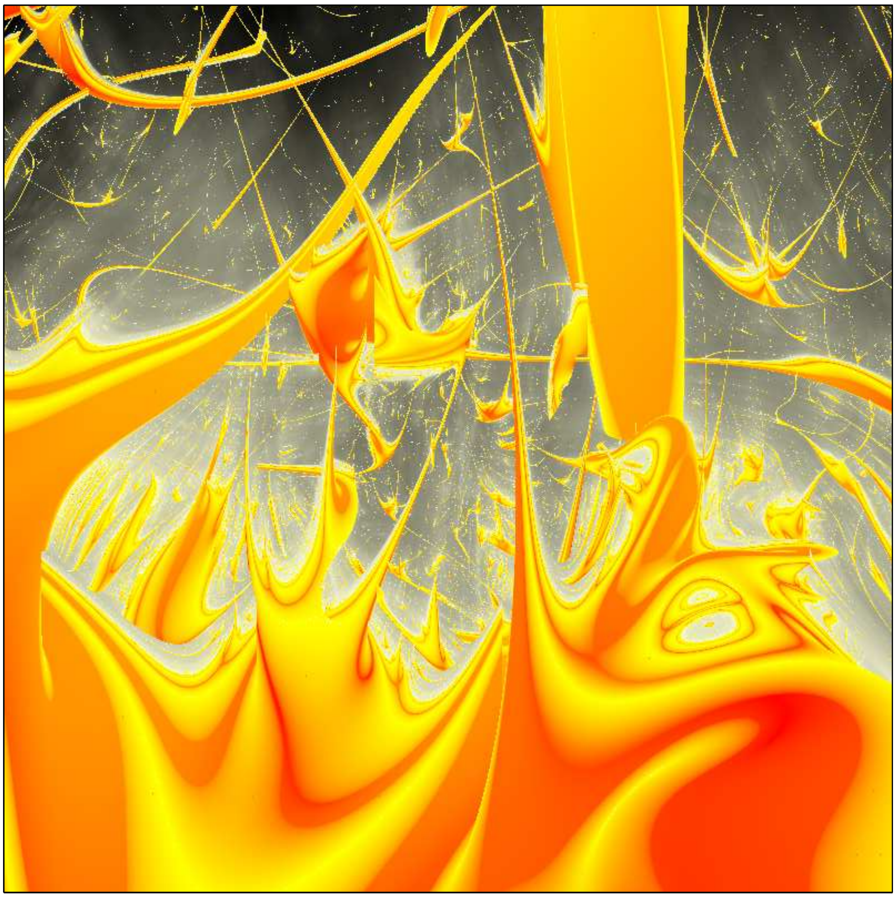
A Journey Through the Dynamical World of Coupled Laser Oscillators



A thesis submitted to the University of Exeter
by Nicholas Blackbeard

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Second Supervisor: Dr Hartmut Erzgräber

Image on front cover: Stability diagram for three coupled laser oscillators. Specifically, it is an expanded view from Fig. 4.9(b)–(c) with a different colour scale; yellow-red = periodic intensity fluctuations, and white-black = chaotic intensity fluctuations. This picture received a prize for the Engineering, Mathematics, and Physical Sciences Image Competition.

A Journey Through the Dynamical World of Coupled Laser Oscillators

Submitted by Nicholas Blackbeard, to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Mathematics, January 2012.

I certify that all material in this thesis which is not my own work has been identified, and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature: ....

Abstract

The focus of this thesis is the dynamical behaviour of linear arrays of laser oscillators with nearest-neighbour coupling. In particular, we study how laser dynamics are influenced by laser-coupling strength, κ, the natural frequencies of the uncoupled lasers, $\tilde{\Omega}_j$, and the coupling between the magnitude and phase of each lasers electric field, α. Equivariant bifurcation analysis, combined with Lyapunov exponent calculations, is used to study different aspects of the laser dynamics. Firstly, codimension-one and -two bifurcations of relative equilibria determine the laser coupling conditions required to achieve stable phase locking. Furthermore, we find that global bifurcations and their associated infinite cascades of local bifurcations are responsible for interesting locking-unlocking transitions. Secondly, for large α, vast regions of the parameter space are found to support chaotic dynamics. We explain this phenomenon through simulations of α-induced stretching-and-folding of the phase space that is responsible for the creation of horseshoes. A comparison between the results of a simple *coupled-laser model* and a more accurate *composite-cavity mode model* reveals a good agreement, which further supports the use of the simpler model to study coupling-induced instabilities in laser arrays. Finally, synchronisation properties of the laser array are studied. Laser coupling conditions are derived that guarantee the existence of synchronised solutions where all the lasers emit light with the same frequency and intensity. Analytical stability conditions are obtained for two special cases of such laser synchronisation: (i) where all the lasers oscillate in-phase with each other and (ii) where each laser oscillates in anti-phase with its direct neighbours. Transitions from complete synchronisation (where all the lasers synchronise) to optical turbulence (where no lasers synchronise and each laser is chaotic in time) are studied and explained through symmetry breaking bifurcations. Lastly, the effect of increasing the number of lasers in the array is discussed in relation to persistent optical turbulence.

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