

University of Exeter

Hierarchy in Honeycombs

Submitted by Christopher Michael Taylor, to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Engineering

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Christopher Michael Taylor

Abstract

The main aim of this project was to examine the effects of introducing hierarchy into honeycombs and determining the variables that preside over the global response of the structure. Specifically to understand how the in and out-of-plane elastic and non-linear plastic properties of honeycombs were affected by hierarchy. Analytical analysis of hierarchical honeycombs has been used to explain and predict the response of finite element simulations validated by experimental investigations.

The early stage of the investigation focused on finding if the elastic modulus could be maintained or improved on an equal density basis due to the introduction of hierarchy. It is clear that honeycombs are sensitive to hierarchical sub-structures, particularly the fraction of mass shared between the super-and sub-structures. Introduction of an additional level of hierarchy without reducing performance is difficult, but was possible by functional grading. Another original result was that it was determined when the sub-structure could be assumed to be a continuum of the super-structure. Meaning the material properties from a single unit sub-cell could be used as the constituent material properties of the super-structure, as in previous work by (Lakes 1993) and (Carpinteri *et al* 2009) for example.

Work investigating the in-plane, non-linear plastic response of hierarchical honeycombs showed that the introduction of hierarchy into honeycombs can have the effect of delaying the onset of elastic buckling, which is a common failure mechanism for low relative density structures. As such it was possible to achieve a marked increase in the recoverable energy absorbed by hierarchical honeycombs prior to elastic buckling or plastic yield. The potential benefits are less *apparent* in higher relative density structures due to the onset of plasticity becoming the first mode of failure. The out-of-plane properties also investigated showed no increase in the elastic properties due to the introduction of hierarchy, but showed a marked increase in the out-of-plane elastic buckling stress of 60% when compared to a conventional hexagonal honeycomb of the same relative density.

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