Long Memory Conditional Volatility and Dynamic Asset Allocation

Submitted by Anh Thi Hoang Nguyen to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Finance in September 2011.

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

(Signature) …………………………………………………………………………………………………
Acknowledgement

I would like to express my deep and sincere gratitude to my supervisor, Professor Richard Harris for his motivation and continuous support of my PhD research. His guidance helped me through all the time of research and writing of this thesis. I could not have imagined having a better supervisor and mentor for my PhD study.

I would also like to thank the Board of Examiners for their constructive comments and suggestions in my viva.

I am grateful to Gilles Zumbach for his useful comments on the analysis of the forecast performance of multivariate long memory volatility models.

I would like to thank the staff at the University of Exeter Business School, the IT officers and the librarians for their help in many different ways throughout my research.

I gratefully acknowledge the funding from the Ministry of Education and Training, Vietnam, that made my PhD work possible.

My time at Exeter was made enjoyable in large part due to many friends and groups that have become a part of my life. Thank you.

Lastly, and most importantly, I would like to thank my family for all their love, understanding and encouragement. To my parents, who raised me with a love of science and have been a constant source of support, and to my husband, who always stands by me and encourages me with his love, I dedicate this thesis.
Abstract

The thesis first evaluates the forecast performance of multivariate long memory conditional volatility models among themselves and against that of short memory conditional volatility models, using the asset allocation framework of Engle and Colacito (2006). While many alternative conditional volatility models have been developed in the literature, my choice reflects the need for parsimonious models that can be used to forecast high dimensional covariance matrices. In particular, I compare the statistical and economic performance of four multivariate long memory volatility models (the long memory EWMA, long memory EWMA-DCC, FIGARCH-DCC and Component GARCH-DCC models) with that of two multivariate short memory volatility models (the short memory EWMA and GARCH-DCC models). The research reports two main findings. First, for longer horizon forecasts, long memory volatility models generally produce forecasts of the covariance matrix that are statistically more accurate and informative, and economically more useful than those produced by short memory volatility models. Second, the two parsimonious long memory EWMA models outperform the other models – both short memory and long memory – in a majority of cases across all forecast horizons. These results apply to both low and high dimensional covariance matrices with both low and high correlation assets, and are robust to the choice of estimation window.

The multivariate conditional volatility models are then analysed further to shed light on the benefits of allowing for long memory volatility dynamics in forecasts of the covariance matrix for dynamic asset allocation. Specifically, the research evaluates the economic gains accruing to long memory volatility timing strategies, using the procedure of Fleming et al. (2001). The research consistently identifies the gains from incorporating long memory volatility dynamics in investment decisions. Investors are willing to pay to switch from the static to the dynamic strategies, and especially from the short memory volatility timing to the long memory volatility timing strategies across both short and long investment horizons. Among the long memory conditional volatility models, the two parsimonious long memory EWMA models, again, generally produce the most superior portfolios. When transaction costs are taken into account, the gains from the daily rebalanced dynamic portfolios deteriorate; however, it is still worth implementing the dynamic strategies at lower rebalancing frequencies. The results are robust to estimation error in expected returns, the choice of risk aversion coefficients and the use of a long-only constraint.
The long memory conditional covariance matrix is inevitably subject to estimation error. The research then employs a factor structure to control for estimation error in forecasts of the high dimensional covariance matrix. Specifically, the research develops a dynamic long memory factor (the Orthogonal Factor Long Memory, or OFLM) model by embedding the univariate long memory EWMA model of Zumbach (2006) into an orthogonal factor structure. The new factor model follows richer processes than normally assumed, in which both the factors and idiosyncratic shocks are modelled with long memory behaviour in their volatilities. The factor-structured OFLM model is evaluated against the six above multivariate conditional volatility models, especially the fully estimated multivariate long memory EWMA model of Zumbach (2009b), in terms of forecast performance and economic benefits. The results suggest that the OFLM model generally produces impressive forecasts over both short and long forecast horizons. In the volatility timing framework, portfolios constructed with the OFLM model consistently dominate the static and other dynamic volatility timing portfolios in all rebalancing frequencies. Particularly, the outperformance of the factor-structured OFLM model to the fully estimated LM-EWMA model confirms the advantage of the factor structure in reducing estimation error. The factor structure also significantly reduces transaction costs, making the dynamic strategies more feasible in practice. The dynamic factor long memory volatility model also consistently produces more superior portfolios than those produced by the traditional unconditional factor and the dynamic factor short memory volatility models.
# Tables of Content

Acknowledgement 2

Abstract 3

Table of Contents 5

List of Tables 9

List of Figures 12

List of Appendices 13

List of Abbreviations 14

Author’s Declaration 16

## CHAPTER 1 INTRODUCTION 17

1.1 Background and Rationale 17

1.2 Research Questions and Scope 20

1.3 Contribution of the Thesis 21

1.4 Structure of the Thesis 23

## CHAPTER 2 THE CLASSICAL ASSET ALLOCATION FRAMEWORK AND COVARIANCE MATRIX ESTIMATORS 25

2.1 The Classical Asset Allocation Framework 26
   2.1.1 Markowitz’s Mean-Variance Portfolio Optimisation Theory 26
   2.1.3 Incorporating Utility Theory – How to Invest Optimally? 31
   2.1.4 Application of the Classical Asset Allocation Theory 33
   2.1.5 Relaxation of the Assumptions 37

2.2 Covariance Matrix Estimators 41
   2.2.1 The Sample Covariance Matrix Estimator 41
   2.2.2 Factor Models 42
      2.2.2.1 The Linear Factor Decomposition 43
      2.2.2.2 Single Factor Models 43
      2.2.2.3 Multifactor Models 44
      2.2.2.4 Practical Implementation and Issues 48
   2.2.3 Shrinkage Models 51
   2.2.4 The Constant Correlation Coefficient Model 52
2.3 Conclusion

CHAPTER 3 THE TIME-VARYING CONDITIONAL VARIANCE-COVARIANCE MATRIX

3.1 Properties of Asset Return Volatility

3.2 Moving Average Models
   3.2.1 The Equally Weighted Moving Average Model
   3.2.2 The Exponentially Weighted Moving Average Model

3.3 GARCH Models
   3.3.1 Univariate GARCH Models
   3.3.1.1 The Basic ARCH Model
   3.3.1.2 The GARCH Model
   3.3.1.3 Other GARCH Models
   3.3.2 Multivariate GARCH Models
   3.3.2.1 Multivariate GARCH Models
   3.3.2.2 The Dynamic Conditional Correlation Model

3.4 Long Memory Volatility Models
   3.4.1 The Fractionally Integrated GARCH Model
   3.4.2 The Hyperbolic GARCH Model
   3.4.3 Component, Break and Regime Switching Volatility Models
      3.4.3.1 The Component GARCH Model
      3.4.3.2 Structural Break Models
      3.4.3.3 Regime Switching Models
   3.4.4 Multivariate Long Memory Volatility Models

3.5 Conditional Volatility Models and Asset Allocation

CHAPTER 4 DATA ANALYSIS

4.1 Data Description

4.2 Evidence of Long Memory in Volatility

CHAPTER 5 LONG MEMORY CONDITIONAL VOLATILITY AND ASSET ALLOCATION

5.1 Multivariate Long Memory Conditional Volatility Models
   5.1.1 The Multivariate LM-EWMA Model
   5.1.2 The Multivariate LM-EWMA-DCC Model
   5.1.3 The FIGARCH(1,d,1)-DCC Model
   5.1.4 The CGARCH(1,1)-DCC Model
   5.1.5 The RiskMetrics EWMA Model
List of Tables

Table 4.1. Summary Statistics for the Two Bivariate Systems .................................................. 89
Table 4.2. Summary Statistics for the International Stock and Bond Returns ......................... 90
Table 4.3. Summary Statistics for the DJIA Components.......................................................... 91
Table 4.4. Fractional Difference Operators for the Two Bivariate Systems ....................... 92
Table 4.5. Autocorrelations and Fractional Difference Operators for the Multivariate Systems ................................................................................................................... 93
Table 5.1. RMSE, MAE and HMSE for the Two Bivariate Systems ........................................ 115
Table 5.2. Mincer–Zarnowitz Regressions for the Two Bivariate Systems ........................... 116
Table 5.3. Comparison of Conditional Volatilities: Bivariate Portfolios ........................... 117
Table 5.4. Comparison of Out-of-Sample Volatilities: Bivariate Portfolios ....................... 118
Table 5.5. Diebold–Mariano Tests of the Stock-Bond Portfolio ........................................... 119
Table 5.6. Diebold–Mariano Tests of the S&P500-DJIA Portfolio ....................................... 120
Table 5.7. Comparison of Volatilities: Multivariate Portfolios ............................................ 121
Table 5.8. Comparison of Conditional Volatilities: Hedging International Portfolios ......... 122
Table 5.9. Comparison of Conditional Volatilities: Hedging DJIA Portfolios .................. 123
Table 5.10. Diebold–Mariano Joint Tests: Hedging Multivariate Portfolios ....................... 124
Table 5.11. RMSE for Longer Horizon Forecasts: Bivariate Systems................................... 125
Table 5.12. Mincer–Zarnowitz Regressions for Longer Horizons: Bivariate Systems ........ 126
Table 5.13. Comparison of Out-of-Sample Volatilities: Bivariate Portfolios with Weekly Rebalancing Frequency ................................................................. 128
Table 5.14. Comparison of Out-of-Sample Volatilities: Bivariate Portfolios with Quarterly Rebalancing Frequency ................................................................. 129
Table 5.15. Comparison of Volatilities: Multivariate Portfolios with Different Rebalancing Frequencies ...................................................................................... 130
Table 5.16. Diebold–Mariano Joint Tests: Hedging DJIA Portfolios with Different Rebalancing Frequencies ...................................................................................... 131
Table 5.17. Comparison of Out-of-Sample Volatilities: Bivariate Portfolios with 5-Year Estimation Window ...................................................................................... 132
Table 5.18. Diebold–Mariano Joint Tests: Hedging DJIA Portfolios with 5-Year Estimation Window ...................................................................................... 133
Table 6.1. The Economic Values of Dynamic Strategies: Daily Rebalanced Bivariate Portfolios .................................................................................................................. 164
Table 6.2. The Economic Values of Dynamic Strategies: Weekly Rebalanced Bivariate Portfolios .................................................................................................................. 165
Table 6.3. The Economic Values of Dynamic Strategies: Monthly Rebalanced Bivariate Portfolios ................................................................. 166
Table 6.4. Performance Fees to Switch from the Short Memory GARCH Volatility Timing Strategy to the Long Memory Volatility Timing Strategy .................. 167
Table 6.5. Performance Fees to Switch from the Short Memory EWMA Volatility Timing Strategy to the Long Memory Volatility Timing Strategy ..................... 168
Table 6.6. Breakeven Transaction Costs of the Bivariate Portfolios ......................... 169
Table 6.7. Estimation Error in Expected Returns: The Sharpe Ratios of the Daily Rebalanced Bivariate Portfolios ................................................................. 170
Table 6.8. Estimation Error in Expected Returns: Relative Performance Fees of the Daily Rebalanced Bivariate Portfolios ................................................................. 171
Table 6.9. Estimation Error in Expected Returns: The Sharpe Ratios of the Weekly Rebalanced Bivariate Portfolios ................................................................. 172
Table 6.10. Estimation Error in Expected Returns: The Sharpe Ratios of the Monthly Rebalanced Bivariate Portfolios ................................................................. 173
Table 6.11. Portfolio Performance of the International Stock and Bond Portfolio ..... 174
Table 6.12. Portfolio Performance of the DJIA Portfolio ............................................ 175
Table 6.13. Average Portfolio Performance of the International Stock and Bond Portfolio with Bootstrap Experiments ................................................................. 176
Table 6.14. Average Portfolio Performance of the DJIA Portfolio with Bootstrap Experiments ................................................................. 177
Table 6.15. Comparison of the Static and the Dynamic Volatility Timing Strategies Using Different Risk Aversion Coefficients: International Stock and Bond Portfolio .............................................................................. 178
Table 6.16. Comparison of the Static and the Dynamic Volatility Timing Strategies Using Different Risk Aversion Coefficients: DJIA Portfolio .............................................................................. 179
Table 6.17. Yearly Performance of the International Stock and Bond Portfolio ........ 180
Table 6.18. Comparison of Rolling Window and Long Memory Volatility Timing ... 181
Table 6.19. Portfolio Performance under the Long-Only Constraint.......................... 182
Table 7.1. Comparison of Out-of-Sample Volatilities ................................................. 201
Table 7.2. Diebold–Mariano Tests of the Hedging Portfolios ....................................... 202
Table 7.3. Portfolio Performance of the International Stock and Bond Portfolio ........ 204
Table 7.4. Portfolio Performance of the DJIA Portfolio ............................................... 205
Table 7.5. Average Portfolio Performance of the International Stock and Bond Portfolio with Bootstrap Experiments ................................................................. 206
Table 7.6. Average Portfolio Performance of the DJIA Portfolio with Bootstrap Experiments........................................................................................................................................ 207

Table 7.7. Comparison of the Volatility Timing and Static Strategies Using Different Risk Aversion Coefficients: International Stock and Bond Portfolio ....................... 208

Table 7.8. Comparison of the Volatility Timing and Static Strategies Using Different Risk Aversion Coefficients: DJIA Portfolio........................................................................ 209

Table 7.9. Comparison with Other Conditional Volatility Models: DJIA Portfolio ....... 210

Table 7.10. Comparison with Other Conditional Volatility Models: International Stock and Bond Portfolio ........................................................................................................ 211

Table 7.11. Comparison of the Factor Models ........................................................................................................................................ 212
List of Figures

Figure 2.1. The Mean-Variance Efficient Frontier........................................................ 54
Figure 2.2. The Mean-Variance Efficient Frontier and The Capital Market Line. ...... 55
Figure 2.3. Utilities and Optimal Portfolios. ................................................................. 56
Figure 2.4. The True, Estimated and Actual Mean-Variance Efficient Frontiers. ....... 57
Figure 2.5. The True and Actual Mean-Variance Efficient Frontiers. ......................... 58
Figure 2.6. The Sharpe Ratios of the Tangency Portfolios. .......................................... 59
Figure 4.1. Autocorrelation of Returns (Black Line), Absolute Returns (Blue Line), and
Squared Returns (Red Line) ................................................................................... 94
Figure 6.1. International Stock and Bond Portfolio: The Sharpe Ratios of the Short
Memory and Long Memory Volatility Timing Strategies. ................................. 183
Figure 6.2. DJIA Portfolio: The Sharpe Ratios of the Short Memory and Long Memory
Volatility Timing Strategies. ................................................................................. 184
Figure 6.3. Year-on-Year Sharpe Ratios of the Static and Long Memory Volatility
Timing Portfolios.................................................................................................. 185
Figure 7.1. Determining the Number of Common Factors. ........................................ 213
Figure 7.2. The Sharpe Ratios and Adjusted Performance Fees of the Bootstrap LM-
EWMA and OFLM4 Portfolios .......................................................................... 214
Figure 7.3. Sensitivity to Estimation Window: The Sharpe Ratios of the Dynamic
Portfolios. .......................................................................................................... 215
Figure 7.4. Average Sharpe Ratios of the Static and Dynamic Factor Long Memory
Portfolios over Years. ....................................................................................... 216
List of Appendices

Equation Section (Next)

Appendix 5.1. LM-EWMA Conditional Covariance Matrix Forecasts ................................................................................................................................. 134

Appendix 5.2. Bayesian Prior Probabilities ................................................................................................................................. 136

Appendix 5.3. Comparison of Out-of-Sample Volatilities: Hedging International Portfolios .............................................................................................................. 137

Appendix 5.4. Comparison of Out-of-Sample Volatilities: Hedging DJIA Portfolios ................................................................................................................................. 138

Appendix 5.5. MAE for Longer HorizonForecasts: Bivariate Systems ................................................................................................................................. 139

Appendix 5.6. Comparison of Out-of-Sample Volatilities: Bivariate Portfolios with Monthly Rebalancing Frequency ................................................................................................................................. 140

Appendix 5.7. Diebold–Mariano Joint Tests: Hedging International Stock and Bond Portfolios with Different Rebalancing Frequencies ................................................................................................................................. 141

Appendix 5.8. Comparison of Out-of-Sample Volatilities: Bivariate Portfolios with Different Estimation Windows and Rebalancing Frequencies ................................................................................................................................. 142

Appendix 5.9. Diebold–Mariano Joint Tests: Hedging DJIA Portfolios with Different Estimation Windows ................................................................................................................................. 144


Appendix 7.1. Comparison of the Orthogonal Factor Long Memory EWMA and the Orthogonal Factor EWMA Models ................................................................................................................................. 217

Appendix 7.2. Comparison of the Orthogonal Factor Long Memory EWMA and the Orthogonal Factor GARCH Models ................................................................................................................................. 219
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>APT</td>
<td>Arbitrage Pricing Theory</td>
</tr>
<tr>
<td>AR</td>
<td>Autoregressive</td>
</tr>
<tr>
<td>ARCH</td>
<td>Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>ARFIMA</td>
<td>Autoregressive Fractionally Integrated Moving Average</td>
</tr>
<tr>
<td>ARIMA</td>
<td>Autoregressive Integrated Moving Average</td>
</tr>
<tr>
<td>ARMA</td>
<td>Autoregressive Moving Average</td>
</tr>
<tr>
<td>BEKK</td>
<td>Baba, Engle, Kraft and Kroner</td>
</tr>
<tr>
<td>BIRR</td>
<td>Burmeister, Ibbotson, Roll and Ross</td>
</tr>
<tr>
<td>CAPM</td>
<td>Capital Asset Pricing Model</td>
</tr>
<tr>
<td>CCC</td>
<td>Constant Conditional Correlation</td>
</tr>
<tr>
<td>CGARCH</td>
<td>Component Generalised Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>CML</td>
<td>Capital Market Line</td>
</tr>
<tr>
<td>CRR</td>
<td>Chen, Roll and Ross</td>
</tr>
<tr>
<td>DCC</td>
<td>Dynamic Conditional Correlation</td>
</tr>
<tr>
<td>DJIA</td>
<td>Dow Jones Industrial Averages</td>
</tr>
<tr>
<td>EGARCH</td>
<td>Exponential Generalised Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>EWMA</td>
<td>Exponentially Weighted Moving Average</td>
</tr>
<tr>
<td>FIGARCH</td>
<td>Fractionally Integrated Generalised Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>GARCH</td>
<td>Generalised Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>Glosten-Jagannathan-Runkle Generalised Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>GMM</td>
<td>Generalised Method of Moments</td>
</tr>
<tr>
<td>GMV</td>
<td>Global Minimum Variance</td>
</tr>
<tr>
<td>GPH</td>
<td>Geweke-Porter-Hudak log periodgram estimator</td>
</tr>
<tr>
<td>HAC</td>
<td>Heteroscedasticity and Autocorrelation Consistent</td>
</tr>
<tr>
<td>HML</td>
<td>High Minus Low</td>
</tr>
<tr>
<td>HMSE</td>
<td>Heteroscedasticity-adjusted Mean Squared Error</td>
</tr>
<tr>
<td>HYGARCH</td>
<td>Hyperbolic Generalised Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>i.i.d</td>
<td>independently identically distributed</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>IC</td>
<td>Information Criterion</td>
</tr>
<tr>
<td>IGARCH</td>
<td>Integrated Generalised Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>LM-EWMA</td>
<td>Long Memory Exponentially Weighted Moving Average</td>
</tr>
<tr>
<td>MAE</td>
<td>Mean Absolute Error</td>
</tr>
<tr>
<td>MS</td>
<td>Moulines-Soulier log periodgram estimator</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Squared Error</td>
</tr>
<tr>
<td>OFLM</td>
<td>Orthogonal Factor Long Memory</td>
</tr>
<tr>
<td>PCA</td>
<td>Principal Components Analysis</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Squared Error</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>Standard &amp; Poor’s 500 Index</td>
</tr>
<tr>
<td>SMB</td>
<td>Small Minus Big</td>
</tr>
<tr>
<td>TGARCH</td>
<td>Threshold Generalised Autoregressive Conditional Heteroskedasticity</td>
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Author’s Declaration

I hereby declare that this thesis incorporates materials that are results of joint research, as follows:

Chapter 5 is based on a paper submitted to the International Journal of Forecasting co-authored with Professor Richard Harris. Professor Richard Harris provided editorial advice and guidance throughout the development of the analysis and the paper. Anh Nguyen carried out the analysis and wrote most of the paper.

Parts of Chapters 6 and 7 are based on a working paper co-authored with Professor Richard Harris. Professor Richard Harris provided editorial advice and guidance throughout the development of the model and the paper. Anh Nguyen developed the model, carried out the analysis and wrote most of the paper.

I am aware of the University of Exeter’s regulation and I certify that I have properly acknowledged the contribution of other researchers to my thesis, and have obtained permission from them to include the above materials in my thesis.

I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work.