AMBIGUITY IN GAMES: THEORY AND EXPERIMENTS

Submitted by

Sara le Roux

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ABSTRACT

Ambiguity arises when a decision maker fails to assign a subjective probability to an event. This failure to attach a subjective probability to an event is caused by a lack of information about the event. Ambiguity provides a gap in the scope of game theory, since the basic assumption of being able to assign meaningful probabilities to one's opponent's actions is no longer valid. It thus opens the debate of how inviduals would react if faced by an ambiguous event.

Risk is a special case of ambiguity, where the decision maker has information about the probabilities of events. There is considerable experimental evidence documenting the fact that individuals show a marked preference for situations in which they face a known level of risk, as opposed to being in a situation where they are faced by an opponent whose strategies are ambiguous. Ambiguity averseness is the tendency of individuals to prefer known risk situations to ambiguous ones.

Although there is extensive experimental literature which shows that ambiguity affects decision making, most of these studies are restricted to single-person decisions. Relatively few experiments test whether ambiguity affects behaviour in games, where individuals interact with each other. The research documented in this thesis aims to study the effect of ambiguity in games. Since many economic problems can be represented as games we believe this research will be useful for understanding the impact of ambiguity in economics.

Moreover, though previous studies have established that ambiguity affects decision making, they do not document the nature of the impact that it has on decision making. It is thus difficult to predict the effect of ambiguity, and the direction in which it will cause behaviour to change. This thesis aims at studying the effect of ambiguity in strategic situations, by analysing individual behaviour in games.

AUTHOR'S DECLARATION

This thesis has been written under the supervision of Prof. David Kelsey and Prof. Dieter Balkenborg, from the Department of Economics, University of Exeter.

Research on the impact of ambiguity on individual behaviour in coordination games and in games of strategic complements and substitutes, has been conducted under the supervision of Prof. David Kelsey. Research on experiments based on signalling games, has been conducted under the supervision of Prof. Dieter Balkenborg.

It may be noted that some of the data collected from the experiments based on signalling games had earlier been studied as part of a Master's dissertation titled "An Experiment on Collective Reputation Effects", that was submitted to the University of Exeter. However, additional experimental sessions and research has been conducted, which has contributed towards this doctoral thesis.

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