Using Transfer Function Analysis in Modelling Biological Invasions

Submitted by

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I certify that all material in this thesis which is not my own work has been identified and that no material is included for which a degree has previously been conferred upon me.

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Abstract

This thesis is made up of seven chapters and two appendices. Chapter 1 provides an introduction whilst Chapter 7 offers a conclusion. In Chapter 2 we provide preliminaries on population projection models and robustness analysis. In Chapter 3 we introduce a stage-structured model in a context of biological invasions. Using a Transfer Function Approach, we provide a detailed analysis of the invasion model where the existence and local stability of all possible equilibria are characterised in terms of the underlying parameters of the model. In Chapter 4, a Lyapunov function approach is used to estimate the basin of attraction for each equilibrium. In Chapter 5, harvesting is incorporated into the model and we specifically examine the effect of harvesting on whether one or both of the species are eliminated. In Chapter 6 we introduce a novel technique to measure the possibility of invasion in non-normal systems where the traditional invasion exponent technique is unreliable.

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