

Analytical and Numerical Studies of Several Fluid Mechanical Problems

Volume 1 of 1

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Abstract

In this thesis, three parts, each with several chapters, are respectively devoted to hydrostatic, viscous and inertial fluids theories and applications. In the hydrostatics part, the classical Maclaurin spheroids theory is generalized, for the first time, to a more realistic multi-layer model, which enables the studies of some gravity problems and direct numerical simulations of flows in fast rotating spheroidal cavities. As an application of the figure theory, the zonal flow in the deep atmosphere of Jupiter is investigated for a better understanding of the Jovian gravity field. High viscosity flows, for example Stokes flows, occur in a lot of processes involving low-speed motions in fluids. Microorganism swimming is such typical a case. A fully three dimensional analytic solution of incompressible Stokes equation is derived in the exterior domain of an arbitrarily translating and rotating prolate spheroid, which models a large family of microorganisms such as cocci bacteria. The solution is then applied to the magnetotactic bacteria swimming problem and good consistency has been found between theoretical predictions and laboratory observations of the moving patterns of such bacteria under magnetic fields. In the analysis of dynamics of planetary fluid systems, which are featured by fast rotation and very small viscosity effects, three dimensional fully nonlinear numerical simulations of Navier-Stokes equations play important roles. A precession driven flow in a rotating channel is studied by the combination of asymptotic analyses and fully numerical simulations. Various results of laminar and turbulent flows are thereby presented. Computational fluid dynamics requires massive computing capability. To make full use of the power of modern high performance computing facilities, a C++ finite-element analysis code is under development based on PETSc platform. The code and data structures will be elaborated, along with the presentations of some preliminary results.

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