

**Integral Projection Models and analysis
of patch dynamics of the reef building
coral *Monstastraea annularis*.**

Submitted by

Heather Rachel Burgess

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Heather Rachel Burgess

Abstract

Over the past 40 years, coral cover has reduced by as much as 80%. At the same time, Coral Reefs are coming under increasing threat from hurricanes, as climate change is expected to increase the intensity of hurricanes. Therefore, it has become increasingly important to understand the effect of hurricanes on a coral population.

This Thesis focuses on the reef-building coral *Montastraea annularis*. This species once dominated Caribbean Coral Reefs, but is fast being replaced by faster growing more opportunistic species. It is important that the underlying dynamics of the decline is understood, if managers stand any chance of reversing this decline.

The aim of this Thesis is to investigate the effect of hurricane activity on the dynamics of the reef-building coral *Montastraea annularis*. To achieve this the Integral Projection Model (IPM) method was adopted and the results compared to those produced using the more traditional method of Population Projection Matrix (PPM) method.

The models were fitted using census data from June 1998 to January 2003, which described the area of individual coral patches on a sample of ramets on Glovers Reef, Belize. Glovers Reef is a marine reserve that lies 30km off the coast of Belize and 15km east of the main barrier reef. Three hurricanes struck Glovers Reef during the study: Hurricane Mitch (October 1998), Hurricane Keith (September 2000) and Hurricane Iris (October 2001).

The data have been divided by two different methods in order to test two research questions, firstly if the initial trauma following a hurricane affects the long term dynamics of a population and, secondly, if the dynamics exhibited during a hurricane varied with hurricane strength.

In this Thesis five main results are shown:

1. All models for all divisions of data are in long term decline.
2. As initial trauma increased, the long term growth rates decreased, conversely the short term extremes increased.
3. Fragmentation is more likely as patch size increased and more likely under stronger hurricanes.
4. Integral Projection Modelling painted a similar picture to Population Projection Matrix models and should be a preferred method of analysis.

5. Interaction of the IPMs can be used to model the changing occurrence of hurricanes under climate change. It is shown that with increased intensity, the population could become extinct 6.3 years sooner.

This research is the first step in modelling coral patch populations by the IPM method. It suggests possible functional forms and compares the results with the PPM method. Further research is required into the biological functions which drive fragmentation, the method by which large patches divide into groups of smaller patches. The conclusions from this Thesis add to the growing body of knowledge concerning the response of coral species to hurricanes, focusing on the importance of understanding patch dynamics, in order to understand colonial dynamics.

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'I can do all this through him who gives me strength' Philippians 4:13.

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