

# On the generation and characterisation of internal micro-architectures

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# Abstract

Open cell micro-architectures are used in a large number of applications, ranging from medical, such as bone scaffolds, to industrial, such as heat transfer structures. Traditionally these structures are manufactured using foaming processes, however advances in additive manufacturing (AM) now allow such structures to be designed computationally and fabricated to a high degree of precision.

In this thesis image-based methods are developed for the purpose of generating periodic micro-architectures based on implicit representations. The algorithms developed are shown to be efficient and robust, allowing for the creation of both surface and volume meshes. Methods are presented for the creation of functionally graded structures allowing for arbitrary variations in density between specifiable volume fractions. These algorithms are further extended for domain conforming applications as well as for internal structures in CAD models. By utilising a hybrid approach, imaging techniques can be exploited for the generation of internal structures in CAD models without de-featuring the original external geometry. The structures of interest are also shown to be manufacturable via selective laser melting (SLM).

The issue of characterisation, for linear elastic properties, is addressed through the use of a novel homogenisation technique. Large multi-scale problems in irregular domains are divided into smaller sub-volumes using established tetrahedral volume meshing techniques. By performing a series of virtual tests on these macroelements their effective properties can be computed and subsequently used in macro-simulations. The technique is shown to yield results in excellent agreement with the often used kinematic uniform boundary conditions (KUBC). It is also shown how these properties may be used for visualising the distribution in properties over a domain.

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