bank’s problem is to see these choices as premature plans for future consumption, liquidation and payback contingent on $r$ on date 0. However, one way or the other, these choices are made free from the uncertainty imposed by the random short-term borrowing rate and hence, they will all be functions of $r$.

The choice of payback $p$ and liquidation $l$ is closely related to the realization of $r$. Recall that in the previous subsection, the optimal allocation yields to $b = B$. This is obvious because without any interest rate uncertainty, given that $R > r$, bank simply uses all available funds and than rolls over the whole debt. This is no more obvious. The information set of the bank on date 1 contains $s, k, b$ from the
previous period and $r$. So, having observed $r$, does the bank choose $l = p = 0$?

There exists two factors in making this decision. First, concern of the bank is to allocate the resources between early and late consumers by smoothing the marginal utilities. As $s$, $k$ and $b$ are chosen before observing $r$, therefore, the optimality conditions imply smoothing of expected marginal utilities. After observing $r$ bank may prefer to do more smoothing by choosing $l$ and $p$ different than zero. Second concern is the trade-off between the resource availability on dates 1 and 2. If the realization of $r$ is too high, the bank may prefer to do some payback on date 1 instead of paying the high interest on date 2 and hence, once again can choose $l$ and $p$ to be different than zero. Furthermore, the initial borrowing on date 0 may not be equal to $B$ anymore given that the distribution of $r$ allows it to be higher than $R$ with positive probability.

Next, we further assume that the short term interest rate on date 1 debt is distributed uniformly in the interval $[1, \bar{r}]$. Recall that, $r$ is the composite rate, i.e. $rb$ shows the total amount that must be paid back in return of borrowing $b$. Therefore, the lower limit is obvious. For the upper limit of possible values for the short-term borrowing rate we assume

$$R < \bar{r} < \frac{R}{\epsilon}, \quad \text{(3.3.15)}$$

First of all, the upper limit of $r$ is greater than the return of the long-term asset $R$ by assumption. This ensures $r$ can be bigger than $R$ with positive probability. Second, we assume $\bar{r} < \frac{R}{\epsilon}$ which means $r$ is smaller than $\frac{R}{\epsilon}$ with probability one. This assumption is critical to ensure bank never liquidates to payback on date 1 whatever the interest rate is.\(^6\) This assumption is reasonable because the analysis in this chapter relies on short-term illiquidity. If the long-term investment is sufficiently illiquid, which means $\epsilon$ is sufficiently small, bank will never liquidate to payback some of its debt on date 1. If we consider the extreme case, where long term asset worths simply nothing when liquidated, it will automatically quit being an option

\[^6^\text{A similar assumption also exists in Hellwig (1994) to ensure that long term investment is never liquidated to invest in short-term asset on date 1.}\]
for payback on date 1, no matter how high the interest rate on loans would be.

Under this set-up Bank’s optimization problem on date 1 will be:

\[
\begin{align*}
\text{max} & \quad U(c_1) + U(c_2) \\
\text{s.t} & \quad c_1 \leq s + \varepsilon l - p \\
& \quad c_2 \leq (s + \varepsilon l - p - c_1) + R(k - l) - r(b - p) \\
& \quad p \leq \min\{b, s + \varepsilon l\} \\
& \quad l \leq k \\
& \quad c_2 \geq c_1 \\
& \quad c_1, c_2, l, p \geq 0
\end{align*}
\] (3.3.16 - 3.3.22)

Once again, we assume \( \lambda = 1/2 \), i.e. the proportions of patient and impatient consumers are equal and recall that \( c_1, c_2 \) are the corresponding consumptions for patient and impatient agents. \( p \) is the amount of the date 0 debt that is paid back in date 1, therefore, \( (b - p) \) amount of debt is rolled over. \( l \) is the amount of long term investment that is liquidated prematurely on date 1 and with early liquidation one unit invested on date 0 only yields \( \epsilon < 1 \).

The general form of the solution to date 1 maximization problem where the bank chooses \( c_1, c_2, l \) and \( p \) is summarized in Preposition 2. On date 1, \( s, k \) and \( b \) are given and \( r \) is realized so the choices will be functions of these variables and other parameters.

**Proposition 2.2:** The optimal values \( c_1^*, c_2^* \), \( l^* \) and \( p^* \) can be characterized by the following conditions:

If \( Rk - r(b - \min\{b, s\}) < s - \min\{b, s\} \)

\[
\begin{align*}
c_1^* &= c_2^* = \frac{s - \min\{b, s\} + Rk - r(b - \min\{b, s\})}{2} \\
l^* &= 0
\end{align*}
\]
Chapter 3. Interest Rate Risk and Financial Fragility

\[ p^* = \min\{b, s\} \]

If \( R_k - r(b - \min\{b, s\}) > s - \min\{b, s\} \)

\[ c_1^* = s + \varepsilon l^* - p^* \]

\[ c_2^* = R(k - l^*) - r(b - p^*) \]

and

\[ 0 < l^* < k \quad \text{if } r < r_1 \]
\[ p^* = 0 \]

\[ l^* = 0 \quad \text{if } r_1 < r < r_2 \]
\[ p^* = 0 \]

\[ l^* = 0 \quad \text{if } r_2 < r < r_3 \]
\[ 0 < p^* < \min\{b, s\} \]

\[ l^* = 0 \quad \text{if } r > r_3 \]
\[ p^* = \min\{b, s\} \]

The first part of the solution shows the case where some of the resources from date 1 are saved to be allocated to late consumers. Note that, transferring resources from date 1 to date 2 is costless by the given storage technology and for optimality, the bank wants to smooth out marginal utilities of early and late consumers. Suppose after observing \( r \) the bank wants to transfer resources from early to late consumers. On the extreme case, it can choose \( l = 0 \) and \( p = \min\{b, s\} \) to maximize date 2 consumption. When it does that, if the resources available on date 1 is still higher than the resources available on date 2, then the bank simply transfers some resources from date 1 to date 2 and equate early and late consumption. Otherwise, the bank allocates the resources that are available on date 1 to early consumers and the resources that are available on date 2 to late consumers which is shown in the second part of the solution. One important feature of the model is that it is easy to transfer resources from early to late consumers but not vice versa. One more unit of repayment on date 1 increases late consumption by \( r > 1 \) units and the storage
technology allows one to one transfer. On the other hand, bringing resources to
date 1 requires premature liquidation of the long-term investment which is costly.
This feature makes the incentive compatibility condition redundant and as long as
the bank chooses consumption optimally late consumption is always greater than or
equal to early consumption (See the Proposition 2.2 below).7

The second part of the solution shows how the realization of \( r \) becomes important
when we have \( Rk \geq s - b \). Different realization of \( r \) yields to four different regions
for optimal choices of the variables. In region 1, it is observed premature liquidation
is only optimal if \( r \) is sufficiently small, i.e. it is below a threshold level \( r_1 \). When
the realization of the short-term borrowing rate is low, the bank chooses to roll-over
all the debt, so \( p^* = 0 \). However, it is so low and late consumption is still so high
that, although it is costly, it chooses to liquidate some of the long-term asset to
increase early consumption in the expense of the late consumption. In region 2,
when \( r \) gets higher and realizes between \( r_1 \) and \( r_2 \), the bank stops liquidating but
still rolls over the whole debt, therefore, \( p^* = l^* = 0 \). When \( r \) reaches to region
3 and realizes between \( r_2 \) and \( r_3 \), the bank starts paying some of its debt back on
date 1, but it does it by using the stored resources as we established by assumption
that it is never optimal to liquidate for the purpose of payback. Therefore, \( l^* = 0 \)
is maintained. Finally, when \( r \) exceeds the third threshold level \( r_3 \), the bank pay
whole debt on date 1 as it becomes too costly to roll over any portion of the debt.
In other words, late consumption becomes too low and the bank tries to transfer
resources from date 1 to date 2. Next, to understand the solution better, we provide
a proof for Proposition 2 and an example solution with a functional form for the
utility function.

**Proof of Proposition 2.2:** On date 1, \( s, k \) and \( b \) are given from date 0 and \( r \)
is realized. Ignore the IC Constraint (3.3.21) for now.

The Lagrangian for the maximization problem (3.3.16) can be written as:

---

7This is a common feature with the Allen and Gale (1998) model, optimality conditions auto-
matically satisfies incentive compatibility condition.
\[ L = U(c_1) + U(c_2) + \gamma_1[s + \varepsilon l - p - c_1] + \gamma_2[s + \varepsilon l - p + R(k - l) - r(b - p) - c_1 - c_2] + \gamma_3[k - l] + \gamma_4[\min\{b, s + \varepsilon l\} - p] \]

where \( \gamma_i \) are the lagrange multipliers. The Kuhn-Tucker necessary conditions for a maximum are:

\[
\begin{align*}
    U'(c_1) - \gamma_1 - \gamma_2 & \leq 0, \quad c_1 [U'(c_1) - \gamma_1 - \gamma_2] = 0 \quad (3.3.23) \\
    U'(c_2) - \gamma_2 & \leq 0, \quad c_2 [U'(c_2) - \gamma_2] = 0 \quad (3.3.24) \\
    \varepsilon (\gamma_1 + \gamma_2) - R \gamma_2 - \gamma_3 & \leq 0, \quad l [\varepsilon (\gamma_1 + \gamma_2) - R \gamma_2 - \gamma_3] = 0 \quad (3.3.25) \\
    -\gamma_1 + \gamma_2 (r - 1) - \gamma_4 & \leq 0, \quad p [-\gamma_1 + \gamma_2 (r - 1) - \gamma_4] = 0 \quad (3.3.26) \\
    s + \varepsilon l - p - c_1 & \geq 0, \quad \gamma_1 [s + \varepsilon l - p - c_1] = 0 \quad (3.3.27) \\
    s + \varepsilon l - p + R(k - l) - r(b - p) - c_1 - c_2 & \geq 0, \quad (3.3.28) \\
    \gamma_2 [s + \varepsilon l - p + R(k - l) - r(b - p) - c_1 - c_2] = 0 \\
    k - l & \geq 0, \quad \gamma_3 [k - l] = 0 \quad (3.3.29) \\
    \min\{b, s + \varepsilon l\} - p & \geq 0, \quad \gamma_4 [b - p] = 0 \quad (3.3.30)
\end{align*}
\]

First of all, Constraint (3.3.18) is binding in any case since what is available in date 2 will be allocated to patient consumers. Therefore, \( \gamma_2 > 0 \) and Constraint (3.3.28) holds with equality.

**Case 1:** Assume Constraint (3.3.17) is not binding which yields \( \gamma_1 = 0 \) and \( U'(c_1) = U'(c_2) = \gamma_2 \).

Then, from Constraint (3.3.28), \( c_1 = c_2 = \frac{s + \varepsilon l - p + R(k - l) - r(b - p)}{2} \) and this is only possible if \( (s + \varepsilon l - p) > R(k - l) - r(b - p) \) with \( \gamma_1 = 0 \). From Constraint (3.3.25) \( (\varepsilon - R) \gamma_2 - \gamma_3 \leq 0 \) which holds with strict inequality because \( (\varepsilon - R) < 0 \) by definition.

This follows \( l = \gamma_3 = 0 \).
Then, from Constraint (3.3.30) \( \gamma_2 (r - 1) - \gamma_4 \leq 0 \). As \( (r - 1) > 0 \) by definition \( \gamma_4 > 0 \) which follows from (3.3.29) \( p = \min\{b, s + \varepsilon l\} \) with \( l = 0, p = \min\{b, s\} \).

To summarize, we have \( c_1 = c_2 = \frac{s - \min\{b, s\} + Rk - r(b - \min\{b, s\})}{2} \), \( l = 0, p = b \) if \( Rk - r(b - \min\{b, s\}) < s - \min\{b, s\} \) which completes the proof of the first part of the proposition.

**Case 2:** Assume (3.3.17) is binding which yields \( \gamma_1 > 0 \) and \( U'(c_1) = \gamma_1 + \gamma_2 \), \( U'(c_2) = \gamma_2 \).

Then, \( c_1 = s + \varepsilon l - p \) and \( c_2 = R(k - l) - r(b - p) \) and this is only possible if \( (s + \varepsilon l - p) < R(k - l) - r(b - p) \) and hence, \( c_1 < c_2 \).

At this point, we proved the optimality conditions automatically satisfies incentive compatibility.

Then, the optimal choices of \( l \) and \( p \) are determined by (3.3.25) and (3.3.26) which were \( \varepsilon U'(c_1) - R U'(c_2) - \gamma_3 \leq 0 \) and \( r U'(c_2) - U'(c_1) - \gamma_4 \leq 0 \).

**Assume internal solution for \( l \):**

\( \gamma_3 = 0 \), \( l < k \) and \( \varepsilon U'(c_1) = R U'(c_2) \) solves for optimal \( l \). Substitute into (3.3.26) to get \( r U'(c_2) - \frac{R}{\varepsilon} U'(c_2) - \gamma_4 \leq 0 \).

Then, given \( r < \frac{R}{\varepsilon} \), \( r U'(c_2) - \frac{R}{\varepsilon} U'(c_2) < 0 \) and \( p = \gamma_4 = 0 \).

**Now, assume internal solution in \( p \):**

\( \gamma_4 = 0 \), \( p < b \) and \( r U'(c_2) = U'(c_1) \) solves for optimal \( p \). Substitute into (3.3.25) to get \( (\varepsilon r - R) U'(c_2) - \gamma_3 \leq 0 \).

Then, given \( r < \frac{R}{\varepsilon} \), \( (\varepsilon r - R) U'(c_2) < 0 \) and \( l = \gamma_3 = 0 \).

**Now, assume \( l = 0 \):**

\( \gamma_3 = 0 \), \( l = 0 \) and \( \varepsilon U'(c_1) < R U'(c_2) \), then \( U'(c_1) < \frac{R}{\varepsilon} U'(c_2) \).

Then, given \( r < \frac{R}{\varepsilon} \), \( r U'(c_2) < \frac{R}{\varepsilon} U'(c_2) \),
and we can have \( p < b \) determined by \( r U'(c_2) = U'(c_1) \) (same as above),
or we can have \( r U'(c_2) < U'(c_1) \) and \( p = 0 \),
or we can have \( r U'(c_2) > U'(c_1) \) and \( p = \min\{b, s\} \).

**Now, assume** \( p = 0 \):

\[
\gamma_4 = 0, \ p = 0 \text{ and } r U'(c_2) < U'(c_1).
\]

Then, given \( r < \frac{R}{\varepsilon} \), \( r U'(c_2) < \frac{R}{\varepsilon} U'(c_2) \),
and we can have \( l < k \) determined by \( U'(c_1) = \frac{R}{\varepsilon} U'(c_2) \) (same as above),
or we can have \( U'(c_1) < \frac{R}{\varepsilon} U'(c_2) \) and \( l = 0 \) (same as above),
or we can have \( U'(c_1) > \frac{R}{\varepsilon} U'(c_2) \) and \( l = k \) (Not Possible, \( c_2 < 0 \)).

**Now, assume** \( l = k \):

\[
\gamma_3 > 0, \ l = k \text{ and } \varepsilon U'(c_1) > R U'(c_2), \text{ then } U'(c_1) > \frac{R}{\varepsilon} U'(c_2).
\]

Then, given \( r < \frac{R}{\varepsilon} \), \( r U'(c_2) < \frac{R}{\varepsilon} U'(c_2) < U'(c_1) \) and we have \( p = 0 \) (Not Possible, \( c_2 < 0 \)).

**Now, assume** \( p = b \):

\[
\gamma_4 > 0, \ p = b \text{ and } r U'(c_2) > U'(c_1).
\]

Then, given \( r < \frac{R}{\varepsilon} \), \( U'(c_1) < r U'(c_2) < \frac{R}{\varepsilon} U'(c_2) \) and we have \( l = 0 \) (same as above).

After considering all possible solutions, we end up with the four possible cases that are summarized in Proposition 2. To complete the proof, to have a better understanding of how consumption, liquidation and payback decisions are made and to see how threshold values for \( r \) are determined, we work with a functional form for the utility function in the next section.

**A Sample Utility Function**

Now, we go one step further and assume
\[ U(c) = \ln(1 + c) \]  
\[ \text{with } U'(c) = \frac{1}{1 + c} \]

The functional form is chosen to satisfy Inada conditions. The log utility function is slightly modified to avoid any negative numbers and make it defined when \( c = 0 \).

Now consider the four regions that are characterized in Proposition 2.

**Region 1**

\( 0 < l^* < k \) and \( p^* = 0 \).

If we denote early and late consumption for Region 1 as \( c_{11} \) and \( c_{21} \):

\[
\begin{align*}
    c_{11} &= s + cl \\
    c_{21} &= R(k - l) - rb
\end{align*}
\]

\[ \varepsilon U'(c_1) = R U'(c_2) \] solves for optimal \( l \),

\[ r U'(c_2) - \frac{R}{\varepsilon} U''(c_2) < 0 \] and \( p = 0 \).

Substituting the marginal utility in Equation (3.3.31) and solving for \( l \) we get

\[ l^* = \frac{1}{2} \left( k + \frac{1 - rb}{R} - \frac{1 + s}{\varepsilon} \right). \]

To have an internal solution in \( l \), we should have a positive \( l \) that solves \( \varepsilon U'(c_1) = R U'(c_2) \).

Therefore, imposing \( l^* > 0 \), we have \( k + \frac{1 - rb}{R} - \frac{1 + s}{\varepsilon} > 0 \) which yields \( r < \frac{R}{s} \left( k + \frac{1}{R} - \frac{1 + s}{\varepsilon} \right) = r_1 \) and that gives us the first threshold level for \( r \).

Then, substituting optimal \( l \) in consumption equations we get:
\[c_{11} = \frac{1}{2} \left[ s - 1 + \epsilon k + \frac{\epsilon(1 - rb)}{R} \right]\]
\[c_{21} = \frac{1}{2} \left[ Rk - 1 - rb + \frac{R(1 + s)}{\epsilon} \right]\]

**Region 2**

\(l^* = 0\) and \(p^* = 0\).

Then, early and late consumption for Region 2 will be:

\[c_{12} = s \quad c_{22} = Rk - rb\]

\(\varepsilon U'(c_1) < R U'(c_2)\) so \(l = 0\).

From this condition, we have \(\frac{\varepsilon}{1 + s} < \frac{R}{1 + Rk - rb}\) which yields \(r > \frac{R}{b} (k + \frac{1 + s}{R} - \frac{1 + s}{\varepsilon}) = r_1\).

Next, we have \(r U'(c_2) < U'(c_1)\) so \(p = 0\). From this condition, we have \(\frac{r}{1 + Rk - rb} < \frac{1}{1 + s}\) which yields \(r < \frac{1 + Rk}{1 + s + b} = r_2\) so we have our second threshold level for \(r\).

**Region 3**

\(l^* = 0\) and \(0 < p^* < \min\{b, s\}\).

Then, early and late consumption for Region 3 will be:

\[c_{13} = s - p^* \quad c_{23} = Rk - r(b - p^*)\]

\(r U'(c_2) - U'(c_1) = 0\) solves optimal \(p\). Substituting the marginal utilities and solving for \(p\), we get \(p^* = \frac{1}{2} (s + b + 1 - \frac{1 + Rk}{r})\).
To have an internal solution in $p$, we should have a positive $p$ that solves $rU'(c_2) = U'(c_1)$.

Therefore, imposing $p^* > 0$, we have $s + b + 1 - \frac{1 + R_k}{r} > 0$, which yields $r > \frac{1 + R_k}{1 + s + b} = r_2$.

Now, we should consider two cases, when $s - b$ is positive and negative to find consecutive threshold levels.

If $s - b > 0$, then $p^* < b$. From this condition, we have $r > \frac{1 + R_k}{1 + s - b} = r_3$, which is the third threshold level for $s - b > 0$.

Next, consider the case $s - b < 0$ and hence, $\min\{b, s\} = s$.

$p^* < s$.

From this condition, we have $r > \frac{1 + R_k}{1 + s - b} = r_3$, which is the third threshold level for $s - b < 0$.

In general, we can write $r_3$:

$$r_3 = \frac{1 + R_k}{1 + |s - b|}$$

Then, substituting optimal $p$ in consumption equations, we get:

$$c_{13} = \frac{1}{2} [s - b - 1 + \frac{1 + R_k}{r}]$$
$$c_{23} = \frac{1}{2} [R_k - 1 + r(1 + s - b)]$$

**Region 4**

$l^* = 0$ and $p^* = \min\{b, s\}$.

Then, early and late consumption for Region 4 will be:

$$c_{14} = s - \min\{b, s\}$$
$$c_{24} = R_k - r(b - \min\{b, s\})$$
Now, we should consider two cases, when $s - b$ is positive and negative to find consecutive threshold levels.

If $s - b > 0$, then $p^* = b$. Next,

\[ c_{14} = s - b \]
\[ c_{24} = Rk \]

Then, since the constraint for $p$ binds, we have $r \cdot U(c_2) > U(c_1)$.

From this condition, we have \( \frac{r}{1 + Rk} > \frac{1}{1 + s - b} \) which yields $r > \frac{1 + Rk}{1 + s - b} = r_3$.

Note that this is the highest possible level for late consumption and lowest possible level for early consumption when $b - s$ is positive. If the early consumption is still higher than the late consumption than for any realization of $r$, the bank applies the first part of the solution presented in Proposition 2 and equates early and late consumptions.

Next, consider the case $s - b < 0$ and hence $\min\{b, s\} = s$.

$p^* = s$, then:

\[ c_{14} = 0 \]
\[ c_{24} = Rk - r(b - s) \]

\( \frac{r}{1 + Rk - rb + rs} > 1 \), which yields $r > \frac{1 + Rk}{1 + b - s} = r_3$.

After this point, when $s - b$ is negative, the first part of the solution in the proposition comes into the picture differently. Recall that the condition in that part states $Rk - r(b - s) < 0$ which yields $r > \frac{Rk}{b - s} = r_4$ which provides another threshold level.
At this level of interest the bank equates early and late consumptions by choosing \( c_1 = c_2 = 0 \) as stated in the first part of Proposition 2. Note that for any higher \( r \), the bank fails to honour at least some of its debt.

The graphical representations of the liquidation, payback and consumption behavior under \( s - b > 0 \) are given in the Figures 3.3.2, 3.3.3 and 3.3.4 below.

When \( s - b > 0 \), we observe the interest rate risk is mainly borne by the early consumers. The risk sharing deposit contract allocates less resources to both consumers as \( r \) increases for the first two regions. However, when it keeps rising, the cost is paid by early consumers due to falling consumption. When \( r \) exceeds a certain level, it becomes optimal for the bank not to roll over the debt and pay as much as it can back on date 1. This lowers the early consumption and raises the late consumption as the payment of the debt is shifted from date 2 to date 1.

Liquidation is efficient only if \( r \) is sufficiently small. This is only for consumption smoothing. A low \( r \) means late consumers will be able to consume more. Therefore,
if \( r \) is too low, liquidation is used to bring some consumption back to date 1. Note that, the liquidation behavior is not affected from the assumption about \( (s - b) \).

Bank only prefers to pay back if \( r \) is sufficiently high. Rolling over is not optimal when roll over cost exceeds a threshold level. Here, as the stored resources are higher than the debt itself, it manages to pay the whole debt back successfully.

The consumption and payback graphs when \( s - b < 0 \) are given next (Figure
A similar behavior is observed when $s - b < 0$, with one particular difference. This time, the liquid funds are not enough to pay the whole debt back. Therefore, as $r$ gets higher, the bank pays back more in Region 3 until it depletes all the liquid funds. Given our liquidation for payback is not optimal assumption, even then some of the debt remains unpaid on date $2$, which means late consumption also falls with $r$ in region 4. If all of the stored goods used for pay back, the early consumption falls to zero. The dashed line indicates the slope of late consumption in region 3 is ambiguous, it may be positive or negative.

### 3.3.4 Date zero problem

On date 0, the bank chooses $s, b, k$. In fact, it can only choose two of them as the endowment $e$ is given and all available resources are either stored or invested. Therefore, we can easily eliminate $s$ as a choice variable using Equation (3.3.2) as a strict equality and imposing:

$$s = e + b - k$$  (3.3.33)
This way $s$ can be eliminated from all consumption levels, plus from the critical values of the random variable $r$ defined in the previous section and the optimization problem can be written in $k$ and $b$ only. The bank decides on how much to invest and borrow on date 0, anticipating how it would behave on date 1 depending on different realizations of the borrowing rate $r$.

From Proposition (2.2), we know early and late consumption, $c_1$ and $c_2$, take different values depending on how $k$ and $b$ are chosen and how $r$ is realized. This means we have to work separately on different regions defined by our choice of $k$ and $b$, because in each of these regions, we maximize a different utility function.

For example, if we choose to borrow nothing on date 0, we will have a deterministic utility function as the realization of $r$ will no longer be a concern. Similarly, if we choose $k$ and $b$ as in the first part of Proposition (2.2) with $e < k$ (or $s > b$), we anticipate that we will pay all of the debt on date 1 and hence once again $r$ will no longer be a concern. Furthermore, when we have to consider $r$ as in the second part of the Proposition (2.2), the three critical values of $r$ also depend on the choice of $k$ and $b$. Therefore, whether these critical points are in or out of the domain $[r, \bar{r}]$ is determined by the choice variables. This defines even more regions to work on. These regions and the utility functions associated with them are outlined below for a specific set of parameter values as an example. In this example, we keep the
functional form $U(c) = \ln(1 + c)$ that we used in identifying date 1 problem and we further assume $R = 2$, $\overline{r} = 2.5$, $\underline{r} = 1$, $e = 10$, $\epsilon = 0.5$, $B = 10$.

First of all, we identify the regions for $s > b$ or equivalently $e < k$. We start with the first part of Proposition (2.2) where $Rk < s - b = e - k$ which yields $k < \frac{e}{1 + R}$. Then, we have $c_1 = c_2 = \frac{e + (R - 1)k}{2}$. As $(R - 1) > 0$, we set $k^* = \frac{e}{1 + R}$ to maximize utility. $k^* = 3.33$, $c_1 = c_2 = 6.67$ and the maximized utility $U = 2(\ln(1 + 6.67)) = 4.07$. Note that, $b$ can take any value such that $0 \leq b^* \leq B$ and $s^* = 6.67 + b$.

Next, we move to the second part of the Proposition (2.2). For this part, we have to identify for what values of $k$ and $b$, the critical points $r_1$, $r_2$, and $r_3$ fall in or out of the domain $[\underline{r}; \overline{r}]$. For this purpose, we consider six different lines for each critical points:

- $r_1 = \overline{r} \implies k = 7.16 + 0.83b$
- $r_1 = \underline{r} \implies k = 7.16 + 1.08b$
- $r_2 = \overline{r} \implies k = 3.33 + 0.66b$
- $r_2 = \underline{r} \implies k = 5.89 + 1.11b$
- $r_3 = \overline{r} \implies k = 3.33$
- $r_3 = \underline{r} \implies k = 5.89$

In addition to these, we have three more conditions:

1) $e \leq k$ to ensure that $s > b$,
2) $k \geq 3.33$ to ensure that we are in the second part of the Proposition (2.2),
3) $b \leq B$ to meet the borrowing constraint.

In line with these conditions, the regions of $k$ and $b$ which we consider are marked in Figure 3.3.7. Each region in the figure is associated with a set of conditions regarding the critical points we derive in the date 1 optimization problem. For example Region A lies above lines 1, 2, 3, 5, 6 and 7 indicating all three critical points are above the upper limit for the random variable $r$. In other words, if the bank chooses $k$ and $b$ in this region, with probability one, the consumption paths of
early and late consumers will lie in the first of the four regions we define in date
1 problem (see Figure 3.3.2). Region A is bounded at the top by line 4 as we are
analyzing the case $s - b > 0$ in this part and hence we are allowed to choose $k < e$.
The positions of three critical points relative to the domain $[r, \bar{r}]$ for each region and
the expected utility function associated with them are outlined below.

Region A:

$\underline{r} < r < r_1$

$\underline{r} < r < r_2$

$\underline{r} < r < r_3$

\[ EU = \frac{1}{\bar{r} - \underline{r}} \int_{\underline{r}}^{\bar{r}} [\ln(1 + c_{11}) + \ln(1 + c_{12})] \, dr \]

Figure 3.3.7: Regions for $s - b > 0$

Region B:

$\underline{r} < r_1 < \bar{r}$

$\underline{r} < r_2 < r_1$

$\underline{r} < r_3 < r_2$

$\underline{r} < r_3 < r_3$
EU = \frac{1}{\tau - \xi} \left\{ \int_{\xi}^{\tau_1} \ln(1 + c_{11}) + \ln(1 + c_{12}) \, dr + \int_{\tau_1}^{\tau} \ln(1 + c_{12}) + \ln(1 + c_{22}) \, dr \right\}

Region C:

r_1 < \xi < \tau
\tau < \xi < r_2
\tau < \xi < r_3
EU = \frac{1}{\tau - \xi} \int_{\xi}^{\tau_1} \ln(1 + c_{12}) + \ln(1 + c_{22}) \, dr

Region D:

r_1 < \xi < \tau
\tau < r_2 < \tau
\tau < \tau < r_3

Region E:

r_1 < \xi < \tau
r_2 < \xi < \tau
\tau < \tau < r_3

Region F:

r_1 < \xi < \tau
\tau < r_2 < \tau
\tau < r_3 < \tau
EU = \frac{1}{1 - \frac{c}{\varepsilon}} \left\{ \int_{r_1}^{r} \left[ \ln(1 + c_{12}) + \ln(1 + c_{22}) \right] dr + \int_{r_2}^{r_3} \left[ \ln(1 + c_{13}) + \ln(1 + c_{23}) \right] dr + \int_{r_3}^{\bar{r}} \left[ \ln(1 + c_{14}) + \ln(1 + c_{24}) \right] dr \right\}

Region G: 
- \bar{r} < r < r_1 < r_2 < r_3 < \bar{r} 

All these regions are analyzed separately to determine which yields the highest value of the objective function. We divide them into sub-regions where necessary. In Figure 3.3.8 the objective function is plotted for Region A (right panel). Note that, we only allow the bundles of k and b that are inside region A. This can be seen on the right panel of the figure. The projection of the utility function on the bk plane is Region A itself.

The analysis of all seven regions for s - b > 0 indicates a maximum at the intersection of regions D and E at b = 10 and k = 10 and the maximized objective is 4.34. The corresponding sub-section of Region E is given in Figure 3.3.9. We evaluate borders in each region and observe the objective function takes the same value due to continuity. However only when b = 0, the optimization problem becomes a different one and to avoid any problems for all regions we assume the lower bound for b is 0.1. In other words, for the sake of simplicity we ignore the choice of any borrowing between 0 and 0.1. The solution for b = 0 is evaluated separately later in this section. The plots for all regions can be found in the Appendix.

For s - b < 0, the bank now has less than full resources to pay all the debt on date 1. Given our no liquidation for pay-back assumption (\bar{r} < \frac{R}{\varepsilon}), this means at
least some portion of the debt is going to be paid on date 2 with the cost of \( r \). We have \( k > e \) as \( b > s \) and also \( r = r_4 = \frac{Rk}{b-s} \), \( c_1 = c_2 = 0 \). Therefore, we do not allow a choice of \( b \) and \( k \) beyond this point, as any interest rate higher than this level will result in some amount of unpaid debt. A positive probability for that makes it possible for the lenders to behave differently and do not extend any funds to the consumers.

In the light of these assumptions, the available regions for \( s - b < 0 \) are presented in the Figure 3.3.10.

Line (8) shows the maximum value that can be chosen for \( k \). The bank is bounded by the resource constraint and the maximum it can invest on date 0 is \( e + b \) choosing \( s = 0 \). It is seen on the Figure 3.3.10 that all feasible values of \( k \) and \( b \) are above Line (12) and hence \( r_2 > \tau \) for all regions. The constraint for \( r_3 \) and \( r_4 \) appear to be redundant for the chosen parameter values. This can be observed in Figure 3.3.11. For all regions where \( k > 10 \), we have \( r_3 > \bar{r} > r \) and \( r_4 > \bar{r} > r \). Therefore, the lenders do not have to worry as the interest rate never goes beyond the point that the bank cannot honour its debt.
The regions and objective function in each region for \( s - b < 0 \) are outlined below.

**Region H:**

\[ r < \bar{r} < r_1 \]
\[ r < \bar{r} < r_2 \]
\[ r < \bar{r} < r_3 \]
\[ r < \bar{r} < r_4 \]

\[ EU = \frac{1}{\bar{r} - r} \int_{r}^{\bar{r}} \left[ \ln(1 + c_{11}) + \ln(1 + c_{21}) \right] dr \]

**Region I:**

\[ r < r_1 < \bar{r} \]
\[ r < \bar{r} < r_2 \]
\[ r < \bar{r} < r_3 \]
\[ r < \bar{r} < r_4 \]

\[ EU = \frac{1}{\bar{r} - r_1} \left\{ \int_{r_1}^{\bar{r}} \left[ \ln(1 + c_{11}) + \ln(1 + c_{21}) \right] dr + \int_{r_2}^{r_1} \left[ \ln(1 + c_{12}) + \ln(1 + c_{22}) \right] dr \right\} \]

**Region J:**

\[ r_1 < \bar{r} < r_2 \]
Figure 3.3.10: Regions for $s-b<0$

\[
EU = \frac{1}{\bar{r}-l} \left[ \int_{l}^{\bar{r}} [\ln(1 + c_{12}) + \ln(1 + c_{22})] dr + \int_{r_1}^{r_2} [\ln(1 + c_{12}) + \ln(1 + c_{22})] dr + \int_{r_2}^{\bar{r}} [\ln(1 + c_{13}) + \ln(1 + c_{23})] dr \right]
\]

Region L: \( r_1 < r < \bar{r} \)
\( r < r_2 < \bar{r} \)
Figure 3.3.11: $r_3$ and $r_4$ for $s - b < 0$

$$ EU = \frac{1}{r_1 - \bar{r}} \left\{ \int_{r_1}^{r_2} \ln(1 + c_{12}) + \ln(1 + c_{22}) \, dr + \int_{r_2}^{\bar{r}} \ln(1 + c_{13}) + \ln(1 + c_{23}) \, dr \right\} $$

Among these five regions, maximum is reached at $b = 10$ and $k = 13.5$ at Region L. The plot of the corresponding part of the objective function is given in Figure 3.3.12.

The maximum points for each region are given in Table 3.3.1. The maximum is reached in Region L which is highlighted in the table.
There is only one more point we have to consider. Neither the part where \( k < 3.33 \) nor the regions we considered above say anything about \( b = 0 \). When the bank chooses not to borrow at all, the aggregate uncertainty is removed from the optimization problem. Then the solution becomes quite straightforward. Lack of aggregate uncertainty leads to zero liquidation and hence the corresponding consumption levels will be \( c_1 = s \) and \( c_2 = Rk \). Then the first order condition implies for the given parameters \( c_1 = 4.25 \) and \( c_2 = 11.5 \). The corresponding value of
the objective function is 4.18. which is smaller than the level we achieve in Region L. That concludes our solution with \( k = 13.5, \, b = 10, \, s = 6.5, \, l = 0 \) and \( p \) is zero for \( r < 1.6 \) and \( 8.75 - \frac{14}{r} \) for \( r > 1.6 \).

### 3.3.5 Characterization of the Optimal Contract and Financial Fragility

The analysis of the bank’s problem on date 0 reveals an optimal borrowing of 10 units which means all the available borrowing is used. Together with 10 units of endowment, the available resources are amount to 20 units. The bank invests 13.5 units of this and store 6.5 units. The contract that maximizes ex-ante expected utility of a typical consumer is then:

\[
\begin{align*}
  c_1(r) &= \begin{cases} 
  6.5 & \text{for } 1 \leq r \leq 1.6 \\
  \frac{14}{r} - 2.25 & \text{for } 1.6 \leq r \leq 2.5
\end{cases} \\
  c_2(r) &= \begin{cases} 
  27 - 10r & \text{for } 1 \leq r \leq 1.6 \\
  13 - 1.25r & \text{for } 1.6 \leq r \leq 2.5
\end{cases}
\end{align*}
\]

The only critical point that is inside the domain \([1, 2.5]\) is \( r_2 \) and \( r_2 = 1.6 \). When \( 1 \leq r \leq 1.6 \), the bank do not pay any debt back on date 1. Once it gets higher, the bank starts to pay some of its debt back on date 1. The consumption paths for early and late consumers under the optimal contract are displayed in Figure 3.3.13.

Note that these consumption levels are associated with the "good" equilibrium in the context of a bank run model. If no runs occur either by depositors or lenders, the bank provides the consumption levels in the contract and honours all of its debt. However, the bank cannot distinguish between the patient and the impatient consumers. Therefore, the payment in the contract is based on when the resources are withdrawn. In other words, the bank promises to pay \( c_1(r) \) to anyone who withdraw early. If more than half of the consumers withdraw on date 1, the bank cannot pay \( c_2(r) \) to the late withdraws and it may not be able to pay all of its debt. Similarly, if some of the lenders refuse to roll-over the loan on date 1, the bank may fail to meet the conditions of deposit contract.
To assess the risk of a run, we once again turn to our measure for financial fragility, which simply is the difference between the short-term assets and short-term liabilities of the bank. From Equation 3.3.14, this is:

\[ f = c_1 + b^* - s^* - \epsilon k^* \]  

(3.3.34)

For the parameter values chosen and the arguments of the optimal contract, this becomes:

\[ f = c_1(r) + 10 - 6.5 - \frac{1}{2} 13.5 = c_1(r) - 3.25 \]  

(3.3.35)

The plot of \( f \) shows that for higher levels of the borrowing rate, maturity mismatch of the bank goes down (Figure 3.3.14). As the risk sharing contract can be written in \( r \), the interest rate risk is bourne by the consumers.
3.4 Conclusion

In the context of our analysis, an ex-ante consumer faces three types of risks:

1) risk of being impatient,
2) interest rate risk due to the random borrowing,
3) risk of a bank-run.

The first two of these risks are modelled explicitly in this chapter. The third-risk emerges only if the consumers choose to share the risk of being impatient and is not modelled in the context of a D&D type bank-run model.

The risk of being impatient is due to the productive investment technology and the costly liquidation. In the case of an autarky where risk sharing does not exist and each consumer acts individually, on date 1, if a consumer finds out she is impatient, she cannot collect the benefits of her investment on date 2 and moreover, she has to pay a cost to bring the date 2 resources to date 1. The contact we derived in
this chapter acts as an insurance for this kind of risk. When all individuals become
together, the ratio of the impatient is known and hence, the uncertainty disappears.
This allows the bank to allocate resources efficiently to maximize utility of a typical
ex-ante consumer.

Whether risk-sharing is optimal or not is a critical issue for the participation of
the consumers. In standard D&D type of models where there exists no aggregate
uncertainty, consumers never lose by pooling the risk. However, when interest
rate risk is introduced, whether risk sharing is optimal or not becomes an issue. To
see that we can simply analyze the autarky solution and see what the consumption
path of an ex-ante consumer will be if she is not engaged in risk-sharing and chooses
to make allocation individually. The autarky levels of early and late consumption
as opposed to optimal risk sharing contract is given in Figure 3.4.1.

![Figure 3.4.1: Optimal Contract versus Autarky](image)

The relative positions of autarky consumption paths are below the risk sharing

---

8The participation issue is tricky in D&D type models because of the bank-run risk. This is
discussed in detail in Chapter Four.
contract for all realizations of $r$. This is quite interesting because in the standard D&D set-up, risk sharing occurs as an ex-ante consumer sacrifices some potential income if she ends being patient, to increase her income when she is impatient. However, in this case, by signing the deposit contract, the consumer guarantees a higher income in both cases (Figure 3.4.1). The main reason for this is the costly liquidation. Risk sharing prevents unnecessary liquidation and leads to a win-win situation for both patient and impatient consumers. As a result, for the considered parameters values, the optimal contract is individually rational and can be implemented.

The second risk we consider is the interest rate risk. This results from the random nature of the borrowing rate, $r$, and from the timing of the decisions. The consumers, individually or collectively, make the decision of borrowing and allocating their resources between storage and investment before observing this rate. The consumption paths of early and late consumers in general show a downward trend. The only exception to this is the region where payback is positive. In this region, late consumption can be increasing in $r$. In our numerical example though, both early and late consumption falls with $r$. Our results do not present a clear picture of who bears higher interest rate risk. In other words, depending on the conditions, the optimal deposit contract can change and with different contracts, the interest rate risk may be shared differently by early and late consumers.

The final risk we consider in our analysis is the risk of a bank-run. Although, this risk is not modelled explicitly, we asses it by looking at the difference between short-term liabilities and assets of the bank. In a standard, D&D model or in a bank-run model with no uncertainty about the interest rate, the measure of financial fragility does not depend on the interest rate. In our analysis, the key difference is that we allow contracting on the uncertain borrowing rate. If this is not possible, financial fragility will be independent of it because all the borrowing and the investment decisions are made before the borrowing rate is observed. However, with the type of contract we have, the payment to early consumers is a function of the borrowing rate. Moreover, the early payment, $c_1$ falls with $r$ and hence financial fragility falls
with \( r \). If we think a higher \( r \) on date 1 as a bad sign for lenders and for depositors, a deposit contract on \( r \) can provide insurance against high \( r \). When \( r \) is higher, the short-term liabilities of the bank will be lower and this means weaker incentives to run for both depositors and lenders.
Chapter 4

Risk Sharing with Private Banking

4.1 Introduction

The idea of risk sharing is based on the desire to smooth consumption under uncertainty. Smoothing the consumption or in other words, reducing the variance of consumption possibilities under different states of nature is preferred only if the consumer is risk averse. A risk averse consumer, due to the existence of a positive risk premium may prefer a payment stream with a lower variance even if the stream also pays lower on average. This idea explains how the basic Diamond and Dybvig (D&D) set up makes the bank viable. A group of consumers who face the risk of ending up with two different income levels in the future, pool their risk and insure themselves against the bad outcome. In other words, they all sacrifice some income from the good state in return of getting more in the bad state. This is no different than a simple insurance contract. Any insurance contract simply lowers the variance of possible income levels in the future and smooths consumption. A risk averse consumer can drive utility out of this and the insurance companies exploit this fact to make profits.

In constructing a typical insurance contract, there exists two main issues. The first one is the size of the pool. In case of insufficient participation, the aggregate uncertainty remains. If we consider a car insurance contract where only a few people shares the risk, all of them can possibly get involved in an accident at the
same time. In that case, the amount of premium they paid would not be enough to pay the damage or each of them has to pay a premium as large as the damage they would face in case of the accident which makes the insurance contract meaningless. Therefore, the pool of risk sharers should be large enough. The sample that was drawn out of the population should be sufficiently large and the proportion of the participants who would require a compensation should be known. The basic D&D model overcomes this issue by assuming it is possible for all the consumers (the whole population) to act together and form a bank to share the risk of needing early consumption and not being able to collect the benefits of the long term project. Although all consumers are ex-ante identical, the proportion of consumers who will suffer from early consumption is constant and known to everyone. Therefore, once the whole population acts collectively, there exists no aggregate uncertainty. The second issue is about the information problems. As it is discussed in the second chapter, moral hazard is a common one. Once insured, it is likely for the participants of the contract to act carelessly or less carefully which may increase their chances of needing a compensation. Therefore, insurance contracts often offer reward or punishment schemes to avoid such behavior. The D&D demand deposit contracts has no such problem as the risk of being impatient is constant, totally random and does not influenced by any type of behavior by the consumer. However, it has got another information problem. Due to being private information, whether the risk of being impatient is realized or not cannot be observed by anyone else. For this reason, a typical insurance contract contingent on the realization of the risk cannot be written. The simple demand deposit contract offered in D&D only specifies payments contingent on when the consumers demand to withdraw their resources from the bank. With the relevant set up, it can implement optimal risk sharing, however, it is also vulnerable to panic based runs.

Another feature of the D&D model and many extensions in the literature, including the ones we consider in the third chapter, is the definition of the bank being a coalition of consumers who desire to share the risk of being impatient. Therefore, the contract is designed in such a way that the bank is eventually dissolved and
all the resources are allocated between the consumers. This is obvious as the aim of the bank is just to implement first best allocation. However, just like any other insurance contract in practice, more realistically, this is not the case. We do not see large groups of people cooperate to share various risks or to form banks. A private company usually coordinates this action, does the risk analysis and offers contracts accordingly. The aim of this company (or companies) is not to implement first-best allocation via risk sharing. It simply offers the contract that maximizes profits. The resulting outcome may or may not be first best depending on the competitive structure of the market as well as the externalities about the cost or information structure. A private bank in the context of the demand deposit contract will change the allocation of the resources. It aims to give the consumers just enough and keeps the rest for profits. The desire for consumption smoothing due to risk aversion makes this possible. Further in this chapter, we present our model, where it is explained in more detail under which conditions positive profits are possible.

This chapter studies optimal demand deposit (risk sharing) contract under private banking. We assume a monopoly bank that maximizes profits. It acts as a principal and offers a demand deposit contract anticipating how a typical ex-ante consumer will behave given that contract. The main contribution of this chapter is the analysis it provides under private banking focusing on financial fragility. It also provides some insights how the consumers and the bank anticipate the risk of a bank run and the influence of this on the deposit contract and hence, the stability of the financial system. Many extensions have been considered in the literature after the basic model was introduced by Diamond & Dybvig (1983). Many key assumptions of the model have been relaxed or abandoned to provide a better understanding for the bank runs. But, as long as the original definition of "the bank" is kept, it remains a formation which only aims to implement optimal risk sharing. However, the recent global liquidity crisis in 2008 proved once again how important the decisions made by financial institutions can be in determining the overall stability of the financial system. The risks they take under uncertainty may end up with causing worldwide economic meltdowns. We believe extending the well-known bank run framework
with private banking also contributes to a better understanding of banking panics and may lead to policy implications that can possibly help maintaining stability in financial markets.

In the next section of this chapter, we discuss some main assumptions about the D&D framework that are important in the context of our model. How they were modified or extended in the literature is summarized and how we considered them in our set up is explained. In the third section, we present our model and results. The final section concludes.

4.2 Extensions to the D&D Model

The Diamond & Dybvig (1983) model led to vast literature about bank runs and deposit insurance. It has been extended in so many ways, relaxing or altering some of its assumptions, introducing new technologies or uncertainties. It provides a relatively simple and comprehensive framework to model the maturity mismatch that a financial institution can bear which may lead to panic based attacks and financial crisis. In this section, we focus on a few features of the D&D model referring to some extensions considered in the literature. Doing so, we aim to provide a better understanding of our motivation and the choices we make in constructing the model we present in the next section.

As mentioned earlier our main extension to the basic set up is the monopoly bank that offers the deposit contracts. When considering a private bank, we inspired from the "industrial organization approach" to banking as it was named by Freixas & Rochet (1997). In this approach, the bank is considered as any industrial organization who buys loans and sells deposits. In other words, it defines banking activity as the production of loan and deposit services. A typical bank faces a cost function which defines the management and organization costs of these services. Then, it chooses the volume of deposits and loans to maximize profits. The Monti-Klein model ((Monti 1972), (Klein 1971)) is the most known example for this
kind of models. In its simplest version, it considers a monopoly bank who faces a downward sloping demand for loans and an upward sloping supply of deposits. Our model also considers a monopoly bank that maximized profits, but it has two distinct features. First, we omit the transaction costs so the only cost item for the bank is the interest it pays over deposits. The second feature is the demand and supply functions that the bank faces. Our bank does not extend any loans so there is no loan demand. Instead, it has a long term investment technology available where it can obtain some benefits out of the deposits it collects. The payoff is constant and the bank can invest as much as it likes. On the supply side of deposits, we do not have an ad-hoc function as in Monti-Klein model. The upward sloping supply of deposits function is derived from a basic D&D set up on the consumers side. In that sense, our model is micro-founded. The trade off that the bank faces is simply between attracting more deposits and paying more interest.

One of the most discussed assumptions of the D&D set up is the sequential service constraint that they define for the panic process. This process is not modeled explicitly or the existence of this feature is not justified. It is only imposed as a realistic assumption how bank serves to depositors. This is explained by Wallace (1988) in the following way:

"Although Diamond and Dybvig seemed aware of the importance of the sequential service constraint, they were vague about why it arises, what in the environment forces banks to deal with their customers sequentially instead of, for example, being able to cumulate withdrawal requests and make payments contingent on the total."

The sequential service assumption is just mentioned to be realistic in D&D model and it is considered to represent some services that the bank provides which are not explicitly modeled. Wallace (1988) extends the D&D model by rationalizing the sequential service constraint. He basically assumes the consumers are isolated when they discover their types and decide either to withdraw or not at random times.
Therefore, there is no possibility of coordination. This is also realistic because, in reality consumers discover their needs in different times and hence they are in a way isolated when they are making the decision of withdrawal.\footnote{Although Wallace’s analysis supports the realistic nature of the sequential service constraint, it also lays doubt on Diamond and Dybvig’s arguments about deposit insurance. See Wallace (1988) for details.}

Calomiris & Kahn (1991), on the other hand, justifies the sequential service constraint by assuming asymmetric information between the bank and the depositors. They consider an environment where the value of the bank’s assets cannot be observed freely by the depositors. The depositors may acquire this information but only at a cost. Therefore, the ones who allocate resources for this purpose become better informed and take their positions on the line to withdraw before the others. This structure which rewards the consumers who invest in information production leads to sequential servicing.

There also exists studies which abandon sequential servicing. Allen & Gale (1998) is a good example. They assume in case of a run, the remaining assets in the bank are divided equally between the consumers who want to withdraw. They justify this by arguing sequential servicing is not realistic. Actually, this assumption is vital for their findings. We believe sequential servicing is more realistic compared to a payment schedule that is considered by Allen & Gale (1998). Both scenarios that are explained to justify sequential servicing above make sense. At least, at the beginning of a bank run agents cannot coordinate and the bank always pays the consumers hoping that the run might end before the failure of the bank at some point. Therefore, the early actors can always benefit and may be able to get their money back before the bank fails. A structure where these early actors have a superior information set is also logical. All in all, this assumption is quite crucial, because it defines what happens in the process of a bank run. When making ex-ante investment or risk sharing decisions, both the bank and the consumers want to figure out what will their expected payoffs be in case of a bank run. The way the bank services the payments in the deposit contract determines these expected payoffs and hence, possible losses for all parties.
Finding it more realistic than the alternatives in the literature, we decide to keep the sequential servicing constraint. In what way the consumers will be paid under this assumption in case of a bank run and the consecutive expected payoffs are derived quite clearly by Diamond & Rajan (2001). Note that their model is quite different. They use the collective action problem under D&D set up as a motivation for the bank to use its special collection ability instead of renegotiating the deposit contract and hence, increasing the overall efficiency in the economy. Moreover, they do not have panic based runs as we consider and the consumers only run when they realize their expected payoff when they run is higher than it is when they wait. However, the assigned probabilities and payoffs they present in case of a run are still valid in case of a panic based run. While constructing the payoff structure under the scenario of a bank run, we benefit from their set up.

Another key feature of the basic D&D set up that can be interpreted as a drawback is how it treats to the possibility of a bank run. The classical D&D model does not propose any mechanism that explains why or how the bank run happens. Figure 4.2.1 displays the basic D&D set up in extensive form. Each allocation shown for patient and impatient consumers refers to one of the possible equilibria. Starting
from left, they are the bad equilibrium, the good equilibrium and the autarky allocations, consecutively. However, the model does not suggest how we end up with the good or bad equilibrium. This is indicated by the question marks on the top-left decision node in the Figure 4.2.1. Moreover, when setting up the assumptions to ensure individual rationality and incentive compatibility, Diamond and Dybvig compare the good equilibrium and the autarky solution. Under the assumptions of the model, the good equilibrium is the social optimum so it is better than the autarky solution. But when making the decision of depositing or not, the ex-ante consumer does not know whether the good or bad equilibrium is going to happen. Therefore, that decision must be based on an expected payoff as the first best allocation cannot be guaranteed. In the basic model though, there exists no explanation about how an ex-ante consumer can consider the possibility of a bank run. How many times the bad equilibrium will occur if the game is played repeatedly many times is ambiguous. Therefore, it is impossible for an ex-ante consumer to derive even an expected payoff once she deposits her resources in the bank.

Obviously Diamond and Dybvig are aware of this problem themselves. They explained this in the following words.

"If we take the position that outcomes must match anticipations, the inferiority of bank runs seems to rule out observed runs, since no one would deposit anticipating a run. However, agents will choose to deposit at least some of their wealth in the bank even if they anticipate a positive probability of a run, provided that the probability is small enough, because the good equilibrium dominates holding assets directly. This could happen if the selection between the bank run equilibrium and the good equilibrium depended on some commonly observed random variable in the economy. This could be a bad earnings report, a commonly observed run at some other bank, a negative government forecast, or even sunspots."
Their explanation rationalizes how people may choose to deposit but it does not offer an explicit way how the agents on date 0 can anticipate a run even with uncertainty. The sunspots argument is used for implying extrinsic uncertainty which is a random phenomena that do not affect tastes, endowments or production possibilities (Cass & Shell 1983).

Note that, runs are determined by the withdrawal decisions of the agents in D&D. So in a way, there exists a mechanism that leads to bank runs. However, the actions of all consumers collectively determines the outcome. From the point of view of a single consumer, there exists no mechanism that she can predict the behavior of the others and hence whether the good or bad equilibrium will prevail. If we consider the sub-games, both equilibria are consistent with the sub-game it is in; i.e. each agents behavior (withdraw or not) is optimal given what others do. However, when the complete equilibrium path is analyzed, the optimality of the initial decision of depositing cannot be verified. This raises an issue about the sub-game perfection of the equilibria presented in D&D set up.

To overcome this problem, a group of studies develop information based panic theories. The basic scenario in these models is about a widely observed macroeconomic variable. The consumers decide weather to withdraw by observing this variable rationally. In Gorton (1988), consumers construct their opinion on the situation of their bank by looking at the economy-wide shocks that they can observe. That leads to bank runs following negative real shocks. Jacklin & Bhattacharya (1988) explain bank runs as an equilibrium phenomenon by assuming two way asymmetric information. As the bank cannot observe depositor type (patient or impatient), the depositors cannot observe the quality of the bank’s assets. Only a few consumers receive information about the situation of the bank and that news being negative is the triggering factor for the bank runs.

Allen & Gale (1998) also studied bank runs as a part of the equilibrium. They show that when the sequential servicing is abandoned, under aggregate uncertainty, panics can implement optimal risk sharing. They introduced an uncertain return
for the long term project. When the contract can be written contingent on this return, the first-best can be implemented without any problems. For non-contingent contracts, runs can implement optimal risk sharing if liquidation is costless. In their model agents decide to run according to the signal they get about the return of the long term project. Their assumption that the remaining resources are allocated equally between agents in case of a run (instead of sequential servicing) allows them to have bank runs when the mentioned return is realized too low and hence to find optimal bank runs. But this only works when early liquidation is costless. Chari & Jagannathan (1988) studied panic based runs where the proportion of early consumers is also stochastic. Depositors can observe the line in front of the bank and base their decision on the length of this line. However, as they do not know the number of the early consumers for sure, a long line may also be due to a high realization of early consumers rather than a panic. Therefore, it is possible that the bank fails, even when there is no reason for a panic or in other words when quality of bank assets is not low.

All these studies propose a mechanism that explains how the consumer decide to withdraw that may eventually lead a bank run. However, most of them does not consider panic based bank runs. The consumers usually run due to a negative realization of a macroeconomic variable. In the context of the D&D model, the bank run occurs when patient agents try to withdraw with a fear of other patient agents will withdraw. This mechanism is constructed quite successfully by Goldstein & Pauzner (2005) via introducing noisy signals in the model. In their model, agents decide to withdraw or not due to a signal they get about the fundamentals in the economy. Although the run process is related to the realization of these fundamentals, it is purely panic based. Each agent when she receives the signal has an idea about what signals do the other agents get as she knows the distribution of the signals around the real value of the stochastic fundamentals. Therefore, she withdraws not because she thinks the fundamentals are low, but because she thinks the others will withdraw. This way, they find a single Bayesian equilibrium for the D&D model where a bank run occurs only if the fundamentals are realized below a
threshold value.

The model presented by Goldstein & Pauzner (2005) has two very important characteristics related to our model. First, it describes a trade-off between benefits from risk sharing and costs of a bank run. They say,

"The main question we ask is, then, whether demand deposit contracts are still desirable when their destabilizing effect is considered. That is whether the short term payment offered by banks should be set above its autarkic level... we analyze the degree of risk sharing provided by the demand deposit contract under the optimal short term payment. We show that, since this contract must trade off the benefit from risk sharing against the cost of bank runs, it does not exploit all the potential gains from risk sharing"

This trade-off comes from the second important feature of the model. The signalling game they consider leads to a probability of a bank run which is an increasing function of the short term payment provided by the bank. Therefore, when the contract involves a higher short term payment for the impatient, this means a higher degree of risk sharing. However, at the same time, it raises the probability of ending up in a bad bank run situation. While the bank chooses the contract, it chooses the optimal short term payment that maximizes the ex-ante consumers welfare.

These two points are related to our model with a private bank. The private bank while offering a contract anticipates how the agents behave in return. The agents deposit in an attempt to share the risk of early consumption but they will evaluate the prospects of a bank run given the contract offered by the bank. These are explained in further detailed in the next section where we present the model.
4.3 The Model

The Diamond & Dybvig (1983) framework shows how banks can provide optimal risk sharing by offering demand deposit contracts when the liquidity needs of the consumers are private information. Their model is based on an investment technology which is productive only if the investors can wait two periods for it to mature. However, initially, the consumers are uncertain about their future liquidity needs. After one period, a certain proportion of the consumers find out they cannot wait for the investment to mature (say due to short-term liquidity needs) and consume early without collecting the benefits of their investment. Risk sharing idea exploits the fact that if the consumers act together or if a central planner collects all the endowments and make the allocation between early (impatient) and late (patient) consumers, there would be no uncertainty and therefore, the market outcome can be improved. They show that under certain assumptions, if the consumers are sufficiently risk averse, the benefits from risk sharing will be positive and the optimal allocation will be different than the market outcome.

This optimal allocation can also be implemented by the market mechanism if a contract with type contingent payments can be written. This risk sharing contract acts as a simple insurance contract against being impatient, where a consumer by accepting it agrees to sacrifice some of her income if she ends up being patient (premium) in return of some additional income, if she discovers she is impatient. Suppose an ex-ante consumer holds 1 unit of endowment without the risk sharing contract. She can consume 1 unit if she ends up being impatient and say $R > 1$ units if she ends up being patient. Let $c_1^*$ and $c_2^*$ be the optimal consumption allocation between the impatient and the patient, respectively. Diamond and Dybvig (D&D) framework makes the necessary assumptions to ensure $1 < c_1^* < c_2^* < R$. If this is the case a simple contract can be written offering $c_1^*$ to the impatient and $c_2^*$ to the patient. This sort of a contract would be offering $c_1^* - 1$ units of compensation for $R - c_2^*$ units of premium. Any consumer would expect such a contract since it provides the first-best allocation as opposed to the autarky allocation so it is
individually rational. There is no incentive compatibility issue as the liquidity needs are observable and the contract is already written contingent on that.

When the liquidity needs are private information on the other hand, such contracts cannot implement the first best allocation. At this point D&D offers demand deposit contracts which only specify the payment conditional on the time of the claim. In other words, a demand deposit contract pays \( c_1^* \) to early withdrawals and \( c_2^* \) to the late withdrawals. As long as the promised payments can be made, individual rationality and incentive compatibility are both satisfied. First, once again the optimal allocation \( c_1^* \) and \( c_2^* \) is superior to the consumption under autarky causing the ex-ante consumer to participate and second, \( c_1^* < c_2^* \) ensures a patient consumer does not mimic an impatient one and withdraw early. However, here a collective action problem arises. If a panic based bank-run occurs and patient consumers also claim early, the bank fails and cannot make the promised payment. Therefore, under a bank-run scenario, neither patient nor impatient consumers are certainly get paid the specified amounts \( c_1^* \) and \( c_2^* \). This sheds doubt on the participation condition on the contract. Although \( (c_1^*, c_2^*) \) is preferred to \( (1, R) \), the contract cannot guarantee these payments and hence, if the consumer believes the probability of a run is sufficiently high, she might as well choose not to accept the demand deposit contract at the beginning.

In this chapter, we analyze risk sharing under a private banking sector. We take the classic D&D set up, so there are three periods. On date 0, we have many ex-ante identical individuals who are uncertain about their future liquidity needs. With a probability of \( \lambda \), they can be impatient and need short-term liquidity. Each individual owns a 1 unit endowment which can be invested in a project which pays 1 unit if collected on date 1 and \( R > 1 \) units if collected on date 2. A one to one storage technology is also available on dates 0 and 1.

We start with a monopoly bank, which offers a demand deposit contract such that for every unit invested in the bank on date 0, it pays either \( c_1 \) on date 1 or \( c_2 \) on date 2. The consumers’ participation to this contract is no longer based on
all or nothing. On date 0, a typical consumer chooses $D$ where $0 < D < 1$, that is the proportion of her endowment she wants to put in the bank. In other words, the consumer evaluates the rates offered in the deposit contract, the options of investment and the risk of being impatient as well as risk of a possible bank-run and decides how much insurance she would like to buy. Therefore, this ratio also shows the intensity of risk sharing.

We can set the problem as a typical principle agent problem where the consumer chooses $D$ and this choice simply satisfies her participation constraint. This choice yields a deposit demand as a function of the rates offered by the bank in the contract. Anticipating this demand function, then the bank as the principle can choose these rates to maximize profits.

**The consumer’s problem:**

The consumer faces two types of risks, the risk of being impatient and the risk of a bank-run. We let $p$ be the probability of a bank-run and for now assume it is exogenously given. If a run occurs all individuals will take their place on the line but only a certain number of them can get paid. Therefore, there exists six possible consumption levels a consumer can end up with.

\[
\begin{align*}
    w_{RIS} &= \text{run-impatient-succeed} = Dc_1 + (1 - D) \\
    w_{RIF} &= \text{run-impatient-fail} = 0 + (1 - D) \\
    w_{RPS} &= \text{run-patient-succeed} = Dc_1 + R(1 - D) \\
    w_{RPF} &= \text{run-patient-fail} = 0 + R(1 - D) \\
    w_{NI} &= \text{no run-impatient} = Dc_1 + (1 - D) \\
    w_{NP} &= \text{no run-patient} = Dc_2 + R(1 - D)
\end{align*}
\]

Considering the corresponding probabilities, the consumer’s objective is to maximize the following expected utility function:
where \( \frac{1}{c_1} \) is the probability of receiving \( c_1 \) when a run occurs and \( U(.) \) is the utility function with \( U' > 0 \) and \( U'' < 0 \).

Optimizing over \( D \) yields to the optimal intensity of risk sharing \( D^*(c_1, c_2) \) which solves the following first order condition.

\[
\begin{align*}
    p \left( \lambda \left[ \frac{1}{c_1} U(w_{RIS}) + (1 - \frac{1}{c_1}) U(w_{RIF}) \right] + (1 - \lambda) \left[ \frac{1}{c_1} U(w_{RPS}) + (1 - \frac{1}{c_1}) U(w_{RPF}) \right] \right) \\
    + (1 - p) \left[ \lambda U(w_{NI}) + (1 - \lambda) U(w_{NP}) \right] 
\end{align*}
\]

(4.3.1)
consumer makes her choice to equate corresponding marginal utilities as she would do for an ordinary insurance contract by evaluating the risks, the premium and the compensation provided by the contract. The risk of a bank-run, on the other hand, just puts extra pressure on the deposit contract.

As a numerical example, we set $U(w) = p w$, $R = 2$, $\lambda = 0.5$ and $p = 0$ (4.3.1). The demand for risk sharing function derived from first order condition in Equation (4.3.2) shows even when bank runs are ruled out, consumer chooses a different level of $D$ to maximize ex-ante expected utility. There are two reasons why a consumer can hesitate depositing her endowment in the bank. First, because of a possible bank run and second because of the possibility of being patient. Depositing your endowment in the bank and then finding out you are patient is just like buying car insurance and then having no accident. From an ex-post point of view, you regret choosing a positive $D$, if you appear to be patient on date 1. This possibility keeps the consumer from depositing all her endowment in the bank even when $p$ is equal to zero. The risk of being impatient $\lambda$ is chosen to be quite high in this example, which increases gains from risk sharing.\(^2\) The demand for risk sharing function in

\(^2\)The standard demand for money function introduces a trade-off between costs of illiquidity and
the Figure 4.3.2 rises quite steeply. It is observed that until the desired rates are achieved, the demand is zero. Once the positive levels of demand is achieved, we have negatively sloped level curves. They indicate a trade-off between $c_1$ and $c_2$ as expected. To keep the same level of demand, a drop in one payment should be compensated by an increase in the other. Given that each consumer endowed with 1 unit of the resource, a 0.2, 0.3 units of increase in one payment is quite significant and raises the demand quite quickly. After some point, the consumer chooses to deposit all her endowment in the bank. We perform some robustness tests by trying different parameter values but the steepness of the demand remains the same. The major change with the different values is the intercept points. For example, for a lower $\lambda$, i.e. the risk of being impatient, the incentive for risk sharing goes down and the positive values for demand can only be achieved for more favorable rates. However, the steepness of the demand function does not change significantly.

The effect of $p$, on the other hand, can be illustrated in another numerical example. For this one, we set $U(.) = \sqrt{w}$, $R = 2$, $\lambda = 0.5$, $c_1 = 1.4$, $c_2 = 1.95$. We assume quite a favorable deposit contract with a generous compensation in return for relatively small premium. Therefore, we can assume a relatively high probability of a bank run, $p$. The Figure 4.3.2 below plots the first order condition for two different values of $p$ where $p = \{0.2, 0.3\}$.

The Figure 4.3.2 shows how an increase in $p$ reduces the demand for risk sharing. When $p = 0.2$, with the proposed deposit contract, the consumer deposits more than 65 percent of her endowment in the bank. However, when $p$ rises to 0.3, $D$ falls drastically. With the increased probability of a run, the consumer only deposits slightly more than 20 percent of her endowment in the banking system. The change seems to be quite substantial. Note that, even when there is no possibility of a run, the consumer pays a premium for compensation in case of being impatient. In case of the opportunity cost of holding money. The three main motives for holding money balances are for transaction purposes, for speculative purposes and for precautionary reasons. The risk of being impatient can best be explained by precautionary motive. The possibility of needing liquidity for an emergency creates a risk of illiquidity. The risk of being impatient can be considered as such a risk. The demand deposits are perfect for handling this kind of risk as they are liquid and gives you the chance to withdraw your resources early if needed.
a bank-run, the gains from risk sharing cannot be collected for certainty. Moreover, even if the risk is not realized and consumer ends up being patient, she can only claim a much lesser income with some probability. Therefore, a 10% increase in the probability of a bank-run is quite significant for the consumer’s utility under risk sharing.

The second order condition also confirms consumer’s problem is concave in $D$, provided that $U'' < 0$.

\[
p((1 - \lambda)\left[\frac{(c_1 - R)^2}{c_1}U''(w_{RPS}) - \frac{R^2(c_1 - 1)}{c_1}U''(w_{RPF})\right] + (1 - p)\left[\lambda(c_1 - 1)^2U''(w_{NI}) + (1 - \lambda)(c_2 - R)^2U''(w_{NP})\right] < 0 \quad (4.3.3)
\]

This can also be seen from the monotonically decreasing plots of the first order condition in the numerical example.

Next, we derive some comparative statics results for the optimal risk sharing decision $D^*(c_1, c_2)$. This will reveal how sensitive is the consumer’s choice to any changes in the parameters of the deposit contract. Let us denote the second deriv-
ative of expected utility function in Equation (4.3.3) by $H$ where $H < 0$ by the second order condition. Total differentiation of the first order condition in Equation (4.3.2) leads to the following results for the slope of $D^*(c_1, c_2)$.

$$
\frac{dD^*}{dc_1} = -\frac{1}{H} \left( p \left[ \frac{1}{(c_1)^2} \left[ \left( U'(w_{RIS}) - U'(w_{RIF}) \right) + \frac{\lambda}{c_1} U''(w_{RIS}) \frac{dw_{RIS}}{dc_1} \right] 
+ (1 - \lambda) \left[ \frac{R}{c_1^2} \left[ U'(w_{RPS}) - U'(w_{RPF}) \right] + \frac{\lambda}{c_1} R U''(w_{RPS}) \frac{dw_{RPS}}{dc_1} \right] 
+ (1 - p) \lambda \left[ U'(w_{NI}) + (c_1 - 1) U''(w_{NI}) \frac{dw_{NI}}{dc_1} \right] \right] \right) 
$$

(4.3.4)

and

$$
\frac{dD^*}{dc_2} = -\frac{1}{H} \left( (1 - p)(1 - \lambda) \left[ U'(w_{NP}) + (c_2 - R) U''(w_{NP}) \frac{dw_{NP}}{dc_2} \right] \right) 
$$

(4.3.5)

Given $H < 0$ the signs of both Equations (4.3.4) and (4.3.5) depends on the sign of the terms in the brackets. When $c_1$ is changed, this affects $D^*$ in two ways. The first one is the response to a possibility of a bank-run which is displayed by the huge term in brackets in Equation (4.3.4) and is multiplied by $p$. In case of a bank-run, a change in $c_1$ affects two things. It is positively related to the amount of payoff consumer gets if she is able to withdraw but it is negatively related to the probability that she is able to get it. Obviously, its affect on the probability causes extra loss of utility and encourages consumer to choose a lower $D$. This negative effect holds regardless of the consumer being patient or impatient. Mathematically, this can be seen from the terms $\frac{1}{(c_1)^2} \left[ \left( U'(w_{RIS}) - U'(w_{RIF}) \right)$ and $\frac{R}{(c_1)^2} \left[ U'(w_{RPS}) - U'(w_{RPF}) \right]$. The effect on the payoff, on the other hand, works differently for the cases of patient and impatient. In case of a bank-run, the consumer can get a possible $c_1$ no matter she is patient or impatient. For every unit of endowment, the consumer chooses to put in the bank, she makes a sacrifice if she ends up being patient in return of a reward if she ends up being impatient. A rise in $c_1$ increases both the reward and
the sacrifice. However, because of the concavity of the utility function, for a higher \( c_1 \) the influence of \( D \) on this reward and sacrifice will be lower. In other words, the utility function will become flatter and the choice of \( D \) has a weaker effect on the increase of the reward and the sacrifice. In case of a reward, this will lead to a tendency to reduce \( D \), whereas in case of the sacrifice, it creates motivation to raise \( D \). Mathematically, these effects are shown by the terms \( c_1^{-1}U''(w_{RIS})\frac{dw_{RIS}}{dc_1} \) and \( c_1^{-R}U''(w_{RPS})\frac{dw_{RPS}}{dc_1} \). The former is negative but the latter is positive. The relative magnitude of these terms depends on \( R \) and \( \lambda \). All in all, there is only one positive term in this first part of the Equation (4.3.4) and it is reasonable to assume this second degree effect is dominated by the other negative terms and therefore, we can conclude the first impact of a higher \( c_1 \) on \( D \) is a negative one due to the increased risk of a loss in case of a bank run.

The second impact of \( c_1 \) on \( D^* \) appears when a bank run does not occur and it shown by the second part of Equation (4.3.4) which is multiplied by \( (1 - p) \). This is basically the effect of risk sharing. A rise in \( c_1 \) means a higher compensation for an impatient consumer and hence, increases risk sharing. The second degree effect \((c_1 - 1)U''(w_{NI})\frac{dw_{NI}}{dc_1}\) is once again negative due to concavity, but it is likely to be dominated by the first degree positive effect of an increase in the compensation offered in the insurance contract. As a result, we established that the total effect of \( c_1 \) on \( D \) is a sum of the negative effect of the risk of bank run and the positive effect of more advantageous risk sharing. For the risk sharing contract to work, we have to have relatively lower probability of a bank run and a sufficiently high risk of being impatient. When these are considered, it is reasonable to assume that \( \frac{dD^*}{dc_1} > 0 \) at least for a wide range of plausible parameters. The negative effect imposed by a lower probability of getting paid in case of a run can be seen as a crowding out effect that reduces the positive effects of the gains from a better insurance contract.

A change in \( c_2 \), on the other hand, is positively related to the choice of risk sharing.

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3Note that, all these effects are due to risk aversion. A rise in \( c_1 \) has no effect on the expected return from \( D \) when a bank-run occurs. With probability \( \frac{1}{c_1} \), consumer gets \( c_1 \) and with probability \( (1 - \frac{1}{c_1}) \), she gets nothing. Hence, my expected payoff is 1 and it is not influenced by \( c_1 \). Therefore, if the consumer were risk neutral, this whole effect would disappear. This can be seen mathematically by considering \( U'' \) to be constant for all levels of payoff and \( U'' \) to be zero.
Both first and second degree effects that can be seen in Equation (4.3.5) are positive. A rise in $c_2$ results in a lower premium for the insurance contract and leads to a higher degree of risk sharing for the consumer. Note that, a change in $c_2$ does not affect consumer’s payoff (in neither state) when a bank-run occurs as even the patient consumers are no longer able to retrieve $c_2$ out of the deposit contract. Therefore, the negative effect that we observed when $c_1$ changes does not exist when $c_2$ is considered.

The bank’s problem:

Having set the consumer’s problem up, we now switch to what the bank does. Anticipating the demand for risk sharing function $D^*(c_1, c_2)$, the bank simply maximizes profits. We assume all the consumers are ex-ante identical, hence, maximizing what can be extracted from one consumer also maximizes aggregate profits. The bank is assumed to be risk neutral i.e. it only maximizes the payoff, however, this does not mean its objective function is linear in $c_1$ and $c_2$. The main advantage of the bank is that it faces no liquidity shock so it can enjoy consumption anytime and can easily exploit the benefits of the long-run project. Therefore, if consumers are sufficiently risk averse or in other words if their risk premium is high enough that they are willing to sacrifice relatively more for a compensation in the state of being impatient, the bank can push $c_1$ and $c_2$ down to make positive profits. The bank’s problem can be written as:

$$\max \ (1 - p) \ [R(1 - \lambda c_1) - (1 - \lambda)c_2] \ D^*(c_1, c_2) \quad (4.3.6)$$

subject to

$$\lambda c_1 \leq 1 \quad (4.3.7)$$

$$1 - \lambda c_2 \leq R(1 - \lambda c_1) \quad (4.3.8)$$

$$c_1 \geq 1 \quad (4.3.9)$$

$$c_2 \leq R \quad (4.3.10)$$

$$c_2 \geq c_1 \quad (4.3.11)$$
CHAPTER 4. RISK SHARING WITH PRIVATE BANKING

The profit function, Equation (4.3.6), is the expected payoff from one unit of deposit multiplied by the deposit demand which amounts to profit extracted from a single representative ex-ante consumer. Constraints (4.3.7) and (4.3.8) are the resource constraints stating that payments of $c_1$ and $c_2$ cannot exceed what is available in the bank on the given date. Conditions (4.3.9) and (4.3.10) make sure chosen $c_1$ and $c_2$ leads to a risk sharing outcome. The final condition (4.3.11) is the incentive compatibility constraint. An interior solution to the bank’s problem under risk sharing is summarized in Proposition 3.1.

Proposition 3.1:

Maximization of Equation (4.3.6) leads to $(c_1^*, c_2^*)$ which is characterized by the following conditions.

$$\frac{D_1(c_1^*, c_2^*)}{D_2(c_1^*, c_2^*)} = \frac{R\lambda}{1 - \lambda}$$  \hspace{1cm} (4.3.12)

and

$$c_2^* \geq c_1^*$$

and Conditions (4.3.9), (4.3.10), (4.3.7) and (4.3.8) with strict inequalities.

Proof of Proposition 3.1:

As our focus is on risk sharing outcome and we already impose that in the consumer’s problem, we can start assuming Equations (4.3.9) and (4.3.10) are slack. Next, Condition (4.3.7) cannot bind because if it binds $c_2^* = 0$ by Condition (4.3.8). Similarly, Equation (4.3.8) cannot bind because that would derive the profits down to zero. Therefore, an interior solution requires all these conditions to hold as strict inequalities. The figure below displays the feasible region for an interior risk sharing equilibrium. The profits will be zero on the line $c_2 = \frac{R}{1-\lambda} - \frac{\lambda R}{1-\lambda} c_1$ since this means everything left from date 1 will be distributed to the patient and hence no matter what $D$ is, the bank makes no profits. The points on the line $c_1 = 1$ can also be
ruled out as a possible equilibrium because up to $c_2 = R$, these points provide no compensation for a positive premium and obviously, demand for risk sharing and hence the profits will be zero on these points as well.

![Figure 4.3.3: The feasible region for a risk sharing equilibrium](image)

The left hand side of Equation (4.3.12) shows the marginal rate of substitution between $c_1$ and $c_2$, while the right hand side is the relative costs. In the context of a standard insurance contract, $\lambda$ is the risk of the accident and it plays a critical role when deciding how much compensation is provided for given premium. In this case, another parameter that influences relative costs is the return of the productive long term technology, $R$. Higher $c_1$ means a higher portion of the long term investment will be liquidated on date 1 and hence lower benefits on date 2. This makes $R$ the opportunity cost of providing a higher $c_1$. Equation (4.3.12) equates the marginal rate of substitution to the relative cost and is simply derived by dividing two first order conditions of the unconstrained optimization problem.

The only ambiguity is about the incentive compatibility constraint (4.3.11) There can possibly be many bundles of $c_1$ and $c_2$ that satisfy condition (4.3.12) but only the ones that are consistent with the other listed conditions. For some of these bundles incentive compatibility constraint may bind and for some it may be slack.
Up to this point, we assume probability of a bank run $p$ is exogenously given. However, more realistically, this may not be the case. Note that, any bank run in this context is panic based since the incentive compatibility constraint makes sure that it is sub-optimal for any patient agent to withdraw knowing others will not. Therefore, the agents may be more likely to panic (and not take the risk of a run) if the date 1 payment $c_1$ is relatively higher.

This is displayed in a detailed model by Goldstein & Pauzner (2005). Their model endogenize the bank-run by introducing a signalling game where every consumer receives an imperfect signal about the bank run. The equilibrium of the signalling game shows that a higher short-term payment $c_1$ leads to a higher chance of a bank run. Then, they find an optimal $c_1$ for risk sharing considering its effect on the probability of a bank run and conclude that this optimal level is lower than the first-best level introduced by the D&D model.

Next, inspiring from Goldstein & Pauzner (2005), we extend our model by assuming the probability of a bank run is now a function of $c_1$.

$$p = p(c_1)$$

where $p' > 0$ for all $c_1$. The function $p$ has a range of $[0, 1]$ and is known to all consumers and the bank. Therefore, given $c_1$ all players know what the probability of a bank run is. With the extension, the expected utility function of the consumer and the profit function of the bank remains the same but now the probability parameter $p$ is a function of $c_1$.

This change made no difference over the consumer’s optimization problem as she optimizes over $D$. However, the slope of the optimal risk sharing $D^*$ with respect to $c_1$ changes as a change in $c_1$ now affects the probability of a bank run as well. The following two terms are now added to the left hand side of (4.3.4).
\[ p'(c_1) \left( \lambda \frac{c_1 - 1}{c_1} \left[ (U'(w_{RIS}) - U'(w_{RIF})) \right] + (1 - \lambda) \left[ \frac{c_1 - R}{c_1} U'(w_{RPS}) - \frac{R(c_1 - 1)}{c_1} U'(w_{RPF}) \right] \right) \]

and

\[ -p'(c_1) [\lambda(c_1 - 1)U'(w_{NI}) + (1 - \lambda)(c_2 - R)U'(w_{NP})] \] 

(4.3.13)

(4.3.14)

Given \( p' > 0 \) both of these terms are negative. Therefore, the extension strengthens the negative effect of \( c_1 \) on \( D^* \) due to the possibility of a bank run which is discussed above. Keeping our assumption that the positive effect dominates and hence, \( \frac{dD^*}{dc_1} > 0 \), it makes the slope flatter. This will basically reduces the marginal benefit of increasing \( c_1 \) from the bank’s point of view.

The second impact of the extension is observed on the equilibrium condition (4.3.12). The new condition can now be written as:

\[
\frac{D_1(c_1^*, c_2^*)}{D_2(c_1^*, c_2^*)} = \frac{R\lambda}{1 - \lambda} + \frac{p'(c_1)[R(1 - \lambda c_1) - (1 - \lambda)c_2]}{(1 - \lambda)(1 - p(c_1))}
\]

The new positive term added increases the relative cost of \( c_1 \). This makes sense as the bank loses everything in case of a run. As a result, in the new extended model, \( c_1 \) has a lower marginal benefit to the bank because the stated crowding out effect is larger on the risk sharing demand and it has a higher relative cost because it increases the probability of a run and accordingly lowers the bank’s expected profit.

Note that, these first order conditions that define optimal values of \( c_1 \) and \( c_2 \) for an interior solution only make sense as long as the profit function is concave and that depends on several conditions such as the shape of the demand function or the form of the utility function. The bank has many options here. All the points defined by the triangle in Figure 4.3.3 are available for the bank. The consumers evaluate the contract in two ways. First, the average magnitude of the offered payments, and second relative positions of \( c_1 \) and \( c_2 \). They desire the payments to be as high.
as possible but, at the same time, because of risk aversion they also care about the distance between the payments. A lower variance of consumption means better consumption smoothing and higher utility. In line with this feature of the contract, in the next sub-section, we analyze complete insurance contracts.

4.3.1 The Degree of Risk Sharing: Complete versus Incomplete Insurance

We mentioned earlier how Goldstein & Pauzner (2005) define the degree of risk sharing by the magnitude of the short term payment. They also point out how higher risk sharing leads to higher financial fragility. The degree of risk sharing in their model is determined by the short-term payment that is intended for the impatient consumer, or \( c_1 \) in terms of our model. The difference in our set-up is that we have partial participation where the consumers can deposit only a portion of their income. So the function \( D(\cdot) \) that the bank faces is another measure for the intensity of risk sharing. As we mention above when bank chooses the consecutive payment in the contract, it considers not only the average magnitude of these payments but also their relative positions against each other. When the bank pushes \( c_1 \) and \( c_2 \) towards each other, it provides better consumption smoothing and even if the expected payment to ex-ante consumer does not change the risk averse consumer tends to deposit more in the bank. Therefore, there might be an opportunity for the bank to raise supply of deposits without increasing costs. This obviously depends on the relative sizes of parameters such as \( \lambda \) or \( R \) and as well as how risk-averse the consumers are. In a model where the probability of a bank-run depends on the short-term payment, offering closer \( c_1 \) and \( c_2 \) may also increase this probability and hence, this must be taken into account as well.

The D&D optimal risk sharing equilibrium is on the right edge of the triangle that defines the feasible region (Figure 4.3.3). On that line, starting from the autarky case \( \{c_1, c_2\} = \{1, R\} \), all the resources left in the bank on date 2 is distributed among the consumers who did not withdraw prematurely. Therefore, the profits
are zero as the bank is only a coalition between consumers that keep no private
profits for itself. Starting from the autarky bundle at which there exists no risk
sharing, as we go down along this line, intensity of risk sharing increases while $c_1$
is going up and $c_2$ is going down. The final point on the line that belongs to the
feasible region is where $c_1 = c_2 = \frac{R}{1 + \lambda(R-1)}$. The points beyond that are simply
not incentive compatible. Note also that the bottom edge of the region which is the
forty five degree line passing through the origin defines all the points where $c_1 = c_2$.

In the context of a typical insurance contract, these points characterize the possible
contracts that offer complete insurance. In other words, these contracts offer the
same level of consumption for the patient and the impatient and hence removes any
uncertainty resulting from the risk of early consumption. Therefore, the right edge
of the triangular region defines all possible risk sharing equilibria under basic D&D
set up where the two extremes are the autarky and full risk sharing outcomes. The
actual optimal risk sharing outcome can be anywhere on this line between these two
points. In case of a private bank, it is very unlikely that a monopoly bank offers a
contract on that line as it will only break even.

Next, we assume that the risk preferences of the consumers is such that the
monopoly bank always offers a complete insurance contract. In terms of our feasible
region, these contracts are characterized by the forty five degree line. This simplifies
our model notably by reducing the choice variables down to one. Now, the bank
chooses a single $c$ between $1$ and $\frac{R}{1 + \lambda(R-1)}$ on the forty five degree line. The possible
payoffs for the consumer are:
Clearly, for the portion of her wealth she deposits in the bank, the consumer no longer cares if she ends up being patient or impatient. She chooses the amount she deposits in the bank considering the size of $c$. Therefore, the function $D(.)$ has only a single argument now. By using a similar analysis, it can be shown an interior solution between 0 and 1 can be found for $D(c)$ which is increasing in $c$ provided that the gains from risk sharing is sufficiently high and the probability of a bank run is sufficiently low. Given this demand for risk sharing function, the profit function of the bank can be written as:

$$ (1 - p)[R - c(1 + \lambda(R - 1))]D(c) $$

(4.3.15)

For simplification, let us denote the constant $(1 + \lambda(R - 1))$ by $A$ hereafter. The first order condition for a maximum is given as:

$$ D'(c)[R - Ac] = AD(c) $$

(4.3.16)

The second order condition can then be written as:

$$ -AD'(c) + D''(c)[R - Ac] - AD'(c) < 0 $$

(4.3.17)
If the function $D$ is concave in $c$, the profit function is concave in $c$ and the second order condition is satisfied. Note that, the function $D$ is bounded between 0 and 1 and it is increasing and concave in $c$ between these borders. How the demand function looks like is displayed in Figure 4.3.4.

The two panels in the Figure 4.3.4 shows the two alternatives for the level of $c$ that the demand function reaches its maximum. Recall $\frac{R}{A}$ is a maximum for $c$ because of the resource constraint. So, the bank chooses $c$ between 1 and $\frac{R}{A}$. However, the demand function may reach its maximum before or after this critical value. Typically, we know for low values of $c$, the demand is zero. For example, if the contract offers $c = 1$, the demand will be zero since even in the autarky case, this payoff can be obtained without any uncertainty. After a threshold value the demand begins to rise and once it reaches to 1; i.e. when the consumer deposits all her endowment it cannot rise anymore no matter how high the offered $c$ is. Accordingly the first order condition given above can be illustrated in a similar Figure 4.3.5.

The upward sloping curve is basically the right hand side of the condition (4.3.16) while the downward sloping one is the left hand side of it. This left hand side hits to zero in two cases. First, if $D = 0$ and second, if $c = \frac{R}{A}$. The two panels of the
Figure 4.3.5: Optimal $c$ under complete insurance

The figure shows these two cases. If the demand reaches to its maximum before $c = \frac{R}{A}$, and stabilizes, it hits to zero at that point (the left panel). If the demand reaches to the maximum after $c = \frac{R}{A}$, then the curve hits to zero at $c = \frac{R}{A}$ (the right panel). Given that the left hand side function is strictly decreasing in $c$, in both cases two curves cut each other between 1 and $\frac{R}{A}$. Note that, we assume in this simplified version probability of a bank run, $p$, is exogenously given and does not depend on $c$. However, if it were increasing in $c$, just like the previous model, we would end up with a lower $c$ because of the extra pressure on the contract. On the other hand, if this effect is too strong (if a rising $c_1$ significantly increases the chance that a bank run occurs), the complete insurance case will not be possible at the first place.

All in all, $c$ can be as large as $\frac{R}{A}$. However, if what we see on the left panel is the case, once the demand hits to 1, there is no point in increasing $c$ any more, so that point will be the maximum $c$ which is possible. Profit maximizing contract under complete insurance is a subset of the possible contracts that we consider in the previous section. Instead of the whole triangular region in Figure 4.3.3, the solution will be on the 45 degree line where $c_1 = c_2$. The actual solution depends on the parameter values such as $R$ and $\lambda$ as well as the risk preferences of the consumers.
4.4 Conclusion

We try to analyze risk sharing with D&D type demand deposit contracts with a private monopolistic bank. The typical bank in the basic D&D model is only a coalition formed by the agents whose sole aim is to maximize ex-ante consumers utility to achieve risk sharing. However, in reality banks are profit seeking organizations and the deposit contract with a private bank can potentially be completely different. The stability of the financial system depends very much on the conditions of the deposit contract, which means a private bank can make a big difference.

The analysis carried out reveals the following results:

1) A profit maximizing bank means many more possible contracts which means examining financial fragility under a standard D&D type bank may lead to an incomplete evaluation of the matter.

2) The risk of a bank run should be accounted in determining the risk sharing preferences of consumers and as well as the deposit contract offered by the private bank. This is also quite critical in terms of financial fragility.

3) Considering the complete insurance case simplifies the problem significantly with a private bank and makes it possible to speculate about how the risk sharing intensity and financial fragility can be affected from different conditions such as the competitive structure of the financial market. The complete insurance assumption is a strong one and any result relying on this assumption should be evaluated cautiously.

Standard risk sharing equilibrium under a D&D bank only allows contracts where all the resources are distributed among early and late consumers. The payment to the early consumers, which is a part of the contract, is a critical part of the bank’s short term liabilities. A private bank can offer a whole new set of contracts which were not available in the standard D&D case. The profit maximizing contract depends on the risk preferences of the bank and the investment opportunities. This new set of possibilities mean potentially different levels of short-term liabilities and hence a different degree of financial fragility. By identifying these points, we show
the bank’s preferences which are ignored in the standard D&D framework can be important for financial fragility. Additionally, the arguments of this contract can determine the incentives for running to the bank. A favorable payment for early consumers can itself increase the probability of a bank run.

The way that the risk of a bank run is anticipated by the agents and the monopolistic bank is very important. We establish a trade off between higher degree of risk sharing and financial fragility. If the agents and bank anticipates the risk of a bank run and how it changes with the parameters of the deposit contract, the profit maximization process leads to a lower short term payment for the impatient and hence, a more stable financial system. This may raise the issues about the moral hazard problem created by deposit insurance or similar guarantees provided by the government to maintain confidence in the financial system. The vulnerability of the system against bank runs makes the bank and agents internalize this risk in their optimization problems. Therefore, the risk of a bank run acts as an automatic stabilizer that disciplines the players in the financial market to be careful when making investments or borrowing funds. When the risk is artificially removed by the government (in our model that obviously means $p = 0$), the short-term payment in the contract goes up particularly because it becomes easier for the bank to attract deposits. As a result, such a policy increases the intensity of risk sharing (as intended by the government). Less money will be kept under the mattress and more will be going into the banking system. However, it might also mean higher financial fragility.

It can be argued that once the deposits are insured the runs will be avoided and therefore, there will not be a rise in financial fragility. In the context of a simple D&D model, this may be right. However, once the bank realizes it is much easier to collect deposits, it may choose to invest in somewhere else other than the safe productive long-term investment of the D&D framework. Moreover, it may not take the necessary steps to avoid a failure, as in that case, the deposits are guaranteed to be paid back by the government. Diamond & Rajan (2001) displays very well how the fragile finance structure for the bank disciplines it to behave and commit to use
its special abilities and create liquidity. All in all, there is a trade off obviously. More risk taking with a deposit contract type of fund raising by the banks means more fragility. On the other hand, when banks rely on bank capital instead of deposits, they may strengthen the financial system, make it less prone to runs. However, this will also reduce liquidity creation by the banks (Diamond & Rajan 2000).

We also consider the case where the banks offer complete insurance contracts only. This makes the model much simpler with a single choice variable, $c$, that the contract is written on. The higher the payment $c$, the higher the demand for risk sharing but the lower the profit margin will be. Under this set up, we can think of a very simple Bertrand game to see how the competition in the deposit market mat affect the equilibrium.

Let us assume we have two banks which are involved in Bertrand competition. There is no product differentiation as both of them offer a simple contract contingent on $c$ for sharing the risk of being an early consumer. The consumers choose the bank that offers the higher rate and if they offer the same rate two banks simply share the deposits equally. This kind of a typical Bertrand game will derive the bank profits to zero as it takes $c$ up as high as $\frac{R}{A}$. An interesting point is that, even if we have the case on the left panel of the Figure 4.3.5, under Bertrand competition even when the demand hits the maximum, the banks keep raising $c$. That is because by charging a slightly higher rate, they can get the whole deposits instead of the half. This continues until profits reach zero.

If we have consumers who prefer complete insurance, the basic D&D first-best risk sharing will be just on the bottom right corner of our feasible triangular region where $c = \frac{R}{A}$. Therefore, we can have turn back to first-best risk sharing by having a more competitive market structure. However, considering the trade off between risk sharing and financial stability, it may be the case that a monopoly bank can offer a stronger financial system by lowering the short-term payment of the deposit contract.
Chapter 5

Conclusion

Today, it has been widely accepted in the literature that extremely mobile international capital is one of the key factors explaining most recent crisis episodes worldwide. The international capital flows, which have a short-term, exceedingly speculative component, are hardly stable and subject to herd behavior. Therefore, the external fragility of an economy is one of the root causes of financial crisis in emerging markets including Turkish experience.

Yeldan (2001b) underlines external fragility as the root cause of Turkish 2000-2001 crisis. He shows the vulnerability of Turkish economy by using the ratio of the short-term foreign debt to Central Bank international reserves as an indicator of fragility. He reports:

"Data at hand disclose that before the crisis (June, 1997) the ratio of short term foreign debt to Central Bank international reserves was on the order of 170% in Indonesia, 150% in Thailand, 90% in Philippines, and 60% in Malaysia. Thus, it could be argued that the value of 60% for this ratio is regarded as a critical threshold from the point of view of international speculation. It is alarming to note that in Turkey this particular ratio has never fallen below the 100 % mark since the opening of capital account in 1989. Thus, the Turkish financial system had been operating constantly under the "danger zone" for the past twelve years as far as this indicator is concerned."
As mentioned several times in this study, literature on the Turkish economy in 1980s and 1990s hold two important developments responsible for this fragility. These are the liberalization of international accounts and deficit financing by domestic debt issuing. These can be named as the "supply" and "demand" sides of the discussed structural fragility.

Financial liberalization leads to availability of international capital forming the supply side but it does not explain by itself the reason why the Turkish economic system demands such a risky source of funding. The high public sector borrowing requirement and the high stock of domestic debt completes the picture by forming the demand side.

Empirical and theoretical evidence for the supply side are plenty in the literature both on Turkey and on the emerging markets in general. Demirguc-Kunt & Detragiache (1999) using a panel show how financial fragility increases the probability of banking crisis. Chang & Velasco (2001), on the other hand, display how financial fragility leads to higher international borrowing opportunities and hence, feeds financial fragility by increasing the short-term liabilities of a bank. But, why do banks take such risks? The domestic banks should be aware of the risks of international short-term borrowing, however, they are willing to take such risks. We believe the conditions that lead to the structural external fragility should be analysed in more detail from the "demand" side.

The Chapters Two and Three of this study contributes to the existing knowledge by explicitly modelling bank behavior under interest rate risk. Note that, the actual risk created by the maturity mismatch is due to the fact that this trade of capital between the foreign lenders and the domestic government is done via the domestic banking system. This makes the bank behavior particularly important.

In Chapter Two, we analyze how the bank chooses between risky and safe ways of finance. This sheds some light on the demand side we discussed above by considering public borrowing. Our model theoretically defines how high rates of public borrowing lead to more risky ways of finance. The theory of international economics simply
explains how capital (if mobile) flows from low to high interest rate market. However, this does not explain how the maturity mismatch formed and why the domestic bank willing to take the risk of extremely volatile international capital. Our model clarifies this issue by stating the effect of the high lending rates (to the government) on the risk premium. The income effect of the high lending rate lowers the risk-premium of the domestic bank and motivates it to take higher risk.

In Chapter Three, we show that the amount of borrowing the bank uses depends on many factors. In Chang & Velasco (2001), the borrowing conditions are extremely favorable and the bank uses all available borrowing. Therefore, financial liberalization by extending the borrowing opportunities increases the financial fragility in their model. Our model, on the other hand, by introducing interest rate risk, models the factors that may lead the bank to borrow more or less, providing some new insights about the demand side of the problem once again. In other words, the availability of short-term borrowing is important but other conditions such as risk preferences of the consumers (and hence the bank) availability of domestic investment opportunities are also critical in determining financial fragility.

Given the structure of the Turkish economy, lowering the interest rates seems to be a win-win situation as high interest rates are held responsible both for the difficulties in debt sustainability and the fragility created by speculative short-term capital movements. Consequently, it has always been the center of the economic policy. However, lowering interest rates to more plausible levels is not an easy task. The public debt stock accumulated in more than two decades and the government’s commitment on the issued debt makes it extremely hard if not impossible. Therefore, trying to push-down the interest rates to strengthen the financial system may not work particularly in the short-run. This should definitely be the target in the long-run for a sound financial system, but there may be other policy options for reducing financial fragility in the short-run. Our model in Chapter Two displays this possibility by pointing out a policy maker who cares about financial stability can trade it with some increase in borrowing costs if he uses a reward scheme that favors the banks who finance their lending to the government from safer sources.
CHAPTER 5. CONCLUSION

Given the key role played by the banking system in the most recent crisis episodes, maintaining financial stability has become one of the main goals of the economic policy all around the world. The 2008 global financial crisis underlined once again how important risk management by financial intermediaries is and reinforces the view that economic policy makers should assess the main risks and vulnerabilities of the financial system and develop alternative policies against these risks by monitoring the financial system as a whole. On the parallel grounds, central banks are reorganising themselves in order to effectively monitor stability.

After the 2000-2001 crisis in Turkey, a broad banking sector restructuring programme has been launched. The main lesson learnt from the crisis is the critical significance of achieving and then monitoring financial stability. Central Bank of Turkey started to publish financial stability reports in 2005. Furthermore, in foreword of the first, Financial Stability Report (2005) the importance of financial stability as a policy objective is explained with the following words:

"...due to the financial crises experienced on the road to globalization, along with financial liberalization and technological improvements, “financial stability” in line with the target of achieving price stability, has become one of the leading policy issues of central banks. While most central banks have been acquiring instrumental independence in implementing monetary policy, the tendency towards assigning the duty of supervising banks to authorities other than central banks has commonly been observed. This, however, has not diminished the importance attached by central banks to assessing financial stability in order to achieve financial stability. On the contrary, the view that price stability and financial stability targets are inseparable has become widespread."

The outline of our model in Chapter Two is in line with this view as it considers a policy maker who has financial stability in his objective function. Although, not mentioned as a policy objective in the report, sustainability of the debt stock is always an important issue in Turkey. When the public debt stock is large and when the government relies mostly on domestic borrowing for the finance of the deficits.
The borrowing rate becomes the foremost important variable in the whole economy. Due to the uncertainties regarding the sustainability of the debt stock, the conditions at which the government borrows is closely monitored by all economic agents. In time it becomes the first measure people consider when shaping their expectations about the future of the economy. In an environment like this lowering the borrowing rate of the government turns out to be the primary aim of the economic policy most of the times. One of the prescriptions that is often offered for this problem is lowering the real interest rates. However, we also show that pushing the borrowing rate down as much as possible only works fine if you can really reduce it. Otherwise, it may be sub-optimal. The government cannot observe how the bank finances its lending to the government. This creates moral hazard if the government is not able to reduce the borrowing rate sufficiently to avoid any risky borrowing by the banks. Our analysis shows that in that case government can optimally trade some financial stability for a rise in borrowing costs.

Another issue that is often emphasized in the context of financial fragility is the ability of financial intermediaries to assess and manage risk. One major problem in Turkey is that the transfer procedure of cheaper international capital from foreign investors to the treasury, who is managing the public debt, loads all the risks on the domestic banking system. If the banking system can transfer some of these risks to its creditors that would definitely reduce the fragility of the system. Banks often make investment and borrowing decisions under uncertainty. This uncertainty causes risk. If the bank’s income depends on this uncertain parameters but its liabilities does not, the bank bears the whole risk by itself which at the same time creates incentives for the lenders to panic in any sign of dispute.

In Chapter Three, we consider a model where one of the two liabilities of the bank, the deposit contract, is a function of the uncertain borrowing rate. This leads to sharing of the interest rate risk among different type of consumers. More surprisingly, this also leads to a reduction in the liquidity risk by indexing the bank’s short-term liabilities to the borrowing rate. On the one hand, when borrowing rate goes up, the bank’s longer term liabilities go up, which may be a reason for
creditors to panic. But the optimal deposit contract we derive shows bank’s short-term liability to the depositors never goes up with the borrowing rate. The fall in the payment to early consumers may also reduce the incentives for a typical depositor to panic.

One other noble feature of our analysis in Chapter Two is that it actually provides evidence such contracts are feasible. Particularly when early liquidation of long-term investments are costly, the consumers will be more willing to share the risk of being impatient and consume early. This is because when they consume early, they not only fail to collect the benefits of the long-term investment but also they pay additional cost for liquidating early. Our model successfully displays a case where the variable rate deposit contract is individually rational, incentive compatible and hence, implementable.

About the issue of endogenizing the risks, the analysis we carry out in Chapter Four also provides some insights. We observe when the commercial bank takes into account that the illiquidity risk is increasing in the short-term payment it provides, profit maximization brings about lower financial fragility automatically. When these risks are artificially removed by guarantees or similar policies, the incentives of the banking system to generate a sounder financial system fades away.

Finally, we believe our analysis in Chapter Four introduces an important extension to the models that analyze financial fragility and financial crisis using D&D type multiple equilibria models. The consideration of a private profit maximizing commercial bank, instead of the standard D&D bank which is formed by the risk sharing individuals, leads to a new set of possible contracts and may affect the degree of financial fragility in the system. In most cases of financial crisis, the profit seeking banks that cannot manage or diversify risk properly play a critical role. They usually try to exploit certain conditions in the financial markets and economic policy towards financial stability often tries to remove these conditions from the market. In doing that, the profit seeking nature of the commercial banks should be taken into account.
Appendix A

Interest Rate Risk and Financial Fragility

A.1 $s - b > 0$

A.1.1 Region A:
A.1.2 Region B:

Subregion B1:
Subregion B2:
A.1.3 Region C:

Subregion C1:
Subregion C2:
A.1.4 Region D:

Subregion D1:
Subregion D2:
Subregion D3:
A.1.5 Region E:
A.1.6 Region F:
A.1.7 Region G:

Subregion G1:
Subregion G2:
A.2 \[ s - b < 0 \]

A.2.1 Region H:

Subregion H1:
Subregion H2:
A.2.2 Region I:

Subregion II:
Subregion I2:
Subregion I3:
A.2.3 Region J:

Subregion J1:
Subregion J2:
A.2.4 Region K:
A.2.5 Region L:

Subregion L1:
Subregion L2:
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