AMBIGUITY IN GAMES: THEORY AND EXPERIMENTS

Submitted by

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature ....

ABSTRACT

Ambiguity arises when a decision maker fails to assign a subjective probability to an event. This failure to attach a subjective probability to an event is caused by a lack of information about the event. Ambiguity provides a gap in the scope of game theory, since the basic assumption of being able to assign meaningful probabilities to one's opponent's actions is no longer valid. It thus opens the debate of how invividuals would react if faced by an ambiguous event.

Risk is a special case of ambiguity, where the decision maker has information about the probabilities of events. There is considerable experimental evidence documenting the fact that individuals show a marked preference for situations in which they face a known level of risk, as opposed to being in a situation where they are faced by an opponent whose strategies are ambiguous. Ambiguity averseness is the tendency of individuals to prefer known risk situations to ambiguous ones.

Although there is extensive experimental literature which shows that ambiguity affects decision making, most of these studies are restricted to single-person decisions. Relatively few experiments test whether ambiguity affects behaviour in games, where individuals interact with each other. The research documented in this thesis aims to study the effect of ambiguity in games. Since many economic problems can be represented as games we believe this research will be useful for understanding the impact of ambiguity in economics.

Moreover, though previous studies have established that ambiguity affects decision making, they do not document the nature of the impact that it has on decision making. It is thus difficult to predict the effect of ambiguity, and the direction in which it will cause behaviour to change. This thesis aims at studying the effect of ambiguity in strategic situations, by analysing individual behaviour in games.

AUTHOR'S DECLARATION

This thesis has been written under the supervision of Prof. David Kelsey and Prof. Dieter Balkenborg, from the Department of Economics, University of Exeter.

Research on the impact of ambiguity on individual behaviour in coordination games and in games of strategic complements and substitutes, has been conducted under the supervision of Prof. David Kelsey. Research on experiments based on signalling games, has been conducted under the supervision of Prof. Dieter Balkenborg.

It may be noted that some of the data collected from the experiments based on signalling games had earlier been studied as part of a Master's dissertation titled "An Experiment on Collective Reputation Effects", that was submitted to the University of Exeter. However, additional experimental sessions and research has been conducted, which has contributed towards this doctoral thesis.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Ambiguity arises when a decision maker fails to assign a subjective probability to an event. Keynes (1921) describes ambiguity being caused because of a lack of information about an event, when the concievable information regarding its occurence could be much more. Ambiguity provides a gap in the scope of game theory, since the basic assumption of being able to assign meaningful probabilities to one's opponent's actions is no longer valid. It thus opens the debate of how inviduals would react if faced by an ambiguous event.

Risk is a special case of ambiguity, where the decision maker has information about the probabilities of events. Considerable experimental evidence documents the fact that individuals show a marked preference for situations in which they face a known level of risk, and can assign probabilities to their opponent's strategies, as opposed to being in a situation where they are faced by an opponent whose strategies are ambiguous. Individuals' tendency to be ambiguity averse, was discovered simultaneously by Fellner (1961) and Ellsberg (1961), who were both working independently.

Although there is extensive experimental literature which shows that ambiguity affects decision making, most of it studies single-person decisions. There are relatively few experiments that test whether ambiguity affects behaviour in games. A game is a stylized situation where a group of individuals are asked to make a number of linked decisions, which together model the economic interactions we face in day to day life.

The research documented in this thesis aims to experimentally analyse the effect of ambiguity in games. Since many economic problems can be represented as games we believe this research will be useful for understanding the impact of ambiguity in economics.

Moreover, there is very little previous experimental research on the impact of ambiguity in strategic situations. Previous studies have established that ambiguity does affect decision-making. However, they do not document the nature of the impact that ambiguity had on decision-making. It is thus difficult to predict what effect ambiguity has, and in which direction ambiguity will cause behaviour to change. This thesis aims at studying the effect of ambiguity in strategic situations, by analysing individual behaviour in games.

Chapter Two provides a summary of the existing literature on ambiguity, ambiguity in games and previous experiments conducted to test ambiguity in games.

Chapter Three reports on experiments conducted to test whether ambiguity influences behaviour in a coordination game. We study the behaviour of subjects in the presence of ambiguity and attempt to determine whether they prefer to choose an ambiguity safe option. We consider a modified version of the Battle of Sexes game which has an added safe strategy available for Player 2. The safe strategy (in our game, option R), is a dominated strategy which would not be played in a Nash equilibrium or selected by iterated dominance.

As in the case of the traditional battle of the sexes games, our game has two Pure Nash equilibria, neither of which is focal. Hence the effect of ambiguity as to which equilibrium strategy will be chosen by the opponent is high, making R (the ambiguity-safe option) attractive for Player 2. Thus, the strategy R which is eliminated under Nash equilibrium, may be chosen in an equilibrium under ambiguity (EUA).

Chapter Four provides a report of experiments run with a set of linked games, to test the theoretical prediction that ambiguity has opposite effects in games of strategic complements and substitutes. A pair of games well suited to testing this hypothesis are the best-shot and weakest-link models of public goods. The games are similar except the weakest link game exhibits strategic complements, whereas the best shot exhibits a game of strategic substitutes. Our hypothesis is that the effect of ambiguity will be to decrease individuals' contributions in the weakest-link version of the game, whereas it will lead to an increase in individuals' contributions in the best-shot version.

In addition, we attempt to ascertain whether subjects' perception of ambiguity differs between a local opponent and a foreign one. Kilka and Weber (2001) find that subjects are more ambiguity-averse when the returns of an investment are dependent on foreign securities than when they are linked to domestic securities. We used a pair of strategic complement/substitute games in which the subject is either matched with a local opponent or a foreign one. Our hypothesis was that subjects will be more ambiguity averse when their opponents are individuals of a foreign country than when they are matched with local opponents.

Chapter Five reports the findings of two series of experiments based on signalling games. The design for the initial experiment was selected by Reinhard Selten. It has the interesting property that the strategically stable outcome (Kohlberg and Mertens (1986)) does not coincide with the outcome of the Harsanyi-Selten solution (Harsanyi and Selten (1988)). The games are complex and as such, standard refinement concepts like the intuitive criterion, or the never-a-weak-best-response criterion, do not help to refine among the equilibria.

The other motive for the design of the experiment, was to analyse whether a change in the reward at a particular terminal node would affect individual behaviour and/or promote better coordination between subjects. Moreover, we wanted to test whether subjects (in the role of the sender of the signal) could work together to build a collective reputation.

The term "collective reputation" basically means that subjects in one role, abstain from a certain action which is in their short run interest (but would harm their opponent), in order to allow for coordination on a mutually beneficial outcome. They thus forego a short run gain, for a long term gain that accrues to both players. We discuss how though observed behaviour cannot completely be explained by Nash, it may be explained using alternative equilibrium concepts such as Quantal Response Equilibrium, Cursed Equilibrium and Equilibrium under Ambiguity.

Chapter Six concludes the thesis.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we discuss the existing literature on ambiguity in theory and experiments. We begin by discussing decision making under risk and models that axiomatise decision making under risk. We then describe how the emergence of ambiguity alters individuals' preferences and present models that axiomatise ambiguity sensitive preferences. We go on to describe the existence of an equilibrium in the presence of ambiguity and end with a review of the existing experimental literature on ambiguity in single person decision problems as well as in games.

2.1 Ambiguity in Theory

2.1.1 Decision making under Risk and Expected Utility Theory

Standard game theory is based on the premise that an individual has the ability to assign subjective probabilities[1] to his opponent's actions. Each individual thus possesses a belief about how his opponent/nature would behave, and is able assign probabilities all the possible events that could take place. This ability to assign probabilities to all possible events, gives rise to a situation where individuals face risky prospects (or gambles). **Expected Utility Theory** (Neumann and Morgenstern (1944)) axiomatises the method by which individuals make choices, when faced by a risky prospect.

The expected utilty of a prospect in the presence of risk, is determined by weighting the utility the individual would receive from each possible event, with the probability with which it is expected to occur. Consider a prospect (x, p), which gives £x with probability p, else £0 with probability $(1-p)$. If the utility of money

[1] A subjective probability is one that is derived from an individual's opinion about the likelihood of an event/outcome.

is represented by the function u, the expected utility of the prospect would be: $p \cdot u(x) + (1-p) \cdot u(0)$.

Individuals may react to risk in three possible ways, they could be risk seeking, i.e., they prefer a prospect to a sure outcome of equal/greater expected value; they could be risk averse, i.e., they prefer a sure outcome of equal/greater expected value; or, they could be risk neutral, i.e., they are indifferent between the two options. Under Expected Utility Theory (EUT), the utility function of a risk-seeking invidual is convex, while that of a risk-averse invidual is concave and a risk-neutral one is linear.

2.1.2 Prospect Theory

Experiments conducted to test individuals' reaction to risk, found that they respond to risk in a fourfold pattern. As expected, individuals were risk averse for losses and risk seeking for gains which had a low probability. However, individuals were found to be risk seeking for losses and risk averse for gains which had a high probability. This fourfold pattern of response towards risk, where individuals are neither purely risk averse nor purely risk seeking, but a combination of both, was modelled by **Prospect Theory** (Kahneman and Tversky (1979), Tversky and Kahneman (1992)).

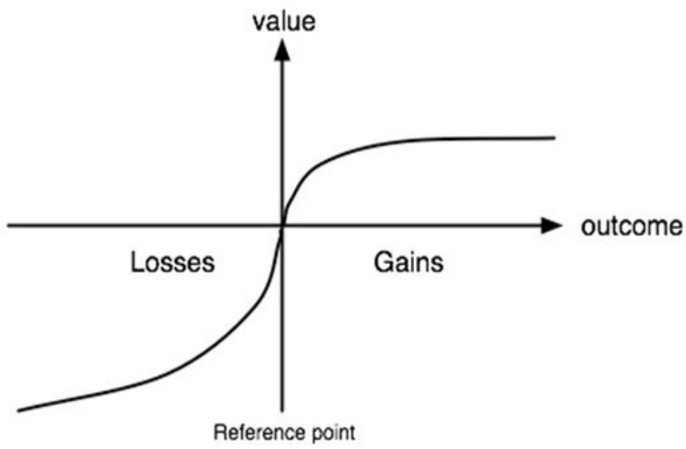
Prospect theory defines gains and losses in terms of monetary outcomes, with respect to a reference point. Thus, when evaluating a prospect (x, p), individuals initially set a reference point. Outcomes (in terms of money) that are less than the reference point are treated as losses and those that are greater, are treated as gains. The overall utility under Prospect Theory is given by the function:

$$U = \sum_{i=1}^{n} w(p_i)v(x_i) = w(p_1)v(x_1) + w(p_2)v(x_2) + ... + w(p_n)v(x_n),$$

where, U is the overall utility of the outcomes $x_1, x_2...x_n$, which occur with probability $p_1, p_2...p_n$, respectively. The weighing function w, captures the tendency of individuals to overreact to small probability events while they underreact to large probability events. The decision weights w are normalised such that $w(0) = 0$ and $w(1) = 1$.

The value function v, passes through the individual's reference point and assigns a value to each outcome with respect to the reference point. As can be seen in Figure 2.1, the value function is asymmetrical i.e., it is *convex for losses* and *concave for gains.* This is because losses hurt more in magnitude, than gains feel good. The S-shaped value function, can be explained simply by saying: increasing the chance of winning a prize by 0.1 has a greater impact, if it changes the probability of winning from 0.9 to 1, rather than when it goes from 0.6 to 0.7 or from 0.3 to 0.4 (Tversky and Kahneman (1992)). Similarly, decreasing the chances of winning a prize from 0.1 to 0, has a greater impact than decreasing it from 0.8 to 0.7. Thus there is greater sensitivity to changes in probabilities which are close to 0 or 1.

Figure 2.1. S-shaped Value Function



However, even though Prospect Theory accounts for loss aversion and the four-fold pattern of response to risk, it still assumes that individuals are aware of the probabilities with which outcomes occur. However, there are certain events that occur as a result of our interactions in a socio-economic setting with other people, to which one cannot assign a meaningful probability.

2.1.3 Ambiguity and the Ellsberg Paradox

Ambiguity occurs when the consequence of a decision is not a single certain outcome, but a number of possible outcomes, to which an individual cannot attach probabilities with surety. When a player fails to assign a subjective probability to the possible outcomes of a decision, we say that he views the situation as ambiguous. According to Keynes (1921), ambiguity is caused by a lack of information about an event, when the concievable information regarding its occurence could be much more.

Ambiguity creates a gap in the scope of classical game theory. The basic assumption that individuals can assign meaningful probabilities to all the possible outcomes is no longer valid, since individuals cannot attach probabilities with certainty. Thus, it throws open the debate about how an individual would react when faced with ambiguity/an ambiguous situation.

Risk is a special case of ambiguity, where the probabilities of events are known. Considerable experimental evidence documents the fact that individuals show a marked preference for situations where they face a known level of risk, and can assign probabilities to their opponent's strategies, as opposed to being in a situation where they are faced by an opponent whose strategies are ambiguous. The fact that individuals tend to be ambiguity averse was discovered simultaneously by both Fellner (1961) and Ellsberg (1961), who were both working independently. Ellsberg's experimental demonstration of the concept of ambiguity aversion gave rise to the "*Ellsberg paradox*", which is described below.

Consider an urn filled with 90 balls, 30 of which are red (R) and the remaining 60 are of an unknown mix of blue (B) and yellow (Y). One ball is drawn at random, and the payoff depends on the colour of the ball drawn and the act you choose. Subjects are asked to choose between acts f, g, f', g' as shown in the table below:

Table 2.1. Acts available in the Ellsberg experiment

	30 balls	60 balls	
Act	Red R	Blue B	Yellow Y
f	£100	0	0
g	0	£100	0
f'	£100	0	£100
g'	0	£100	£100

Subjects who are asked to choose between f and g, generally prefer f because of the definite $\frac{1}{3}$ chance of winning £100 to the ambiguous act g, but when asked to choose between f' and g', the same subjects prefer g' which gives a $\frac{2}{3}$ chance of winning £100, again avoiding the ambiguous act f'.

These choices cannot be represented as maximising expected utility with respect to a standard subjective probability distribution π. Choosing f rather than g implies $\pi(R) > \pi(B)$. However preferring g' to f' implies $\pi(B \cup Y) > \pi(R \cup Y)$. Given the standard additivity properties of probabilities, i.e. $\pi(R \cup Y) = \pi(R) + \pi(Y)$, these two inequalities are inconsistent. The inconsistency would not arise however, if we represented beliefs by a non-additive set function ν. In this case it is possible that $\nu(R \cup Y) \neq \nu(R) + \nu(Y)$, which could be compatible with the choices in the Ellsberg paradox.

2.1.4 Choquet Expected Utility

Non-additive beliefs were introduced in Schmeidler (1989)'s seminal paper on **Choquet Expected Utility** (CEU). In CEU, an outcome is evaluated by a weighted sum of utilities, but unlike EUT the weights used depend on the acts. In the case of non-additive beliefs, if an event is thought to be unlikely to occur, its complement is not necessarily a certain event. It is however believed to have a much greater chance of occuring than the former event. We give a brief explantion of CEU below.

Let S be the set of mutually exclusive and exhaustive states of nature and $\mathcal{A}$ be the set of events, such that $\mathcal{A}$ is the subset of S, and $S, \phi \in \mathcal{A}$. Let $\mathcal{C}$ be the set of consequences in terms of payoffs or outcomes, such that $\mathcal{D} \in C$ indicates a subset of consequences. Additionally, let $\mathcal{F}$ be the set of acts such that, a consequence $f(s)$ will be the outcome of choosing an act f when s is the true state that materialises.

A function $v : \mathcal{A} \rightarrow [0, 1]$ is a capacity if $v(\phi) = 0$ and $v(S) = 1$. Moreover, the capacity v is monotonic if $A \supset B \Rightarrow v(A) \geq v(B)$, i.e., if event B is a subset of event A, an individual would believe that A is more likely to occur than B. Intuitively, suppose A describes the event that a number greater than 10 is chosen and B describes the event that an even number greater than 10 is chosen. One can note that B is a subset of A and that A is more likely to occur than B.

A capacity v would be an ordinary probability measure if it is additive, such that $v(A \cup B) = v(A) + v(B)$, for all disjoint sets A and B. Moreover, a capacity v is convex (resp. concave) if for all A, $B \subseteq S$, $v(A \cup B) + v(A \cap B) \geq v(A) + v(B)$ (resp. $v(A \cup B) + v(A \cap B) \leq v(A) + v(B)$), where A and B are events contained in the universal set S.

If there exists a capacity v on $\mathcal{A}$, n possible states: $s_1 ... s_n$ and a utility function $U : \mathcal{C} \rightarrow \mathbb{R}$ such that $U(f(s_1)) \geq ... \geq U(f(s_n))$ then the CEU of an act f is:

$$\mathrm{CEU}(f) = \sum_{i=1}^{n-1} \left(U(f(s_i)) - U(f(s_{i+1})) \right) v(\{s_1, ..., s_i\}) + U(f(s_n)).$$

Sarin and Wakker (1992) provide an intuitive exposition of Schmeidler's model. They extend the CEU model in a way that preserves additivity in probability for events in the presence of risk, while permitting non-additivity for ambiguous events. The basic concepts on which the model is built, are described below.

Let $\mathcal{F}$ be the set of acts, such that $\mathcal{F}^{ua} \subset \mathcal{F}$, is the set of all unambiguous acts; and $\mathcal{F}^{a} \subset \mathcal{F}$, is the set of all ambiguous acts. An act f is constant act, if it gives the same consequence $f(s) = \alpha$, for all states s, where $\alpha \in \mathcal{C}$. A constant act is referred to in terms of its resultant consequence. Thus, if $f(s) = \alpha$ for all s, then the act would be referred to as α.

The binary relation $\succeq$, gives the decision maker's preferences over acts. Thus, $f \succeq g$ implies that the decision maker weakly prefers act f to g. Similarly, $f \preceq g$ implies that act g is weakly preferred to f. Moreover, $f \succ g$ implies that act f is strictly preferred to g, while $f \sim g$ implies indifference between acts f and g.

An act $f_A h$ refers to an act which results in consequences $f(s)$ for all $s \in A$ and $h(s)$ for all $s \in S \setminus A$. For example, $\alpha_A \beta$ refers to a constant act, that gives an outcome α whenever event A takes place, else an outcome β (when A does not take place). Moreover, if there exist outcomes $\alpha \succ \beta$ such that $\alpha_A \beta \succeq \alpha_B \beta$, it would mean that event A is more likely to take place than event B. Intuitively, the former act gives an outcome α if event A occurs, otherwise β. The latter act gives an outcome α if event B occurs, otherwise β. Since α is preferred to β, we can conclude that $\alpha_A \beta$ would only be preferred to $\alpha_B \beta$, if A was more likely to occur than B.

A *preference interval* P is a subset of consequences D, such that if α, γ are elements of P and $\alpha \succeq \beta \succeq \gamma$, then β is also an element of P (Fishburn (1982)). A *cumulative consequence set* is a set of consequences P, such that if $\alpha \in P$ and $\beta \succeq \alpha$, then $\beta \in P$. Intuitively this implies that if a is an element of P, but β is weakly preferred to α, then β is also an element of the set P.

The CEU for a cardinal, bounded, nonconstant utility function U on $\mathcal{C}$ and a unique capacity v on $\mathcal{A}$, is maximised by the preference relation $\succeq$ if the following postulates (See Sarin and Wakker (1992)) are satisfied:

1. *Weak ordering.*

   This means that an individual has complete and transitive preferences over acts, such that if f is weakly preferred to g $(f \succeq g)$ and g is weakly preferred to h $(g \succeq h)$, it implies that f would be weakly preferred to h $(f \succeq h)$, for all acts f, g and h.

2. *For all events A and acts f, g, h, h' where $f_A h$, $g_A h$, $f_A h'$, $g_A h' \in F^{ua}$:*

   $$f_A h \succeq g_A h \Longleftrightarrow f_A h' \succeq g_A h'.$$

   This is also referred to as the *sure-thing principle.* Note that the acts $f_A h$, $g_A h$, $f_A h'$ and $g_A h'$ are elements of the set of unambiguous acts. Intuitively, act $f_A h$ gives consequences $f(s)$ if event A occurs, else $h(s)$, and so on. If a decision maker prefers $f_A h$ to $g_A h$, the sure-thing principle implies that he would also weakly prefer act $f_A h'$ to act $g_A h'$, and vice versa. The Ellsberg paradox suggests that the sure-thing principle applies to unambiguous events, but not to ambiguous events.

3. *For all acts $f \in F$, consequences α, β and events $A \in A$,*

   $$\alpha \succeq \beta \Rightarrow \alpha_A f \succeq \beta_A f.$$

   Both acts $\alpha_A f$ and $\beta_A f$ give consequence $f(s)$, if event A does not occur. If a decision maker weakly prefers consequence α to β, it would imply that he would prefer an act that gives him α if event A, occurs rather than one that gave him consequence β.

Moreover, if A is an unambiguous event and f is an unambiguous act, then if the decision maker prefers $\alpha_A f$ to $\beta_A f$, it would also imply that he preferred consequence α to β.

4. *For all acts f, g, and cumulative consequence sets P,*

$$f \succeq g \text{ whenever } f^{-1}(P) \succeq g^{-1}(P).$$

As described before, P is a set of consequences where, if α is an element of P, but the decision maker weakly preferred β to α, then it implied that β was also an element of the same set P. This postulate states that if a decision maker weakly prefers an act f that results in a consequence from set P, to another act g which also leads to a consequence from set P, it would imply that he weakly prefers act f to act g.

5. *Consequences such as α, β exist, such that $\alpha \succ \beta$.*

Intuitively, this means that the decision maker can have strict preferences over consequences, such that he strictly prefers getting α to getting β.

6. *Continuity: If $f \succ g$, where $f \in F^{ua}, g \in F$,* and $\alpha \in C$, *then for all elements in the set of unambiguous events A^{ua}, there exists a partition $(A_1, ..., A_m)$ of S, such that $\alpha_{A_j} f \succ g$ for all j, and the same is also true with $\prec$ in place of $\succ$.*

Consider a decision maker who strictly prefers unambiguous acts to ambiguous ones. Then there would be a set of unambiguous acts $(A_1, ..., A_m)$, such that the decision maker strictly prefers getting α if A_j occurs (else f), to the ambiguous act g. Moreover, the reverse is also true, i.e., for some values of the consequence α, the ambiguous act g is preferred to $\alpha_{A_j} f$.

7. *For nonempty $f-$convex events A, and $f, g \in F$,*

$$f(s)_A f \succeq g \text{ for all } s \in A \Rightarrow f \succeq g,$$

and the same is also true for $\preceq$ *in place of* $\succeq$ *.*

An $f - convex$ set is one such that, given s, s'' are elements of the set of events A, and s' is an element of S, then if $f(s)$ is strictly preferred to $f(s')$ and $f(s')$ is strictly preferred to $f(s'')$, it implies that s' is also an element of A ($f(s) \succ f(s') \succ f(s'') \Rightarrow s' \in A$). Intuitively, Postulate 7 states that if, an act $f(s)_A f$, that gives consequences $f(s)$ if the event in $f - convex$ set A occurs, is weakly preferred to an act g, then it implies that the act f is weakly preferred to g, for all elements of A. Similarly, if a decision maker prefers the act g to $f(s)_A f$, then g is preferred to f for all elements of the $f - convex$ set A.

In the case of the set of unambiguous events $\mathcal{A}^{ua}$, if the above 7 assumptions are satisfied, the capacity is additive and convex.

As seen above, Sarin and Wakker (1992) provide an intuitive extention of Schmeidler (1989)'s model, thus permitting subjects to attach nonadditive probabilities to ambiguous events, while also giving conditions under which CEU gets reduced to SEU (for unambiguous events).

Individuals can be either optimistic or pessimistic in their outlook to ambiguity. A decision maker with an optimistic outlook would over-estimate the likelihood of a good outcome; whereas a pessimistic decision maker would over-estimate the likelihood of a bad outcome. For some capacities, CEU preferences are compatible with a multiple priors approach.

Convex capacities are used to model a pessimistic outlook to ambiguity, while concave capacities model an optimistic outlook. Let μ be a convex capacity on S for any $\alpha \in [0, 1]$, where α is the level of optimism/pessimism towards ambiguity. A decision maker's attitude to ambiguity is measured by α, with $\alpha = 1$ denoting pure optimism and $\alpha = 0$ denoting pure pessimism. If the decision maker has $0 < \alpha < 1$, he is neither purely optimistic nor purely pessimistic, but reacts to ambiguity in a partly pessimistic way by putting a greater weight on bad outcomes and in a partly optimistic way by putting a greater weight on good outcomes. Consider a capacity v defined by:

$$v(a) = \alpha\mu(A) + (1-\alpha)[1 - \mu(S\backslash A)].$$

This is termed a *JP-capacity* and allows for both CEU and multiple prior forms (Jaffray and Philippe (1997)).

For a capacity v on S, the core of the capacity v ($core(v)$), is a set of probability distributions that yield a higher probability for every event than v. If there is no ambiguity regarding the probability of an event, the decision maker's capacity v would be additive and the core would consist of a single probability distribution.

If $\alpha \in [0,1]$, μ is a convex capacity and v is a JP-capacity, the CEU of an act (f) is given by:

$$\text{CEU}(f) : \int u(a)dv = \alpha min_{p\in\ core(\mu)} \int u(a(s))dp(s) + (1-\alpha)max_{p\in\ core(\mu)} \int u(a(s))dp(s),$$

where α is the ambiguity attitude of the decision maker and the core of μ is the set of priors that describe his ambiguity (Jaffray and Philippe (1997)).

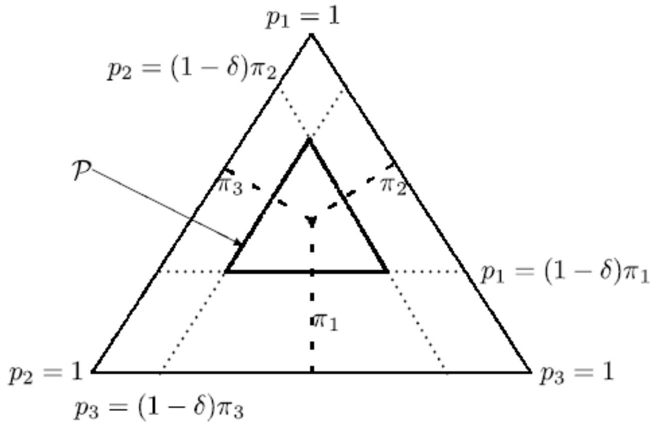
2.1.5 Neo-additive Capacities

Neo-additive capacities were introduced by Chateauneuf, Eichberger, and Grant (2007). The neo-additive capacity is a special case of a JP-capacity, with a convex capacity μ, such that $\mu(E) = (1-\delta)\pi(E)$ for all events $E \neq S$, where π is a probability distribution on S and $0 \leq \delta \leq 1$. Intuitively, π can be thought to be the decision maker's belief. However, given that the decision maker faces ambiguity, π is an ambiguous belief. The level of confidence that the decision maker possesses in the belief is modelled by $(1-\delta)$, where $\delta = 1$ denotes complete ignorance and $\delta = 0$ denotes no ambiguity. The set of priors is given by

$$\mathcal{P} = core(\mu) = \{p \in \Delta(S) |\ p(E) \geq (1-\delta)\pi(E)\}.$$

An intuitive explanation of the core of the neo-additive capacity can be made using Figure 2.2 (Eichberger and Kelsey (2009)). We consider the case where the set S consists of three possible states, $S = \{s_1, s_2, s_3\}$. Let each state s_i occur with probability p_i. The corners of the outer triangle would then correspond to the

Figure 2.2. Core of a Neo-additive Capacity



scenarios where each state occurs with complete certainty. For instance, at the top of the triangle, s_1 occurs with probability $p_1 = 1$, while $p_2 = p_3 = 0$. Similarly at the bottom-right corner of the triangle, s_3 occurs with $p_3 = 1$, while $p_1 = p_2 = 0$, and so on.

As we move away from the point at which $p_3 = 1$ along the line connecting it to the point where $p_1 = 1$, the probability of s_3 keeps diminishing slowly, while that of s_1 rises. Along this line, $p_1 = (1-\delta)\pi_1$, i.e., the probability of s_1 taking place is weighed by $(1-\delta)$, the ambiguous belief of the decision maker. Thus, as we move towards the top of the triangle, $\delta \to 0$, such that at the peak, $p_1 = 1$. The intuition is the same for the other two sides of the outer triangle.

The belief $p_i = (1-\delta)\pi_i$, holds along all points on a line drawn parallel to the outer triangle (in Figure 2.2, these are shown as the dotted lines). The inner triangle (which is formed by the intersection of the three dotted lines), forms the set of priors $\mathcal{P}$, which represents the core of the neo-additive capacity μ.

Chateauneuf, Eichberger, and Grant (2007) consider a set of events $\mathcal{E}$, which is subset of the set S. $\mathcal{E}$ is further divided into three subsets of events, on the basis of the likelihood of their occurence:

1. $\mathcal{N}$ *or the set of "null" events* - This is the set of events that are considered impossible to take place, for example, $\phi \in \mathcal{N}$. If an event $A \in \mathcal{N}$, then for any event B which is a subset of A, $B \in \mathcal{N}$. Moreover, if both events A and B are elements of $\mathcal{N}$, then $A \cup B \in \mathcal{N}$.

2. *$\mathcal{U}$ or the set of "universal" events* - This is the set of events that are considered certain to take place. Intuitively, the complement of every act that is included in the null set is an element of $\mathcal{U}$, such that $\mathcal{U} = \{E \in \mathcal{E} : S \backslash E \in \mathcal{N}\}$.

3. *$\mathcal{E}^*$ or the set of "essential" events* - This is the set of events that are considered neither certain nor impossible, i.e. $\mathcal{E}^*$ contains all the events not included in the previous two sets.

Mathematically the Hurwicz capacity of an event E, given the set of null events $\mathcal{N}$ and $\alpha \in [0,1]$ (where α is the degree of optimism) is:

$$\mu_\alpha^{\mathcal{N}}(E) = \left\{ \begin{array}{c} 0 \text{ if } E \in \mathcal{N} \\ \alpha \text{ if } E \notin \mathcal{N} \text{ and } S \backslash E \notin \mathcal{N} \\ 1 \text{ if } S \backslash E \in \mathcal{N} \end{array} \right\}.$$

The Hurwicz capacity is modelled such that it is a convex combination of two capacities: one capacity that attaches complete ambiguity to everything except the universal set, while the other attaches complete confidence to everything except the null set. Thus, if an event is from the universal set, $\mu^{\mathcal{U}}(E) = 1$, else $\mu^{\mathcal{U}}(E) = 0$. Similarly if an event is from the null set, $\mu^{\mathcal{N}}(E) = 0$, else $\mu^{\mathcal{N}}(E) = 1$. It is simple to note that $\mu^{\mathcal{U}}$ is the complement of $\mu^{\mathcal{N}}$, or $\mu^{\mathcal{U}} = \overline{\mu}^{\mathcal{N}}$. Given this framework, the Hurwicz capacity models an individual's response to an event E in a partly optimistic way and a partly pessimistic way, such that

$$\mu_\alpha^{\mathcal{N}}(E) = \alpha\mu^{\mathcal{N}} + (1-\alpha)\overline{\mu}^{\mathcal{N}}. \tag{2.1}$$

In addition, the Hurwicz capacity $\mu_\alpha^{\mathcal{N}}(E)$ can be modified to reflect δ, or the degree of ambiguity as well. Given an additive probability distribution π on (S, ε) and $\delta, \alpha \in [0,1]$ where δ is the degree of ambiguity and α is the level of optimism/pessimism towards ambiguity, the neo-additive capacity $v(E \backslash \mathcal{N}, \pi, \delta, \alpha)$ for a given set of null event $\mathcal{N} \subset \varepsilon$ is:

$$v(E \backslash \mathcal{N}, \pi, \delta, \alpha) := (1-\delta)\pi(E) + \delta\mu_x^{\mathcal{N}}(E), \tag{2.2}$$

and the Choquet expected value of a function f, given the neo-additive capacity $v(E\backslash\mathcal{N}, \pi, \delta, \alpha)$ is:

$$\text{CEU}(f) : \int f\ dv = (1-\delta)\, E_\pi [f] + \delta \left(\alpha \cdot \max\left\{x : f^{-1}(x) \notin \mathcal{N}\right\} + (1-\alpha) \cdot \min\left\{y : f^{-1}(y) \notin \mathcal{N}\right\}\right).$$

It can be noted that varying the values of π, δ, and α would lead to special cases of the Choquet integral, as follows:

1. The case of expected utility, i.e., when $\delta = 0$.
2. The case of perfect optimism, i.e., when $N = (\phi)$, $\delta > 0$, $\alpha = 1$.
3. The case of perfect pessimism, i.e., when $N = (\phi)$, $\delta > 0$, $\alpha = 0$.
4. The case of Hurwicz criterion, i.e., when $N = (\phi)$, $\delta = 1$, $\alpha \in (0, 1)$.

Similar to the JP-capacity, neo-additive capacities also satisfy both CEU and multiple-priors, i.e., capacities can be made convex/concave. From (2.1) and (2.2) we have,

$$\begin{aligned} v(E\backslash\mathcal{N}, \pi, \delta, \alpha) \quad &: \quad = (1-\delta)\pi(E) + \delta\mu_x^{\mathcal{N}}(E) \\ \text{and } \mu_x^{\mathcal{N}}(E) \quad &= \quad \alpha\mu^{\mathcal{N}} + (1-\alpha)\overline{\mu}^{\mathcal{N}} \end{aligned}$$

$$\text{so } v = \alpha[(1-\delta)\pi + \delta\mu_1^{\mathcal{N}}] + (1-\alpha)[(1-\delta)\pi + \delta\overline{\mu}_1^{\mathcal{N}}],$$

where π reflects the subjective belief of the decision-maker, $(1-\delta)$ reflects the degree of confidence he attaches to the subjective belief he holds ($\delta = 0$ would reflect complete confidence), $\delta\alpha$ and $\delta(1-\alpha)$ reflect pessimism and optimism, respectively. Thus, $v = \alpha\rho + (1-\alpha)\overline{\rho}$, where $\rho = [(1-\delta)\pi + \delta\mu_1^{\mathcal{N}}]$ is concave, and $\overline{\rho}$ is its dual and hence convex (Chateauneuf, Eichberger, and Grant (2007)).

2.1.6 Maxmin Expected Utility

An alternative to Choquet integration was put forth by Gilboa and Schmeidler (1989), in the form of Maxmin Expected Utility. Gilboa and Schmeidler (1989) explore and build on a statement found in Wald (1950), which states that when an a priori distribution of the set of events Ω is unknown to the decision-maker, it would be reasonable to consider a minimax solution. In this subsection, we briefly describe the Maxmin Expected Utility (MMEU) model and its result.

Let X be a non-empty set of outcomes and Y be the set of probability distributions over X. Intuitively, the elements of Y attach probability distributions to elements in X, such that the sum of all the possible distributions equals unity. Y may be thought of as the set of random outcomes.

Let S be the set of all the possible states of nature, such that S is a non-empty set and $\sum$ is an algebra-subset of the events contained in S. Let L be the set of acts. L_0 is a subset of L and contains the set of finite step functions from S to Y. L_c denotes the constant acts in L_0. L is a convex subset of Y^s and contains convex combinations that are performed pointwise, such that for f and g in Y^s and α in $[0, 1]$: $\alpha f + (1 - \alpha)g = h$, where $h(s) = \alpha f(s) + (1 - \alpha)g(s)$ for all $s \in S$.

A binary relation $\geqq$ over L gives the decision maker's preferences, such that $\geqq$ satisfies the following axioms:

A.1. *Weak Order - a) For all f and g in L, either $f \geqq g$ or $g \geqq f$.*

b) For all f, g and h in L : If $f \geqq g$ and $g \geqq h$ then $f \geqq h$.

This axiom means that an individual has complete preferences over acts, such that either act f is weakly preferred to act g, or vice versa. Moreover, the decision maker has transitive preferences, such that if f is weakly preferred to g and g is weakly preferred to h, it would imply that f is weakly preferred to h, for all acts f, g and h.

A.2. *Certainty-Independence - For all f, $g \in L$, $h \in L_c$ and $\alpha \in]0, 1[$:*

$$f > g \text{ iff } \alpha f + (1 - \alpha)h > \alpha g + (1 - \alpha)h.$$

This axiom is less restrictive than the standard independence axiom, since it is easier to think of mixtures of act f and g with a constant act h, rather than an arbitrary one. Given that both mixtures of acts contain the constant act h with the same probability $(1-\alpha)$, if a decision maker strictly prefers αf to αg, it would imply that he strictly prefers act f to act g.

A.3. *Continuity - For all $f, g, h \in L$: if $f > g$ and $g > h$, then there are α and β in $]0,1[$ such that*

$$\alpha f + (1-\alpha)h > g \textit{ and } g > \beta f + (1-\beta)h.$$

This axiom allows for continuity, i.e., it implies that small changes in acts result in small changes in the decision maker's preferences.

A.4. *Monotonicity - For all $f,\ g \in L$: if $f(s) \geqq g(s)$ on S, then $f \geqq g$.*

Put simply, if act f is strictly preferred to act g, irrespective of the true state $s \in S$ that materialises, then the decision maker always prefers act f to g.

A.5. *Uncertainty Aversion - For all $f, g \in L$ and $\alpha \in]0,1[$: $f \backsimeq g$ implies*

$$\alpha f + (1-\alpha)g \geqq f.$$

This axiom assumes that decision makers are ambiguity averse and would prefer to hedge across acts f and g. Thus, this axiom "smoothes" the distribution of utility over states.

A.6. *Non-degeneracy - Not for all, but for some f,$g \in L$, $f \geqq g$.*

The decision maker weakly prefers some acts over others.

Given these properties of the binary preference relation $\geqq$, Gilboa and Schmeidler (1989) state that the following conditions are equivalent:

1. $\geqq$ *satisfies assumptions $A.1 - A.5$ for $L = L_0$.*

2. *There exists an affine function* $u : Y \to R$ *and a non-empty, closed and convex set* C *of finitely additive probability measures on* $\sum$ *such that:* $f \geqq g$ *iff* $min_{P \in C} \int uof \ dP \geqq min_{P \in C} \int uog \ dP$ *(for all* $f, g \in L_0$*). Moreover, this function* u *is unique up to a positive transformations and the set* C *is unique if assumption* $A.6$ *is added to* 1).

Thus, we consider a decision maker who has a known utility function u, that allows him to calculate his utility from two acts f and g, given a set of probability measures. The decision maker calculates the minimum expected utility he would get from each act, i.e. the worst case scenario, and (being rational) prefers the act that gives him the higher utility. He thus maximizes the minimum expected utility from the two acts.

Intuitively, in the absence of information regarding probabilities, maxmin expected utility (MMEU) allows the decision maker to believe that a range of probabilities are possible. If the decision maker has a von Neumann-Morgenstern type of utility function and a convex set C of non-unique subjective probabilities, every action would have an interval of expected possible utilities attached to it. MMEU predicts that if there is an action a, such that it's minimum possible expected utility is greater than that of another action b, then the action a is preferred over action b.

If the convex set C, consists of a number of probability distributions, MMEU uses maxmin to choose between alternatives. However, when C contains a unique probability distribution, MMEU coincides with SEU, since only a single probability distribution exists for the decision-maker. As such, MMEU is a hybrid of SEU and maxmin.

2.1.7 Equilibrium under Ambiguity

In a Nash equilibrium, players are believed to behave in a manner that is consistent with the actual behaviour of their opponents. They perfectly anticipate the actions of their opponent and can thus provide a best response to it in the form of their own action.

Neumann and Morgenstern (1944) propose another equilibrium for zero-sum games in which players are completely ignorant of their opponent's behaviour. They suggest that in the absence of complete information regarding the opponent's behaviour, players consider what the worst outcome for all their strategies would be and then choose the strategy which yields the highest payoff among the worst outcomes (maximin).

However, in the case of ambiguity and non-additive beliefs, the Nash equilibrium idea of being able to have consistent beliefs about one's opponent's action and thus being able to play an optimum mixed strategy as a response to these beliefs, no longer holds. Dow and Werlang (1994) were the first to allow for players to have non-additive beliefs about their opponent's strategy choice. They assume that players opt for pure strategies and that in equilibrium, the beliefs about these pure strategies are best responses to the opponent's actions.

Consider a game with i players and a finite pure strategy set S_i. Each player i's beliefs about the opponent's behaviour is represented by a capacity v_i on S_{-i}, which is the set of strategy combinations which all other players excluding i could choose. Given non-additive beliefs, the expected payoff that a player i could earn from a strategy s_i is determined by using the Choquet integral.

Unlike Nash equilibrium where a player attaches additive probabilities to his opponent's actions, in the presence of ambiguity players need a "*support*" to be attached to their capacities. A support is a decision maker's belief of how his opponent will act. The support of a capacity is the smallest set of the opponent's strategies, whose complement has capacity zero, i.e., the player expects it to be infinitely less likely that this strategy set is used, but not entirely null. It must be kept in mind that even though the player might attach a measure zero to these strategies, he is not completely certain that the opponent will not use this particular strategy.

Let P_i be the non-additive belief and A_i be the pure strategy set for Player i. Further, let a_i be the support of the non-additive belief P_i. Thus, a_1 would denote the support for the non-additive belief P_1, over a pure strategy set A_1 for Player 1, and so on.

A pair of non-additive beliefs P_1 over A_1 and P_2 over A_2, is an Equilibrium under Ambiguity (EUA) if there exist supports for (P_1, P_2) such that:

1. a_1 maximises the expected utility of Player 1, given that P_2 represents Player 2's beliefs and strategies for Player 1, for all a_1 in the support of P_1

2. a_2 maximises the expected utility of Player 2, given that P_1 represents Player 1's beliefs and strategies for Player 2, for all a_2 in the support of P_2 (Dow and Werlang (1994)).

Thus, an EUA can exist no matter how ambiguity averse a player is, since agents take into account payoffs relative to the worst strategy of their opponent, while deciding their own actions. Moreover, it allows a subject to play the same game differently against different opponents, since in every instance the beliefs regarding the opponent's strategies would change. The above definition of an EUA is also compatible with bounded rationality justifications.[2] .

Klibanoff (1993) and Lo (1996) provide an alternative approach to equilibrium under ambiguity, which is consistent with MMEU. Players are allowed to have beliefs which are represented by multiple sets of additive probability distributions, on the basis of which they choose mixed strategies. Moreover, it is assumed that there is a strict preference among players to randomise between strategies, when they are indifferent to pure strategies.

2.1.8 Equilibrium under Ambiguity in N-Player Games

Eichberger and Kelsey (2000), extend the Dow and Werlang (1994) approach to n-player games. They assume that all players employ maximising behaviour given their beliefs. Furthermore, they assume that players behave consistently with their beliefs, i.e., no player expects his opponent to choose an action which is not a best response given their beliefs. This best response of player i given the beliefs or capacity v_i, is denoted $R_i(v_i) = \arg\max \{P_i(s_i, v_i) | s_i \in S_i\}$.

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[2] Bounded rationality is the idea individuals make decisions based on the limited amount of information they possess and the cognitive ability of their thought process, given the restrictions on their time. Thus, they might not make the universally optimal decision, but optimise given the resources at their disposal.

Given the maximising behaviour and consistency on the part of the players, a set of capacities $(v_1^*, ..., v_I^*)$ is an EUA, if for each player $i \in I$ there exists a support v_i^* such that this supp $v_i^* \subseteq \times_{j \in I \setminus \{i\}} R_j(v_j^*)$, i.e., v_i^* is the best response for player i, given all the other players' best responses to it (Eichberger and Kelsey (2000)).

For simplicity, we consider a 2×2 matrix game with two players J and K. Each player has a pure strategy set, such that $S_J = \{s_1, s_2\}$ and $S_K = \{t_1, t_2\}$ for Players J and K, respectively. The payoff matrix is in Table 2.2.

Table 2.2. Equilibrium under Ambiguity

		Player K	
		t_1	t_2
Player J	s_1	a_{11}, b_{11}	a_{12}, b_{12}
	s_2	a_{21}, b_{21}	a_{22}, b_{22}

Since each player has two strategies, the capacity representing J's beliefs can be denoted as: $v_J : (q_{t1}, q_{t2})$,where q_{t1} is J's belief of the probability with which K will choose t_1, and q_{t2} is J's belief of the probability with which K will choose t_2. Note that both $q_{t1} \geq 0$, $q_{t2} \geq 0$, and $q_{t1} + q_{t2} \leq 1$.

Similarly, for K: $v_K : (q_{s1}, q_{s2})$, where q_{s1} is K's belief of the probability with which Player J will choose s_1, and q_{s2} is K's belief of the probability with which J will choose s_2. Again, $q_{s1} \geq 0$, $q_{s2} \geq 0$, and $q_{s1} + q_{s2} \leq 1$.

The supports of these capacities is denoted:

$$\text{supp } v_J = \left\{ \begin{array}{ll} \{t_1\}, \{t_2\} & \text{for } q_{t1} = 0,\ q_{t2} = 0, \text{ i.e., } J \text{ believes } K \text{ would choose neither action.} \\ \{t_1\} & \text{for } q_{t1} > 0,\ q_{t2} = 0, \text{ i.e., } J \text{ believes } K \text{ is more likely to pick } t_1. \\ \{t_2\} & \text{for } q_{t1} = 0,\ q_{t2} > 0, \text{ i.e., } J \text{ believes } K \text{ is more likely to pick } t_2. \\ \{t_1, t_2\} & \text{for } q_{t1} > 0, q_{t2} > 0, \text{ i.e., } J \text{ believes } K \text{ could pick either option.} \end{array} \right\}$$

and

$$\text{supp } v_K = \left\{ \begin{array}{ll} \{s_1\}, \{s_2\} & \text{for } q_{s1} = 0,\ q_{s2} = 0, \text{ i.e., } K \text{ believes } J \text{ would choose neither action.} \\ \{s_1\} & \text{for } q_{s1} > 0,\ q_{s2} = 0, \text{ i.e., } K \text{ believes } J \text{ is more likely to pick } s_1. \\ \{s_2\} & \text{for } q_{s1} = 0,\ q_{s2} > 0, \text{ i.e., } K \text{ believes } J \text{ is more likely to pick } s_2. \\ \{s_1, s_2\} & \text{for } q_{s1} > 0, q_{s2} > 0, \text{ i.e., } K \text{ believes } J \text{ could pick either option.} \end{array} \right\}.$$

Moreover, the Choquet Integral of the capacities are calculated as under:

$$P_J(s_i, v_J) = \left\{ \begin{array}{l} a_{i1} \cdot q_{t1} + a_{i2} \cdot (1 - q_{t1}) \text{ for } a_{i1} > a_{i2} \\ a_{i1} \text{ for } a_{i1} = a_{i2} \\ a_{i2} \cdot q_{t2} + a_{i1} \cdot (1 - q_{t2}) \text{ for } a_{i1} < a_{i2} \end{array} \right\},$$

and

$$P_K(t_i, v_K) = \left\{ \begin{array}{l} b_{i1} \cdot q_{s1} + b_{i2} \cdot (1 - q_{s1}) \text{ for } b_{i1} > b_{i2} \\ b_{i1} \text{ for } b_{i1} = b_{i2} \\ b_{i2} \cdot q_{s2} + b_{i1} \cdot (1 - q_{s2}) \text{ for } b_{i1} < b_{i2} \end{array} \right\}.$$

Given this basic setup, three cases may be considered:

Case 1. Where one of the two players (or both) has a dominant strategy. In this case there will be a unique Nash equilibrium.

Case 2. Where neither player has a dominant strategy. In this case, there is a unique mixed-strategies Nash equilibrium.

Case 3. Where neither player has a dominant strategy. In this case, there are three Nash equilibria - two Pure Nash and one with mixed strategies.

The three cases are discussed in detail below.

Case 1. Where one of the two players (or both) has a dominant strategy. If $a_{11} > a_{21}$ and $a_{12} > a_{22}$, then s_1 is the dominant strategy for J. If in addition, $b_{11} > b_{12}$ and $b_{21} > b_{22}$, then K has a dominant strategy as well, namely t_1. Thus, there is a unique Nash equilibrium (s_1, t_1).

However, if K does not have a dominant strategy, while s_1 is the dominant strategy for J, then the unique Nash is determined by whether or not K believes J will use his dominant strategy. For instance, if the 2×2 game is as under:

Table 2.3. Equilibrium under Ambiguity Case 1

		Player K	
		t_1	t_2
Player J	s_1	$3, 8$	$3, 7$
	s_2	$0, 0$	$0, 7$

For Player J, $P_J(s_1, v_J) = 3$ and $P_J(s_2, v_J) = 0$, therefore his best response would be $R_J(v_J) = \{s_1\}$. For Player K, $P_K(t_1, v_K) = 8 \cdot q_{s1}$ and $P_K(t_2, v_K) = 7$. If K believes that $q_{s1} < \frac{1}{2}$, her best response would be $R_K(v_K) = \{t_2\}$. Thus if

$(q_{s1}, q_{s2}) = (\alpha, 0)$ is believed to be played with $\alpha < \frac{1}{2}$, and $(q_{t1}, q_{t2}) = (0, \beta)$ is believed to be played with with $\beta > 0$, the supports for (v_J^*, v_K^*) are supp $v_J^* = \{t_2\}$ and supp $v_K^* = \{s_1\}$.

Hence if K is not certain that J will definitely play his dominant strategy, she will stick with her safe strategy t_2, that guarantees her the payoff of 7. The equilibrium under ambiguity in this case does not coincide with the Nash.

Case 2. Where neither player has a dominant strategy and there is a unique mixed-strategies Nash equilibrium. An example of this case is the matching pennies game as under:

Table 2.4. Equilibrium under Ambiguity Case 2

		Player K	
		t_1	t_2
Player J	s_1	$1, 0$	$0, 1$
	s_2	$0, 1$	$1, 0$

As can be seen above, neither player has a dominant strategy - both players are indifferent between their strategies. The support thus consists of the full strategy set. The equilibrium under ambiguity is (v_J^*, v_K^*) where $v_J^*(s_1) = v_J^*(s_2) = \alpha$, i.e., J plays either/both of his strategies with some probability α and similarly, K plays either/both of her strategies with some probability β or $v_K^*(t_1) = v_K^*(t_2) = \beta$. The supports for (v_J^*, v_K^*), are supp $v_J^* = \{t_1, t_2\}$ and supp $v_K^* = \{s_1, s_2\}$.

The expected payoffs of J are $P_J(s_1, v_J^*) = 1 \cdot v_J^*(s_1) + 0 \cdot (1 - v_J^*(s_1)) = \alpha$ and $P_J(s_2, v_J^*) = 1 \cdot v_J^*(s_2) + 0 \cdot (1 - v_J^*(s_2)) = \alpha$. Therefore his best response would be $R_J(v_J^*) = \{s_1, s_2\}$. Using the same logic, The expected payoffs of K are $P_K(t_1, v_K^*) = 1 \cdot v_K^*(t_1) + 0 \cdot (1 - v_K^*(t_1)) = \beta$ and $P_K(t_2, v_K^*) = 1 \cdot v_K^*(t_2) + 0 \cdot (1 - v_K^*(t_2)) = \beta$. Therefore her best response would be $R_K(v_K^*) = \{t_1, t_2\}$.

The equilibrium under ambiguity is similar to the Nash equilibrium in this case, since both players are indifferent between the strategies available to them. However, the key difference is that there is no certain probability α or β with which either of the players must play their strategies.

Case 3. Where neither player has a dominant strategy and there are three Nash equilibria - two Pure Nash and one with mixed-strategies. For instance, let us consider the game below:

Table 2.5. Equilibrium under Ambiguity Case 3

		Player K	
		t_1	t_2
Player J	s_1	$1,2$	$1,1$
	s_2	$0,0$	$2,1$

In this case, there are three Nash equilibria - two Pure Nash (s_1, t_1) and (s_2, t_2), and one with mixed strategies $((\frac{1}{2}s_1, \frac{1}{2}s_2), (\frac{1}{2}t_1, \frac{1}{2}t_2))$. However, there are four types of equilibrium under ambiguity:

$$\text{i) } (v_J^*, v_K^*) : \left\{ \begin{array}{c} q_{s1} > \frac{1}{2},\ q_{s2} = 0,\ \text{supp } v_J^* = \{t_1\},\ R_J(v_J^*) = \{s_1\} \\ q_{t1} > 0,\ q_{t2} = 0,\ \text{supp } v_K^* = \{s_1\},\ R_K(v_K^*) = \{t_1\} \end{array} \right\}$$

Here J's expected payoffs are, $P_J(s_1, v_J) = 1$ and $P_J(s_2, v_J) = 2 \cdot q_{t2}$. If he believes $q_{t2} = 0$, his best response would be $R_J(v_J^*) = \{s_1\}$. For K, $P_K(t_1, v_K) = 2 \cdot q_{s1}$ and $P_K(t_2, v_K) = 1$. If she believes $q_{s1} > \frac{1}{2}$, her best response would be $R_K(v_K^*) = \{t_1\}$. The support for these capacities is: supp $v_J^* = \{t_1\}$ and supp $v_K^* = \{s_1\}$. The equilibrium under ambiguity in this case coincides with the first pure Nash equilibrium (s_1, t_1).

$$\text{ii) } (v_J^*, v_K^*) : \left\{ \begin{array}{c} q_{s1} = 0,\ q_{s2} > 0,\ \ \text{supp } v_J^* = \{t_2\},\ R_J(v_J^*) = \{s_2\} \\ q_{t1} = 0,\ q_{t2} > \frac{1}{2},\ \text{supp } v_K^* = \{s_2\},\ R_K(v_K^*) = \{t_2\} \end{array} \right\}$$

Here J's expected payoffs are, $P_J(s_1, v_J) = 1$ and $P_J(s_2, v_J) = 2 \cdot q_{t2}$. If he believes $q_{t2} > \frac{1}{2}$, his best response would be $R_J(v_J^*) = \{s_2\}$. For K, $P_K(t_1, v_K) = 2 \cdot q_{s1}$ and $P_K(t_2, v_K) = 1$. If she believes $q_{s1} = 0$, her best response would be $R_K(v_K^*) = \{t_2\}$. The support for these capacities is: supp $v_J^* = \{t_2\}$ and supp $v_K^* = \{s_2\}$. The equilibrium under ambiguity in this case coincides with the second pure Nash equilibrium (s_2, t_2).

$$\text{iii) } (v_J^*, v_K^*) : \left\{ \begin{array}{c} q_{s1} = \frac{1}{2},\ q_{s2} > 0,\ \text{supp } v_J^* = \{t_1, t_2\},\ R_J(v_J^*) = \{s_1, s_2\} \\ q_{t1} > 0,\ q_{t2} = \frac{1}{2},\ \text{supp } v_K^* = \{s_1, s_2\},\ R_K(v_K^*) = \{t_1, t_2\} \end{array} \right\}$$

Here J's expected payoffs are, $P_J(s_1, v_J) = 1$ and $P_J(s_2, v_J) = 2 \cdot q_{t2}$. If he believes $q_{t2} = \frac{1}{2}$, he earns the same payoff whether he chooses s_1 or s_2, and so he is indifferent between his strategies. His best response would be $R_J(v_J^*) = \{s_1, s_2\}$. For K, $P_K(t_1, v_K) = 2 \cdot q_{s1}$ and $P_K(t_2, v_K) = 1$. If she believes $q_{s1} = \frac{1}{2}$, she earns the same payoff whether she chooses t_1 or t_2, and so she is indifferent between her strategies. Her best response would be $R_K(v_K^*) = \{t_1, t_2\}$. The support for these capacities is: supp $v_J^* = \{t_1, t_2\}$ and supp $v_K^* = \{s_1, s_2\}$. The equilibrium under ambiguity in this case is similar to the mixed strategies Nash equilibrium, however no definite probabilities can be attached to the strategies. Any strategy combination may be chosen by the players.

$$\text{iv) } (v_J^*, v_K^*) : \left\{ \begin{array}{c} q_{s1} \in \left[0, \frac{1}{2}\right],\ q_{s2} = 0,\ \text{supp } v_J^* = \{t_2\},\ R_J(v_J^*) = \{s_1\} \\ q_{t1} = 0,\ q_{t2} \in \left[0, \frac{1}{2}\right],\ \text{supp } v_K^* = \{s_1\},\ R_K(v_K^*) = \{t_2\} \end{array} \right\}$$

Here J's expected payoffs are, $P_J(s_1, v_J) = 1$ and $P_J(s_2, v_J) = 2 \cdot q_{t2}$. If he believes $q_{t2} \in \left[0, \frac{1}{2}\right]$, he is uncertain about which equilibrium will be played. His best response would be $R_J(v_J^*) = \{s_1\}$, since this guarantees him a sure payoff of 1. For K, $P_K(t_1, v_K) = 2 \cdot q_{s1}$ and $P_K(t_2, v_K) = 1$. If she believes $q_{s1} \in \left[0, \frac{1}{2}\right]$, she too is uncertain about which equilibrium will be played by the opponent. Her best response would be $R_K(v_K^*) = \{t_2\}$, since this is the safe strategy for her and guarantees her a certain payoff of 1. The support for these capacities is: supp $v_J^* = \{t_2\}$ and supp $v_K^* = \{s_1\}$. The equilibrium under ambiguity in this case is (s_1, t_2), with each player choosing the ambiguity safe action.

The above analysis shows that an equilibrium under ambiguity may exist even if players' possess non-additive beliefs that are incompatible with Nash equilibrium.

2.2 Ambiguity in Experiments

Strategic aversion to ambiguity, makes ambiguous games worthy of being studied as a separate class of games. Studying the attitude of individuals to ambiguity in games would not only provide a better understanding to the limitations of the SEU theory as it stands today, but also provide an empirical justification for the theoretical work that has been done so far in modeling individuals' attitudes to ambiguity.

In particular, one expects ambiguity to be highest in one-shot normal form games, which makes it a logical starting point to study the effect of ambiguity in games. Here we survey previous literature related to the effect of ambiguity.

2.2.1 Papers studying Ambiguity in Single-Person Decision Problems

A *"competence hypothesis"* was proposed by Heath and Tversky (1991), in which they argue that the decision maker's attitude to ambiguity is determined by how competent he feels in the situation he faces. They find that if individuals feel competent in certain areas, they prefer ambiguous gambles to lottery tickets (thereby displaying ambiguity loving behaviour). On the other hand, individuals preferred lottery tickets to ambiguous gambles in areas where they were not competent (thereby displaying ambiguity averse behaviour).

Fox and Tversky (1995), proposes a *"comparative ignorance"* hypothesis, according to which, if an individual evaluates an ambiguous and an unambiguous bet simultaneously, it undermines his feeling of competence, leading to an attitude of ambiguity aversion. However, if the same person were to evaluate the ambiguous and unambiguous bets individually, his ambiguity aversion would decrease.

They explain a fall in ambiguity aversion, by saying that people feel more competent in evaluating an ambiguous bet in isolation, rather than jointly evaluating it in the presence of the unambiguous bet. According to Fox and Tversky (1995), the popular Ellsberg phenomenon and resultant ambiguity averse behaviour is inherently present only in comparative contexts and would not arise when uncertain prospects are evaluated independent of each other.

Chow and Sarin (2001), test the Fox and Tversky (1995) result in order to ascertain whether ambiguity aversion does indeed disappear in a non-comparative context. They find that in their experiments, subjects always price a known bet higher than an ambiguous one and thus, ambiguity averse behaviour is prevalent in both comparative as well as non-comparative contexts. However, the difference in prices between the known bet and the ambiguous one was found to be higher in the comparative context, than under independent evaluation.

Even though ambiguity aversion has been accepted in most cases, some studies have found evidence of the prevalence of an ambiguity loving attitude among individuals. Binmore, Stewart, and Voorhoeve (2011), attempt to test whether subjects are indeed ambiguity averse. They investigate whether the apparent ambiguity averse behaviour, predominantly reported by a number of papers, can be captured by the Hurwicz criterion.

They report that subject behaviour in experiments conducted by them is inconsistent with the Hurwicz criterion. Instead, they find that the principle of insufficient reason[3] has greater predictive power with respect to their data, than ambiguity averse behaviour.

Charness, Karni, and Levin (2012), test whether individuals display a non-neutral attitude towards ambiguity. In particular, they investigate whether subjects who are given a chance to interact, can persuade others to change their ambiguity attitude. They find that though a number of their subjects displayed an incoherent attitude towards ambiguity, a majority of subjects displayed ambiguity neutral preferences. A small minority of subjects (smaller than the number of subjects who were ambiguity-incoherent) displayed ambiguity averse and ambiguity seeking behaviour. More interestingly, they find that if subjects are allowed to interact with each other, given the right incentives, ambiguity neutral subjects often manage to convince ambiguity seeking and ambiguity incoherent subjects to change their mind and follow ambiguity neutral behaviour.

2.2.2 Papers studying Ambiguity in Ellsberg Urn Experiments

Halevy (2005), extends the standard Ellsberg type experiment to demonstrate that ambiguity preferences are associated with compound objective lotteries. The study finds that the subject pool can be divided into two groups of people. The first group consists of those who are ambiguity neutral and can reduce compound

[3]:Let there be $n > 1$ mutually exclusive possibilities. According to the principle of insufficient reason, if the n possibilities are indistinguishable except in name, then the decision maker should assign each a probability equal to $\frac{1}{n}$.

For example, a fair dice has 6 faces, labeled from 1 to 6. If the dice is thrown, it must land on one of the six possibilities. According to the principle of insufficient, we must assign each of the possible outcomes a probability of $\frac{1}{6}$.

objective lotteries, i.e., they have behaviour which is consistent with SEU. The second group consists of people who display different preferences over ambiguity and compound lotteries and are consistent with models that capture ambiguity seeking/averse preferences. They conclude that there is no unique theory that can capture all the different preference patterns observed in a given sample. As such, the experimental findings of Halevy (2005) are consistent with Epstein (1999), where ambiguity aversion is defined as a behaviour that is not probabilistically sophisticated and thus cannot be aligned with a specific functional form or model.

Dynamic consistency and consequentialism are the two key links between conditional and unconditional preferences. Dynamic consistency entails that a decision made ex-ante, remains unchanged if preferences are updated. For instance, suppose there are two acts f and g, and an event E. Consider a decision maker who prefers f to g if event E occurs and also prefers f to g if E does not occur. Then the decision maker always prefers f to g, i.e., he does not make a decision that will be reversed if information about event E is made known ex-post.

Consequentialism entails that only valid outcomes (that are still possible) are taken into account once preferences are updated. Intuitively, if a decision maker is informed that the event E has occured, while making his conditional preferences, he should only be concerned with the ambiguity surrounding subevents of E. Individuals who display the Ellsberg paradox cannot be both dynamically consistent as well as consequentialist.

Dominiak, Dürsch, and Lefort (2009), test individual behaviour in a dynamic Ellsberg urn experiment, to test whether individuals behave in a manner that is dynamically consistent and consequentialist. Subjects were presented with an urn containing 30 balls, 10 of which were known to be yellow, the remaining an unknown mix of blue and green. They were then asked to choose whether they would prefer winning if a yellow ball is drawn or winning if a blue ball is drawn.

Once they had stated their choice to the earlier question, subjects were told that a ball had been drawn from the urn and that it was not green. They were then asked again whether they preferred winning if a yellow/blue ball was drawn. Depending on the choices made in the follow-up question, subjects were judged as being dynamically consistent/consequentialist/both. They find that subject behaviour is more in tune with consequentialism than with dynamic consistency. Moreover, they find that subjects who are initially ambiguity-neutral when faced with a static Ellsberg urn, cannot be described by SEU theory when faced by the dynamic version of the Ellsberg urn.

Eliaz and Ortoleva (2011), study a three-colour Ellsberg urn in which they have increased the level of ambiguity. Subjects face ambiguity on two accounts: the unknown proportion of balls in the urn as well as the size of the prize money. In their experiment, both winning and the amount that the subject could possibly win were both perfectly correlated - either positively or negatively, depending on which of the two treatments was run by them. In the experiment, most subjects preferred betting in the positively correlated treatment rather than the negative one. Moreover, subjects also showed a preference for a gamble when there was positively correlated ambiguity, as opposed to a gamble without any ambiguity.

Another Ellsberg experiment that allows for an additional source of ambiguity is studied by Eichberger, Oechssler, and Schnedler (2011). They consider a two-colour Ellsberg experiment and insert an additional element of ambiguity in terms of the money the subject wins in the various outcomes. In the standard treatment, if the colour drawn matches the colour chosen by the subject, he receives an envelope marked with an equal sign (=), and if it does not match he receives an envelope with an unequal sign ($\neq$). He is aware that the (=) envelope contains €3 and the ($\neq$) envelope contains €1. This standard treatment is referred to as O, or open envelope.

In the second treatment called the S or sealed envelope treatment, subjects know that one envelope contains €3 and one contains €1, but do not know which envelope contains which amount. In the third treatment called R or the random treatment, subjects are told that the amount in the envelope will be determined by the toss of a fair coin, once they have made their choice of colour for the bet on the urn.

Treatment O, is the standard Ellsberg experiment. In treatment R, winning €3 or €1 depends totally on the toss of the coin and so the subject faces equal odds of winning either amount. Treatment S, is different from the other two treatments in that subjects are not sure how much they would win, even if they won. They should thus, be indifferent between the ambiguous urn and the known one. Subjects were asked to choose an urn and the colour of the ball they would like to bet on. In addition, they could state that they are indifferent between the known urn and the unknown one, as well as being indifferent between a green ball and a blue one. In case of indifference, subject were assigned to the unknown urn/blue ball options.

Eichberger, Oechssler, and Schnedler (2011), find that 30 of the 48 (62%) subjects preferred the known urn in treatment O, which is similar to the standard Ellsberg result. In treatment R, when subjects should be indifferent between the ambiguous urn and the known one since their payment depends on the flip of a coin, 25 of the 48 (52%) subjects preferred the known urn. In treatment S, 19 (40%) subjects preferred the known urn, 17 (35%) preferred the ambiguous one, while 12 (25%) stated they were indifferent between the two. It can be noted that significantly fewer subjects preferred the known urn to the ambiguous one in treatment S where there was additional ambiguity, when compared to treatment O, the standard Ellsberg case.

Liu and Colman (2009), presented subjects with gambles that were modelled as either modified Ellsberg urn choices or as marketing strategy decisions. The subjects had to choose between ambiguous and risky gambles, under single as well as multiply repeated choice conditions. It was found that subjects chose the ambiguous gambles more often in repeated choice conditions than they did in single-choice conditions. Moreover, the number of subjects choosing risky single choices and ambiguous repeated choices exceeded the number of subjects who preferred ambiguous single choices and risky repeated choices. One of the reasons given to explain this behaviour, is that if subjects believed that luck was loaded against them in single events, they might have felt safer in the repeated conditions.

2.2.3 Papers studying Ambiguity in Games

Colman and Pulford (2007), explain the concept of ambiguity aversion as a state that arises as a result of a pessimistic response to uncertainty, mainly driven by the loss of decision confidence. They argue that people tend to become anxious and less confident while making decisions in the face of ambiguity, as opposed to known-risk situations. In a series of experiments, they found that individual responses differed between ambiguous and risky versions of the game being studied. Players did not respond to ambiguity by simply equating it to riskiness, but showed a marked preference to avoid ambiguity whenever the option of doing so was provided to them.

Keller, Sarin, and Sounderpandian (2007) investigate whether individuals deciding together as pairs (termed dyads in their paper) display ambiguity averse behaviour. Participants were initially asked to state how much they were willing to pay for six monetary gambles. Five of the six gambles put before the subjects involved ambiguity, while the sixth involved no ambiguity.

Once the particpants had all disclosed their individual willingness to pay, they were randomly paired with another subject and each pair had to re-specify how much they were willing to pay for the six gambles. It was found that the pairs displayed risk averse as well as ambiguity averse preferences. It was observed that the willingness-to-pay among pairs of individuals deciding together, was lower than the average of their individual willingness-to-pay for gambles. They thus conclude that ambiguity averse behaviour is prevalent in group settings.

Keck, Diecidue, and Budescu (2012), conduct an experiment in which subjects made decisions individually, as a group, and individually after interacting and exchanging information with others. Subjects were asked to make binary choices between sure sums of money and ambiguous and risky bets. They found that individuals are more likely to make ambiguity neutral decisions after interacting with other subjects. Moreover, they find that ambiguity seeking and ambiguity averse preferences among individuals are eliminated by communication and interaction between individuals; and as such, groups are more likely to make ambiguity neutral decisions.

Ivanov (2009), discusses the findings of a series of experiments on one-shot normal form games run to distinguish between eighteen types of players. A person was classified on the basis of his attitude to ambiguity - as being either ambiguity averse, ambiguity neutral, or ambiguity loving; on the basis of his attitude to risk - as being risk averse, risk neutral or risk loving; and whether he played strategically or naively.

Each type was modelled on the basis of the dimension they fell in. Ambiguity loving, neutral and averse individuals were modelled as having subadditive, additive and superadditive beliefs, respectively. Risk loving, neutral and averse individuals were modelled as having convex, linear and concave utility functions, respectively. A person who played in a naive manner was modelled as having a uniform belief in every game he played, whereas if he played strategically, his beliefs were different for every game and were thus unrestricted.

The study finds that about 32% of the subjects taking part in the experiment were ambiguity loving, as opposed to 22% who were ambiguity averse. The majority of subjects (46%) were found to be ambiguity neutral. While being tested on the basis of their attitude to risk, 62% of the subjects were found to be risk averse, 36% to be risk neutral, and a mere 2% were risk loving. 90% of the subjects played in a strategic manner, while 10% played naively.

One of the questions Ivanov (2009) raises, is the fact that there are more subjects who are ambiguity loving/neutral, than those who are ambiguity averse, given that on average a majority of them play strategically. This is attributed to players' altruistic behaviour, i.e., they played in a manner that would maximise the sum of both players' payoffs. This may be because a player is willing to compromise with his opponent, in order to do well himself.

Nagel, Heinemann, and Ockenfels (2009) consider strategic uncertainty in one-shot coordination games with strategic complementarities. In the study conducted by them, they elicit certainty equivalents for situations where a subject's payoff depends on his opponents' behaviour. In each coordination game a subject had a choice between a safe amount X, which was allowed to vary such that $X \leq$ €15, and an option where the payoff was dependant on his opponent's decision. In the uncertain option, a subject could earn €15 if at least a fraction $k \in (0, 1]$ of his opponent's chose the same option as him, else he earned nothing. Subjects were found to choose the safe option when X was large, while they chose the uncertain option for small values of X. The point at which a subject switched from the safe option to the uncertain one, was interpreted as his certainty equivalent for strategic uncertainty. This is analogous to situations in which the risk attitude of a subject is measured with respect to lotteries.

2.2.4 Papers studying Ambiguity in Public Goods Games

In a public goods game the dominant strategy, predicts zero contribution to the public good, since the linear payoff function of the public good lies below the payoff the player would get from investing in the private good. However empirical findings show that in one-shot games and initial stages of finitely repeated games, players contribute between $40\% - 60\%$ of their initial endowment to the public good, which is halfway between the free-riding level and the Pareto optimal level. It was also noted that the contributions made towards the public good decreased in subsequent rounds and increased if there was any form of interaction between the players (Davis and Holt (1993)).

The fact that there is positive contribution towards the public good despite a Nash prediction of free-riding, could be accrued to altruistic behaviour by the subjects (Ledyard (1995)), or to decision errors on the part of the subjects because they are not clear about the rules of the game (Andreoni (1995)). The third reason for the increased contribution to the public good was put forth by Eichberger and Kelsey (2002), who study the effect of ambiguity on the voluntary provision of public goods.

Eichberger and Kelsey (2002) show that when the production function for public goods is concave or there is diminishing marginal utility of public goods, ambiguity-aversion causes public good provision to be above Nash equilibrium level.[4] More generally they show the deviation from the Nash equilibrium depends on the nature of strategic interactions taking place, i.e., on the basis of whether the game being played was one of strategic substitutes or complements.

[4] These results will apply even in the presence of increasing returns to scale in production provided there is sufficiently strong diminishing marginal benefit from public goods.

Voluntary provision of public goods may be considered as an example of a game of strategic substitutes with positive externalities. If one individual contributes more, this lowers the marginal product of other people's donations. Ambiguity aversion causes a given individual to over-weigh bad outcomes. In this case, a bad outcome is when others make low donations. When others' donations are low, the marginal benefit of a donation by the given individual is high. Thus the expected effect of ambiguity would be to increase the perceived marginal benefit of donations by a given individual. If all have similar perceptions of ambiguity, total donations will rise. Hence an increase in ambiguity is expected to raise both individual and total donations.

Di Mauro and Castro (2008) conduct a set of experiments designed to test the Eichberger and Kelsey (2002) hypothesis that it is ambiguity that causes an increase in contribution towards the public good, and not altruism. In order to negate the chance that altruism, or a feeling of reciprocation prompted the subjects' actions, the subjects were informed that their opponent would be a virtual agent and the opponent's play was simulated by a computer. It was explained to them that their opponent's (i.e., the computer's) contributions would not be affected in any way by the subject's contribution to the public good. The subjects were thus made to understand that there should be no expectation of reciprocity from the virtual player.

Subjects played in two scenarios, one with risk, the other with ambiguity. It was noted that contributions were significantly higher when the situation was one of ambiguity. These results showed that there was indeed evidence that ambiguity significantly affects the decisions made by individuals, in a manner that depends directly on the strategic nature of the game in consideration, and not altruism.

Another paper that tests whether ambiguity affects individual behaviour in a game setting is Eichberger, Kelsey, and Schipper (2008), who study strategic ambiguity in games experimentally. They studied games in which subjects faced either a granny, who was described as being ignorant of economic strategy, a game theorist, who was described as a successful professor of economics, or another student as an opponent. It was conjectured that subjects would view the granny as a more ambiguous opponent than the game theorist. They find that subjects did find the granny to be the more ambiguous opponent, and this affected their decision choices.

Ambiguity averse actions were chosen significantly more often by subjects against the granny, as opposed to against the game theorist, irrespective of whether the game was one of strategic complements, strategic substitutes or one with multiple equilibria. When the level of ambiguity the subjects faced while playing the granny was compared to the level of ambiguity the subjects faced playing other students, it was found that the players still found the granny a more ambiguous opponent.

Subjects were also found to react to variations in the level of ambiguity, which was tested by altering the cardinal payoff in the game while keeping the ordinal payoff structure unchanged. It can thus be seen that subjects react not only to ambiguity on the part of the opponent being faced, but also to subtle changes in the payoff structures of the experiment being conducted (Eichberger, Kelsey, and Schipper (2008))

CHAPTER 3

AN EXPERIMENTAL STUDY ON THE EFFECT OF AMBIGUITY IN A COORDINATION GAME

3.1 Introduction

This chapter reports an experimental study of the impact of ambiguity in games. Although there is extensive experimental literature which shows that ambiguity affects decision making, most of it studies single-person decisions. There are relatively few experiments that test whether ambiguity affects behaviour in games. A game is a stylized way of representing a situation where a group of individuals have to make a number of linked decisions and thus forms a model of many economic interactions. Games provide a valuable setting in which economic models can be replicated in the laboratory. Since many economic problems can be represented as games we believe this research will be useful for understanding the impact of ambiguity in economics.

Moreover, there is very little previous experimental research on the impact of ambiguity in strategic situations. Previous studies have established that ambiguity does affect decision-making. However, they do not document the nature of the impact that ambiguity had on decision-making. It is thus difficult to predict what effect ambiguity has, and in which direction ambiguity will cause behaviour to change. The research documented in this chapter aims to experimentally test comparative statics of ambiguity in games.

Table 3.1. Battle of Sexes Game

		Player 2		
		L	M	R
Player 1	T	0, 0	300, 100	$50, x$
	B	100, 300	0, 0	$55, x$

We consider a Battle of Sexes game which has an added safe strategy available for Player 2 (See Table 3.1). The value of x, which is the safe option available to

Player 2, varies every round in the range $60 - 260$. For some values of x, the safe strategy (in our game, option R) is dominated by a mixed strategy of L and M, and thus would not be played in a Nash equilibrium and would be deleted under iterated dominance.

The traditional battle of the sexes games has two pure Nash equilibria, neither of which is focal. Hence, we expect ambiguity to be high due to the potential multiplicity of equilibria and to the fact that our games were one shot. The effect of ambiguity as to which equilibrium strategy will be chosen by the opponent is high, making R (the ambiguity-safe option) attractive for Player 2. Thus, even if strategy R is not played in Nash equilibrium, it may be chosen in an equilibrium under ambiguity (EUA).

The ambiguity safe strategy is never chosen in Nash equilibrium for the parameter values considered by us. Moreover for some values of x, our games are dominance solvable and R is not part of the equilibrium strategy. Despite this, we find that R is chosen quite frequently by subjects in our experiment. While the behaviour of the Row Player, is consistent with expected behaviour of randomizing $50 : 50$ between her strategies, the Column Player shows a marked preference for avoiding ambiguity and choosing his ambiguity-safe strategy. Thus, ambiguity influences behaviour in the games.

During the experiment, we alternated the Battle of Sexes games with Ellsberg urn type decision problems. This had the dual aim of erasing the short term memory of subjects, so that decisions on previous rounds did not affect subsequent behaviour; and providing an independent measure of subjects' ambiguity-attitudes. Moreover, we wished to test if there was a correlation between ambiguity-averse behaviour in the game and ambiguity-attitude in the single person decision problem.

In the Ellsberg urn rounds, subjects were presented with an urn containing 90 balls, of which 30 were Red, and the remainder an unknown proportion of Blue or Yellow and asked to pick a colour to bet on. The payoff attached to Red (the balls whose proportion was known) was varied in order to obtain an ambiguity threshold.

We did not observe a notable correlation between ambiguity aversion in the Battle of Sexes game, and ambiguity-attitude in the Ellsberg urn.

It is interesting to note however, that people appeared to be more ambiguity-averse in a two-person game experiment, as opposed to single-person decision problems. This may be because in single person decision problems a proxy for ambiguity is introduced by the experimenter, using an artificial device such as the Ellsberg urn. However in games, ambiguity is created by the other subjects taking part in the experiment and hence there is no need for the experimenter to introduce a proxy for ambiguity. Behaviour in the financial market is dependant on other people, and games can be used to effectively model such economic conditions. Natural disasters on the other hand, are more like single person decision problems.

3.1.0.0.1 Organisation of the Chapter In Section 2, we describe the theory being tested in the experiments. Section 3 describes the experimental design employed, Section 4 consists of data analysis and results, Section 5 reviews related literature and Section 6 provides a summary of results together with future avenues of research.

3.2 Preferences and Equilibrium under Ambiguity

3.2.1 Modelling Ambiguity

The Ellsberg paradox is a violation of the Subjective Expected Utility (SEU) Savage (1954). One version of the paradox is explained below. Consider an urn filled with 90 balls, 30 of which are red (R) and the remaining 60 are of an unknown mix of blue (B) and yellow (Y). One ball is drawn at random, and the payoff depends on the colour of the ball drawn and the act you choose. Subjects are asked to choose between acts f, g, f', g' as shown in the Table 3.2 (Pay-offs in Experimental Currency Units - ECU):

Subjects are asked to choose between f and g, generally prefer f because of the definite $\frac{1}{3}$ chance of winning 100 ECU to the ambiguous act g, but when asked

Table 3.2. The Ellsberg Options

	30 balls	60 balls	
Act	Red R	Blue B	Yellow Y
f	100	0	0
g	0	100	0
f'	100	0	100
g'	0	100	100

to choose between f' and g', the same subjects prefer g' which gives a $\frac{2}{3}$ chance of winning 100 ECU, again avoiding the ambiguous act f'.

These choices cannot be represented as maximising expected utility with respect to a standard subjective probability distribution π. Choosing f rather than g implies $\pi(R) > \pi(B)$. However preferring g' to f' implies $\pi(B \cup Y) > \pi(R \cup Y)$. Given the standard additivity properties of probabilities, i.e. $\pi(R \cup Y) = \pi(R) + \pi(Y)$, these two inequalities are inconsistent. The inconsistency would not arise however, if we represented beliefs by a non-additive set function ν. In this case it is possible that $\nu(R \cup Y) \neq \nu(R) + \nu(Y)$, which could be compatible with the choices in the Ellsberg paradox.

Non-additive beliefs were first introduced and used by Schmeidler (1989). He proposed a theory called **Choquet Expected Utility (CEU)**, where outcomes are evaluated by a weighted sum of utilities, but unlike EUT the weights used depend on the acts. An intuitive exposition of Schmeidler's model, which reformulates Savage's axioms is given in Sarin and Wakker (1992). The model is extended such that it preserves additivity in beliefs for events in the face of risk, while permitting non-additivity for ambiguous events.

The CEU model also categorises individuals' response to ambiguity. Individuals can be either optimistic or pessimistic in their outlook towards ambiguity. An optimistic outlook would over-estimate the likelihood of a good outcome - inducing one to bid on a gold mine, with the hope that it would make one very rich. On the other hand, a pessimistic outlook would over-estimate the likelihood of a bad outcome - such as losing all your wealth in a bad investment.

CEU uses capacities to model optimistic and pessimistic outlooks to ambiguity. A capacity v is convex (resp. concave) if for all A and $B \subseteq S$, $v(A \cup B) + v(A \cap B)$

$\geq v(A) + v(B)$, (resp. $v(A \cup B) + v(A \cap B) \leq v(A) + v(B)$), where A, B are events contained in the universal set S. In CEU, convex (resp. concave) capacities are used to model a pessimistic (resp. optimistic) outlook to ambiguity.

Neo-additive capacities were introduced by Chateauneuf, Eichberger, and Grant (2007), and are so named because they are additive over non-extreme outcomes and are convex combinations of an additive capacity and a special capacity. A neo-additive capacity represents an ambiguous belief about an additive probability distribution π, with δ determining the size of the set of probabilities around π and thus measures the decision maker's ambiguity. His/her attitude to ambiguity is represented by the parameter α, with higher values of α corresponding to greater ambiguity-aversion.

Consider a two-player game with a finite set of pure strategies S, such that s_i is the player's own strategy and s_{-i} denotes the set of possible strategy profiles for i's opponents. The payoff function of player i is denoted $u_i(s_i, s_{-i})$. The functional form of preferences may be represented as:

$$V_i\left(s_i; \pi_i, \alpha_i, \delta_i\right) = \delta_i \alpha_i M_i\left(s_i\right) + \delta_i\left(1 - \alpha_i\right) m_i\left(s_i\right) + \left(1 - \delta_i\right) \int u_i(s_i, s_{-i}) d\pi_i(s_{-i}), \tag{3.1}$$

where $M_i\left(s_i\right) = \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ and $m_i\left(s_i\right) = \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$.[1]

Intuitively, π can be thought to be the decision maker's belief. However, he is not sure of this belief, hence it is an ambiguous belief. His confidence about this belief is modelled by $(1 - \delta_i)$, with $\delta = 1$ denoting complete ignorance and $\delta = 0$ denoting no ambiguity. His attitude to ambiguity is measured by α_i, with $\alpha = 1$ denoting pure optimism and $\alpha = 0$ denoting pure pessimism. If the decision-maker has $0 < \alpha < 1$, he is neither purely optimistic nor purely pessimistic (i.e., ambiguity-averse), but reacts to ambiguity in a partly pessimistic way by putting a greater weight on bad outcomes and in a partly optimistic way by putting a greater weight on good outcomes.

[1] Note that Chateauneuf, Eichberger, and Grant (2007) write a neo additive capacity in the form $\mu(E) = \delta\alpha + (1 - \delta)\pi(E)$. We have modified their definition to be consistent with the rest of the literature where α is the weight on the minimum expected utility.

3.2.2 Equilibrium under Ambiguity

In this section we present an equilibrium concept for strategic games with ambiguity. In a Nash equilibrium, players are believed to behave in a manner that is consistent with the actual behaviour of their opponents. They perfectly anticipate the actions of their opponent and can thus provide a best response to it in the form of their own action. In the case of ambiguity, represented by non-additive beliefs, however, the Nash idea of having consistent beliefs regarding the opponent's action and thus being able to play an optimum strategy as a response to these beliefs, needs to be modified. We assume that players choose pure strategies, and that in equilibrium the beliefs about these pure strategies are best responses to the opponent's actions.

Consider a game with 2 players and a finite pure strategy sets S_i, $i = 1, 2$. Each player i's beliefs about the opponent's behaviour is represented by a capacity v_i on S_{-i}, which is the set of strategy combinations which his/her opponent could choose. Given neo-additive beliefs, the expected payoff that a player i could earn from a strategy s_i, is determined by equation (3.1),

$$V_i\left(s_i; \pi_i, \alpha_i, \delta_i\right) = \delta_i \alpha_i M_i\left(s_i\right) + \delta_i\left(1 - \alpha_i\right) m_i\left(s_i\right) + \left(1 - \delta_i\right) \int u_i(s_i, s_{-i}) d\pi_i(s_{-i}), \tag{3.2}$$

where $M_i\left(s_i\right) = \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ and $m_i\left(s_i\right) = \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$.

Unlike the scenario of Nash equilibrium where a player could attach a set of additive probabilities to his opponent's actions, in the presence of ambiguity, beliefs are represented by capacities. The support of a capacity is a player's belief of how the opponent will act. Formally, the support of a neo-additive capacity, $\nu\left(A\right) = \delta\alpha + \left(1 - \delta\right)\pi\left(A\right)$, is defined by $\operatorname{supp}\left(\nu\right) = \operatorname{supp}\left(\pi\right)$. Thus the support of a neo-additive belief is equal to the support of its additive component. This definition is justified in Eichberger and Kelsey (2011).

Definition 1 *A pair of neo-additive capacities (ν_1^*, ν_2^*) is an Equilibrium Under Ambiguity (EUA) if for $i = 1, 2$, $\operatorname{supp}\left(\nu_i^*\right) \subseteq R_{-i}(\nu_{-i}^*)$, where R_i denotes the best-*

response correspondence of player i given that his/her beliefs are represented by ν_i is defined by

$$R_i(\nu_i) = R_i(\pi_i, \alpha_i, \delta_i) := \text{argmax}_{s_i \in S_i} V_i\left(s_i; \pi_i, \alpha_i, \delta_i\right).$$

This definition of equilibrium is taken from Eichberger, Kelsey, and Schipper (2009), who adapt an earlier definition in Dow and Werlang (1994). These papers show that an EUA will exist for any given ambiguity-attitudes for the players. In games, one can determine π_i endogenously as the prediction of the players from the knowledge of the game structure and the preferences of others. In contrast, we treat the degrees of optimism, α_i and ambiguity, δ_i, as exogenous. In equilibrium, each player assigns strictly positive likelihood to his/her opponent's best responses given the opponent's belief. However, each player lacks confidence in his/her likelihood assessment and responds in an optimistic way by over-weighting the best outcome, or in a pessimistic way by over-weighting the worst outcome.

Alternative approaches to equilibrium with ambiguity can be found in Klibanoff (1993) and Lo (1996). They model players as having preferences which satisfy the axioms of maxmin expected utility (MMEU, Gilboa and Schmeidler (1989)). Players are allowed to have beliefs which are represented by multiple sets of conventional probability distributions. As such, players can have mixed strategies that are chosen from these multiple sets of additive probability distributions. They model ambiguity aversion as a strict preference among players to randomise between strategies when they are indifferent to pure strategies.

3.3 Experimental Model

3.3.1 Battle of the Sexes Game

In this section, we explain the games used in our experimental sessions. There are similar to the standard battle of the sexes game, except that they have been modified by giving the column player an extra option, which is the ambiguity-safe strategy R. We shall adopt the convention that male pronouns he, his etc. denote the row player (also known as Player 1), while female pronouns denote the column player or Player 2.[2]

3.3.1.1 Nash Equilibrium

The base game (without the secure option R) has two pure Nash equilibria (T, M) and (B, L), neither of which is focal. We believe this may be a cause of ambiguity. Even if a given player believes his/her opponent will play a Nash equilibrium strategy, there may be ambiguity about which of the two possible equilibrium strategies (s)he will choose. Ambiguity-aversion makes R, which is the safe option, attractive for Player 2. When $x = 60$, the secure strategy R is dominated by a mixed strategy and hence is not played in Nash equilibrium or iterated dominance equilibrium.

Theorem 1 *The game has the following Nash equilibria:*

1. When $0 \leq x \leq 75$, *there are 3 equilibria:* $(T, M), (B, L)$ *and* $(\frac{3}{4} \cdot T + \frac{1}{4} \cdot B, \frac{3}{4} \cdot L + \frac{1}{4} \cdot M)$;[3]

2. When $75 < x \leq 100$, *there are 3 equilibria:* $(T, M), (B, L)$ *and* $(\frac{x}{100} \cdot T + \frac{100-x}{100} \cdot B, \frac{1}{61} \cdot M + \frac{60}{61} \cdot R)$;

3. When $100 < x < 300$, *there is a unique equilibrium:* (B, L).

1. $0 \leq x < 75$ **:** By inspection (T, M) and (B, L) are pure strategy Nash equilibria. For x in this range, R is dominated by $\frac{3}{4} \cdot L + \frac{1}{4} \cdot M$, which yields an expected pay-off of 75 no matter what player 1 chooses. When she plays $\frac{3}{4} \cdot L + \frac{1}{4} \cdot M$, her payoff from this strategy is given by $300p(1 - q) + 100(1 - p)q = 75$. This gives her an expected payoff of 75, regardless of the strategy chosen by Player 1. The strategy R is thus dominated by a mixed strategy and hence cannot be played in Nash equilibrium. Player 1 is indifferent between T and B when $300(1 - p) = 100p$, $p = 3/4$. There are 3 equilibria: (T, M), (B, L) and $(\frac{3}{4} \cdot T + \frac{1}{4} \cdot B, \frac{3}{4} \cdot L + \frac{1}{4} \cdot M)$.

2. $75 < x \leq 100$: For x in this range, (T, M) and (B, L) remain pure strategy Nash equilibria. Player 2 is indifferent between M and R when: $100q = x$ or $q =$

[2] Of course this convention is for convenience only and bears no relation to the actual gender of subjects in our experiments.

[3] The notation $\frac{3}{4} \cdot T + \frac{1}{4} \cdot B$ denotes the mixed strategy where T is played with probability $\frac{3}{4}$ and B is played with probability $\frac{1}{4}$.

$x/100$. Player 1 is indifferent between T and B when: $300p+50(1-p) = 55(1-p)$, or $p = 1/61$. There are 3 equilibria: (T, M), (B, L) and $(\frac{x}{100}\cdot T + \frac{100-x}{100}\cdot B, \frac{1}{61}\cdot M + \frac{60}{61}\cdot R)$.[4]

3. $100 < x < 300$: For this range, M is dominated for Player 2 by R. Once M is eliminated, Player 1 will never play T, which is his dominated strategy. He thus plays B. The best response for Player 2 to make to B, is to play L. In this case there is a unique Nash equilibrium: (B, L) which satisfies iterated dominance. □

3.3.1.2 Ambiguity Aversion

Ambiguity about the behaviour of Player 1 would make the secure option R more attractive for Player 2. Note that the best response to R, is for Player 1 to play B. Hence of the two possible Nash (T, M) and (B, L), the latter may be more robust to ambiguity.[5]

We assume that the beliefs of the players may be represented by neo-additive capacities and that players are purely pessimistic towards ambiguity, i.e., $\alpha = 0$.

Theorem 2 *The game has the following Equilibria under Ambiguity:*

1. when $0 \leq x \leq (1-\delta)75$, *there are 3 equilibria,* (T, M), (B, L) *and* $(\frac{3}{4}T + \frac{1}{4}B, \frac{3}{4}L + \frac{1}{4}M)$*;*

2. when $(1-\delta)75 < x \leq (1-\delta)100$, *there are 3 equilibria:* (T, M), (B, L) *and* $(\frac{x}{(1-\delta)100}T + \frac{(1-\delta)100 - x}{(1-\delta)100}B, \frac{1}{61}M + \frac{60}{61}R)$*;*

3. when $(1-\delta)100 < x < (1-\delta)300$, *there is a unique equilibrium:* (B, L)*;*

4. when $x > (1-\delta)300$, *there is a unique equilibrium:* (B, R).

1. $0 \leqslant x \leqslant (1-\delta)75$: In this range there are two EUA in pure strategies and one in mixed strategies. In the pure equilibria, the supports of the equilibrium beliefs are given by (T, M) and (B, L). Consider the first of these. Define ν^1 by $\nu^1 = (1-\delta)\,\pi_M(A)$, where π_M is the additive probability on S^2 defined by $\pi_M(A) = 1$ if $M \in A$, $\pi_M(A) = 0$ otherwise. Similarly define Player 2's beliefs ν^2 by $\nu^2 =$

[4] Consider what would happen if Player 2 mixes between L and R. Player 2 will be indifferent between L and R when: $300(1-q) = x$ or $q = \frac{300-x}{300}$. Player 1 is indifferent between T and B when: $100p + 55(1-p) = 50(1-p)$, or $p = -\frac{5}{95}$. It is impossible for a probability to be negative, hence there can be no such equilibria.

[5] Theorem 2 confirms that (B, L) is an equilibrium for a greater parameter range than (T, M).

$(1-\delta)\,\pi_T(A)$. By definition $\operatorname{supp}\nu^1 = M$ and $\operatorname{supp}\nu^2 = T$. Denote this equilibrium by $\langle T, M\rangle$.

By similar reasoning we may show that there exists a pure equilibrium where $\operatorname{supp}\nu^1 = L$ and $\operatorname{supp}\nu^2 = B$, which we denote by $\langle B, L\rangle$.

Now consider the mixed equilibria. Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\,\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\,\tilde{\pi}^2$. Player 2's Choquet expected pay-offs are given by, $V^2(L) = 300\,(1-\delta)\,\tilde{\pi}^2(B)$, $V^2(M) = 100\,(1-\delta)\,\tilde{\pi}^2(T)$ and $V^2(R) = x$. If $V^2(L) < x \leqslant (1-\delta)75$ then $\tilde{\pi}^2(B) < \frac{1}{4}$, which implies $\tilde{\pi}^2(T) > \frac{3}{4}$. Hence $V^2(M) = 100\,(1-\delta)\,\tilde{\pi}^2(T) > (1-\delta)75 \geqslant x$. Thus R cannot be a best response for Player 2, hence $\tilde{\pi}^1(R) = 0$. Consequently in any mixed equilibrium 2's strategies are L and M.

In a mixed equilibrium Player 2 must be indifferent between L and M, hence,

$$\begin{aligned} V^2(L) &= V^2(M) \Longleftrightarrow 300\,(1-\delta)\,\tilde{\pi}^2(B) = 100\,(1-\delta)\,\tilde{\pi}^2(T) \\ &\Leftrightarrow\ 300\left(1-\tilde{\pi}^2(T)\right) = 100\tilde{\pi}^2(T) \Leftrightarrow \tilde{\pi}^2(T) = \frac{3}{4}. \end{aligned}$$

In this equilibrium $V^2(L) = V^2(M) = 75\,(1-\delta)$. Similarly we may show that for Player 1 to be indifferent between T and B, we must have $\tilde{\pi}^1(L) = \frac{3}{4}$ and $\tilde{\pi}^1(M) = \frac{1}{4}$.

Thus in the mixed equilibrium $\tilde{\nu}^1 = (1-\delta)\,\tilde{\pi}^1$ with $\tilde{\pi}^1(L) = \frac{3}{4}$ and $\tilde{\pi}^1(M) = \frac{1}{4}$ and $\operatorname{supp}\tilde{\nu}^1 = \{L, M\}$ while $\tilde{\nu}^2 = (1-\delta)\,\tilde{\pi}^2$ with $\tilde{\pi}^2(T) = \frac{3}{4}$ and $\tilde{\pi}^2(B) = \frac{1}{4}$, with support $\{T, B\}$. In this equilibrium $V^2(L) = V^2(M) = 75\,(1-\delta)$. By an abuse of notation we shall denote this equilibrium by $\left\langle \frac{3}{4}T + \frac{1}{4}B, \frac{1}{4}L + \frac{3}{4}M)\right\rangle$.

2. $(1-\delta)75 < x < (1-\delta)100$: In this range, there are two EUA in pure strategies: (T, M) and (B, L). The proof for these EUA in pure strategies is similar to Part a. above.

In addition, there is a mixed strategy equilibrium. Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\,\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\,\tilde{\pi}^2$.

Player 2's Choquet expected pay-offs are given by, $V^2(L) = 300\,(1-\delta)\,\tilde{\pi}^2(B)$, $V^2(M) = 100\,(1-\delta)\,\tilde{\pi}^2(T)$ and $V^2(R) = x$. L cannot be a best response for Player

2, hence $\tilde{\pi}^1(L) = 0$.[6] Consequently in any mixed equilibrium 2's strategies are M and R.

Player 2 is indifferent between M and R when:

$$\begin{aligned} V^2(M) &= V^2(R) \Leftrightarrow 100(1-\delta)\tilde{\pi}^2(T) = x \\ &\Leftrightarrow \tilde{\pi}^2(T) = \frac{x}{(1-\delta)100}. \end{aligned}$$

Similarly, Player 1's Choquet expected payoff is given by: $V^1(T) = 300(1-\delta)\tilde{\pi}^1(M) + 50(1-\delta)\tilde{\pi}^1(R)$ and $V^1(B) = 55(1-\delta)\tilde{\pi}^1(R)$. Player 1 is indifferent between T and B when:

$$\begin{aligned} V^1(T) &= V^1(B) \\ &\Leftrightarrow 300(1-\delta)\tilde{\pi}^1(M) + 50(1-\delta)\tilde{\pi}^1(R) = 55(1-\delta)\tilde{\pi}^1(R) \\ &\Leftrightarrow 300\tilde{\pi}^1(M) = 5(1-\tilde{\pi}^1(M)) \Leftrightarrow \tilde{\pi}^1(M) = \frac{1}{61}. \end{aligned}$$

Thus in the mixed equilibrium $\tilde{\nu}^1 = (1-\delta)\tilde{\pi}^1$, with $\tilde{\pi}^1(M) = \frac{1}{61}$ and $\tilde{\pi}^1(R) = \frac{60}{61}$ and $\operatorname{supp}\tilde{\nu}^1 = \{M, R\}$, while $\tilde{\nu}^2 = (1-\delta)\tilde{\pi}^2$ with $\tilde{\pi}^2(T) = \frac{x}{(1-\delta)100}$ and $\tilde{\pi}^2(B) = \frac{((1-\delta)100)\,-\,x}{(1-\delta)100}$, with support $\{T, B\}$. In this equilibrium $V^2(M) = V^2(R) = x$. The mixed strategy equilibrium is $\left\langle \frac{x}{(1-\delta)100}T + \frac{((1-\delta)100)\,-\,x}{(1-\delta)100}B, \frac{1}{61}M + \frac{60}{61}R \right\rangle$.

3. $(1-\delta)100 < x < (1-\delta)300$: Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\tilde{\pi}^2$. Player 2's Choquet expected pay-offs are given by, $V^2(L) = 300(1-\delta)\tilde{\pi}^2(B)$, $V^2(M) = 100(1-\delta)\tilde{\pi}^2(T)$ and $V^2(R) = x$, where $(1-\delta)100 < x < (1-\delta)300$.

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[6] Consider what would happen if Player 2 mixes between L and R. For Player 2 to be indifferent between L and R :

$$\begin{aligned} V^2(L) &= V^2(R) \Leftrightarrow 300(1-\delta)\tilde{\pi}^2(B) = x \\ &\Leftrightarrow \tilde{\pi}^2(B) = \frac{x}{300(1-\delta)}. \end{aligned}$$

Player 1 is then indifferent between playing T and B when,

$$\begin{aligned} V^1(T) &= V^1(B) \Leftrightarrow 50(1-\delta)\tilde{\pi}^1(R) = 100(1-\delta)\tilde{\pi}^1(L) + 55(1-\delta)\tilde{\pi}^1(R) \\ &\Leftrightarrow 100\tilde{\pi}^1(L) = -5(1-\tilde{\pi}^1(L) \Leftrightarrow \tilde{\pi}^1(L) = -\frac{5}{95}. \end{aligned}$$

It is impossible for a belief to be negative, hence there can be no such equilibria.

For x in this range, $V^2(R) > V^2(M)$ for any beliefs of Player 2, hence $\tilde{\pi}^1(M) = 0$. Player 1's Choquet expected pay-offs are given by, $V^1(T) = 50(1-\delta)\tilde{\pi}^1(R)$ and $V^1(B) = 100(1-\delta)\tilde{\pi}^1(L) + 55(1-\delta)\tilde{\pi}^1(R)$. B yields a higher Choquet expected payoff than T for any beliefs of Player 1, with support contained in $\{L, R\}$. For Player 2, L is the best response to B. In this case there is a unique EUA: $\langle B, L\rangle$.

4. $x > (1-\delta)300$: Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\tilde{\pi}^2$. Player 2's Choquet expected pay-offs are given by, $V^2(L) = 300(1-\delta)\tilde{\pi}^2(B)$, $V^2(M) = 100(1-\delta)\tilde{\pi}^2(T)$ and $V^2(R) = x$, where $x > (1-\delta)300$.

For x in this range, R strictly dominates both L and M for any beliefs of Player 2, hence $\tilde{\pi}^1(L) = \tilde{\pi}^1(M) = 0$. Player 1's best response is to play B, with $\operatorname{supp}\nu^1 = R$. There is a unique EUA: $\langle B, R\rangle$. $\square$

In the above analysis, players are presumed to be uniformly ambiguity averse, Assume $\delta = \frac{1}{2}$, which is in line with the findings of Kilka and Weber (2001). Then (a) occurs for $0 \leqslant x \leqslant 37.5$, (b) occurs for $37.5 \leqslant x \leqslant 50$, (c) occurs for $50 \leqslant x \leqslant 150$ and (d) occurs for $150 \leqslant x$. The testable hypothesis that arises from the analysis, is that while Nash equilibrium predicts that R cannot be chosen in the range $37.5 < x < 50$ or $150 < x < 300$, EUA predicts R can be chosen in these ranges.

3.3.2 Ellsberg Urn Experiments

The Battle of Sexes game was alternated with single person decision problems regarding an Ellsberg Urn. Subjects were presented with an urn containing 90 balls, of which 30 were Red, and the remainder an unknown proportion of Blue or Yellow. Subjects were asked to pick a colour, and a ball was drawn from the urn. If the colour of the ball matched the colour chosen by the subject, it entitled the subject to a prize. The decisions put to the subjects took the following form:

"An urn contains 90 balls, of which 30 are Red. The remainder are either Blue or Yellow.

Which of the following options do you prefer?

a) Payoff of y if a Red ball is drawn.

b) Payoff of 100 if a Blue ball is drawn.

c) Payoff of 100 if a Yellow ball is drawn."

Payoff "y" attached to the option *Red* was changed from round to round, with $y = 95$, 90 or 80, to measure the ambiguity threshold of subjects. In addition, we also put before subjects the classic case of Ellsberg Paradox, when $y = 100$, as described in table 6. If $y = 100$, this has a similar structure to the above experiments.

3.4 Experimental Design

The Battle of Sexes game and Ellsberg Urn problem described above were used in two series of paper-based experiments, one conducted at St. Stephen's College in New Delhi, India, and the other at the Finance and Economics Experimental Laboratory in Exeter (FEELE), UK.

Sessions 1 and 2 consisted of 20 subjects each. Sessions 3 and 4 consisted of 18 and 22 subjects respectively. In total there were 80 subjects who took part in the experiment, 38 of which were females and 42 were males. We were also interested in whether or not participants had a mathematical background - of those taking part in the sessions, 45 studied a quantitative subject such as Biochemistry, Electronic Engineering or Astrophysics, while 35 studied a non-quantitative subject such as History, Philosophy, or International Relations. Each session lasted a maximum of 45 minutes.

Subjects were allowed to read through a short but comprehensive set of instructions at their own pace, following which the instructions were also read out to all the participants in general. The subjects were then asked to fill out some practice questions to test their understanding of the games, before the actual set of experimental questions were handed out. At the start of the experiment, subjects were randomly assigned the role of either a Row Player or a Column Player for the purpose of the Battle of Sexes game, and remained in the same role throughout the rest of the experiment.

The experiment consisted of 11 rounds, starting with a decision regarding a Battle of Sexes game, which was then alternated with an Ellsberg Urn decision, such that there were in total 6 Battle of Sexes rounds and 5 Ellsberg urn decisions to be made. Each subject had to choose one option per round: Top/Bottom if they were a Row Player or Left/Middle/Right if they were a Column Player, and in case of the Ellsberg urn rounds Red, Blue or Yellow.

The values of x, the ambiguity-safe payoff available to the Column Player that were used for the Battle of Sexes game rounds were: $230, 120$, 200, 170, 260, 60 (in that order). In the first three Ellsberg urn rounds, the pay-offs attached to drawing a Blue or Yellow ball were held constant at 100, while those attached to drawing a Red ball varied as 95, 90, 80. The last two Ellsberg urn rounds consisted of the classic case of the Ellsberg paradox, where subjects had to choose between a payoff of 100 for a Red or 100 for a Blue ball, followed by a choice between a payoff of 100 for drawing a Red/Yellow ball or 100 for drawing a Blue/Yellow ball.

Once subjects had made all 11 decisions, a throw of dice determined *one* Battle of Sexes round and *one* Ellsberg urn round for which payments were to be made. Row Players' decisions were matched against the Column Players according to a predetermined matching, and pay-offs were announced.

Rather than using a real urn we simulated the draw from the urn on a computer.[7] The computer randomly assigned the number of blue and yellow balls in the urn so that they summed to 60, while keeping the number of red balls fixed at 30 and the total number of balls in the urn at 90. It then simulated an independent ball draw for up to 30 subjects. If the colour of the ball drawn by the computer matched that chosen by the subject, it entitled him to the payoff specified in the round chosen for payment.

The total earnings of a subject was the sum of a show-up fee, payoff earned in the chosen Battle of Sexes round and payoff earned in the chosen Ellsberg urn round. Average payment made to Indian subjects was $Rs.420$ (£6 approximately), and to Exeter subjects was £7.40.

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[7] The computer simulated urn can be found at the following link: http://people.exeter.ac.uk/dk210/Ellsberg-110708.xls.

3.5 Data Analysis and Results

3.5.1 Behaviour of the Row Player in the Battle of Sexes Rounds

In the Battle of Sexes rounds of the experiment, the task of the Row Player was to choose between T and B. In the mixed equilibrium, the Row Player randomises $\frac{3}{4} : \frac{1}{4}$ between T and B. However we find that the Row Player randomises more closely to $50 : 50$ in the experiments. See Table 3.3 and Figure 3.1 , for a summary of the Row Player's behaviour.

Table 3.3. Summary of Row Player Behaviour

	$x = 60$		$x = 120$		$x = 170$		$x = 200$		$x = 230$		$x = 260$	
Top	20	50%	22	55%	18	45%	26	65%	24	60%	23	58%
Bottom	20	50%	18	45%	22	55%	14	35%	16	40%	17	43%
Σ	40		40		40		40		40		40	

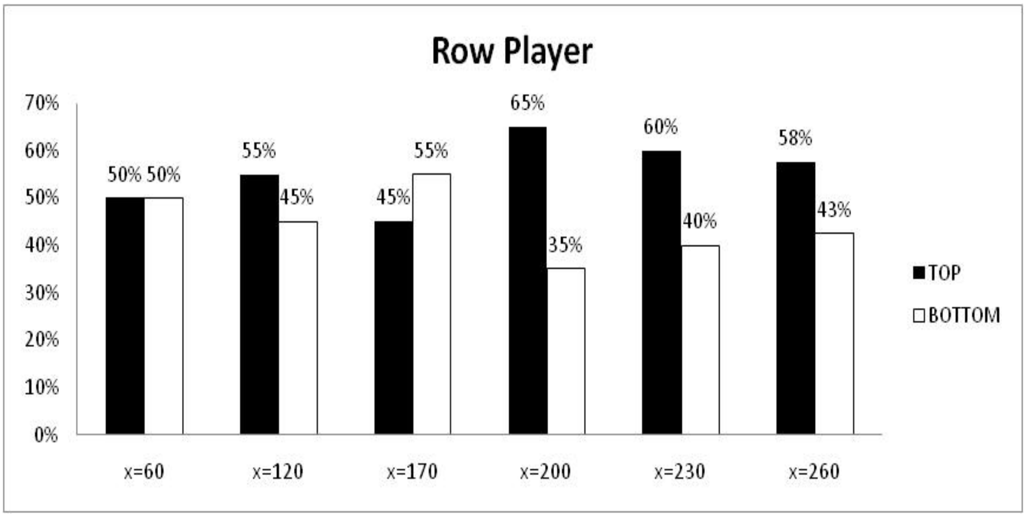
We conducted a binomial test with the null that the Row Player randomises $50 : 50$ between T and B, for each value of x. We fail to reject this hypothesis for each individual session even at a 10% level of significance. When tested for each value of x on the whole (as a sum of all sessions combined), we fail to reject the null for all the values of x, except when $x = 200$, where we reject the null at 5%. In this case, the Row Player plays T significantly more often than B. This is puzzling, since B would be the best response to the Column Player choosing R.

We conducted a chi-squared test with the null hypothesis that the Row Player chooses T and B with equal probability ($H_0 : prob(T) = 0.5$, $prob(B) = 0.5$) versus the alternative that this is not true ($H_1 : prob(T) \neq 0.5$). The null is rejected at 1% level of significance, for each value of x.

3.5.2 Behaviour of the Column Player in the Battle of Sexes Rounds

In the Battle of Sexes rounds of the experiment, the task of the Column Player was to choose between L, M and the ambiguity-safe option R. See Table 3.4 and Figure 3.2, for a summary of the Column Player's behaviour.

Figure 3.1. Summary of Row Player Behaviour



When $x = 60$ one might expect the Column player to pick L, since R is dominated and L has a much higher maximum pay-off than M. As seen in Table 4 most subjects do indeed choose L. However, even at this low value of x, where the ambiguity-safe option R is dominated by randomisation between the other strategies available to the subject, a significant 30% of subjects still choose it. This is analogous in a game-setting to the results of Dominiak and Schnedler (2011), who found aversion to objective ramdomisation in the presence of ambiguity in single-person decisions.

What is more interesting to note however, is that the number of subjects playing R, steadily increases from 28% to 98% for $120 \leq x \leq 260$. Nash equilibrium predicts that R cannot be chosen for any of these values, but it is the clear choice of a majority of subjects in the presence of ambiguity, as seen in Figure 2.

Table 3.4. Summary of Column Player Behaviour

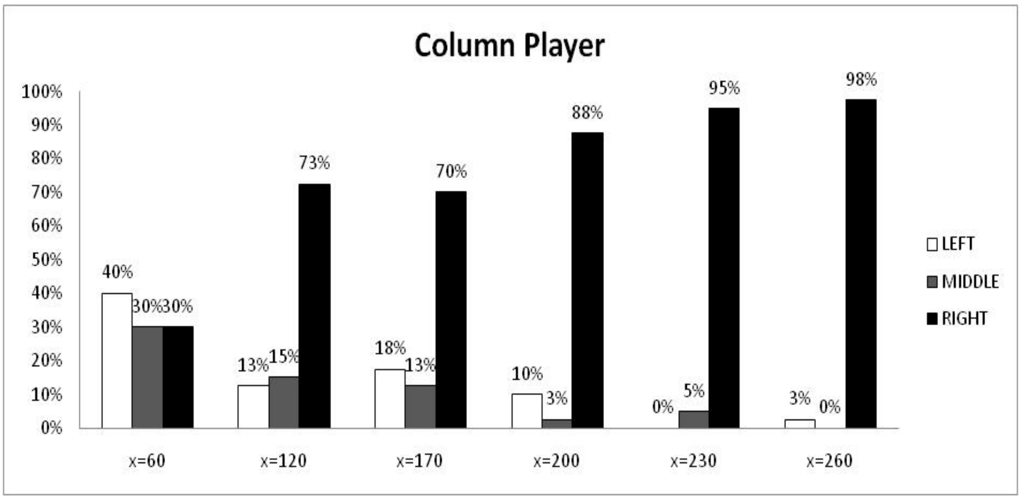
	$x = 60$		$x = 120$		$x = 170$		$x = 200$		$x = 230$		$x = 260$	
Left	16	40%	5	13%	7	18%	4	10%	0	0%	1	3%
Middle	12	30%	6	15%	5	13%	1	3%	2	5%	0	0%
Right	12	30%	29	73%	28	70%	35	88%	38	95%	39	98%
Σ	40		40		40		40		40		40	

We conducted a binomial test with the null that the Column Player chooses R as often as he does $L + M$ ($H_0 : prob(Right) = 0.5$, $prob(Left + Middle) = 0.5$), against the alternative that she plays R more often than both $L + M$ combined

($H_1: prob(Right) > prob(Left + Middle)$), for each value of x.[8] We reject the null at a 1% level of significance for all the values of x in the range $120 - 260$. This leads us to the conclusion that subjects play R significantly more often than both L and M combined, at a 1% level of significance.

A chi-squared test with the null hypothesis that the Column Player chooses R and $L + M$ with equal probability ($H_0: prob(Right) = 0.5$, $prob(Left + Middle) = 0.5$) versus the alternative that this is not true ($H_1: prob(Right) \neq 0.5$) is also rejected at 1% level of significance, since R is chosen significantly more often.

Figure 3.2. Summary of Column Player Behaviour



We ran a probit regression to ascertain what factors influenced subjects in choosing R more often than L or M. Dummy variables were defined to capture the characteristics of the data such as: $Math = 1$, if the subject was doing a Quantitative degree ($Math = 0$, for degrees like English, History, Philosophy, Politics etc.); $Male = 1$, if Gender is male (0, otherwise); $Delhi = 1$, if the session was run in India (0 for Exeter); x_60, x_120, x_170, x_200, x_230, $x\_260 = 1$, depending on the value "x" took in that particular round.

[8] The binomial test was conducted for each value of x except $x = 60$, where EUA predicts that the column player can play L. It may be noted that for $x = 60$, subjects play $L + M$ more than 50% of the time.

A probit regression of $Right$ on $Math$, $Male$, and the various $x-value$ dummies x_120, x_170, x_200, x_230, x_260, has a chi-square ratio of 75.55 with a p-value of 0.0001, which shows that our model as a whole is statistically significant.[9]

All the variables in the probit regression were individually statistically significant. We see that if the subject had a quantitative degree, the z-score increases by 0.538, making him more likely to pick R. If the subject is male, the z-score decreases by 0.402, hence males are less likely to opt for the ambiguity-safe option R than females. When $x = 120$: the z-score increases by 1.16, $x = 170$: the z-score increases by 1.08, $x = 200$: the z-score increases by 1.75, $x = 230$: the z-score increases by 2.27, $x = 260$: the z-score increases by 2.57; more than the base which is $x = 60$. Thus, as the value of x increases, the subject is more likely to pick the ambiguity-safe option.

3.5.3 Player Behaviour in the Ellsberg Urn Rounds

The Ellsberg Urn rounds were alternated with the Battles of Sexes rounds. This was designed to test whether there was a correlation between ambiguity-averse behaviour in the game and ambiguity attitude in single person decision problems.[10]

Subjects were offered an Urn that held 90 balls - 30 of which were Red, the remaining an unknown proportion of $Blue$ and $Yellow$. They were then asked to choose between winning a payoff 95 experimental currency units (ECU) (90 and 80 ECU respectively, in subsequent rounds) if they picked Red, or a payoff of 100 ECU if they picked $Blue$ or $Yellow$; and the colour picked by them matched the colour of the ball drawn from the Urn.

As can be seen in Table 4.16, subjects chose $Blue$ and $Yellow$ coloured balls (the ambiguous option) more often than they chose Red.[11] We had expected to observe

[9] An initial probit regression, showed that the dummy variable for location ($Delhi/Exeter$) was not significant, showing that behaviour of Indian subjects was very similar to the Exeter subjects. Thus, the location dummy variable was dropped and the model was re-run without it.

[10] We would like to thank Peter Dursch, whose suggestions helped the design of the experiment.

[11] The data for $y = 100$ is from the classic Ellsberg paradox round. It is not completely comparable as subjects were not given the option of choosing yellow. Thus it is not included in the data analysis below.

that subjects who chose *Right* (the ambiguity-safe option) in the Battle of Sexes rounds, would choose *Red* (the colour with the unambiguous number of balls) in the Urn rounds. However, the observed correlation was weak.

Table 3.5. Summary of Player Behaviour in Ellsberg Urn Rounds

	Red = 100		Red= 95		Red = 90		Red= 80	
Red	57	71%	32	40%	22	28%	24	30%
Blue	23	29%	37	46%	36	45%	39	49%
Yellow	0	0%	11	14%	22	28%	17	21%
Σ	80		80		80		80	

One notable feature of this data is the low level of ambiguity-aversion compared to previous studies. In the case where $y = 100$ our results are comparable to the previous literature. For lower values of y, subjects have to pay an monetary penalty to avoid ambiguity. Even small penalties produced a large drop in the number of subjects choosing the unambiguous option.

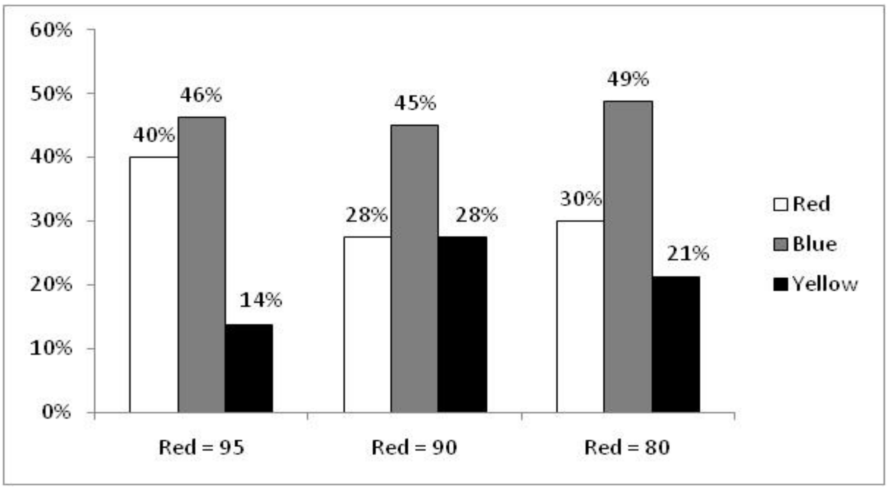
Of the 80 subjects that took part in the experiment, only 12 subjects always chose *Red*, 11 chose *Red* twice, 20 chose *Red* once, and a significant 37 subjects never chose *Red* - always opting for either *Blue* or $Yellow$, the ambiguous options.

It is interesting to note that even in the round where the payoff attached to *Red* was 80 *ECU*, a large minority (30%) still chose the safe option, despite facing a substantial monetary penalty. Of the 12 subjects who always picked *Red*, 3 are Row Players and so not relevant to our discussion. The remaining 9 are Column Players: 7 of these always chose the ambiguity-safe combination of $Right - Red$ (not considering their choice when $x = 60$), while 2 chose $Left/Middle/Right$ while always picking Red.

We conducted a binomial test with the null *Red* was chosen often as $Blue + Yellow$ combined ($H_0 : prob(Red) = 0.5$, $prob(Blue + Yellow) = 0.5$), against the alternative that $Blue + Yellow$ was chosen more often ($H_1 : prob(Blue + Yellow) > prob(Red)$). We reject the null at a 5% level of significance when the payoff attached to $Red = 95$, and at 1% level of significance when $Red = 90$ & 80. Looking at subject choices on the whole, over the three rounds, we can reject the null at a 1% level of significance.

Thus, the ambiguous options *Blue* and *Yellow* are chosen significantly more often than *Red*, which leads us to speculate whether the penalty for choosing *Red* was set too high or whether subjects are mildly ambiguity-seeking in the Ellsberg urn rounds, even though they appear to be ambiguity-averse in the Battle of Sexes rounds. A probit regression run to investigate whether gender, location or degree subject affected subjects' choice of *Blue* and *Yellow* was inconclusive and none of these explanatory variables was found to be significant.

Figure 3.3. Summary of Player Behaviour in Ellsberg Urn Rounds



3.5.4 Classic Ellsberg Paradox Rounds

In the last two Ellsberg Urn rounds, subjects were offered an Urn that held 90 balls - 30 of which were *Red*, the remainder an unknown proportion of *Blue* and *Yellow*. They were then asked to choose between winning a payoff 100 *ECU* if they picked *Red*, or a payoff of 100 *ECU* if they picked *Blue*; and the colour picked by them matched the colour of the ball drawn from the Urn. Once they had made this choice, they were asked to choose between winning a payoff 100 *ECU* if they picked either a *Red* or *Yellow* ball, or a payoff of 100 *ECU* if they picked either a *Blue* or *Yellow* ball; and the colour picked by them matched the colour of the ball drawn from the Urn.

Table 3.6. Player Behaviour in Classic Ellsberg Paradox Rounds

Choice	Response	
Red followed by Red/Yellow	19	24%
Red followed by Blue/Yellow	38	48%
Blue followed by Blue/Yellow	16	20%
Blue followed by Red/Yellow	7	9%

As can be seen from Table 3.6, a majority of the subjects preferred *Red* to *Blue*, and then the choice $Blue/Yellow$ over $Red/Yellow$. 38 of the 80 (48%) subjects that took part in the experiment chose *Red* followed by $Blue/Yellow$, thus displaying the Classic Ellsberg Paradox. 7 (9%) subjects that took part in the experiment chose *Blue* followed by $Red/Yellow$, which indicates ambiguity preference. Looking strictly at the Column Players who display the Ellsberg Paradox[12]: 16 (67%) subjects always chose the ambiguity-safe option $Right-$ but these people do play $Blue/Yellow$ when the payoff attached to $Red = 95, 90, 80$, while 8 (33%) play a mixture of $Right/Left/Middle$.

3.6 Related Literature

3.6.1 Papers on Games

While the current study focusses on the effect of ambiguity in a Battle of Sexes game, Eichberger and Kelsey (2002), study how ambiguity affects the voluntary provision of public goods. They show that when the production function for public goods is concave ambiguity-aversion causes public good provision to be above Nash equilibrium level.[13] More generally they show the deviation from the Nash equilibrium depends on the nature of strategic interactions taking place, i.e., on the basis of whether the game being played was one of strategic substitutes or complements.

Voluntary provision of public goods may be considered as an example of a game of strategic substitutes with positive externalities. If one individual contributes more, this lowers the marginal product of other people's donations. Ambiguity-aversion

[12] We do not consider $x = 60$, where R is a dominated strategy.

[13] These results will apply even in the presence of increasing returns to scale in production provided there is sufficiently strong diminishing marginal benefit from public goods.

causes a given individual to over-weigh bad outcomes. In this case, a bad outcome is when others make low donations. When others' donations are low the marginal benefit of a donation by the given individual is high. Thus the expected effect of ambiguity would be to increase the perceived marginal benefit of donations by a given individual. If all have similar perceptions of ambiguity, total donations will rise. Hence an increase in ambiguity is expected to raise both individual and total donations.

Di Mauro and Castro (2008) conduct a set of experiments designed to test the Eichberger and Kelsey (2002) hypothesis that it is ambiguity that causes an increase in contribution towards the public good, and not altruism. In order to negate the chance that altruism, or a feeling of reciprocation prompted the subjects' actions, the subjects were informed that their opponent would be a virtual agent and the opponent's play was simulated by a computer. Subjects played in two scenarios, one with risk, the other with ambiguity. They find that contributions were significantly higher when the situation was one of ambiguity. These results are similar to our findings and showed that there was indeed evidence that ambiguity significantly affects the decisions made by individuals, in a manner that depends directly on the strategic nature of the game in consideration.

Another paper that tests whether ambiguity affects individual behaviour in a game setting is Eichberger, Kelsey, and Schipper (2008), who study strategic ambiguity in games experimentally. They studied games in which subjects faced either a granny, who was described as being ignorant of economic strategy, a game theorist, who was described as a successful professor of economics, or another student as an opponent. It was conjectured that subjects would view the granny as a more ambiguous opponent than the game theorist. They find that subjects did find the granny to be the more ambiguous opponent, and this affected their decision choices. In our paper, even though subjects are paired against other opponents (and not a granny), we find subjects display similar ambiguity averse behaviour.

Colman and Pulford (2007) explain the concept of ambiguity aversion as a state that arises as a result of a pessimistic response to uncertainty, mainly driven by a loss of decision confidence. They argue that people tend to become anxious and less confident while making decisions in the presence of ambiguity. They found that individual responses differed between ambiguous and risky versions of the game being studied. They find that players did not respond to ambiguity by simply equating it to riskiness, but showed a marked preference to avoid ambiguity whenever the option of doing so was provided to them. This is consistent with our findings that when an ambiguity-safe option is made available to subjects, they show a marked preference for it.

3.6.2 Papers on Ellsberg Urns

The Ellsberg urn experiments conducted by us investigated whether there was any correlation between ambiguity-averse behaviour in the game and ambiguity attitude in single person decision problems. Moreover, we were interested in evaluating whether there was a threshold beyond which ambiguity-averse individuals became ambiguity-neutral (or seeking) in their preferences. Although there exists a number of experimental studies related to Ellsberg urns, we do not find any that is similar to our experiment.

Eliaz and Ortoleva (2011), study a three-colour Ellsberg urn in which they have increased the level of ambiguity. Subjects face ambiguity on two accounts: the unknown proportion of balls in the urn as well as the size of the prize money. In their experiment, both winning and the amount that the subject could possibly win were both perfectly correlated - either positively or negatively, depending on which of the two treatments was run by them.

In the experiment, most subjects preferred betting in the positively correlated treatment rather than the negative one. Moreover, subjects also showed a preference for a gamble when there was positively correlated ambiguity, as opposed to a gamble without any ambiguity. This behaviour of the subjects is compatible with our results, where we find that more subjects were willing to gamble on $Blue/Yellow$ which were the ambiguous choices rather than on Red.

Another Ellsberg experiment that allows for an additional source of ambiguity is studied by Eichberger, Oechssler, and Schnedler (2011). They consider a two-colour

Ellsberg experiment and insert an additional element of ambiguity in terms of the money the subject wins in the various outcomes. In the standard treatment, if the colour drawn matches the colour chosen by the subject, he receives an envelope marked with an equal sign (=), and if it does not match he receives an envelope with an unequal sign ($\neq$). He is aware that the (=) envelope contains €3 and the ($\neq$) envelope contains €1. This standard treatment is referred to as O, or open envelope.

In the second treatment called the S or sealed envelope treatment, subjects know that one envelope contains €3 and one contains €1, but do not know which envelope contains which amount. In the third treatments called R or the random treatment, subjects are told that the amount in the envelope will be determined by the toss of a fair coin, once they have made their choice of colour for the bet on the urn.

Treatment O, is the standard Ellsberg experiment. In treatment R, winning €3 or €1 depends totally on the toss of the coin and so the subject faces equal odds of winning either amount. Treatment S, is different from the other two treatments in that subjects are not sure how much they would win, even if they won. They should thus, be indifferent between the ambiguous urn and the known one. Subjects were asked to choose an urn and the colour of the ball they would like to bet on. In addition, they could state that they are indifferent between the known urn and the unknown one, as well as being indifferent between a green ball and a blue one. In case of indifference, subject were assigned to the unknown urn/blue ball options.

Eichberger, Oechssler, and Schnedler (2011), find that 30 of the 48 (62%) subjects preferred the known urn in treatment O, which is similar to the standard Ellsberg result. In treatment R, when subjects should be indifferent between the ambiguous urn and the known one since their payment depends on the flip of a coin, 25 of the 48 (52%) subjects preferred the known urn. In treatment S, 19 (40%) subjects preferred the known urn, 17 (35%) preferred the ambiguous one, while 12 (25%) stated they were indifferent.

It can be noted from Eichberger, Oechssler, and Schnedler (2011), that significantly fewer subjects preferred the known urn to the ambiguous one in treatment S where there was additional ambiguity, when compared to treatment O, the standard

Ellsberg case. This is analogous to our finding that fewer subjects preferred *Red* to the ambiguous choices.

Dynamic consistency and consequentialism are the two key links between conditional and unconditional preferences. Dynamic consistency entails that a decision made ex-ante, remains unchanged if preferences are updated. Consequentialism entails that only valid outcomes (that are still possible) are taken into account once preferences are updated. Individuals who display the Ellsberg paradox cannot be both dynamically consistent as well as consequentialist. Dominiak, Dürsch, and Lefort (2009), use a dynamic version of the three colour Ellsberg experiment, and find that violations of consequentialism are more common than violations of dynamic consistency. Moreover, they find that subjects who are initially ambiguity-neutral when faced with a static Ellsberg urn, cannot be described by SEU theory when faced by the dynamic version of the Ellsberg urn.

Fox and Tversky (1995), compare ambiguity preferences in comparative as well as non-comparative contexts and conclude that ambiguity averse preferences cannot be seen in non-comparative contexts. According to them, the popular Ellsberg phenomenon and resultant ambiguity averse behaviour is inherently present only in comparative contexts and do not arise when uncertain prospects are evaluated independent of each other.

Chow and Sarin (2001), test the Fox and Tversky (1995) result in order to ascertain whether ambiguity aversion does indeed disappear in a non-comparative context. They find that in their experiments, subjects always price a known bet higher than an ambiguous one and thus, ambiguity averse behaviour is prevalent in both comparative as well as non-comparative contexts. However, the difference in prices between the known bet and the ambiguous one was found to be higher in the comparative context, than under independent evaluation.

3.6.3 Preference for Randomisation

There has been a debate on whether ambiguity-aversion induces a strict preference for objective randomisation. Raiffa (1961) argues that in the classic Ellsberg paradox, ambiguity can be turned into risk by objectively randomising between betting on Blue and betting on Yellow with equal probabilities. This would suggest that an ambiguity-averse individual should have a strict preference for randomisation. However Eichberger and Kelsey (1996) show that in the CEU model, if the capacity is convex, individuals must be indifferent to randomisation. The experimental literature which has tested these results has typically found *aversion* to objective ramdomisations in the presence of ambiguity, see Dominiak and Schnedler (2011).

In the context of games, the analogous question is whether a strategy which is dominated by a mixed strategy will be played. Choosing the mixed strategy is analogous to displaying a preference for randomisation. Choosing a dominated strategy which gives a certain pay-off is analogous to displaying aversion or indifference to randomisation. As Table 3.4 indicates 30% of column players choose R even when it is dominated by a mixed strategy. Thus we see a similar pattern to the experiments on single person decisions with evidence that many subjects display indifference to randomisation.

Liu and Colman (2009), presented subjects with gambles that were modelled as either modified Ellsberg urn choices or as marketing strategy decisions. The subjects had to choose between ambiguous and risky gambles, under single as well as multiply repeated choice conditions. Similar to subject behaviour in our experiment, it was found that subjects chose the ambiguous gambles more often in repeated choice conditions than they did in single-choice conditions. Moreover, the number of subjects choosing risky single choices (gambles) and ambiguous repeated choices exceeded the number of subjects who preferred ambiguous single choices and risky repeated choices.

One of the reasons given by Liu and Colman (2009) to explain this behaviour, is that if subjects believed that luck was loaded against them in single events, they might have felt safer in the repeated conditions. This is consistent with the behaviour seen in our experiments, where subjects believed that good and bad luck would balance out in the long run and thus, randomising between $Blue/Yellow$ was better than choosing Red.

3.7 Conclusions

The Nash equilibrium prediction that R cannot be chosen for $150 < x < 300$, was not observed in our experiments. R, which is selected by EUA, was the choice of a majority of subjects when $120 < x < 260$. There was also a significant minority of subjects choosing R when $x = 60$. Thus, there is sufficient indication for us to conclude that ambiguity does indeed affect the play in the coordination game.

We expected to observe a correlation between ambiguity-averse behaviour in the game and ambiguity attitude in the Ellsberg Urn decision problem. However, we observed only a limited relation between the two choices. On the whole, subjects displayed more ambiguity-aversion in Battle of Sexes rounds than in the Ellsberg Urn rounds. This suggests that subjects perceive a greater level of ambiguity in a two-person coordination game, than a single person decision problem.

This might be because in the absence of information, subjects use the principle of insufficient reason and attach a $50 : 50$ probability to the remaining 60 blue and yellow balls left in the urn. The principle of insufficient reason thus implies that the probability distribution attached to the Red, Blue and Yellow balls in the urn is $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. The ambiguity aversion perceived in the Battle of Sexes game is not sufficient to overcome this assumption. More generally it suggests that perceptions of ambiguity and even attitudes to ambiguity depend on context. Hence it may not be possible to measure ambiguity-attitude in one context and use it to predict behaviour in another context.

It is interesting to note that there is a growing consensus that subjects find more ambiguity regarding real events as opposed to simulations of/actual Ellsberg urns. It was found that when Ellsberg-type problems were put to students in a class-environment[14], a large proportion of PhD-level students were ambiguity-neutral, while a large proportion of MBA-level students displayed ambiguity-seeking behaviour. However, when asked whether they preferred a payoff of \$100 if the US President elected in 2016 was a Democrat (or not a Democrat) or if a fair coin came

[14] These observations are as recorded by Gilboa (2011), in a discussion on experiments on ambiguity in Ellsberg experiments.

up heads when tossed on the day of the election, a large proportion of the students preferred betting on the coin.

One of the reasons put forth to explain this divergence in behaviour is that it is easy to be Bayesian in an Ellsberg experiment or that the phrasing of the Ellsberg problem might lead to it being treated as a gamble. However, when asked to make a decision regarding a realistic scenario such as predicting the next President of the US, the students have no "natural" prior. A realistic scenario then is better at revealing ambiguity aversion on the part of the subject.

Parallels can be drawn between this discussion and the data we observe from our experiment, whereby subjects clearly display ambiguity-averse behaviour when put in the scenario of the coordination game while they fail to do so in the Ellsberg urn rounds. Subjects might be treating the Ellsberg urn rounds as a gamble, where they readily take a chance. However, when faced with the task of coordinating with another participant in the environment of a one-shot game with no previous learning, the subjects have no natural prior on the basis of which to make their decisions. The Column Player thus selects the strategy that gives a definite payoff of x irrespective of the Row Player's decision.

One can note that our results support the Dow and Werlang (1994) model of equilibrium under ambiguity where in the presence of ambiguity players choose their safe strategy, rather than the model of Lo (1996). Lo's equilibrium predictions coincide with the Nash for games with only pure equilibria. Thus for many of our game experiments Lo's predictions coincide with Nash equilibrium. Hence for these experiments EUA appears to predict the implications of ambiguity better.

CHAPTER 4

DRAGON SLAYING AND DYKE BUILDING - HOW DOES AMBIGUITY AFFECT INDIVIDUAL BEHAVIOUR?

4.1 Introduction

This paper reports an experimental study into whether individuals' decision-making is affected by ambiguity. There exists an extensive literature that shows that ambiguity affects individual decision making. However, most of this literature documents single person decision-making. There are relatively few experiments that analyse ambiguity in a game setting (Eichberger, Kelsey, and Schipper (2008); Kelsey and Le Roux (2012)). Games provide a valuable setting in which economic models can be replicated in the laboratory, and are thus a valuable tool that can be used to understand how ambiguity affects decision-making.

In the experiments we run a set of linked games to test the theoretical prediction that ambiguity has opposite effects in games of strategic complements and substitutes. Eichberger and Kelsey (2002), study a public good game with Knightian uncertainty, and explain the deviation from the expected Nash equilibrium on the basis of the inherent nature of strategic interactions taking place, i.e., on the basis of whether the game being played was one of strategic substitutes or complements, and the role of externalities (postive or negative) in the game.

A player's best response is an increasing (resp. decreasing) function to his opponent's actions, in case of a game of strategic complements (resp. substitutes). In the case of strategic substitutes, increasing the level of ambiguity would cause a shift in equilibrium strategies in a Pareto improving direction, whereas for strategic complements, an increase in ambiguity would cause a shift in equilibrium, away from the ex-post Pareto optimal level. Thus it was hypothesised that ambiguity had an adverse effect in the case of games with strategic complements, but was helpful in attaining a Pareto efficient outcome in the case of games with strategic substitutes (Eichberger and Kelsey (2002)).

A pair of games well suited to testing this hypothesis are the best-shot and weakest-link models of public goods. Public goods are those goods which once supplied can be consumed by everybody, irrespective of individual contribution. The usual assumption made regarding the socially available amount of the public good, is that it is the function of the sum of all individual contributions made by members of a community.

An alternative possibility is the weakest link version of public good provision, where the amount of the public good that is socially available is equivalent to the minimum contribution made by an individual in the community. This may be represented as: $u_i(x_i, x_{-i}) = \min\{x_1, ..., x_n\} - cx_i$, where x_i denotes the contribution of individual i and c denotes the marginal cost of the contribution.

Consider a small island community that must build dykes to protect itself from flooding in a storm. The success in holding back the storm waters will depend on the minimum height or strength of the different sections of the dyke around the island. Similarly, a weakest-link problem is observed when trying to prevent the spread of infectious diseases, combatting the entry of illegal drugs into a country, or providing security on the borders of a country during war-time.

In the best shot version of the public good game, the socially available amount of the public good is equivalent to the maximum contribution made by an individual in the community. This may be represented as: $u_i(x_i, x_{-i}) = \max\{x_1, ..., x_n\} - cx_i$, where x_i denotes the contribution of individual i and c denotes the marginal cost of a contribution.

Consider a medieval village that is beseiged by a dragon. It is only the knight endeavouring to slay the dragon, who bears the cost - in this case, the chance that he will be burnt to a crisp by the dragon. However, if the dragon is slayed, the benefits of a dragon-free village are enjoyed equally by all the village folk! Another example of a best-shot problem is the research into finding a cure for the common cold. The payoff of the best outcome (i.e., a cure) will be available to everyone.

The games are similar except the weakest link game exhibits strategic complements, whereas the best shot exhibits a game of strategic substitutes. Henceforth, we refer to these games as the effort rounds. Our hypothesis is that the effect of ambiguity will be to decrease individuals' contributions in the weakest-link version of the game, whereas it will lead to an increase in individuals' contributions in the best-shot version of the game.

Kilka and Weber (2001) find that subjects are more ambiguity-averse when the returns of an investment are dependent on foreign securities than when they are linked to domestic securities. We used a pair of strategic complement/substitute games in which the subject is either matched with a local opponent or with a foreign one. The foreign opponent was intended to be the analogy of the foreign securities used in the Kilka and Weber (2001) paper. Our hypothesis was that subjects will be more ambiguity-averse when their opponents are individuals of a foreign country than when they are matched with local individuals. In order to test this hypothesis, we recruited subjects both locally at the University of Exeter as well as overseas in St. Stephens College, India.

In addition we also alternated the main games with Ellsberg Urn type decision problems to evaluate whether individuals display ambiguity averse, ambiguity neutral or ambiguity seeking behaviour. This was done in order to test whether there was any difference in ambiguity attitude between the games and the single person decision problems. Moreover, it allowed us to elicit an independent measure of subjects' ambiguity-attitudes.

We find that behaviour of the subjects is consistent with our hypothesis and that ambiguity perceived by subjects does indeed lead to a decrease (resp. increase) in contributions in the weakest link (resp. best shot) game. However, though subjects display ambiguity aversion on the whole, the level of ambiguity does not become more pronounced when they are matched against a foreign opponent.

There are several reasons that might explain why the level of ambiguity remains unchanged against the foreign subject, one being that subjects may view a foreign student as akin to any other local student. Given the effects of globalisation, media, social networking and growing international student numbers, it is understandable that political borders do not divide subjects as much as they did in the past.

Another interesting observation from the data is that even though subjects display ambiguity-averse behaviour when faced by other opponents (whether local or foreign), they often display ambiguity seeking behaviour when faced by nature/in single-person decision situations. This is consistent with an earlier study (Kelsey and Le Roux (2012)), where subjects showed differences in ambiguity attitudes based on the scenario they were facing.

We believe that subjects perceive greater ambiguity when their payoffs depend on the decisions of other people, rather than nature which is uncontrollable. These differences in ambiguity attitude would explain why people are more concerned with fluctuations in the financial market - which is dependent on other people, but appear to discount the seriousness of possible natural disasters - which are beyond anyone's control.

4.1.0.0.1 Organisation of the Chapter In Section 2, we describe the theory being tested in the experiments. Section 3 describes the experimental design employed, Section 4 consists of data analysis and results, Section 5 reviews related literature and Section 6 provides a summary of results together with future avenues of research.

4.2 Preferences and Equilibrium under Ambiguity

4.2.1 Modelling Ambiguity

The Ellsberg paradox is a violation of the Subjective Expected Utility (SEU) Savage (1954). We describe a version of the paradox that is used by us in our experiments. Consider an urn which contains 90 balls - 30 of which are labelled X and the other 60 are labelled an unknown mix of Y and Z. A ball is drawn at random and the subject's payoff depends on the letter on the ball drawn and the act chosen by him/her. Subjects are asked to choose between acts f, g, f', g' as shown in the table below (Pay-offs below are shown in terms of Experimental Currency Units - ECU):

Table 4.1. Acts available to Subjects

	30 balls	60 balls	
Act	X	Y	Z
f	100	0	0
g	0	100	0
f'	100	0	100
g'	0	100	100

Subjects when asked to choose between f and g, generally prefer f . This is because of the definite $\frac{1}{3}$ chance of winning 100 ECU when compared to the ambiguous act g. However, when the same subjects are asked to choose between f' and g', they prefer g' which gives a $\frac{2}{3}$ chance of winning 100 ECU rather than f', again avoiding the ambiguous act.

These choices are not consistent with maximising expected utility subject to a standard probability distribution π. Opting for act f rather than g would imply that $\pi(X) > \pi(Y)$. However choosing g' over f', would then imply that $\pi(Y \cup Z) > \pi(X \cup Z)$. Given the standard additivity properties of probabilities, i.e., $\pi(X \cup Z) = \pi(X) + \pi(Z)$, these two inequalities are inconsistent!

This inconsistency could be solved by representing beliefs by a non-additive set function ν. Non-additive set functions allow that $\nu(X \cup Z) \neq \nu(X) + \nu(Z)$, which would be compatible with the choices in the Ellsberg paradox.

Non-additive beliefs were first introduced and used by Schmeidler (1989). He proposed a theory called **Choquet Expected Utility (CEU)**, where outcomes are evaluated by a weighted sum of utilities, but unlike Expect Utility Theort (EUT) the weights used depend on the acts. An intuitive exposition of Schmeidler's model, which reformulates Savage's axioms is given in Sarin and Wakker (1992). The model is extended such that it preserves additivity in beliefs for events in the face of risk, while permitting non-additivity for ambiguous events.

A special class of capacities, termed neo-additive capacities, was introduced by Chateauneuf, Eichberger, and Grant (2007), to model optimistic and pessimistic outlooks to ambiguity. An optimistic outlook would over-estimate the likelihood of a good outcome - inducing one to take part in a lottery, with the hope of a large prize. On the other hand, a pessimistic outlook would over-estimate the likelihood of a bad outcome - such as losing all your wealth in a bad investment.

A capacity v is convex (resp. concave) if for all A and $B \subseteq S$, $v(A \cup B) + v(A \cap B) \geq v(A) + v(B)$, (resp. $v(A \cup B) + v(A \cap B) \leqslant v(A) + v(B)$), where A, B are events contained in the universal set S. In CEU, convex capacities are used to model a pessimistic outlook to ambiguity, while concave capacities model an optimistic outlook.

Consider a two-player game with a finite set of pure strategies S, such that s_i is the player's own strategy and s_{-i} denotes the set of possible strategy profiles for i's opponents. The payoff function of player i is denoted $u_i(s_i, s_{-i})$. The functional form of preferences may be represented as:

$$V_i = \delta \left[(1-\alpha) \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i}) + \alpha \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i}) \right] + (1-\delta) \cdot \mathbf{E}_\pi u_i(s_i, s_{-i}), \quad (4.1)$$

where $\mathbf{E}_\pi u_i(s_i, s_{-i})$, is the conventional expectation taken with respect to the probability distribution π.[1]

Intuitively, π can be thought to be the decision maker's belief. However, he is not sure of this belief, hence it is an ambiguous belief. His confidence about this belief is modelled by $(1 - \delta)$, with $\delta = 1$ denoting complete ignorance and $\delta = 0$ denoting no ambiguity. His attitude to ambiguity is measured by α, with $\alpha = 1$ denoting pure optimism and $\alpha = 0$ denoting pure pessimism. If the decision-maker has $0 < \alpha < 1$, he is neither purely optimistic nor purely pessimistic (i.e., ambiguity-averse), but reacts to ambiguity in a partly pessimistic way by putting a greater weight on bad outcomes and in a partly optimistic way by putting a greater weight on good outcomes.

4.2.2 Equilibrium under Ambiguity

In this section we present an equilibrium concept for strategic games with ambiguity. In any Nash equilibrium, players are believed to behave in a manner that is

[1]Note that Chateauneuf, Eichberger, and Grant (2007) write a neo additive capacity in the form $\mu(E) = \delta\alpha + (1-\delta)\pi(E)$. We have modified their definition to be consistent with the rest of the literature where α is the weight on the minimum expected utility.

consistent with the actual behaviour of their opponents. They perfectly anticipate the actions of their opponent and can thus provide a best response to it in the form of their own action. In the case of ambiguity, represented by non-additive beliefs, however, the Nash idea of having consistent beliefs regarding the opponent's action and thus being able to play an optimum strategy as a response to these beliefs, needs to be modified. We assume that players choose pure strategies, and that in equilibrium the beliefs about these pure strategies are best responses to the opponent's actions.

Consider a game with 2 players and a finite pure strategy sets S_i, $i = 1, 2$. Each player i's beliefs about the opponent's behaviour is represented by a capacity v_i on S_{-i}, which is the set of strategy combinations which his/her opponent could choose. Given neo-additive beliefs, the expected payoff that a player i could earn from a strategy s_i, is determined by equation (4.1),

$$V_i\left(s_i; \pi_i, \alpha_i, \delta_i\right) = \delta_i \alpha_i M_i\left(s_i\right) + \delta_i\left(1 - \alpha_i\right) m_i\left(s_i\right) + \left(1 - \delta_i\right) \int u_i(s_i, s_{-i}) d\pi_i(s_{-i}), \tag{4.2}$$

where $M_i\left(s_i\right) = \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ and $m_i\left(s_i\right) = \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$.

Unlike the scenario of Nash equilibrium where a player could attach a set of additive probabilities to his opponent's actions, in the presence of ambiguity, beliefs are represented by capacities. The support of a capacity is a player's belief of how the opponent will act. Formally, the support of a neo-additive capacity, $\nu\left(A\right) = \delta\alpha + \left(1 - \delta\right) \pi\left(A\right)$, is defined by $\operatorname{supp}\left(\nu\right) = \operatorname{supp}\left(\pi\right)$. Thus the support of a neo-additive belief is equal to the support of its additive component. This definition is justified in Eichberger and Kelsey (2011).

Definition 2 *A pair of neo-additive capacities* (ν_1^*, ν_2^*) *is an Equilibrium Under Ambiguity (EUA) if for* $i = 1, 2$, $\operatorname{supp}\left(\nu_i^*\right) \subseteq R_{-i}(\nu_{-i}^*)$.
Here R_i *denotes the best-response correspondence of player* i *given that his beliefs are represented by* ν_i, *and is defined by* $R_i(\nu_i) = R_i(\pi_i, \alpha_i, \delta_i) := \operatorname{argmax}_{s_i \in S_i} V_i\left(s_i; \pi_i, \alpha_i, \delta_i\right)$.

This definition of equilibrium is taken from Eichberger, Kelsey, and Schipper (2009), who adapt an earlier definition in Dow and Werlang (1994). These papers show that an EUA will exist for any given ambiguity-attitude of the players. In games, one can determine π_i endogenously as the prediction of the players from the knowledge of the game structure and the preferences of others. In contrast, we treat the degrees of optimism, α_i and ambiguity, δ_i, as exogenous. In equilibrium, each player assigns strictly positive likelihood to his/her opponent's best responses given the opponent's belief. However, each player lacks confidence in his/her likelihood assessment and responds in an optimistic way by over-weighting the best outcome, or in a pessimistic way by over-weighting the worst outcome.

Alternative approaches to equilibrium with ambiguity can be found in Klibanoff (1993) and Lo (1996). They model players as having preferences which satisfy the axioms of maxmin expected utility (MMEU, Gilboa and Schmeidler (1989)). Players are allowed to have beliefs which are represented by multiple sets of conventional probability distributions. As such, players can have mixed strategies that are chosen from these multiple sets of additive probability distributions. They model ambiguity aversion as a strict preference among players to randomise between strategies when they are indifferent to pure strategies.

4.3 Experimental Model

In this section, we shall explain the games used by us in the experimental sessions and discuss the Nash equilibria as well Equilibrium under Ambiguity (EUA) for each game. We represent the preferences of a player in a game with neo-additive capacities. We will first look at the effort games, followed by the coordination games and finally have a brief look at the Ellsberg decision problems being studied by us. Henceforth we will use male pronouns he, his etc. to denote the Row Player, while female pronouns she, hers etc. will be used to denote the Column Player.[2]

[2] This convention is for the sake of convenience only and does not bear any relation to the actual gender of the subjects in our experiments.

4.3.1 Effort Games

In the effort rounds, we use a set of linked games to test our hypothesis that ambiguity has opposite effects in games of strategic complements and substitutes. The weakest link game exhibits strategic complements, while the best shot game exhibits a strategic substitutes game. Our hypothesis is that ambiguity will lead to a decrease (resp. increase) in individuals' contributions in the weakest link (resp. best shot) game.

Figure 4.1. Payoff Matrix for the Weakest Link Game

		Opponent's Effort					
		100	**110**	**120**	**130**	**140**	**150**
My Effort	**100**	50	50	50	50	50	50
	110	45	55	55	55	55	55
	120	40	50	60	60	60	60
	130	35	45	55	65	65	65
	140	30	40	50	60	70	70
	150	25	35	45	55	65	75

The task given to the subjects was to choose an effort level from the set $E = \{100, ..., 150\}$. They were informed that the cost of exerting an effort (c) was 50% of the effort exerted, i.e., $c = 0.5$. In the case of the weakest link game, the payoff of the subject would thus be: $u_i\,(x_i, x_{-i}) = \min\{100, ..., 150\} - 0.5x_i$, where x_i denotes the contribution of individual i and $c = 0.5$ is the marginal cost of the contribution. The final payoff matrix (after subtracting costs) was provided to the subjects and can be seen in Figure 4.1. In the best shot game scenario, the payoff of the subject was: $u_i\,(x_i, x_{-i}) = \max\{100, ..., 150\} - 0.5x_i$, where x_i denotes the contribution of individual i and $c = 0.5$ denotes the marginal cost of a contribution. As before, the final payoff matrix (after subtracting costs) for this scenario was provided to the subjects and can be seen in Figure 4.2.

The Nash equilibrium of the weakest link game is for both players to coordinate on any one of the six effort levels available in E, thus $\{(e_1^*, e_2^*) \in E^2 \mid e_1^* = e_2^*\}$. As a result, there are multiple Nash equilibria possible on which the subjects can coordinate. Given this multiplicity of equilibria, it is understandable that there would ambiguity among the subjects about which effort level they should opt for/coordinate on. The equilibrium action under ambiguity would be for a subject to choose an

Figure 4.2. Payoff Matrix for the Best Shot Game

		Opponent's Effort					
		100	110	120	130	140	150
My Effort	100	50	60	70	80	90	100
	110	55	55	65	75	85	95
	120	60	60	60	70	80	90
	130	65	65	65	65	75	85
	140	70	70	70	70	70	80
	150	75	75	75	75	75	75

Figure 4.3. Two-Player Representation of the Best Shot Game

		Column Player					
		100	110	120	130	140	150
Row Player	100	50, 50	60, 55	70, 60	80, 65	90, 70	100, 75
	110	55, 60	55, 55	65, 60	75, 65	85, 70	95, 75
	120	60, 70	60, 65	60, 60	70, 65	80, 70	90, 75
	130	65, 80	65, 75	65, 70	65, 65	75, 70	85, 75
	140	70, 90	70, 85	70, 80	70, 75	70, 70	80, 75
	150	75, 100	75, 95	75, 90	75, 85	75, 80	75, 75

effort of 100, which gives him a definite or ambiguity-safe payoff of $50ECU$ (See Figure 4.1). Selecting an effort level of 100, frees the subject from having to depend on his opponent's choice and/or having to achieve perfect coordination in their chosen effort levels.

As can be seen in Figure 4.3, the best shot game has two pure Nash equilibria: $\{(e_1^*, e_2^*) = ((100, 150), \ (150, 100)\}$. The Nash thus predicts that one of the players will exert the highest effort level (in our case $150)$, while the other will free-ride and choose the lowest effort available to him (in our case 100). Here again, we have multiple Nash equilibria and it is expected that subjects would be ambiguous about which one to choose. If the level of ambiguity about the opponent is high, the equilibrium action under ambiguity is to choose highest effort level, i.e., 150, since this provides the player with a constant payoff irrespective of the opponent's decision.

The equilibrium actions chosen in the weakest link and best shot games are consistent with our initial hypotesis that that ambiguity would lead to subjects decreasing (resp. increasing) their effort levels in case of the weakest link (resp. best shot) game.

4.3.2 Coordination Games

The coordination games used in the experimental sessions can be seen in Figure 4.4. Games ($SC1$) and ($SC2$) (as labelled in Figure 4.4) were used in Round 5 and 9, respectively, and are games with strategic complements games and positive externalities. Games ($SS1$) and ($SS2$), were the strategic substitutes and negative externalities games used in Round 7 and 11, respectively.

Figure 4.4. Coordination Games

(SC1)	Left	Middle	Right
Top	90, 85	90, 95	90, 0
Centre	0, 90	110, 95	110, 0
Bottom	5, 105	105, 95	105, 110

(SC2)	Left	Middle	Right
Top	115, 95	115, 100	115, 0
Centre	0, 95	135, 100	135, 0
Bottom	5, 105	105, 100	105, 135

(SS1)	Left	Middle	Right
Top	0, 0	70, 0	170, 150
Centre	100, 70	165, 165	165, 150
Bottom	150, 170	150, 165	150, 150

(SS2)	Left	Middle	Right
Top	95, 5	5, 0	5, 90
Centre	0, 5	100, 100	100, 90
Bottom	90, 95	90, 100	90, 90

Theorem 3 *In the case of games with strategic complements and positive (resp. negative) externalities, the equilibrium strategy under ambiguity of an agent i with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.*

We assume that both players are ambiguity averse, i.e. $\alpha = 0$, and have preferences that can be represented by neo-additive capacities. In order to illustrate the theorem, we use the 3x3 game in Table 4.2 , which echoes Game ($SC2$) used by us in our experiment.

Suppose $0 < a < b < c < d < e < f$, then the 3x3 game given by Table 4.2 has one pure Nash eqilibribrium: (C, M). Moreover, if we order the strategy spaces as

Table 4.2. An example of a Strategic Complements Coordination Game

	L	M	R
T	e, b	e, c	$e, 0$
C	$0, b$	f, c	$f, 0$
B	a, d	d, c	d, f

follows: $T < C < B$ and $L < M < R$, the game is one of strategic complements and positive externalities.

Let the Row Player have the following beliefs : $v^{RP}(L) = 1 - \delta^{RP}$ and $v^{RP}(M, R) = 0$. Then the Choquet expected payoff for the Row Player would be:

$$\begin{aligned} V^{RP}(T) &= e \\ V^{RP}(C) &= f\delta^{RP} \\ V^{RP}(B) &= a + (d-a)\delta^{RP}. \end{aligned}$$

Thus, T is the best response for the Row Player if $\delta^{RP} \geq \frac{e}{f}$. Intuitively, this means that if the Row Player is sufficiently ambiguous about the opponent's behaviour, he would opt for T, which is the ambiguity safe option.

Similarly, if the Column Player has the following beliefs : $v^{CP}(B) = 1 - \delta^{CP}$ and $v^{CP}(T, C) = 0$. Then the Choquet expected payoff for the Column Player would be:

$$\begin{aligned} V^{CP}(L) &= d + (b-d)\delta^{CP} \\ V^{CP}(M) &= c \\ V^{CP}(R) &= f\left(1 - \delta^{CP}\right). \end{aligned}$$

Thus, M is the best response for the Column Player if $\delta^{CP} \geq \frac{f-c}{f}$. Intuitively, this means that if the Column Player is sufficiently ambiguous about the opponent's behaviour, he would opt for M, which is the ambiguity safe option.

Hence, the best response for both players in a game with strategic complements and positive externalities, given sufficient ambiguity is one that decreases ambiguity.

□

Theorem 4 *In the case of games with strategic substitutes and negative (resp. positive) externalities, the equilibrium strategy under ambiguity of an agent with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.*

We assume that both players are ambiguity averse, i.e., $\alpha = 0$, and have preferences that can be represented by neo-additive capacities. In order to prove this by illustration, we use the 3x3 game in Table 4.3 , which echoes Game ($SS2$) used by us in our experiment.

Table 4.3. An example of a Strategic Substitutes Coordination Game

	L	M	R
T	c, a	$a, 0$	a, b
C	$0, a$	d, d	d, b
B	b, c	b, d	b, b

Suppose $0 < a < b < c < d$, then the 3x3 game given by Table 4.3 has one pure Nash eqilibribrium: (C, M). Moreover, if we order the strategy spaces as follows: $T > C > B$ and $L > M > R$, the game is one of strategic substitutes and negative externalities.

Let the Row Player have the following beliefs : $v^{RP}(L) = 1 - \delta^{RP}$ and $v^{RP}(M, R) = 0$. Then the Choquet expected payoff for the Row Player would be:

$$\begin{aligned} V^{RP}(T) &= c + (a - c)\,\delta^{RP} \\ V^{RP}(C) &= d\delta^{RP} \\ V^{RP}(B) &= b. \end{aligned}$$

Thus, B is the best response for the Row Player if $\delta^{RP} \geq \frac{b}{d}$. Intuitively, this means that if the Row Player is sufficiently ambiguous about the opponent's behaviour, he would opt for B, which is the ambiguity safe option.

Let the Column Player have the following beliefs : $v^{CP}(T) = 1 - \delta^{CP}$ and $v^{CP}(C, B) = 0$. Then the Choquet expected payoff for the Column Player would be:

$$V^{CP}(M) = d\delta^{CP}$$
$$V^{CP}(R) = b.$$

The Column Player would thus prefer R if, $\delta^{CP} \geq \frac{b}{d}$. Intuitively, this means that if the Column Player is sufficiently ambiguous about the opponent's behaviour, he would opt for R, which is the ambiguity safe option.

Hence, the best response for both players in a game with strategic substitutes and negative externalities, given sufficient ambiguity is one that decreases ambiguity. $\square$

Games (1) and (2) are games with strategic complements games and positive externalities. This can be verified if we fix the order $T < C < B$ and $L < M < R$. Both games have one pure Nash equilibrium: (C, M). The equilibrium under ambiguity for these games is (T, M).

Game (3) is a strategic substitues game with negative externalities and multiple Nash equilibria, if we fix $T > C > B$ and $L > M > R$. The game has three pure Nash equilibria: (T, R), (C, M) and (B, L), none of which are focal. The equilibrium under ambiguity for this game is (B, R). It can be noted that the equilibrium under ambiguity (B, R) is Pareto-dominated by the Nash equilibrium (C, M).

Game (4) is a strategic substitues game with negative externalities if we fix $T > C > B$ and $L > M > R$. The game has a unique Nash equilibrium: (C, M). The equilibrium under ambiguity for this game is (B, R).

4.3.3 Ellsberg Urn Experiments

The game rounds were alternated with single person decision problems regarding an Ellsberg Urn. Subjects were presented with an urn containing 90 balls, of which 30 were labelled X, and the remainder were an unknown proportion of Y or Z balls. Subjects were asked to pick a letter, and a ball was drawn from the urn. If the letter of the ball drawn matched the letter chosen by the subject, it entitled the subject to a prize. The decisions put to the subjects took the following form:

"An urn contains 90 balls, of which 30 are labelled X. The remainder are either Y or Z.

Which of the following options do you prefer?

a) Payoff of y if an X ball is drawn.

b) Payoff of 100 if a Y ball is drawn.

c) Payoff of 100 if a Z ball is drawn."

Payoff "y" attached to the option X was changed from round to round, with $y = 95,\ 90, 80, 100,\ 105$ (in that order), to measure the ambiguity threshold of subjects.

In our Ellsberg urn experiments, we use balls labelled X, Y and Z, rather than following the traditional practice of using Red, Blue and Yellow coloured balls.[3] This is because in a previous set of experiments conducted by us (Kelsey and Le Roux (2012)), we used the traditional Ellsberg Urn setup and found that subjects often chose Blue (the ambiguous option), simply because they had a fondness for the colour blue. Similarly, we found a large number of Chinese subjects chose Red, because it was considered "auspicious" in Chinese culture. In this study we use balls labelled X, Y and Z, in order to avoid any such trivial decisions being made.

4.4 Experimental Design

The games described above were used in paper-based experiments, conducted at St. Stephen's College in New Delhi, India, and at the Finance and Economics Experimental Laboratory in Exeter (FEELE), UK. The experiments were conducted with three different treatments - under the first treatment, subjects were only matched with locally recruited subjects; under the second treatment, Exeter subjects were only matched with subjects recruited in India; and under the final treatment, subjects were matched against both internationally as well as locally recruited subjects, for the purpose of payment.

[3]In the traditional Ellsberg urn setup, the urn would contain Red, Blue and Yellow coloured balls. The number of Red balls in the urn would be known, while the remaining Blue and Yellow coloured balls would be ambiguous in number.

Treatments 1 and 3 consisted of 60 subjects each and Treatment 2 had 61 subjects. In total there were 181 subjects who took part in the experiment, 81 of whom were males and the remaining 100 were females. We were also interested in whether or not participants had a quantitative background - 59 of the subjects had studied a quantitative subject such as Biochemistry, Electronic Engineering or Astrophysics, while 122 of the subjects had studied a non-quantitative subject such as History, Philosophy, or International Relations. Each session lasted a maximum of 45 minutes including payment.

Subjects first read through a short, comprehensive set of instructions at their own pace, following this the instructions were also read out to all the participants in general. The subjects were asked to fill out practice questions to check that they understood the games correctly. Once the practice questions had been answered and discussed, the actual set of experimental questions were handed out to the subjects. Subjects were randomly assigned the role of either a Row Player or a Column Player at the beginning of the experiment, for the purpose of matching in the coordination games, and remained in the same role for the rest of the experiment.

The experiment consisted of 11 rounds, starting with a decision regarding a strategic complements/substitutes game, which was then alternated with an Ellsberg Urn decision, such that there were a total of 6 game rounds and 5 Ellsberg urn rounds. Each subject had to select one option per round: An effort level in case of the effort rounds, Top/Centre/Bottom if they were a Row Player or Left/Middle/Right if they were a Column Player, and in case of the Ellsberg urn rounds X, Y or Z. In the Ellsberg urn rounds, the pay-offs attached to drawing a Y or Z ball were held constant at 100, while those attached to drawing an X ball varied as 95, 90, $80, 100, 105$.

Once subjects had made all 11 decisions, a throw of dice determined *one* game round and *one* Ellsberg urn round for which subjects would be paid. We picked one round at random for payment in order to prevent individuals from self-insuring against payoff risks across rounds (See Charness and Genicot (2009)). If all rounds count equally towards the final payoff, subjects are likely to try and accumulate a high payoff in the first few rounds and then care less about how they decide in the following rounds. In contrast, if subjects know that they will be paid for a random round, they treat each decision with care. Players' decisions were matched according to a predetermined matching, and pay-offs were announced.

Instead of using a real urn we used a computer to simulate the drawing of a ball from the urn.[4] The computer randomly assigned the number of Y and Z balls in the urn so that they summed to 60, while keeping the number of X balls fixed at 30 and the total number of balls in the urn at 90. The computer then simulated an independent ball draw for each subject. If the label of the ball drawn by the computer matched that chosen by the subject, it entitled him to the payoff specified in the round chosen for payment.

The total earnings of a subject was the sum of a show-up fee, payoff earned in the chosen game round and payoff earned in the chosen Ellsberg urn round. Average payment made to Indian subjects was $Rs.440$ (£5.50 approximately), and to Exeter subjects was £6.50. The maximum payment made to an Indian subject was $Rs.600$ (£7.50 approximately), and to Exeter subjects was £8.40.

4.5 Data Analysis and Results

4.5.1 Behaviour in Effort Rounds

The task of the subjects in effort rounds was to choose an effort level from the set $E = \{100, ..., 150\}$. The cost of exerting an effort (c) was 50% of the effort exerted, i.e., $c = 0.5$. The Nash equilibrium of the weakest link game is for both players to coordinate on any one of the six possible effort levels. As a result, there are multiple Nash equilibria on which the subjects can coordinate. In the best shot game, the Nash equilibrium predicts that one of the players will exert the highest effort level (in our case 150) , while the other will free-ride and choose the lowest effort available to him (in our case 100).

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[4] The computer simulated urn can be found at the following link: http://people.exeter.ac.uk/dk210/Ellsberg-110708.xls.

The equilibrium action under ambiguity is to choose the lowest effort level, i.e., 100, in the case of the weakest link game and to choose the highest effort level, i.e., 150, in the case of the best shot game. In both scenarios, the equilibrium action under ambiguity provides the player with a constant payoff irrespective of the opponent's decision. Moreover, it is expected that ambiguity would lead to subjects reducing their effort levels in case of the weakest link game, while increasing their effort levels in case of the best shot game.

4.5.1.0.2 Treatment I In this treatment, subjects were matched against other locally recruited subjects only. Goeree and Holt (2001), study a minimum effort coordination game similar to this treatment, where subjects faced a marginal cost of either $c = 0.1$ or $c = 0.9$. They found that for low marginal costs ($c = 0.1$), subjects choose high effort levels and for high marginal costs ($c = 0.9$), a majority of subjects choose low effort levels.

In our game with $c = 0.5$, we find that 22% (13) of the subjects chose an effort level of 100 in the weakest link game round. This is the effort level at which the subject has a constant payoff which is independent of the opponent's action, and is the equilibrium action under ambiguity (See Table 4.4). Moreover, 65% (39) of the subjects chose an effort level between $100 - 120$, i.e., the lower end of the spectrum of effort choices. This confirms our hypothesis that the effect of ambiguity would lead to subjects reducing their effort levels, when compared to Nash predictions. A small number of subjects (9), chose the maximum effort level 150. However, they were in a very small minority.

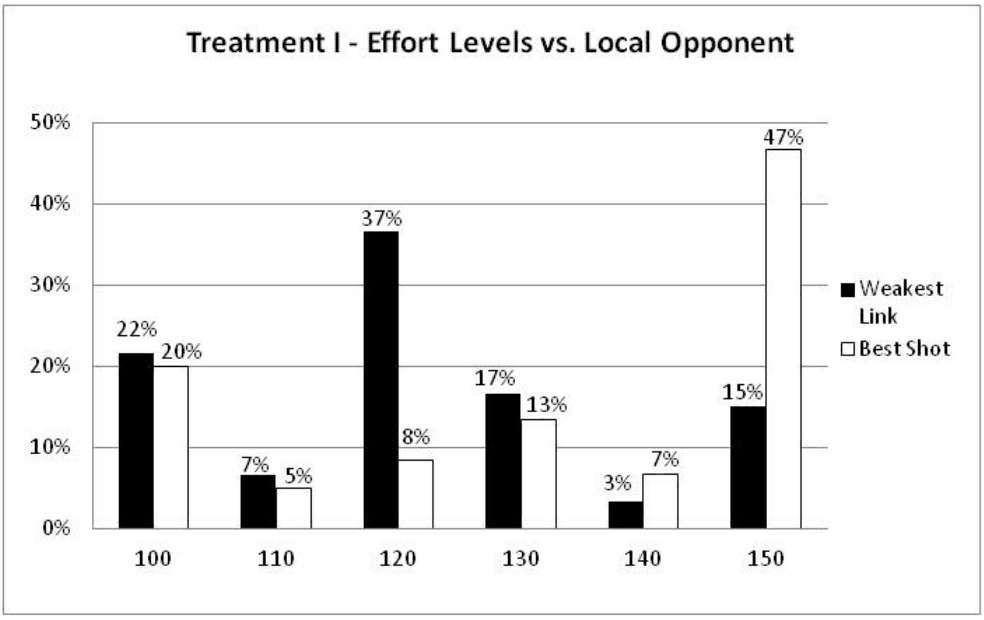
In the best shot game round, we find that 47% (28) of the subjects chose the effort level 150 (the equilibrium action under uncertainty). Moreover, ambiguity has led to subjects increasing their effort levels with 67% (40) of the subjects choosing an effort level in the high range of $130 - 150$.

While analysing the manner in which people switch effort levels between the two scenarios, we find that 55% (33) of the subjects switch from a low effort level

Table 4.4. Treatment I - Effort Levels vs. Local Opponent

	Weakest Link		Best Shot	
100	13	22%	12	20%
110	4	7%	3	5%
120	22	37%	5	8%
130	10	17%	8	13%
140	2	3%	4	7%
150	9	15%	28	47%
$\sum$	60		60	

Figure 4.5. Treatment I - Subject Behaviour in Effort Rounds



in the weakest link round to a higher effort level in the best shot game (Please see Table 4.5). These subjects display ambiguity averse behaviour, which is in line with Eichberger and Kelsey (2002). Interestingly, we find that 25% (15) of subjects display a preference for ambiguity, choosing a high effort level in the weakest link game and then switching to a low effort level in the best shot round. We also note that 20% (12) of subjects did not change their chosen effort levels between the two rounds - these subjects could be displaying ambiguity neutral behaviour.[5]

[5] Alternatively, unchanged effort levels might be caused by subjects were trying to be consistent. Another trivial reason could be that, there are subjects who having chosen an effort level in the previous round, do not want to go to the trouble of thinking again and stick with their previous decision.

Table 4.5. Switching Effort Levels between Weakest Link and Best Shot Game

	Treatment I		Treatment II	
Low to High	33	55%	28	46%
High to Low	15	25%	20	33%
Constant	12	20%	13	21%
$\sum$	60		61	

4.5.1.0.3 Treatment II In this treatment, subjects were matched with foreign opponents only. In the weakest link round, only 8% (5) of the subjects chose effort level 100 (See Table 4.6). Even though the ambiguity-safe effort level has been chosen by a small minority, the effect of ambiguity can be seen in the sizeable 59% (36) of subjects who have chosen the lower end of the effort spectrum $100 - 120$.

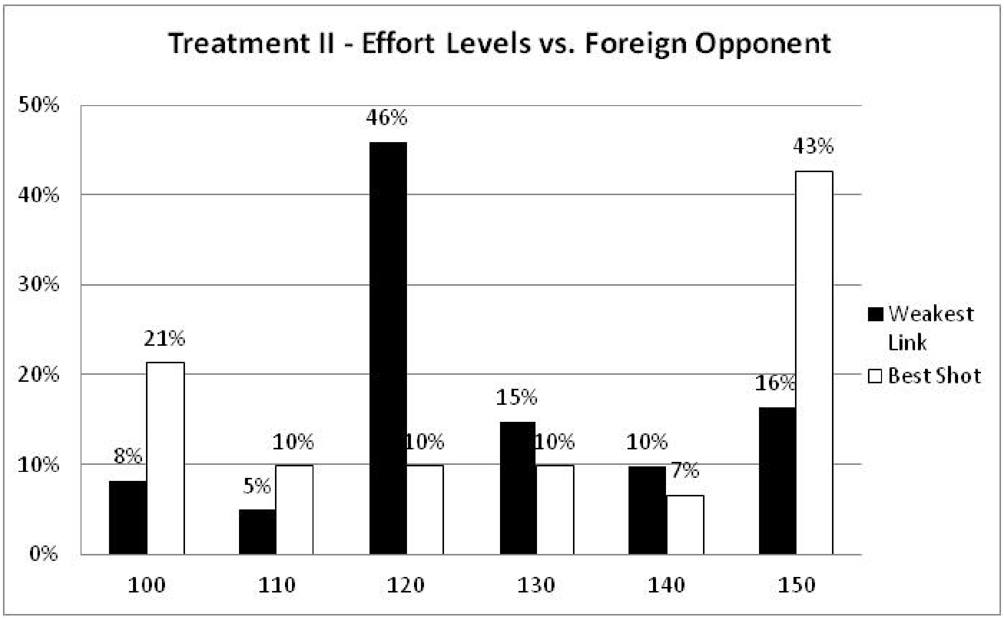
In the best shot game, 43% (26) of the subjects chose an effort of 150, which is the equilibrium action under ambiguity, while 59% (36) of the subjects chose in the high effort range of $130 - 150$. It is clear that ambiguity is resulting in efforts being concentrated at the lower end, in case of the weakest link game; and at the higher end, in case of the best shot game (See Figure 4.6).

Table 4.6. Treatment II - Effort Levels vs. Foreign Opponent

	Weakest Link		Best Shot	
100	5	8%	13	21%
110	3	5%	6	10%
120	28	46%	6	10%
130	9	15%	6	10%
140	6	10%	4	7%
150	10	16%	26	43%
$\sum$	61		61	

Table 4.5 , summarises the manner in which people switch effort levels between the two scenarios. We find that 46% (28) of the subjects switch from a low effort level in the weakest link round to a higher effort level in the best shot game, displaying ambiguity averse behaviour. Moreover, 33% (20) of subjects display ambiguity seeking behaviour, choosing a high effort level in the weakest link round followed by a lower effort level in the best shot game, while 21% (13) of subjects do not change their chosen effort levels between the two rounds.

Figure 4.6. Treatment II - Subject Behaviour in Effort Rounds



4.5.1.0.4 Treatment III In this treatment, subjects were matched with both local as well as foreign opponents to check whether this would have an impact (if any) on the level of ambiguity perceived by them, and their attitude towards such ambiguity. Table 4.7 and Figure 4.7, provide a summary of subject behaviour in this treatment.[6]

In the weakest link round, 27% (16) of the Exeter subjects chose an effort level of 100 against a local subject while 28% (17) chose it against the Indian subject. The difference in the number of people choosing the lowest effort level vs. the foreign opponent is very marginal. On the whole, 58% (35) of the subjects chose an effort level between $100 - 120$, i.e., the lower end of the spectrum of effort choices vs. the local subject, while 53% (32) chose an effort in that range against the foreign subject. There are more people choosing a low effort level vs. the local subject than vs. the foreign subject.

Another point that may be noted, is that 22% (12) of subjects chose 150 (the highest effort) against the foreign subject. Subjects had been told that the foreign

[6]Henceforth, in Treatment III tables, a Local Subject is referred to as L.S. and a Foreign Subject is referred to as F.S.

opponents were recruited at one of the India's most famous colleges, which had produced a number of distinguished alumni - subjects may have perceived this as a signal that the Indian subjects would be more willing to exert a greater effort and coordinate on a higher joint payoff, because of this background information.

Table 4.7. Treatment III - Effort Levels vs. both Local Subject and Foreign Subject

	Weakest Link vs. L.S.		Weakest Link vs. F.S.		Best Shot vs. L.S		Best Shot vs. F.S.	
100	16	27%	17	28%	17	28%	21	35%
110	3	5%	3	5%	4	7%	4	7%
120	16	27%	12	20%	6	10%	1	2%
130	10	17%	8	13%	4	7%	4	7%
140	7	12%	7	12%	5	8%	4	7%
150	8	13%	13	22%	24	40%	26	43%
$\sum$	60		60		60		60	

In the best shot game, we find that 40% (24) of the subjects chose 150 vs. the local subject, while 43% (26) chose it vs. the foreign subject. Moreover, while 55% (33) of the subjects chose in the high effort range of $130 - 150$, when making a choice against the local subject, 57% (34) chose this effort range against the foreign subject. Again, we find that 28% (17) and 35% (21) of subjects chose effort level 100 against the local and foreign subjects respectively.

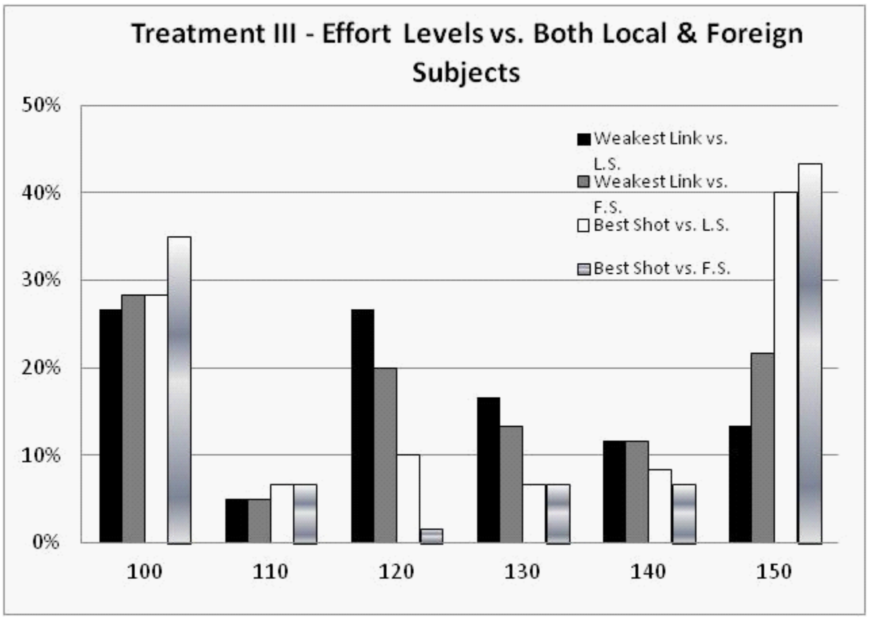
Even though we do not see a huge disparity in the effort choices versus the local and foreign oppoenent, Figure 6 shows that ambiguity does explain (most of) the deviations from Nash equilibrium. In the case of the weakest link game, most responses are concentrated towards the lower end of the spectrum between $100-120$, while in case of the best shot game, responses are concentrated towards the high end, i.e., at 150.

4.5.2 Behaviour in Coordination Game Rounds

4.5.2.1 Row Player Behaviour

Henceforth, SC1 refers to Round 5, SC2 refers to Round 9, SS1 refers to Round 7 and SS2 refers to Round 11. The task of the Row Player in the coordination game rounds was to choose between *Top* (T), *Centre* (C) and *Bottom* (B). Games $SC1$,

Figure 4.7. Treatment III - Subject Behaviour in Effort Rounds



$SC2$ and $SS2$ have one pure Nash equilibrium: (C, M), while $SS1$ has three pure Nash equilibria: (T, R), (C, M) and (B, L). The equilibrium action under ambiguity for the Row Player is to choose T in case of games $SC1$ and $SC2$ and to choose B in case of games $SS1$ and $SS2$.

4.5.2.1.1 Treatment I This treatment consisted of matching subjects against other locally recruited subjects only and as such was the base treatment.[7] See Table 4.8 and Figure 4.8, for a summary of Row Player behaviour in Treatment I.

We find that of the 30 row players who took part in this treatment, 63% (19) and 73% (22) of subjects chose the ambiguity-safe strategy T in $SC1$ and $SC2$, respectively. In comparison, only 13% (4) and 17% (5) of subjects opted for C, which is the choice under Nash.

We conducted a binomial test with the null that the ambiguity-safe option T is played as often as $C + B$ ($H_0 : prob(Top) = 0.5$, $prob(Centre + Bottom) = 0.5$), against the alternative that T was played more often than $C + B$ ($H_1 : prob(Top) >$

[7] A probit regression showed that the dummy variable for location (*Delhi*/*Exeter*) does not have a significant impact on choosing the ambiguity safe option. This implies that the behaviour of Indian subjects was very similar to the Exeter subjects, when they are matched against other local opponents. Thus for the purpose of analysing subject behaviour in Treatment I, we can combine the responses of the Delhi subjects with the Exeter subjects without loss of efficiency.

$prob(Centre + Bottom)$).[8] We reject the null at a 5% level of significance for $SC1$ and at a 1% level of significance for $SC2$. Subjects choose the ambiguity-safe option significantly more often than either of the other two options available to them, in the strategic complement games.

Table 4.8. Treatment I - Row Player Behaviour vs. Local Opponent

	SC1		SC2		SS1		SS2	
Top	19	63%	22	73%	3	10%	2	7%
Centre	4	13%	5	17%	12	40%	8	27%
Bottom	7	23%	3	10%	15	50%	20	67%
$\sum$	30		30		30		30	

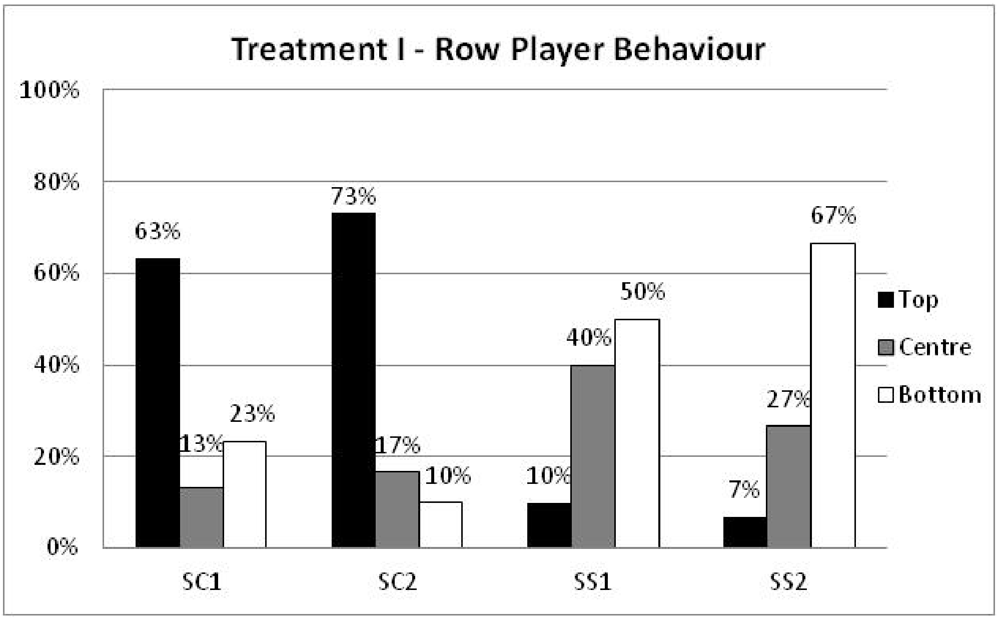
In the strategic substitutes game $SS1$, we find that 40% (12) of subjects, chose C (which is a choice under Nash) while 50% (15) of subjects chose B (which is the choice under EUA). It is interesting to note that when multiple Nash equilibria are present, 40% of the subjects seem to be selecting the Nash $(C, M)-$ which gives an equal payoff to both players. This might indicate that fairness constraints affect these subjects more than ambiguity. However in $SS2$, 67% (20) of subjects chose the ambiguity-safe strategy B, while only 27% (8) subjects chose C which is the choice under Nash.

We conducted a binomial test with the null that the ambiguity-safe option B is played as often as $T + C$ (H_0 : $prob(Bottom) = 0.5$, $prob(Top + Centre) = 0.5$), against the alternative that T was played more often than $C + B$ (H_1 : $prob(Bottom) > prob(Top + Centre)$).[9] We fail to reject the null for $SS1$ where subjects play B as often as $T + C$, but reject the null at 5% for $SS2$, where subjects play the ambiguity-safe option B more often than either of the alternatives.

[8] Henceforth, we shall refer to this as Binomial Test A, when using the same null and alternative hypothesis as described here.

[9] Henceforth, we shall refer to this as Binomial Test B, when using the same null and alternative hypothesis as described here.

Figure 4.8. Treatment I - Row Player Behaviour vs. Local Opponent



4.5.2.1.2 Treatment II This treatment consisted of matching Exeter subjects against an Indian opponent. Subjects were told that the same experiments had been run in India and that they would be matched up against an Indian subject whose responses had been already collected. Cultural studies conducted in the past have shown that western societies are individual-oriented, while Asian cultures tend to be collectivist. Members of Asian cultures have larger social networks that they can fall back upon in the event of an emergency/loss. This makes them more risk/ambiguity-seeking than their western counterparts (Weber and Hsee (1998)). As such, we expected that subjects would be more ambiguous when matched against opponents who are from a different socio-cultural background than themselves. See Table 4.9 and Figure 4.9, for a summary of Row Player behaviour in Treatment II.

We find that of the 30 row players who took part in this treatment, 85% (25) and 87% (26) of subjects chose the ambiguity-safe strategy T in $SC1$ and $SC2$, respectively. In comparison, only 7% (2) and 13% (4) of subjects opted for C, which is the choice under Nash. When compared to the base treatment, it is clear that subjects perceived greater ambiguity in this situation and a clear majority chose to play the ambiguity-safe option.

Binomial Test A (null and alternative as described in Treatment I), can be rejected at 1% for both $SC1$ as well as $SC2$. Subjects chose the ambiguity safe option significantly more often than either of the other two options available to them.

Table 4.9. Treatment II - Row Player Behaviour vs. Foreign Opponent

	SC1		SC2		SS1		SS2	
Top	25	83%	26	87%	2	7%	0	0%
Centre	2	7%	4	13%	13	43%	10	33%
Bottom	3	10%	0	0%	15	50%	20	67%
$\sum$	30		30		30		30	

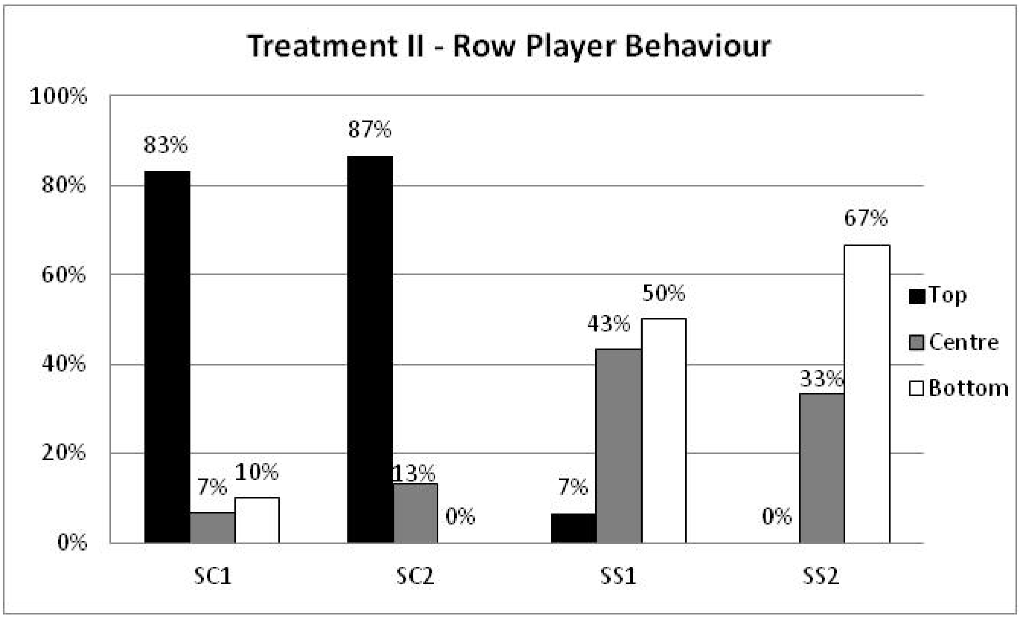
In the strategic substitutes game $SS1$, we find that 43% (13) of subjects, chose C (which is a choice under Nash) while 50% (15) of subjects chose B (which is the choice under EUA). In the presence of multiple Nash equilibria, 43% of the subjects select the Nash (C, M) rather than the other Nash eqilibrium options (T, R) or (B, L). Even in this treatment where we perceive heightened ambiguity on the part of the subjects, about half of them opt for the Nash outcome which would result in equitable payoffs for both players. In the second strategic subtitutes game, $SS2$, 67% (20) of subjects chose the ambiguity-safe strategy B, while 33% (10) subjects chose C which is the choice under Nash.

Binomial Test B cannot be rejected for $SS1$, where subjects play B as often as $T + C$. However, we do reject the null at a 5% level of significance for $SS2$, where subjects play the ambiguity-safe option B more often.

4.5.2.1.3 Treatment III In Treatment III, subjects were asked to make decisions versus both the local as well as the foreign opponent. They were allowed to choose different actions against the foreign oppoent and the domestic one. See Table 4.10 and Figure 4.10, for a summary of Row Player behaviour in $SC1$ and $SC2$.

In $SC1$, 67% (20) of subjects chose the ambiguity-safe strategy T against the local subject (L.S.), while and 63% (19) of subjects chose it against the foreign subject (F.S). Fewer subjects chose the ambiguity-safe option against the foreign

Figure 4.9. Treatment II - Row Player Behaviour vs. Foreign Opponent



opponent than against the local opponent. In comparison, the Nash was played by 13% (4) and 10% (3) of subjects versus local and foreign opponent, respectively.

In $SC2$, 67% (20) and 50% (15) of subjects chose the safe strategy T against the local and foreign opponent respectively. It can be noted again that fewer subjects pick the ambiguity safe option against the foreign opponent. In comparison, 27% (8) and 33% (10) of subjects opted for C, the choice under Nash.

Table 4.10. Treatment III - Row Player Behaviour vs. both Local Subject and Foreign Subject

	SC1 vs. L.S.		SC1 vs. F.S.		SC2 vs. L.S.		SC2 vs. F.S.	
Top	20	67%	19	63%	20	67%	15	50%
Centre	4	13%	3	10%	8	27%	10	33%
Bottom	6	20%	8	27%	2	7%	5	17%
$\sum$	30		30		30		30	

As before, we conducted Binomial Test A for the Row Player. We reject the null at 5% for both $SC1$ and $SC2$, when looking at responses against the local opponent. However, we fail to reject the null when analysing choices against the foreign opponent. Subjects choose the ambiguity-safe option significantly more often against the local opponent than the foreign subject.

Figure 4.10. Treatment III - Row Player Behaviour vs. Both Local and Foreign Subjects

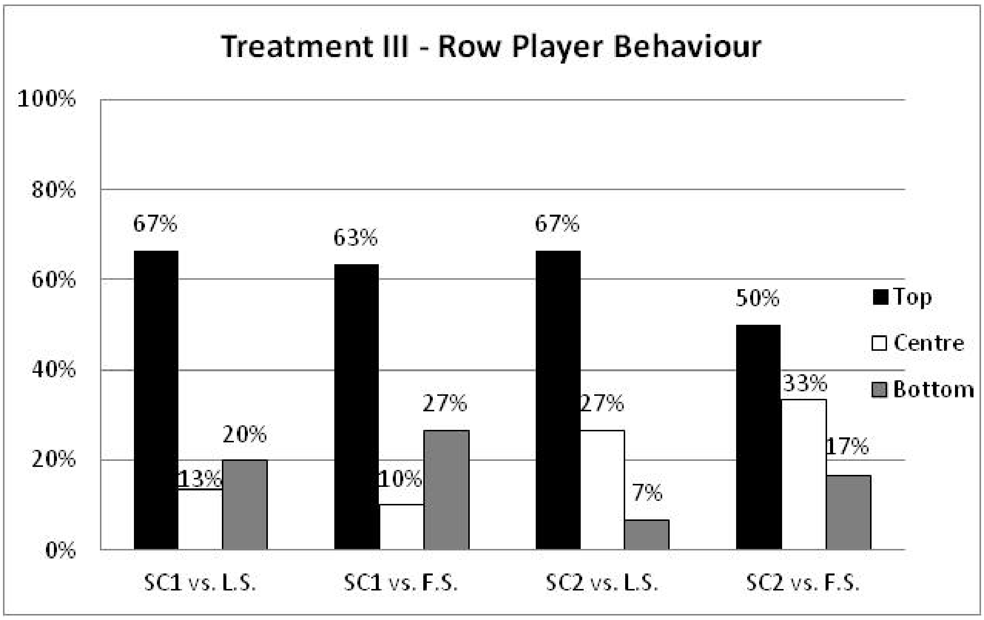


Table 4.11. Treatment III - Row Player Behaviour vs. both Local Subject and Foreign Subject

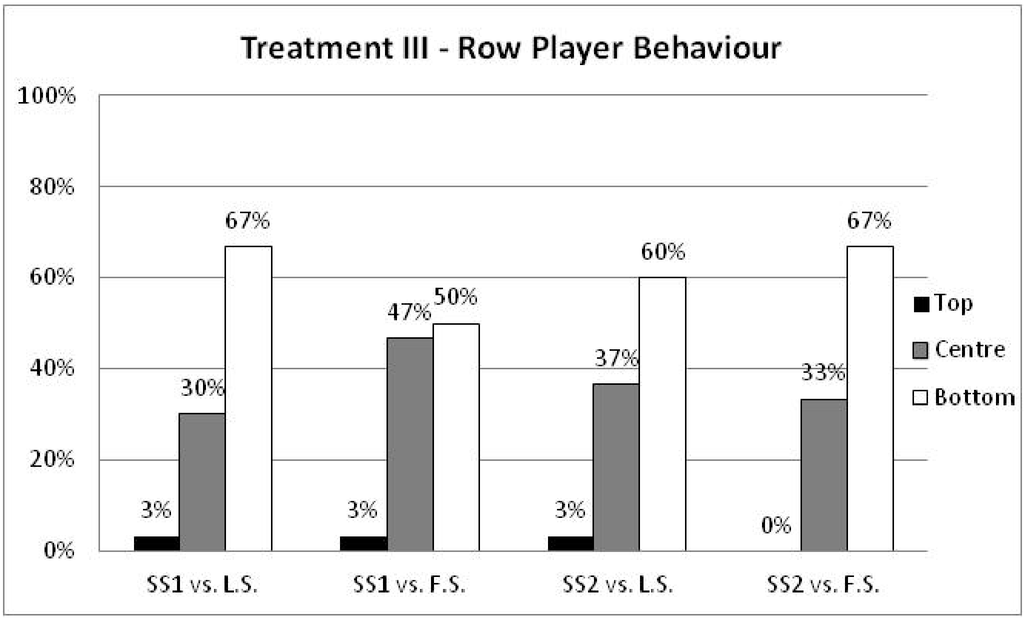
	SS1 vs. L.S.		SS1 vs. F.S.		SS2 vs. L.S.		SS2 vs. F.S.	
Top	1	3%	1	3%	1	3%	0	0%
Centre	9	30%	14	47%	11	37%	10	33%
Bottom	20	67%	15	50%	18	60%	20	67%
$\sum$	30		30		30		30	

We find similar behaviour in the strategic substitutes game $SS1$: 67% (20) and 50% (15) of subjects chose B, the choice under EUA, against the local and foreign subject respectively. Again, fewer subjects took the ambiguity-safe option versus foreign subject than against Indian subject. See Table 4.11 and Figure 4.11, for a summary of Row Player behaviour in $SC1$ and $SC2$.

In contrast, in $SS2$, 60% (18) and 67% (20) of subjects chose the ambiguity-safe strategy B against the local and foreign opponent respectively. The number of subjects choosing the ambiguity-safe option against the foreign subject is slightly larger in this round - though very slightly.

Binomial Test B cannot be rejected for $SS1$ when subjects are faced by a foreign opponent, but can be rejected at a 5% level of significance for choices against the local subject. When analysing decisions for $SS2$, we reject the null at 5% level of significance for decisions pertaining to the foreign subject; but fail to reject it against local subject.

Figure 4.11. Treatment III - Row Player Behaviour vs. Both Local and Foreign Subjects



It can be noted that the behaviour in $SS2$ supports our hypothesis that when faced by both the foreign subject and the local subject simultaneously, the safe act would be taken more often against foreign subject. One of the reasons for not picking the ambiguity-safe option more often against the foreign subject, may be that subjects were trying to be dynamically consistent when making their choices. Another reason for this behaviour could be that subjects could see the other local subjects sitting in the experimental laboratory, whereas the foreign subject seemed very far away. They thus chose to play cautiously against the local subject, while taking their chances against the foreign subject.

4.5.2.2 Column Player Behaviour

The task of the Column Player in the coordination game rounds was to choose between *Left* (L), *Middle* (M) and *Right* (R). Recall, that games $SC1$, $SC2$ and $SS2$ have one pure Nash equilibrium: (C, M), while $SS1$ has three pure Nash equilibria: (T, R), (C, M) and (B, L). The equilibrium action under ambiguity for the Columnn Player is to choose M in case of games $SC1$ and $SC2$ and to choose R in case of games $SS1$ and $SS2$.

4.5.2.2.1 Treatment I

In this treatment, subjects were only matched against other locally recruited subjects and as such was the base treatment. See Table 4.12 and Figure 4.12, for a summary of Column Player behaviour in Treatment I.

We find that of the 30 column players who took part in this treatment, 70% (21) and 87% (26) of subjects chose the Nash strategy M in $SC1$ and $SC2$, respectively.[10] We conducted a binomial test with the null that M is played as often as $L + R$ ($H_0 : prob(Middle) = 0.5, prob(Left+Right) = 0.5$), against the alternative that M was played more often than $L+R$ ($H_1 : prob(Middle) > prob(Left+Right)$).[11] We reject the null at a 5% level of significance for $SC1$ and at a 1% level of significance for $SC2$. Subjects choose the Nash option significantly more often than either of the other two options available to them, in the strategic complement games.

Table 4.12. Treatment I - Column Player Behaviour vs. Local Opponent

	SC1		SC2		SS1		SS2	
Left	6	20%	4	13%	2	7%	1	3%
Middle	21	70%	26	87%	4	13%	6	20%
Right	3	10%	0	0%	24	80%	23	77%
$\sum$	30		30		30		30	

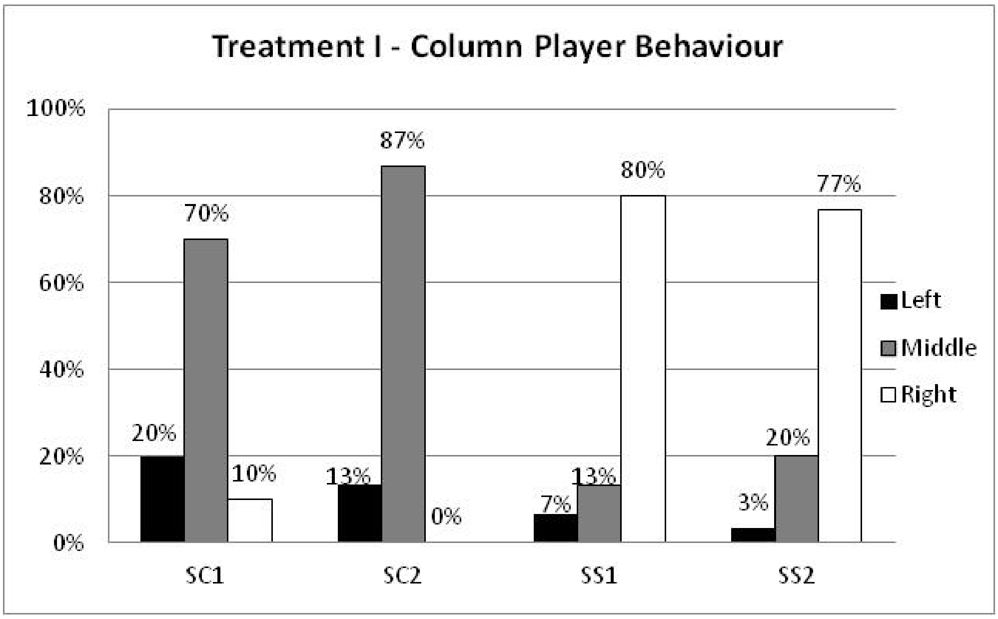
In the strategic substitutes games $SS1$ and $SS2$, we find that 80% (24) and 77% (23) of subjects choose the ambiguity-safe strategy R. In comparison, only 13% (4) and 20% (6) of subjects chose M, which is the choice under Nash. A binomial test with the null that the ambiguity-safe option R is played as often as $L + M$ ($H_0 : prob(Right) = 0.5$, $prob(Left + Middle) = 0.5$), against the alternative that R was played more often than $L + M$ ($H_1 : prob(Right) > prob(Left + Middle)$)[12], is rejected at 1% for $SS1$ and at 5% for $SS2$.

[10] Note in the case of $SC1$ and $SC2$, the equilibrium action under ambiguity coincides with the Nash strategy, for the Column player.

[11] Henceforth, we shall refer to this as Binomial Test C, when using the same null and alternative hypothesis as described here.

[12] Henceforth, we shall refer to this as Binomial Test D, when using the same null and alternative hypothesis as described here.

Figure 4.12. Treatment I - Column Player Behaviour vs. Local Opponent



4.5.2.2.2 Treatment II In this treatment, Exeter subjects were only matched against an Indian opponent. Subjects were informed they would be paired against an Indian subject whose responses had already been collected. As mentioned before, we expect subjects to be more ambiguous when matched against opponents who are from a different socio-cultural background than themselves, as compared to the base treatment. See Table 4.13 and Figure 4.13, for a summary of Column Player behaviour in Treatment II.

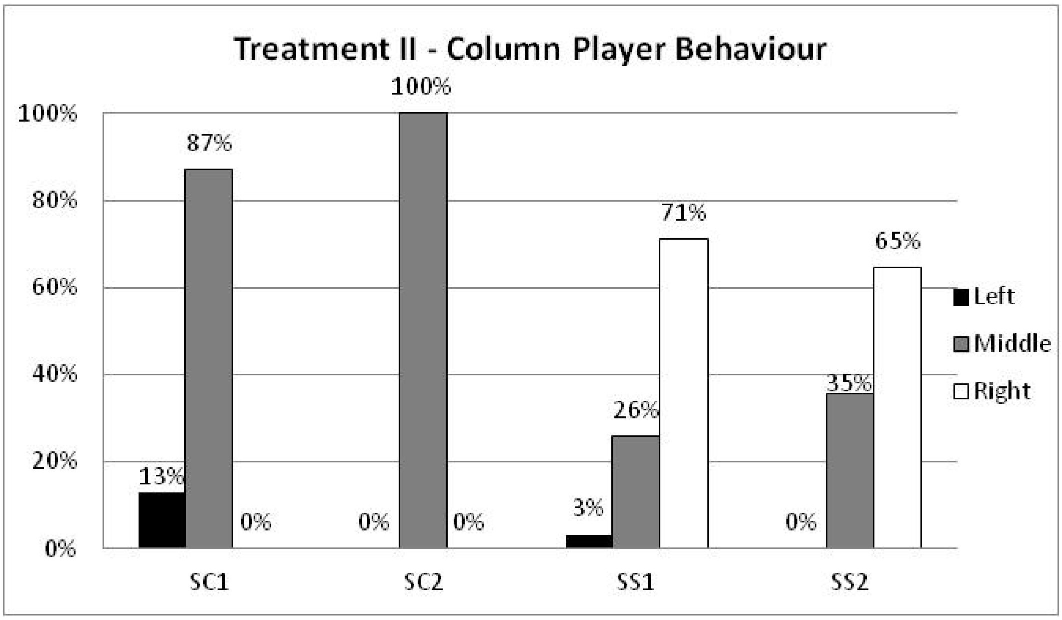
In rounds $SC1$ and $SC2$, of the 31 column players who took part in this treatment, 87% (27) and 100% (31) of subjects choose the Nash strategy M, respectively. Binomial Test C can be rejected at a 1% level for both $SC1$ as well as $SC2$.

Table 4.13. Treatment II - Column Player Behaviour vs. Foreign Opponent

	SC1		SC2		SS1		SS2	
Left	4	13%	0	0%	1	3%	0	0%
Middle	27	87%	31	100%	8	26%	11	35%
Right	0	0%	0	0%	22	71%	20	65%
$\sum$	31		31		31		31	

In games $SS1$ and $SS2$,we find that 71% (22) and 65% (20) of subjects choose the ambiguity-safe strategy R. In comparison, 26% (8) and 35% (11) of subjects chose M, which is the choice under Nash. Binomial Test D is rejected at 1% for $SS1$ and at 5% for $SS2$.

Figure 4.13. Treatment II - Column Player Behaviour vs. Foreign Opponent



When faced by foreign opponents, subjects did indeed choose the ambiguity-safe option significantly more often than the other actions available to them. Moreover, we encouraged subjects to write a short account at the end of the experiment, about their reactions and what they were thinking about when they made their choices during the experiment. A number of subjects concluded that they preferred to stick with a safe (but definite) payoff rather than take a chance and lose out, since they were not sure what prompted the foreign opponent's decision choices. It was clear that the situation was perceived by them as being ambiguous, and they were willing to forego the possibility of getting a higher payoff, in order to get a certain payoff.

4.5.2.2.3 Treatment III In this treatment, subjects were matched against both local as well as foreign opponents and as such, we expect the ambiguity perceived by the subjects to be higher in the case of a foreign opponent. See Table 4.14 and Figure 4.14, for a summary of Column Player behaviour in $SC1$ and $SC2$.

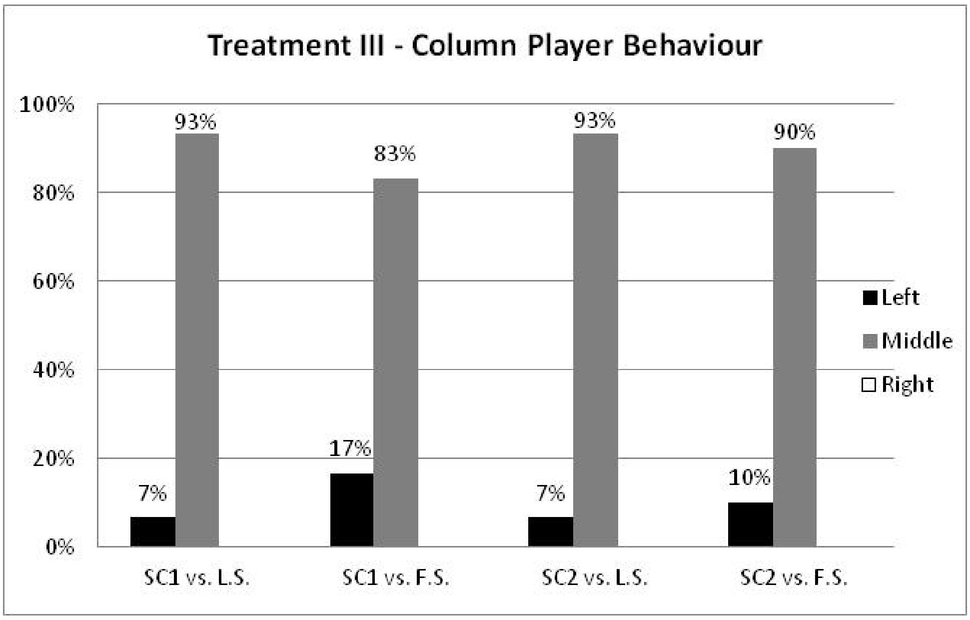
In $SC1$, of the 30 column players took part in this treatment, 93% (28) and 83% (25) of subjects chose M (the Nash strategy) against the local and the foreign opponent, respectively. It is clear that a large majority of the subjects are choosing the Nash; however, fewer subjects choose it against the foreign subject. In $SC2$, 93% (28) and 90% (27) of subjects chose the Nash against the local and foreign subject respectively.

We conducted Binomial Test C and reject the null at a 1% level of significance for both $SC1$ as well as $SC2$, irrespective of whether the subject was faced by a local subject or a foreign one.

Table 4.14. Treatment III - Column Player Behaviour vs. both Local Subject and Foreign Subject

	SC1 vs. L.S.		SC1 vs. F.S.		SC2 vs. L.S.		SC2 vs. F.S.	
Left	2	7%	5	17%	2	7%	3	10%
Middle	28	93%	25	83%	28	93%	27	90%
Right	0	0%	0	0%	0	0%	0	0%
$\sum$	30		30		30		30	

Figure 4.14. Treatment III - Column Player Behaviour vs. Both L.S. and F.S.



In game $SS1$,we find that 70% (21) and 77% (23) of subjects chose the ambiguity-safe strategy R, against the local and foreign subject respectively. In comparison, 27% (8) and 23% (7) of subjects chose M, which is the choice under Nash. In $SS2$, half the subjects (15) chose the ambiguity-safe strategy while the other half chose the Nash against the local opponent. When faced with the foreign opponent, 60% (18) chose the ambiguity-safe option while 40% (12) chose the choice under Nash.

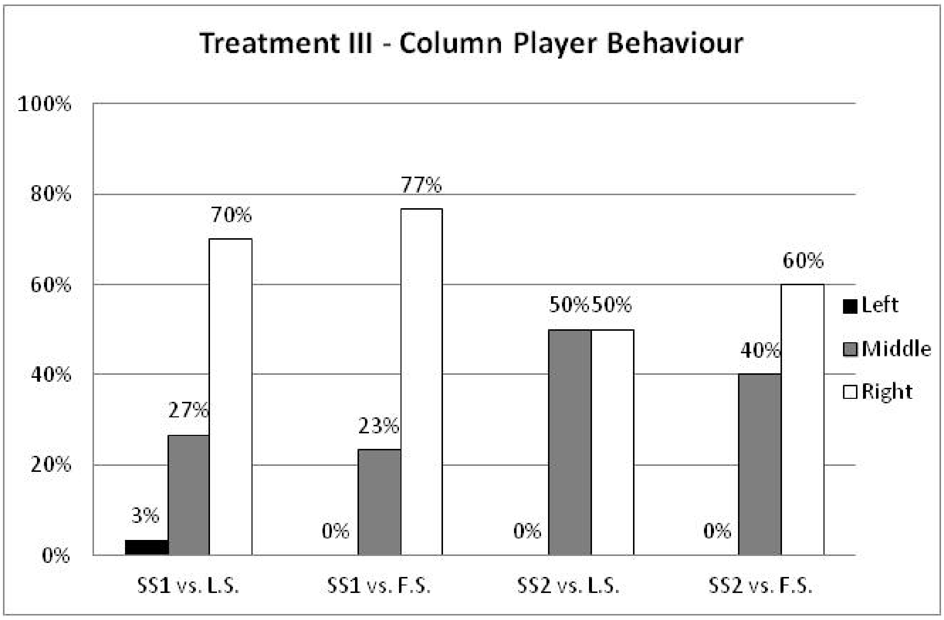
It can be noted that in both the strategic substitutes rounds, the ambiguity-safe option was chosen more often against the foreign subject.

As before, we conduct Binomial Test D and reject the null at a 5% level of significance for decisions against the local opponent and at a 1% level for decisions against an foreign opponent for $SS1$. We fail to reject the null for $SS2$, since the decisions are very close to the $50 - 50$ mark. However, it is clear in both $SS1$ as well as $SS2$, that the ambiguity-safe option is chosen more often against the foreign subject.

Table 4.15. Treatment III - Column Player Behaviour vs. both Local Subject and Foreign Subject

	SS1 vs. L.S.		SS1 vs. F.S.		SS2 vs. L.S.		SS2 vs. F.S.	
Left	1	3%	0	0%	0	0%	0	0%
Middle	8	27%	7	23%	15	50%	12	40%
Right	21	70%	23	77%	15	50%	18	60%
$\sum$	30		30		30		30	

Figure 4.15. Treatment III - Column Player Behaviour vs. Both L.S. and F.S.



4.5.3 Behaviour in Ellsberg Urn Rounds

The strategic complement and substitute games were alternated with Ellsberg Urn decisions, in order to elicit an ambiguity threshold of the subjects. Moreover, it enabled us to evaluate whether the ambiguity of subject remained consistent between single player decision-making situations and situations where they were faced by ambiguity created by interaction with other players.

In the Ellsberg urn rounds, subjects were offered an urn containing 90 balls, of which 30 were labelled X, while the remaining were an unknown mix of Y or Z. Subjects were asked to pick a letter and if this matched the letter of the ball drawn from the urn, they would earn a payoff. The payoff attached to Y and Z balls was 100 ECU, and the payoff attached to X balls was 95, $90, 85, 100$ or 105 ECU, depending on the round being played.

As can be seen in Table 4.16 , when the payoff attached to X was 100 (the standard Ellsberg urn decision problem), a large majority of subjects, i.e., 73% (133) of subjects chose X, while 27% (48) chose to bet on Y and Z.[13] This result is consistent with previous Ellsberg urn studies, with most subjects displaying ambiguity-averse behaviour by choosing X, which is the known proportion of balls in the urn.

When there is a premium attached to X, i.e., the payoff on X is 105 ECU while the payoff for choosing Y or Z is 100 ECU, a majority of 73% (132) of subjects opt for X. However, what is more interesting to note is that 27% (49) of subjects opt for $Y + Z$. This is very interesting because these subjects are willing to take a cut in payoff, in order to choose Y or Z - the balls whose proportion is unknown! We believe this captures ambiguity-seeking behaviour on the part of the subjects.

Table 4.16. Subject Behaviour in Ellsberg Urn Rounds

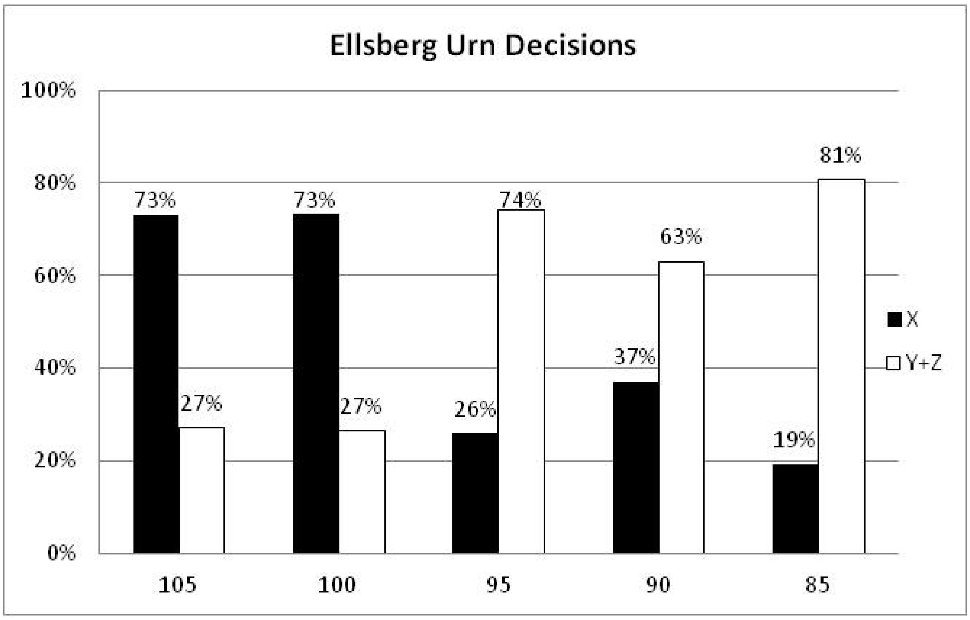
	$X = 105$		$X = 100$		$X = 95$		$X = 90$		$X = 85$	
X	132	73%	133	73%	47	26%	67	37%	35	19%
$Y + Z$	49	27%	48	27%	134	74%	114	63%	146	81%
$\sum$	181		181		181		181		181	

As can be seen from Figure 4.16, even a small cut in the payoff of X from 100 to 95 ECU, leads to a big jump in the number of subjects choosing $Y + Z$. When

[13] We consider the sum of the people who chose Y and Z, rather than the number of people who chose Y or Z balls individually, in order to negate any effect of people choosing Y just because it appeared before Z on the choice set.

$X = 95$, 74% (134) of subjects choose $Y + Z$. This goes up substantially to 81% (146) of subjects choosing $Y + Z$, when the payoff of $X = 85$. Most subjects are not ambiguity-averse enough to take a cut in payoff, in order to continue choosing X. It is interesting to note here that 19% (35) of subjects chose X, even when $X = 85$, thus displaying strong ambiguity-averse behaviour.

Figure 4.16. Subject Behaviour in Ellsberg Urn Rounds



We conducted a binomial test with the null hypothesis that X was chosen as often as $Y + Z$ combined (H_0 : $prob(X) = 0.5$, $prob(Y + Z) = 0.5$), against the alternative that $Y + Z$ was chosen more often (H_1 : $prob(Y + Z) > prob(X)$). We fail to reject the null when $X = 105$ and $X = 100$, at these payoffs subjects choose X significantly more often than $Y + Z$.[14] However, for $X = 95$, 90 and 85, the null is rejected at a 1% level of significance. In these rounds subjects prefer the ambiguous choice, i.e., $Y + Z$ balls.

On the whole, subjects seem to prefer "betting" on Y and Z. Responses gathered from the subjects showed that subjects viewed the urn rounds as "gambles", since the computer could have picked any of the three options and Y or Z balls could have been more in number than X balls, that were capped at 30 balls. The subjects thus displayed an optimistic attitude towards ambiguity, choosing $Y + Z$ rather

[14]This is significant at a 1% level of significance.

than X. Moreover, some subjects treated these rounds as based on luck rather than reasoning.[15]

4.6 Related Literature

4.6.1 Papers on Games

The study that is closest to our experiment in the existing literature, is the theoretical paper by Eichberger and Kelsey (2002). They find that in a game with positive (resp. negative) externalities, ambiguity prompts a player to put an increased (resp. decreased) weight on the lowest of his opponent's actions. The marginal benefit that the player gets as a result of his own action then, gets decreased (resp. increased) in the case of a game with strategic complements (resp. substitutes). In the presence of positive externalities, players often have the incentive to use a strategy below the Pareto optimal level, and so, the resultant Nash equilibrium is inefficient.

In the case of strategic substitutes, increasing the level of ambiguity would cause a shift in equilibrium strategies towards a Pareto efficient outcome, whereas for strategic complements, an increase in ambiguity would cause a shift in equilibrium, away from the ex-post Pareto efficient outcome. Thus it was hypothesised that ambiguity had an adverse effect in case of games with strategic complements, but was helpful in attaining a Pareto efficient outcome in the case of games with strategic substitutes (Eichberger and Kelsey (2002)). Ambiguity thus causes a decrease in equilibrium actions in a game of strategic complements and positive externalities or one that consists of the reverse case, i.e., strategic substitutes and negative externalities.

Di Mauro and Castro (2008) conduct a set of experiments designed to test the Eichberger and Kelsey (2002) hypothesis that it is ambiguity that causes an increase in contribution towards the public good, and not altruism. In order to negate the chance that altruism, or a feeling of reciprocation prompted the subjects' actions,

[15] One subject in particular noted that– "The urn question is pure luck, because majority of the unmarked balls are either Y or Z, and choosing either is a gamble."

the subjects were informed that their opponent would be a virtual agent and the opponent's play was simulated by a computer.

Subjects played in two scenarios, one with risk, the other with ambiguity. It was noted that contributions were significantly higher when the situation was one of ambiguity. The results showed that there was indeed evidence that ambiguity was the cause of increased contribution, as hypothesised by Eichberger and Kelsey (2002), and not altruism. This is akin to the results found in our paper that ambiguity significantly affects the decisions made by individuals, in a manner that depends directly on the strategic nature of the game in consideration.

Another paper that studies strategic ambiguity in games experimentally, is Eichberger, Kelsey, and Schipper (2008). While in our paper we look at subject's behaviour when faced with local and foreign opponents, they studied games in which subjects faced either a granny (who was described as being ignorant of economic strategy), a game theorist (who was described as a successful professor of economics), or another student as an opponent. The key hypothesis being tested was that ambiguity has the opposite effect in games of strategic complements and substitutes.

Ambiguity averse actions were chosen significantly more often against the granny than against the game theorist, irrespective of whether the game was one of strategic complements, strategic substitutes or one with multiple equilibria. When the level of ambiguity the subjects faced while playing the granny was compared to the level of ambiguity the subjects faced playing against each other, it was found that the players still found the granny a more ambiguous opponent.

The paper also tested whether ambiguity had the opposite effect in games of strategic complements and substitutes. Similar to our study where we found that ambiguity had opposite effects depending on the strategic nature of the game, Eichberger, Kelsey, and Schipper (2008) too conclude that comparative statics broadly supported the theoretical prediction. Subjects were also found to react to variations in the level of ambiguity, which was tested by altering the cardinal payoff in the game while keeping the ordinal payoff structure unchanged. It can thus be seen that subjects react not only to ambiguity on the part of the opponent being faced, but also to subtle changes in the payoff structures of the experiment being conducted.

Nagel, Heinemann, and Ockenfels (2009) consider strategic uncertainty in one-shot coordination games with strategic complementarities. In the study conducted by them, they elicit certainty equivalents for situations where a subject's payoff depends on his opponents' behaviour. In each coordination game a subject had a choice between a safe amount X, which was allowed to vary such that $X \leq$ €15, and an option where the payoff was dependant on his opponent's decision. In the uncertain option, a subject could earn €15 if at least a fraction $k \in (0, 1]$ of his opponent's chose the same option as him, else he earned nothing. Subjects were found to choose the safe option when X was large, while they chose the uncertain option for small values of X. The point at which a subject switched from the safe option to the uncertain one, was interpreted as his certainty equivalent for strategic uncertainty. This is analogous to situations in which the risk attitude of a subject is measured with respect to lotteries.

While our study concentrated on investigating individual behaviour in the presence of ambiguity, Keller, Sarin, and Sounderpandian (2007) investigate whether individuals deciding together as pairs (termed dyads in the paper) display ambiguity averse behaviour. Participants were initially asked to state how much they were willing to pay for six monetary gambles. Five of the six gambles put before the subjects involved ambiguity, while the sixth involved no ambiguity.

Once the participants had all disclosed their individual willingness to pay, they were randomly paired with another subject and each pair had to re-specify how much they were willing to pay for the six gambles. It was found that the pairs displayed risk averse as well as ambiguity averse preferences. It was observed that the willingness-to-pay among pairs of individuals deciding together, was lower than the average of their individual willingness-to-pay for gambles. They thus conclude that ambiguity averse behaviour is prevalent in group settings.

In the experiments conducted by us, we did not allow subjects to interact with each other. We believed that any interaction between individuals would reduce the level of ambiguity they would perceive, when asked to make decisions against each other. In contrast, Keck, Diecidue, and Budescu (2012), conduct an experiment in which subjects made decisions individually, as a group, and individually after interacting and exchanging information with others. Subjects were asked to make binary choices between sure sums of money and ambiguous and risky bets.

Keck, Diecidue, and Budescu (2012) found that individuals are more likely to make ambiguity neutral decisions after interacting with other subjects. Moreover, they find that ambiguity seeking and ambiguity averse preferences among individuals are eliminated by communication and interaction between individuals; and as such, groups are more likely to make ambiguity neutral decisions.

Ivanov (2009), discusses the findings of a series of experiments on one-shot normal form games run to distinguish between eighteen types of players. A person was classified on the basis of his attitude to ambiguity - as being either ambiguity averse, ambiguity neutral, or ambiguity loving; on the basis of his attitude to risk - as being risk averse, risk neutral or risk loving; and whether he played strategically or naively. A person who played in a naive manner was modelled as having a uniform belief in every game he played, whereas if he played strategically, his beliefs were different for every game and were thus unrestricted.

Ivanov (2009) finds that about 32% of the subjects taking part in the experiment were ambiguity loving, as opposed to 22% who were ambiguity averse. The majority of subjects (46%) were found to be ambiguity neutral. While being tested on the basis of their attitude to risk, 62% of the subjects were found to be risk averse, 36% to be risk neutral, and a mere 2% were risk loving. 90% of the subjects played in a strategic manner, while 10% played naively. These results are opposite to ours, since we find more subjects who are ambiguity averse than those who are ambiguity seeking, in the game rounds.

The study by Ivanov (2009) questions the fact that there are more subjects who are ambiguity loving/neutral, than those who are ambiguity averse, given that on average a majority of them play strategically. This is attributed to players' altruistic behaviour, i.e., they played in a manner that would maximise the sum of both players' payoffs. This may be because a player is willing to compromise with his opponent, in order to do well himself.

4.6.2 Papers on Weakest Link/Best Shot Games

The weakest link game was introduced by Huyck, Battalio, and Beil (1990). They study tacit coordination in a weakest link game, and conclude that it is unlikely that a payoff-dominant equilibrium would be chosen in a one-shot game or in repeated play. Moreover, when there are a large number of players attempting to coordinate in a repeated game, it results in a secure but inefficient equilibrium being reached.

Our results in the weakest link round are consistent with their conclusions. We find that 59% (142) of subjects chose an effort level in the range $100 - 120$, which would result in a payoff-dominated equilibrium being reached. Furthermore, even though our game consisted of only two subjects coordinating (and not a large number of players), we found that 21% (51) of subjects chose an effort level of 100, which would have resulted in a secure but inefficient equilibrium.

Harrison and Hirshleifer (1989), compare contribution to a public good in a sealed bid as well as a sequential game. They implement all three possible versions of the game - standard summation, weakest link and best shot, in order to ascertain which of the three formats results in the greatest free-riding. They find that both sealed bid as well as the sequential game treatments, confirmed their hypothesis that the underprovision of the public good expected under the standard summation format, is mitigated under the weakest link format, but aggravated under best shot version.

The conclusions of our study are in direct contrast to those of Harrison and Hirshleifer (1989). Our hypothesis was that individuals would reduce their effort levels in the weakest link game (i.e., more free-riding) and increase their effort levels in the best shot game (i.e., less free-riding). We found that 55% (33) and 46% (28) of subjects in Treatment I and II switched from a low effort level in the weakest link round to a higher effort level in the best shot game (Please see Table 4.5). Thus, our findings our opposite to those of Harrison and Hirshleifer (1989), who found that the underprovision of the public good is greater in the best shot format and lower in the weakest link format.

Goeree and Holt (2001) studies, a set of games which initially conform to Nash predictions when tested experimentally. However, they note that a small change in payoffs leads to a large change in observed subject behaviour and Nash predictions. In particular, they study a minimum effort coordination game where subjects could choose an effort from the set $E = \{110, ..., 170\}$ at a marginal cost of either $c = 0.1$ or $c = 0.9$.

Recall that the Nash equilibrium of this game is that subjects coordinate on the same effort level $\{(e_1^*, e_2^*) \in E^2 \mid e_1^* = e_2^*\}$. Goeree and Holt (2001) find that for low marginal costs ($c = 0.1$), subjects choose high effort levels and for high marginal costs ($c = 0.9$), a majority of subjects choose low effort levels. They conclude that this concentration of subject choice at the lower (resp. higher) end of the effort spectrum is caused by the high (resp. low) marginal cost of the effort.

In our games, we use $c = 0.5$, and find that subjects' effort choices do not depend on the cost of the effort, but on the effect of ambiguity, given the strategic nature of the game being played. This can be seen in Table 4.5, where even though the marginal cost of the effort is constant at $c = 0.5$, subjects switch their effort levels depending on whether it is a weakest link or a best shot game.

Eichberger and Kelsey (2011) provide further arguments based on ambiguity, to explain the concentration of observations at the lower (and higher) end of the spectrum found by Goeree and Holt (2001). The best outcome in a minimum effort game is for both subjects to choose the highest effort level. Consider a scenario where both players are coordinating on an effort level other than the lowest. Each of the players can increase his marginal benefit by $\delta a + (1 - \delta)$ by reducing 1 unit of contribution, thereby saving on marginal cost c. Thus, if $c > \delta a + (1 - \delta)$, it would be rational to reduce contributions to the lowest possible level.

If however they do not coordinate in this manner, both players could increase their marginal benefit by δa, at a marginal cost of c.[16] Hence, it would be rational to increase contributions to the maximum possible effort level, if $\delta a > c$. However, ambiguity reduces the perceived marginal benefit of increasing one's effort, since

[16] δ and α are as defined in Section 4.2.1.

the benefit from the effort would only be received in the ambiguous event that one's opponent plays a high strategy as well.

Moreover one can note that, if α and δ lie in the range given by Kilka and Weber (2001), such that $0.38 \geq \delta a \geq 0.16$, the prediction made by Eichberger and Kelsey (2011) would explain subject behaviour as observed by Goeree and Holt (2001), reasonably well.

4.6.3 Papers on Ellsberg Urns

The Ellsberg urn experiments conducted by us investigated whether there was any correlation between ambiguity-averse behaviour in the game rounds and ambiguity attitude in single person decision problems. Moreover, we wanted to evaluate whether there was any threshold at which individuals switched from being ambiguity averse to being ambiguity neutral (or seeking) in their preferences.

Eliaz and Ortoleva (2011), study a three-colour Ellsberg urn with increased ambiguity, in that the amount of money that subjects can earn also depends on the number of balls of the chosen colour in the ambiguous urn. The subjects thus face ambiguity on two accounts: the unknown proportion of balls in the urn as well as the size of the prize money.

In their experiment, both winning and the amount that the subject could possibly win were both perfectly correlated - either positively or negatively, depending on which of the two treatments was run by them. In the experiment, most subjects preferred betting in the positively correlated treatment rather than the negative one. Moreover, subjects also showed a preference for a gamble when there was positively correlated ambiguity, as opposed to a gamble without any ambiguity. This behaviour of the subjects, is compatible with our findings that subjects preferred betting on Y/Z where there was ambiguity, rather than on X, the known choice.

Binmore, Stewart, and Voorhoeve (2011), attempt to test whether subjects are indeed ambiguity averse. They investigate whether the apparent ambiguity averse behaviour, predominantly reported by a number of papers, can be captured by the Hurwicz criterion. They report that subject behaviour in experiments conducted by them is inconsistent with the Hurwicz criterion. Instead, they find that the principle of insufficient reason has greater predictive power with respect to their data, than ambiguity averse behaviour.

Our results are consistent with the findings of Binmore, Stewart, and Voorhoeve (2011), since we find that subjects are not willing to pay even a moderate penalty to avoid ambiguity in the Ellsberg urn rounds where the payoff attached to X were $95/90/85ECU$. This might be because in the absence of information, subjects use the principle of insufficient reason and attach a $50:50$ probability to the remaining 60 Y and Z balls left in the urn. The principle of insufficient reason would imply that the probability distribution attached to the X, Y and Z balls in the urn is $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. It would thus be rational to choose Y or Z and get a payoff of $100ECU$, than to choose X and suffer a penalty, i.e., get payoffs $95/90/85ECU$.

In our experiments we did not allow the subjects to communicate or interact with each other. Charness, Karni, and Levin (2012), test whether individuals display a non-neutral attitude towards ambiguity, and given a chance to interact, can subjects persuade others to change their ambiguity attitude. They find that though a number of their subjects displayed an incoherent attitude towards ambiguity, a majority of subjects displayed ambiguity neutral preferences.

A small minority of subjects (smaller than the number of subjects who were ambiguity-incoherent) displayed ambiguity averse and ambiguity seeking behaviour. More interestingly, they find that if subjects are allowed to interact with each other, given the right incentives, ambiguity neutral subjects often manage to convince ambiguity seeking and ambiguity incoherent subjects to change their mind and follow ambiguity neutral behaviour (Charness, Karni, and Levin (2012)).

Halevy (2005), extends the standard Ellsberg type experiment to demonstrate that ambiguity preferences are associated with compound objective lotteries. The study finds that the subject pool can be divided into those who are ambiguity neutral and reduce compound objective lotteries, i.e., they have behaviour which is consistent with SEU; and those who fail to reduce compound lotteries. The latter category of individuals display different preferences over ambiguity and compound lotteries, and are consistent with models that capture ambiguity seeking/averse preferences.

It is concluded (in the study), that there is no unique theory that can capture all the different preference patterns observed in a given sample.

As such, the experimental findings of Halevy (2005) are consistent with Epstein (1999), where ambiguity aversion is defined as a behaviour that is not probabilistically sophisticated and thus cannot be aligned with a specific functional form or model.

4.7 Conclusions

Subject behaviour was found to be consistent with our hypothesis. We find that in the presence of ambiguity, subjects choose low effort levels in the weakest link game and high effort levels in the best shot round. Moreover, we find that on average, 51% (61) of the subjects who took part in Treatments I and II, display ambiguity averse behaviour[17]; 29% (35) of subjects display ambiguity seeking behaviour[18]; and 20% (25) of subjects do not change their chosen effort levels between the two rounds.

In the coordination games we find that subjects do indeed choose the equilibrium action under ambiguity more often than either of the other actions available to them. Thus, on the whole subjects display ambiguity averse preferences when making decisions in two-person game scenarios. We expected the subjects to display a greater level of ambiguity-averse behaviour when faced by a foreign opponent. However, though we observe ambiguity averse behaviour on the whole in the games, we fail to see an escalation in the level of ambiguity when subjects are faced with foreign opponents.

This is quite a curious finding, as one would expect that the ambiguity-safe option would be chosen more often against the foreign subject and not otherwise. Moreover, it is interesting to note that our findings are opposite to those of Kilka and Weber (2001), who found that subjects are more ambiguity-averse when the

[17] These subjects initially choose a low effort level in the weakest link game followed by a higher effort level in the best shot game.

[18] These subjects initially choose a high effort level in the weakest link game followed by a lower effort level in the best shot game.

returns of an investment are dependent on foreign securities than when they are linked to domestic securities.

Nonetheless, there may be several reasons that might explain why the level of ambiguity when facing a foreign subject may remain unchanged. One of the reasons for subjects not choosing a more ambiguity-safe action against the foreign subject than against the local subject (in Treatment III), may be that they wanted to be dynamically consistent in their choices. If this were the case, they would put extra effort into choosing the same action against both opponents.

In addition, one can note that decisions regarding financial markets are much more complex than the act of dealing with other people. This may explain part of the heightened "ambiguity" captured by Kilka and Weber (2001), where the subjects were asked to choose between an investment dependent on foreign securities or one linked to domestic securities. It is easier for subjects to conceptualise another person whom they may be faced against, rather than investments in known/unknown financial markets. Follow-up experiments may be run, where subjects are given a choice of whether they would like to face a foreign opponent in a coordination game, or invest in a foreign security.

Eichberger, Kelsey, and Schipper (2008), found that subjects were more ambiguous about the behaviour of the granny, as opposed to that of the game theorist. The level of ambiguity when faced by another student, was similar to the level of ambiguity when faced by the granny. Given the reaction of these subjects, we were suprised to see that our subjects did not react with as much ambiguity to the foreign opponent. We believe that this might be because in the Granny Experiment, the game theorist provided a stark contrast in terms of rationality to the granny. In comparison, in our experiment we only had an agent providing ambiguity (the foreign opponent) to the situation, but no agent to provide the analogy to the game theorist. It might be worth introducing an analogy to the game theorist, in future experiments to check if this causes any change in behaviour on the part of the subjects.

Another reason for subjects choosing the same action against both foreign and local opponents, may be that some students were afraid that if they chose a different option against the foreign subject, they might appear racist.[19] In an attempt to appear fair, subjects may have chosen the same option against both opponents. We could avoid this complication in future experiments, by comparing different groups of a similar race, such as African-Americans and Africans.

Moreover, it may have been the case that subjects viewed the foreign student as akin to any other local student. This is not that difficult to understand. Globalisation and increased media awareness, together with the spreading tentacles of social networking and escalating international student numbers, have ensured that a foreign subject (in this case those from India) is not an unknown quantity any more. There are not many parts of the world, that hold the kind of ambiguity for us today, as there were in the past.

In future experiments, we could have treatments where subjects are allowed to choose which opponent they would like to face, local or foreign. Furthermore, we could check if they are willing to pay a penalty in order to avoid facing the foreign opponent. It would be interesting if subjects were willing to pay a penalty to avoid an ambiguous foreign opponent, since we find little evidence of willingness to pay a penalty, to avoid the ambiguous balls in the Ellsberg urn experiments conducted by us.

In the Ellsberg Urn rounds we find that for $X = 105$ and $X = 100$ subjects prefer to opt for X rather than Y or Z, but even the smallest reduction on the payoff attached to X leads to subjects choosing Y or Z (which is the ambiguous choice). When the payoff attached to X was 95, 90, or 85, $Y + Z$ was chosen significantly more often than X. We notice that the subjects are unwilling to bear even a small penalty in order to stick with X balls (the unambiguous choice).

Thus, even though subjects displayed ambiguity averse preferences when faced by other opponents (whether local or foreign), they displayed ambiguity seeking

[19] This was part of an overheard conversation between subjects, who were talking to each other at the end of the experiment.

preferences in the single-person decision situations. This is consistent with our earlier study (Kelsey and Le Roux (2012)), where we found that the ambiguity-attitude of subjects was dependent on the scenario they were facing. It might be interesting to elicit subjects' preferences on whether they would like to face an opponent or an Ellsberg urn.

It is our belief that subjects find it more ambiguous to make decisions against other people than against the random move of nature, over which everyone is equally powerless. This might even explain why people are more concerned with scenarios involving political turmoil or war - situations dependent on other people, but appear to discount the seriousness of possible natural disasters - which are beyond anyone's control.

CHAPTER 5

DEVIATIONS FROM EQUILIBRIUM IN AN EXPERIMENT ON SIGNALLING GAMES

5.1 Introduction

In this chapter we provide a summary of results of two series of experiments that were run based on a modified signalling game. The experiments were computer-based, such that the games were presented graphically to the subjects on a screen. The design for the initial experiment was selected by Reinhard Selten. It has the interesting property that the strategically stable outcome (Kohlberg and Mertens (1986)) does not coincide with the outcome of the Harsanyi-Selten solution (Harsanyi and Selten (1988)). However, it is a complex game insofar as standard refinement concepts like the intuitive criterion, or the never-a-weak-best-response criterion, do not help to refine among the equilibria. The second motive for the design was to analyse, how the change in the reward at a particular terminal node would affect behaviour.

In the initial set of experiments, we found that the strategically stable equilibrium is never a good description of the data. A strategically stable equilibrium is one that satisfies the conditions of backward and forward induction, iterated dominance and invariance.[1] While behaviour in some of the sessions converged to the Harsanyi-Selten outcome, there were systematic deviations from the equilibrium behaviour.

Casual observations and discussions with participants suggested that a "*collective reputation*" effect[2] might be at work within the random matching framework in

[1]Invariance suggests that the stable sets of a game, that are selected by backward/forward induction and iterated dominances, are also projections of the stable sets of a larger game in which it could be embedded. In our game this would mean that the stable set of a signalling game T, is also the stable set of a bigger signalling game (T') in which it has been embedded. As such, this ensures that the feasibility of players' strategies is preserved.

[2]This term is credited to Reinhard Selten.

which our basic games were played. The term "*collective reputation*" basically means that subjects in the role of one player, abstain from a certain action which is in their short run interest (but would harm their opponent), in order to allow for coordination on a mutually beneficial outcome. They thus forego a short run gain, for a long term gain that accrues to both players.

Moreover in the experiments, subjects were not only given the result of their own play, but were also given information about all the other parallel plays that were conducted simultaneously. Our hypothesis was that a reduction of this information would make it harder to build up a collective reputation.

In the second set of experiments we modified the initial signalling game to make the mutually beneficial outcome more attractive and hence give a stronger incentive to build a collective reputation. In addition, we varied the information on past outcomes given to subjects. It was conjectured that more information would make it easier to coordinate on the mutually beneficial outcome. However, though we do find systematic violations from equilibrium behaviour similar to those in the initial series of experiments, we do not find evidence that varying the amount of information affects play.

5.1.0.0.4 Organisation of the Chapter In Section 2, we provide a brief review of previous literature on signalling games and refinement criteria. Section 3 describes the signalling games being tested in the experiments and their normative solutions. Section 4 describes the experimental design employed, Section 5 consists of data analysis and results, Section 6 provides alternative equilibrium concepts that might explain some of the observed player behaviour and Section 7 summarises the results and conclusions.

5.2 Previous Literature

5.2.1 Signalling Games and Refinement Criteria

Signalling games have been used to study core strategic issues that arise due to the economics of information, in particular the case where there is asymmetric information. General equilibrium theories break down in the wake of asymmetric information. When parties taking part in an exchange of goods and services have unequal access to information, equilibrium-forming market mechanisms get upset. In particular, Michael Spence's model of job market signalling was seminal in studying signalling games with multiple equilibriums, where the equilibrium reached would determine whether the market was efficient or inefficient (See Spence (1973)).

Signalling games model economic scenarios characterised by asymmetric information. In a standard signalling game, there are two parties. One of the parties is informed about the prevailing state of nature, while the other party remains uninformed. The informed party must take an action which is observed by the uninformed party, who then draws certain inferences from the observed action and responds by taking a suitable action of his own.

A formal signalling game may be described by the following rules:

1. Nature draws a type t of Player 1 from a finite set T, according to some probability distribution π, such that $\pi(t) > 0$, for all t.

2. Player 1, is informed about the type of player t that he is and selects a message m, from a finite set M.

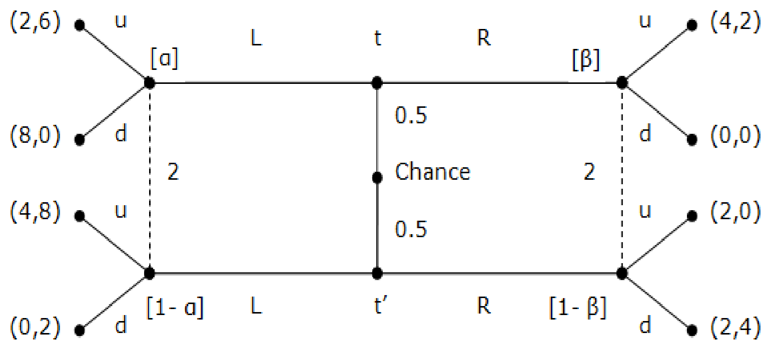
3. Player 2, does not observe the type of player t, but does observe the message sent out by him, i.e., Player 2 can observe m. On the basis of this observation, he selects a response a, from a finite set A.

4. The payoffs to Players 1 and 2 are $u(t, m, a)$ and $v(t, m, a)$, respectively (van Damme (1991)).

The rules of the game are assumed to be common knowledge to both players, such that both players know the sextuple (T, M, A, π, u, v), but asymmetry arises since only Player 1 knows his type, Player 2 does not (van Damme (1991)). Such a game is called a signalling game, since the action of Player 1 acts as a signal of his type to Player 2, on the basis of which, Player 2 develops certain beliefs about the type of Player 1 he faces.

An example of a signalling game can be seen in Figure 5.1, where Player 1 is informed of nature's move, while Player 2 is ignorant of it. Player 2's beliefs are

Figure 5.1. Signalling Game (Peters (2008))



shown by the numbers between the square brackets at his decision nodes (Peters (2008)). There can be two types of Player 1 in the game, t and t', depending on chance's move, where each type occurs with a 50% probability. Each of the players has four strategies. The strategy set of Player 1 is $\{LL, LR, RL, RR\}$, whereas the strategy set of Player 2 is $\{uu, ud, du, dd\}$, where L, R, u and d represent the decision to go left, right, up or down respectively. The strategic form of the game is seen in Table 5.1, where the best replies of a given player have been marked with an asterisk. The two pure strategy Nash equilibria that emerge as a result of this analysis are (RL, uu) and (LL, ud).

Table 5.1. Strategic Form of Signalling Game

	uu	ud	du	dd
LL	$3, 7^*$	$3^*, 7^*$	$1, 4$	$4, 1$
LR	$2, 3$	$2, 5$	$5^*, 0$	$5^*, 2$
RL	$4^*, 5^*$	$2, 4$	$2, 2$	$0, 1$
RR	$3, 1$	$1, 2^*$	$3, 1$	$1, 2^*$

The Nash equilibrium (RL, uu), is only consistent if $\alpha = 0$ and $\beta = 1$. Assuming these beliefs, uu would be the best reply for Player 2. Hence, (RL, uu) is a perfect Bayesian equilibrium[3], provided $\alpha = 0$ and $\beta = 1$. (RL, uu) is a *separating equilibrium*. Each type of Player 1, plays a different action and hence, (RL, uu),

[3] A perfect Bayesian equilibrium requires that the players' beliefs should be such that:

1. players act rationally given their beliefs
2. the conditional probabilities attached to the nodes in an information set are consistent with the combination of strategies being considered.

In the games we consider here, refinement concepts of perfect Bayesian Nash equilibrium, sequential equilibrium and perfect equilibrium all coincide (Fudenberg and Tirole (1991)) separates or distinguishes between the two types of Player 1. The equilibrium action of Player 1 thus signals his type, and the equilibrium is '*information revealing*' (Peters (2008)).

The other Nash equilibrium (LL, ud) is consistent with the belief that $\alpha = \frac{1}{2}$. This can be seen in that each type of Player 1 plays L, so that Player 2 assigns each decision node in the left information set a probability of $\frac{1}{2}$. Since $\alpha = \frac{1}{2}$, u is the best reply to Player 1 that Player 2 can make in his left information set. On the other hand, the beliefs $(\beta, 1 - \beta)$ have not been restricted since the probability of the right information set being reached in equilibrium is 0. The beliefs regarding β should however be modelled such that, at Player 2's right information set, d is the optimal response. The expected payoff of Player 2, resulting from playing d should be at least equal to the expected payoff he would receive were he to play u, so $4(1 - \beta) \geq 2\beta$ or $\beta \leq \frac{2}{3}$. Thus, with $\alpha = \frac{1}{2}$ and $\beta \leq \frac{2}{3}$, (LL, ud) is a *pooling equilibrium* as it pools both types of Player 1, without providing any type-relevant information about his specific type (Peters (2008)).

Literature on game theory and signalling games includes other refinement criteria as well. One such restriction put forth by Cho and Kreps (1987) is the *intuitive criterion*. The intuitive criterion considers a signalling game with a perfect Bayesian equilibrium. The Cho-Kreps intuitive criterion holds that if one type of Player 1 could not possibly improve his equilibrium payoff by deviating, while it might be possible for another type of Player 1 to gain by deviating, then it would be sensible to believe that the deviation from equilibrium is made by the type who stands to gain.

Refinement concepts such as the intuitive criterion are useful while building receiver beliefs in response to unexpected behaviour. The only scenario in which the intuitive criterion does not place restrictions on the beliefs of Player 2, is when all the possible types of Player 1 get excluded. Additionally, the intuitive criterion lacks bite when analysing a separating equilibrium (like the (RL, uu) separating equilibrium discussed in the example above), since in this case the beliefs of Player 2 are wholly determined by the optimal equilibrium actions of Player 1.

The intuitive criterion can be applied to the pooling equilibrium (LL, ud), of the previous example. The equilibrium payoff of Player 1-type t is 2. Player 1 has the incentive to deviate by picking R, since he would earn a higher payoff of 4 by doing so. The equilibrium payoff of Player 1-type t' is 4 and such a Player 1 would get at most 2 if he were to deviate and pick R instead. According to the intuitive criterion, Player 2 would assign zero probability to Player 1-type t' deviating to R. The intuitive criterion thus implies that $\beta = 1$, in which case (LL, ud) can no longer be a pooling equilibrium.

Camerer (2003) provides a critique of the intuitive criterion, in that it ties the possibility of a deviator earning an out-of-equilibrium payoff (which is higher than the equilibrium payoff), with the plausibility of a Player 2's beliefs about which type of Player 1 will deviate. However, the intuitive criterion fails to give a definitive answer in case more than one type of Player 1 gains by deviating. One type of Player 1 may have a greater incentive to deviate if the response of Player 2 to the deviation makes it more lucrative for him to deviate. The intuitive criterion is silent when it comes to analysing which type of Player 1 is more likely to deviate.

Banks and Sobel (1987) introduce the concept of *divinity,* which is the property of one type of Player 1 having a greater incentive to deviate than the other. Divinity requires that Player 2's beliefs should assign a greater weight to the type of Player 1, whose deviation-supporting belief set is larger. In other words, while the intuitive criteria simply divides the set of types of Player 1 into those that might deviate and those that would never deviate – divinity divides the set of those who might deviate into the those that deviate more often than the others. Divinity thus requires more reasoning than the intuitive criterion. *Universal divinity* takes the reasoning a step further by concluding that if the response to deviating is much higher for one type of Player 1 than another, Player 2 must believe that the deviation came from the more likely type, with complete certainty.

Another refinement technique is the *never-a-weak-best-response* (NWBR) criterion. The rationale of the NWBR criterion is that if it is not possible to apply universal divinity, because the set of Player 2's responses does not make it strictly more profitable for one type of Player 1 to deviate more than the other, there would still be one type of Player 1 for whom the deviation payoff was just as profitable as the equilibrium payoff, while the other type of Player 1 preferred the equilibrium payoff to the deviation payoff (Camerer (2003)).

The last refinement to be discussed here is *stability*. Roughly speaking, a stable equilibrium requires that there is an equilibrium close to the candidate equilibrium, for every possible *tremble*[4] of strategies. The stability concept guarantees a stable equilibrium and is the closest that game theorists come to finding a Holy Grail theory.

5.2.2 Experiments on Signalling Games

Laboratory experiments provide empirical data and insights that throw a whole new light on questions that arise in game theory. It would be difficult, if not near impossible, for a situation to arise naturally in an economy, where one could observe a sequential, pooled, or separating equilibrium being formed. On the other hand, it is possible to use financial rewards during experiments, in a way that would motivate participants to reach an equilibrium.

Previous experimental work on signalling games concentrated on the predictive power of refinement concepts (See Brandts and Holt (1992), Brandts and Holt (1993), Banks, Colin, and David (1994)). The analysis in these papers concentrated on pure strategy equilibria, but in our case the strategically stable equilibrium is mixed. Brandts and Holt (1992), argue that deciding which refinements are appropriate while analysing any given signalling game, can only be determined on the basis of subjective opinion regarding the rationality of individuals taking part in the game. Empirical work that tests the validity of such arguments would pave the way towards more advanced and streamlined refinements.

Banks, Colin, and David (1994), conducted a series of experiments that aimed at testing whether subjects chose refined subsets of Nash equilibria in signalling

[4] A tremble takes place when a player who is faced with a number of actions to choose between, decides to take a particular action, but through inattention or a slip of the hand/pen/tongue takes another action instead (Kreps (1990)).

games. The experiments consisted of simple signalling games where the sender was informed of a randomly-drawn type and then chose a message. The receiver had information about the message but not of the sender's type, and had to choose an action. The experiment showed that about 70% of the message-action pairs chose Nash equilibrium outcomes. Subjects had a tendency to choose a more refined equilibrium in some games, but no specific refinement could be predicted. In some games where a pooled equilibrium was predicted, senders preferred to separate rather than pool, since it gave them a higher payoff.

Banks, Colin, and David (1994) noted that no single, simple decision criteria such as minimax or principle of insufficient reason could explain why senders chose non-Nash and unrefined messages. However, when several criteria select a particular message, senders picked it out about 90% of the time. The conclusion they arrived at was that if equilibria are consistent with several different criteria, they were more likely to be played. In our experiments we find that behaviour differs systematically from the Nash equilibria of the game and cannot be explained by any one simple decision criteria.

Brandts and Holt (1996), believe that as economists we should take the process of learning and adjustment towards equilibrium seriously. In their paper they use adjustment theories to model naïve Bayesian learning in signalling games, where subjects learn and adapt in an unfamiliar environment. They find that when standard equilibrium assumptions fail to offer explanations for behaviour patterns observed during experiments, computer simulations of Bayesian learning and adjustment can prove to be useful. More recent experiments study how changing a game or deciding in teams affects behaviour in signalling games can be seen in Cooper and Kagel (2003) and Cooper and Kagel (2005). These papers find that teams play more strategically than individuals especially when there are changes in payoffs that change the equilibrium outcome. Moreover, they find that teams exhibit positive learning transfer far more than individual subjects. In our experiment we test to see whether subjects undergo any learning and though we found some evidence for learning, it was not strong.

5.3 Experimental Model

In this section, we shall explain the games used in the experimental sessions and discuss the Nash equilibrium of each game. Henceforth we will use male pronouns he, his etc. to denote Player 1, while female pronouns she, hers etc. denote Player 2.[5]

Figure 5.2. Game S

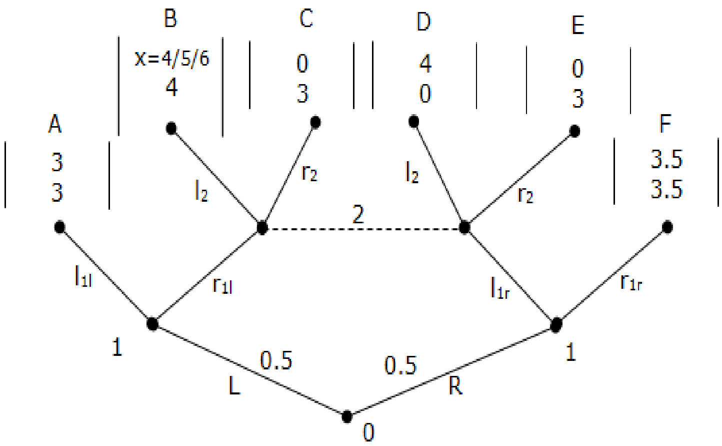
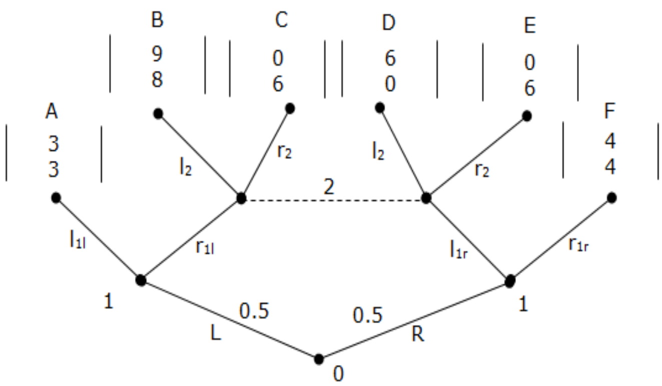


Figure 5.3. Game T



The experiments are based on the two signalling games shown in Figures 5.2 and 5.3. Both signalling games have the following structure: The two players have a choice between a strategically safe and a strategically risky option. The game is one of incomplete information in which Player 1 can be of two possible types, $1a$ or $1b$, with equal probability.

[5]This convention is for the sake of convenience only and does not bear any relation to the actual gender of the subjects in our experiments.

Player 1 chooses first, followed by Player 2. Player 1 can either end the game (the strategically safe option) or give the move to Player 2 (the strategically risky option). Player 2 can then choose between a strategically safe option which gives type-independent payoffs and a strategically risky option, which gives type-dependent payoffs.

In the first set of experiments (related to the S game), we varied the payoff "x" of type $1a$ at the terminal node B. The value attached to x could be $4, 5$ or 6.As can be seen from Figure 5.2, Player 1 would only take the strategically risky option if Player 2 does so as well. Player 2 would like to take the strategically risky option only if she faces type $1a$ and the strategically safe option against type $1b$.

5.3.1 Normative Analysis

In this section, we work exclusively with behaviour strategies. Both games have two Nash equilibrium components. The first component consists of Nash equilibria where both types of Player 1 take the strategically safe option and Player 2 chooses the strategically safe option with a sufficiently high probability, namely at least with probability $\frac{x-3}{3}$ in Game S (with $x = 4, 5, 6$) and at least probability $\frac{2}{3}$ in Game T. This component contains, the Nash equilibrium where all players and types take their strategically safe option with certainty. The latter is uniformly stable and can be shown to be the equilibrium selected by the theory of Harsanyi and Selten (1988).

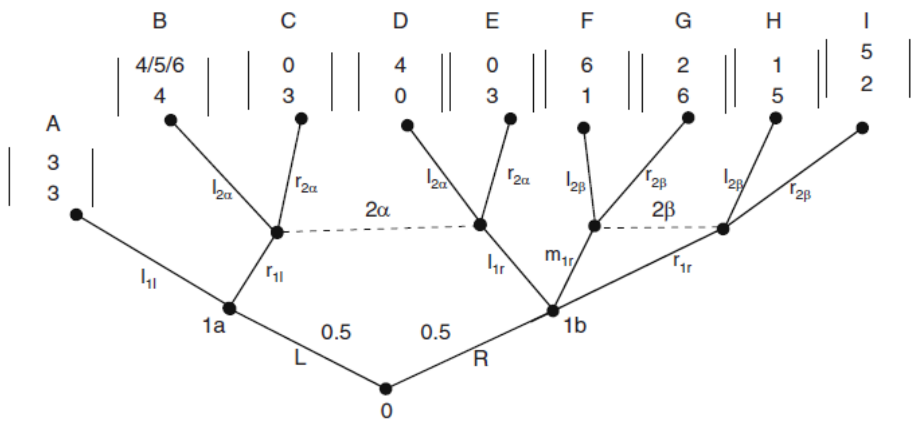
The second component consists of a single equilibrium where Player $1a$ takes the strategically risky option with certainty, while type $1b$ and Player 2 randomise. Thus, in Game S type $1b$ chooses the strategically safe option with probability $\frac{2}{3}$ and Player 2 chooses the strategically safe option with probability $\frac{1}{8}$. In Game T type $1b$ chooses the strategically safe option with probability $\frac{2}{3}$ and Player 2 chooses the strategically safe option with probability $\frac{1}{3}$. Conditional on her information set being reached, Player 2 believes she faces type $1b$ with probability $\frac{1}{4}$. This equilibrium component can be shown to be the only strategically stable component of Nash equilibria in the sense of Kohlberg and Mertens (1986).

The purpose of the first set of experiments (Game S) was to test the two equilibrium refinements against each other. We expected the Harsanyi-Selten solution to arise for the parameter value $x = 4$, but did not rule out that terminal node B would be reached more often if x was increased. In the new version (Game T), it was more attractive for both types of Player 1 to choose the strategically risky choice (but we made it more attractive for type $1a$ than for type $1b$). We hence expected that Player 2's information set would be reached substantially more often in the new experiment.

5.3.2 The extended games

For most part of the experiments we used the extended models, S' and T' (See Figures 5.4 and 5.5), which were modified versions of the basic games S and T, respectively. In essence type $1b$'s strategically safe option was replaced with a 2×2 game with a unique equilibrium, which had the same expected payoffs as the strategically safe option in the basic game.[6]

Figure 5.4. The Game S'



In the game S', we used a 2×2 game with unique mixed strategy equilibrium, where both players choose *Right* with probability $\frac{5}{8}$. In the game T', we used a prisoner's dilemma game where *Right* was the dominant strategy for both players. Since the strategically safe choice of type $1b$ is reached with positive probability in the Nash equilibria of both basic games S and T, the Nash equilibria of the extended games are obtained by replacing the strategically safe strategy of type $1b$ with the

[6]The 2×2 game was added following the strategically safe choice of type 1b, but then moves were coalesced.

equilibrium strategy from the 2×2 game. In addition, Player 2's behaviour strategy is amended with her choice at the new information set. See Table 5.2.

Figure 5.5. The Game T'

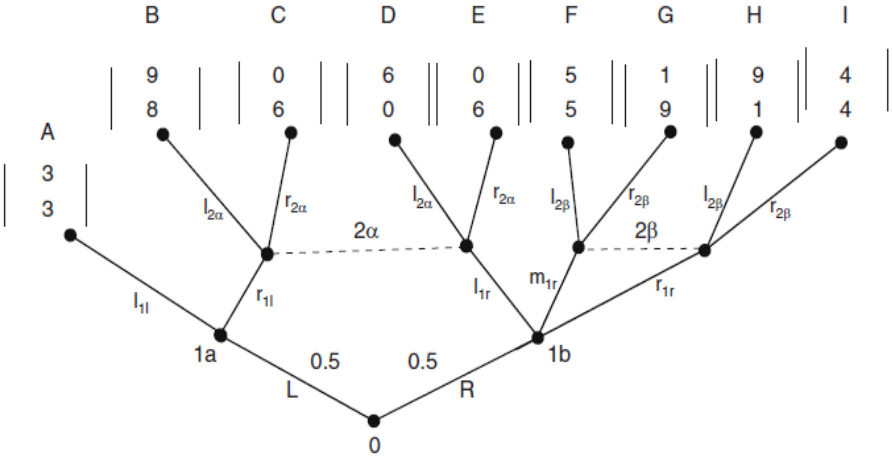


Table 5.2. Probabilities in the Nash equilibriat

	r_{1l}	m_{1r}	r_{1r}	$r_{2\alpha}$	$r_{2\beta}$
First Component Game S'	0	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{x-3}{x}$	$\frac{5}{8}$
Second Component Game S'	1	$\frac{3}{12}$	$\frac{5}{12}$	$\frac{1}{8}$	$\frac{5}{8}$
First Component Game T'	0	0	1	$\frac{2}{3}$	1
Second Component Game T'	1	0	$\frac{2}{3}$	$\frac{1}{3}$	1

5.3.3 Normative Analysis with Trembling Refinements

A tremble takes place when a player who is faced with a number of actions to choose between, decides to take a particular action, but through inattention or a slip of the hand/pen/tongue takes another action instead (Kreps (1990)). The normative analysis with trembling refinements for both Games S and T is described below.

5.3.3.1 S-Game

Let $\varepsilon_{i,s}$ be the trembling probability of agent i for action s. We require $\varepsilon_{i,l}+\varepsilon_{i,r} < 1$ for all i. Moreover, $\varepsilon_{2,s} < \frac{1}{8}$ and so on. Let the probability of type $1a$ and type $1b$ taking action right be p and q respectively. The probability that type $1a$ trembles and chooses left thus is $\varepsilon_{1a,l}$. The probability that type $1b$ trembles and chooses left is $\varepsilon_{1b,l}$. The probability of Player 2 taking action right is ρ.

Player 2 is indifferent when type $1a$ chooses r with maximal probability $1-\varepsilon_{1a,l}$, such that: $\frac{\frac{1}{2}\times\left(1-\varepsilon_{1a,l}\right)\times 4}{\frac{1}{2}\times\left(1-\varepsilon_{1a,l}\right)+\frac{1}{2}(1-q)} = 3 \Leftrightarrow 4\left(1-\varepsilon_{1a,l}\right) = 3\left(2-\varepsilon_{1a,l}-q\right)$ and $q = \frac{2+\varepsilon_{1a,l}}{3}$.

Player 2 is indifferent when type $1b$ chooses l with probability $\varepsilon_{1b,l}$, such that: $\frac{\frac{1}{2}\times 4\times p}{\frac{1}{2}p+\frac{1}{2}\varepsilon_{1b,l}} = 3 \Leftrightarrow 4p = 3\left(p+\varepsilon_{1b,l}\right)$ and $p = 3\varepsilon_{1b,l}$.This is possible only when $\varepsilon_{1a,r} \leq 3\varepsilon_{1b,l}$, for instance when $\varepsilon_{1a,r} = \varepsilon_{1b,l}$.

Type $1a$ is indifferent if[7]: $3 = (1-\rho)\,x \Leftrightarrow \rho = 1-\frac{3}{x}$, while Type $1b$ is indifferent if: $4(1-\rho) = 3.5 \Leftrightarrow 0.5 = 4\rho \Leftrightarrow \rho = \frac{1}{8}$

The Trembling Equilibria thus is:

1. Player $1a$ goes right with maximal probability. Player 2 and Player $1b$ mix to make each other indifferent. This equilibrium exists for all sufficiently small perturbations:

$$\left\{\varepsilon_{1a,l}l + \left(1-\varepsilon_{1a,l}\right)r, \left(1-\frac{2+\varepsilon_{1a,l}}{3}\right)l + \frac{2+\varepsilon_{1a,l}}{3}r, \frac{7}{8}l + \frac{1}{8}r\right\}.$$

2. Player $1a$ and Player 2 make each other indifferent. Player $1b$ enters with minimal probability. This equilibrium exists only for $3\varepsilon_{1b,l} \geq \varepsilon_{1a,r}$:

$$\left\{\left(1-3\varepsilon_{1b,l}\right)l + 3\varepsilon_{1b,l}r, \varepsilon_{1b,l}l + \left(1-\varepsilon_{1b,l}\right)r, \frac{3}{x}l + \left(1-\frac{3}{x}\right)r\right\}.$$

3. Both Player $1a$ and $1b$ stay out with maximal probability and Player 2 chooses right. If the condition for existence is satisfied with a strict inequality, this is a strict equilibrium and hence the unique primitive formation of the perturbed game. Existence requires again $3\varepsilon_{1b,l} \geq \varepsilon_{1a,r}$:

$$\left\{\left(1-\varepsilon_{1a,r}\right)l + \varepsilon_{1a,r}r, \varepsilon_{1b,l}l + \left(1-\varepsilon_{1b,r}\right)r, \varepsilon_{2,l}l + \left(1-\varepsilon_{2,l}\right)r\right\}.$$

For r to be the best reply for Player 2 we need $3 \geq 4\frac{\frac{1}{2}\varepsilon_{1a,r}}{\frac{1}{2}\varepsilon_{1a,r}+\frac{1}{2}\varepsilon_{1b,l}}$ or $3\varepsilon_{1b,l} \geq \varepsilon_{1a,r}$, as claimed.

5.3.3.2 T-Game

Let $\varepsilon_{i,s}$ be the trembling probability of agent i for action s. We require $\varepsilon_{i,l}+\varepsilon_{i,r} < 1$ for all i. Moreover, $\varepsilon_{2,s} < \frac{1}{3}$ etc. The probability of Player $1a$ and Player $1b$ taking

[7] Recall $x \geq 4$

action right is, p and q respectively. The probability that Player 1a trembles and chooses left is $\varepsilon_{1a,l}$. The probability that Player 1b trembles and chooses left is $\varepsilon_{1b,l}$. The probability of Player 2 taking action right is ρ.

Player 2 is indifferent when type 1a chooses r with maximal probability $1-\varepsilon_{1a,l}$: $\frac{\frac{1}{2}\times\left(1-\varepsilon_{1a,l}\right)\times 8}{\frac{1}{2}\times\left(1-\varepsilon_{1a,l}\right)+\frac{1}{2}(1-q)} = 6 \Leftrightarrow 8\left(1-\varepsilon_{1a,l}\right) = 6\left(2-\varepsilon_{1a,l}-q\right)$ and $q = \frac{2+\varepsilon_{1a,l}}{3}$.

Player 2 is indifferent when type 1b chooses l with probability $\varepsilon_{1b,l}$: $\frac{\frac{1}{2}\times 8\times p}{\frac{1}{2}p+\frac{1}{2}\varepsilon_{1b,l}} = 6 \Leftrightarrow 8p = 6\left(p+\varepsilon_{1b,l}\right)$ and $p = 3\varepsilon_{1b,l}$.This is possible only when $\varepsilon_{1a,r} \leq 3\varepsilon_{1b,l}$, for instance when $\varepsilon_{1a,r} = \varepsilon_{1b,l}$.

Type 1a is indifferent if: $3 = (1-\rho)9 \Leftrightarrow \rho = \frac{2}{3}$, while Type 1$b$ is indifferent if: $6\left(1-\rho\right) = 4 \Leftrightarrow \rho = \frac{1}{3}$.

Equilibria:

1. Player 1a goes right with maximal probability. Player 2 and type 1b mix to make each other indifferent. This equilibrium exists for all sufficiently small perturbations:

$$\left\{\varepsilon_{1a,l}l + \left(1-\varepsilon_{1a,l}\right)r, \left(1-\frac{2+\varepsilon_{1a,l}}{3}\right)l + \frac{2+\varepsilon_{1a,l}}{3}r, \frac{2}{3}l + \frac{1}{3}r\right\}.$$

2. Player 1a and Player 2 make each other indifferent. Player 1b enters with minimal probability. This equilibrium exists only for $3\varepsilon_{1b,l} \geq \varepsilon_{1a,r}$:

$$\left\{\left(1-3\varepsilon_{1b,l}\right)l + 3\varepsilon_{1b,l}r, \varepsilon_{1b,l}l + \left(1-\varepsilon_{1b,l}\right)r, \frac{1}{3}l + \frac{2}{3}r\right\}.$$

3. Both Player 1a and 1b stay out with maximal probability and Player 2 chooses right. If the condition for existence is satisfied with a strict inequality, this is a strict equilibrium and hence the unique primitive formation of the perturbed game. Existence requires again $3\varepsilon_{1b,l} \geq \varepsilon_{1a,r}$.

$$\left\{\left(1-\varepsilon_{1a,r}\right)l + \varepsilon_{1a,r}r, \varepsilon_{1b,l}l + \left(1-\varepsilon_{1b,r}\right)r, \varepsilon_{2,l}l + \left(1-\varepsilon_{2,l}\right)r\right\}.$$

For r to be the best reply for player 2 we need $6 \geq 8\frac{\frac{1}{2}\varepsilon_{1a,r}}{\frac{1}{2}\varepsilon_{1a,r}+\frac{1}{2}\varepsilon_{1b,l}}$ or $3\varepsilon_{1b,l} \geq \varepsilon_{1a,r}$, as claimed.

Thus for both games S and T,we observe the following for equilibrium refinement:

1. For $3\varepsilon_{1b,l} < \varepsilon_{1a,r}$ there is a unique equilibrium in the perturbed game and it is not close to the isolated Nash equilibrium (l, r, r). The latter is hence not part or a strategically stable set.

2. In the uniformly perturbed game with $\varepsilon_{1b,l}\varepsilon_{1a,r} = \varepsilon > 0$ the strategy combination

$$((1-\varepsilon)\, l + \varepsilon r, \varepsilon l + (1-\varepsilon)\, r, \varepsilon l + (1-\varepsilon)\, r)\,,$$

   is the unique strict equilibrium of the perturbed game in standard form. One sees immediately that the game has no other primitive formations. The initial candidate in the selection procedure in Selten / Harsanyi thus starts with this equilibrium point as the unique solution candidate, which is hence selected. Taking the limit $\varepsilon \to 0$, we see that (l, r, r) is the Harsanyi Selten solution for the game.

5.4 Experimental Design

The games above were used in two series of experiments, one conducted at the Bonn Laboratory of Experimental Economics[8], and the other at the Finance and Economics Experimental Laboratory in Exeter (FEELE). The games were computer-based and the extensive games were shown to the subjects graphically on the computer screen. The subjects decided by highlighting their choice in the extensive form on the screen. Throughout the experiment, games were repeated in a uniform random matching environment with 6 subjects in the role of Player 1 and 6 subjects in the role of Player 2. Subjects remained in the same role as long as the game remained unchanged.[9]

[8] These experiments were conducted under the supervision of Reinhard Selten.

[9] In Bonn, subjects were assigned a role at the start of game S' and stayed in that role till the end of the experiment.

In Exeter, subjects were assigned a role at the start of game T and remained in that role for all subsequent rounds that game T was played. Roles were re-assigned at the start of game T' and

The first set of experiments consisted of 9 sessions of the game S', with the value of x varied as $x = 4, 5, 6$ in three sessions each. After the initial random allocation of roles, subjects played the game S', in strictly sequential order for 50 rounds. There was a short break after period 25. In the final 5 rounds, called the *Tournament* rounds, subjects had to submit strategies for the extensive game. Each strategy of a player was then evaluated against the strategies of all the players in the opposite role and the subject received the average payoff from all matchings. Thus, we used the strategy method, where subjects first learn to play the game sequentially and then submit complete strategies.

In the second set of experiments, subjects played the simpler game T in the first 25 periods and then switched to the more complex game T' which was played sequentially for the next 25 rounds and the final 5 *Tournament* rounds. In some sessions we gave subjects only the results of their own play while in other sessions we gave them additional information regarding the other 5 parallel plays that occured simultaneously. We wanted to study whether this affected subject behaviour in any manner. In the Bonn experiments we had always given information on all plays to the subjects. In the Exeter experiments we gave this full information only in 5 of the 10 sessions conducted.[10]

The experimental sessions lasted about $3\frac{1}{2}$ hours in Bonn and about $2\frac{1}{2}$ hours in Exeter. Average payment per subject was about £12 in Exeter.

5.5 Data Analysis and Results

5.5.1 Player 1 Behaviour at Information Set 1a

We evaluate how often the strategically safe option was taken by Player 1 at information set $1a$ in Table 5.3.[11] In the old set of experiments, when $x = 4$ or 5,

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subjects remained in that role for all subsequent rounds that game T' was played. Subjects may or may not have been assigned the same role in T' as they had in T.

[10] However, we did not see any indication that this difference of information mattered.

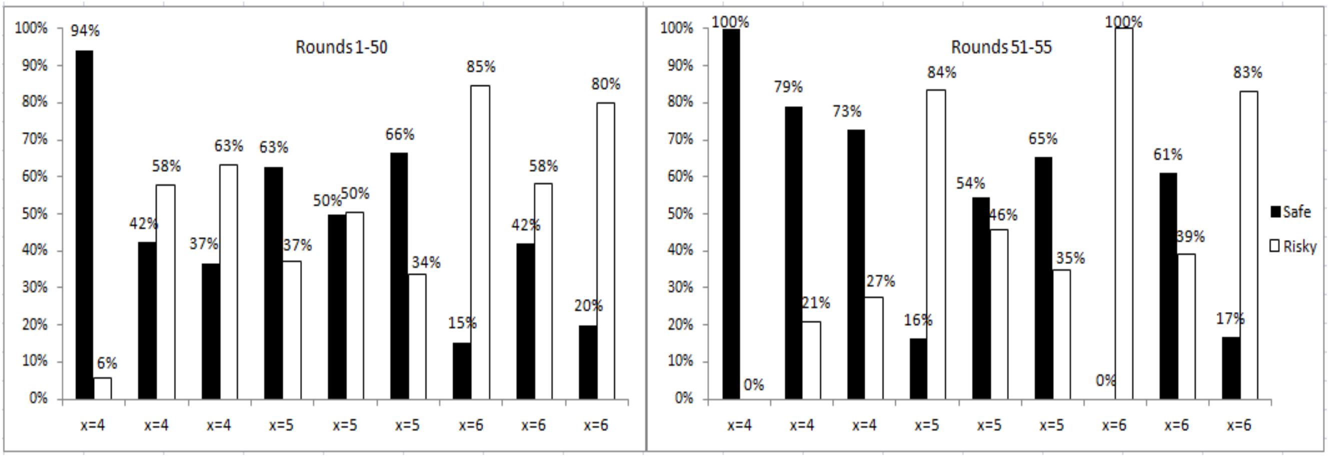
[11] We calculated the average and standard deviation of the percentage of times Player 1 chose left (strategically safe option) at information set 1a, relative to the number of times this information set was reached for each session. The following tables are calculated in a similar manner.

one can note that Player 1 preferred to take his strategically safe option, which gave a definite payoff of 3, rather than risking a payoff of 0 were Node C to be reached. Play changed dramatically in the $x = 6$ version of the game, with a majority of Player 1s selecting their strategically risky option (See Figure 5.6).

Table 5.3. Observed Frequency of Strategically Safe Option at Information Set 1a

	Old Experiments		New Experiments	
	Average	Std. Dev.	Average	Std. Dev.
Rounds $1-25$	0.515437	(0.26552)	0.208411	(0.1274)
Rounds $26-50$	0.43788	(0.28489)	0.265718	(0.2042)
Rounds $51-55$	0.517491	(0.33409)	0.19606	(0.15302)

Figure 5.6. Player 1a Behaviour in Old Experiments



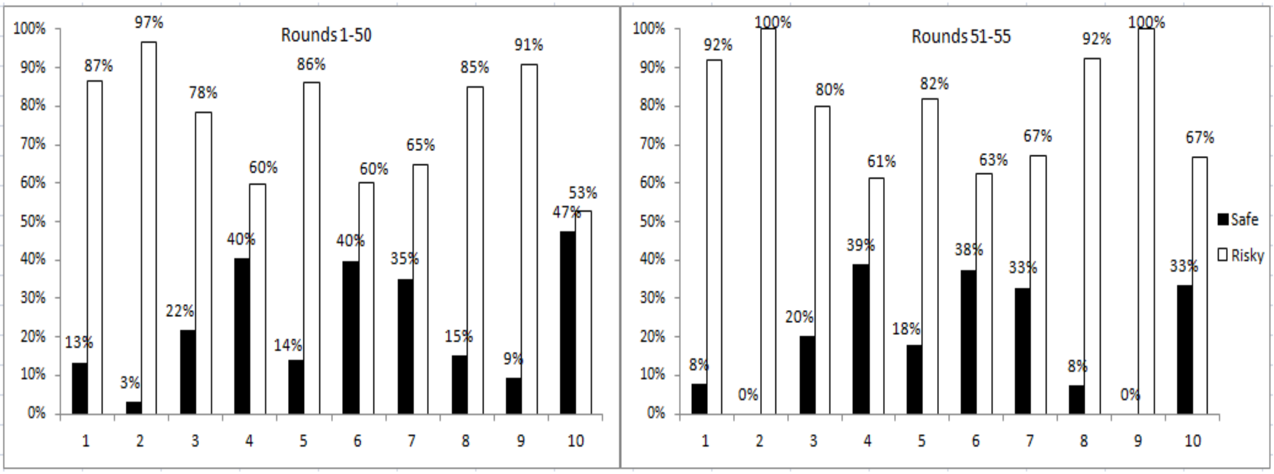
A Mann-Whitney U test shows that the strategically safe option is taken significantly more often in Rounds $1-50$ in the sessions where Player 1's payoff at terminal node B was 4 or 5, as compared to when it was 6. Moreover, in each session of the game where $x = 6$, Player 1 takes the safe option less often at information set 1a in Rounds $26-50$, compared to Rounds $1-25$.

As found in many experiments where players have an outside option there is a substantial fraction of subjects who take it (see Cooper, DeJong, Forsythe, and Ross (1990)). We see here for Rounds $1-50$ that the strategically safe option is taken in at least 15% of the plays. We only find one session (with $x = 6$) where the strategically safe option is practically not taken in Rounds $26-50$ and Rounds $51-55$ of the experiment. In only one session (with $x = 4$) the strategically safe option is almost always taken (in 94% of plays), for all the others it ranges between

15 − 67%. The results are qualitatively the same for the tournament periods 51 − 55. Thus, behaviour is overall not consistent with either of the two Nash equilibrium components of the game where the strategically safe option is taken either with probability 0 or with probability 1.

Comparing the old and new experiments, the strategically safe option is foregone significantly more often in the new experiments, as we expected (See Figure 5.7). This is significant by a Mann-Whitney U test conducted separately for Rounds 1 − 50 and tournament periods 51 − 55.[12] In the new set of experiments the percentage with which the strategically safe option is taken is below 50% in each session, and separately for Rounds 1 − 25, 26 − 50 and tournament periods 51 − 55, with just one exception for period 26 − 50 (always significant by a sign test).

Figure 5.7. Player 1a Behaviour in New Experiments



Arguably in the new set of experiments, Player 1 did not take the strategically risky option often enough at his information set 1a. Given the observed frequencies with which Player 2 chose her strategically risky option at information set 2α, he would have made a gain in each session. More precisely, $[(9 * B\%) - 3]$ is positive for each session[13], where for a given session $B\% = \frac{\text{(number of times B is reached)}}{\text{(number of times B or C are reached)}}$. In contrast, this "gain" varies considerably for the sessions in the old experiment.

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[12]The test is highly significant for Rounds 1 − 25, but not for Rounds 26 − 50. Thus, the original stronger incentive for Player 1 to take the strategically risky option gets somewhat dampened by experience.

[13]By session we mean Rounds 1-50 or Rounds 51-55

However, players are not simply irrational. The number of times the strategically risky option is taken is highly correlated with the gain to be made. This is significant at a 5% level of significance, using Spearman's Rank Correlation Test for both the new and old experiment sessions.[14]

5.5.2 Player 1 Behaviour at Information Set 1b

In both the old and the new sets of experiments, Player 1 always chose the strategically risky option (left) significantly less often than the 33% predicted by the strategically stable Kohlberg-Mertens Nash equilibrium (See Table 5.4).[15] This is significant by a sign test for each individual session and part of the old experiment – Rounds $1-25$, $26-50$ and $51-55$ – separately. In the new set of experiments game T' was used in Rounds $26-55$. Here, Player 1 chose the strategically safe option (right) at information set $1b$ significantly more often than both left and middle, in each part of the experiment (Rounds $26-50$ and $51-55$) separately.

Table 5.4. Observed Frequency of Strategically Risky Option at Information Set 1b

	Old Experiments		New Experiments	
	Average	Std. Dev.	Average	Std. Dev.
Rounds $1-25$	0.055979	(0.04221)	0.220375	(0.09273)
Rounds $26-50$	0.049949	(0.03563)	0.103184	(0.06487)
Rounds $51-55$	0.056884	(0.05144)	0.098769	(0.08681)

In the old set of experiments, left was never chosen in more than 12% of the cases for both Rounds $1-25$ and $26-50$, and 16% for the tournament periods $51-55$ (See Figure 5.8). For the new experiments, the corresponding percentages of choosing left are 40%, 22% and 27% for Rounds $1-25$, $26-50$, $51-55$, respectively (See Figure 5.9).

We calculated various proxies for the gains Player 1 could have made at information set $1b$ by going left rather than right. These gains were sometimes positive and

[14]Except for Rounds 51-55 in the new set of experiments which just misses the 5% level of significance.

[15]This is the average and standard deviation of the percentage of times Player 1 chose left (strategically risky option) at information set 1b, relative to the number of times this information set was reached for each session.

Figure 5.8. Player $1b$ Behaviour in Old Experiments

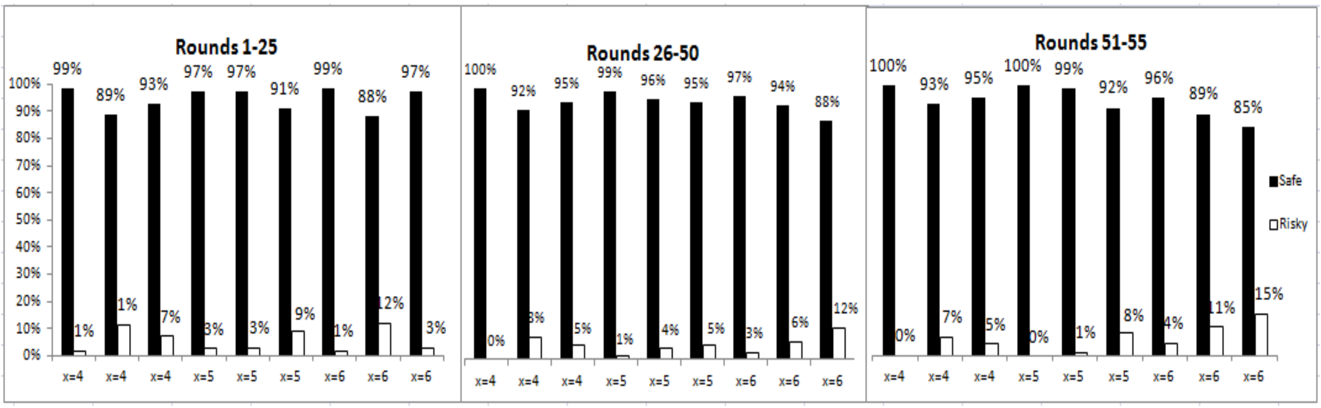
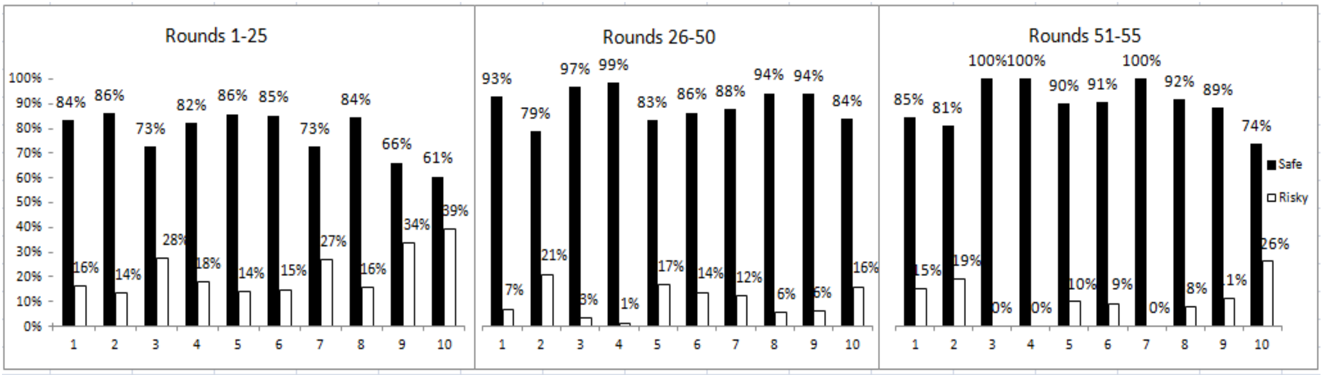


Figure 5.9. Player $1b$ Behaviour in New Experiments



sometimes negative, varying greatly from session to session. We never found any significant correlation between the percentages of times Player $1b$ chose left and the gains. Subjects simply seemed to be reluctant to take the strategically risky option, which would be consistent with the aim to build up a collective reputation.

5.5.3 Observed frequency for Information Set 2α

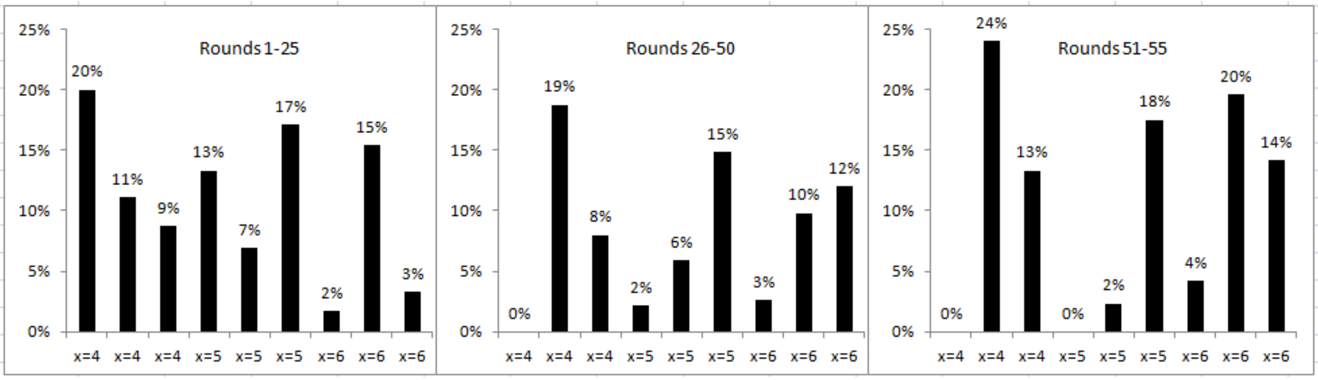
Since Player 1 rarely chooses his strategically risky (left) option at information set $1b$, the relative frequency with which the right node in information set 2α is reached is significantly below 25% (See Table 5.5, Figures 5.10 and 5.11). A sign test shows this is true for Rounds $1-25$, $26-50$ and $51-55$ in the sessions of the old experiment and Rounds $26-50$ and $51-55$ in the sessions of the new experiment.

This is consistent with a collective reputation effect and it would thus make sense for Player 2 to select her strategically risky option. Even for Rounds $1-25$ in the sessions of the new experiment, the percentages are close to 25% or below.

Table 5.5. Observed Frequency of Right Node being reached at Information Set 2a

	Old Experiments		New Experiments	
	Average	Std. Dev.	Average	Std. Dev.
Rounds $1-25$	0.108521	(0.06229)	0.213985	(0.0844)
Rounds $26-50$	0.082285	(0.06245)	0.126246	(0.08414)
Rounds $51-55$	0.118831	(0.0875)	0.094254	(0.08086)

Figure 5.10. Old Experiments - Observed Frequency of being in the Right Node at Set 2α



5.5.4 Player 2 Behaviour at Information Set 2α

In the new set of experiments, Player 2 chooses the strategically safe option significantly more often than $33\frac{1}{3}\%$ – which is the maximal probability in a Nash equilibrium, when information set 2α is not reached (See Figure 5.12). This is significant in Rounds $1-25$ and $51-55$ by a Sign test; and for Rounds $26-50$ by a Wilcoxon Rank Test.[16] On average, the strategically risky option is taken in 60% of the cases, well below the $66\frac{2}{3}\%$ required by the Kohlberg-Mertens strategically stable Nash equilibrium.

Given these averages[17], it makes sense for Player 1 to choose right at both information sets $1a$ and $1b$, which is roughly consistent with actual behaviour. However, this is an overview on average behaviour in all the sessions. In some of individual sessions the percentage of Player 2 choosing left is well above $66\frac{2}{3}\%$ and in these cases Player 1 would have an incentive to choose left at information set $1b$.

[16] Significance tests were carried out on the percentage of times left is taken minus $\frac{1}{3}$.

[17] This is the average and standard deviation of the percentage of times Player 2 chose left (strategically risky option) at information set 2α, relative to the number of times this information set was reached for each session.

Figure 5.11. New Experiments - Observed Frequency of being in the Right Node at Set 2α

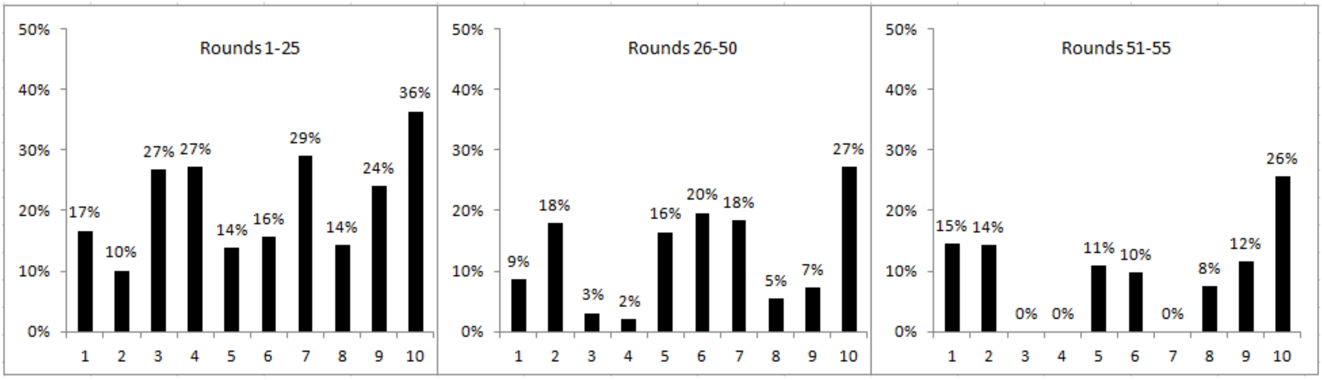
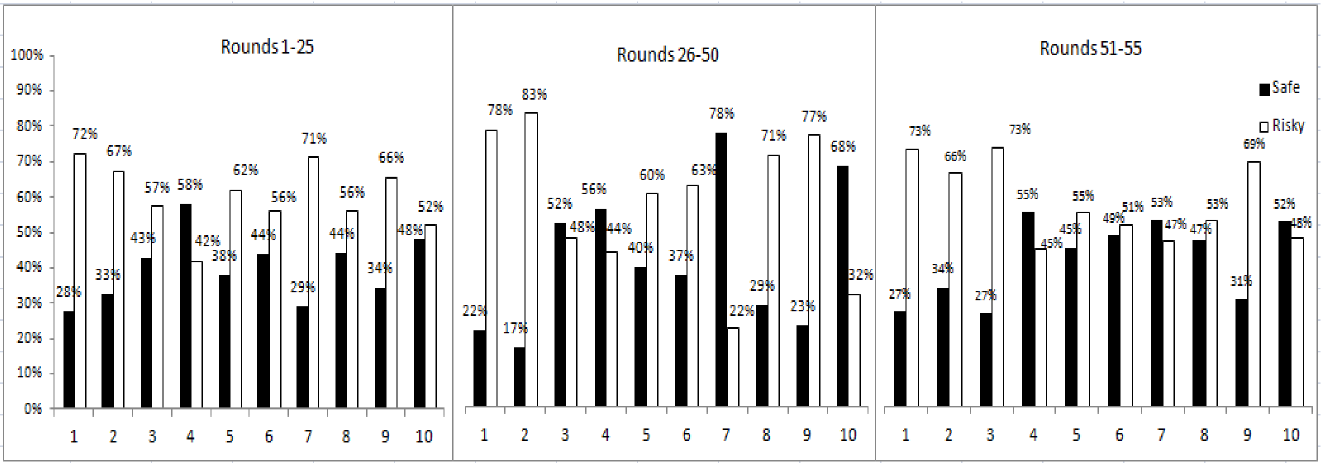


Figure 5.12. Player 2 Behaviour at Information Set 2α - New Experiments



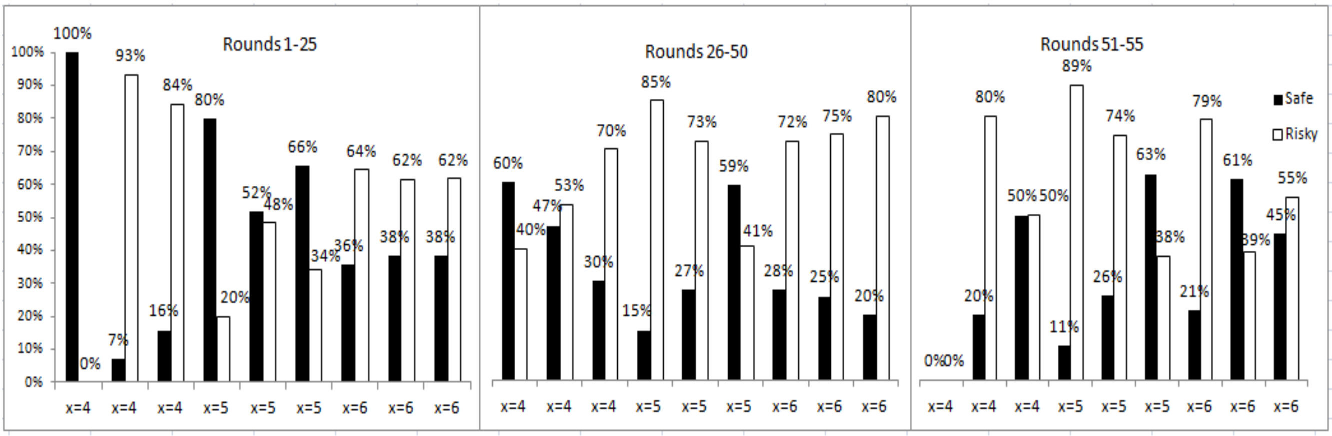
In the old experiments, information set 2α is reached less often. The first session may be taken as an example of the Harsanyi-Selten solution being played. In this session, information set 2α is only reached 5 times in Rounds $1-25$ and $26-50$, and never in the final part. Disregarding this session, it is significant by a sign test that the strategically risky option is chosen in at least $33\frac{1}{3}\%$ of the cases. However, the percentages are significantly below the 87.5% required by strategic stability - with the one exception of 93% in Rounds $1-25$ and 89.5% in Rounds $51-55$ (See Figure 5.13).

We wanted to analyse whether Player 2 learned from her experience (from past plays at information set 2α) and did not just make choices randomly. Since information set $2a$ was not reached very often in the old experiments, we restrict this analysis to the new experiments. In order to analyse whether learning took place, we checked whether a player changed her behaviour more often after a "failure" than after a "success". There are two ways in which Player 2 could make a failure –

Table 5.6. Observed Frequency of Strategically Risky Option at Information Set 2a

	Old Experiments		New Experiments	
	Average	Std. Dev.	Average	Std. Dev.
Rounds $1-25$	0.519377	(0.29827)	0.601518	(0.09404)
Rounds $26-50$	0.653417	(0.1657)	0.577839	(0.20669)
Rounds $51-55$	0.631131	(0.20074)	0.580699	(0.11262)

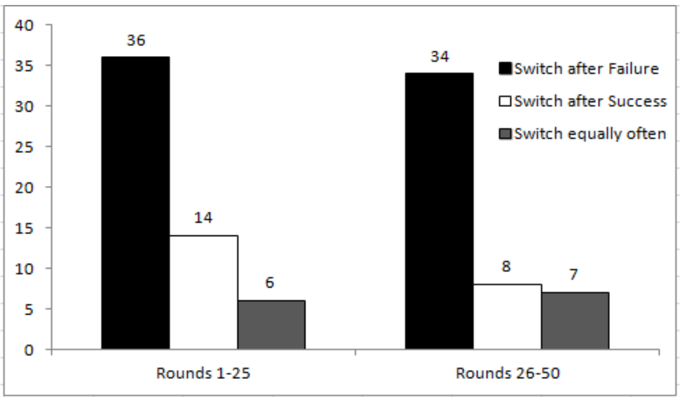
Figure 5.13. Player 2 Behaviour at Information Set 2α - Old Experiments



1. if she chooses right and the left node of information set 2α is reached, resulting in a payoff of 6 instead of 8.

2. if she chooses left and the right node of information set 2α is reached, resulting in a payoff of 0 instead of 6.

Figure 5.14. Observed Direction of Switching

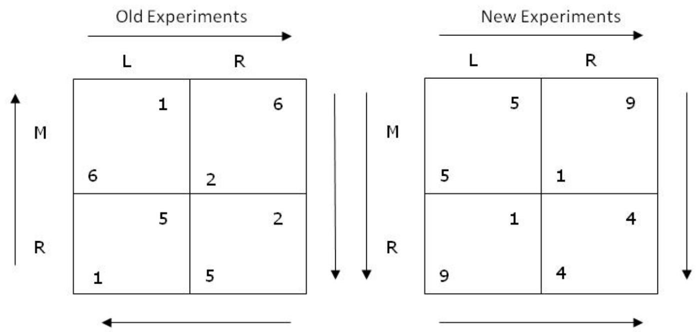


We counted how often each subject switched after a failure/success in Rounds $1-25$ and $26-50$ and took the difference of the two percentages. Individuals for whom the difference was zero or for whom the information set was never reached were disregarded. In Rounds $1-25$, there were 36 individuals who switched more often after failure, 14 who switched more often after success and 6 who switched equally often (See Figure 5.14). For Rounds $26-50$, the corresponding numbers were 34, 8 and 7. Sign tests based on these numbers would indicate that most subjects switch more often after failure than success. Thus, there is reasonable evidence of learning at this information set.

5.5.5 Player Behaviour in the embedded 2x2 game

In the extended models S' and T', type $1b$'s strategically safe option was replaced with a 2×2 game with a unique equilibrium, which had the same expected payoffs as the strategically safe option in the basic game. The 2×2 games used in the experiments can be seen in Figure 5.15.

Figure 5.15. 2×2 Games used in the Experiments



In the new experiments, the Prisoner's Dilemma type 2×2 game was embedded into the signalling game in Rounds $26-50$ and $51-55$. As expected, both players choose their dominant action (right) more often than the dominated action (See Table 5.7). However, the percentages with which the dominated action is chosen are not negligible and can be as high as 23% in Rounds $26-50$ and 32% in Rounds $51-55$ in individual sessions (See Figure 5.16).[18]

Averaged over all sessions, Player 1 chooses the dominated action more often than Player 2, but a Wilcoxon Rank test does not yield significant results. For Rounds $26-50$, the percentage of times the dominated action was chosen by Player 1 and Player 2 is positively correlated (correlation coefficient $= 0.34$). However,

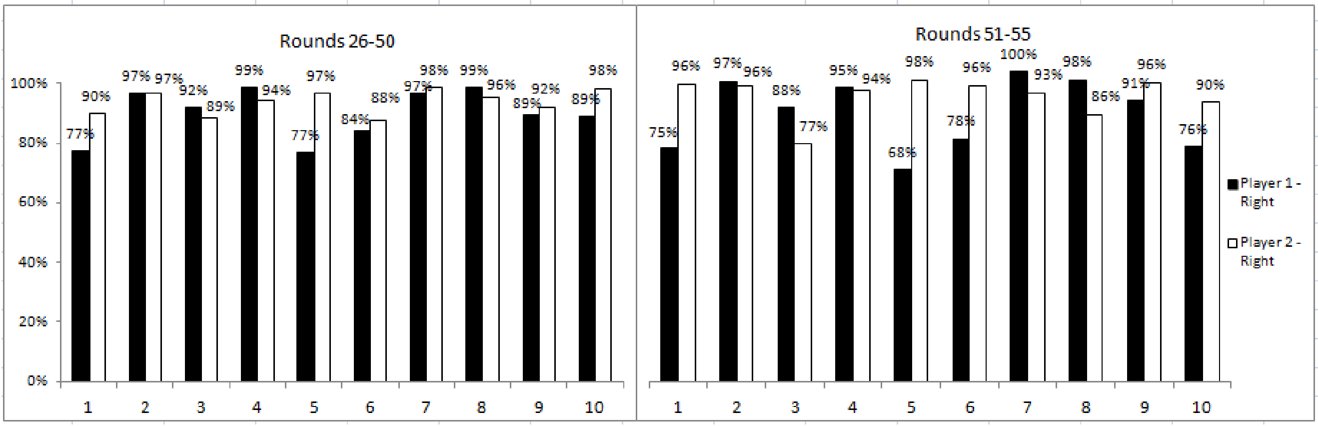
[18]The fractions are calculated relative to the number of times Nature chose right and Player 1 did not choose left.

Table 5.7. Observed Frequency of Right being chosen in the 2x2 game - New Experiment

	Player 1		Player 2	
	Average	Std. Dev.	Average	Std. Dev.
Rounds $26-50$	0.899071	(0.08269)	0.937936	(0.0406)
Rounds $51-55$	0.866204	(0.11283)	0.921962	(0.06393)

the correlation between the number of times the dominated action was chosen in the tournament periods 51-55 is negative (correlation coefficient $= -0.25$). It is interesting to observe that there is no significant difference between the number of times Player 1 chose left and the number of times he chose the dominated action middle at information set $1b$.

Figure 5.16. Observed Frequency of Dominant Action - New Experiment

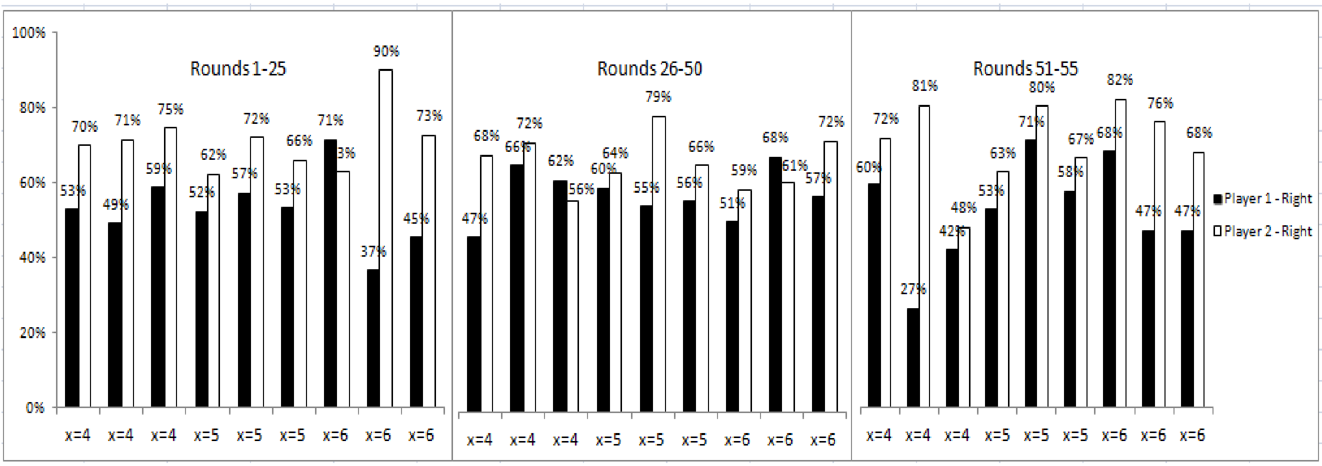


In the old experiments, we embedded a 2×2 game with a unique mixed-strategy Nash equilibria into the signalling game S and used the resulting game S' in all the rounds. The percentages of strategy choices in the 9 sessions are roughly comparable with the mixed-strategy equilibrium (See Table 5.8), but as in many experiments with such 2×2 games (see for instance, Goerg, Chmura, and Selten (2008) and the literature they cite), one has strong own-payoff effects. For the main part of the experiment, periods $1-50$, the percentages with which right is chosen are significantly below the equilibrium values for Player 1 and above for Player 2 (by a sign test). This finding is consistent with the predictions made by the alternative solution concepts for such 2×2 games in Goerg, Chmura, and Selten (2008).[19]

Table 5.8. Observed Frequency of Right being chosen in the 2x2 game - Old Experiment

	Player 1		Player 2	
	Average	Std. Dev.	Average	Std. Dev.
Rounds 1 – 25	0.530085	(0.09553)	0.713404	(0.08242)
Rounds 26 – 50	0.578689	(0.06828)	0.663489	(0.07171)
Rounds 51 – 55	0.525137	(0.13771)	0.708392	(0.10936)

Figure 5.17. Observed Frequency of Dominant Action - Old Experiment



5.6 Alternative Equilibrium Concepts

As seen from the preceding section on data analysis, subject behaviour observed in the experimental data does not match with Nash predictions. In this section we examine alternative equilibrium concepts that might provide a better explanation for the observed behaviour. For the sake of simplicity, we restrict analysis in this section to Game T, since including Game S would require a separate analysis for each case of $x = 4/5/6$. However, the analysis can be intuitively extended to Game S, since both games are similar in essence.

[19]In the impulse balance equilibrium, right is chosen with probability $\frac{1}{2}$ by Player 1 and with probability $\frac{2}{3}$ by Player 2. In the action sampling equilibrium, right is chosen with probability 0.56 by Player 1 and with probability 0.66 by Player 2.

5.6.1 Quantal Response Equilibrium

One possible explanation for observed behaviour could be that although the strategy choice of subjects is dependant on the expected utility of their strategies, these choices are based on a quantal choice model. Moreover, the subjects make the assumption that their opponents use a similar quantal choice model to choose between their strategies as well. A proposed explanation for such an equilibrium mechanism was put forth by McKelvey and Palfrey (1995), when they introduced the concept of a Quantal Response Equilibrium (QRE) for normal form games.

The best response functions of a player, when based on a quantal choice model, becomes stochastic or probabilistic in nature rather than being deterministic. QRE makes allowance for the fact that often players may make infinitesimal errors such that, *better* responses are more likely to be played than *worse* responses and best responses are no longer observed with complete certainty.

The difference between QRE and Nash equilibrium, is that it replaces the perfectly rational expectations assumption of Nash equilibrium, with an equilibrium that makes allowances for noise, imperfectness and irrational behaviour of subjects. Mckelvey and Palfrey (1998) extends the QRE approach to include extensive form games, where behavioural strategies are used to reach an equilibrium. Players employee Bayes' rule and QRE strategies to calculate their expected payoffs and assume that opponents do the same.

QRE assumes that the probability that a player selects a decision, is a smooth, increasing function of the payoff that the player would earn from that decision. Thus, if the expected payoffs of a column player from his actions $Right$ and $Left$ are $\pi^e(R)$ and $\pi^e(L)$, then the logit probability for choosing a strategy R (resp. L) would be: $p_R = \frac{\exp(\lambda\pi^e(R))}{\exp(\lambda\pi^e(L)) + \exp(\lambda\pi^e(R))}$ (resp. $p_L = (1 - p_R)$). Similarly, if the row player's expected payoffs from Up and $Down$ are $\pi^e(U)$ and $\pi^e(D)$, then the logit equilibrium probability for choosing a strategy U (resp. D) would be $p_U = \frac{\exp(\lambda\pi^e(U))}{\exp(\lambda\pi^e(D)) + \exp\exp(\lambda\pi^e(U))}$ (resp. $p_D = (1 - p_U)$).

It can be noted that the denominator ensures that the probability determined from the above equation lies between 0 and 1. A rationality/error parameter is introduced in the form of λ. When $\lambda \to \infty$, the highest payoff is selected with complete certainty, i.e., probability 1. Thus $\lambda \to \infty$, signifies perfect rationality and the resultant QRE coincides with the Kohlberg-Mertens Nash equilibrium. On the other hand, when $\lambda \to 0$, players become completely irrational and are equally likely to play all strategies.

We estimated a Quantal Response Equilibrium for the signalling game T. As can be seen in Table 5.9, at $\lambda = 0$, each strategy choice is equally likely. On the other end of the spectrum, we note that at $\lambda = 1032471$ (i.e., as $\lambda \to \infty$), the QRE predictions match with the Nash equilibrium. In our experiments we observed that Player $1a$, chose R with probability 0.8, while Player $1b$ did so with probability 0.78. The closest value of λ that would reflect this behaviour is at $\lambda = 0.955$. One drawback of the software tool[20] used by us is that it assumes that the parameter λ is the same for both Players 1 and 2. However, at $\lambda = 0.955$, Player 2 is expected to mix 50 : 50, which does not match our data. A closer match to observed Player 2 behaviour of mixing 0.6 : 0.4 between her choices l and r, is seen $\lambda = 2.264$.

Table 5.9. Estimated Quantal Response Equilibrium for Game T

λ	$Pr(1a, L)$	$Pr(1a, R)$	$Pr(1b, L)$	$Pr(1b, R)$	$Pr(2, l)$	$Pr(2, r)$
0	0.5	0.5	0.5	0.5	0.5	0.5
0.955679	0.200686	0.799314	0.2729091	0.729091	0.494012	0.505988
2.264230	0.004194	0.995806	0.292784	0.707216	0.601751	0.398249
1032471	0	1	0.333333	0.666667	0.666667	0.333333

Thus, a rationality level or $\lambda = 0.955$ on the part of Player 1 would explain observed behaviour in the experiments. As mentioned above, if players are using a quantal choice model to make their decisions, it would take very high levels of rationality on the part of both Player 1 and 2, to reach the Nash equilibrium.

5.6.2 Cursed Equilibrium

An observation in common-value auctions is the phenomenon termed the "winner's curse". Winner's curse alludes to bidders' tendency to overbid, which results in a winning bid that exceeds the value of the good. The reason for this over-bidding

[20] QRE was calculated using Gambit, which is a software tool for game theory. McKelvey, McLennan, and Turocy (2010)

is that bidders do not fully realise the fact that lower bids by their opponents signal that the object is of lower value than earlier perceived. Players thus underestimate the correlation between their opponents' actions and their information. For example, there is no accurate method to judge the true value of a diamond mine. Consider a mine whose true value is £20 million. Bidders might guess that the value of the diamond mine is between $\pounds 15 - 40$ million. Thus, the bidder who bid $\pounds 40$ millon at the auction would win, but later find that the mine was not worth as much.

Eyster and Rabin (2005) generalise this behaviour in the form of a "Cursed Equilibrium" model, which assumes that subjects update incorrectly at their information sets, such that rather than having a separating equilibrium with type-specific actions, all types of players play the same action with some positive probability. Thus, players in the cursed equilibrium model make their choices based on maximising their own expected payoffs, subject to incorrect beliefs of their opponent's actions. The degree to which a player is "cursed" is denoted by a parameter $\chi \in [0, 1]$, where χ is the probability that all types of opponent's play the same action, while $(1 - \chi)$ is the probability with which they play their true type-specific equilibrium action.

A variant of the lemons problem may be used to illustrate the concept of a cursed equilibrium. Consider a simultaneous-move lemons game where both agents (the buyer and the seller) must announce their decision to trade a car concurrently and a sale only takes place if they both agree to trade. The seller knows whether the car he is selling is a lemon (worth $\pounds 0$) or a peach (worth $\pounds 6000$ to the buyer and $\pounds 5000$ to the seller). The buyer is willing to spend up to $\pounds 2500$ for the car, but cannot perceive the true value of the car $-$ and so assigns a $50 - 50$ probability that a car offered for sale is a lemon or a peach. If the buyer is rational, he would realise that the seller would only be willing to trade the car for $\pounds 4000$, if it was a lemon. Thus, a rational buyer would refuse to buy the car. However in such a scenario, a cursed buyer may agree to trade.

Consider a $\chi-$cursed buyer who believes that with probability χ, both types of sellers agree to trade. The cursed buyer then believes that the probability the car is a peach is: $\chi \cdot \frac{1}{2} + (1 - \chi) \cdot 0 = \frac{\chi}{2}$. It's worth to him would be $\frac{\chi}{2} \cdot 6000 = 3000\chi$. Thus all buyers cursed with a $\chi \geq \frac{5}{6}$, would make the trade and find that they bought a lemon (Eyster and Rabin (2005)).

If we use a similar logic to our game, consider what would happen if Player 2 did not update correctly at her information set and is instead a cursed-Player 2. In such a situation if Player 2 has $\chi = 1$, i.e., she is fully cursed, she would believe she faces both types of Player 1 with equal probability. Given that Player 2 expects to face both types of Player 1 with equal probability, the expected payoff from left (for Player 2) would be: $8 \cdot \frac{1}{2} = 4$. Thus, Player 2 makes a higher expected payoff $(= 6)$ if she goes right and would never choose to go left. The degree of cursedness causes Player 2 to infer nothing from Player 1a's signals and the result of this would be that Player 1a would stop taking his strategically risky action.

On the other hand, if χ is allowed to vary such that $\chi \in [0, 1]$, we would have a partially cursed Player 2 who believes that:

- with some probability χ, both Player $1a$ and $1b$ play their strategically risky action. Thus, if she is called to make a decision at Information Set 2α, she faces both types with equal probability.

- with probability $(1 - \chi)$, Player $1a$ and $1b$ choose their type-specific equilibrium action – Player $1a$ chooses his strategically risky option, while Player $1b$ chooses his strategically safe option. Thus, if she is called to make a decision at Information Set 2α, she faces Player $1a$ with certainty.

Hence, the overall probability that Player 2 faces Player $1a$ is: $\chi \cdot \frac{1}{2} + (1 - \chi) \cdot 1 = \left(1 - \frac{\chi}{2}\right)$. In this scenario, the expected payoff of choosing left (for Player 2) would be: $\left(1 - \frac{\chi}{2}\right) \cdot 8$, while the expected payoff of choosing right is 6. It would make sense for Player 2 to choose left, if $\left(1 - \frac{\chi}{2}\right) \cdot 8 > 6$. Thus, Player 2s cursed with $\chi < \frac{1}{2}$ would choose left, while all others would choose right.

The above discussion might explain why the Player 1s in some sessions preferred to take their strategically safe option (the degree of cursedness of Player 2 did not make it worthwhile for Player $1a$ to choose right), while in other sessions we see a large number of Player $1a$ and 2s taking their strategically risky options (here

Player 2 though cursed, still goes left with a sufficiently high probability). Cursed equilibrium thus fits better with most of our data than any of the Nash equilibria - but only for highly specific values of χ, or updating of beliefs.

5.6.3 Equilibrium under Ambiguity

When a player fails to assign a subjective probability to the actions of his opponent, ambiguity arises. It is quite easy to note that in our game, if Player 1 does not take his strategically safe option, he does not know with what subjective probability Player 2 will choose left.[21] He thus faces ambiguity about Player 2's choices and his response to this ambiguity may be postive (ambiguity-seeking), negative (ambiguity-averse), or neither (ambiguity-neutral).

Ambiguity averse behaviour was first identified by Ellsberg (1961). The Ellsberg paradox documents subjects' preference for a definite chance of winning and thus, their subsequent tendency to avoid ambiguous acts. Ambiguity averse behaviour leads to choices are that not consistent with maximising expected utility and give rise to probabilities that do not always sum up to 1.

This inconsistency was solved by representing beliefs by a non-additive set function ν. Non-additive set functions allow that $\nu(X \cup Z) \neq \nu(X) + \nu(Z)$. Non-additive beliefs were first introduced and used by Schmeidler (1989). He proposed a theory called Choquet Expected Utility (CEU), where outcomes are evaluated by a weighted sum of utilities, but unlike Expect Utility Theory (EUT) the weights used depend on the acts. A special class of capacities, termed neo-additive capacities, was introduced by Chateauneuf, Eichberger, and Grant (2007), to model optimistic and pessimistic outlooks to ambiguity.

A capacity v is convex (resp. concave) if for all A and $B \subseteq S$, $v(A \cup B) + v(A \cap B) \geq v(A) + v(B)$, (resp. $v(A \cup B) + v(A \cap B) \leqslant v(A) + v(B)$), where A, B are events contained in the universal set S. In CEU, convex capacities are used to model a pessimistic outlook to ambiguity, while concave capacities model an optimistic outlook. Consider a two player game with a finite pure strategy set S_i,

[21] For the purpose of this section, the choices available to Player 1 are either strategically safe or strategically ambiguous (not risky).

$i = 1, 2$. Each player i's beliefs about the opponent's behaviour is represented by a capacity v_i on S_{-i}, which is the set of strategy combinations which his/her opponent could choose. Given neo-additive beliefs, the expected payoff that a player i could earn from a strategy s_i, is determined by equation:

$$V_i\left(s_i; \pi_i, \alpha_i, \delta_i\right) = \delta_i \alpha_i M_i\left(s_i\right) + \delta_i\left(1 - \alpha_i\right) m_i\left(s_i\right) + \left(1 - \delta_i\right) \int u_i(s_i, s_{-i}) d\pi_i(s_{-i}), \tag{5.1}$$

where $M_i\left(s_i\right) = \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ and $m_i\left(s_i\right) = \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ and $d\pi_i(s_{-i})$, is the conventional expectation taken with respect to the probability distribution π.[22]

Intuitively, π can be thought to be the decision maker's belief. However, he is not sure of this belief, hence it is an ambiguous belief. His confidence about this belief is modelled by $(1 - \delta)$, with $\delta = 1$ denoting complete ignorance and $\delta = 0$ denoting no ambiguity. His attitude to ambiguity is measured by α, with $\alpha = 1$ denoting pure optimism and $\alpha = 0$ denoting pure pessimism. If the decision-maker has $0 < \alpha < 1$, he is neither purely optimistic nor purely pessimistic (i.e., ambiguity-averse), but reacts to ambiguity in a partly pessimistic way by putting a greater weight on bad outcomes and in a partly optimistic way by putting a greater weight on good outcomes.

In the case of ambiguity and non-additive beliefs, the Nash equilibrium idea of having consistent beliefs regarding the opponent's action and being able to play an optimum strategy as a response to these beliefs, no longer holds true and needs to be modified. Given neo-additive beliefs and expected payoffs determined by equation 5.1, the support of a capacity is a player's belief of how the opponent will act. Formally, the support of a neo-additive capacity, $\nu\left(A\right) = \delta\alpha + \left(1 - \delta\right)\pi\left(A\right)$, is defined by $\operatorname{supp}\left(\nu\right) = \operatorname{supp}\left(\pi\right)$.

Definition 3 *A pair of neo-additive capacities* (ν_1^*, ν_2^*) *is an Equilibrium Under Ambiguity (EUA) if for* $i = 1, 2$, $\operatorname{supp}\left(\nu_i^*\right) \subseteq R_{-i}(\nu_{-i}^*)$.

[22] Note that Chateauneuf, Eichberger, and Grant (2007) write a neo additive capacity in the form $\mu(E) = \delta\alpha + (1 - \delta)\pi(E)$. We have modified their definition to be consistent with the rest of the literature where α is the weight on the minimum expected utility.

Here R_i denotes the best-response correspondence of player i given that his beliefs are represented by ν_i, and is defined by $R_i(\nu_i) = R_i(\pi_i, \alpha_i, \delta_i) := \text{argmax}_{s_i \in S_i} V_i(s_i; \pi_i, \alpha_i, \delta_i)$.

This definition of equilibrium is taken from Eichberger, Kelsey, and Schipper (2009), who adapt an earlier definition in Dow and Werlang (1994). These papers show that an EUA will exist for any given ambiguity-attitude of the players. In games, one can determine π_i endogenously as the prediction of the players from the knowledge of the game structure and the preferences of others. In contrast, we treat the degrees of optimism, α_i and ambiguity, δ_i, as exogenous. In equilibrium, each player assigns strictly positive likelihood to his/her opponent's best responses given the opponent's belief.

Table 5.10. Normal Form Game T

	$Left$	$Right$
LL	$9,3$	$3,9$
LR	$7,7$	$7,7$
RR	$13,12$	$4,10$
RL	$15,8$	$0,12$

Given this framework of ambiguity and an equilibrium under ambiguity, assume that Player 1 is ambiguity averse and his beliefs can be modelled as neo-additive capacities. Consider the normal form of Game T in Table 5.10, where Player 1 is seen as the row player and Player 2 as the column player. The strategy LL is dominated by RR. We can thus eliminate LL, since CEU preferences will never select a strategy which is dominated by a pure strategy.

The Choquet expected payoff of Player 1 for the three strategies available to him, given a capacity v, can be calculated as under:

$$\begin{aligned} V(LR, v) &= 7 \\ V(RR, v) &= 4 + 9 \cdot v(\{L\}) \\ V(RL, v) &= 15 \cdot v(\{L\}). \end{aligned}$$

Suppose that v is constructed such that $V(LR, v) < V(RL, v)$. Intuitively this would mean that the Choquet expected payoff of RL, which gives 15 some of the time and 0 otherwise, is greater than that of LR, which gives a definite payoff of 7.

Then there would exist a capacity v' that is more ambiguous than v, such that $v'(\{L\}) < v(\{L\})$. If there exists such a capacity v', then $V(LR, v') > V(RL, v')$, i.e., getting a definite payoff of 7 is valued more than getting a payoff of 15, which is higher but uncertain. The necessary and sufficient case for this to be true is that $v(\{L\}) > \frac{7}{15} > v'(\{L\})$.

Hence, the best response given the more ambiguous capacity v', is for Player 1 to always choose his strategically safe action. Thus, in the presence of ambiguity, Player 1s who are ambiguity averse would prefer their strategically safe option, rather than taking a chance and facing ambiguous payoffs.

5.7 Results and Conclusions

We conducted an experiments based on extensive form signalling games, which had 2×2 games embedded in them. The initial set of experiments was conducted with the aim of testing which of two competing theories of equilibrium selection or refinement theories better described behaviour. We find systematic deviations from both types of theories.

The second set of experiments was conducted in order to test for the appearance of a collective reputation. However, though we did not find any statistically significant evidence that would substantiate the hypothesis of collective reputation, we do note a number of interesting observations about subject behaviour in the games.

- At information set $1a$, we find that a significant number of players take the strategically safe option, even if it would pay well to forego it. This is often observed in games with outside options and could simply be explained by risk avoidance or low aspiration levels. In addition, this may be explained by ambiguity averse behaviour, reactions to cursed-Player 2 behaviour and the fact that players would need to be highly rational (quantal response parameter $\lambda \to \infty$) for them to play the Nash with certainty. On the whole, behaviour was fairly consistent with payoff maximisation, such that actions yielding higher average payoffs were taken more often.

- Behaviour in the embedded 2×2 games is comparable with previous literature on experimental results of 2×2 games played in isolation. In the Prisoner's Dilemma type game, subjects chose the dominant actions more often. However, the percentage of times that the dominated action was chosen was not negligible. For the game with the unique mixed-strategy Nash equilibrium observed frequencies tended to be in a 10% range around the equilibrium. Deviations from the Nash equilibrium tended to be in the direction of behavioural concepts which adjust for the own-payoff effect (Goerg, Chmura, and Selten (2008)).

- At information set $1b$, players rarely take left option. This holds independent of payoffs and player behaviour at all the other information sets. One of the reasons why this might happen is that the potential gains from taking this action are not very high. In fact, in many sessions it would not pay given the behaviour of Player 2 – but even when it would pay, the action left is not taken.

  This may be some indication that the subjects in the role of Player 1 are trying to collectively build up the reputation that they do not take this action at $1b$, in order not to destroy a cooperation which leads to the outcome at node B (when information set $1a$ is reached). This may or may not point towards the attempt to build a collective reputation.

  Alternatively, the potential threat of a payoff of 0 may deter Player 1 from taking the action left. Both middle and right always yield non-negative profits. *Right* guarantees a payoff of 4 in Game T, while middle secures a minimum payoff of 2 in Game S.

- At information set 2α, the percentage of times that play was in the right decision node is systematically below $\frac{1}{4}$. Thus, it would maximise Player 2's payoff if she chose her strategically risky choice.

- At information set 2α, left is typically chosen in at least 33% of the cases. Often this percentage is much higher although rarely above 67%. There is some evidence for learning at this information set but it is not strong. This may be because there is a substantial fraction of subjects who do not understand the strategic situation very well and choose both actions equally often, for instance, by always taking the highlighted choice randomly selected by the computer.[23] Such subjects would bias observed frequencies towards $50 - 50$ and the behaviour of the other players may not fully compensate for this "irrationality".

In the previous section we discussed how though observed behaviour cannot completely be explained by Nash, it may be explained using alternative equilibrium concepts such as Quantal Response Equilibrium, Cursed Equilibrium and Equilibrium under Ambiguity. There might be a number of other models of bounded rational behaviour that explain our findings, however we leave them for future research.

Appendix

Table 5.11. Old Experiment: Terminal Nodes Reached in Rounds 1-25

	A	B	C	D	E	F	G	H	I
1 $(x = 4)$[24]	75	0	4	0	1	11	22	10	27
2 $(x = 4)$	15	59	5	8	0	12	20	6	25
3 $(x = 4)$	30	44	8	4	1	7	19	9	28
4 $(x = 5)$	66	3	10	0	2	9	24	17	19
5 $(x = 5)$	46	13	14	1	1	10	22	11	32
6 $(x = 5)$	53	9	20	3	3	10	19	11	22
7 $(x = 6)$	21	37	21	1	0	4	16	22	28
8 $(x = 6)$	38	26	18	6	2	4	34	2	20
9 $(x = 6)$	24	37	21	0	2	9	27	9	21

[23] Our programme initially highlights each choice at the relevant information set with equal probability. Subjects can then change which choice is highlighted with the left/right cursor keys. Once the desired choice is highlighted, subjects decide on it by pressing the Enter key.

Table 5.12. Old Experiment: Terminal Nodes Reached in Rounds 26-50

	A	B	C	D	E	F	G	H	I
1 $(x = 4)$	72	2	3	0	0	13	26	10	24
2 $(x = 4)$	51	13	13	4	2	2	21	17	27
3 $(x = 4)$	27	32	14	3	1	13	15	19	26
4 $(x = 5)$	32	38	7	1	0	13	16	13	30
5 $(x = 5)$	28	34	14	3	0	8	24	7	32
6 $(x = 5)$	50	10	13	1	3	15	17	10	31
7 $(x = 6)$	3	54	20	1	1	11	24	18	18
8 $(x = 6)$	27	33	13	5	0	9	14	19	30
9 $(x = 6)$	7	55	11	5	4	8	21	11	28

Table 5.13. Old Experiment: Terminal Nodes Reached in Rounds 51-55

	A	B	C	D	E	F	G	H	I
1 $(x = 4)$	91	0	0	0	0	10	26	15	38
2 $(x = 4)$	72	16	3	4	2	10	51	6	16
3 $(x = 4)$	69	12	14	3	1	22	25	20	14
4 $(x = 5)$	15	68	8	0	0	16	26	17	30
5 $(x = 5)$	50	31	11	1	0	5	20	12	50
6 $(x = 5)$	62	13	20	2	5	13	20	13	32
7 $(x = 6)$	0	72	19	3	1	3	24	12	46
8 $(x = 6)$	58	17	20	1	8	8	32	10	26
9 $(x = 6)$	16	44	35	7	6	13	25	10	24

Table 5.14. New Experiment: Terminal Nodes Reached in Rounds 1-25

	A	B	C	D	E	F	G	H	I
1	17	45	15	7	5	61	0	0	0
2	4	53	27	7	2	57	0	0	0
3	10	32	28	15	7	58	0	0	0
4	26	16	24	7	8	69	0	0	0
5	4	43	25	6	5	67	0	0	0
6*	28	31	23	5	5	58	0	0	0
7*	15	37	17	17	5	59	0	0	0
8*	14	40	26	3	8	59	0	0	0
9*	9	48	25	15	8	45	0	0	0
10*	30	23	26	17	11	43	0	0	0
$\sum$	157	368	236	99	64	576	0	0	0

Table 5.15. New Experiment: Terminal Nodes Reached in Rounds 26-50

	A	B	C	D	E	F	G	H	I
1	2	49	14	5	1	2	16	6	55
2	1	61	12	13	3	0	2	2	56
3	24	29	34	2	0	2	3	5	51
4	33	21	26	0	1	0	1	4	64
5	17	38	23	6	6	0	14	2	44
6*	35	24	17	8	2	1	9	7	47
7*	36	7	33	4	5	0	2	1	62
8*	10	50	19	2	2	0	1	3	63
9*	5	48	16	5	0	1	7	5	63
10*	43	9	23	5	7	0	7	1	55
$\sum$	206	336	217	50	27	6	62	36	560

Table 5.16. New Experiment: Terminal Nodes Reached in Rounds 51-55

	A	B	C	D	E	F	G	H	I
1	7	59	23	11	3	2	17	1	57
2	0	64	32	10	6	0	2	3	63
3	19	55	20	0	0	1	9	19	57
4	31	22	27	0	0	0	5	6	89
5	16	38	35	7	2	0	26	2	54
6*	39	35	30	2	5	0	15	3	51
7*	25	24	27	0	0	0	0	7	97
8*	7	45	41	4	3	0	2	11	67
9*	0	59	25	7	4	1	7	2	75
10*	32	28	36	13	9	0	15	6	41
$\sum$	176	429	296	54	32	4	98	60	651

CHAPTER 6

CONCLUSION

The aim of this thesis was to document the effect of ambiguity on individual decision-making, especially in situations where individuals interact with other people. We conducted experiments based on games, that were aimed at studying individual behaviour in games. We find that there is indeed sufficient evidence that ambiguity affects decision-making in strategic situations. Moreover, the level of ambiguity individuals face is context-dependant rather than constant across scenarios. We provide a brief summary of our experiments below.

In Chapter Three, we reported the findings of experiments that were conducted to test whether ambiguity influences behaviour in a coordination game. We studied the behaviour of subjects in the presence of ambiguity, in order to determine whether they prefer to choose an ambiguity safe option.

We found that though this ambiguity safe strategy is not played in either Nash equilibrium or iterated dominance equilibrium, it is indeed chosen quite frequently by subjects. This provides evidence that ambiguity aversion influences behaviour in games. While the behaviour of the Row Player was consistent with randomising between her strategies, the Column Player showed a marked preference for avoiding ambiguity and choosing the ambiguity safe strategy.

We expected to observe a correlation between ambiguity averse behaviour in the games and ambiguity attitude in the Ellsberg Urn decision problem. However, we observed only a limited relation between the two choices. On the whole, subjects displayed more ambiguity aversion in Battle of Sexes rounds than in the Ellsberg Urn rounds. This suggests that subjects perceive a greater level of ambiguity in a two-person coordination game, than in a single person decision problem.

Moreover, we note that the results of our experiments are in support of the Dow and Werlang (1994) model of equilibrium under ambiguity, where in the presence of ambiguity players choose their safe strategy, rather than the model of Lo (1996). Lo's equilibrium predictions coincide with the Nash for games with only pure equilibria. Thus, for our game experiments Lo's predictions coincide with Nash equilibrium, while EUA appears to predict the implications of ambiguity better.

In Chapter Four, we reported the findings of experiments conducted to test whether ambiguity had opposite effects on individual behaviour, in games of strategic complements and strategic substitutes. Moreover, we studied whether subjects' perception of ambiguity differed between a local opponent and a foreign one.

Subject behaviour was found to be consistent with our hypothesis, and ambiguity does indeed have opposite effects in strategic complements and strategic substitutes games. Moreover, in the coordination games we find that subjects choose the equilibrium action under ambiguity more often than either of the other actions available to them. Thus, on the whole subjects display ambiguity averse preferences when making decisions in two-person game scenarios.

We expected subjects to display a greater level of ambiguity averse behaviour when faced by a foreign opponent. However, though we observe ambiguity averse behaviour on the whole in the games, we fail to see an escalation in the level of ambiguity when subjects are faced with foreign opponents. Moreover, in the Ellsberg Urn rounds we find that subjects are unwilling to bear even a small penalty in order to stick with the unambiguous choice of ball.

Thus, even though subjects displayed ambiguity averse preferences when faced by other opponents (whether local or foreign), they fail to display sufficient aversion to ambiguity in the single-person decision situations. This is consistent with our findings in Chapter Three, where we found that the ambiguity-attitude of subjects was context-dependent.

In Chapter Five, we report the findings of two series of experiments based on signalling games. We wanted to test two equilibrium concepts against each other, to see which fitted subject behaviour better; and to check whether subjects in the role of the Player 1 could work together to build a collective reputation. However, though subject behaviour did not match either of the Nash equilibria, we did not find any statistically significant evidence that would substantiate the hypothesis of collective reputation.

We discussed how though observed behaviour cannot completely be explained by Nash, it may be explained using alternative equilibrium concepts such as Quantal Response Equilibrium, Cursed Equilibrium and Equilibrium under Ambiguity. However, since the game is one of repeated matching and allows players to update their beliefs, it is more complex to analyse than one-shot games in the presence of ambiguity. There might be a number of other models of bounded rational behaviour that explain our findings, however we leave them for future research.

6.0.0.0.1 Future Research Plans In order to further study the effects of ambiguity on decision-making, we plan to test subject behaviour in settings that compare different groups of a similar race, such as African-Americans and Africans. The purpose of this would be to eliminate racial elements that might affect decision-making, and concentrate on ambiguous beliefs and its affects.

In our experiments we found that though subjects chose ambiguity safe options in the game rounds, they were unwilling to pay even a small penalty to avoid the ambiguous option in Ellsberg urn rounds. In future experiments, we plan to test whether this unwillingness to pay a penalty extends to situations where subjects are given a choice of paying a penalty, in order to avoid facing an ambiguous opponent.

Additionally, so far we have only found experimental evidence that ambiguity does affect decision-making. However, we do not know whether individuals make optimal decisions in the presence of ambiguity. In future studies, we would like to run experiments that test the optimality of individuals' decisions in the face of ambiguity. One possible way of testing this, would be to study how individuals react to ambiguous events such as climate change catastrophes.

There exists a great deal of ambiguity surrounding climate change and the possibility that this climate change could at any point trigger a catastrophe that would cause wide-scale damage. The questions that we would like to ask are: Given the ambiguity surrounding a catastrophic event taking place, are people sufficiently concerned in order to insure themselves against it? Moreover, if given the opportunity to protect themselves against such a catastrophe, do individuals sufficiently insure themselves against it?

Lastly, testing the optimality of individuals' decisions in the face of ambiguity raises an additional question. Should a benevolent dictator/the State intervene, if individuals fail to make optimal decisions in the face of ambiguity? Can State intervention ensure a better outcome? We hope to answer these and other questions, in future research papers.

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