

**Modelling Light Transport Through Biological
Tissue Using the Simplified Spherical Harmonics
Approximation**

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Declaration

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Abstract

Optical Tomography is a medical imaging modality that can be used to non-invasively image functional changes within the body. As near-infrared light is highly scattered by biological tissue, the process of image reconstruction is ill-posed and, in general is also under-determined. As such, model based iterative image reconstruction methods are used. These methods require an accurate model of light propagation through tissue, also known as the forward model.

The diffusion approximation (DA) to the radiative transport equation is one of the most widely used forward models. It is based on the assumption that scattering events dominate over absorption events resulting in a diffuse light distribution. This is valid in cases with low absorption coefficients or large geometries (greater than a few scattering lengths). In many cases, however, such as in small animal imaging where the source-detector separation is small, this assumption is not valid and so a higher-ordered approximation is required.

In this thesis, a three-dimensional frequency domain forward model based on the simplified spherical harmonics (SP_N) approximation to the radiative transport equation is introduced. By comparison with a Monte-Carlo model, the SP_N approximation is shown to be more accurate than the DA, especially in regions near to the sources and detectors and the increase in accuracy is greater in cases with stronger absorption. This is particularly important for bioluminescent imaging of small animals which involve both small geometries and strong absorption. Due to the asymptotic nature of the

SP_N approximation, the highest ordered model was not necessarily the most accurate, but all models with $N>1$ were more accurate than the DA.

The SP_N based forward model has also been implemented into an image reconstruction algorithm. Despite the fact that the SP_N approximation does not combine the scattering coefficient and anisotropy factor into a single variable, as is the case in the DA, it was found that it is not possible to reconstruct them uniquely. The SP_N based models were shown to be able to reconstruct optical maps with greater accuracy than the DA. However, due to the increased number of unknowns to be recovered, the SP_7 based reconstructed images contained significant artefact and cross-talk.

Finally, a SP_N -Diffusion hybrid model was developed in which the SP_N model was used in the regions near to the source and the DA elsewhere. This model provides the increase of accuracy of the SP_N models in the regions where the DA is insufficient, whilst retaining the computational efficiency of the DA. It was shown that the hybrid model leads to increased accuracy not only in the regions solved using the SP_N model, but also in the DA based regions where as in a pure DA model, the errors near the source were propagated throughout the domain. It is also shown that the hybrid model can be solved in half the time of the full SP_N model.

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List of publications

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Journal papers

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