Tracking the Early Number Skills Performance of 5- to 7-Year-Old Students: A Longitudinal Study

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ABSTRACT

This longitudinal study tracks how 5- to 7-year-olds perform with early number skills. The aim of this study is to diagnose at-risk mathematics students by distinguishing the skills that, if not mastered by the end of Kindergarten, lead to greater difficulty in mathematics in 1st grade. This study's methodology is mixed as it follows an exploratory and inductive path in light of its use of a hypothesis, an interpretive path in light of its interest in the individual student, and a positivist path in light of its focus on developing rules from analyzed data. An oral diagnostic test based on a comprehensive collection of early number skills was used to test students as Kindergarteners and again as 1st graders. The test results created benchmarks, revealing how the majority of the students performed with early number skills. The test results also revealed that each early number skill is highly, moderately, or minimally predictive in terms of student placement by the end of 1st grade. When comparing the individual skill scores of each Kindergarten student to his/her total test results of 1st grade, the predictive power of each skill emerged. Performing poorly with skills that are minimally predictive did not seem to have an impact on how the Kindergarten student finished in 1st grade; performing poorly with moderately predictive skills had a greater impact on 1st grade placement; performing poorly with highly predictive skills in Kindergarten increased the likelihood that the student would finish in the lower attaining group in 1st grade. A third result of the test showed that certain skills serve as preconditions for other skills; success with certain skills usually meant success with other skills. These connections between skills point to a learning model called in this study "simultaneous pathways," indicating that there are connections between certain skills, and that students can be learning on several pathways simultaneously. The impact of the predictive power of early number skills is that diagnosis becomes more effective. Early diagnosis means early remediation which may prevent at-risk students from falling further behind their peers. The benchmarks developed by this research will help teachers assess their students because they will know the general skill level of Kindergarteners and 1st graders. This oral diagnostic test informs curriculum development. If test results show that students are missing the skills that are highly predictive, teachers can address those gaps in order to insure mastery.

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If, because of this work, some students become more successful in mathematics, it is due to all of you. Thank you.

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CHAPTER ONE
INTRODUCTION

> Some children have difficulty in learning to count. Number problems start here. If they are not overcome, then they can lead to a vicious circle in the acquisition of more advanced arithmetical skills that rely on counting for their establishment (Daniels & Anghilieri, 1995, p.125).

The area of study for this dissertation is on early number skills and young children. Why this combination? "Counting skills are fundamental to learning base-10 concepts. Early difficulties in counting portend later difficulties with arithmetic operations" (Jordan, Kaplan, Olah, & Locuniak, 2006, p.154). Therefore, the focus of this study is to understand, through the use of an oral diagnostic test, young children's ability with early number skills in order to distinguish the skills that, if not mastered, might lead to greater difficulty in achieving mathematical competence.

The significance of the study in relation to the existing literature is as follows: although some researchers write about early number skills as a group (e.g., Gelman & Gallistel, 1978; Kamii, 1982; Fosnot & Dolk, 2001), much of the research on these skills views them individually. Yet early number skills appear to be neither learned nor used in isolation; they seem to intertwine. For example, the act of counting objects effectively seems to involve several skills simultaneously; rote counting, one-to-one correspondence, organized counting, and cardinality are all in use, as children appear to need to know how to recite the number words, how to designate one number to one object, how to differentiate between the counted and the to-be-counted, and how to know what the total is. Therefore, developing a comprehensive collection of early number skills seems to be necessary in order to have a thorough knowledge of how young children perform with them.

A second point of significance is that although there is a great deal of research done on assessment in all areas of education, there does not seem to be a diagnostic tool to specifically assess early number skills in an efficient and informative way for teachers to use in an everyday classroom environment. A number of diagnostic tools exist but, as will be discussed below, none that assess early number skills in an efficient way and for the same purpose as the tool developed for this study. Knowing how to evaluate young students in early number skills is a crucial component in teaching. Without knowing exactly how their students are performing in these skills, teachers cannot create a meaningful curriculum, nor can they remediate effectively. With a diagnostic tool that informs teachers about their students, the teachers have a means to guide their teaching. "Diagnostic assessment through which learning difficulties may be scrutinized and classified enables appropriate help and guidance to be provided" (Daniels & Anghilieri, 1995, p.60).

The oral diagnostic test used in this study allows teachers to assess students during class, as opposed to withdrawing the child out of the classroom for a private interview, as it is short and relatively easy to administer. Alongside the research carried out in this study in relation to children's early number skills, an assessment tool that had been created nine years ago, and piloted extensively during those nine years was used, and is described in greater detail below.

Other diagnostic tools exist (e.g., Ginsburg and Baroody 1990, TEMA; Moomaw, 2008, APCBM-Math; Van Luit et al, 1994, Utrecht Early Mathematical Competence Test; Early Mathematics Diagnostic Assessment, 2002), but they serve in other ways than the oral diagnostic test used for this study. Ginsburg and Baroody's TEMA, Moomaw's APCBM-Math, and the Early Mathematics Diagnostic Assessment are administered one-on-one, as is the test used for this study, but they seem to cover more skills than those on which this study focused, as the tasks go from early subitizing to multiplication, and include other skills such as identifying numerals and combining quantities. The point of the diagnostic test used for this study is that the focus is just on early number skills. Van Luit et al's Utrecht Early

Mathematical Competence Test is quite comprehensive. Although it is also administered in the same fashion as is the one used for this study, it takes much longer to administer, which can impose a hardship both on classroom teachers who might lack the time to do so, and on students who might lack the focus required for such a long list of tasks. The diagnostic test used for this study takes approximately ten minutes to administer, which means that a classroom teacher can diagnose a class of children relatively quickly. This efficiency is a key feature of this test, as are its informativity concerning early number skills, as well as its validity and reliability.

Therefore, it seems necessary to create a diagnostic tool for early number skills to evaluate how the students perform with them in order to inform teaching. This diagnostic tool needs to be short enough for a teacher to administer easily and focused enough on early number skills to be informative. It also needs to be inclusive enough to have tasks on “Piagetian logical abilities but also procedural counting knowledge…It is not good practice to look for a single deficient arithmetic ability…It is important to build models that include the several markers for arithmetic development and then investigate the interactions between those components” (Stock, Desoete, & Roeyers, 2010, p.261). The other diagnostic tests described above do not include “Piagetian logical abilities” tasks, such as conservation of number and hierarchical inclusion, as the oral diagnostic test used for this study does. These latter tasks illuminate important information concerning the performance of young children with early number skills. "Even after controlling for differences in working memory, logical abilities in 6-year-old children remain a strong predictor for arithmetic abilities 16 months later" (Stock et al, 2010, p.251). To focus on these tasks alone, however, as Piaget did, ignores "the importance of counting" (Stock et al, 2010, p.251). To include them in a test along with counting tasks, as the test for this study does, creates a "model (that) could serve as a framework for a better understanding of the development of arithmetic abilities" (Stock et al, 2010, p.251).

In addition, although researchers note how children typically perform with various early number skills (e.g., Fuson, 1988; Fischer, 1992; Clements, 2004), there does not seem to be specific information on the impact of poor performances with early number skills. Researchers might write about how high young children can rote count (e.g., Geary, 1994), or how large a group of objects young children can subitize (e.g., Baroody & Gatzke, 1991), or how old children tend to be when they have mastered conservation of number (e.g., Ginsburg, 1989), but they do not seem to write about what happens when students perform below the majority of their peers. Knowing both how most young children perform with each of the early number skills, and what might happen if a child's performance is below that of his/her peers serve to inform the implementation of a mathematics curriculum, as well as the use of intervention. Therefore, developing both benchmarks and "red flags"—information on what most young children can do with early number skills, and on what the lack of mastery of particular skills might mean—seem necessary in order to teach mathematics effectively. As Dowker (2005) writes, "It may be difficult to ameliorate or prevent numeracy difficulties without a greater understanding of their nature" (p.1).

Last, although researchers have written about what should be in a mathematics curriculum for young children (e.g., Reys & Reys, 2010), and have written about various learning models (e.g., Simon, 1995), there do not seem to be specific guidelines for each grade or agreement on how these early number skills are learned. Standards for mathematics tend to come in grade bands rather than for each grade (e.g., NCTM, 2000). This structure makes it hard for teachers of each grade to know exactly what is expected of their students. As Fuson (2004) writes, "Grade-band goals such as used by NCTM and by states can lead to excessive review and non-mastery at any grade level because the grade-band specification is taken to mean "do at each Grade 3, 4, and 5” rather than to mean “do well somewhere within the three Grades 3, 4, and 5” (p.109).

The mathematics standards for the state of Maine (Maine’s Learning Results, 2007) are also written in one grade band for pre-Kindergarten to 2nd grade. From 3rd grade on, the standards are grade specific, but for the younger grades, what teachers should teach and when they should teach it is not clearly specified. It is clear only what should be taught during these grades, but not when and to what level. For example, the standard for whole number is written for grades pre-Kindergarten to 2nd grade—an age span that ranges from 4 years old to 8 years old. One of the standards states that students should “read and write numbers to 1000 using numerals.” This broad and vague standard is of little help to teachers of these grades, as they might not know what would be appropriate for their class. In addition to the lack of clarity is the lack of important early number skills themselves. Only place value, addition, and subtraction are listed for number for these grades. There is no mention of counting with or without objects, nor is there mention of any of the other early number skills listed below in Table 1. The local standards are no different as they were also derived from the state standards. For many years, teachers in the local schools have requested clearly written mathematics standards and benchmarks. The impetus for this study was partly driven by this need from local teachers. (One addendum: since the submission of this dissertation in April, the Common Core Standards—the national standards for America—were published in June, 2010. These standards are written for each grade, and specify succinctly what mathematics should be taught and to what level of mastery. As a result, Maine's Learning Results are in the process of being phased out, with the Common Core being phased in for use in creating classroom curriculum and state assessments).

In addition to this ambiguity of performance indicators, there is disagreement about how to structure learning because of the various differing learning models. One learning model suggests that skills be taught hierarchically (Denvir & Brown, 1986); another learning model suggests that skills be taught in a broad path (Van den Heuvel-Panhuizen, 2001). Therefore, creating specific mathematical guidelines for each grade, as well as a learning model that fits how these skills are learned seems again necessary in order to teach mathematics effectively.

My ultimate goal was to find the mathematically at-risk students to provide immediate and effective remediation so that they can rise to grade-level early in their schooling before the gap becomes too great to overcome. Why? When I was a mathematics consultant, I saw struggling older students who had gaps in their ability. When I assessed them for early number skills, I found that these students had not mastered some of them. For example, Alan (name changed), a 5th grader (11 years old) who struggled with whole-number computation, could not count above 109. When asked what the next number was, he replied, “200,” counted to 209, and then said 300. Toby (name changed), a 4th grader (10 years old), also struggled with computation, and was found to not have yet mastered conservation of number. From these and many other students who behaved similarly, I developed a hypothesis that if early number skills were not sufficiently mastered at a young age, then later mathematical concepts could become much more difficult to perform.

The way to test this hypothesis was to create and use a diagnostic tool based on a comprehensive collection of early number skills from which I could develop benchmarks and possible red flags for Kindergarteners and 1st graders. I was not the only one interested in such a tool. The teachers with whom I worked asked me for a diagnostic test to use with their students to learn more about on what level they are. These teachers spent a great deal of time and expertise assessing literacy levels in their students, but did not have a similar tool to do the same for mathematics. As a mathematics education consultant, I often saw a great disparity between literacy assessment and mathematics assessment. Baroody and Ginsburg's (1990) observations below were the same as mine.

> With reading instruction, there is usually an effort made to tailor instruction to children's readiness, their rate of progress, and their individual learning style. Reading instruction often takes place in small groups where the teacher can closely monitor a child's errors, provide corrective feedback, and otherwise monitor progress and adjust the training. Children do not graduate to more advanced readers until they have demonstrated a measure of competence with more basic readers. In contrast, mathematics instruction is frequently done in a large group and practiced alone without direct feedback. However, even among children just beginning school, there are a wide range of individual differences. Kindergarteners and first graders are far from uniform in their informal mathematical knowledge and readiness to master formal mathematics (p. 59).

Creating and using the oral diagnostic tool used in this longitudinal study for three years with two different cohorts would allow me to see how children perform with early number skills over two years, and find out which skills, if not mastered, caused the students to lose ground mathematically. I want to improve mathematics teaching, especially remedial, so that students can create a sturdy mathematics foundation that can continue to be built upon successfully. "Robust mathematical learning by all young children is a necessary base for later learning and is necessary to keep children from falling permanently behind in mathematics" (Fuson, 2004, p.105). Jordan et al. (2006) agrees: “If children’s learning needs can be identified early on, we may be able to design interventions that prevent failure in math” (p.154).

This information would impact on our understanding of learning models, curriculum development, assessment, and teacher training. Using a learning model that matched the early number skills and how students learned them would increase the effectiveness of mathematics teaching. An effective learning model would help with curriculum development and the accompanying assessment. Perhaps even more important, this information would increase the content level of teacher training. Teachers often asked me what their students should know at each grade level, and how they would know if a particular gap in skills is problematic or not. These teacher-raised questions show not only a desire to become a better mathematics teacher, but also show gaps in teacher-training. As Higgins (1973) points out below, the mathematics teacher needs to not only know the subject matter thoroughly, but must also know how students learn, as well as what each student already knows.

> Best of all is the teacher who sees himself not as a lecturer but as a traffic monitor at the center of a great intersection. The intersection is formed by the students in his class; the traffic is traffic of ideas which are being actively tried, manipulated, changed, and tested…The teacher who acts as a traffic monitor in ideas must know far *more* subject matter than the lecturer, and he must have planned for many different contingencies that may be triggered by students’ ideas (p.87).

My personal and professional commitment to this research, combined with the gaps in research described above formed the key questions for this study.

1. What are the most fundamental early number skills?
2. According to the oral diagnostic tool used for this study, how do most exiting Kindergarteners and 1st graders perform with these early number skills?
3. Are there early number skills that can predict how Kindergarten students will perform in 1st grade?
4. How might this understanding of early number skills impact on the teaching of early number skills, specifically with learning models, curriculum development, assessment, and teacher training?

How was this study done? It is a longitudinal study of two groups of Kindergarteners and 1st graders, with each group studied for two years. The benefits of a longitudinal study were clear: "Longitudinal research is critical for understanding how math difficulties develop and change over time" (Jordan et al., 2006, p.153). The results of the oral diagnostic test were then analyzed in order to answer the research questions posed above.

The boundaries of the research are twofold: subject matter and age limits. No research was done on addition or subtraction skills, and no research was done with preschoolers or 2nd graders. Since, as noted above, children's difficulties with computation seem to start not with computation but with a faulty mastery of early number skills, it appears to be those early number skills that need the primary focus, rather than computation skills. As for the age limit, optimally the at-risk students would be located by the end of Kindergarten, and brought up to grade level by the end of 1st grade. If left later than this, closing the mathematical gap becomes much more difficult. "It is vital that we increase the mastery by all through Grade 2 because at present many children leave Grade 2 already so far behind that it is difficult to catch up" (Fuson, 2004, p.106). In terms of younger children, preschool-age children could also be tested and the data analyzed because possible remediation could begin even earlier, ensuring fewer at-risk Kindergarteners. However, that remains to be done in a further study.

The structure of this thesis is as follows: In the second chapter, there is a review of the literature. The various learning models are reviewed as are the way certain researchers have grouped early number skills. Also discussed in that chapter is each individual skill: how it is defined, how children perform with it, and how it interrelates to other skills. In the third chapter, the methodology and methods are examined. The choice of the use of the scientific paradigm is explained, as is how the oral diagnostic test is used. Validity, reliability and ethical concerns are all discussed. The fourth chapter describes and analyzes the data. The findings are explained and illustrated with student examples. The implications and recommendations of the data are explored in the fifth chapter. The data analysis is compared to what was written about the research in the second chapter, and the impact of the data analysis is described in terms of learning models, curriculum development, mathematics assessment, and teacher training. Finally, the study and its limitations are summarized in the conclusion, the sixth chapter.

CHAPTER TWO
LITERATURE REVIEW

Introduction

This literature review has four sections: first is an overview of various learning models; second is a survey of the way several researchers have grouped early number skills; third is an examination of each individual early number skill used in this research; and fourth is a discussion of how these skills contribute to the development of mathematical competency through their interrelatedness.

The complex issue of how early number skills are developed needs to be explored. One question that arose during this study was, "Are early number skills learned in a sequential way, one after another, or are they learned in a simultaneous way, one along with another?" This question centers on what learning model is useful and accurate for early number skills. Though this complicated question can potentially intertwine both how children learn and how mathematics should be taught, the focus in this study is primarily on what early number skills children have, or do not have, as they exit Kindergarten and first grade.

Counting skills have been researched numerous times and a great deal has been written about several of them as individual skills. Often, too, researchers have grouped a few skills together, seeing them as “principles,” or “core skills,” or “interrelated aspects.” However, these sets are at times in disagreement with each other by including some skills but excluding others. This research aims to create a comprehensive diagnostic tool for assessing the foundations early number skills in a form that is both manageable in terms of time to implement, and informative in terms of data collected for classroom teachers to use. Which skills to include is a crucial issue that will be discussed later in this chapter.

Creating a collection involves examining each skill both individually and collectively. Each skill needs to be explored individually to learn what researchers have written about them. These skills do not stand alone, however, so examining the interrelatedness of early number skills follows naturally, as well as exploring how each contributes to the development of mathematical competency.

A Review of Learning Models:

The Hierarchical Model

Some researchers write that mathematical skills come in a certain order and involve a hierarchy: one skill will not be learned until the skill before it has been learned. Daniels and Anghilieri (1995) write of Gagne’s work (1965) in which he “attempted to identify elements of simple tasks which function as elements of more complex tasks and proposed learning hierarchies of ordered skills with prerequisite sub skills” (Daniels & Anghilieri, 1995, p.50). Klahr and Wallace (1973) also wrote about a hierarchy of skills in which “cognitive skills are built on earlier skills, new knowledge being acquired when integration of learned skills permits increasing generalization to new situations” (Daniels & Anghilieri, 1995, p.51). Denvir and Brown (1986) also “attempted to identify a framework which describes pupils’ order of acquisition of number concepts” (Daniels & Anghilieri, 1995, p.53). Treffers (1987) describes this hierarchy as “vertical planning” which “is based on the notion that the ‘lower’ activity offers a necessary basis of experience for the ‘higher’ activity” (p.62). Orton (1992) goes so far as to warn that “if the teacher does not identify all of the stages in the pyramid, and omits some, pupils will become confused somewhere in the middle of the hierarchy” (p.52).

Many of the early number skills do seem to contain an inherent order; the skills appear to build on each other according to both mathematical logic and cognitive development. For example, rote counting (counting without objects) would seem to precede counting with objects because one needs to know the number words before one can count objects. There are other skills that would seem to follow suit, such as hierarchical inclusion and part-whole relationships. With the former skill, students learn that numbers nest: that within the quantity of eight lies the quantities of seven, and six, and five, et cetera. With the latter skill, students learn that a whole quantity can be broken into parts: that the quantity of eight can be broken into parts of one and seven, or two and six, et cetera. It would seem that these skills follow a hierarchical order: that counting objects seems impossible without knowing number names, and breaking a whole into parts seems impossible without knowing that lesser amounts reside in larger amounts. But as Dowker (2005) asks,

> If arithmetic is made up of numerous subcomponents, the question arises: can these be ordered in a hierarchy? Are there some skills that are always prerequisites for other skills, in the sense that one must learn to perform skill A before one can perform skill B? (p.29).

A Review of Learning Models:

The Learning Trajectory Model

Dowker decides no: "Some subskills do facilitate the development of other skills, but there are few subskills that must invariably precede other skills" (p.29). If so, then the hierarchical model of skills might not be the only possible structure. There seem to be faults in the hierarchical model, for it would imply that children learn one skill at a time, and learn it to mastery before going onto the next skill. This certainly does not happen; children do not learn to rote count perfectly into high numbers before attempting any other mathematics skill. Therefore, another model of skill development must clearly exist, one less sequential and more simultaneous. Could learning happen on a horizontal basis, as opposed to vertical basis, such that skills can be learned at the same time? Certain skills can promote growth in other skills; for example, a child's ability to rote count can increase as s/he encounters more objects to count. This understanding points to a more simultaneous model of skill acquisition rather than a sequential model for learning.

One possible model comes from Simon (1995). He describes his "hypothetical learning trajectory" (p.135) in which the teacher creates a goal for student learning but recognizes that this "learning trajectory" is "hypothetical" because the student's learning path "is not knowable in advance" (p.135). Because the student's path of learning is unpredictable, Simon's learning model is less strictly hierarchical and more flexible. He criticizes "traditional instruction" for "focusing on one skill or idea at a time" (p.138). Having a "learning goal that defines the direction" (p.136) is crucial, but at the same time, since "students' thinking and understanding will evolve in the context of the learning activities" (p.136), the learning model needs to be flexible, and teachers need to have a "well-developed map of the mathematical conceptual area" (p.139).

Van den Heuvel-Panhuizen (2001) also writes of a more simultaneous model rather than a hierarchical model of learning, and like Simon, describes it as a trajectory.

> A learning-teaching trajectory puts the learning process in line, but at the same time it should not be seen as a strictly linear, singular step-by-step regime in which each step is necessarily and inexorably followed by the next. A learning-teaching trajectory should be seen as being broader than a single track (p.7).

This image of learning allows for the idea that "multiple skills can be learned simultaneously and that different concepts can be in development at the same time" (Van den Heuvel-Panhuizen, 2001, p.7).

For Fosnot and Dolk (2001), Simon's hypothetical learning trajectory is "too linear" (p.17). One would expect Fosnot and Dolk to find Van den Heuvel-Panhuizen's learning-teaching trajectory to be somewhat better, but it is still not "messy" (p.17) enough. Fosnot and Dolk "prefer instead the metaphor of a landscape" (p.17). Fosnot and Dolk (2001) write that the "landscape of learning" is not a straightforward path, but rather "the paths twist and turn; they cross each other, are often indirect" (p.18).

> Children do not construct each of these ideas and strategies in an ordered sequence. They go off in many directions as they explore, struggle to understand, and make sense of their world mathematically (Fosnot & Dolk, 2001, p.18).

However, my own observations suggest that children do seem to learn some skills before they learn others, and more complicated skills seem to come later in children's cognitive development. This realization then points the learning model direction back to the hierarchical model of learning. Perhaps the answer lies in a combination of the models. Perhaps some skills are built on a hierarchical model and are learned in a more vertical fashion, whereas some skills are learned in a more horizontal fashion, resembling the landscape image. Perhaps there exists even a third option for certain skills which, when learned, serve to influence other skills both vertically and horizontally.

Grouped Early Number Skills

As can be seen from Table 1 below, several researchers have created groups of early number skills. Each group is different and no two groups have the same skills, nor does any group have all the skills in one collection. What follows is a brief description of each group of skills.

Table 1: Six researchers' grouped early number skills, arranged from oldest to most recent

Gelman & Gallistel	**Kamii**	**Wright et al.**	**Fosnot & Dolk**	**Clements**	**Cross et al.**
1978	1982	2000	2001	2004	2009
		Subitizing	Subitizing	Subitizing	
Rote counting (stable word order)		Rote Counting		Rote counting	Rote counting
	Conservation of number		Conservation of number		
One-to-one correspondence			One-to-one correspondence	One-to-one correspondence	One-to-one correspondence
			Organized counting		
Cardinality			Cardinality	Cardinality	Cardinality
Abstraction					
Order irrelevance					
	Hierarchical inclusion		Hierarchical inclusion		
			Part/Whole relationships		
		Written numerals			Written numerals

Gelman and Gallistel's (1978) "counting principles" is the oldest of the above groups. They argued that "the counting behavior of young children was guided by (these) five implicit principles" (Geary, 1994, p.24).

> The one-one correspondence principle emphasizes that only one number word can be assigned to each counted object. Implicit knowledge of this rule would be reflected not in the use of the standard counting sequence, but by the child's use of different number words to tag each separate item…The stable-order principle would be reflected in the child's counting, if the child used the same sequence of number words for counting different

> sets of objects…The cardinality principle is reflected in the child's understanding that the number associated with the last counted item has a special meaning…The abstraction principle refers to the child's awareness of what is countable. That is, it does not guide the act of counting in itself, but rather defines the domain onto which counting can be applied…The final principle, order irrelevance, reflects the child's understanding that no matter what order the items are counted in, from left to right or right to left, the result is the same" (Geary, 1994, p.24).

Although these counting skills are important, they are only part of what needs to be examined when learning about how young children perform with early number skills. As can be seen from Table 1, other researchers list additional skills. In order to create a more complete picture of the child as mathematician, one needs to examine not just counting skills but all the early number skills. Counting skills form the core of early number skills, but do not form the entirety. Other skills seem to come into play in the development of mathematical competency, and would indicate the necessity of examination. Indeed, the other researchers note these additional skills, as described below.

Yet, there is also an outer boundary for this research when examining early number skills, and that would be arithmetic skills, such as basic addition, subtraction, and place value. Written numerals, listed by Wright, Martland, and Stafford (2000) and Cross, Woods, & Schweingruber (2009), also fall outside the boundary for this research. Being able to write numerals is certainly an important skill, but appears to be more a matter of fine motor control and handwriting than early number skills; therefore writing numerals is not part of the group of skills used in this research.

Kamii (1982) writes extensively about conservation, hierarchical inclusion, and the construction of number in general. In terms of rote counting, however, Kamii writes that it should not be taught explicitly (p.36). She speaks instead of the need to create an environment in which children can construct their understanding of number. Kamii agrees that rote counting is a crucial skill but that hierarchical inclusion plays a greater part in mathematical understanding:

> Counting is not unimportant. It is, in fact, essential for children to learn to count if they are to go on to addition. Research shows, however, that ability to say the number words is one thing and using this skill is quite another thing (p.33).

Children have to “assimilate number words into the mental structure (known as hierarchical inclusion)” (p.36) in order to make full sense of rote counting, and use it as “the most desirable tool” (Kamii, 1982, p.36).

On the other hand, Wright et al. (2000) writes extensively of rote counting but calls it “Forward Number Word Sequence” because it is “merely saying a sequence of number words” (p.27). He focuses deeply on the importance of being able to recite numbers both forwards and backwards. Subitization and numeral recognition are also considered by Wright et al. to be crucial parts of early number skills, but again, as other researchers have noted, there are more skills to be considered.

When writing of early number skills, Fosnot and Dolk (2001) describe several of them as “big ideas:” one-to-one correspondence, organized counting, cardinality, hierarchical inclusion, part/whole relations, and conservation. Big ideas are described as “‘the central, organizing ideas of mathematics—principles that define mathematical order’ (Schifter & Fosnot, 1993, p.35). As such, they are deeply connected to the structures of mathematics” (Fosnot & Dolk, 2001, p.11). Subitization is also listed, but as a “landmark strategy.” Strategies are ways, or methods, of dealing with big ideas. Fosnot and Dolk’s list is quite full, and several items overlap with Gelman & Gallistel and Cross et al., yet they do not mention rote counting.

In another grouping of skills, Clements (2004) writes that

> Early numerical knowledge has four interrelated aspects: instantly recognizing and naming how many items of a small configuration (subitizing; e.g., “That’s two crackers”), learning the list of number words to at least ten, enumerating objects (i.e., saying number word in correspondence with objects), and understanding that the last number word said when counting refers to how many items have been counted (p.20).

Thus Clements focuses on subitization, rote counting, one-to-one correspondence, and cardinality, but does not mention conservation, organized counting, or hierarchical inclusion. He does write about part/whole relationships as a skill that children "can develop" (p.22), but does not list it as one of the "interrelated aspects" of early number skills as Fosnot or Kamii would.

Another more recent list, developed by the Committee on Early Childhood Mathematics (Cross, Woods, & Schweingruber, 2009), describes "The Number Core" which consists of four "mathematical aspects (that) involve culturally specific ways that children learn to perceive, say, describe/discuss, and construct numbers" (p.126). These four items are: cardinality, number word list, one-to-one correspondence, and written number symbols. These four items are certainly crucial, and the authors' statement that "connecting counting and cardinality is a milestone in children's numerical learning path" (p.126) reiterates what other researchers have written (Fuson, 1992a; Payne & Huinker, 1993; Sophian, 2007). Yet once again other researchers have written about more early number skills that need to be acknowledged.

Thus, for this research, the above groups of counting skills became amalgamated and enlarged into a comprehensive collection of early number skills. There seemed to be general agreement that subitization, rote counting, conservation of number, cardinality, and hierarchical inclusion were important. Organized counting and part-whole relationships, however, did not have as many in agreement as to their importance, but seemed to me to be crucial. Without organized counting, it is easy to lose track of what has been counted. “The lack of effective keeping-track strategies may result in skipping an item or items, or counting an item or items more than once” (Baroody & Wilkins, 1999, p.53). When counting objects, one-to-one correspondence and cardinality are not enough to get an accurate count. A system of organization seems to be crucial, and is one of the early number skills that should be assessed to get a full picture of the child as mathematician. The concept of part-whole relationships also seems to be crucial, as it means that a whole can be broken into parts. With part-whole relationships, a child will know that the whole of eight can be split into two parts, such as one and seven. With the understanding of part-whole relationships, the child will find the beginnings of arithmetic much more accessible.

> (Acquiring)…part-whole understanding of number…is a major advancement in children’s conceptual knowledge of number. Now children can develop many relationships among numbers because they can think of a number as both a whole amount and as being comprised of smaller groups or parts (Payne & Huinker, 1993, p.49).

Thus the collection of early number skills examined in this research is as follows:

- subitization
- rote counting
- conservation of number
- one-to-one correspondence
- organized counting
- cardinality
- hierarchical inclusion
- part-whole relationships

The next section of this literature review will examine each of these skills to see how they are defined, and how, according to other researchers, children have performed with each of them.

Early Number Skills:

Subitization

Subitizing "is 'instantly seeing how many.' From a Latin word meaning *suddenly*, subitizing is the direct perceptual apprehension of the numerosity of a group" (Clements, 1999, p.400). Children seem to be able to subitize at an early age. Indeed, infants seem to be able to note the difference in small quantities (Wynn, 1995), but certainly cannot name them until much later when they have learned number words. Geary (1994) writes:

“These findings indicate that a sensitivity to numerosity, at least of arrays of up to three or four items is likely to be innate (Gelman, 1990)” (p.5). Although research has been done about infants’ subitizing, this study focuses on later subitization in connection with number words. Subitization that occurs earlier than that is purely perceptual, as Clements (2004) writes:

> Infants can discriminate among and match very small configurations (one to three) of objects…This provides an early perceptual basis for number, but it is not yet ‘number knowledge’ (p.17).

The ability to subitize develops “naturally, without the culturally specific supports, in all constitutionally normal children…The form of subitizing in children is probably invariant across sociocultural contexts, because it is a perceptual process that requires no special cultural supports” (Klein & Starkey, 1988, p.21). There is a difference, however, between noting the size of quantities and being able to name them. Baroody (2004) writes, “Children typically start recognizing small collections of one to about four items and identifying them reliably with a number word between 2 and 4 years of age” (p.184). Subitizing seems to appear before the advent of counting. “Some children could subitize sets of one or two but were not able to count them. (Fitzhugh, 1978) concluded that subitizing is a necessary precursor to counting” (Clements, 1999, p.401).

The amount that can be subitized is small, from one to five objects. Indeed, Fischer (1992) states that amounts over three are not actually subitized but rather known through an “automatized procedure” (p.203). “This procedure may involve decomposition of the set into smaller, directly apprehendable units” (p.203). In other words, amounts of one to three are directly apprehendable, but amounts of four or five are, Fischer (1992) writes, apprehendable only by decomposing the group into directly apprehendable units of two and two, or two and three (p.207). However, Baroody and Gatzke (1991) write that children can “immediately recognize the exact numerosity of sets up to 5 items but not sets of 6 to 12 items” (p.59).

Researchers are generally in agreement about how children perform with subitization. When using groups of one to three, young children perform similarly well. However, when using slightly larger groups of four or five or even six, the performance changes. “The amount of time (needed to subitize) rose sharply as the size of the set increased beyond three” (Siegler, 1986, p. 278). This increase in time could be due to the need to switch to an automatized procedure.

> Usually, when presented with more than five objects, other mental strategies must be utilised. For example, we might see a group of six objects as two groups of three. Each group of three is instantly

> recognised, then very quickly (virtually unconsciously) combined to make six (Way, 2005).

The increase in time could also vary depending on the formation of the objects. Fischer (1992) found that when larger amounts were placed in "canonical geometric patterns, such as 4 in a square or 5 in the die configuration," children could subitize faster than when such amounts were in a random placement (p.206).

Early Number Skills:

Rote Counting

Researchers also seem to agree that another crucial early number skill developed in young children is that of rote counting—counting without objects. Rote counting is simply reciting the number words, without relating the words to any objects. Baroody and Wilkins (1999) write: "At first, oral counting may be nothing more than a 'sing-song'—a pattern of sounds uttered without any apparent purpose" (p.51). Indeed, Fuson (1988) writes that initially children do not realize that the number words are separate entities, but rather hear and recite them as a single word—onetwothree—much like children do with "LMNOP" when singing the alphabet song. However, in time children learn that rote counting numbers is more than a song, but rather symbolizes items in their world. Children "develop an appreciation of the ways that these number words can be used for counting and measurement" (Aubrey, 1997, p.21), or "as a way of determining the relation between two sets" (Sophian, 2007, p.34).

How well do young children perform with rote counting? Aubrey (1997) describes a study of rote counting and four to five year olds in which the range was dramatic: "Counting ranged from four to more than 100, with a mean length of sixteen, with 80 per cent of children counting to at least ten, 48 per cent of children counting to within the range eleven to twenty, and a further 15 per cent to within the range twenty-one to thirty" (p.24). Payne (1993) agreed, writing that "most children can count by rote to 10 or 20 when they enter kindergarten" but he adds a crucial note, "with ability strongly affected by opportunities to practice" (p.46). In other words, home environment is another ingredient to add to the study of how young children perform with rote counting, but that is beyond the scope of this study.

The arbitrariness of the English language is another factor in how children perform with rote counting. Baroody and Wilkins (1999) note that the English language inconsistencies cause young children to "overextend their counting rules (i.e., make rule-governed errors such as '…nineteen, ten-teen, eleven-teen…' or '…twenty-nine, twenty-ten, twenty-eleven…')"

(p.53). Baroody and Wilkins continue that "it is not until first grade that many children recognize that the decade series parallels the single-digit sequence…and master the decades" (p.53). Fuson (1992a) agrees with Baroody and Wilkins' age and ability statements, but note that for some children, "it takes…a very long time to learn the decade words themselves in their correct order" (p.132). Geary (1994) also agrees, writing that "The child's acquisition of (counting) skills…is a slow and often difficult process. For most children this process spans a 6-year-period, from the ages of 2 to 8 years" (p.13).

Why does it take so long to learn to rote count? Thompson (1997) writes that American children perform poorly in rote counting when compared to Asian children, and he attributes that standing to the differences in the languages, rather than to cultural differences. In English as well as in Asian languages, the numbers one to ten are distinct and arbitrary, and children simply have to memorize them without the help of any pattern that might give the string of words more sense. In Asian languages, however, the words after ten reflect place value sense, in that each number word starts with the word for ten and adds the word for the correct number of ones: ten one, ten two, ten three. In English, on the other hand, the arbitrariness of number words continues after ten with such words as "eleven" and "twelve" which show no relation to what they actually symbolize. “Thirteen” and “fifteen” also hide their relation to ten and three or ten and five by distorting the words for three and five. “Fourteen,” “sixteen,” and “seventeen,” et cetera, clearly state the ones words, but “teen” is not clearly “ten.”

The problems do not end at twenty. The decade names in English use “ty” for “ten,” again not clearly reflecting the number value. The tens words are also often irregular—“twen-ty,” “thir-ty”—which again only serve to emphasize arbitrariness in the counting sequence. In Asian languages, the decade names use the same words as in the original one to ten sequence: two ten, three ten, et cetera. The same word for ten is thus used repeatedly in the teens and in all the numbers up to 100—a total of ninety times—as opposed to the use of the word ten in English, which is used for a total of once in counting from one to 100. This list of language inconsistencies causes great difficulty for young children, in that they “have to memorize a long sequence of seemingly unrelated number names before the patterns become visible” (Thompson, 1997, p.124).

Early Number Skills:

Conservation of Number

Conservation of number, a concept developed by Jean Piaget (1952) from his research on how children understand number, means knowing that the amount of a group of objects does not change even if the appearance of the group of objects changes. When children cannot conserve number, they notice that the appearance has changed in some manner, whether a line of objects has been elongated or moved to a side. That change in appearance is what drives the children's answers. Non-conserving children fail to notice that nothing has been done to the set of objects; none has been added nor subtracted. When children have mastered conservation of number, they no longer believe that the quantity changes when its appearance changes. Young children, however, "rely on appearances in making judgment of number…When one of the sets is manipulated so as to change only its appearance, children fail to recognize that the number is the same…" (Ginsburg, 1989, p.19). However, if the number of objects is small enough to be subitized, then children will seem to be able to conserve, but are actually only subitizing. As Kamii (1982) writes,

> Piaget referred to small numbers up to four or five as perceptual numbers because small numbers such as 'oo' and 'ooo' can easily be distinguished at a glance, perceptually. When seven

> objects are presented, however, it is impossible to distinguish ‘ooooooo’ from ‘oooooooo,’ for example, by perception alone (p.2).

According to some researchers, children perform consistently with conservation of number. Kamii (1982) writes, “Young children do not conserve number before five years of age” (p.15). Ginsburg (1989) also writes that “cultural differences affect the ages at which children attain conservation” (p.17). What happens in the home can affect how quickly a child constructs the internal relationships necessary to conserve number. Generally, however, “some children may master it as early as 4 or 5, others at 6; still others not until 8 or 9” (Ginsburg, 1989, p.17). Ginsburg and Opper (1988) also write of the long timeframe it takes to learn to conserve number: that “from about 4 to 7 years,” the child is in the early stages of learning how to conserve numbers (p.154).

Some researchers, however, disagree with that time frame. Gelman and Gallistel (1978) devised an experiment in which much younger children watched as small groups of objects were disarrayed but these children believed that the quantity did not change:

> Gelman claims that children as young as 3 years understand the invariance of small numbers—that is when there are three objects or fewer on display. They appear to understand that displacing the objects in such as array does not affect its numerosity in the way that adding or subtracting objects does (Hughes, 1986, p.21).

However, this conclusion does not hold because of the small amount of the array. As seen from the research described above in the section on subitization, as well as from Kamii's comment from Piaget, children as young as three can subitize small amounts, such as three. Therefore, the children in Gelman and Gallistel's study were not conserving—they were subitizing.

Children learn conservation of number in stages over time. As Kamii (1982) writes, "Conservation is not achieved overnight" (p.5). The first stage is that of the non-conserver: the child who believes that the two equal quantities do not have the same amount after the appearance of one of them has changed. The second stage is that of the transitional conserver: the child who is not sure and needs to count to verify his/her perceptions. The counting might confirm that the amounts are the same, or the counting might be manipulated by the child (with faulty one-to-one correspondence, for example) so that the child believes the amounts are now different. The third stage is that of the conserver: the child who might find the question absurd since the answer is so clear. "To children who had constructed the logico-mathematical structure of number, the answer was so obvious that counting was superfluous" (Kamii, 1982, p.18).

Fuson, Secada, and Hall (1980) thought that counting was not superfluous in conservation, and that the children who could not conserve could be taught to do so by teaching them to count or match the displaced objects. Their hypothesis was that children would not normally think to use counting during the conservation task: "The child might simply fail to produce a strategy in a situation even though that strategy would in fact help him solve the problem" (p.3). Dowker (2005) agrees, writing that children do not "understand that one-to-one correspondence is a better cue than length for such comparisons" (p.82) as is required in the conservation task. However, the results of Fuson et al.'s experiment showed that not only is counting "not necessary for correct performance on a conservation task," it was not "sufficient" since a quarter of the children in the study counted but still "answered the conservation questions incorrectly" (Fuson et al., 1980, p.8). Another issue with the results of the study is that, as noted above, children who count when performing the conservation task are not fully conservers but rather transitional in that they are on the cusp between their perception and their logic. Fuson et al. (1980) recognizes that, and notes:

> Piaget was chiefly concerned with conservation as a test of children's conviction of the logical necessity of the maintenance of equivalence over the transformation. The use of an empirical strategy to establish the post-transformation equivalence in particular cases does not reflect such a conviction of logical necessity (p.8).

Neither one-to-one correspondence nor matching are good “cues” for the conservation of number task, but rather the use of logic is the best cue, which tells children that since no item was added or subtracted, the amount must stay the same.

Donaldson (1978) is also concerned with the validity of Piaget’s conservation task and asks “(What) actually happens when a ‘non-conserving’ response occurs?” (p.63). Donaldson goes on to answer that the child is more concerned with the experimenter than the experiment. Since the experimenter says, “Now watch this,” as s/he displaces the counters, the child is inclined to expect that something happens that demands a different answer, Donaldson believes, and therefore responds with a “non-conserving response.” In a similar vein, Gelman and Greeno (1989) agree, bringing attention to “*interpretative competence*, which includes understanding the rules of conversation for different social contexts” (Baroody, 1993, p.418). Since one “rule of everyday conversation is ‘Do not repeat what is already known by the listener’ (Gelman & Greeno, 1989, p.146), (children) avoid a social gaffe by…changing their answer” (Baroody, 1993, p.418). In other words, both Donaldson and Gelman and Greeno believe that children pitch their answers according to what they think the experimenter wants them to say. Both Donaldson and Gelman and Greeno seem to believe that children are not paying attention to the mathematics task in front of them but are rather paying closer attention to social cues.

> What Donaldson has taught us…is that at the heart of the experimental situation is a child who is actively trying to make his or her own sense of the situation—and in particular, trying to understand, from what the adult says and does, and from how the materials are manipulated, what the adult's motives and intentions might be. Crucially, the child's interpretation of these factors might be quite different from that intended by the adult (Grieve & Hughes, 1990, p.3).

Baroody (1993) disagrees with this claim from Gelman and Greeno, and by extension, from Donaldson:

> Even if children thought it unnecessary to repeat what was obvious, why would they choose to make an obviously incorrect (statement)? After all, stating a patently false answer contravenes an arguably more important social norm: 'Tell the truth.' It seems unlikely that children would prefer to be viewed as untruthful or willful, rather than modestly impolite (p.419).

It also seems unlikely that virtually all non-conserving children should respond similarly, if not the same, when the task is administered. The similarity of non-conservers' responses to the task, done over so many years and places since Piaget developed it, cannot be attributed to simply being polite.

Would children perform better if the conservation task had only one row of counters rather than two? As Dowker (2005) notes, "(The) traditional two-row number conservation task requires the child not only to understand invariance, but also to be able to compare the numerosities of two sets" (p.82). Bergeron and Herscovics (1990) tried this idea with five and six year olds—to use only one row of counters, rather than two— to determine "the child's perception of the invariance of plurality with respect to dispersion and contraction" (p.130). However, both the dispersion and the contraction tasks showed that children of this age "depend on their visual perception of the objects in their conception of plurality" (Bergeron & Herscovics, 1990, p.130).

Early Number Skills:

One-to-one Correspondence

Counting objects, as opposed to simply reciting the counting words without objects, requires skills in addition to knowing the number words. One-to-one correspondence requires that "each countable is paired with one and only one sequence word" (McEvoy, 1989, p.108). One-to-one correspondence comes with some sort of physical contact with the objects, usually through touching each one with a finger. "To count objects, children learn to coordinate this list of words with pointing or moving objects that tie each word said in time to an object to be counted" (Clements, 2004, p.19).

Children perform at a wide span of ability with one-to-one correspondence. Two features need to be in place before one-to-one correspondence can be mastered: rote counting and enough fine motor control to be able to point to objects clearly. Ginsburg (1989) writes that "the average 4-year-old can count up to about nine objects without error; the 5-year-old about 20; 6-year-olds, about 28" (p.38). As noted above, children's rote counting ability exceeds these numbers at these ages, but that is to be expected: "Typically, children can say more number words than they can count things" (Ginsburg, 1989, p.35). Analyzing the act of one-to-one correspondence illuminates why it is so difficult for young children to master. As McEvoy (1989) writes, one-to-one correspondence actually refers to

> two sets of correspondences: correspondence in time between a word and the pointing act, and correspondence in space between pointing and the object…Accurate production of number words, plus their coordination in time with pointing and in space with objects, allow considerable scope for error…(The) coordination of verbal, visual and motor components in the execution of a count poses considerable processing demands on the young child (p.108).

Early Number Skills:

Organized Counting

As crucial as one-to-one correspondence is when counting objects, organized counting is equally important. One-to-one correspondence simply means assigning one number to one object. But without having some system of knowing what one has counted and not counted, one-to-one correspondence will not lead to an accurate count. Initially,

> Children see no need to rearrange grouped objects with a clear beginning and end, and thus they often recount the same objects many times. As they begin to see the need for organization as a way to keep track, and as they encounter larger groups of objects, they begin to find ways to organize their counting…(Fosnot & Dolk, 2001, p.64).

As Fosnot and Dolk note, some system of organization is necessary so that the counting does not continue *ad infinitum*. Systems vary, as Thompson (1997) writes:

> There are 'visual counters' who 'point' with their eyes; 'digital counters' who point with their fingers; 'touch counters' who touch the objects but do not displace them; and 'physical partitioners' who move the objects where possible while counting them (p.128).

What tends to work most effectively for organized counting is the latter—some kind of physical movement that shows clearly what has and has not been counted—"since physical partitioners make fewer errors than children in the other categories" (Thompson, 1997, p.128).

Organized counting is not an easy task and is fraught with mistakes. As Thompson (1997) writes, "To be successful in their counting, children have to coordinate the recitation of the number words with the physical act of pointing while at the same time ensuring that each object is counted once and only once" (p.128). Fuson (1992a) writes that "5-year-olds still make many recount or uncounted object errors on large, disorganized arrangements having 10-30 objects" (p.133). Ginsburg's (1989) description of a young child without organized counting trying to count is quite common:

> The child's procedure was to point to each candy in its original location; she did not bother to push any candies aside after counting them. Consequently, she forgot which were counted and which were not and ended up counting several candies twice and several not at all; as a result, she got different results each time (p.33).

Early Number Skills:

Cardinality

Cardinality is another skill involved in counting objects, and it means knowing that the last number recited while counting a group of objects signifies the total amount. "The cardinal principle is the simple recognition that the last digit in a counting sequence has a special status in that it represents a property of the collection as a whole rather than of the last element (Wagner & Walters, 1982, p.149). Learning one-to-one correspondence and organized counting are the first two crucial steps in counting items in a group, but knowing how many one has counted is the whole point to the exercise, yet is not immediately obvious to young children. "They also need to realize that the number that they have assigned to the last object tells them how many there are in the collection" (Thompson, 1997, p.129).

However, once again, children perform at varying degrees of ability. Initially, children use what Fuson calls the "last-word rule"—simply stating a number at the end of their count, a number that does not necessarily reflect an accurate count. "Many children who do answer a 'how many?' question with the last counted word seem to have constructed only a last-word rule, in which that last word does not refer to the whole set and does not refer to the numerosity of that set" (Fuson, 1992a, p.134). Achieving accuracy with one-to-one correspondence, organized counting, and cardinality depends on the amount of objects. The larger the set, the more likely "children may use a last-word rule while producing neither correct sequences nor correct correspondences" (Fuson, 1992a, p.135). Another response to the question, "how many?" might be to recount the entire collection. If a child responds in this way, s/he has "no grasp of the cardinal principle…Such a child seems to view counting as a discrete activity which has no end-product" (Maclellan, 1997, p.36). Dowker (2005) cautions that

> The cardinal word principle should not…be regarded as something that one either has completely or not at all. As with most mathematical concepts and skills, the cardinal word principle may be used in some situations before it is used in others and with smaller set sizes before it is used for larger set sizes (p.76).

Early Number Skills:

Hierarchical Inclusion

The next two skills described—hierarchical inclusion and part-whole relationships—involve understanding that a whole can be split into parts. This understanding goes against what children have been working on until this moment: that of creating a whole, or a group, out of parts, or single units. Once children understand cardinality, they recognize that single items in a group can be called by one name, and this name signifies the total amount of the items. That name defines a set that children cannot break apart until they acquire hierarchical inclusion. Hierarchical inclusion requires that children recognize that smaller amounts nest inside of a larger amount; if one has eight, one also has seven, and six, and five, et cetera. Although adults recognize this as obvious, young children often do not, and instead believe that since they have counted eight, they have eight and only eight, not seven or six or five. "As soon as the child's attention is given to the part, the whole is forgotten" (Copeland, 1970, p.90).

This mobility of thought is not achieved in very young children. Kamii (1982) writes that "it is precisely what four-year-olds cannot do" (p.13). Copeland (1970) writes that children achieve hierarchical inclusion by age "six or seven or during the time children are usually in first grade" (p.91). Frye, Braisby, Lowe, Maroudas, and Nicholls' work (1989) also suggests that preschool children find hierarchical inclusion difficult. They studied how "20 children ranging in age from 3.9-4.11" would perform with three different questions: "How many counters are there? Are there (X) counters here? Please give me (X) counters" (p.1161). The first question involves cardinality, but the other two involve hierarchical inclusion since the researchers asked for quantities that were one less than the total number of counters. "Overall, the children were able to answer the how-many question" (p.1161) showing that cardinality was within reach of children that age. However, with the latter two questions that involved hierarchical inclusion, the children "performed poorly" (p.1161). The researchers attributed the poor results with the fact that the questions made "more demands" on the children's understanding of hierarchical inclusion.

This is a reasonable response on the part of the researchers, considering the age of the children and the type of study done. Were there other factors that might have caused the children to falter on the latter questions? The language of the second question seems clear enough, yet since the children counted a group of objects, found a total, and were asked that total, then asking whether there is one less than the total could seem confusing to the children who might respond with an adamant negative. The third question seems easy enough to garner a correct response from the children, yet being able to correctly count out a total is a difficult task, as seen in the descriptions of one-to-one correspondence, organized counting, and cardinality above. It might be possible that the children "performed poorly" on the latter two questions due to those reasons, as opposed to a faulty understanding of hierarchical inclusion.

Early Number Skills:

Part-whole Relationships

Part-whole relationships appears to continue the thinking started by hierarchical inclusion. In hierarchical inclusion, children recognize that numbers nest within a whole; in part-whole relationships, children recognize the parts that make up the whole: that seven and one reside within eight, or two and eight reside within ten.

> Composing and decomposing are combining and separating operations that allow children to build concepts of ‘parts’ and ‘wholes.’ For example, children can develop the ability to recognize that the numbers two and three are ‘hiding inside’ five, as are the number four and one (Clements, 2004, p.22).

Children seem to be able to reasonably master part-whole relationships later than the skills listed above. Riley, Greene, and Heller (1983) believe that “young children’s inability to solve missing-addend word problems and equations has been taken as evidence that they lack a part-whole concept (Baroody & Wilkins, 1999, p.60). Kamii (1982) writes that “the mental structure of number…is not structured sufficiently before seven-and-one-half years of age to permit (children) to know that all consecutive numbers are connected by the operation of ‘+ 1’” (p.16).

The Contribution of Early Number Skills to the Development of Mathematical Competency

Subitization contributes to the development of mathematical competency in several ways. Baroody and Wilkins (1999) write that “subitizing different arrangements of a collection…may lead children to the important realization that collections can have the same number despite appearances" (p.55). Thus, subitizing can facilitate the development of conservation of number. Wagner and Walters (1982) similarly note that, "it is in realizing that certain transformations (like spreading out, or rearranging) do not change the value (output) of a subitized array, whereas others (like adding or subtracting elements) do, that the child comes to an understanding of important quantificational principles such as conservation" (p.138). Cardinality can come from subitizing as well. "When children count, subitizing the number in the set both encourages and reinforces understanding of the cardinal principle that the last number word is the same as the number the child recognizes" (Clements & Sarama, 2008, p.363). Part-whole relationships is also facilitated by subitizing. Since children can subitize small groups of objects such as two and three, as well as five, they can start to see that two plus three equals five, and thus learn the beginnings of addition (Sarama & Clements, 2008, p.398). Fuson (1992b) agrees, writing that "children 'see' the addends and the sum, as in 'two olives and two olives make four olives'" (p. 248).

Rote counting contributes to the development of mathematical competency by being the precursor of counting objects. This is a "vital connection," writes Cross, et al. (2009), "(Children) first connect saying the number word list with 1-1 correspondences to begin counting objects" (p.126). However, children cannot correctly count objects if they cannot correctly recite the number words in order. As Thompson (1997) writes, "Knowing only the first seven counting words is of limited use if you are counting ten objects" (p.125).

Like subitization and rote counting, conservation of number seems to be a crucial step in the development of mathematical competency. If children do not master conservation of number, "their world of number must be very chaotic indeed. If quantity is seen to change whenever mere physical arrangement is altered, then the child fails to appreciate certain basic constancies or invariants in the environment" (Ginsburg & Opper, 1988, p.141). Until children can conserve number, they depend not on the timeless structure of mathematics but rather on the shifting cloud of perception. "(Perceptual factors) are not yet sufficiently controlled by mental actions which can compensate for misleading information" (Ginsburg & Opper, 1988, p.147). As children learn to count and as they develop a sense of how numbers relate to each other, they start to depend more on this mathematical knowledge instead of perceptual knowledge. However, before this shift in thinking happens, a child will still depend on "his eyes" (Kamii, 1982, p.17). Fuson (1992a) agrees: "In situations in which information obtained from perceptual strategies conflicts with that obtained from the quantitative strategies of counting or matching, children will use the information from perceptual strategies" (p.137).

When children learn to conserve number, they can then make cognitive decisions, as opposed to perceptual decisions. "This level of conservation ability is a measure of the type of intellectual structures the child has developed" (Wadsworth, 1971, p.77). Wadsworth continues: "(Conservation) abilities will not emerge until cognitive structures evolve that make true conservation responses possible" (p.87). In order for children to be able to "make true conservation responses," they need to be able to recognize that a quantity can be both transformed and reversed: it can be changed, and also changed back. "Around the age of 6 or 7 the child learns to conserve number. Concurrently he *decenters* his perceptions, attends to the *transformations*, and *reverses* operations" (Wadsworth, 1971, p.79). This growth in children's cognitive structure allows them to "penetrate the tasks of addition and subtraction" (Maclellan, 1997, p.38). Therefore, conservation of number contributes to the development of mathematical competence because of the cognitive structural growth that manifests it. It is the intellectual growth that children have to do in order to conserve number that allows them to then go on to more complicated forms of mathematics.

Children who cannot conserve number have not developed the logico-mathematical understanding that will enable them to function competently in mathematics.

One-to-one correspondence contributes to the development of mathematical competency by taking the "song" of rote counting and applying it to the real world. Children learn that the numbers recited in rote counting can be used as useful labels to learn more about their immediate world. "By counting (usually in concert with a caregiver) the number of peas on the plate…young children are exposed to a critical feature of the counting context, which is that there is coordination between a number word and the countable" (Maclellan, 1997, p.34). Although one-to-one correspondence is crucial, it requires mastery of organized counting and cardinality as well to be fully useful.

Organized counting contributes to the development of mathematical competency because simply applying a number to an item is not enough for an accurate count. Recognizing that one has to keep track of what has been counted and what still needs to be counted is an important mathematical idea. Organized counting allows children to stop counting when they are done labeling each item. This ceasing of counting shows a child's recognition that counting has an objective—to know how many items are in a group.

This recognition can lead to cardinality; children realize that counting brings knowledge of “how many” and is not simply a diverting activity involving recitation and pointing.

Cardinality contributes to the development of mathematical competency by allowing children to create a group. This group can then be added (or subtracted) from another group, thus beginning the work of arithmetic. Clements (2004) calls cardinality the “capstone of early numerical knowledge and the necessary building block for all further work with number and operations” (p.19). Without cardinality, children would simply be left with counting for counting’s sake. With cardinality, “children begin to understand numerical equivalence, and to add and subtract in certain situations” (Fuson, 1992a, p.134). Payne and Huinker (1993) agree, writing that,

> Once children begin making connections between counting and cardinality, they can begin to ‘think in groups’ and make an important advance in their number concepts. By thinking in groups, children see relationships between numbers and are more flexible in dealing with quantities (p.48).

Hierarchical inclusion contributes to the development of mathematical competency in the same way that conservation of number does: it shows a certain level of intellectual growth that will allow for greater complexity in mathematical thought. In order for children to achieve hierarchical inclusion, they have to have the mobility of thought that allows them to reverse their thinking: “I have counted eight, but within my group of eight is another group of seven.” As Kamii (1982) writes, “In order to compare the whole with a part, the child has to do two opposite mental actions at the same time—cut the whole into two parts and put the parts back together into a whole” (p.13). This mobility of thought will now facilitate more complex mathematical thinking with part-whole relationships. “Until this reversibility of thought is achieved…addition…cannot be learned. It is the achieving of reversibility of thought (from whole to parts to whole again) that constitutes a logical or intellectual action as contrasted to the perceptual or prelogical, which is based on sensory experience” (Copeland, 1970, p.91).

Part-whole relationships contribute to the development of mathematical competence because once the knowledge of part-whole relationships becomes established, the knowledge of addition and subtraction is not far behind. “Part-whole understanding of number provides a stronger conceptual base for addition and subtraction strategies” (Payne & Huinker, 1993, p.51). Baroody (2004) concurs and adds:

> A part-whole concept may be the foundation for understanding the following more advanced concepts of number: (a) place-value representation (e.g., the whole 123 can be decomposed into the parts 1 one hundred, 2 tens, and 3 ones…(b) common fractions (in the representation a/b, the numerator ‘a’ indicates the number of equal-size parts of a whole of interest, and the

> denominator ‘b’ indicates the total number of equal parts into which the whole is subdivided), and (c) ratios (p.200).

Geary (1994) also sees part-whole relationships as the doorway to computation, and calls this skill “the hallmark” of young children’s mathematical thinking (p.13).

> The understanding that numbers can be represented by groups of other numbers is an essential step in conceptually understanding the addition and subtraction of relatively large numbers (p.13).

Summary

In this chapter, possible learning models have been described, ranging from a linear, hierarchical, sequential model, to a more flexible trajectory, to an even broader bandwidth, and finally to a landscape model. Also described in this chapter were the grouped early number skills that come from various researchers and show particular philosophies about early number skills. Individual skills have been defined; student performance in each of them has been explained; and the skills’ interrelatedness and contributions to the development of mathematical competency have been examined. This notion of interrelatedness cycles back to the notion of learning models. Can these early number skills be indeed hierarchical if they are so interwoven? Or is there yet another learning model that grows out of this research that is perhaps a combination of a hierarchy and a trajectory and a landscape?

The assemblage of skills used for this research comes from an amalgamation of these grouped skills, creating a complete collection of early number skills. This collection forms the heart of this research, and is used in an oral diagnostic test with exiting Kindergarteners and first graders. Why and how this oral diagnostic test is used is discussed in the following chapter on methodology. Chapter four explains the data as it is collected and analyzed, and chapter five focuses on the implications and recommendations from the data analysis. Finally, chapter six summarizes and expands on the conclusions drawn from this study.

CHAPTER THREE
METHODOLOGY

Introduction

Both the method chosen to gather the data for this study, and the methodology behind that choice are crucial, as together, they "determine the nature of the findings of research" (Opie, 2004, p.17). They also help answer the research questions posed in chapter one, the introduction. In this chapter, these sections follow: an exploration and justification of the paradigms used, as well as an account of their limitations; a description of the sample; a report of the use and administration of the oral diagnostic test used for this research; an examination of the validity and reliability of this oral diagnostic test, as well as a consideration of the ethical dimensions of the research; and finally, a discussion of the limitations of this method on this data collection.

The Scientific Paradigm

The aim of this study was to analyze how young children perform with early number skills, and from that analysis, identify the skills that predict how students will perform in 1st grade in mathematics. What might be the appropriate methodology then for this study? One methodology that might fit this conception of the study is positivism, which "strives for objectivity, measurability, predictability, controllability, patterning, the construction of laws and rules of behavior, and the ascription of causality" (Cohen, Manion, & Morrison, 2000, p.28). Cohen et al. continues that with positivism, the research is generally done "from the outside" (p.35), and is done to "(explain) behaviour, (seek) causes, (and generalize) from the specific" (p.35). With positivism, "it is accepted that...people are the *objects* of educational research, notwithstanding their uniqueness as one from another and from the other objects of the natural world…(and where) only educational phenomena…that are *observable* through experience can validly be considered as knowledge" (Morrison, 2007, p.21). According to this description, the "people" were exiting Kindergarteners and first graders and the "educational phenomena" were the data about early counting skills: analyzing how exiting Kindergarteners and first graders perform with early number skills, and what patterns can be found amongst those children who cannot perform as well as their peers with early number skills.

Adhering to positivism would have located the study in the scientific paradigm, in which "the search for generalizations (enables predictions about) future educational outcomes" (Ernest, 1998, p.35). Morrison (2007) agrees, and states that through the positivist approach and the scientific paradigm, the "findings can be *generalized* beyond the location of their project" (p.23). Laws, or predictions, can be made based on the analysis of the data. "Once general laws have been derived, the scientific research paradigm adopts a top-down perspective, using the general to deduce predictions about particular instances or observations" (Ernest, 1998, p.34). This ability to predict was initially a goal of this research: to be able to predict who might have difficulty in mathematics based on the skill level shown with early number skills. However, the ability to predict became necessarily limited due to the nature of the research (as can be seen below). Issues such as the size and the nature of the sample population, as well as the validity and reliability of the research, affected the generalizability and predictability of the study. From the data, I could suggest relationships between skills, but I could not generalize or predict with certainty.

Another aspect of the scientific paradigm is that the research should be able to be replicated by another person. "It should not matter who does the research, provided that others are as 'expert' as they are in applying the scientific method" (Morrison, 2007, p.22). Since I created it in 2001, the oral diagnostic test used for this study has been used by over 400 pre-service and in-service teachers, but only after I had taught them both about early number skills and how to use this particular oral diagnostic test. Typically we would spend several hours going through each task on the test, role modeling the language and the actions used for giving this oral diagnostic test. These pre-service and in-service teachers would then use this test on a few of their own students, and write up their results. The teachers' results with the oral diagnostic test, when compared to this study's results, appear to be similar; Kindergarteners and 1st graders performed consistently no matter who was giving the oral diagnostic test. Yet the teachers' data were never thoroughly analyzed; therefore, this study fills that gap. In terms of inter-tester reliability, however, no data exists because the students tested did not have the same testers. In other words, the students that were tested by the pre- and in-service teachers were not also tested by me. Therefore, although the results from the teachers' data mirror my results for this age group, there would need to be additional tests to claim inter-tester reliability.

Another issue exists: that of intra-tester reliability, which differs from the reliability that comes from others using the test and having the same results. Intra-tester reliability would mean that the results for one group would be the same with the same tester if done again. This would imply that the tester would remain consistent with how the test is administered and with how the student answers are understood. Although it is impossible to test for intra-tester reliability because that would potentially change the results if the same group of students received the test more often than other groups of students, I as tester made every effort to be consistent with how I administered the test and how I understood the student answers. (See p. 206 for more discussion on this issue).

But before others can use the same tool, and before findings can be generalized must first come the hypothesis. In the scientific paradigm, the hypothesis "is a statement indicating a relationship (or its absence) between two or more of the chosen elements and stated in such a way as to carry clear implications for testing" (Cohen et al., 2000, p.17). Cohen continues that it is from the hypothesis "that much research proceeds, especially where cause-and-effect…relationships are being investigated" (p.14). For this research, the hypothesis was that if certain early number skills were not sufficiently mastered by the end of Kindergarten, then succeeding in mathematics in 1st grade would become much less likely.

However, this was not a hypothesis to prove but rather to investigate. This makes the methodology of the study fall less in the explanatory mode of the scientific paradigm and more in the exploratory mode. The scientific paradigm applies truths to an instance, and then deductively draws out the implications of the truths already given. Although I might have wanted this study to do just that, the data collection called for another methodology. I was not able during the data collection to record the ages of the students exactly, nor was I able to record the students' socio-economic status. These gaps in the data caused the study to be less purely scientific and more uncertain about in its place in that methodology. As I took the data and tried to induce meaning from it, this method put the study into the exploratory and inductive paradigm. Indeed, according to Cohen and Crabtree (2006), this method put the study into the interpretive paradigm as well, in which "(f)indings or knowledge claims are created as an investigation proceeds… (M)eanings are emergent from the research process" (The Interpretivist Paradigm section, third and eighth bullets).

Before further discussion of this particular research, a description of the drawbacks of the scientific paradigm is in order. Ernest (1998) writes that the paradigm itself is a limitation.

> The weakness of this (scientific) paradigm is that it involves simplifying the phenomena described, and its application is too often based on unquestioned assumptions. All persons and human situations and contexts are unique and individual, but the scientific research paradigm treats whole classes of individuals or events as identical, or at least indistinguishable (p.36).

The focus on the group, as opposed to the individual, could distort the conclusions. What "children" can do is quite different from what a "child" can do; we potentially lose the subject when we become objective. To account for this potential problem with the paradigm, specific individual student examples are described and analyzed in chapter four, the data analysis.

What this means is that the process of the scientific paradigm excludes intention by the individual. “The precise target of the anti-positivists’ attack has been science’s mechanistic and reductionist view of nature which, by definition, excludes notions of choice, freedom, individuality, and moral responsibility” (Cohen et al., 2000, p.17). What is the researcher missing by only examining the externalities of the group? The researcher is potentially glossing over the differences of the individual responses in order to find the similarities of the group, and use those to make generalizations that might mistakenly highlight a small corner of a falsely constructed reality. By so carefully “restricting, simplifying and controlling variables” (Cohen et al., 2000, p.19), researchers who use this paradigm are running the risk of creating a world of their own making. “Where positivism is less successful, however, is in its application to the study of human behavior where the immense complexity of human nature and the elusive and intangible quality of social phenomena contrast strikingly with the order and regularity of the natural world” (Cohen et al., 2000, p.9). Because this research focuses on both the group and the individual, this drawback is potentially minimized. Certainly the group result is analyzed, but the individuals are as well in all their "immense complexity." This research therefore has several elements of the scientific research paradigm, but also includes some elements that are more interpretive in nature.

The Interpretivist Paradigm

The interpretive paradigm tends to focus on the subjective nature of the individual and the way s/he interprets his/her experience.

> Up until the 1960s, the 'scientific method' was the predominant approach to social inquiry, with little attention given to qualitative approaches such as participant observation. In response to this, a number of scholars across disciplines began to argue against the centrality of the scientific method. They argued that quantitative approaches might be appropriate for studying the physical and natural world, they were not appropriate when the object of study was people. Qualitative approaches were better suited to social inquiry (Cohen & Crabtree, 2006, Five Common Paradigms section, para. 5 and 6).

From this stance, the interpretive investigator develops a "multi-faceted (image) of human behavior as varied as the situations and contexts supporting them" (Cohen et al., 2000, p.23), and thus seeks to interpret human behavior from the basis of the individual. The interpretive stance is, therefore, one of subjectivity, in which both the researcher and the researched must be examined. "(The) interpretivist paradigm posits that researchers' values are inherent in all phases of the research process" (Cohen & Crabtree, 2006, The Interpretivist Paradigm section, para. 2). Hence, my belief that not acquiring early number skills in Kindergarten would lead to difficulties in 1st grade was present throughout this study. This subjectivity continues, as the data analysis must be viewed in the same light. Therefore, "all interpretations are based in a particular moment. That is, they are located in a particular context or situation and time" (Cohen & Crabtree, 2006, The Interpretivist Paradigm, fifth bullet). What is learned from these data cannot be separated from the locale in which the study took place.

The positivist paradigm, on the other hand, posits itself in objectivity, and believes that "human behavior is essentially rule-governed" (Cohen et al., 2000, p.22), and thus seeks to generalize human behavior from the basis of the group. The aim of this particular research was to know how young children as a collective perform with early number skills. However, it can be questioned whether that is possible, given the emphasis this study also places on the test results of each student.

In addition, when people research people, one cannot be rid of all the variables, nor can one fully control all the elements that might make the study less than purely scientific, even though one might want to do that. For example, my own bias towards this test might come through to taint the results. I had developed this oral diagnostic tool as a teaching tool, not as a research tool. I saw it as useful, as did the teachers with whom I worked. That stance is not an objective place from which to start research. I had developed a personal belief in this tool, and this belief stops the study from being purely objective, and stops it from being fully in the positivist paradigm. “Positivism is objectivist through and through” (Crotty, 1998, p.27). Crotty continues that “What turns (a) study into a positivist piece of work is not the use of quantitative methods but the attribution of objectivity…” (p.41).

Therefore, this study does not fall neatly into the positivist paradigm because at times this study is not purely objective or scientific. Rather, this study’s methodology is mixed as it follows a positivist path in light of its focus on developing rules from analyzed data, an exploratory and inductive path in light of its use of a hypothesis, and an interpretive path in light of its interest in the individual student. Using mixed methodologies for this study makes sense as it involves both the study of the individual’s behavior with the desire to create rules from that behavior. Crotty (1998) describes the difference between the two paths that positivism and intrepretivism walk: "(Science) is looking for consistencies, regularities, the 'law' (nomos) that obtains. In the case of human affairs…we are concerned with the individual (idios) case" (p.67). However,these two paths can converge, as can be seen when Crotty describes what Max Weber, considered the founder of interpretivism, believed: "(O)ne scientific method should apply to these two forms of science and should cater for both nomothetic and idiographic inquiry" because "general covering laws may explain human behavior as well as natural phenomena" (Crotty, 1998, p.68). By its very nature, with its dual foci on general laws and the individual, this study does not fit neatly into one methodology, but rather borrows from several.

A Description of the Sample

Who was chosen to participate in this study, and why was that group chosen? The grades chosen were exiting Kindergarteners who were then tested a year later as exiting 1st graders. The age for the Kindergarteners was five or six years old, and the 1st graders were six or seven years old. The focus was to remain on the youngest children in public school, in order to fully learn their ability with early number skills, with the possibility of early remediation in mind. These children came from the largest local elementary school in my district (see chapter four, the data analysis, for the detailed breakdown of the population). Within this particular school, three of the five Kindergarten classes were chosen to be the sample population, for a total of close to fifty children for each cohort. Two cohorts were studied: the exiting Kindergarteners of 2006 (and then again as exiting first graders of 2007), and the exiting Kindergarteners of 2007 (and then again as exiting first graders of 2008). This combination—two cohorts of three classes each—provided an initial sample of 93 children.

> (While) sample size does matter, of at least equal importance is the way that the sample is drawn…(The) sample should be as big as you can manage within the practical constraints and the resources available to you (Fogelman & Comber, 2007, p.136).

Figure 1: Timeline of study

	Spring, 2006	Spring, 2007	Spring, 2008
Cohort 1:	Kindergarteners →	1st graders	
Cohort 2:		Kindergarteners →	1st graders

I tested the children for two to three weeks in the spring of 2006, 2007, and 2008. I would be at the school most mornings each week for two-three hours, depending on the classrooms' schedules. In 2006, I tested exiting

Kindergarteners; in 2007, I tested these same children as exiting first graders, and also tested the next cohort of exiting Kindergarteners; in 2008, I tested this latter group again as exiting first graders. The first Kindergarten group had 46 children, which dropped to 37 children in 1st grade. The second Kindergarten group had 47 children, which dropped to 41 children in 1st grade. The total amount of Kindergarteners tested was 93, and the total amount of 1st graders tested was 78. The attrition was largely due to families moving away from the area.

Table 2: Amount of children in each grade and cohort

	Kindergarten	**1st grade**
Cohort 1	46 children	37 children
Cohort 2	47 children	41 children
Total children	**93 Kindergarteners**	**78 1st graders**

Although other classes could have been chosen, either from the same school or from other schools in the district, the population would have remained relatively similar; the students for each Kindergarten class in the district are interviewed by the Kindergarten teachers when they are in preschool and then selected for each class so that the same representative mix is in each class. (When deciding which children will go into each class, teachers look for a balance of academic ability, behavioral issues, and gender). Choosing three classes that were close in proximity to each other and to me helped ease the geographical burden of one oral diagnostic tester and 93 children. This kind of sample choice is called "convenience" or "opportunity" sampling. "The researcher…chooses the sample from those to whom she has easy access" (Cohen et al., 2000, p.102). (The other two Kindergarten classes were not tested because they were housed in a separate building not easily accessed, nor was there room in that building to sit separately with a student to administer the test). At the class level, however, no more sampling was done; all the students from each class were chosen to be tested, rather than being hand-picked within each class. This was to avoid sampling bias: "(Failure) to collect answers from everyone selected to be in the sample is a…potential source of bias" (Fowler, 2009, p.14).

This kind of sampling is called "nonprobability sampling." As such, the generalizations drawn from this study will be limited, as the sample population "does not represent the wider population, it simply represents itself" (Cohen et al., 2000, p.102). In order to evaluate this sample, the "process by which it was selected" (Fowler, 2009, p.19) needs to be examined to see "how well the sample frame corresponds to the population (this) researcher wants to describe" (Fowler, 2009, p.19).

How does this sample succeed and how does it fall short? It succeeds in that it focuses on the right age group and it uses two different cohorts over three years in order to increase the validity and reliability of the study. However, the children all come from the same rural area and thus do not represent urban or suburban children. Also, the children of this survey are all taught with the same constructivist math program ("Investigations in Number, Data, and Space," Dale Seymour, NY, 1998), and thus do not represent children who are taught by teachers using a more traditional mathematics approach.

The Use of the Oral Diagnostic Test

In order to test this study's hypothesis, to fully see how young children perform with early number skills, children need to be assessed, and these data need to be analyzed for patterns or "general laws." For this research, therefore, a one-on-one oral diagnostic test with exiting Kindergarteners and exiting 1st graders was used to collect data to find those patterns. Due to the nature of the sample, however, the patterns must necessarily be confined to the group itself, as noted above.

> (The) survey is the appropriate approach to use when systematically collected and comparable data are needed which can be obtained directly from a (relatively) large number of

> individuals. More specifically, a survey is the most advisable methodology…where data are required in a standardised form and are not available from other sources and where the researcher wishes to explore quantifiable differences between groups or relationships between variables (Fogelman and Comber, 2007, p.128).

The oral diagnostic test has many features in common with a survey approach, which Kerlinger (1986) proposes has "the advantage of wide scope: a great deal of information can be obtained from a large population" (p.387). A survey allows for answers from a large group of people, thus a substantial enough set of data is obtained to justify the creation of "general laws." Even more, a survey allows for the data to be explored in multiple ways, either for "frequency counts" or "simple correlations" (Fogelman & Comber, 2007, p.128).

But what kind of survey would be optimal? The population surveyed in this research was young, and the research needed an approach that would elicit information in a systematic and clear way. A paper-and-pencil standardized test would not have been possible, since many of these children could neither read nor write. The information could have been gathered just through observations—watching these young children perform early number skills—but observations alone can be unmanageable when working with a large population. Observations are also often not consistent enough to use in a collection of information: "Observation is useless if what the investigator is interested in does not happen with any frequency" (Ginsburg, 1997, p.48).

An oral diagnostic test, therefore, provides the data numbers that a survey provides. How, then, to create and shape the oral diagnostic test? Since discovering how young children performed with early number skills was the aim of the research, having the children actually do tasks was primary. Also, the oral diagnostic test could not be unstructured, as then the data collection would be too amorphous to analyze. Therefore, the oral diagnostic test script needed to be both task-based and reproducible. The key was to create a tool that would allow for the same tasks to be presented to each child, using the same language, but would also allow for periodic questions to be asked in case the child's response was unclear.

This oral diagnostic test (described in detail below) was a series of tasks using objects as well as several purely verbal questions. If a student answered in a way that was unclear, or if a student gave conflicting answers, I followed up with a question in order to clarify the student's answer. As Kerlinger (1986) writes, "An (oral diagnostic test…is) especially suitable for research with children. An (oral diagnostic tester) can know whether the (child)…does not understand a question and can, within limits, repeat or rephrase a question" (p. 440). However, because I wanted this oral diagnostic test to be reliable and the data to be comparable, I followed its script carefully, using the same language each time for each task as well as for the clarifying follow-up questions.

This kind of oral diagnostic test differs from what is known as the clinical interview, in which "the examiner begins with some common questions but, in reaction to what the child says, modifies the original queries, asks follow-up questions, challenges the child's response, and asks how the child solved various problems and what was meant by a particular statement or response" (Ginsburg, 1997, p.2). The clinical interview might indeed be able to "enter the child's mind" (Ginsburg, 1997, p.40) but would not have necessarily provided data that is reliable or quantifiable for this particular research. The standardized oral diagnostic test, on the other hand, can do just that. As Ginsburg, et al. (1983) writes, "The main aim of the standard (tool) is to produce reliable rankings of individuals on some characteristic" (p.17). Therefore each time this oral diagnostic test was administered, the script remained the same; each child was asked the same questions and performed the same tasks.

The oral diagnostic test used for this research (see Appendix 1 for the actual script) was the main instrument of data collection. The early number skills that were to be investigated were all included in the script. The data collected from these questions form the basis of this study. "These questions are then to be considered as items in a measurement instrument, rather than as mere information-gathering devices" (Kerlinger, 1986, p.440).

How, then, to order the tasks? The decision to place tasks in a certain order was partly practical in that tasks have to come in some sort of order, as they cannot be offered simultaneously. The decision was also partly theoretical in that the order of the tasks on the oral diagnostic test is loosely developmental. For example, researchers (e.g., Gelman, 1990; Geary, 1994) describe subitization and rote counting as two of the earliest learned skills, as opposed to hierarchical inclusion and part-whole relationships. Therefore, these two skills came first and second, respectively. Other researchers (e.g., Fuson et al, 1984; Thompson, 1997)) list one-to-one correspondence, organized counting, and cardinality as appearing to make a logical order as well, with skills that seemed to be both used and learned in that order. Hierarchical inclusion and part-whole relationships according to yet other researchers (e.g., Clements, 2004; Geary, 1994) seemed to work similarly, as skills that are also both used and learned in an order. Conservation of number did not appear to fit into an order as neatly, but seemed to benefit by the learning of rote counting, thus followed it. The tasks' final order was the same as listed in the literature review above:

- Subitization
- Rote Counting
- Conservation of Number
- One-to-One Correspondence
- Organized Counting
- Cardinality
- Hierarchical Inclusion
- Part-whole Relationships

As noted in the previous chapter, this understanding was based on the hierarchical learning model, as opposed to other learning models. Thus, offering them in what might be thought of as a hierarchical order is one possible solution. Whether children actually learn these early number skills in a particular linear order as opposed to a more simultaneous manner is a matter of research and debate, and something this research will help to investigate.

Pilot Study and Development of the Oral Diagnostic Test

Before examining how this oral diagnostic test was administered, an examination of its background is in order. The oral diagnostic test used in this study was six years in the making, starting in 2000. The first step towards it was in realizing that the act of counting was a more complex area of mathematics than might seem at first glance, and that fully understanding all the skills involved was necessary in order to teach effectively. This realization came from Ma's work (1999), which states that teachers need to develop a "knowledge package" (p.19) for each area of mathematics to "promote a solid learning of a certain topic" (p.19). Developing a knowledge package for early number skills involved both reading (e.g., Fosnot & Dolk, 2001; Kamii, 1982; Kliman and Russell, 1998) and working with teachers and students. "(A) fully developed and well-organized knowledge package about a topic is a result of deliberate study" (Ma, 1999, p.22). (See Appendix 1: The Knowledge Package for Counting).

As mentioned in chapter one, the teachers with whom I worked as consultant wanted this knowledge package in a form they could use with their students. Thus, the knowledge package was shaped into an oral diagnostic test that could be used to evaluate individual students. (See Appendix 2: Interview Questions for Assessing Counting). The same skills listed on the knowledge package were used in the oral diagnostic test, in a question-and-task form. A script was created that included what the teacher would ask for each task, and what students might say in response. This script form allowed for consistency in the administration of the test, in how the question was asked, how the task was set up, and how the tester rated the response of the student. After creating the test, it was used as part of my work as consultant and as university professor of mathematics education. (See Table 3 below). It was also used in three pilot study papers for this degree: one that tested exiting Kindergarteners, another which followed up these same students as exiting 1st graders, and a third on the methodology behind the latter paper. These papers, as well as the work done with the in-service and pre-service teachers, helped become the basis for this study.

Table 3: Timeline of the use of the oral diagnostic test

2001	2002	2002-2006	2006-2007	2007-2008
Read Ma's book and developed knowledge package for early number skills	Used knowledge package with teachers; developed into oral diagnostic tool	Used oral diagnostic tool with in-service teachers and with pre-service teachers	Used oral diagnostic tool for two pilot papers on the 1st cohort: first as Kinder-garteners and then as 1st graders	Described oral diagnostic tool for methodology paper as precursor to this study, and used this tool with 2nd cohort

The results from the pilot papers were different from those from this study (see Chapter Five, "Discussion"), and were different than each other because of the way the data were analyzed. For the first pilot paper, the data were analyzed by examining how most Kindergarteners performed with early number skills, and then by what the Kindergarten “strugglers” had in common. For the second pilot paper, the data were analyzed by examining how most 1st graders performed with early number skills, and then by how the Kindergarten “strugglers” fared in 1st grade. From these analyses, two lists were created that described how most Kindergarteners and 1st graders performed with each early number skill, and a third list was created from the data that described what Kindergarten “strugglers” had in common; in other words, how lower attaining Kindergarteners performed with early number skills. Finally, a fourth list was created that described how those “strugglers” performed in 1st grade, and what they had in common.

This form of analyzing the data revealed two sets of conclusions: one after the first pilot paper, and another after the second pilot paper. The initial conclusion was that those students who could not conserve number were the low attainers in Kindergarten. After the data for the 1st graders were analyzed, however, the conclusion changed to include that those who could not rote count to 100 as well as not be able to conserve were the low attainers in 1st grade. (See Table 4 below). The combination of these two skills seemed to be an indicator for lack of success in mathematics for this age student.

Table 4: Pilot Papers' Conclusions:

First Pilot Paper—on exiting Kindergarteners	Second Pilot Paper—on exiting 1st graders (same cohort as the first pilot paper)
The low attainers had lack of conservation of number in common.	The low attainers had lack of conservation of number and could not rote count to 100 in common.

For this study, however, the method used to analyze the overall data from both cohorts changed, because it became clear that investigating the "predictive power" of each skill, rather than investigating what just the lower attaining students had in common, would be more conclusive because the entire sample could be used. For the pilot papers, only the data from the lower attaining students had been examined, which had not taken account of the rest of the data. In addition, this change in analysis—to focus on the skills rather than the individual students—was more in keeping with the original hypothesis: that if certain early number skills were not sufficiently mastered by the end of Kindergarten, then succeeding in mathematics in 1st grade would become much less likely.

Thus, the tool itself was not significantly altered in the interest of keeping the longitudinal study reliable. Rather, between the pilot papers and the dissertation study, it was the data analysis that became dynamic instead of static. The oral diagnostic tool remained static in order to be able to compare the data from each cohort but the focal point of the data analysis became dynamic as it changed from focusing on the commonalities of the low attaining students at the end of each grade to the commonalities of the score results of each skill at the end of each grade. By focusing on the score results instead of the individual students, the trends of the data were clearer and more substantial. Rather than relying on the results of the few low attaining students, the entire sample was analyzed for their score results. Because the method used to analyze the data changed, the conclusions also changed from the conclusions of the pilot papers, as can be seen below in the Discussions chapter.

Administration of the Oral Diagnostic Test:

Overview

The children were pleased for the most part to participate in this oral diagnostic test, but were the expected mix of personalities—some energetic, some quiet, some talkative, some silent, and some confident. After some initial questions: what is your name, can you spell that for me, how old are you, when is your birthday, do you like math, do you do math at home or only in school—we would get started on the diagnostic tasks. The whole oral diagnostic test took about 10-15 minutes per child. Each student's task response was scored, and the student's total test score was recorded. These individual task scores and the total test scores are analyzed in chapter 4 and discussed in further detail in chapter 5. (See Appendices 4-8 for all the scores).

Administration of the Oral Diagnostic Test:

Subitization

For the subitization task, counters were placed in four groups: one group of two counters, one group of three, one group of four, and one group of five. These groups were randomly placed. I asked the child to point as fast as s/he could, without counting, to the group that has four in it, then three, then five, then two. I then asked the child to close his/her eyes while I rearranged the groups, and repeated the task. I watched carefully for how the child did the task, noting especially the child's eye movements. The three usual responses were: pointing without hesitation to all the groups; hesitating for the larger groups of four and five while counting with his/her eyes; or counting all groups with his/her eyes or finger.

Administration of the Oral Diagnostic Test:

Rote Counting

Rote counting—counting without objects—was the second task on the oral diagnostic test. For this task, I asked the child to count for me as high as s/he could go, by ones, out loud, without using objects. I listened for the correct order, for hesitation at the decades, for numbers skipped, and for when the child stopped on his/her own. When s/he stopped, I asked if s/he knew what number came next. Sometimes the child continued to count, but often the child was finished.

Administration of the Oral Diagnostic Test:

Conservation of Number

For conservation of number task, I used Piaget's classic task: first I lined up two rows of seven counters each. Seven was used because it is a number of counters larger than what can easily be subitized, according to Fischer, 1992, and Baroody & Gatzke, 1991. I asked each child, "Do my two rows of counters have the same number of counters in each or does one row have more or less counters in it?" After the child answered, I asked, "How do you know?" Then, while the child watched, I pushed one row of counters together, removing all spaces between the counters, and elongated the second row, enlarging the spaces between the counters, and asked, "Do my two rows of counters have the same number of counters in each or does one row have more or less counters in it?" After the child answered, I again asked, "How do you know?" The responses to each question fell into three groups: the conserver's answer ("The groups are the same"); the transitional conserver's answer ("I'm not sure—I have to count"); and the non-conserver's answer ("The longer line has more").

Administration of the Oral Diagnostic Test:

One-to-one Correspondence, Organized Counting, and Cardinality

The tasks involving counting with objects came next: one-to-one correspondence, organized counting, and cardinality. I asked the child to count the group of objects I placed before him/her. I used a large amount—twenty objects (the fourteen objects from the conservation task, with six added)—and placed them in a deliberately disorganized group, with some objects on top of each other, in order to see if the child would organize the objects. I asked the child to count these objects out loud for me. I watched for one-to-one correspondence: did the child use one and only one number for each object? I watched for a system of organization: did the child push aside the ones s/he had counted in order to know what was still left to be counted? I listened to the counting to see if it was accurate. When the child was done, I asked, "How many do you have?" This question uncovers the child's understanding of cardinality. Did s/he understand that the last number s/he recited indicated the total amount in the group? I listened for whether the child knew without hesitation how many s/he had, or whether s/he had to recount, or whether s/he did not understand the question.

Administration of the Oral Diagnostic Test:

Hierarchical Inclusion

The hierarchical inclusion and part-whole relationships tasks came next. For the hierarchical inclusion task, I asked the following question directly after the cardinality question: "If you have 20 counters right here, do you also have 19 counters right here?" I listened for whether the child knew that to be true or not. If a child understood hierarchical inclusion, s/he answered in the affirmative: "Yes, I have 19 if I take away one," or "Yes, I have 19, and 18, and 17, et cetera." If the child did not understand hierarchical inclusion, s/he was quite certain that s/he had twenty and only twenty counters. When asked, "How do you know," the explanations ranged from complete understanding ("I know that 19 is less than 20") to uncertainty ("I'm not sure") to complete certainty about the wrong answer ("I don't have 19, I only have 20").

Administration of the Oral Diagnostic Test:

Part-whole Relationships

I then continued to assess for part-whole relationships understanding by asking if the child knew two numbers that make five, and two numbers that make ten. I listened for whether the child knew any pairs for either number. For both questions, I watched for how the child solved this problem. Did s/he know any pairs automatically? Did s/he use fingers or counters to help?

Validity and Reliability of the Oral Diagnostic Test

Validity in an oral diagnostic test involves making sure each item or task is "measuring what we think we are measuring" (Kerlinger, 1986, p.417). Content validity takes this definition one step further: "The instrument must show that it fairly and comprehensively covers the domain or items that it purports to cover" (Cohen et al., 2000, p.109). To ensure content validity, "each item must be judged for its presumed relevance to the property being measured” (Kerlinger, 1986, p.418). If, for example, rote counting is being measured, how might the task be structured to achieve the closest match to what the child can actually do? What also must be asked is how relevant is rote counting to early number skills? If those questions produce answers that show that the task and the way it is used in the oral diagnostic test are consistent with what is known about rote counting and early number skills, then the content validity is proven. For this research, each item in the oral diagnostic test was chosen carefully based on what research said about early number skills, particularly from the works of Kamii and Housman (2000), Fosnot and Dolk (2001), Baroody and Wilkins (1999), and Ginsburg (1989). Each task was developed according to research and field experience to produce the closest match between the question asked and the answer produced.

Time and experience impacted on this oral diagnostic test and increased its validity. As noted above, the entire oral diagnostic test was piloted for several years informally during my years as math consultant and university professor, and more formally for three years during the background work leading up to this full study. Thus, the language for each task was shaped and reshaped during this piloting period. For example, when I set up the second part of the conservation task (when the two rows of chips are spaced differently), I learned to ask, “Do my two rows of chips have the same *number of chips* in each row, or does one row have more or less chips in it?” When I simply asked, “Are my two rows of chips the same,” I was not sure if the children were focusing on length rather than on number. Including the words “number of chips” was important in order to avoid confusion with length and clarify the focus on number of items.

The language of the hierarchical inclusion task was also changed in order to increase the clarity of the question for the children. Initially, I asked, “If you have 20 counters, do you also have 19 counters?” However, I was concerned that the language of the task was confusing, and that the children might think I was asking about an additional group—that I was asking about a group of 20 and a *separate* group of 19. Therefore, I changed the question to: “If you have 20 counters *right here*, do you also have 19 counters *right here*?” Clarifying instructions to the children also became necessary when the initial instructions prompted confusing responses from the students. For example, for rote counting, asking a child to simply count often produced either silent counting or skip counting. Therefore, I had to change the instructions to ask the child to count out loud and by ones.

External validity can also be increased by using two forms of triangulation. “Respondent triangulation” requires “asking the same questions of many different participants" (Bush, 2007, p.100). The sample chosen to be tested has to be large enough "to ensure that the people studied are representative of the wider population to which generalizations are desired" (Bush, 2007, p. 99). This population came in two cohorts, the first with 46 children and the second with 47 children. The other form of triangulation was through time. As noted above, each group was followed for two years. "Longitudinal studies collect data from the same group at different points in the time sequence" (Cohen et al., 2000, p.113). Longitudinal studies also allow "investigators (to) attempt to establish causal relationships" (Cohen et al., 2000, p.176). The fact that the study was longitudinal helps ensure the external validity of the research.

Reliability in an oral diagnostic test means that if it were repeated, the results would be similar, if not identical. "A reliable (oral diagnostic test)… should give more or less the same results each time it is used with the same person or group" (Bush, 2007, p. 92). Wragg (2002) asks, "Would two (oral diagnostic testers) using the schedule or procedure get similar results? Would an (oral diagnostic tester) obtain a similar picture using the procedures on different occasions?" (p.156). Ginsburg (1997) also asks, "Are responses to an (oral diagnostic test) stable over time or across children?" (p.168). As noted earlier, the oral diagnostic test was used for several years by both pre-service teachers in my university math methods education course, and by in-service teachers in their own classrooms, for a total of over 400 people. The results were consistent in both cases, as the data revealed in countless student papers and faculty discussions.

Kerlinger (1986) notes that reliability is improved when the task explanations are written clearly so as to “reduce errors of measurement” (p.415). Because this oral diagnostic test was used so often and by so many people, the language of the oral diagnostic tester and the directions to the oral diagnostic tester for each task were continually edited until the final version produced clarity and thus reliability. The continual use of this oral diagnostic test and the subsequent editing of the tasks and the language mean that the reliability increased, to the point that it can now “be administered by different clinicians to different children in different contexts” and yield similar results (Goldin, 1998, p.53).

Not only was the language revised but it was also “tightly structured” in order to maintain reliability. “Fowler (1993) emphasizes the need to ensure that all interviewees are asked the same questions in the same way if the procedure is to be reliable. This can work only if the interview schedule is tightly structured, with the properties of a questionnaire” (Bush, 2007, p.93). Each child was asked the same questions, using the same order of tasks as well.

Ethical Considerations

The ethical dimension of research needs careful consideration, especially when children are involved. Both the British Educational Research Association (BERA) (2004) and the American Educational Research Association (AERA) (2000) agree on the importance of informed consent and confidentiality. These issues, as well as those of privacy and anonymity, become heightened because the children "are unable to give voluntary consent entirely on their own" (Diener & Crandell, 1978, p.47) and because they are particularly vulnerable due to their youth. In order to acquire consent from children, researchers must actually turn to the adults who are responsible for them, namely parents and principals. "In the case of children…a parent should give permission for participation, preferably in writing, after reading or hearing details of the study" (Diener & Crandell, 1978, p.47). For this research, a permission slip was sent to the home of each child being tested, describing the study and the potential impact on the child (see Appendix 8). Each child then brought the signed slip back to his/her classroom, where the teacher collected and held them for me. No child was tested if the slip did not get returned, nor was a child tested if the slip was returned but the parent did not give permission. (Only one parent refused to allow the child to participate in the study).

Although the parents were obviously not the ones being tested, they were the ones to speak for their children, who were too young to make the decision alone. With children, it is the parents who need to be informed and make the decision. “Researchers must…seek the collaboration and approval of those who act in guardianship (e.g. parents)” (BERA, 2004, p.7). Yet the children also had a chance to object to the oral diagnostic test. Each time the teacher and I selected a child to be tested, the child was always informed that I was going to do math with them and then asked if s/he wanted to participate. No child ever objected to coming with me to do the oral diagnostic test. Why were there no objections? There might not have been any objections due to a genuine desire on the part of the students to participate, or it might have been due to a feeling from the students that they could not object to the situation. This is a difficult situation, as BERA (2004) points out:

> In the case of participants whose age, intellectual capability or other vulnerable circumstance may limit the extent to which they can be expected to understand or agree voluntarily to undertake their role, researchers must fully explore alternative ways in which they can be enabled to make authentic responses (p.7).

In this case, informing the parents and the teachers, asking the student if s/he wanted to participate, and waiting for the student's response were the ways the students were "enabled to make authentic responses." Recognizing that the students might feel unable to dissent was important because "educational researchers should not use their influence over subordinates, students, or others to compel them to participate in research" (AERA, 2000, p.4).

Diener and Crandell (1978) write of the importance of clearing permission with those responsible for the children if the study was to be done in a school, such as teachers and principals. "To use schoolchildren as subjects, it is necessary to obtain permission from the…school" (p.47). For this research, I spoke with the principal of the school and the three teachers of the classes I was using. The purpose of the study and the logistics of the oral diagnostic testing were all discussed and approved before the research began. In addition, since I had been using this oral diagnostic test informally since 2000 in the same school and in the classes with the same three Kindergarten teachers and the same 1st grade teachers, the principal and the teachers were all quite familiar with the logistics of the testing and the permission slips.

One of the logistics of the oral diagnostic testing was where it would be held. The ethical consideration is that of privacy: "Settings are an important determinant of privacy, and they vary along a continuum from very private…to completely public" (Diener & Crandell, 1978, p.57). During the piloting of the oral diagnostic tests, they were conducted according to the same continuum, depending on the time of day and the availability of space. The most private space was a small, usually unused classroom that was a distance from the classroom. This space gave complete privacy, but was inconvenient, and often not available. The most public space was in the same classroom as the other students. This space was always available, but did not afford the necessary privacy for effective oral diagnostic tests, as the students were distracted at times by what was happening in the classroom around them. The disadvantages of these spaces became clear during the pilot study mentioned above, and convinced me of the need for an accessible but as private as possible space to use for this research. That turned out to be the hallway directly outside the classroom, where a desk and chairs were set up. This space gave enough privacy for focused oral diagnostic tests, but was also convenient and always available. This became the space of choice during the data collection period for this research.

Another ethical consideration is that of confidentiality. This is a matter of privacy also—not of the setting but rather of the identity. The children tested needed to be anonymous in the data collection and analysis in order to protect them from possible embarrassment. For this research, however, because this study is a longitudinal one, and the children are followed for two years, their identity was crucial in order to maintain that later identification. Therefore, the names of the children were listed on each oral diagnostic test in order to know who was tested so that they could be contacted for an oral diagnostic test the following year. What was also on each oral diagnostic test was a number that was used for identification and to connect the oral diagnostic test to the total data collection. The total data collection listed each student by number rather than by name so that the results of each student could be monitored distinctly but also anonymously. As Diener and Crandell (1978) write, “The progress of schoolchildren may be followed over several years in a longitudinal study. To keep track of individuals, names must be connected to the data. But this information must remain within the confidential limits of the research team” (p.67). This confidentiality and anonymity was promised to the parents, teachers, and the principal during the informed consent period.

One last issue of ethics to consider is that of potential harm. In this case, potential harm was minimal, as the children “emerge(d) from their research experience unharmed” and the research’s “risks are minimal, understood by the participants, and accepted as reasonable" (Diener & Crandell, 1978, p.23). Because of the use of informed consent, the children knew that they were going to do "some math;" because of the use of privacy, the children were not exposed to potentially embarrassing moments; and because of confidentiality, the children's identity was and is hidden from others. In addition, the actual test took only ten minutes; therefore the amount of time the children were affected by it was negligible.

Limitations of the Data Collection

Weaknesses exist in any research. As Bush (2007) writes, "(The) research could always have been better grounded, the subjects more representative, the researcher more knowledgeable, the research instruments better formulated, and so on" (p.102). The weaknesses range from the general to the specific in any research, and certainly here. What follows is a catalog of this research's limitations.

The desire to generalize to a whole population using data of a necessarily limited sample is potentially dangerous, or at least simplistic. As noted above, one of the reasons of using the scientific paradigm is to create general laws based on the analysis of the data collected. For this research, the point is to develop a sense of what young children are capable of doing with early number skills and to examine the commonalities amongst those who cannot perform as well as their peers. However, the results cannot be called representative because of the sample size. Krejcie and Morgan (1970) suggest that for the amount of students that were available for testing (170 Kindergarteners), 118 should have been tested (Cohen et al., 2000, p.94). 93 were tested for this study, falling short of 118, therefore the sample cannot be called truly representative.

Yet, testing students in a longitudinal study has value in that the results can suggest what other Kindergarteners can do. As Dowker (2005) writes, "The best way of investigating whether certain skills are prerequisites for other skills is to carry out longitudinal studies" (p. 29), which is how this research was designed. Cohen et al. (2000) lists several benefits of a longitudinal study, in that it "separates real trend from chance occurrence;" it is "useful for charting growth and development;" and the "sampling error (is) reduced as the study remains with the same sample over time" (p.178). These benefits appear to have occurred in this study. However, because the actual sample is a non-probability one of convenience, rather than a probability sample following stricter rules of randomness, the ability to generalize is substantially limited.

Would the results of the data be different if collected in another part of the world? Yes, quite possibly—testing urban and/or suburban children as opposed to rural students might yield different results. Children of different ethnic backgrounds as well as children using different math curricula might also respond differently to the oral diagnostic test. These are issues that are worthy of future research to find out how different, if at all, children's performances with early number skills might be. These issues also limit the generalizability of the predictions, as complete confidence that all children will perform the same way is eroded if other populations might be different than the sample population.

Using an oral diagnostic test is also rife with issues. By definition, an oral diagnostic test comes with an oral diagnostic tester. This in itself is a potential limitation. "One of the difficulties with the (oral diagnostic test)…is the (oral diagnostic tester), because he is part of the measuring instrument" (Kerlinger, 1986, p.488). This "relative stranger, the clinician" (Goldin, 1998, p.58) can make the child's responses less accurate than if the child is interacting with someone familiar. As Kerlinger (1986) writes, "This apartness may affect the respondent so he talks to, and interacts with, the (oral diagnostic tester) in an unnatural manner" (p.387). For this research, the children tested did not know me, but I went to some lengths to set each child at ease, with some relaxing banter (“Great tee-shirt! I see you’re a fan of the Red Sox!”) and reassuring explanations (“We get to do some math together so I can learn more about kids and math”).

Time can be a burden and a limitation in research. “The major shortcoming of the (oral diagnostic test) and its accompanying schedule is practical. (Oral diagnostic tests) take a lot of time” (Kerlinger, 1986, p.440). However, as I felt that the only way to truly get the knowledge I needed for the research would be to work one-on-one with each child, I felt the time taken was necessary.

Another limitation of this data collection is timing, which can be crucial, as Goldin (1998) notes: “The oral diagnostic test itself may be taking place at a moment when the child is alert, tired, hungry, distracted, or excited. On the one hand, the child might prefer to be back in his or her regular class with friends or might, on the other hand, be looking forward to an interesting break from the classroom routine” (p.58). I tried to test children in the first hours of the school day because that is when the children were more available (as opposed to the afternoon when the children were at recess or other special classes such as art or music). I also felt that the beginning of the day was when the children would be more alert. I did indeed see students who were distracted by other student activity. More frequently, however, the students seemed ready to focus on the test, and seemed to regard going with me as a treat, even if they did not know why they were going:

Student: "Boy, oh boy, I can't wait to do this! I haven't done this in ages!"

Me: "What is it you think we're going to do?"

Student: "I don't know!"

Summary

In this chapter, both the methodology chosen and the method used for this research have been discussed. In terms of methodology, no one paradigm fits perfectly. Rather, the scientific paradigm fits the study as it looks for rules; the interpretive paradigm fits the study as it examines individuals; and the whole study fits an exploratory and inductive orientation as opposed to an explanatory and deductive orientation as the study tries to find the truth through data analysis.

The method used was an oral diagnostic test consisting of eight tasks of early number skills, and the reasons why that method was chosen as well as how it was administered are examined. The sample used in this longitudinal study was described as being a convenience sample of two cohorts of exiting Kindergarteners and 1st graders over the course of three years: the first cohort of 46 children was tested in the spring of 2006 and again in 2007; the second cohort of 47 children was tested in the spring of 2007 and again in 2008. This test was administered on a one-on-one basis with each child twice, once when the child was in Kindergarten and again when the child was in first grade.

Also discussed in this chapter was the validity and reliability of the oral diagnostic test, as well as the ethical considerations such as informed consent, privacy, confidentiality, and potential harm of the research. Concluding this chapter was an examination of the limitations of the data collection. What follows in chapter four is an examination of the data collected over three years. Chapter five discusses the data analysis, and then finally, chapter six examines the conclusions drawn from this work.

CHAPTER FOUR
DATA ANALYSIS

Introduction

This chapter focuses on the analysis of the data collected for this study. Before the actual analysis, however, comes the examination of the sample itself. Then the data are analyzed in the following stages: first, an explanation of how the data were organized; second, an overview of the data as a whole in broad terms; third, a deeper look at the data in order to examine the parts in detail; and finally, an interpretation of the analysis. This chapter is followed by an exploration of the deeper implications of the data in terms of both teaching and assessment.

Description of the Sample

The data analyzed in this chapter have been collected from the oral diagnostic test given to two cohorts. Cohort 1 was tested in the spring of 2006 and again a year later in 2007, and Cohort 2 was tested in the spring of 2007 and again a year later in 2008. Both groups came from the same school in a rural part of western Maine. The elementary school is in a town of 7,000, and the town itself is the county seat and home to a small liberal arts university. Therefore, the population is a mix of rural poor and professional middle class. The three classes chosen for this study all used the same math program, which focuses on hands-on activities that are constructivist in nature, as opposed to a more traditional textbook approach.

In terms of gender, the first cohort had an even split of 23 (50%) males and 23 (50%) females in Kindergarten, dropping to 18 (49%) males and 19 (51%) females in 1st grade. The second cohort had a much less even divide of 28 (60%) males and 19 (40%) females, evening up slightly after attrition to 23 (56%) males and 18 (44%) females in 1st grade.

Table 5: Gender of children in each grade and cohort

	Males in Kindergarten	**Females in Kindergarten**	**Males in 1st grade**	**Females in 1st grade**
Cohort 1	23 males (50%)	23 females (50%)	18 males (49%)	19 females (51%)
Cohort 2	28 males (60%)	19 females (40%)	23 males (56%)	18 females (44%)
Total children	**51 males (55%)**	**42 females (45%)**	**41 males (53%)**	**37 females (47%)**

The ages of the Kindergarten children in Cohort 1 were closely split between 5 and 6 year olds, with 21 5-year-olds (46%) and 23 6-year-olds (50%), with two 7-year-olds as well (4%). (In the United States, children are at times retained for a second year in the same grade because it is thought they will succeed to a greater extent than if allowed to continue to the next grade—thus the 7-year-olds in Kindergarten). However, in 1st grade, attrition brought the number of the younger group well down below the older group: 13 6-year-olds (35%) and 23 7-year-olds (62%), with only one of the oldest children still in school (3%). The ages in Cohort 2 were quite different, being weighted by the older group: 15 5-year-olds (32%), 30 6-year-olds (64%), and 2 7-year-olds. Attrition did not change Cohort 2 as much as it had Cohort 1: 16 6-year-olds (39%), 25 7-year-olds (61%), and the oldest children were gone altogether.

Table 6: Age of children in each Kindergarten cohort

	5-year-olds in Kindergarten	**6-year-olds in Kindergarten**	**7-year-olds in Kindergarten**
Cohort 1	21 (46%) children	23 (50%) children	2 (4%) children
Cohort 2	15 (32%) children	30 (64%) children	2 (4%) children

Table 7: Age of children in each 1st grade cohort

	6-year-olds in 1st grade	**7-year-olds in 1st grade**	**8-year-olds in 1st grade**
Cohort 1	13 (35%) children	23 (62%) children	1 child (3%)
Cohort 2	16 (39%) children	25 (61%) children	0 (0%) children

The exact ages of the children were not recorded, nor was that information available due to issues of parental consent. Information about the socio-economic status of each child was also not available. Therefore, the information from this data is lacking in important specificity that curtails the generalizability of the conclusions. The specific age of each child, and the average for each cohort especially, both contribute to an understanding of the performance of each cohort. If one cohort performed with greater ability than the other, it could have been due to the age difference.

Organizing the Data

Data were compiled using the oral diagnostic test described in the methodology chapter. Percentages were calculated according to the student performances for each skill. List 1 and List 2 below show the percentage of students performing at each task level as well as the points given for each task level.

List 1: Percentage of Kindergarteners for each early number skills task

Levels of task mastery are listed from highest level to lowest level.
Numbers on the left indicate points given for each level of task mastery.
Percentages indicate the number of Kindergarteners in both cohorts in this study performing this task correctly, out of a total of 93 students.

<u>Subitization:</u>

5—Can subitize all groups (2, 3, 4, 5): 44%
3—Can subitize only groups of 2 and 3: 45%
1—Needs to count all groups with eyes: 11%

<u>Rote Counting by 1's:</u>

5—Can count to 100 or higher: 43%
3—Can count from 21 to 99: 45%
1—Can count from 1 to 20: 12%

<u>Conservation of Number:</u>

5—Conserves (recognizes that the quantity has not changed): 32%
3—Transitional (needs to count the objects in order to perform this task): 25%
1—Does not conserve (does not recognize the quantity has not changed): 43%

<u>One-to-one Correspondence:</u>

5—Has one-to-one correspondence: 90%
1—Does not have one-to-one correspondence: 10%

<u>Organized Counting:</u>

5—Organizes objects while counting into counted and not-yet-counted: 71%
1—Does not organize objects while counting: 29%

<u>Cardinality:</u>

5—Knows how many are in the group when done counting: 81%
3—Needs to recount in order to know how many are in the group: 15%
1—Has no way of knowing how many are in the group: 4%

<u>Hierarchical Inclusion:</u>

5—Knows that there is one less than the total in the group: 65%
1—Does not know that there is one less than the total in the group: 35%

<u>Part-whole Relationships</u>:

5—Can list two addends for both 5 and 10: 34%
3—Can list two addends for either 5 or 10: 15%
1—Cannot list addends for either 5 or 10: 51%

List 2: Percentage of 1st Graders for each early number skills task

Levels of task mastery are listed from highest level to lowest level.
Numbers on the left indicate points given for each level of task master.
Percentages indicate the number of 1st graders in both cohorts in this study performing this task correctly, out of a total of 78 students.

<u>Subitization:</u>

5—Can subitize all groups (2, 3, 4, 5): 60%
3—Can subitize only groups of 2 and 3: 38%
1—Needs to count all groups with eyes: 1%

<u>Rote Counting by 1's:</u>

5—Can count to 100 or higher: 93%
3—Can count from 21 to 49: 6%
1—Can count from 1 to 20: 1%

<u>Conservation of Number:</u>

5—Conserves (recognizes that the quantity has not changed): 58%
3—Transitional (needs to count the objects in order to perform this task): 25%
1—Does not conserve (does not recognize the quantity has not changed): 17%

<u>One-to-one Correspondence:</u>

5—Has one-to-one correspondence: 96%
1—Does not have one-to-one correspondence: 4%

<u>Organized Counting:</u>

5—Organizes objects while counting into counted and not-yet-counted: 91%
1—Does not organize objects while counting: 9%

<u>Cardinality:</u>

5—Knows how many are in the group when done counting: 94%
3—Needs to recount in order to know how many are in the group: 4%
1—Has no way of knowing how many are in the group: 2%

<u>Hierarchical Inclusion:</u>

5—Knows that there is one less than the total in the group: 81%
1—Does not know that there is one less than the total in the group: 19%

<u>Part-whole Relationships</u>:

5—Can list two addends for both 5 and 10: 82%
3—Can list two addends for either 5 or 10: 9%
1—Cannot list addends for either 5 or 10: 9%

These lists show percentages for the total number of students in each grade: 93 students in Kindergarten; 78 students in 1st grade. However, because this study was longitudinal, there was attrition due to students moving out of the area. This loss of fifteen students will necessarily change the results of the data. Because of the need to compare the same students in Kindergarten and in 1st grade, the rest of the data analysis will focus on the remaining 78 students in Kindergarten who continue into 1st grade (unless noted otherwise).

The task points ranged from 1 point for not being able to perform the task at all, to three points for partially completing the task, to five points for successfully completing the task. For example, if the child did not have cardinality, s/he scored one point. If the child needed to recount in order to answer the cardinality question, s/he scored three points. If the child answered the cardinality question correctly and without recounting, s/he scored five points. The total for eight tasks on the test could be as low as eight points and as high as forty points. For both grades in Figures 2 and 3, test totals were listed in order from lowest to highest. Figures 2 and 3 below show the individual results of each child in each grade divided into the lower attaining group and the higher attaining group.

Figure 2: Individual Kindergarten test results divided into lower attaining group and higher attaining group

Lower Attaining Group								Higher Attaining Group					
14	16	18	20	22	24	26	28	30	32	34	36	38	40
				22	24	26	28	30	32	34	36	38	40
				22	24	26	28	30	32	34	36	38	40
				22	24	26	28	30	32	34	36	38	40
				22	24	26	28	30			36	38	40
				22	24	26	28	30			36	38	
					24	26	28	30			36	38	
					24	26	28	30			36	38	
						26	28					38	
						26	28						
						26	28						

Figure 3: Individual 1st grade test results divided into lower attaining group and higher attaining group

Lower Attaining Group							Higher Attaining Group		
24	26	28	30	32	34	36	38	40	40
	26	28	30	32	34	36	38	40	40
		28	30	32	34	36	38	40	40
		28	30	32	34	36	38	40	40
				32	34	36	38	40	40
				32	34	36	38	40	40
				32	34	36	38	40	
				32	34	36	38	40	
				32		36	38	40	
						36	38	40	
							38	40	
							38	40	
							38	40	
							38	40	
							38	40	
							38	40	
							38	40	

The scoring method used for this oral diagnostic test was arbitrary, thus the total scores listed in Figures 2 and 3 are arbitrary as well. It is acknowledged that another point system could have been used, which would have yielded another set of scores. However, these scores serve an important task: they identify which students performed successfully on the early number skills tasks, and which did not. These arbitrary points and scores are a way of organizing these particular data. When this test is used with other students, the data could be organized into more than two groups; rather, there might be three or four groups, depending on the spread of the scores. Assigning students to "lower attaining" and "higher attaining" groups serves to further identify these students, but only in order to target remedial skill work, not to permanently label them. In addition, students can be moved in and out of the groups, depending on their performance with remedial and other classroom work. There are benefits in being in attainment groups in that the work will be better scaffolded and directed to particular needs, but the groups should remain flexible, and it is anticipated that in a classroom situation, the work should not be limited or limiting.

Using the greatest percentage for each task, I created a composite picture of how most Kindergarteners and 1st graders in this study performed with early number skills.

Table 8: Greatest percentage for each task for Kindergarten and 1st grade

Task	**Kindergarten (n=78)**	**1st Grade (n=78)**
Subitization	46% (36 students could subitize 2 and 3)	60% (47 students could subitize all groups)
Rote Count (to 100)	47% (37 students)	92% (72 students)
Conservation of Number	40% (31 students were non-conservers)	58% (45 students were conservers)
One-to-one Correspondence	90% (71 students)	96% (75 students)
Organized Count	69% (54 students)	91% (71 students)
Cardinality	81% (63 students)	94% (73 students)
Hierarchical Inclusion	64% (50 students)	81% (63 students)
Part-whole Relationships	46% (36 students had no addend pairs)	82% (64 students had addend pairs for 5, 10)

What became immediately obvious was that the percentages illuminated which task was easiest for the most students in each grade, and which was most difficult. If ordered from highest to lowest percentage, the order of the tasks changed from the order on the oral diagnostic test, as can be seen from Tables 9 and 10.

Table 9: Kindergarten's original and revised order of tasks

Original order of tasks	**Rank in test**	**Revised order of tasks**	**Rank in test**
Subitization	1	One-to-one Correspondence	4
Rote Counting	2	Cardinality	6
Conservation of Number	3	Organized Counting	5
One-to-one Correspondence	4	Hierarchical Inclusion	7
Organized Counting	5	Rote Counting	2
Cardinality	6	Subitization	1
Hierarchical Inclusion	7	Part-whole Relationships	8
Part-whole Relationships	8	Conservation of Number	3

Table 10: 1st grade's original and revised order of tasks

Original order of tasks	Rank in test	Revised order of tasks	Rank in test
Subitization	1	One-to-one Correspondence	4
Rote Counting	2	Cardinality	6
Conservation of Number	3	Rote Counting	2
One-to-one Correspondence	4	Organized Counting	5
Organized Counting	5	Part-whole Relationships	8
Cardinality	6	Hierarchical Inclusion	7
Hierarchical Inclusion	7	Subitization	1
Part-whole Relationships	8	Conservation of Number	3

Interpretation of the Revised Order of Tasks

This change in order is noteworthy for this particular study. The original order on the test was developed because I had thought, due to both research and field experience, that the tasks were developmental in nature, i.e., that subitizing was learned by children first, and then rote counting was learned, and then conservation of number, et cetera. However, Table 8 brought to light the fact that some tasks were mastered by more children than others, indicating a different order of learning, or perhaps no order at all, considering the landscape model noted earlier in the literature review chapter. Thus, after organizing the data, I revised the order of the tasks for both the data analysis below and for the revised test (see Appendix 3), based on the highest percentages of the children tested.

This revised order shows that certain tasks are mastered by more children than others. For example, as can be seen in Table 9, in Kindergarten, more children are more successful with one-to-one correspondence than with subitization or conservation of number. For both grades, subitization and conservation of number are shown to be mastered by fewer children and therefore received lower percentages. However, part-whole relationships, which received a low percentage for Kindergarten, appears to be an easier task for 1st graders, and therefore received a higher percentage, as can be seen in Table 11.

Table 11: Comparison of Kindergarten and 1st grade's revised order of tasks to the original order on the test

Revised order of tasks for Kindergarten	Rank in test	Revised order of tasks for 1st grade	Rank in test
One-to-one Correspondence	4	One-to-one Correspondence	4
Cardinality	6	Cardinality	6
Organized Counting	5	Rote Counting	2
Hierarchical Inclusion	7	Organized Counting	5
Rote Counting	2	Part-whole Relationships	8
Subitization	1	Hierarchical Inclusion	7
Part-whole Relationships	8	Subitization	1
Conservation of number	3	Conservation of number	3

Certain tasks remained stable from grade to grade: one-to-one correspondence, cardinality, and conservation of number. The former two seem the easiest to master for both grades, whereas the latter skill seems the hardest to master.

Data Overview for Tasks Receiving One or Five Points

After this initial organization of the data, each student's early number skill task score at the end of Kindergarten was compared to his/her final standing at the end of 1st grade. What I was looking for was the "predictive power" of each skill: how well could the performance on a particular skill in Kindergarten predict the overall performance in 1st grade? If the performance on the skill in Kindergarten did not seem to predict whether the student would finish in the lower or higher attaining group in 1st grade, the skill was labeled "minimally predictive." If the performance on the skill in Kindergarten was able to somewhat predict placement in 1st grade, the skill was labeled "moderately predictive." If the performance on the skill in Kindergarten was able to strongly predict placement in 1st grade, then the skill was labeled "highly predictive."

Although the term "predictive power" is and will be used throughout this study, it is recognized that there are limitations to the use of this term. First, this oral diagnostic test, although used twice for each student, is still only one measure of assessment. In order to fully understand how a student can perform with early number skills, a teacher would need to use several measures of assessment, such as observations, interviews, running records, et cetera, over a period of time. The oral diagnostic test from this study creates a "snapshot" of a student's ability; it is a tool to be used alongside of other assessment tools, such as the ones listed above. Generalizations about the predictive power of any skill, or of the student's ability, would be limited if the oral diagnostic test would be were used only once, or without conjunction to other forms of assessments. When used more frequently (as it is for this study), however, or when used in conjunction to other forms of assessment, the clarity of the snapshot provided by this test increases.

Each of the 78 students was tracked for each skill from Kindergarten into 1st grade. The data charts (see Appendices 3, 4, 5, and 6) show that each student was numbered, and all of the student's points from the oral diagnostic test were recorded, along with the total points given for the test. This was done twice: once in Kindergarten, and again in 1st grade. The student's placement at the end of each grade—either in the lower attaining group or the higher attaining group—was also recorded. Thus, by following each student through Kindergarten into 1st grade, tallies could be created for each possible Kindergarten task score and 1st grade placement. Tables 10-34 show the results from these tallies for each task: they show how many students received each set of points (1, 3, or 5 points) and where these students finished in 1st grade (in the lower attaining group or the higher attaining group).

Predictive power was defined as follows: if the difference between the percentages of the children in the lower attaining group and the higher attaining group was from 50%-50% to 59%-41%, then the skill is labeled "minimally predictive." In other words, if, for example, there was a 41% likelihood that a student would finish in the lower attaining group by the end of 1st grade after receiving a particular score for a skill task in Kindergarten, as well as a 59% likelihood that another student would end up in the higher attaining group by the end of 1st grade after receiving the same score on the same task, the task was rated "minimally predictive." If the difference between the percentages for the lower attaining group and the higher attaining group was from 60%-40% to 69%-31%, then the skill is labeled "moderately predictive." If the difference between the percentages for the lower attaining group and the higher attaining group was from 70%-30% to 100%-0%, then the skill is labeled "highly predictive." A skill rated "highly predictive" meant that receiving a particular score in Kindergarten for a skill task strongly suggested a student's placement in 1st grade in either the lower attaining group or the higher attaining group.

Figure 4: Range of Predictive Power Scores

Minimally Predictive	Moderately Predictive	Highly Predictive
50%-50% to 59%-41%	60%-40% to 69%-31%	70%-30% to 100%-0%

The term "predictive" is not used here in the standard statistical sense. Since the data were gathered using a convenience sample, the ability to use probabilistic methods on the data is curtailed. Therefore, the further ability to predict in the empirical sense is curtailed as well. Instead, the term predictive is used to allow teachers to use the data to gain understanding of the student's ability in comparison to his/her peers, with the objective of an early identification of the need for remediation.

What follows below is an examination of student results for each individual skill. In the tables below, the column labeled "Skill level in Kindergarten" shows how many students were able to perform, or not perform, each task. The next two columns show how many of those students with those points finished either in the lower or higher attaining group by the end of 1st grade.

Thus, the predictive power of each skill can be seen by the comparison of the individual student's points at the end of Kindergarten versus the individual student's overall standing at the end of 1st grade. Did a poor performance in a skill in Kindergarten indicate a poor standing a year later in 1st grade? Conversely, did a strong performance in a skill in Kindergarten indicate a strong performance a year later in 1st grade?

For example, as can be seen in Appendix 3, student #3 scored five points for one-to-one correspondence, showing s/he successfully performed this task. This student finished 1st grade in the lowest attaining group. Student #4 also scored five points for one-to-one correspondence but finished in the higher attaining group by the end of 1st grade. Student #7 scored one point for one-to-one correspondence and finished in the lower attaining group by the end of 1st grade. All students were examined the same way, and the results were tallied and computed to find the percentages to determine the predictive power of each skill.

For all of the skills, each student received either 1 point, indicating s/he was not able to perform this skill successfully, or 5 points, indicating s/he was able to perform this skill successfully. These scores were analyzed first (see Tables 12-20). However, several of the skills have an intermediate level, for 3 points, indicating that the student has performed the skill partly but not entirely successfully. Tables 21-26 examine the intermediate point results.

One-to-one Correspondence

Table 12: One-to-one correspondence results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No one-to-one correspondence	6 students (86%)	1 student (14%)
Yes one-to-one correspondence	32 students (45%)	39 students (55%)

One-to-one correspondence has no intermediate points. The student can either perform this skill successfully—giving each object one and only one number—or s/he cannot. Few students could not perform one-to-one correspondence successfully by the end of Kindergarten: 71 out of 78 students performed this skill successfully (91%) (see Table 12). Only seven students were not able to perform one-to-one correspondence (9%). However, out of those seven students, six of them ended up in the lower attaining group of 1st grade. Conversely, having one-to-one correspondence did not consistently predict success in 1st grade. The students who could perform this skill successfully ended up relatively equally spread in both the bottom and top halves of 1st grade, with a slight leaning towards the higher attaining group.

The conclusions for one-to-one correspondence are:

- With *no* one-to-one correspondence, a student will more likely end up in the *lower attaining group* in 1st grade.

- However, *with* one-to-one correspondence, a student is somewhat more likely to end up in the *higher attaining group* in 1st grade.

Therefore, one-to-one correspondence is a **highly** predictive skill if **missing**, and a **minimally** predictive early number skill if **present**.

Cardinality

Table 13: Cardinality results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No cardinality	0 students (0%)	2 students (100%)
Yes cardinality	29 students (46%)	34 students (54%)

Cardinality does have an intermediate level that is not apparent in Table 13. When a student needs to recount in order to answer the question, "How many do you have," then that response earns the student three points instead of the five points s/he would receive for an immediate and correct response (see Table 21). In terms of the responses in Table 13, almost all students performed cardinality successfully (97%), with only two students unable to perform the task successfully (3%). The successfully performing students ended up relatively equally spread in the bottom and top halves in 1st grade, with a slightly larger percentage of students in the higher attaining group.

The conclusions for cardinality are:

- Only two students did *not* have cardinality, but they still ended up *in the higher attaining group* in 1st grade. As there are only two students in this category, the results might be considered as outliers.

- Students who *had* cardinality were somewhat more likely to end up *in the higher attaining group* in 1st grade.

Therefore, cardinality is a **minimally** predictive early number skill.

Organized Counting

Table 14: Organized counting results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No system of organization	15 students (63%)	9 students (37%)
Yes system of organization	23 students (43%)	31 students (57%)

Organized counting, like one-to-one correspondence, has no intermediate level. A student either has a successful system of organizing objects while counting in order to know what has been counted and what has not, or s/he does not have such a system and instead counts randomly, often miscounting because of the lack of organization. Unlike one-to-one correspondence, however, organized counting has many more students who could not successfully do this task. Out of the 78 students represented in Table 14, 24 of them (31%) did not have organized counting. Out of that group of 24, most finished in the lower attaining group in 1st grade. Those who did have a system of organization ended up relatively equally spread in the lower and higher attaining groups in 1st grade, with a slight increase in the percentage of students in the higher attaining group.

The conclusions for organized counting are:

- With *no* system of organization for counting, students were more likely to finish *in the lower attaining group* of 1st grade.

- However, *with* a system of organization for counting, students were somewhat more likely to finish *in the higher attaining group* by the end of 1st grade.

Therefore, organized counting is a **moderately** predictive early number skill if **missing**, and a **minimally** predictive skill if **present**.

Hierarchical Inclusion

Table 15: Hierarchical inclusion results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No hierarchical inclusion	18 students (64%)	10 students (36%)
Yes hierarchical inclusion	20 students (40%)	30 students (60%)

Hierarchical inclusion has no intermediate level. The student either knows that smaller numbers nest inside larger numbers, so that if she has twenty objects, s/he also has nineteen objects—or the student thinks that s/he has twenty and only twenty objects. In terms of Table 15, more students had hierarchical inclusion than not: 50 students (64%) to 28 students (36%). Having hierarchical inclusion seemed to help student finish in the higher attaining group by the end of 1st grade.

The conclusions for hierarchical inclusion are:

- With *no* hierarchical inclusion, a student is more likely to finish *in the lower attaining group* by the end of 1st grade.

- *With* hierarchical inclusion, a student is more likely to finish *in the higher attaining group* by the end of 1st grade.

Therefore, hierarchical inclusion is a **moderately** predictive early number skill if **present or missing**.

Rote Counting

Table 16: Rote counting results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Counts to **20**	5 students (83%)	1 student (17%)
Counts to **100 or up**	8 students (22%)	29 students (78%)

Rote counting has an intermediate level of counting from 21 to 99, for which a student would receive three points (see Table 20). In terms of Table

16, most (37 students—86%) could count to at least 100. Counting that high enabled 78% of those students to finish in the higher attaining group by the end of 1st grade. Conversely, only being able to count to 20 (or less) seemed to cause those students to finish in the lower attaining group by the end of 1st grade.

The conclusions for rote counting are:

- *Not* being able to count past 20 means a student is most likely to finish *in the lower attaining group* by the end of 1st grade.

- *Being* able to count to 100 or above means a student is most likely to finish *in the higher attaining group* by the end of 1st grade.

Therefore, rote counting is a **highly** predictive early number skill if **missing or present.**

Subitization

Table 17: Subitization results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No subitizing any amount	7 students (100%)	0 students (0%)
Yes subitizing all groups (2, 3, 4, 5)	12 students (34%)	23 students (66%)

Subitization, like cardinality, does have an intermediate level. When a student can subitize groups of two and three successfully, but not groups of four and five, then s/he receives three points (see Table 23). In terms of the responses in Table 17, most of the students (83%) could subitize all groups. Of the students who could not subitize (17%), all of them finished in the lower attaining group in 1st grade. Of those who could successfully subitize all groups, many although not most finished in the higher attaining group in 1st grade.

The conclusions for subitization are:

- With *no* ability to subitize, a student is most likely to finish *in the lower attaining group* by the end of 1st grade.

- However, *with* the ability to subitize all groups, a student is likely to finish *in the higher attaining group* by the end of 1st grade.

Therefore, subitization is a **highly** predictive early number skill if **missing**, and a **moderately** predictive skill if **present**.

Part-whole relationships

Table 18: Part-whole relationships results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No addend pairs for 5 or 10	27 students (75%)	9 students (25%)
Yes addend pairs for **both** 5 and 10	5 students (18%)	23 students (82%)

Part-whole relationships does have an intermediate level. When a student could make an addend pair for five or ten but not both, then s/he received three points (see Table 24). In terms of Table 18, being able to make pairs for both five and ten clearly helped students finish in the higher attaining group by the end of 1st grade, whereas not being able to make any pairs caused students to finish in the lower attaining group by the end of 1st grade.

The conclusions for part-whole relationships are:

- *Without* part-whole relationships, a student is most likely to finish *in the lower attaining group* of 1st grade.
- *With* part-whole relationships, a student is most likely to finish *in the higher attaining group* of 1st grade.

Therefore, part-whole relationships is a **highly** predictive early number skill if **missing or present**.

Conservation of Number

Table 19: Conservation of number results for 1 and 5 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Non-conserver	23 students (74%)	8 students (26%)
Conserver	10 students (37%)	17 students (63%)

Conservation of number does have an intermediate level. When asked if the newly rearranged rows of counters have the same amount of counters in each row or if one row has more or less counters than the other, some students cannot say outright that the rows are the same in number or that they are not the same in number. Instead, these students need to count each row before answering. These students are called "transitional conservers" and receive three points (see Table 25). In terms of Table 19, the students are close to being equally divided between the high and low groups, between being conservers (27 students—47%) or non-conservers (31 students—53%). However, being able to conserve helped 17 out of 27 students finish in the higher attaining group by the end of 1st grade, and not being able to conserve caused a large group—23 out of 31—to finish in the lower attaining group by the end of 1st grade.

The conclusions for conservation of number are:

- *Without* conservation, a student is most likely to finish *in the lower attaining group* by the end of 1st grade.

- *With* conservation, a student is more likely to finish *in the higher attaining group* by the end of 1st grade.

Therefore, conservation is a **highly** predictive early number skill if **missing** and a **moderately** predictive skill if **present**.

To sum up the above analysis, each early number skill can be described as minimally, moderately, or highly predictive of 1st grade placement. Some skills can also be described in two ways, depending on whether the skill is present or missing, as can be seen in Table 20.

Table 20: Overview of each skill's predictive power for one point (cannot perform skill) or five points (successfully performs skill)

Skill	**Minimal**	**Moderate**	**High**
One-to-one	5		1
Cardinality	1 and 5		
Organized	5	1	
Hier. Inclusion		1 and 5	
Rote counting			1 and 5
Subitization		5	1
Part-whole			1 and 5
Conservation		5	1

However, this is not the complete picture as several of these skills have an intermediate level that need to be analyzed.

Data Overview for Tasks Receiving Three Points

The early number skills with intermediate levels are as follows: cardinality, rote counting, subitization, part-whole relationships, and conservation of number. As with the skill tables above, each student's individual skill task points in Kindergarten was compared to his/her final placement in 1st grade. These data can tell us more about the role of the intermediate level. Is receiving three points on a skill task more like receiving one point or five points? In other words, is succeeding on the intermediate level more like completely having the skill or more like not having the skill at all? Some intermediate level results show that having partial success with that skill is close to total success, whereas other intermediate level results show that having partial success is more like not having the skill at all.

Cardinality

Table 21: Cardinality results for 3 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Recounts for cardinality	9 students (69%)	4 students (31%)

Having to *recount* for cardinality means that a student will most likely finish in the lower attaining group by the end of 1st grade (see Table 21). Therefore, recounting is more like *not* having cardinality (1 point) than having cardinality (5 points). The intermediate score for cardinality is **moderately** predictive.

Rote Counting

Table 22: Rote counting results for 3 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Counts from **21 to 99**	25 students (71%)	10 students (29%)

Not being able to count to 100 means that a student will most likely finish in the lower attaining group by the end of 1st grade (see Table 22). Therefore, being able to rote count from 1 to as high as 99 but not all the way to 100 is more like *not* being able to count past 20 (1 point) than being able to count to 100 or more (5 points). The intermediate level for rote counting is **highly** predictive.

Subitization

Table 23: Subitization results for 3 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Subitizes **groups of 2 and 3** but not 4 and 5	19 students (53%)	17 students (47%)

Being able to subitize groups of *2 and 3* but not 4 and 5 means that a student will likely finish in the lower attaining group by the end of 1st grade (see Table 23). Therefore, being able to subitize groups of 2 and 3 but not 4 and 5 is more like *not* being able to subitize any groups (1 point) than being able to subitize all groups (5 points). The intermediate level for subitization is **minimally** predictive.

Part-whole Relationships

Table 24: Part-whole relationships results for 3 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Makes an **addend pair for either 5 or 10** but not both	6 students (43%)	8 students (57%)

Being able to make an addend pair for *only* 5 or 10 rather than both means that a student will likely finish in the higher attaining group by the end of 1st grade (see Table 24). Therefore, being able to partially complete the part-whole relationships task is more like being able to fully complete the task (5 points) than not being able to complete it at all (1 point). The intermediate level for part-whole relationships is **minimally** predictive.

Conservation of Number

Table 25: Conservation of number results for 3 points

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Transitional conserver	6 students (30%)	14 students (70%)

Being a *transitional* conserver means that a student will most likely finish in the higher attaining group by the end of 1st grade (see Table 25). Therefore, having to count in order to perform the conservation task successfully is more like *having* conservation of number (5 points) than not having conservation (1 point). The intermediate level for conservation of number is **highly** predictive.

To sum up the above analysis, several early number skills have an intermediate level that can be described as minimally, moderately, or highly predictive of 1st grade placement (see Table 26). Some of these intermediate levels are more in line with scoring one point; some of these intermediate levels are more in line with scoring five points. In other words, sometimes being able to perform only part of the skill is close to not being able to perform it at all, and sometimes being able to perform only part of the skill is close to being able to perform it completely successfully.

Table 26: Overview of each skill's predictive power for three points (can partially perform skill)

Skill	**Minimal**	**Moderate**	**High**
Cardinality		Recounts	
Rote counting			Counts 21-99
Subitization	Subitizes 2 and 3		
Part-whole	Pair for 5 or 10		
Conservation			Transitional

Interpretation of the Predictive Power of Early Number Skills

Combining the results of Tables 20 and 26 into Table 27 below completes the picture of early number skills' predictive power.

Table 27: Overview of each skill's predictive power for one (cannot perform skill), three (can partially perform skill), and five points (successfully performs skill)

Skill	**Minimal**	**Moderate**	**High**
One-to-one	5		1
Cardinality	1, 5	3	
Organized	5	1	
Hier. Inclusion		1, 5	
Rote counting			1, 3, 5
Subitization	3	5	1
Part-whole	3		1, 5
Conservation		5	1, 3

This table shows that the early number skills that most Kindergarteners can perform successfully tend to have minimal to moderate predictive power. Namely, one-to-one correspondence, cardinality, and organized counting all fall under the minimal and moderate columns, with the exception of one-to-one correspondence when it is missing. This table also shows that the last three skills on the table—rote counting, conservation of number, and part-whole relationships—tend to have moderate to high predictive power, with the exception of part-whole relationships when it can be done only partially. Subitization and hierarchical inclusion fall in the moderate column for predictive power.

Table 27 indicates that if most Kindergarteners can perform an early number skill successfully, then not being able to do it has adverse consequences. For example, one-to-one correspondence is a skill that most Kindergarteners (90% of 78 students) perform competently. Doing so has minimal predictive power; a student who performs one-to-one correspondence successfully could finish in either the lower or the higher attaining group by the end of 1st grade. However, not being able to perform one-to-one correspondence competently has high predictive power; a student who did not have one-to-one correspondence almost always finished in the lower attaining group of 1st grade.

Organized counting tells a different story. Being able to perform successfully with this skill is common to most Kindergarteners; 69% of 78 Kindergarteners performed it successfully. Therefore its predictive power is minimal when the skill is present. When the skill is missing, however, the predictive power does not jump to high as with one-to-one correspondence. Rather it moves to moderate. It seems that not having organized counting is not as detrimental as not having one-to-one correspondence.

Conversely, if most Kindergarteners cannot fully perform a particular early number skill, then doing it successfully also has consequences, most often beneficial. For example, with part-whole relationships, being able to make addend pairs for both five and ten was beyond the reach of close to half (46% of 78 students) of the Kindergarteners. Being able to do so, therefore, proved to be both beneficial and predictive, as 82% (out of 78 students) of those who could make addend pairs for both five and ten finished in the higher attaining group by the end of 1st grade.

Rote counting is considered highly predictive no matter how many points a student receives. The data indicate that if a student can count to 99 or less, s/he will most likely finish in the lower attaining group by the end of

1st grade. Conversely, if a student can count to 100 or higher, s/he will most likely finish in the higher attaining group by the end of 1st grade. These trends again follow the idea stated above: if most Kindergarteners cannot do a particular skill, namely counting to 100 or higher, being able to do so yields beneficial consequences. Only 37 (47%) of the 78 Kindergarteners could count to 100 or higher. In 1st grade, 29 students of those same students (78%) finished in the higher attaining group by the end of 1st grade.

Data Overview of Combined Skills

What happens when the results of early number skills are combined? Does that change the original predictive power of the skills when each was analyzed individually? Combining the results of two skills shows to have some impact on their predictive power. In the following tables, the "Skill level in Kindergarten" column lists the four possible combinations of the skills: not having either of them, having one but not the other, the reverse of the latter, and having both of them. The next two columns, "Lower attaining group in 1st Grade" and "Higher attaining group in 1st Grade," record how the students finished in 1st grade, depending on which combination of skill level they had.

Table 28: Combined results for Rote Counting and Conservation

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Counts **1-99** and **No** conservation	16 students (89%)	2 students (11%)
Counts **1-99** and **Yes** conservation (or transitional)	14 students (61%)	9 students (39%)
Counts **100 up** and **No** conservation	8 students (62%)	5 students (38%)
Counts **100 up** and **Yes** conservation (or transitional)	1 student (4%)	23 students (96%)

When combining the results of rote counting and conservation of number, the percentages of the high and low points increase (see Table 28). When compared to the percentages of just rote counting to 20 or to 99, or just non-conserving (see Table 29), the combination of rote counting to 99 and non-conserving is higher.

Table 29: One point (cannot perform skill) and three points (can partially perform skill) results for rote counting and non-conservation

Skill level in Kindergarten (n=78)	Lower attaining group in 1st grade (n=78)	Higher attaining group in 1st grade (n=78)
Counts 1-20	5 students (83%)	1 student (17%)
Counts 21-99	25 students (71%)	10 students (29%)
Non-conserver	23 students (74%)	8 students (26%)
Counts 1-99 and Non-conserver	**16 students (89%)**	**2 students (11%)**

A similar increase happens when comparing just high rote counting (to 100 or higher) or just conserving to the combination of high rote counting and conserving. The predictive power of these two early number skills increases when combined.

The conclusions for combining rote counting and conservation are:

- *Counting to 99 or below* ***and*** *not having conservation* means a student will most likely finish *in the lower attaining group* in 1st grade.
- *Counting to or above 100* ***and*** *having conservation* means a student will most likely finish *in the higher attaining group* in 1st grade.
- *Counting to 99 or below* ***or*** not having conservation means a student will likely finish *in the lower attaining group* in 1st grade.

Therefore, not being able to count to 100 and not having conservation is a **highly** predictive combination, as is the opposite: being able to count to 100 or beyond and having conservation. Not being able to count to 100, and/or not having conservation drop the predictive power of the combination down to **moderate**.

Rote counting and conservation of number are both highly predictive when examined singly and become even more so when combined. What happens when combining two early number skills that are minimally predictive when examined individually? Cardinality is minimally predictive whether missing or present. One-to-one correspondence is also minimally predictive, but only when present; it is highly predictive when missing. When these two skills are combined, one-to-one correspondence's highly predictive power impacts on cardinality, but the inverse does not seem to be as true (see Table 30). For example, not having one-to-one correspondence keeps most students in the lower attaining group in 1st grade, whether the students have cardinality or not. Once students have one-to-one correspondence, the addition of cardinality seems to help students finish in the higher attaining group in 1st grade, but the percentage increase is small.

Table 30: Combined results for one-to-one correspondence and cardinality

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No one-to-one correspondence and **no** cardinality (or recounts)	2 students (100%)	0 students (0%)
No one-to-one correspondence and **yes** cardinality	4 students (80%)	1 student (20%)
Yes one-to-one correspondence and **no** cardinality (or recounts)	7 students (54%)	6 students (46%)
Yes one-to-one correspondence and **yes** cardinality	25 students (43%)	33 students (57%)

Therefore, the conclusions for the combination of one-to-one correspondence and cardinality are:

- With *no* one-to-one correspondence and *no* cardinality, a student is most likely to finish *in the lower attaining group* in 1st grade (although with only 2 students in this category, the result may be an outlier).

- With *no* one-to-one correspondence but *with* the ability to either recount for cardinality or with complete cardinality, a student is still most like to finish *in the lower attaining group* in 1st grade.

- *With* one-to-one correspondence and *some, all, or no* cardinality, a student could potentially end up *in either the higher or lower attaining group*.

As shown earlier, one-to-one correspondence is a **highly** predictive skill if **missing** (as can also be seen in Table 12), but a **minimally** predictive skill if **present**, as is cardinality.

One-to-one correspondence and cardinality are for the most part minimally predictive skills. What happens for two skills that are deemed moderately predictive? Hierarchical inclusion is moderately predictive if missing or present. Conservation of number falls into two columns: it is highly predictive for non-conservers and for transitional conservers, but for conservers it is moderately predictive.

Table 31: Combined results for hierarchical inclusion and conservation

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No hierarchical inclusion and **no** conservation	12 students (92%)	1 student (8%)
No hierarchical inclusion and **yes** conservation (or transitional)	6 students (40%)	9 students (60%)
Yes hierarchical inclusion and **no** conservation	10 students (67%)	5 students (33%)
Yes hierarchical inclusion and **yes** conservation (or transitional)	10 students (29%)	25 students (71%)

When examining the results of the combination of these two skills, it seems that being able to conserve, or not, impacts hierarchical inclusion to a much greater extent than the latter impacts the former. For example, when the skill results are examined individually (see Table 32), not having hierarchical inclusion causes students to finish in the lower attaining group: 64% in the lower attaining group; 36% in the higher attaining group. However, as can be seen in Table 31, the addition of successfully performing conservation flips that ratio so that more students finish in the higher attaining group rather than below: 40% lower attaining group; 60% higher attaining group. The lack of conservation, combined with the lack of hierarchical inclusion also increased the percentage of those in the lower attaining group, more than when viewed individually. In addition, the reverse is true: being able to successfully perform both skills increased the percentage of those in the higher attaining group, more than when viewed individually.

Table 32: Comparative scores for hierarchical inclusion and conservation

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No hierarchical inclusion	18 students (64%)	10 students (36%)
Non-conserver	23 students (74%)	8 students (26%)
No hierarchical inclusion and no conservation	**12 students (92%)**	**1 student (8%)**
Yes hierarchical inclusion	20 students (40%)	30 students (60%)
Conserver (or transitional)	16 students (34%)	31 students (66%)
Yes hierarchical inclusion and conserver (or transitional)	**10 students (29%)**	**25 students (71%)**

The conclusions for the combination of hierarchical inclusion and conservation are:

- With *no* hierarchical inclusion and *no* conservation, a student is most likely to finish *in the lower attaining group* in 1st grade.
- With *no* hierarchical inclusion but *with* conservation, a student is likely to finish *in the higher attaining group* in 1st grade.
- Similarly, *with* hierarchical inclusion but *no* conservation, a student is likely to finish *in the lower attaining group* in 1st grade.

- With *both* skills, a student is most likely to finish *in the higher attaining group* in 1st grade.

Therefore, the combination of **not having both** skills or of **having both** skills is **highly** predictive. When combined, however, it seems that conservation is the skill that more clearly determines placement in 1st grade, rather than hierarchical inclusion.

When combining a minimally predictive skill with a highly predictive skill, the former seems to have little impact on the latter. For example, the combination of cardinality (a minimally predictive skill) with rote counting (a highly predictive skill) shows that cardinality did not change the percentages of rote counting, as can be seen in Table 33.

Table 33: Combined results for cardinality and rote counting

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No cardinality (or recounts) and **counts 1-99**	8 students (73%)	3 students (27%)
No cardinality (or recounts) and **counts to 100 up**	1 student (25%)	3 students (75%)
Yes cardinality and **counts 1-99**	22 students (73%)	8 students (27%)
Yes cardinality and **counts 100 up**	7 students (21%)	26 students (79%)

The results for no cardinality and low counting are virtually the same as the results for yes cardinality and low counting (see Table 33). In other words, the addition of the ability to know how many are in the group, even after recounting, did not boost more students to the higher attaining group. In fact, the addition of cardinality did not change the percentages of rote counting in either direction, as can be seen by Table 34.

Table 34: Comparison of just rote counting to cardinality and rote counting

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
Counts 1-99	30 students **(73%)**	3 students **(27%)**
Yes cardinality and counts 1-99	22 students **(73%)**	8 students **(27%)**
Counts to 100 up	8 students **(22%)**	29 students **(78%)**
Yes cardinality and counts 100 up	7 students **(21%)**	26 students **(79%)**

The conclusions for the combination of cardinality and rote counting are:

- With *no* cardinality and *low* counting, a student is most likely to finish *in the lower attaining group* in 1st grade.
- With *no* cardinality but *with* high counting, a student is likely to finish *in the higher attaining group* in 1st grade.
- *With* cardinality but with *low* counting, a student is likely to finish *in the lower attaining group* in 1st grade.

- With *both* skills, a student is most likely to finish *in the higher attaining group* in 1st grade.

Therefore, the combination of **not having both** skills or of **having both** skills is **highly** predictive. When combined, however, it seems that rote counting is the skill that more clearly determines placement in 1st grade, rather than cardinality.

Subitzing and part-whole relationships are two skills that are spread throughout the predictive powers; neither is fully predictive at any strength. What happens when they are combined? It seems that when combined (see Table 35), the predictive powers are still spread from minimal to moderate to high.

Table 35: Combined results for subitization and part-whole relationships

Skill level in Kindergarten (n=78)	Lower attaining group in 1st Grade (n=78)	Higher attaining group in 1st Grade (n=78)
No subitizing (or subitizes 2 and 3) and **cannot** make pairs for 5 or 10	20 students (83%)	4 students (17%)
No subitizing (or subitizes 2 and 3) and **makes pairs** for either 5 or 10 or both	6 students (32%)	13 students (68%)
Yes subitizes all groups and **cannot** make pairs for 5 or 10	7 students (58%)	5 students (42%)
Yes subitizes all groups and **makes pairs** for either 5 or 10 or both	5 students (22%)	18 students (78%)

In this combination, part-whole relationships becomes the dominant partner. Although not having as well as having both skills are both highly predictive combinations, when part-whole relationships is missing, the student is likely to finish in the lower attaining group in 1st grade even when able to subitize all groups (see Table 35). The opposite is also true: when part-whole relationships is present and subitization is missing, the student is more likely to finish in the higher attaining group in 1st grade.

The conclusions for the combination of subitization and part-whole relationships are:

- With *no* subitization and *no* part-whole relationships, a student is most likely to finish *in the lower attaining group* in 1st grade.

- With *no* subitization but *with* part-whole relationships, a student is likely to finish *in the higher attaining group* in 1st grade.

- *With* subitization but with *no* part-whole relationships, a student is likely to finish *in the lower attaining group* in 1st grade.

- With *both* skills, a student is most likely to finish *in the higher attaining group* in 1st grade.

Therefore, the combination of **not having both** skills or of **having both** skills is **highly** predictive. When combined, however, it seems that part-whole relationships is the skill that more clearly determines placement in 1st grade, rather than subitization.

Interpretation of Combined Skills

Table 36: Predictive power results for combined skills

Combined Skills	Minimally Predictive	Moderately Predictive	Highly Predictive
Rote counting/ Conservation		Counts 1-99 and conserves; Counts to 100 but does not conserve	Counts 1-99 and conserves; Counts to 100 and conserves
One-to-one/ Cardinality	Has one-to-one but no cardinality; Has one-to-one and cardinality		No one-to-one and no cardinality; No one-to-one but has cardinality
Hierarchical inclusion/ Conservation		Does not have hier. inc. but can conserve; Has hier. inc. but cannot conserve	Does not have hier. inc. and cannot conserve; Does have hier. inc. and can conserve
Cardinality/ Rote counting			Every combination
Subitization/ Part-whole relationships	Can subitize but does not have part-whole pairs	Cannot subitize but does have part-whole pairs	Cannot subitize and has no part-whole; Can subitize and does have part-whole pairs

Analyzing combined skills in Table 36 shows that there are more ways to predict how a Kindergartener might perform in 1st grade than gauging a solitary skill result in Kindergarten against the total result in 1st grade. When combining skills, certain skills seem to dominate, bringing other skills up or down. A highly predictive skill appears to dominate the results of a minimally predictive skill, increasing the chances that a student will finish in the bottom or higher attaining group by the end of 1^{st} grade, such as the combination of cardinality and rote counting. A minimally predictive skill does not seem to offer enough counter balance to off set the highly predictive skill. In addition, moderately predictive skills seem to increase their predictive powers when combined, such as hierarchical inclusion and conservation.

Why might this happen? In order to create these tables, the results of each of the 78 Kindergarten students were examined, not for one skill but for two skills. Each student's end result in 1^{st} grade was also recorded. Thus, trends emerged: a student who performed poorly on one skill and poorly on a second skill seemed to increase his/her chance of finishing it the lower attaining group in 1^{st} grade. The opposite was also true: if a student performed well on one skill and also on another, his/her chance of finishing in the higher attaining group in 1^{st} grade seemed to increase.

These tables also show how powerfully influential some skills can be in terms of propelling a student towards success or failure. Both rote counting and one-to-one correspondence seem to be dominant skills. With rote counting, both counting to 99 or lower and counting to 100 or higher propel students either to the bottom or the higher attaining group in 1st grade, no matter what skill with which it is paired. One-to-one correspondence also propels students to the lower attaining group in 1st grade, but only when it is missing. Students with one-to-one correspondence do not seem to be affected by its presence in terms of where they finish at the end of 1st grade.

Student Examples

Examining the test results of individual Kindergarteners and comparing them to how the students finish in 1st grade reveals further the predictive power of certain early number skills. For example, students #3 and #5 from Cohort 1 had the following scores in Kindergarten (see Table 37):

Table 37: Student comparison with early number skill scores in Kindergarten and resulting placement in 1st grade

Early number skill	**Student #3 from Cohort 1**	**Student #5 from Cohort 1**
One-to-one correspondence	5	5
Cardinality	5	1
Organized counting	5	1
Hierarchical Inclusion	1	1
Rote Counting	1	5
Subitization	3	5
Part-whole relationships	1	1
Conservation of number	1	3
Total score from test in Kindergarten	22	22
Placement at the end of Kindergarten	Lower attaining group	Lower attaining group
Placement at the end of 1st grade	Lower attaining group	Higher attaining group

Student #3 performed well with the first three tasks, but poorly from then on (see Table 37). This student scored in the lower attaining group in Kindergarten. By the end of 1st grade, this student was still in the lower attaining group, as could have been predicted by the low results in rote counting, conservation of number, and part-whole relationships. The high results this student had in Kindergarten were all of minimal predictive power, meaning that receiving high points in certain skills was not enough to secure a higher spot in 1st grade.

Student #5, however, scored poorly in two of the minimally predictive skills: cardinality and organized counting (see Table 37). Those low results mattered less than the high score in a highly predictive skill combination: rote counting and conservation. As can be seen in Table 28, the combination of being able to count to 100 (receiving 5 points) and being a transitional conserver (receiving 3 points) becomes a highly predictive skill pair. Thus, student #5 was in the lower attaining group in Kindergarten, but jumped to the higher attaining group in 1st grade. In conclusion, these two students received the same total results at the end of Kindergarten and finished in the same placement, but because the predictive power of the skills varies, each student ended up in completely different places by the end of 1st grade. Thus, the overall score on the test does not foretell placement in 1st grade, but rather the skill scores do.

Two other students offer a similar kind of comparison (see Table 38):

Table 38: Student comparison with early number skill levels in Kindergarten and resulting placement in 1st grade

Early number skill	**Student #32 from Cohort 1**	**Student #38 from Cohort 1**
One-to-one correspondence	5	5
Cardinality	5	5
Organized counting	5	5
Hierarchical Inclusion	5	5
Rote Counting	3	3
Subitization	3	5
Part-whole relationships	1	1
Conservation of number	3	1
Total score from test in Kindergarten	30	30
Placement at the end of Kindergarten	Higher attaining group	Higher attaining group
Placement at the end of 1st grade	Lower attaining group	Lower attaining group

These two students in Table 38 received the same amount of points in Kindergarten and finished the year in the higher attaining group of the sample. Yet, by the end of 1st grade, both students finished in the lower attaining group of the sample. Could this movement have been predicted from their test results? Both students received five points for the first four tasks, which means they were successful in performing these tasks. However, all four of these tasks are either minimally or moderately predictive, meaning that one might not be able to predict how these students will finish in 1st grade from these results. But both students received lower points for rote counting, part-whole relationships, and conservation of number, which are all highly predictive. Performing poorly with these tasks in Kindergarten tends to predict a poor placement by the end of 1st grade. These two students finished in the lower attaining group by the end of 1st grade quite possibly because they could not count to 100, they could not make addend pairs for both five and ten, and they could not successfully conserve number.

In addition, the combination of subitization and part-whole relationships seems to have an impact for both students. Student #32 received 3 points for subitization and 1 point for part-whole relationships. That combination is highly predictive (see Table 35), and indeed the student finishes in the lower attaining group of 1st grade. Student #38 received 5 points for subitization and 1 point for part-whole relationships, which is a minimally predictive pair, but still tends to cause students to finish in the lower attaining group in 1st grade, which is what this student did.

However, the predictive power of skills can sometimes be misleading, as can be seen from Tables 39 and 40 below.

Table 39: Student comparison with early number skill scores in Kindergarten and resulting placement in 1st grade

Early number skill	**Student #26 from Cohort 2**	**Student #34 from Cohort 2**
One-to-one correspondence	5	5
Cardinality	5	5
Organized counting	5	1
Hierarchical Inclusion	1	5
Rote Counting	5	5
Subitization	3	3
Part-whole relationships	1	3
Conservation of number	1	1
Total score from test in Kindergarten	26	28
Placement at the end of Kindergarten	Lower attaining group	Lower attaining group
Placement at the end of 1st grade	Higher attaining group	Higher attaining group

Both students in Table 39 received 5 points for rote counting and 1 point for conservation of number, meaning they could count to 100 but could not conserve number. According to Table 28, this is a moderately predictive pair and should have caused these students to finish in the lower attaining group in 1st grade. Yet they did not. What caused them to rise to the top of the class in 1st grade? Perhaps because the combination of skills is only moderately predictive, we need to look elsewhere for possible clues. Being able to count to 100 alone is considered highly predictive; perhaps that skill is enough to help these two students succeed in 1st grade.

However, this idea is confounded by the following two students:

Table 40: Student comparison with early number skill scores in Kindergarten and resulting placement in 1st grade

Early number skill	Student #36 from Cohort 2	Student #46 from Cohort 1
One-to-one correspondence	5	5
Cardinality	5	5
Organized counting	5	5
Hierarchical Inclusion	5	1
Rote Counting	5	5
Subitization	3	5
Part-whole relationships	5	5
Conservation of number	1	1
Total score from test in Kindergarten	34	32
Placement at the end of Kindergarten	Higher attaining group	Higher attaining group
Placement at the end of 1st grade	Lower attaining group	Lower attaining group

Both students in Table 40 finished Kindergarten in the higher attaining group but ended 1st grade in the lower attaining group. What might have caused this movement? Like the students in Table 39, these students received 5 points for rote counting and 1 point for conservation of number. Again, this is a moderately predictive skill pair, which tends to cause students to finish in the lower attaining group in 1st grade, as these students did. However, the combination of subitization and part-whole relationships is highly predictive if both skills are present, which they are for student #46, and tend to cause students to finish in the higher attaining group in 1st grade. Yet this student did not finish in the top but rather the lower attaining group in 1st grade. Evidently, for these two students, high rote counting was not enough to sustain finishing in the higher attaining group in 1st grade, nor were being successful with part-whole relationships. Perhaps with these students, not being able to conserve was enough to cause them to drop to the lower attaining group in 1st grade, as conservation of number is highly predictive when missing.

Perhaps, however, there is another reason for the very different results of the four students above, and that is the vagaries of data analysis. These predictive powers are based on percentages of a sample. They are not foolproof, nor is the oral diagnostic test foolproof, nor is the tester foolproof. There are limitations to this method, and they will be discussed in further detail in the next chapter.

Summary

In conclusion, the data analysis shows that each early number skill has predictive power. This predictive power serves to foretell what direction the student might take in 1^{st} grade. The predictive power ranges from minimal to moderate to high, and can increase when combined with other skills. Student examples showed this movement; students finished in either the top or the lower attaining group in 1^{st} grade depending on their scores on the skills. Student examples also showed that this system can be either limited or incorrect. The implications of this analysis will be discussed in the next chapter in relation to the research examined in the literature review chapter.

CHAPTER FIVE
DISCUSSION

Introduction

In the data analysis chapter, we have seen what the benchmarks for early number skills are for Kindergarteners and for 1st graders (see Lists 1 and 2). We have also seen what the predictive power of each individual skill is (see Table 27), and how that predictive power might change when one skill is combined with another skill (see Table 36). We have seen that the predictive power can indicate how a student might perform in 1st grade, based on his/her individual skill score in Kindergarten (see Tables 12-25). Performing well—or poorly—on a particular skill in Kindergarten can mean the difference between finishing in the higher or lower attaining group by the end of 1st grade. This information answered the second and third of the research questions posed in chapter one, the introduction: "According to the oral diagnostic tool used for this study, how do most exiting Kindergarteners and 1st graders perform with these early number skills? Are there skills that can predict how Kindergarten students will do in 1st grade?" What needs to be answered still is how this information from the data analysis compares to what researchers have noted about these skills, and how this information compares to the rest of the original research questions posed in the introduction.

In this chapter the data analysis will be discussed in light of the way researchers have grouped skills. This section will help answer the first research question posed in chapter one, the introduction: “What are the most fundamental early number skills?” The interrelatedness of the skills will then be analyzed, and the various learning models will be discussed in light of both this interrelatedness and what the researchers noted in the literature review chapter. Finally, implications and recommendations for the teaching of early number skills will be explored in terms of creating a Kindergarten mathematics curriculum, revising and using the oral diagnostic test, and training teachers. These last two sections will help answer the final research question posed in chapter one: ”How might this understanding of early number skills impact on the teaching of early number skills, specifically with learning models, curriculum development, assessment, and teacher training?”

The Data Analysis Compared to Researchers’ Grouped Skills

In the literature review chapter, it was noted that several researchers grouped skills, focusing on those that contributed to the foundation of mathematics and the development of a successful mathematician. No group included all the skills that were used for this study; each group of skills excluded at least one of the skills used in the oral diagnostic test. When those groups of skills are viewed in light of the data above, the gaps seem puzzling.

Wright et al. (2000) and Kamii (1982) do not include one-to-one correspondence but the other four researchers do include it in their list of grouped skills. However, one-to-one correspondence was seen in the data analysis to be a highly predictive skill if missing. The data also showed that by the end of Kindergarten, most children performed successfully with one-to-one correspondence, and that if present, one-to-one correspondence was a minimally predictive skill. It seems that one-to-one correspondence is both accessible to most Kindergarteners in this study, and crucial for the formation of competency in mathematics, as can be seen by the results of Table 12 in the data analysis chapter. This evidence would suggest that it should be on the list of early number skills because of that high predictiveness.

Wright et al. (2000) and Kamii (1982) also do not include cardinality but the other four researchers do include it, often noting its importance in the development of a foundation of mathematics. As Dowker (2005) writes, “(The) cardinal word principle…may indeed be a true prerequisite for many other arithmetical abilities” (p.76). The evidence from this study bears this out, as can be seen below in the section on the interrelatedness of the skills, in which several skills are found to be “prerequisites” for other skills, or as they are called in this study, “preconditions” (see below). According to the data analysis, cardinality seems to be a precondition for another skill, namely part-whole relationships.

Yet in the analysis above, cardinality did not figure as a highly predictive skill. Rather it appeared to be a moderately predictive skill when partially missing, and a minimally predictive skill when present or missing. Kindergarteners who demonstrated success with cardinality, and they are in the majority at 81%, do not seem to be affected by it in terms of their placement in 1st grade. Like one-to-one correspondence, cardinality was accessible to most Kindergarteners in this study, but unlike the former skill, cardinality, when missing, did not seem to have a similar impact in their placement in 1st grade. Yet because of its effect on part-whole relationships as a precondition, this study suggests that it be on the list of early number skills.

Fosnot and Dolk (2001) are the only ones who list organized counting in their list. In this study, organized counting appeared to be moderately predictive when missing, and only minimally predictive when present. It seems to have much less of an impact on the development of mathematical competency than the other skills above. However, because of its contribution to the efficiency of counting with objects, and because of its moderately predictive power, it should be on the list of early number skills.

Similarly, hierarchical inclusion clearly contributes to the development of mathematical competency, as noted by Copeland (1970). Yet it was only listed by Fosnot and Dolk (2001) and Kamii (1982). Hierarchical inclusion was seen in the data analysis to be a moderately predictive skill when both missing and present (see Table 15), meaning that when students had not mastered it, they were more likely to finish in the lower attaining group in 1st grade, and when they had mastered it, they were more likely to finish in the higher attaining group in 1st grade. Hierarchical inclusion also seems be a precondition for part-whole relationships, as shown below, which is a highly predictive skill and appears to be necessary for success in 1st grade. There seems to be less chance for that success without hierarchical inclusion. Therefore, based on this study, hierarchical inclusion should be on the list of early number skills.

Both Fosnot and Dolk (2001) and Kamii (1982) exclude rote counting from their lists. They recognize the importance of several other skills, but seem to pay minimal attention to rote counting. However, as can be seen by the data analysis, rote counting is a highly predictive skill when missing, partially missing or present. How high a student can count seemed to help determine his/her placement in 1st grade in this study. Rote counting appears to contribute to the development of a successful mathematician, as can be seen by the results of Tables 16 and 22 in the data analysis chapter, and, based on those results, should be on the list of early number skills.

Fosnot and Dolk (2001), Wright et al. (2000), and Clements (2004) list subitization, and the others do not. In this study, subitization appeared to be highly predictive when missing, moderately predictive when present, and minimally predictive when partially achieved. Subitization's high predictiveness alone could qualify it to be on the list of early number skills. In addition, it also serves as a precondition for cardinality, conservation, and part-whole relationships, as will be seen below. This gives subitization even more reason to be on the list of early number skills because of its effect on the skills and the students as seen in this study.

Only Fosnot and Dolk (2001) list part-whole relationships. Part-whole relationships was shown in the data analysis to be a highly predictive skill when missing or present, and moderately predictive when partially achieved. It seems to have an impact on the students who do not achieve it, as can be seen by Tables 18 and 24 in the data analysis, and, based on that evidence, it seems part-whole relationships should therefore be on the list of early number skills.

Only Fosnot and Dolk (2001) and Kamii (1982) include conservation of number in their lists. However, what became clear from the data analysis is that conservation of number is a highly predictive skill when missing. If children in this study could not conserve by the end of Kindergarten, they were likely to finish in the lower attaining group in 1st grade. Conservation of number appears to contribute to the development of a successful mathematician, as can be seen by the results of Tables 19 and 25 in the data analysis chapter, and, based on that evidence, should be on the list of early number skills.

The Interrelatedness of Early Number Skills

The data analysis shows another aspect of early number skills and that is their interrelatedness. This was noted in the literature review chapter in that researchers often saw a particular skill contributing to the development of mathematical competency. Dowker (2005) noted that some skills "facilitate the development of other skills" (p.29). In the literature chapter, several researchers describe how each skill contributes to the development of mathematics, and the data from this research bear out these connections. I call this contribution a "precondition." Certain skills seem to serve as preconditions for other skills, which in turn serve as preconditions of yet other skills. For early number skill A to serve as a precondition for early number skill B, it would need to be present in most of the students who have mastered skill B. It would also show in that those who have mastered skill A would not have necessarily mastered skill B, and the data would reveal that discrepancy.

For example, as described in the literature review chapter, subitization was seen by Wagner and Walters (1982), Clements and Sarama (2008), and Fuson (1992a) to contribute to several early number skills: cardinality, conservation, and part-whole relationships. How does this compare with the data analysis? Of the 30 (out of 93) Kindergarten students who received five points on conservation of number, only two (6%) could not subitize at all. The implication of these data is that a student who can conserve will most likely also be able to subitize. The same holds true for cardinality. Of the 75 (out of 93) Kindergarten students who received five points on cardinality, only seven (9%) could not subitize any amount. Again the implication of these data is that a student who has mastered cardinality will most likely also be able to subitize. For part-whole relationships, out of the 32 (out of 93)

Kindergarteners who received five points, none (0%) could not subitize at all. Thus, the evidence from this study agrees with the above researchers in connecting subitization to cardinality, conservation of number, and part-whole relationships.

Does this mean that being able to subitize facilitates cardinality, conservation, and part-whole relationships? Not necessarily. Of the 10 (out of 93) Kindergarten students who could not subitize, six (60%) had cardinality whereas only two (20%) had conservation of number and only one (10%) could perform the part-whole relationships task, and only partially at that. It seems that not being able to subitize has a greater impact on conservation and part-whole relationships than on cardinality. This makes subitization a likely first step towards conservation and part-whole relationships but not necessarily so for cardinality; not having subitization seems less of an obstacle towards achieving cardinality than towards achieving conservation and part-whole relationships. Yet once students can perform cardinality and conservation and part-whole relationships successfully, they also seem likely to be able to perform subitization successfully, making subitization a precondition for these skills. In other words, it seems that subitization does not necessarily cause cardinality, conservation, and part-whole relationships, but it is quite likely to be present when those skills have been mastered because of its connection to them.

This idea of a precondition works with other skills as well. One-to-one correspondence seems to serve as a precondition for organized counting. Of the 66 (out of 93) Kindergarteners in this study who performed organized counting successfully, only 2 (3%) could not perform one-to-one correspondence. The reverse is not true: of the 84 (out of 93) Kindergarteners who could perform one-to-one correspondence, 20 (24%) could not perform organized counting. In other words, one-to-one correspondence seems to serve as a precondition for organized counting, as can be seen in that very few students could organize their counting without one-to-one correspondence. Therefore, it appears that one-to-one correspondence will most likely be present when organized counting is present. This confirms Thompson's (1997) idea: that to be "successful in their counting," students need to have mastered both one-to-one correspondence and organized counting.

Cardinality also seems to serve as a precondition for part-whole relationships. Out of the 32 (out of 93) Kindergarteners who performed the part-whole relationships task successfully, only 2 (6%) of them could not perform the cardinality task. Out of the 64 (out of 78) 1st graders who performed the part-whole relationships task successfully, only 1 (2%) of them could not perform the cardinality task. This makes sense because conceptually, cardinality seems to be a crucial part of early number skills. Without it, children would see counting as merely an activity for its own sake, rather than a means to an end. Cardinality allows students to start to think in groups, and therefore, seems to need to be present in order for students to have part-whole relationships. The data analysis shows that cardinality seems to be a precondition needed for part-whole relationships to flourish.

Two other skills seem to serve as preconditions for part-whole relationships: hierarchical inclusion and conservation of number. Of the 32 (out of 93) Kindergarten students who have performed the part-whole relationships task successfully, 29 of them (91%) have also performed the hierarchical inclusion task successfully. Of the 60 (out of 93) Kindergarten students who have mastered hierarchical inclusion, only 37 of them (62%) could perform part-whole relationships either partially (can make addend pairs for either five or ten) or fully (can make addend pairs for both five and ten). Of the 33 (out of 93) Kindergarten students who could not perform hierarchical inclusion successfully, 24 of them (73%) also could not perform part-whole relationships successfully. It seems that if children do not grasp the concept of hierarchical inclusion, they will be less able to function competently with part-whole relationships. The implication of this data is that children appear to need to learn hierarchical inclusion in order to be successful with part-whole relationships. Hierarchical inclusion marks the “reversibility of thought” (Copeland, 1970), without which the ability for further computation remains elusive. Both skills are crucial in the development of mathematical competency, as noted by Payne and Huinker (1993) and Baroody (2004).

Like hierarchical inclusion, conservation of number also seems to serve as a precondition for part-whole relationships. Of the 32 (out of 93) Kindergarten students who performed the part-whole relationships task successfully, 18 (56%) of them were conservers. Conversely, of the 40 (out of 93) Kindergarten students who could not conserve, only five (13%) of them performed the part-whole relationships task successfully. In other words, when a student in this study had mastered part-whole relationships, s/he most likely had also mastered conservation.

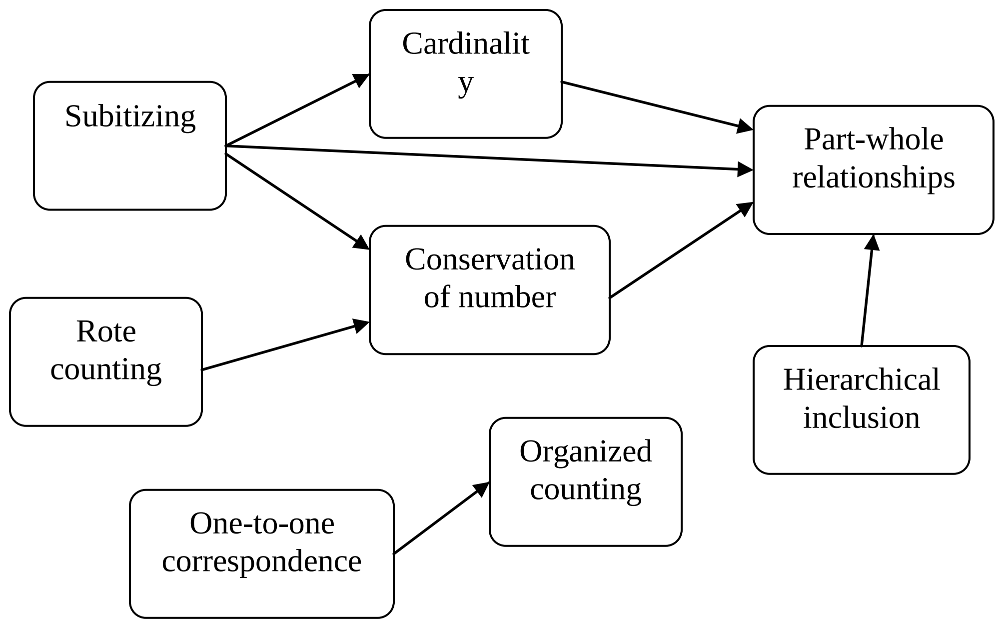
When a student had not mastered conservation of number, part-whole relationships suffered as a result. Therefore, the evidence from this study agrees with Maclellan (1997) when he states that it is conservation of number that allows students to understand “the tasks of addition and subtraction" (p.38), which is what part-whole relationships initiates. This connection between conservation of number to part-whole relationships is surprising, for it had seemed that the importance of conservation was its conceptual power, and that it was not directly linked to any other early number skill. Wadsworth (1971) had noted that conservation of number showed a level of intellectual growth but did not necessarily serve another skill. The data analysis seems to show, however, that conservation of number does serve as a precondition for part-whole relationships.

What serves as a precondition for conservation of number? Subitization was noted above as one early number skill to do so, but rote counting also seems to serve as a precondition for conservation. Of the 30 (out of 93) Kindergarten students who could conserve, 16 (53%) of them could rote count to 100 or beyond. The inverse was not true: of the 40 (out of 93) Kindergarten students who could count to 100 or beyond, 16 (40%) of them had conservation of number, and 14 (35%) of them could not conserve. Thus, conservation of number did not serve as a precondition for rote counting. If a student in this study could conserve, s/he could most likely count high, but being able to count high in Kindergarten did not guarantee conservation. If a Kindergarten student could not rote count high but could only count to 20, s/he most likely could not conserve either. Out of the ten (out of 93) Kindergarten students who could not count past 20, only two (20%) of them could conserve. This confirms what Kamii (1982) wrote about counting and conservation:

> A child who does not have the (mental) structure of number uses the best thing he can think of to make quantitative judgments, namely space. When he has constructed the structure of number, however, the space occupied by the objects becomes irrelevant (p.16).

In conclusion, certain skills seem to serve as preconditions for other skills, which in turn sometimes serve as preconditions for yet other skills. The data often shows that when skill B is mastered, skill A has been mastered as well. The opposite is not true, which shows that preconditions are not commutative.

Figure 5: The preconditions of early number skills



The concept of preconditions illustrated in Figure 5 seems like the hierarchical learning model discussed in the literature review chapter, and yet it has its differences. How are the concept of precondition and the hierarchical learning model similar, and how are they different?

The Learning Models in Light of the Data Analysis and Preconditions

Various learning models were discussed in the literature review chapter: the hierarchical model, the learning/teaching trajectory model, and the landscape model. These models will be reviewed in light of the data analysis and preconditions.

The concept of a precondition initially appears to be similar to the hierarchical learning model. The hierarchical learning model focuses on the idea that one skill serves as a “prerequisite sub skill” for the later skill (Daniels & Anghilieri, 1995). This concept presumes that each former skill is a “simple task” leading to a more “complex task” (Gagne, 1965). The concept of a precondition as described above does seem like it is the same, i.e., one-to-one correspondence seems to lead to organized counting. The precondition idea implies that if skill B is mastered, skill A will most likely be mastered as well. Conversely, if skill A is mastered, skill B might not yet be mastered.

However, there are differences between the hierarchical learning model and the preconditions concept. One difference is that with the precondition concept, skill A is not considered a “sub skill,” in that it is not necessarily less complex than skill B, nor is it “simple.” Rather, skill A appears to serve as a precondition for skill B because it provides the learning necessary for skill B to be mastered. One-to-one correspondence is not less complex than organized counting, nor is it a simple task. However, the concept of one-to-one correspondence seems to be a necessary precondition for organized counting. Students seem to need to realize that each object receives one and only one number before the concept of organized counting can emerge. Once students realize that each object receives one and only one number, then the realization that each object that has been counted should not be counted again and must be pushed aside can flourish.

The other difference between the concept of a precondition and the hierarchical learning model is that the latter seems to be a linear vision of mathematics, whereas the former is not. The language of the hierarchical learning model is revealing: “pyramid” (Orton, 1992), and “vertical planning” (Treffers, 1987). The precondition concept, on the other hand, simply means that skills appear to nest within other skills. This means that the learning of different skills can be simultaneous. Therefore, the hierarchical learning model is not sufficient for understanding children’s learning. The data of this study shows instead interrelationships between more than two skills at a time.

With this in mind, then another learning model that has some similarity to the precondition concept is the hypothetical learning trajectory from Simon (1995). Simon’s model seems reasonable in light of the data from this study in that he recognizes that students’ learning “evolves” as they gather more skills. This hypothetical learning trajectory also focuses on the role of the teacher in that the teacher needs to have a conceptual map of the mathematics his/her students are learning in order to guide their instruction. This is a crucial idea and forms a key point of this study. Knowing what the early number skills are, how well young children perform with them, and how to find those who are not succeeding are all vital for effective teaching.

Yet Simon's model does not seem to be grounded in the data of how students perform with early number skills, as the concept of preconditions is, but rather it starts with a "conjecture" (Simon & Tzur, 2004, p.100). Simon asks, "What activity, currently available to the students, might be the basis for the intended lesson?" (p.96). Simon's theory is more of a way to plan curriculum by including a goal and possible tasks. The hypothetical learning trajectory becomes more of a linear collection of tasks geared towards creating curriculum, rather than a map of specific skills that are linked by preconditions, geared towards understanding the paths of mathematical learning.

Thus, Simon's hypothetical learning model can be seen as too linear, similar to the hierarchical learning model. It does not seem to allow for interrelationships to be acknowledged. Van den Heuvel-Panhuizen (2001) proposes increasing the width of Simon's trajectory to allow for the simultaneous learning of a variety of skills. The concept of the precondition fits this idea well. As was noted above, several skills interweave with others:

subitization and rote counting seem to be preconditions for conservation of number; conservation of number, cardinality, and hierarchical inclusion seem to be preconditions for part-whole relationships; et cetera. The learning of early number skills follows not a slim, linear path but a much broader, "multi-lane highway." Van den Heuvel-Panhuizen (2008) acknowledges that "multiple skills can be learned simultaneously and that different concepts can be in development at the same time…In short, there is sufficient reason to talk about a learning-teaching trajectory having a certain bandwidth" (p.14). However, there is no mention of these "multiple skills" and "different concepts" being interrelated, only that they can be learned at the same time.

Does the concept of preconditions, therefore, fit the landscape model of Fosnot and Dolk (2001)? Their model is the broadest of all because they believes that learning is "messy." However, Fosnot and Dolk's model does not seem to include any sort of structured, ordered sequence. Fosnot and Dolk do write of a developmental progression—"the construction of some essential big ideas" (p.11)—but do not seem to specify connections between skills or big ideas. The data analysis appears to reveal that, as described above, certain skills do seem connected to others and serve as preconditions for those skills. Certain mathematical understandings do seem to help other mathematical understandings to be mastered. The data analysis seems to show that learning is not entirely messy; it does follow a variety of paths, however short they might be. The “messiness” is not entirely removed because not all students follow each pathway in complete synchronicity. In general, however, the preconditions seem to point to connections that are important to keep in mind when teaching early number skills.

In conclusion, the learning model that seems to emerge from this particular data is called *simultaneous pathways*, with the idea that some pathways are longer than others, but all include complex tasks that serve as preconditions for other complex tasks. Simultaneous pathways create a learning model of students learning several skills at the same time, each skill building on the preconditions serving it. These pathways are frequently interconnected because of the relationships between the skills. Each student will progress on each pathway at his/her own rate, and will be learning on several pathways simultaneously. The teacher needs to be aware not only of all of these conceptual pathways, but also where each student is on each pathway.

It is not to say that the three other learning models are of no use, but rather that the data from this study suggests a possible refinement of the existing models. The simultaneous pathways learning model both widens and interlinks the paths of the hierarchical learning model and of the hypothetical learning trajectory model, and adds structure to the amorphousness of the landscape learning model. It is out of the data of this study that the concept of preconditions emerges; it is out of the concept of preconditions that the new simultaneous pathways learning model emerges. Thus, as opposed to the other learning models, the simultaneous pathways learning model is based on empirical data. With Gagne's hierarchical learning model, logic creates the map of learning beforehand; with the hypothetical learning trajectory, the teacher uses a conjecture and student response to activities create the learning map; with the simultaneous pathways model, the data create the map. The implications and recommendations of this learning model will be discussed below.

Implications and Recommendations for the Teaching of Early Number Skills

The discussion of recommendations from these data includes: an overview of a Kindergarten mathematics curriculum; the revisions of the original oral diagnostic test, and the use of it in assessment; and the training of teachers to develop their conceptual map of mathematics. The predictive power of each early number skill gives an indication as to how it fits into a

Kindergarten mathematics curriculum. The oral diagnostic test needs to be revised based on the data analysis, and can be used for different benefits at various times of the year. The conceptual map of mathematics for teachers is crucial for effective teaching and curriculum development.

Implications and Recommendations: Creating a Kindergarten Mathematics Curriculum

How might a Kindergarten curriculum be designed with these data in mind? Which skills should be used, and why? Creating a Kindergarten mathematics curriculum based on these data would involve giving more time to certain skills that are not usually a part of an early childhood mathematics curriculum, as well as making sure the usual early number skills are taught to mastery. For example, one-to-one correspondence is typically part of a Kindergarten mathematics curriculum. Students usually spend a good deal of time counting objects. As could be seen by the data, mastering one-to-one correspondence was crucial in order to finish in the higher attaining group by the end of 1^{st} grade. As could also be seen by the data, most Kindergarteners in this study had mastered it by the end of the school year (84 out of 93 students—90%). One recommendation would be for one-to-one correspondence to be taught until mastery, but then since it drops to being minimally predictive once mastered, to not spend unnecessary time counting objects. Once students have mastered one-to-one correspondence, they might not need to spend more time on it. However, since one-to-one correspondence is a precondition for organized counting, students should be assessed for both skills before decreasing the time spent on counting objects. If a student has mastered both skills, then most likely s/he may be better served by focusing on other early number skills.

Like one-to-one correspondence, cardinality is a typical part of the Kindergarten mathematics curriculum and was mastered by most Kindergarteners in this study by the end of the school year (75 out of 93 students—81%). Unlike one-to-one correspondence, cardinality is a minimally predictive skill whether missing or present. Thus, performing well or poorly with cardinality will not guarantee placement in 1st grade. Does that mean that cardinality is not important as an early number skill? This is doubtful. Clements (2004), Fuson (1992a), and Payne and Huinker (1993) all agree that cardinality is both the "capstone" and the "building block" of numerical understanding. If so, then how are we to understand the above data in light of what is said about cardinality?

Conceptually, cardinality is a crucial part of early number skills. Without it, children would see counting as merely an activity for its own sake, rather than a means to an end. Cardinality also allows students to start to think in groups. Cardinality should certainly be a part of the curriculum. But how much of a focus should cardinality be if it is mastered by most and is minimally predictive? One might think it should receive less time than other skills. With most Kindergarteners in this study achieving success with it, it might not be necessary to devote a great deal of time to the concept. However, as noted earlier, cardinality serves as a precondition for part-whole relationships. Because of its effect on this skill, cardinality needs to be a regular part of the curriculum so that there are many opportunities for students to master it in order to go on to master part-whole relationships.

Organized counting is minimally predictive and does not seem to serve as a precondition for other skills. This early number skill resembles cardinality in the data analysis in that most Kindergarteners in this study achieved success with this skill (66 out of 93 students—71%). Yet performing the task of organized counting successfully is not enough to ensure placement in the higher attaining group in 1st grade. Performing successfully with this skill does not seem to impact on the development of mathematical competence. The data show that those who have organized counting finish in either the lower or the higher attaining group by the end of 1st grade. Without organized counting, however, the predictive power rises to moderate, indicating that some young children are affected negatively when not being able to count in an organized manner.

Organized counting tends to play a minor role in a Kindergarten mathematics curriculum. However, because it plays a crucial role in effective counting with objects, it needs to be a focus until students master it. Organized counting seems to be an important skill in the development of mathematical competency because of the efficiency it produces, yet it is only without it that children are adversely affected. Like cardinality, organized counting seems to need to be a part of the curriculum because of its importance in developing mathematical competence.

Hierarchical inclusion is moderately predictive when both missing and present. Fewer Kindergarteners (60 out of 93 students—65%) in this study were able to perform well with hierarchical inclusion than with cardinality or organized counting. What does this say about this skill? Hierarchical inclusion marks the "reversibility of thought" (Copeland, 1970), without which the ability for further computation remains elusive. Therefore, not only is it necessary to teach hierarchical inclusion, but it seems it is also necessary to teach it before or certainly during the teaching of part-whole relationships. Before children can fully learn specific basic addend pairs, they seem to need to know more about the concept of addend pairs. They need to understand the concept that within a number lies other numbers, and that within a whole lies parts.

Hierarchical inclusion is not a usual part of the Kindergarten mathematics curriculum. Perhaps that is because it seems to be a concept that is not taught explicitly but rather constructed after enough exposure. Since it is moderately predictive if missing or present, it might not seem to be a crucial part of the curriculum. However, as it appears to serve as one of part-whole relationships' preconditions, it needs to become a more integral part of the curriculum in order for students to have multiple opportunities for mastery.

Subitization is also not a usual part of the Kindergarten mathematics curriculum, perhaps because the usefulness of this skill is not easily seen. But as can be seen by the data analysis, it is highly predictive when missing. It also seems to serve as a precondition for cardinality, conservation of number, and part-whole relationships. Based on this evidence, subitization should be a vital part of the curriculum until students have mastered it. What amounts should be subitized? As noted earlier in the literature review chapter, there is some controversy as to whether amounts over three are actually subitized as opposed to computed with an "automatized procedure" (Fischer, 1992). If the amounts for subitizing in the oral diagnostic test had been limited to groups of two and three, 83 out of 93 Kindergarteners (89%) and 77 out of 78 1st graders (99%) would have succeeded. With those scores, subitization would have passed one-to-one correspondence as the highest score in 1st grade, and would have been a close second in Kindergarten. However, as other researchers (e.g., Baroody & Gatzke, 1991) believe that amounts up to six can be subitized, the task on the oral diagnostic test included amounts of four and five.

Since it was noted in the data analysis chapter that students who subitized only amounts of two and three were more likely to finish in the lower attaining group of 1st grade (as well as those who could not subitize any group), and that students who could subitize groups of four and five finished in the higher attaining group in 1st grade, it becomes clear that practicing subitizing larger amounts is beneficial. Fischer (1992) might indeed be right—that only amounts of two and three can be subitized and larger amounts are subitized with an automated procedure—but Baroody and Gatzke (1991) seem also to be right in that larger amounts are possible. The data analysis shows that not only are these larger amounts possible but they are also desirable in order to finish in the higher attaining group in 1st grade.

The implication of rote counting's highly predictive power seems to be that it serves as a primary focus in the Kindergarten mathematics curriculum. Being the precondition for conservation of number adds to rote counting's importance in the curriculum. As Piaget and Szeminska (1952) write, "Our hypothesis is that the construction of number goes hand-in-hand with the development of logic" (p.viii). Rote counting is always a part of the Kindergarten mathematics curriculum but often in an unspecified way. Teachers know that students need to learn to count, but how high? Even the National Council of the Teachers of Mathematics Principles and Standards for School Mathematics (2000) does not specify. It simply states that "Throughout the early years, teachers should regularly give students varied opportunities to continue to develop, use, and practice counting…" (p.80). As quoted earlier, Payne and Huinker (1993) note that counting to ten is a routine task for most Kindergarteners. Indeed, if the rote counting task stopped at ten, then all of the children in Kindergarten in this study would have been successful. However, the task was to count as high as possible, and counting higher seems to have been an obstacle for most of the children. As with subitization, it is clear from the data that most children can successfully do the lesser task—in this case, count to ten. However, again as with subitization, those who went further—in this case, counting to 100—had a much greater chance of finishing in the higher attaining group by the end of 1st grade. Thus, the data reveals that rote counting to 100 or beyond appears to benefit students.

Conservation of number is a highly predictive skill when missing or partially present, and it seems to serve as a precondition for part-whole relationships. These are two reasons to include conservation of number in the Kindergarten mathematics curriculum. Conservation of number, like hierarchical inclusion, is not explicitly taught, nor should it be. Conservation should not be taught just to get a right answer on the task itself. Kamii (1982) writes that

> For educators…it is absurd to train children to give higher-level answers on this task. The reason is that performance on this task in one thing, and the development of the underlying mental structure…is quite another thing. Educators must foster the development of this structure, rather than trying to teach children to give correct surface answers on the conservation task (p.16).

Instead, teachers should be looking for opportunities to ask logical thinking questions about quantities. Indeed, Kamii (1982) continues,

> Some (teachers) conclude that nonconservers must be taught to conserve number…(Direct) teaching of the conservation task is a misapplication of Piaget's research" (p. 1)…However, I do not draw the pedagogical implication that the only thing the teacher can do is to sit back and wait. There are certain things the teacher can do to encourage children to think actively (to put things into relationships) thereby stimulating the development of this mental structure (p.18).

Mastering conservation of number requires the use of logic, rather than the use of subitizing or counting. As noted by Kamii (1982), once students construct the logic of number, the whole conservation task seems obvious and somewhat silly. This construction of logic seems to be a necessary focus in a Kindergarten mathematics curriculum. It involves asking students to reason mathematically and to communicate that reasoning. Without that development of logic, students have to rely on perception and on counting when confronted with the conservation of number task. Using eyes and fingers to count show the "absence of logical certitude" (Kamii, 1982, p.18). Students need to recognize the stability of number through the power of logic in order to see that a change in appearance does not change quantity. As noted by Wadsworth (1971), this potent understanding allows children to decenter perceptions, attend to transformations, and reverse operations—all of which are necessary for mathematical competency.

Part-whole relationships, like rote counting and conservation of number, is highly predictive both when missing and when present. This means that not being able to make addend pairs increases the chances of finishing in the lower attaining group by the end of 1st grade, while being able to make addend pairs increases the chances of finishing in the higher attaining group by the end of 1st grade. An implication of this data is that part-whole relationships should be an integral part of the Kindergarten mathematics curriculum. However, part-whole relationships also has several preconditions: subitization, cardinality, conservation of number, and hierarchical inclusion. Therefore, it behooves Kindergarten teachers to focus on these preconditions first, and introduce part-whole relationships later in the year.

Students need to understand the concept of part-whole relationships: that a whole can be broken into parts. This requires reversibility of thought: "Reversibility refers to the ability to mentally do opposite actions simultaneously—in this case, to cut the whole into two parts and reunite the parts into a whole" (Kamii, 1982, p.13). However, since "some first graders honestly believe that 5 + 5 = 10, but others only recite these numbers because they are told to (Kamii, 1982, p.21)," teachers need to make sure that part-whole relationships is not taught through memorization. Rather, students will be better served if they come to this skill through the logic of hierarchical inclusion. An early emphasis on mathematical notation and the overuse of flashcards defeat the growth of logico-mathematical knowledge.

Implications and Recommendations: Revising and Using the Oral Diagnostic Test

It is because of the data analysis that the order of the early number skills on the original oral diagnostic test has been revised (see Appendix 1 and 2 for both the original and revised versions). The revisions removed certain early number skills that did not prove to be as illuminating about student mathematical ability as the remaining skills. In addition, the revisions changed the layout of the oral diagnostic test in order to facilitate its use.

The three early number skills tasks that have been removed from the original oral diagnostic test are skip counting, compensation, and unitizing. The remaining skills have been rearranged in order from the highest percentage to the lowest percentage for each task, according to the Kindergarten results. The tasks now include the scores for circling, rather than spaces for checkmarks. The test also includes space for marking the total points received as well as the class midpoint—the score that falls in the middle of the lower attaining group and the higher attaining group—if the test is being used to assess an entire class. This allows the teacher to see immediately how the student compares to the rest of the class.

Some of data analysis results and ensuing revisions were surprising. Because of the inherent difficulties with fine motor control in young children, as noted by Fuson (1984), it is surprising that one-to-one correspondence seems to be the easiest of the tasks. It is also surprising that cardinality seems to be easier for most than organized counting. One might think that children need to be able to count objects accurately before they can understand the concept of the "total." Since subitization is so often considered "innate" (Gelman, 1990), one might not think that subitizing all groups would be so difficult for young children. Last, it is surprising that hierarchical inclusion seems to be easier for young children than rote counting to 100 because hierarchical inclusion is an abstract concept rarely discussed, and rote counting is a common activity for young children with their parents and teachers.

What is not especially surprising is the fact that part-whole relationships and conservation of number are the lowest scoring tasks for Kindergarteners. Part-whole relationships is a complicated concept, in that it seems to have several preconditions: subitization, cardinality, conservation of number, and hierarchical inclusion. In addition, two out of those four preconditions—cardinality and conservation of number—also seem to have preconditions: cardinality has subitization as its precondition, and conservation of number has subitization and rote counting as its precondition. It appears that much needs to be learned before part-whole relationships can be mastered. In terms of conservation of number, as noted in the literature review, children tend to become conservers around the age of seven, which is older than most Kindergarteners.

Some tentative conclusions as to why the results turned out this way can be found in the way the oral diagnostic test tasks were formed. For instance, if the students were asked to count to a lower number, as stated above, the success rate for rote counting might have been higher. If students had been asked to subitize only two and three, the success rate might have been higher. If students had to organize fewer objects while counting, the success rate might have been higher. If the number used for hierarchical inclusion were lower, the success rate might have been higher. These possible factors that might be influential could serve as the beginnings of further research.

Yet there are other possible reasons for the results of this study. As mentioned in chapter four on the data analysis, neither the percentages in the analysis nor the test nor the tester are foolproof but are all subject to human error or inconsistencies. For example, although certain skills have been labeled highly predictive, there is still the chance that students will finish 1st

grade otherwise than predicted. That chance increases as the predictive power decreases. As noted above, the oral diagnostic test is not foolproof in that the tasks could have been incorrectly designed and therefore the results might be misleading. Finally, the test is dependent on the judgment of the tester. Deciding how many points a student should receive for a task performance can be difficult. If a student does not organize his/her counting on the first attempt but does so on the second attempt, how many points should that student receive? This is a moment when, as noted by Cohen et al. (2000) in chapter three on methodology, that "the immense complexity of human nature and the elusive and intangible quality of social phenomena contrast strikingly with the order and regularity of the natural world." Perhaps this conflict is unavoidable in that as much as the tester wants consistency, the human subjects cannot necessarily provide that.

However, the tester can strive to be consistent with the scoring of the test with all students. The teacher cannot let one student try a task several times and not let others do the same. The teacher cannot encourage one student to rote count higher but not do the same for other students. If the teacher is consistent with the way s/he uses the test, then the results will have a greater chance of being reliable.

A certain amount of consistency can also emerge when the oral diagnostic test is used regularly. However, one crucial note needs to be made: this oral diagnostic test is only one form of assessing students. In order to fully evaluate the progress of students, multiple measures of assessment need to be made throughout the school year; one test alone is not enough to fully judge a student. There are too many reasons that a student's score might not be accurate. Therefore, using the test multiple times during the year, along with other forms of assessment, will give the teacher a much deeper and more reliable understanding of his/her students. There are advantages to using this test three times a year: at the beginning of the year, during the middle of the year, and at the end of the year. If this test is used in the beginning of Kindergarten, teachers can see how their class as a whole is performing, as well as how each individual is performing with early number skills. The teacher can then shape the curriculum according to these scores. For example, as noted earlier, subitization is a highly predictive skill if missing. If the majority of the class does not have this skill at the beginning of the year, the teacher can start immediately providing activities to facilitate the learning of subitization. Conversely, one-to-one correspondence is a minimally predictive skill when present. If the teacher notes through the scores on the test that the class has mastered this skill, the teacher can decrease the time spent on this skill, and focus instead on other more highly predictive skills.

If this test is used in the middle of the year, the teacher has another opportunity to assess the class as a whole in order to again shape the curriculum, and assess individuals in order to provide remediation effectively. If this oral diagnostic test is used at the end of Kindergarten, teachers have a way of pinpointing the at-risk students, based on their scores. Remediation can start in Kindergarten, or during the summer, or right at the beginning of 1st grade.

Both Kindergarten and 1st grade can use the same test in order to inform teaching, shape curriculum, and evaluate individual students. The use of the test increases the chances of early and effective remediation for those who are in need. Since it was noted above that these skills seem to be learned in several simultaneous pathways, it is helpful to have the test to highlight which pathway has been completed, and which is in need of guidance. The results of the test will show clearly which skills have been mastered and which have not, allowing the teacher to make informed decisions about possible remediation. The results will vary not only with each class but each time the test is given. With the results of the test, the teacher will know which individual needs more help with rote counting, or with conservation of number, or with subitization, et cetera. The teacher will also be able to see if more time is needed by the whole class for a particular skill.

The oral diagnostic test does not have to be given to the entire class each time. It is helpful to use when a teacher needs more information about a particular student, either for purposes of remediation or for extra challenge. If the teacher senses that a student has mastered most or all of these skills, using the test to verify this impression is helpful. If the teacher is correct, then the student will be better served if the teacher knows to move onto other areas to be learned.

The data analysis also showed that certain skills when combined increase their predictiveness. What is the implication of this result? After scoring his/her class' results on the oral diagnostic test, the teacher has further information about those scores, based on the results of the combined skills' tables. An overall result from this test can tell the teacher where the student placed in relation to his/her peers. However, this overall result cannot tell the teacher as much about the future of this student as the individual or combined skill scores can. As noted in the student examples in the data analysis chapter, some student test results looked adequate at times. Yet the longitudinal data revealed that the student can perform poorly a year later. The data results for combined skills add to the information provided by individual skills. When test results are examined closely for both individual skill scores and combined skill scores, teachers will have a far greater sense of how the student is and will be performing.

Implications and Recommendations: Training Mathematics Teachers

In order to move on to other areas to be learned, however, the teacher needs to be well trained in mathematics. Learning as much as possible about the mathematics one is teaching is the goal. Learning higher levels of mathematics, as is often done at teacher training institutes or universities, does not necessarily prepare teachers for teaching Kindergarten or 1st grade mathematics. As noted earlier, teachers need to develop a conceptual map of mathematics, especially the area of mathematics s/he is teaching. Fosnot and Dolk (2001) note that, "In the United States, teacher education programs have added more and more mathematics courses" (p.171). These courses are college level mathematics, with perhaps a mathematics education course as well. However, in the Netherlands, teacher training is different in mathematics. There, "students take some seven or eight courses in mathematics education, courses geared towards a deep understanding of the mathematical topics *they will be teaching"* (Fosnot & Dolk's italics, p.172). It is this kind of training that enables teachers to develop a conceptual map of mathematics for the students they are teaching.

Having a deep understanding of the math they are teaching means that teachers can use this oral diagnostic test to its fullest potential. They can evaluate the results and have a plan of action depending on the scores of their students. They can have a thorough knowledge of the mathematical pathways their students are taking, and they can have a solid understanding of how their students learn as well so as to develop and implement activities that give students the richest opportunities to learn these early number skills. If teachers do not have a full understanding of early number skills, then simply using the oral diagnostic test is not enough. Teachers can evaluate their students, but if they do not know how to proceed, then there is little point to using the assessment. As Ma (1999) writes, "Given that their own schooling does not yet provide future teachers with sound mathematical competence, their base for developing solid teaching knowledge is weakened" (p.146).

Summary

In this chapter, the results of the data analysis were compared to the researchers' thoughts described in the literature review chapter. The groupings of early number skills according to the researchers were examined, and the revision of the original oral diagnostic test was described. The interrelatedness of the skills and the preconditions concept were both examined, as were the various other learning models along with the new learning model that emerged from the data analysis, called "simultaneous pathways."

The implications and recommendations from the data analysis were described. The impact of both the predictive powers of the early number skills and their preconditions on the development of a Kindergarten mathematics curriculum were examined. Teachers can use this information to create an effective Kindergarten mathematics curriculum. Teachers can also use the revised oral diagnostic test, along with other forms of assessment, to evaluate the progress of their students in order to decide on possible extra challenges or needed remediation. First, however, teachers need to be well trained in elementary mathematics—in the mathematics that they teach—in order to develop a complete conceptual map of mathematics.

Without that map, teachers might not know what to do with their students' results with the oral diagnostic test.

What follows is the conclusion to this research, which describes the overall discussion of this study, as well as possible future studies based on this work.

CHAPTER SIX
CONCLUSIONS

Introduction

This study concludes with reviewing the aim and discoveries of this research. This chapter discusses how these research discoveries relate to the original questions posed in the introduction and how they impact on the teaching of mathematics to young children. Last, the kind of changes that could be made to the research and the kind of future research issues that could be explored are discussed.

The Aim of This Study

The aim of this study was to find a way to diagnose at-risk students in order to provide remediation early enough in their schooling to prevent them from falling further behind. Unlike literacy, mathematics has no system of a grade-by-grade evaluation, and minimal specifics in the benchmarks for each grade. As Bredekamp (2004) writes, “The NCTM (2000) standards list mathematics accomplishments that cover the broad range of prekindergarten through second grade…(For) specific mastery goals to truly be useful guides for teachers, they need to be more closely connected to age/grade levels” (p.79). Therefore, an important goal was to create mathematics benchmarks for Kindergarten and 1st grade. How most students in those grades performed with early number skills became the first step for this research. Only by knowing what most Kindergarteners and 1st graders can do mathematically can one find those students who are not performing at grade level.

The Discoveries of This Study

The results of the oral diagnostic test revealed what the majority of the students could do with early number skills. Most of the 93 Kindergarteners could perform one-to-one correspondence, cardinality, and organized counting successfully. Two-thirds of the total Kindergarteners had mastered hierarchical inclusion, and slightly more than half of the students could perform the part-whole relationships task successfully or partially successfully. Less than half the total Kindergarteners could subitize all groups or rote count to 100 or beyond. Slightly less than a third of the total Kindergarteners had mastered conservation of number.

In 1st grade, almost all of the 78 students could perform one-to-one correspondence, cardinality, and organized counting successfully. The percentage of success with hierarchical inclusion and part-whole relationships jumped dramatically as most students had success with these skills. Rote counting was also much easier for 1st graders as most of them could count to 100 or beyond. Close to two-thirds of the 1st graders could subitize all groups, but conservation still proved problematic, with only slightly over half of the children being able to conserve number.

These test percentages are interesting and important for they provide a clear picture of the performance with early number skills for young children. But it is the individual task scores that proved to show more dramatically how children would perform. Once the test results had been tallied, they were ranked from lowest to highest, and divided into a lower attaining group and a higher attaining group for each grade. Then each student's test result could show how s/he performed against his/her peers. This showed that some students finished Kindergarten in the lower attaining group, and a year later, finished 1st grade still in the lower attaining group. Conversely, some students finished Kindergarten in the higher attaining group, and a year later were still in the higher attaining group. However, some students moved from lower to higher or vice versa. The question became: could that movement have been predicted from the student's Kindergarten test scores? If so, then the oral diagnostic test would prove to be a way to expose at-risk students.

The answer seemed to be yes—the movement from Kindergarten to 1st grade could at times be predicted. The total test results from Kindergarten showed only in which attaining group the student finished by the end of the year, but the individual skill scores revealed much more. By comparing the individual skill scores of Kindergarten to the total test results of 1st grade, what emerged was the predictive power of each skill. Each skill was discovered to be highly, moderately, or minimally predictive in terms of placement for the student by the end of 1st grade. If the skill was highly predictive when missing, then the student who could not perform that skill would most likely finish in the lower attaining group by the end of 1st grade. If the skill was moderately predictive when missing, then the student who could not perform that skill would probably finish in the lower attaining group by the end of 1st grade, but the results were not as definitive as from a skill that was highly predictive. If the skill was minimally predictive when missing, the student had an equally likely chance of finishing either in the higher or the lower attaining group by the end of 1st grade.

In addition, certain skills when combined with other skills increased their predictive power. Missing one particular skill might cause the student to finish in the lower attaining group by the end of 1st grade, but missing it in conjunction with missing another would most certainly cause the student to fall behind. The research showed that highly predictive skills increased the predictiveness of minimally or moderately predictive skills, and, similarly, that minimally predictive skills held little sway over moderately or highly predictive skills.

However, these combinations foretold another discovery that emerged from the research, and that is of preconditions. Certain skills were seen to serve as preconditions for other skills. The former skills were not sub-skills in that they were not less complex than the latter skills. Rather, what was found was that success with the latter skills usually meant success with the former skills. For example, if a student had mastered organized counting, s/he most likely had mastered one-to-one correspondence, but the reverse did not hold true. If the student had mastered one-to-one correspondence, s/he had not necessarily mastered organized counting.

How These Discoveries Relate to the Original Research Questions

The discovery of the predictive power of early number skills can be used to fulfill the original intent of the study. After using the oral diagnostic test at the end of Kindergarten, teachers can locate students who score poorly on highly predictive skills. These students can be given remediation so that they can begin to come up to grade level. The test can also be given at the beginning of the year and during the middle of the year, along with other forms of assessment, to assess the levels of both the whole class and each student. If students are missing the skills that are highly predictive, teachers can shift or increase different areas in the curriculum to address the gaps in mastery. The predictive power of early number skills can help address the issue of the lack of diagnostic ability in mathematics; they can help teachers diagnose their students more effectively. The benchmarks developed by this research will also help teachers because they will know the general skill level of Kindergarteners and 1st graders. However, teachers should always use multiple measures to assess their students. The oral diagnostic test is a useful snapshot of the student ability with early number skills, but it is only one snapshot. Other forms of assessment are necessary to form a full understanding of the student's mathematical ability.

The discovery of the preconditions can be used to explain learning models as discussed in the literature review. The preconditions point to a combination of a hierarchical learning model as well as a learning/teaching trajectory model. This model is called in this study "simultaneous pathways." In this model, certain skills seem to be mastered only if another skill is also mastered. Although this image of simultaneous pathways sounds hierarchical, it is not, in that the preconditional skills are not less complex than the skills they serve. Although this image of simultaneous pathways sounds like the learning/teaching trajectory, it is not, in that there is a broader swath of skills being learned at any one time than the image of a trajectory implies. Although this image of simultaneous pathways sounds like a messy landscape model, it is not, in that there is more structure than the landscape allows.

The Impact of Preconditions on the Teaching of Mathematics to Young Children

Preconditions can be used to inform curriculum development. Knowing that some skills serve as preconditions to other skills should increase the need to spend time on them in class. For example, subitization would not usually be a substantial part of a Kindergarten mathematics curriculum. In the NCTM Principles and Standards (2000), subitization is noted for supporting “the development of visually grouping objects as a strategy for estimating quantities” (p.80), but it is not listed as a performance indicator in the number standard. However, subitization seems to serve as a precondition for cardinality, conservation of number, and part-whole relationships. When students succeed with any of these latter three skills, they also tend to succeed with subitization. Therefore, increasing the amount of time spent on subitization experiences for students will presumably help students become successful in mastering subitization, and by extension, cardinality, conservation of number, and part-whole relationships.

The simultaneous pathways learning model also impacts on the teaching of mathematics to young children. Teachers need to have in mind that students will be learning several early number skills at the same time, as the students will be on several pathways as they develop competence in mathematics. Teachers need to be aware not only of all the different pathways students can take, but also where each student is on his/her pathway.

This information about early number skills can be used to shape how teachers view the teaching of number to young children. The more subject knowledge teachers have, the stronger they are in the teaching of mathematics. “Knowledge of what young children can do and learn, as well as specific learning goals, are necessary for teachers to realize any vision of high-quality early childhood education” (Clements, 2004, p.9). Teachers need to have a high degree of subject matter knowledge as well as pedagogical knowledge. The discoveries in this research can help teachers understand more about each skill, as well as how each skill interacts with every other skill. The discoveries in this research can also help teachers have a stronger understanding of how to develop a cohesive mathematics curriculum.

> Knowledge of mathematics is obviously fundamental to being able to help someone else learn it. In order to select and construct fruitful tasks and activities for their pupils, as well as interpret and appraise pupils' ideas flexibly, teachers must understand the mathematical concepts and procedures themselves" (Ball, 1988, p.12).

Finally, it can help teachers know how to diagnose students to find those at-risk early in their schooling. The mathematical benchmarks developed from this research will help teacher know what to expect from their students. The predictive power of the skills will help teachers be aware of the dangers of not mastering certain skills by the end of Kindergarten. These tools can help teachers know how to evaluate their students' progress and know when to provide remediation.

> Are children making expected progress?...To answer (this) question, we must know what the expected standard is, we must know what to do to help children achieve it, and we must know how to assess what children have learned (Bredekamp, 2004, p.81).

Possible Changes that Could be Made to the Research Process

One change that could be made is to remove from the oral diagnostic test the skills that seem to matter less in diagnosis, and focus instead on the skills that seem to matter more. The oral diagnostic test could be streamlined so that it takes less time but still includes the most important and telling early number skills. The skills that could be excluded for this purpose would be: skip counting by twos, fives, and tens; compensation; and unitizing. Each of these skills is important and reveals more about the mathematical ability of the student, but does not necessarily reveal new information. However, they might be of use when diagnosing older students. Each of the remaining skills reveals new information about the student and would need to be included in the test.

Another change that could be made is to change the kind of sample used to one that is more statistically reliable. Using the convenience sample worked well enough for this study, but could not be considered statistically generalizable. For teachers to feel completely confident in this research, the sample needs to be one that is more statistically reliable than a convenience sample.

Possible Issues for Further Research

There are several possible issues that could be researched that stem from this study. One area to pursue would be: How do these same children fare now in mathematics in school? Are the students who were at-risk at the end of Kindergarten still at-risk? A way to answer these questions would be to follow up these same students and examine their scores from the standardized tests they have done in school in mathematics. These test scores would show whether the students have continued to finish in the lower attaining group of their grade or whether they have improved their mathematical competency.

Another area to pursue would be: How would students from other backgrounds, such as urban or suburban, perform with this test? Would students with different ethnic backgrounds perform the same? Would the benchmarks remain constant, or would students from a different environment have greater ability with the same skills? Another difference worth investigating would be whether students who have a more traditional, text-book curriculum for mathematics in Kindergarten perform differently on this same test. These are some of the variables that need to be teased out before stating with total confidence how Kindergarteners and 1st graders perform with early number skills. It is quite possible that students with different home and/or school environments could perform better or worse than the students of this study did.

A third question to pursue would be: What kind of oral diagnostic test could be used to do the same procedure with students in other grades? It would be helpful to create an evaluation system to diagnose students in each grade to find those at risk in order to provide effective remediation. Students can begin to falter at any grade, and providing immediate and effective remediation can help rectify the issue before it becomes insurmountable. Having a complete system of evaluation for each grade could help. Deciding on the skills to evaluate would be the first step; how to evaluate these same skills would be the second step; and analyzing the data to create benchmarks for each grade would be the third step.

One last question to pursue would be: How might using a different paradigm get at different information? A different paradigm and a different methodology might reveal new and important information. Rather than a study that mixes paradigms (such as this one), using a purely interpretive paradigm for doing a case study of several families could show how much the home environment factors into the development of mathematical competency in young children. How much the parents encourage their children to count or explore number could be seen with this kind of study, and could show ways in which parents, and not just teachers, could help young children in mathematics. This kind of study would bypass the limitation of the scientific paradigm which objectifies the child in favor of children. An interpretive study that focuses on the child in his/her natural family surroundings would use interviews and observations in order to create dialogue between the researcher and the child (as well as the parents). This would lead to an informed understanding (Cohen and Crabtree, 2006) of the influence of the home in the child's learning of mathematics.

Summary

My hope is that this study can help illuminate an area of mathematics that appears to be somewhat simple but is actually quite complicated, and that this illumination will ultimately help teachers and their students. Teachers can use this study to begin or continue the process of developing a conceptual map of early number skills in order to develop curriculum, teach effectively, assess efficiently, and remediate successfully.

> Mathematicians need to understand a problem only for themselves; math teachers need both to know the math and to know how 30 different minds might understand (or misunderstand) it. Then they need to take each mind from not getting it to mastery. And they need to do this in 45 minutes or less" (Green, 2010, p.7).

Using the oral diagnostic test used in this study can show teachers not only what early number skills need to be part of a mathematics curriculum, but also to what degree students need to master each skill. In addition, teachers can use the test to assess individual students as well as a full class in order to inform their teaching.

If at-risk students can be identified and given effective remediation early in their schooling, these students will be able to keep pace with their peers, rather than falling increasingly behind. If all young students can be helped to master early number skills, they will be stronger mathematicians as they progress through school. These early number skills form a crucial foundation to mathematics. As Baroody and Wilkins (1999) write,

> Indeed it could be argued that the construction of counting concepts and skills is the single most important element in preschoolers' mathematical development. Not only are counting competencies essential everyday 'survival skills' in their own right, they provide a basis for the development of number and arithmetic concepts and skills (p.51).

Certainly, more research is necessary to be assured of standardized and confident results. But if this study can help young children develop a solid mathematical foundation, and if this study can help teachers develop a more complete conceptual map of mathematics, then it will have done its job.

APPENDICES

Appendix 1: The Knowledge Package for Counting

Subitizing—means knowing the total of a small group of objects without counting. (Comes from the Latin meaning suddenly—like subito).

- This is important because every time a child stops to count, s/he stops thinking. (Think of what happens when one stops writing in order to try to spell a word—often one loses one's train of thought because of this pause).
- Also, when a child can see a group of 2, 3, 4, or 5 as a group rather than as discrete objects, s/he has started to unitize (see below).
- As teachers, we need to expose students to activities of subitizing in order to build the skill. "Quick Images" is a great way to do this.
- For older children, subitizing is necessary when working with arrays. Students might be tempted to count each square one-by-one, but can be shown that subitizing is faster. Cover part of each array to show only a small row that can be subitized, then skip count.
- Subitizing is also important for conservation. Showing cards that have the same number of dots on them but arranged differently is a way to expose students to the need to conserve—that there is the same number even though each card looks different.
- Subitizing also helps with addition pairs because once one addend is subitized, one can count on, rather than counting from one.

Rote Counting—means knowing the "song of counting."

- It is important that a child be able to count fluently. However, rote counting does not mean that those numbers have any meaning to the child. Attaching meaning to the numbers is called Rational Counting.
- Younger children need to know the song of counting up to 10, then 100, then 1000. Older children need to know the song of counting up to 10, 000, then 100, 000, then 1, 000, 000.
- Counting games, in which children "sing the song" from whatever number you choose are helpful. These are helpful, rather than counting objects, because we want children to get the song down first, and deal with higher numbers than we can have objects for. With older children, start at a high number. With younger children, start lower—but no matter what the age, count over a "hump"—a place where the place value shifts.

Notes:

Rational counting:

One-to-one correspondence—means knowing that there is one number for one object.

- A child who has one-to-one correspondence will be able to match up item with item, or match a number to an object. (A child might still "double count"—count an item twice—but this is a matter of organization).
- Young children need lots of experience counting small groups of objects to make this cognitive leap. Make every opportunity during the day to count everyday objects—snack items, books on the shelf, coats hanging by the door, etc.

Organized counting—means knowing that one needs to keep track of one's counting by some system: moving objects to the side, or into a line, etc.

- Older children will need to organize their counting when working with larger amounts, like grouping by tens.
- Give all students opportunities to count large amounts of items to practice organizing their counting.

Cardinality—means knowing that the last number counted is the amount of objects in the group.

- This is an important skill because then the child can continue mathematizing, rather than recounting.
- It is also important because children then understand that number means amount—something meaningful rather than simply the song of counting, and can move onto the next skill of hierarchical inclusion.
- In order to assess this skill, ask a student—after s/he has counted a group of objects—how many s/he has. If the child knows right away how many s/he has, then cardinality has been achieved.

Hierarchical Inclusion—means knowing that the quantity one has counted also has one less than the total.

- This means that numbers "nest" within each other. ("If I have 11, I also have 10").
- In order to assess this skill, after ascertaining cardinality, ask the student, "If you have…then do you also have…?" Make sure the latter amount is only one or two away from the total, otherwise the student has to make too large a leap, and might not answer correctly.
- This skill leads to an understanding of part/whole relationships and compensation and is therefore really important for older students when learning number pairs.

Notes:

Part/whole relationships and Compensation.

- The former skill means that the child knows, for example, that 9 can be made by 8 and 1.
- Compensation means that 7 and 2 make 9 because one can take one from 8 (which makes 7), and put it on the 1 to make 2.
- These concepts are crucial for becoming fluent in computation.

Conservation—means understanding that appearances do not change quantity.

- Conservation is important in all areas of math: number, geometry, measuring, data, etc. Older students struggle with conservation when turning a 10 x 6 array into a 5 x 12 array, wondering if each array has the same amount of squares.
- Younger children struggle with conservation in data when the daily question has differently spaced kidpins on either side, wondering if the side where the pins are spaced far apart has more than the side where the pins are close together.
- All children need many experiences with changing appearances of objects in order to make the cognitive leap that the number has remained the same.

Unitizing—means knowing that one can count in groups, that ten objects can become one group.

- When a child counts objects (as opposed to rote counting) by 2's, 5's, etc., s/he is unitizing.
- Activities that focus on natural groups of things to be counted help students unitize—like counting eyes (2's), or fingers in the class (5's). When using counters, try making towers first, rather than counting individual cubes.
- For older students, skip counting is the unitizing song, but that does not mean that children understand that each number has to have that number of items; "2, 4, 6, 8" means that there are two items, then four items, then six, then eight items.
- Unitizing is part of place value—when we compose units of ten, we are creating the next higher value.

Notes:

Appendix 2: Interview Questions for Assessing Counting

Student Name______________________________ **Date**___________________

(T = Teacher; S = Student)

Subitizing—knows total of group of objects under 6 without counting:

T: Make 4 groups of beans, with 2, 3, 4, and 5 beans in a group, in a random order. Ask student to point to each group of (2, 3, 4, or 5) without counting. Then mix up the bean groups, make them again, and repeat the task.

S points correctly to all groups without hesitating: __________
S hesitates for larger groups (4, 5): __________
S hesitates for all groups, counting with eyes: __________
Comments:

Rote Counting—knows how to count by rote fluently:

T: Ask: "Count out loud by ones for me as high as you can."

S counts to high numbers fluently: __________
S counts to 10 fluently but stumbles after that: __________
S cannot count to 10: __________

If student is a fluent counter, try having him/her count by 2's, 5's, and/or 10's.

S counts by 2's, 5, and 10's fluently: __________
S counts by ________fluently up to_________: __________
S cannot skip count: __________
Comments:

Conservation—knows that appearances do not change quantity:

T: Place two evenly spaced rows of 7 beans in front of the student and ask, "Do these two rows have the same number of beans in them, or does one row have more beans or fewer beans?" Then, while the student watches, push the beans of one row together and elongate the other row of beans. Ask, "Do these rows have the same number of beans in them or does one row have more beans or fewer beans?"

S says, "They're still equal." (conserver) __________
S says, "I'm not sure; I have to count." (transitional conserver) __________
S says, "No, that one has more because it's longer." (non-conserver) __________
Comments:

Rational Counting—the next 4 tasks are done one right after each other based on the initial counting below. The student only has to count once:

T: Place 10-25 counters (depending on the age and rote counting ability of the child) in a clump in front of the student. Ask: "Would you count these out loud for me?"

One-to-One Correspondence—knows there is one number for one object:

S gives one number to one object: __________
S double counts or misses counters when counting: __________
Comments:

<u>**Organized Counting**—knows to keep track of what is counted:</u>
S touches each item once, showing a system of organization: __________
S cannot count in an organized manner: __________
Comments:

<u>**Cardinality**—knows that the last number counted is the number of objects in the group:</u>
As soon as the student has finished counting, T asks, "How many do you have?"
S tells correct number without hesitation: __________
S needs to recount: __________
S has no strategy for knowing: __________
Comments:

<u>**Hierarchical Inclusion**—knows that numbers "nest" within a total:</u>
T asks: "If you have x# of counters, do you also have x – 1# counters?"
S says, "Yes, and I also have (lists some or all lower #'s): __________
S says, "No! I have (tells final quantity counted):" __________
Comments:

<u>**Part/Whole Relationships**—knows a number can be made by adding or subtracting numbers:</u>
T asks, "Can you tell me two numbers that add up to (5? 10? 15? depending on the age of the student):
S gives a combination: __________
S cannot come up with a combination: __________
Comments:

<u>**Compensation**—knows that numbers can be composed in many ways:</u>
T asks, "Can you think of other pairs that add up to that same number?"
S says, "If a + b = c, then (a – 1) + b = c." __________
S finds other pairs randomly: __________
S cannot find other pairs: __________
Comments:

<u>**Unitizing**—knows how to group counters so that the correct number is in each group when skip counting:</u>
T asks a fluent counter to count 15-20 counters by groups.
S counts counters by 2's, or 5's, etc: __________
S skip counts by rote but pushes only 1 aside: __________
Comments:

Appendix 3: Revised Oral Diagnostic Test

Student Name______________________________ **Date___________________**
(S = Student) Total Score____________________ Class Midpoint___________

One-to-one Correspondence—knows there is one number for one object:

Place 20 counters in a clump in front of the student. Ask: "Would you count these out loud for me?"

S gives one number to one object: 5
S double counts or misses counters when counting: 1
Comments:

Cardinality—knows that the last number counted is the number of objects in the group:

As soon as the student has finished counting, ask, "How many do you have?"

S tells correct number without hesitation: 5
S recounts before telling the total: 3
S has no strategy for knowing: 1
Comments:

Organized Counting—knows to keep track of what is counted:

Watch while S counts objects to see if S is showing a system of organization while counting:

S counts in an organized manner, keeping track of what has been counted: 5
S does not count in an organized manner: 1
Comments:

Hierarchical Inclusion—knows that numbers "nest" within a total:

Ask: "If you have 20 counters right here, do you also have 19 counters right here?"

S says, "Yes, and I also have (lists some or all lower numbers)": 5
S says, "No! I have 20": 1
Comments:

Revised Oral Diagnostic Test, p.2

Student Name______________________________ Date___________________

<u>**Rote Counting**—knows how to count without objects fluently:</u>

Ask: "Count out loud by ones for me as high as you can."

S counts to 100 or above:	5
S counts to 99 or below:	3
S counts to 20 or below:	1

<u>Comments</u>**:**

<u>**Subitization**—knows total of group of objects under 6 without counting:</u>

Make 4 groups of beans, with 2, 3, 4, and 5 beans in a group, in a random order. Ask student to point to each group of (2, 3, 4, or 5) without counting. Then mix up the bean groups, make them again, and repeat the task.

S points correctly to all groups without hesitating:	5
S hesitates for larger groups (4, 5):	3
S hesitates for all groups, counting with eyes:	1

Comments:

<u>**Part-whole Relationships**—knows a whole number can be split into parts:</u>

Ask, "Can you tell me two numbers that add up to 5? Can you tell me two numbers that add up to 10?

S gives a combination for both 5 and 10:	5
S gives a combination for 5 or 10:	3
S cannot come up with a combination for either sum:	1

Comments:

<u>**Conservation of Number**—knows that appearances do not change quantity:</u>

Place two evenly spaced rows of 7 beans in front of the student and ask, "Do these two rows have the same number of beans in them, or does one row have more or less beans?" Then, <u>while the student watches</u>, push the beans of one row together and elongate the other row of beans. Ask, "Do these two rows have the same number of beans in them or does one row have more or less beans?"

S says, "They're still equal." (conserver)	5
S says, "I'm not sure; I have to count." (transitional conserver)	3
S says, "No, that one has more because it's longer." (non-conserver)	1

Comments:

Appendix 4: Cohort 1—Kindergarten '06 Data & 1st Grade '07 Scores

	K-06	K-06	K-06	K-06	K-06	K-06	K-06	K-06	K-06	1-07
I.D. #	One-to-one	Card.	Org.	H.I.	Rote Count	Sub.	Part-whole	Cons	Score/ Place	Score/ Place
3	5	5	5	1	1	3	1	1	22—LA	30—LA
4	5	5	5	5	5	3	1	1	30—HA	38—HA
5	5	1	1	1	5	5	1	3	22—LA	38—HA
6	5	5	5	5	5	5	5	5	40—HA	Gone
7	1	3	1	5	1	3	1	1	16—LA	32—LA
8	1	5	1	5	3	5	3	5	28—LA	32—LA
9	5	5	5	5	3	5	1	1	30—HA	Gone
10	1	1	1	1	3	1	1	1	10—LA	Gone
11	5	5	1	1	3	3	1	5	24—LA	30—LA
12	5	3	1	1	3	5	1	3	22—LA	34—LA
13	5	1	1	5	1	1	1	1	16—LA	Gone
14	5	3	1	5	3	5	3	1	26—LA	34—LA
15	5	5	1	5	3	5	3	3	30—HA	38—HA
16	5	5	5	5	3	5	5	3	36—HA	Gone
17	5	5	1	5	3	1	1	1	22—LA	36—LA
18	5	5	5	5	5	3	5	5	38—HA	40—HA
19	5	3	1	5	5	5	5	1	30—HA	36—LA
20	5	5	5	1	3	5	1	5	30—HA	36—LA
21	5	5	5	1	5	5	5	5	36—HA	38—HA
22	5	5	1	5	1	3	1	5	26—LA	36—LA
23	5	5	5	5	5	5	5	5	40—HA	38—HA
24	5	5	5	5	5	5	5	3	38—HA	40—HA
25	5	5	5	1	5	5	1	1	28—LA	26—LA
26	5	5	5	1	3	3	1	1	24—LA	32—LA
27	5	5	5	1	3	5	3	5	32—HA	38—HA
28	5	5	5	5	3	5	5	5	38—HA	32—LA
29	5	5	5	1	5	5	1	5	32—HA	40—HA
30	5	5	5	5	5	5	5	3	38—HA	40—HA
31	5	5	5	5	5	5	5	5	40—HA	40—HA
32	5	5	5	5	3	3	1	3	30—HA	36—LA
33	5	5	5	1	3	5	5	5	34—HA	38—HA
34	5	5	5	1	1	5	1	3	26—LA	40—HA
35	5	5	5	5	3	3	1	1	28—LA	Gone
36	5	5	5	1	3	3	1	1	24—LA	34—LA
37	5	5	5	5	5	5	5	5	40—HA	38—HA
38	5	5	5	5	3	5	1	1	30—HA	34—LA
39	5	5	5	1	3	3	1	3	26—LA	34—LA
40	5	3	5	1	3	5	1	5	28—LA	34—LA
41	5	5	5	5	3	3	5	5	34—HA	Gone
42	5	5	5	1	3	3	1	1	24—LA	28—LA
43	5	5	1	5	3	1	1	5	26—LA	30—LA
44	1	3	1	1	1	3	3	1	14—LA	28—LA
45	1	5	1	1	3	1	1	1	14—LA	Gone
46	5	5	5	1	5	5	5	1	32—HA	32—LA
47	5	5	5	1	3	3	1	5	28—LA	Gone
48	5	5	1	5	5	5	5	5	36—HA	40—HA

Appendix 5: Cohort 1—1st Grade '07 Data

I.D. #	1--07 1-1	1--07 Card.	1--07 Rote Count	1--07 Org.	1--07 Part-whole	1--07 H.I.	1--07 Sub.	1—07 Cons.	1-07 Score/ Placement
3	5	3	5	5	1	5	3	3	30—LA
4	5	5	5	5	5	5	3	5	38—HA
5	5	5	5	5	5	5	3	5	38—HA
6	***	***	***	***	***	***	***	***	Gone
7	5	5	1	5	5	5	5	1	32—LA
8	5	5	5	5	3	5	3	1	32—LA
9	***	***	***	***	***	***	***	***	Gone
10	***	***	***	***	***	***	***	***	Gone
11	5	5	5	5	1	1	3	5	30—LA
12	5	5	5	5	5	1	3	5	34—LA
13	***	***	***	***	***	***	***	***	Gone
14	5	5	3	5	5	5	5	1	34—LA
15	5	5	5	5	5	5	3	5	38—HA
16	***	***	***	***	***	***	***	***	Gone
17	5	5	5	5	5	5	3	3	36—LA
18	5	5	5	5	5	5	5	5	40—HA
19	5	5	5	5	5	5	3	3	36—LA
20	5	5	5	5	5	1	5	5	36—LA
21	5	5	5	5	5	5	3	5	38—HA
22	5	5	5	5	3	5	3	5	36—LA
23	5	5	5	5	5	5	3	5	38—HA
24	5	5	5	5	5	5	5	5	40—HA
25	5	1	5	5	3	1	3	3	26—LA
26	5	5	5	5	5	1	3	3	32—LA
27	5	5	5	5	5	5	3	5	38—HA
28	1	5	5	1	5	5	5	5	32—LA
29	5	5	5	5	5	5	5	5	40—HA
30	5	5	5	5	5	5	5	5	40—HA
31	5	5	5	5	5	5	5	5	40—HA
32	5	5	5	5	1	5	5	5	36—LA
33	5	5	5	5	5	5	3	5	38—HA
34	5	5	5	5	5	5	5	5	40—HA
35	***	***	***	***	***	***	***	***	Gone
36	5	5	5	5	5	5	3	1	34—LA
37	5	5	5	5	5	5	5	3	38—HA
38	5	5	5	5	5	1	5	3	34—LA
39	5	5	5	1	5	5	3	5	34—LA
40	5	5	5	5	3	1	5	5	34—LA
41	***	***	***	***	***	***	***	***	Gone
42	5	5	5	1	5	1	3	3	28—LA
43	5	1	5	1	5	5	3	5	30—LA
44	5	5	3	5	5	1	3	1	28—LA
45	***	***	***	***	***	***	***	***	Gone
46	1	3	5	5	5	5	5	3	32—LA
47	***	***	***	***	***	***	***	***	Gone
48	5	5	5	5	5	5	5	5	40—HA

Appendix 6: Cohort 2—Kindergarten '07 Data & 1st Grade '08 Scores

	K-07	K-07	K-07	K-07	K-07	K-07	K-07	K-07	K-07	1-08
I.D. #	One-to-one	Card.	Org.	H.I.	Rote count	Sub.	Part-whole	Cons	Score/ Place	Score/ Place
3	5	5	5	5	1	5	1	1	28—LA	Gone
4	5	5	1	5	5	5	5	5	36—HA	40—HA
5	1	5	1	5	3	3	1	1	20—LA	24—LA
6	5	5	5	5	3	3	1	1	28—LA	36—LA
7	5	5	5	1	5	3	3	1	28—LA	32—LA
8	5	3	1	1	3	3	1	1	18—LA	36—LA
9	5	5	5	5	5	3	5	5	38—HA	40—HA
10	5	5	1	5	3	3	3	3	28—LA	28—LA
11	5	5	5	1	3	1	1	3	24—LA	32—LA
12	5	5	5	5	3	3	3	1	30—HA	38—HA
13	5	5	5	1	5	3	1	1	26—LA	36—LA
14	5	5	5	5	5	3	5	3	36—HA	40—HA
15	5	3	1	5	3	5	5	3	30—HA	38—HA
16	5	3	5	1	3	1	3	1	22—LA	26—LA
17	5	5	5	5	5	3	5	3	36—HA	40—HA
18	1	5	1	5	3	5	1	3	24—LA	34—LA
19	5	3	1	1	5	3	3	3	24—LA	40—HA
20	5	5	1	1	3	5	1	1	22—LA	28—LA
21	5	5	5	1	3	3	1	3	26—LA	38—HA
22	5	3	1	5	3	5	1	5	28—LA	38—HA
23	5	5	5	1	5	3	1	1	26—LA	Gone
24	5	5	5	1	3	3	3	3	28—LA	40—HA
25	5	3	1	5	5	3	1	3	26—LA	38—HA
26	5	5	5	1	5	3	1	1	26—LA	38—HA
27	5	5	5	5	3	3	1	1	28—LA	Gone
28	1	5	5	5	5	5	3	5	34—HA	38—HA
29	5	3	5	5	1	5	1	1	26—LA	Gone
30	5	5	5	5	5	5	5	5	40—HA	40—HA
31	5	5	5	5	5	5	5	5	40—HA	40—HA
32	5	5	5	5	5	3	5	3	36—HA	38—HA
33	5	3	5	5	3	1	1	1	24—LA	32—LA
34	5	5	1	5	5	3	3	1	28—LA	40—HA
35	5	5	5	5	5	3	5	1	34—HA	38—HA
36	5	5	5	5	5	3	5	1	34—HA	36—LA
37	5	5	5	5	5	5	5	3	38—HA	40—HA
38	5	5	5	5	5	5	1	1	32—HA	40—HA
39	5	5	5	5	3	3	5	5	36—HA	40—HA
40	5	5	5	5	1	1	1	5	28—LA	34—LA
41	5	5	5	5	5	5	5	1	36—HA	40—HA
42	5	5	5	5	5	3	5	5	38—HA	40—HA
43	5	5	5	1	1	3	1	3	24—LA	Gone
44	5	5	5	5	5	3	5	5	38—HA	40—HA
45	5	5	5	5	3	1	1	1	26—LA	32—LA
46	1	5	5	5	5	3	1	1	26—LA	30—LA
47	5	5	5	5	5	5	5	3	38—HA	Gone
48	5	5	5	5	5	3	5	5	38—HA	36—LA
49	5	1	5	5	3	5	3	1	28—LA	40—HA

Appendix 7: Cohort 2—1st Grade ’08 Data

I.D. #	1-08 1-1	1-08 Card.	1-08 Rote Count	1-08 Org.	1-08 Part-whole	1-08 H.I.	1-08 Sub.	1-08 Cons.	1-08 Score/ Place
3	***	***	***	***	***	***	***	***	Gone
4	5	5	5	5	5	5	5	5	40—HA
5	1	3	5	1	3	5	5	1	24—LA
6	5	5	5	5	5	5	5	1	36—LA
7	5	5	5	5	1	5	5	1	32—LA
8	5	5	5	5	5	5	3	3	36—LA
9	5	5	5	5	5	5	5	5	40—HA
10	5	5	5	5	1	1	3	3	28—LA
11	5	5	3	5	5	1	3	5	32—LA
12	5	5	5	5	5	5	3	5	38—HA
13	5	5	5	5	5	5	3	3	36—LA
14	5	5	5	5	5	5	5	5	40—HA
15	5	5	5	5	5	5	3	5	38—HA
16	5	5	5	5	1	1	1	3	26—LA
17	5	5	5	5	5	5	5	5	40—HA
18	5	5	5	1	5	5	5	3	34—LA
19	5	5	5	5	5	5	5	5	40—HA
20	5	5	5	1	5	1	5	1	28—LA
21	5	5	5	5	5	5	5	3	38—HA
22	5	5	5	5	3	5	5	5	38—HA
23	***	***	***	***	***	***	***	***	Gone
24	5	5	5	5	5	5	5	5	40—HA
25	5	5	5	5	5	5	5	3	38—HA
26	5	5	5	5	5	5	5	3	38—HA
27	***	***	***	***	***	***	***	***	Gone
28	5	5	5	5	5	5	5	3	38—HA
29	***	***	***	***	***	***	***	***	Gone
30	5	5	5	5	5	5	5	5	40—HA
31	5	5	5	5	5	5	5	5	40—HA
32	5	5	5	5	5	5	3	5	38—HA
33	5	5	5	5	5	1	5	1	32—LA
34	5	5	5	5	5	5	5	5	40—HA
35	5	5	5	5	5	5	5	3	38—HA
36	5	5	5	5	5	5	5	1	36—LA
37	5	5	5	5	5	5	5	5	40—HA
38	5	5	5	5	5	5	5	5	40—HA
39	5	5	5	5	5	5	5	5	40—HA
40	5	5	3	5	5	5	3	3	34—LA
41	5	5	5	5	5	5	5	5	40—HA
42	5	5	5	5	5	5	5	5	40—HA
43	***	***	***	***	***	***	***	***	Gone
44	5	5	5	5	5	5	5	5	40—HA
45	5	5	3	5	1	5	3	5	32—LA
46	5	5	5	5	3	1	5	1	30—LA
47	***	***	***	***	***	***	***	***	Gone
48	5	5	5	5	5	5	5	1	36—LA
49	5	1	5	5	3	5	3	1	40—HA

Appendix 8: Complete Data of 93 Kindergarteners and 78 1st Graders

Gender	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
Males	23—50%	18—41%	28—60%	23—56%	51—55%	41—53%
Females	23—50%	19—51%	19—40%	18—44%	42—45%	37—47%
Total	46	37	47	41	93	78

Age	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
5 yr olds	21—46%		15—32%		36—39%	
6 yr olds	23—50%	13—35%	30—64%	16—39%	53—57%	29—37%
7 yr olds	2—4%	23—62%	2—4%	25—61%	4—4%	48—62%
8 yr olds		1—3%		0—0%		1—1%

1-to-1	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
Has 1-1	41—89%	35—95%	43—91%	40—98%	84—90%	75—96%
No 1-1	5—11%	2—5%	4—9%	1—2%	9—10%	3—4%

Cardinality	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
Has card.	37—80%	33—89%	38—81%	40—98%	75—81%	73—94%
Recounts	6—13%	2—5%	8—17%	1—2%	14—15%	3—4%
No card.	3—7%	2—5%	1—2%	0—0%	4—4%	2—2%

Org. count	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
Has org.	30—65%	33—89%	36—77%	38—93%	66—71%	71—91%
No org.	16—35%	4—11%	11—23%	3—7%	27—29%	7—9%

Hier. inc.	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
Has h. i.	25—54%	28—76%	35—74%	35—85%	60—65%	63—81%
No h. i.	21—46%	9—24%	12—26%	6—15%	33—35%	15—19%

Rote count	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
1-20	6—13%	1—3%	5—11%	0—0%	10—11%	1—1%
21-49	21—46%	2—5%	12—25%	3—7%	34—36%	5—6%
50-99	4—9%	0—0%	5—11%	0—0%	9—10%	0—0%
100-149	12—26%	33—89%	23—49%	30—73%	35—38%	63—81%
150-200	3—7%	1—3%	2—4%	8—20%	5—5%	9—12%

Subitizing	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
2, 3, 4, 5	26—57%	16—43%	15—32%	31—76%	41—44%	47—60%
2, 3	15—33%	21—56%	27—57%	9—22%	42—45%	30—38%
None	5—10%	0—0%	5—11%	1—2%	10—11%	1—1%

Part-whl.	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
Both	15—33%	30—81%	17—36%	34—83%	32—34%	64—82%
Only 5	2—4%	0—0%	6—13%	2—5%	8—9%	2—3%
Only 10	3—6%	4—10%	3—6%	1—2%	6—6%	5—6%
No pairs	26—57%	3—8%	21—45%	4—10%	47—51%	7—9%

Cons. of #	*K '06*	*1st gr. '07*	*K '07*	*1st gr. '08*	*Total K*	*Total 1st*
Conserves	19—41%	23—62%	11—23%	22—53%	30—32%	45—58%
Transition.	9—20%	9—24%	14—30%	11—27%	23—25%	20—25%
Non-con.	18—39%	5—14%	22—47%	8—20%	40—43%	13—17%

Appendix 9: Permission Slip to Parents

Dear Parents,

I am MSAD #9's Math Consultant, and am also a graduate student for a doctorate in Math Education. For my studies, I plan to do research on counting skills in Kindergarteners. **I write to ask your permission to interview your child for this project**. The interview takes about ten to fifteen minutes. The results will remain anonymous, as I am only interested in the age and gender of your child. Your child's teacher has assured me that your child will not miss any important class work.

I plan to share my findings with the Kindergarten teachers of MSAD #9 in order to increase our expertise in teaching math.

Please sign the form below and return to your child's teacher by Friday, April 14, 2006.

Thank you for your help in this matter.

Victoria Cohen

****

PLEASE RETURN TO YOUR CHILD'S TEACHER BY
FRIDAY, APRIL 14, 2006

I do give permission for my child____________________________________
to be interviewed by Victoria Cohen concerning counting skills.

I do not give permission for my child_________________________________
to be interviewed by Victoria Cohen concerning counting skills.

Parent signature____

Date_____________________________

Thank you for your help!

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