

Mathematical modelling for bidirectional motor-mediated motility in a fungal  
model system

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# Abstract

In *Ustilago maydis* hyphae, bidirectional transport of early endosomes (EEs) occurs on microtubules (MTs) that have plus and minus ends. The transport is powered by kinesin-3 towards the plus ends of MTs and dynein towards the minus ends. Experiments show an accumulation of dynein at the MT plus end.

To investigate the mechanism of this accumulation, I consider two extended asymmetric simple exclusion principle (ASEP) models for the bidirectional transport of dynein in this thesis. In the simpler two-lane model, collision between opposite-directed motors is excluded whereas the more sophisticated 13-lane model takes into account that the MT usually consists of thirteen protofilaments. The presence of multi protofilaments allows dynein to avoid collision with kinesin by changing protofilaments, a behaviour that has been experimentally described. Both models are supplied by quantitative data obtained in *U. maydis* by live cell imaging and suggest that the stochastic behaviour of dynein can account for half of dynein motors in the accumulation at the MT plus end. Moreover, for the two-lane model, by using a mean field approximation, I give an analytical approximation for the accumulation size which shows linear dependence on the flux. In contrast, this dependence is nonlinear in the 13-lane model and appears to be associated with a phase transition leading to a “pulsing state”.

Accompanied experimental studies have shown that *U. maydis* contains a complex MT array and that kinesin-3 moves early endosomes along antipolar MT bundles. In order to better understand the bidirectional EE motility, I extend the two-lane ASEP to model bidirectional transport along an antipolar MT bundle. In this model, the MTs are coupled at minus ends where organelles can switch MTs on which they move. By a mean-field approximation and numerical simulations, I investigate how the switching affects phases of density profiles as well as the type of motor that dominates the active transport in the bundle.

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