

Tracking the Early Number Skills Performance
of 5- to 7-Year-Old Students: A Longitudinal Study

Submitted by Victoria Joy Cohen
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I certify that all material in this dissertation which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

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ABSTRACT

This longitudinal study tracks how 5- to 7-year-olds perform with early number skills. The aim of this study is to diagnose at-risk mathematics students by distinguishing the skills that, if not mastered by the end of Kindergarten, lead to greater difficulty in mathematics in 1st grade. This study's methodology is mixed as it follows an exploratory and inductive path in light of its use of a hypothesis, an interpretive path in light of its interest in the individual student, and a positivist path in light of its focus on developing rules from analyzed data. An oral diagnostic test based on a comprehensive collection of early number skills was used to test students as Kindergarteners and again as 1st graders. The test results created benchmarks, revealing how the majority of the students performed with early number skills. The test results also revealed that each early number skill is highly, moderately, or minimally predictive in terms of student placement by the end of 1st grade. When comparing the individual skill scores of each Kindergarten student to his/her total test results of 1st grade, the predictive power of each skill emerged. Performing poorly with skills that are minimally predictive did not seem to have an impact on how the Kindergarten student finished in 1st grade; performing poorly with moderately predictive skills had a greater impact on 1st grade placement; performing poorly with highly predictive skills in Kindergarten increased the likelihood that the student would finish in the lower attaining group in 1st grade. A third result of the test showed that certain skills serve as preconditions for other skills; success with certain skills usually meant success with other skills. These connections between skills point to a learning model called in this study "simultaneous pathways," indicating that there are connections between certain skills, and that students can be learning on several pathways simultaneously. The impact of the predictive power of early number skills is that diagnosis becomes more effective. Early diagnosis means early remediation which may prevent at-risk students from falling further behind their peers. The benchmarks developed by this research will help teachers assess their students because they will know the general skill level of Kindergarteners and 1st graders. This oral diagnostic test informs curriculum development. If test results show that students are missing the skills that are highly predictive, teachers can address those gaps in order to insure mastery.

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