# MODELLING OF COORDINATING PRODUCTION AND INVENTORY CYCLES IN A MANUFACTURING SUPPLY CHAIN INVOLVING REVERSE LOGISTICS

Submitted by

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

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# Abstract

In today's global and competitive markets selling products at competitive prices, coordination of supply chain configuration, and environmental and ecological consciousness and responsibility become important issues for all companies around the world. The price of products is affected by costs, one of which is inventory cost. Inventory does not give any added value to products but must be kept in order to fulfill the customer demand in time. Therefore, this cost must be kept at the minimum level. In order to reduce the amount of inventory across a supply chain, coordination of decisions among all players in the chain is necessary. Coordination is needed not only for a two-level supply chain involving a manufacturer and its customers, but also for a complex supply chain of multiple tiers involving many players. With increasing attention being placed to environmental and ecological consciousness and responsibility, companies are keen to have a reverse supply chain where used products are collected and usable components remanufactured and reused in production to minimize negative impacts on the environment, adding further complexity to decision making across a supply chain.

To deal with the above issues, this thesis proposes and develops the mathematical models and solution methods for coordinating the production inventory system in a complex manufacturing supply chain involving reverse logistics and multiple products. The supply chain consists of tier-2 suppliers for raw materials, tier-1 suppliers for parts, a manufacturer who manufactures and assembles parts into finished products, distributors, retailers and a third party who collects the used products and returns usable parts to the system. The models consider a limited contract period among all players, capacity constraints in transportation units and stochastic demand. The solution methods for solving the models are proposed based on decentralized, semi-centralized and centralized decision making processes.

Numerical examples are used by adopting data from the literature to demonstrate, test, analyse and discuss the models. The results show that centralised decision making process is the best way to coordinate all players in the supply chain which minimise total cost of the supply chain as a whole. The results also show that the selection of the length of limited horizon/ contract period will be one of the main factors which will determine the type of coordination

(decentralised, centralised or semi-centralised) among all players in the supply chain. We also found that the models developed can be viewed as generalised models for multi-level supply chain by examining the models using systems of different tiers from the literature. We conclude that the models are insensitive to changes of input parameters since percentage changes of the supply chain's total cost are less than percentage changes of input parameters for the scenarios studied.

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***I specially dedicate this thesis to my big family*** **(*my mother and father, my wife and children, my parents in law, my brothers and sisters and others*).**

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**Jonrinaldi** and Zhang, D.Z., An integrated production and inventory model for a whole manufacturing supply chain involving reverse logistics with finite horizon period, OMEGA (2012), http://dx.doi.org/10.1016/j.omega.2012.07.001

**Jonrinaldi** and Zhang, D.Z., 2012, Optimal integrated production and inventory cycles over an Entire green manufacturing supply chain network with coordination, *Journal of Japan Industrial Management Association (JIMA),* Vol. 62 No. 6E. http://www.jimanet.jp/journal/journal-of-jima/paper/vol62.

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# Chapter 1

# Introduction

## 1.1 Introduction

In this chapter, an introduction to overall study is given. In section 1.2, the background of the research is summarised. We discuss major issues which are related to this research. The statement of the problem in this research including some literatures which are related to the problem is described in section 1.3. The proposed approach in this research is described in section 1.4. The aims and objectives of the research are presented in section 1.5. Section 1.6 outlines thesis organisation. A concise summary of the thesis chapters is presented in section 1.7.

## 1.2 Background

In today's global and competitive markets, selling products at competitive prices and quality, coordination of supply chain configuration, and environmental and ecological consciousness and responsibility have become important issues for all companies around the world. The global competitive market forces all companies around the world to sell their products at competitive prices in order to win competition in which they participate. All these aspects will affect the sustainability of the companies.

To achieve competitive prices, every company has to conduct the production and operations of their products in effective ways and at efficient costs. One of the costs affecting the price of products is inventory costs. Inventory cost is a number of costs not giving added value to the product but must be presented to assure that the product can fulfill the customer demand at any time at which the customer needs it so that this cost must absolutely be presented but must be at the minimum level. Based on data surveys in United States (U.S.) from 2006-2010 total inventories values per year are \$ 306,792 millions, \$ 315,011 millions, \$ 330,826 millions, \$ 274,286 millions and \$ 297,824 millions, respectively for durable goods (motor vehicle, furniture, construction materials, electronics products, electrical products, machinery equipments) with ratio to sales being 10.80%, 10.87%, 11.67%, 11.89% and 11.38%, respectively[1]. Inventory costs commonly consist of ordering cost, carrying or holding cost (Tersine, 1994). However, in complex environments and for complex products, there are also other costs which are included in the inventory cost such as a stock out cost, backorder cost, pipeline cost, transportation cost and deterioration cost. Therefore, they must be reduced as much as possible as they affect the price of products. To reduce the inventory costs, a company needs to manage inventory level. Inventory management includes when the products are ordered and how many products will be ordered per order cycle. Increasing in order quantity makes inventory cost higher, however decreasing in order quantity may not fulfill the customer demand at all so that customer service level is low. Therefore, we need to determine the optimal order cycle time and order quantity to minimize the total inventory cost.

Due to competition on price which has been described above, most companies in the world do not manage all process of producing the products from raw materials to finished products by themselves. High investment to the facilities used to produce the products and short life cycle of the products are their considerations not to do that. Many companies just produce some parts and order other parts from other companies to be manufactured and assembled into finished products, especially in the electrical and manufacturing industries. To achieve success in market, each company needs to collaborate or partner with others. To reduce the system cost of this collaboration an integrated coordination between companies is absolutely necessary. The group of companies which has the cooperation between them is named as a supply chain. A typical simple supply chain can consist of three levels of companies, a buyer, a manufacturer and a supplier. The buyer orders the product from the manufacturer and then the manufacturer orders raw materials from the supplier to be manufactured and assembled into products. Simchi-Levi et al. (2007) defined supply chain management as a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements..

In the context of inventory management, each player in the supply chain has to manage their inventory in order to minimize their own costs. The buyer manages their inventory in order to minimize their own cost. Similarly, the

[1] 2012 US Census Bureau

manufacturer and the supplier do the same way. Since the decisions due to order quantity and order or production cycle time among them may be different coordination of the supply chain is needed. The buyer can coordinate their orders with the manufacturer and the supplier simultaneously or they can coordinate their decisions in a sequential process, i.e., the buyer coordinates its inventory decisions with the manufacturer and then the manufacturer coordinates its inventory decisions with the supplier. The objective of this coordination is to minimize the system or supply chain inventory cost. This inventory system is named as multi-echelon inventory system. However, coordination in a supply chain is not only limited to this problem. The coordination of the supply chain can be more than three levels (the buyer, the manufacturer, and the supplier) and three players. It can be a complex supply chain consisting of multiple suppliers, the manufacturer, multiple distributors and multiple buyers managing multiple items (raw materials, parts and products). It also includes other issues such as transportation cost and reverse logistics. Therefore, a comprehensive approach is needed to manage the supply chain as a whole.

Finally, due to environmental and ecological consciousness and responsibility, competitive pressure, shortened life cycle, collaboration and smart use of resources in supply chain are becoming more important. Companies are trying to reuse, remanufacture and recycle used products to reduce the negative impacts on environment as well as the costs of the product. Many products such as metal scrap brokers, waste paper recycling, car parts remanufacturing, reusable packaging, electronics scrap recycling and deposit system for soft drinks bottles are examples for this. In these cases the recovery of used products is economically more attractive than disposal. Based on surveys in UK, carbon dioxide emissions in 2008 reach 228,137 kilo tones for industry and commercial. Local authority collected waste disposal for period 2009/10 reaches 32,496 thousand tones (49% landfill, 11% incineration with energy recovery, 39% recycled/ composted) and recycling of household waste for period 2007/08 reaches 1537.2 kilograms per household per year[2].

In addition to enhanced environmental performance and a ‘green’ image, product recovery may also prove beneficial due to savings in material, manufacturing, and disposal costs. Hence, reverse manufacturing and logistics problems which are strongly related to all players in the supply chain to achieve

[2] 2011 UK Office for National Statistics

the competitive price and environmental and ecological consciousness and responsibility, are critical problems to companies around the world.

## 1.3 Statement of Problem

As mentioned before, inventory cost influences significantly the cost of producing the product of a company as well as the supply chain total cost. Therefore, every company in the supply chain has to manage integrally with other companies' inventory to minimize the supply chain cost and achieve the higher performance of the supply chain with competitive prices. To minimize supply chain cost regarding the inventory cost, many models have been developed to determine the optimal inventory since the first classical economic order quantity (EOQ) was found by Harris (1915) and derived independently by Wilson (1934). This model has been used widely in inventory management to determine order quantity and interval of the products and control the inventory positions over time period. However, this model can just be applied in a single facility or stage or company. In today's competitive environment where each company must collaborate with others in the supply chain to achieve high performance this model has to be extended to apply in such system.

To address this interesting issue many models have been developed to solve the problem. Goyal (1976) firstly developed and extended the EOQ model to be applied in coordinating production and inventory system in the system. The model only considered one buyer and one vendor with infinite production rate of the product of the manufacturer. This is the simplest model for coordinating production and inventory decisions in the supply chain. Since this model has some assumptions which is restrictive in nature, subsequent works have been carried out to relax and eliminate such assumptions such as Banerjee (1986) who considered finite production rate in the model and Goyal (1988) who extended Banerjee's model by implementing the integer multiple of order cycle time of the retailer (buyer) to obtain the production cycle time of the manufacturer.

The increase of interest and extension in the supply chain such as the level of coordination, a complex supply chain, have forced practitioners and researchers to develop an inventory model which can be applied for such a system. A complex supply chain can consist of suppliers, manufacturer, distributors, retailers and a third party collecting used products from end customers. Simultaneously with the level of the coordination, many aspects such as multiple items and products, finite lifetime and deteriorating products, reverse logistics, transportation costs, contract period among all players in the supply chain, credit option and quantity discount and delay in payments have played an important role in the supply chain coordination and integration. Little research has been carried out to model an integrated production and inventory system for such a supply chain.

Again, unfortunately, the researches only considered part of these aspects and part of a complex supply chain such as Chung et al. (2008) who developed a model to determine optimal policy in a multi-echelon supply chain inventory system with remanufacturing. This model had considered multi-player in supply chain that are supplier, manufacturer, retailer and the third-party recycle dealer returning used products to the manufacturer for remanufacturing. But, in this model, there is only single product which is processed from material supplied by the supplier. Moreover, returned used products are also remanufactured by manufacturer only. In fact, there are also some used products which are returned to the supplier for repairing or reproducing again. Also, Gou et al. (2008) developed a model to determine an optimal joint inventory for an open-loop reverse logistics. This model focuses on an open-loop reverse supply chain, which includes a single centralized returns center and multiple local collection points. This model only considered returned used products. Joint inventory policy for the new products was not considered in the model, whereas used products and new products must be integrally considered in a model to result lower costs and the best performance for the supply chain.

For a multi-level integrated production and inventory model without reverse logistics, little research has been carried out. Chung and Wee (2007) considered a three-level supply chain with backordering. Also Ganeshan (1999) considered three-level supply chain with multiple retailers, one warehouse and multiple suppliers. Jaber and Goyal (2008) developed a model with multiple suppliers, a vendor and multiple buyers. Chen and Kang (2007) and Zhang et al. (2007) considered one vendor and one buyer. Kim et al. (2006) also considered a two-level supply chain in the model including different aspects such as quality of product and quantity discounts. However, these models considered only some aspects and the part of a complex supply chain.

As briefly described above, most research which has been carried out in the area of integrated production and inventory model in complex multi-level supply chains just considered part of the complex system and/or some aspect of that system as we have mentioned above. A typical complex supply chain involving reverse logistics consists of many levels of the players and aspects. The typical complex supply chain can consists of tier-2 suppliers who produce multiple raw materials supplied to tier-1 suppliers, tier-1 suppliers who produce multiple parts or components supplied to a manufacturer, the manufacturer who manufactures and assembles parts into multiple finished products and delivers them to distributors, distributors who deliver finished products to retailers, retailers who sell finished products to end customers, and the third party collecting reusable used products from end customers and returning parts and materials back to the manufacturer and/or suppliers (tier-1 and/or tier-2) depending on the condition of the used products. If used products only need to be manufactured or reused, they will be returned to manufacturer, otherwise they will be returned to suppliers to be recycled into new products. The coordination of such a system is the problem of coordinating production and inventory system among all players in a complex manufacturing supply chain in order to minimize the system inventory cost. Fig. 1.1 illustrates the system under consideration.

Similarly as with simpler supply chain systems, the problem of coordinating a typical complex supply chain could be described as follows:

*Given a manufacturing company that manufactures and assembles many types of finished products and given two levels of suppliers that supply the raw materials and parts, distributors that deliver finished products to retailers, retailers that sell products to end customers, and the third party who collects the used products to be returned back to the system, the manufacturer and/or the suppliers, determine*:

- *When should items, i.e., raw materials, parts, finished products, involving reverse logistics, be produced and/or ordered by a company from other companies in the supply chain system in a finite horizon period?*
- *How many items should the companies order and/or produce for every order and/or production cycle time?*
- *How many units of transportation should they use to deliver the items from upstream level to downstream for the new materials, parts and products, and from the third party to the system for returned parts and used products?*

such that the supply chain's total inventory cost including transportation and the third party's inventory cost is minimized.

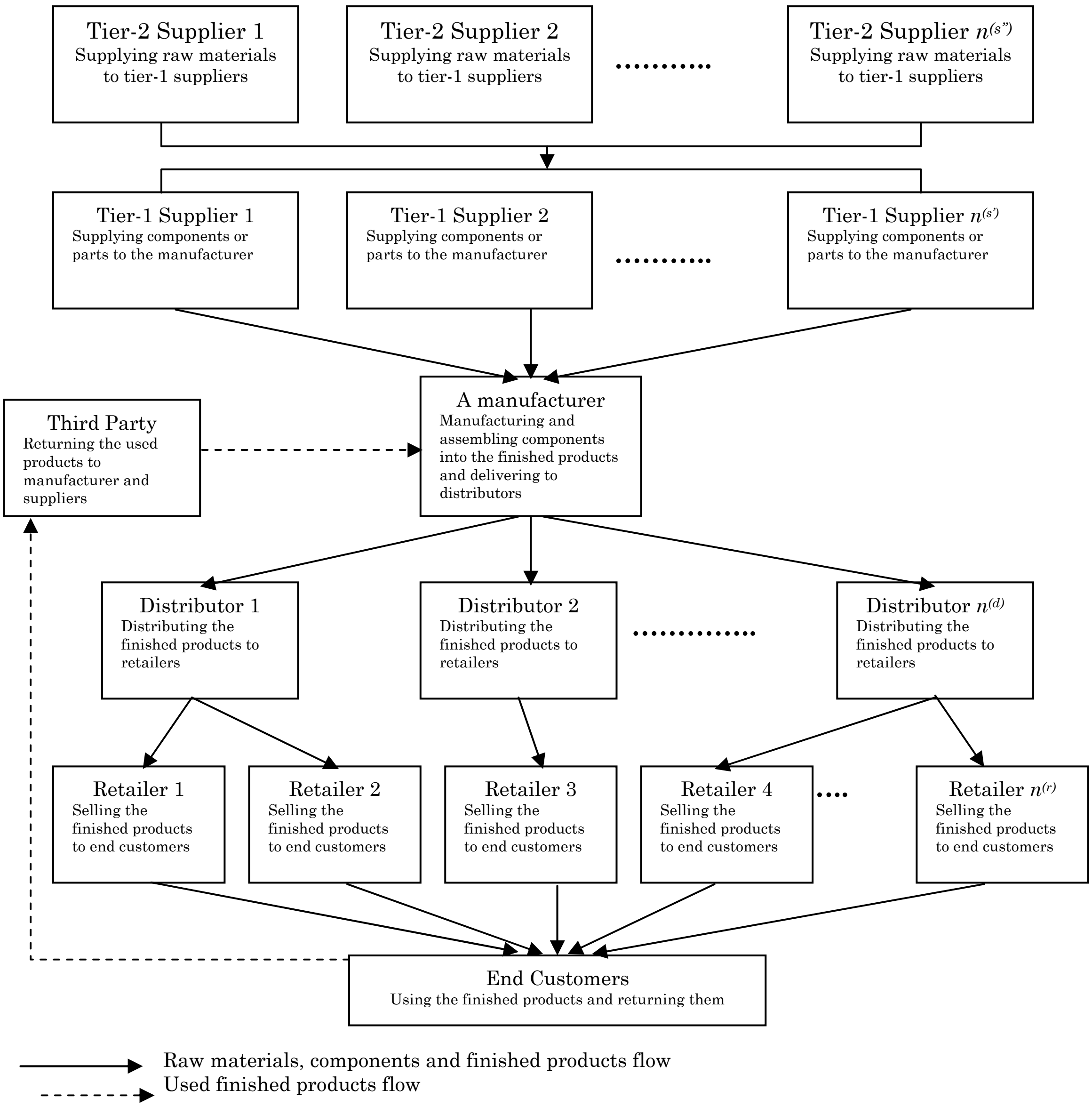


Figure 1.1 Network representation of a complex manufacturing supply chain with reverse logistics

The supply chain's total inventory cost is the sum of each company's inventory cost commonly consisting of ordering cost, setup cost and holding cost. The ordering cost is the cost to order items from other companies. The cost includes any costs for ordering product such as vendor analysis, writing an order, receiving material, inspecting material, and any costs in order transaction. The setup cost is the cost to prepare the production process of the items. The holding/carrying cost is any costs related to investment in inventory and maintenance of product in warehouse. This cost includes cost of capital, tax, insurance, product handling, warehouse, deterioration of products. The transportation cost is any cost incurred related to delivering items from one location to another.

## 1.4 Proposed Approach

The approach proposed for determining the optimal solution to coordinating production and inventory of a typical complex supply chain involving reverse logistics, consisting of tier-2 suppliers, tier-1 suppliers, the manufacturer, distributors, retailers and a third party collector of used products, to minimize the whole supply chain's total cost is based on mathematical modelling. The model is built to describe every relevant cost incurred by every company or player in the supply chain to manufacture and supply items (raw materials, parts and products) in the limited contract period among all players subject to capacity constraints of the transportation units. We model functions of every relevant cost incurred by every company based on the classical economic order quantity (EOQ) and economic production/ batch quantity (EPQ) and works that have been carried out in previous research, with parameters and decision variables related to those of other companies. The sum of all companies' total relevant cost functions including transportation cost is the whole supply chain's total cost function.

In order to solve the model, we propose and develop solution methods based on centralized, semi-centralized and decentralized (sequenced) decision making processes. Under centralized decision making process, the optimal solution of the model for the whole supply chain is obtained simultaneously. We use a mixed integer nonlinear programming method to solve such a problem. Under a decentralized decision making process, the optimal solution of the model is obtained by a sequencing process. Downstream level players of the supply chain solve their problems first and upstream level players follow this process in sequence by adopting the optimal solution of immediate downstream level players. Semi-centralized decision making process is the combination of both processes described above.

## 1.5 Aim and Objectives of the Research

The aim of this research is to establish models to determine coordinated and integrated production and inventory decisions in a complex manufacturing supply chain involving reverse logistics in the limited contract period among all players subject to capacity constraints of the transportation units.

The objectives of this research are as follows:

1. to build mathematical models for coordinating and integrating production and inventory decisions among all players in a complex manufacturing supply chain

involving reverse logistics in order to minimize the supply chain's total cost in the limited contract period subject to capacity constraints of transportation units, and

2. to develop and propose solution methods of such models based on centralized, semi-centralized and decentralized decision making processes.

## 1.6 Thesis Organisation

An overview of the thesis chapters is given as follows:

- Chapter 1: Introduction. The first chapter introduces the research, describes the context of supply chain management, inventory management and reverse logistics, summarizes the statement of the problem and proposed approach of the study, sets the aims and objectives of the study and overviews the structure of the thesis.
- Chapter 2: Supply Chain Management. In this chapter, concepts about Supply Chain (SC) and Supply Chain Management (SCM) are defined. Formal definition of SCM is given and some key issues in supply chain management are discussed, especially on inventory management and distribution network configuration and strategies.
- Chapter 3: Managing Inventory in Supply Chains: a Literature Review. In this chapter, the literature review about production inventory models in supply chain management is provided. The integrated production and inventory models in supply chain management is divided into three categories, buyer-vendor coordination models including a single buyer and a single vendor, a single vendor and multiple buyers, multiple vendors and a single buyer, three-level supply chain coordination models and multi-level supply chain coordination models. The review of models involving reverse logistics in the supply chains is also provided.
- Chapter 4: Mathematical Modelling of Inventory System in a Complex Manufacturing Supply Chain. In this chapter, we build mathematical models of the integrated production and inventory model in the whole manufacturing supply chains. We summarize and describe the assumptions and notations of the model and then formulate the mathematical models. The models are derived based on two policies, independent and coordinated policies. Under independent policy, each player determines their optimal decisions without considering other companies. Under coordinated policy, all players in the supply chain determine their optimal decision integrally and simultaneously.

- Chapter 5: Considering Reverse Logistics, Transportation, and Limited Horizon Period in the Manufacturing Supply Chain Model. The aim of this chapter is to consider and involve aspects which have not been included yet in the previous chapter in the model. They are reverse logistics, transportation costs which are separated from order processing cost caused by the capacity constraint of the transportation units, and limited contract period among all players. These aspects result in different optimal solutions as well as different total cost of each player and of the whole supply chain.
- Chapter 6: Solution Methods. In this chapter, we describe the solution methods developed and proposed. We divide them based on three decision making processes into three solution procedures, centralized, semi-centralized and decentralized decision making processes.
- Chapter 7: Analysis and Discussion. In order to test and validate the models, we test the models with two numerical examples. We analyse the results obtained. To examine the validity and the generality of the model we test the models using different data from the literature.
- Chapter 8: Conclusions and Future Work. In the last chapter, the conclusions of this research are summarized. Finally, suggestions for future work of this research are proposed.

## 1.7 Summary

This introduction chapter provides background of this research describing important issues relating to this research such as competitive prices, coordination in the supply chain and environmental and ecological consciousness and responsibility. The specific problem of this research is described as: given a complex manufacturing supply chain consisting of tier-2 suppliers, tier-1 suppliers, the manufacturer, distributors, retailers and the third party, how to manage integrally a production and inventory system involving reverse logistics to minimise the supply chain's total cost in a limited horizon/ contract period subject to capacity constraints of transportation units. The aim of the research is defined as: to establish models to determine coordinated and integrated production and inventory decisions in a complex manufacturing supply chain involving reverse logistics subject to the limited contract period among all players and to capacity constraint of the transportation units. The objectives of the research are stated and thesis organization is given. Finally a concise summary of each chapter is provided.

# Chapter 2

# Supply Chain Management

## 2.1 Introduction

The aim of this chapter is to give an overview of the supply chain management concepts and theory including definitions and key issues in managing supply chains. In section 2.2, the formal definitions of supply chain and supply chain management are given. We also present the objectives of the supply chain. Then, three levels of decision making including strategic level, planning level and operations level are discussed. We also discuss three macro processes to manage the flows of products, information and funds in the supply chain. Lastly, we mention some reasons and challenges about why supply chains are difficult to manage and integrate.

Next, we discuss some key issues in supply chain management in section 2.3. Those key issues are distribution network configuration, inventory control, production sourcing, supply contracts, distribution strategies, supply chain integration and strategic partnering, outsourcing and offshoring strategies, product design, information technology and decision-support systems, customer value, and smart pricing. Some of the key issues such as inventory management, transportation decisions in distribution network configuration are considered in this thesis. Lastly, section 2.4 summarises the chapter.

## 2.2 Definitions

In today's global markets, every company is always in competitive environments. They have to sell products and/ or services at competitive prices and quality and with short lead time. On the other hand, the reduction of product life cycles, the increased expectations of customers, and the high investment involved in manufacturing products from raw materials and delivering to end customers have forced each company to collaborate with others in producing and delivering products. The collaboration among companies establishes a supply chain. Chopra and Meindl (2004) defined a supply chain as consisting of all companies involved, directly and indirectly, in fulfilling a customer request. The supply chain not only includes the manufacturer and suppliers, but also transporters, warehouses, distributors, retailers, and customers themselves. Mentzer et al. (2001) with a little difference also defined a supply chain as a set of three or more entities (organizations or individuals) directly involved in the upstream and downstream flows of products, services, finances, and/ or information from a source to a customer. The supply chain is also referred to as the logistic network (Simchi-Levi et al., 2007). Within each organisation, supply chain includes all functions involved. These functions include, but are not limited to, new product development, marketing, operations, distribution, finance, and customer service. A supply chain is dynamic and involves the constant flow of information, product, and funds between different stages. The supply chain stages are illustrated in Fig. 2.1.

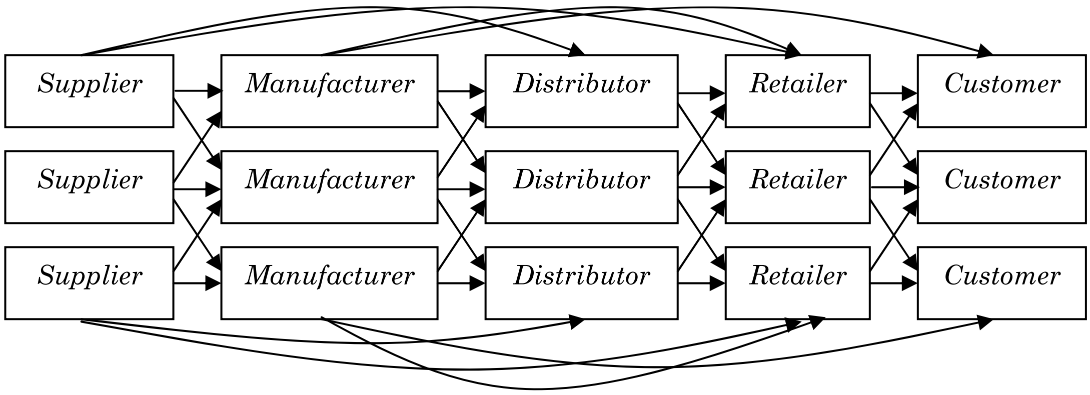


Figure 2.1: Supply chain stages. Source: Chopra and Meindl (2004), page 5

Fig. 2.1 shows the general stages of the supply chain. For a typical supply chain we do not need to present all stages described above. The design of the typical supply chain will depend on both customers' needs and the roles of the stages involved. A typical supply chain could consist of raw material suppliers, component suppliers, a manufacturer, distributors, retailers and end customers. Raw materials are procured from suppliers supplying to other level suppliers producing parts/ components. The components and parts are then supplied to the manufacturer to produce products which are shipped to warehouses for intermediate storage. Finally, the products are shipped to retailers or customers. Since many companies are involved in a supply chain, effective strategies must consequently take into account interactions at various levels in the supply chain to reduce cost and improve service levels.

To create an effective supply chain, we need to manage the supply chain to achieve the objective of the supply chain. The objective of a supply chain is to maximize the overall value generated. Supply chain management (SCM) could be defined as follows (Simchi-Levi et al., 2007):

"SCM is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements."

This definition leads to several observations. First, supply chain management considers every facility that has an impact on cost and plays a role in making the product to conform to customer requirements, from supplier and manufacturing facilities through warehouses and distribution centres to retailers and stores. Second, the objective of supply chain management is to be efficient and cost-effective across the entire system; total system wide costs, from transportation and distribution to inventories of raw materials, work-in process and finished products, are to be minimised. Finally, supply chain management encompasses the company's activities at many levels, from strategic level through the tactical to the operational level. For a general and wider system, Mentzer et al. (2001) defined supply chain management (SCM) as the systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole.

In order to successfully manage the supply chain, decisions are related to the flow of information, product and funds. These decisions fall into three categories or phases depending on frequency and time frame of each decision. These three categories are explained as follows:

- *Supply chain strategy or design.* In this category, a company makes decisions about how to structure the supply chain over the next several years. These decisions are what the chain's configuration will be, how resources will be allocated, and what processes each stage will perform. They include the location and capacities of production and warehousing facilities, the products to be manufactured or stored at various locations, the modes of transportation to be made available along different shipping legs, and the type of information system to be utilized. The time period for decisions in this category is a long-term planning horizon (1 year or more).

- *Supply chain planning.* In this category, the time period for decisions is a quarter to a year (medium-term planning horizon). Companies make decisions about demand in different markets through forecasting. Planning phase includes which markets will be supplied from which locations, the subcontracting of manufacturing, the inventory policies to be followed, and the timing and size of marketing promotions. Companies must include uncertainty in demand, exchange rates, and competition over this time horizon in their decisions. Planning establishes parameters within which a supply chain will function over a specific period of time. As a result of the planning phase, companies define a set of operating policies that govern short-term operations.
- *Supply chain operations.* The time horizon for this category is weekly or daily. Companies make decisions regarding individual customer orders. At the operational level, strategy and planning policies are already defined. The goal of this phase is to handle incoming customer orders in the best possible manner such as allocating inventory or production to individual orders, setting a date when an order is to be filled, generating pick lists at a warehouse, allocating an order to a particular shipping mode and shipment, setting delivery schedules of trucks, and placing replenishment orders.

The design, planning and operation of a supply chain have a strong impact on overall profitability and success.

Since a supply chain is a sequence of processes and flows that take place within and between different stages and combine to fill a customer need for a product there are two different ways to view the processes performed in the supply chain (Chopra and Meindl, 2004).

- *Cycle view.* In this view, the processes in a supply chain are divided into a series of cycles, each performed at the interface between two successive stages of the supply chain. This view of the supply chain is very useful when considering operational decisions because it clearly specifies the roles and responsibilities of each member of the supply chain.
- *Push/pull view.* The processes in the supply chain are divided into two categories depending on whether they are executed in response to a customer order or in anticipation of customer orders. Pull processes are initiated by a customer order whereas push processes are initiated and performed in anticipation of customer orders.

There are three macro processes to manage the flow of information, product, and funds required to generate, receive, and fulfil a customer request (Chopra and Meindl, 2004). The three macro processes are described as follows:

- Customer Relationship Management (CRM). All processes that focus on the interface between the company such as the manufacturer and its customers. This macro process aims to generate customer demand and facilitate the placement and tracking of orders. It includes processes such as marketing, sales, order management, and call centre management.
- Internal Supply Chain Management (ISCM). All processes which are internal to the company. It aims to fulfil demand generated by the CRM process in a timely manner and at the lowest possible cost. ISCM processes include the planning of internal production and storage capacity, preparation of demand and supply plans, and internal fulfilment of actual orders.
- Supplier Relationship Management (SRM). All process that focus on the interface between the company and its suppliers. It aims to arrange for and manage supply sources for various goods and services. SRM processes include the evaluation and selection of suppliers, negotiation of supply terms, and communication regarding new products and orders with suppliers.

| Supplier | Company | Customer |
|---|---|---|
| **SRM** | **ISCM** | **CRM** |
| • Source<br>• Negotiate<br>• Buy<br>• Design Collaboration<br>• Supply Collaboration | • Strategic Planning<br>• Demand Planning<br>• Supply Planning<br>• Fulfilment<br>• Field Service | • Market<br>• Sell<br>• Call Centre<br>• Order Management |

Figure 2.2: Supply chain macro process. Source: Chopra and Meindl (2004), page 17

As mentioned in the definition of SCM integration among supply chain entities is the key factor for significantly reducing costs and improving service levels. Unfortunately, supply chain integration is difficult for the following reasons (Simchi-Levi et al., 2007):

1. Supply chain strategies cannot be determined in isolation. They are directly affected by another chain that most organizations have, the development

chain that includes the set of activities associated with new product introduction. At the same time, supply chain strategies also should be aligned with the specific goals of the organizations, such as maximising market share or increasing profit. Therefore, to manage the supply chain needs a comprehensive and integrated approach.

2. It is challenging to design and operate a supply chain so that total system-wide costs are minimised, and system-wide services levels are maintained. Indeed, it is frequently difficult to operate a single facility so that costs are minimised and service level is maximised. The difficulty increases exponentially when an entire system is being considered. The process of finding the best system-wide strategy is known as global optimisation.
3. Uncertainty and risk are inherent in every supply chain; customer demand can never be forecast exactly, travel times will never be certain, and machines and vehicles will break down. Recent industry trends, including outsourcing, offshoring, and lean manufacturing that focus on reducing supply chain costs, significantly increase the level of risk in the supply chain.

Furthermore, since a supply chain consists of many different entities with own objectives it is difficult to find the best system-wide or global optimal solution. Some factors make this a challenging problem.

1. A supply chain might be a complex network of facilities and organizations. These organisations might be dispersed over a large geography, and in many cases, all over the globe. They should find the best supply chain strategy for a particular company.
2. Different facilities in the supply chain frequently have different, conflicting objectives. For example, suppliers typically want manufacturers to commit themselves to purchasing large quantities in stable volumes with flexible delivery dates. On the other hand, although most manufacturers would like to implement long production runs, they need to be flexible to their customers' needs and changing demands. Thus, the suppliers' goals might be in direct conflict with the manufacturers' desire for flexibility. Similarly, the manufacturers' objective of making large production batches typically conflicts with the objective of both warehouses and distribution centres to reduce inventory. To make matters worse, this latter objective of reducing

inventory levels typically implies an increase in transportation costs because if inventory levels reduced the number of orders increase so that transportation costs will increase too.

3. A supply chain is a dynamic system that evolves over time. Not only do customer demand and supplier capabilities change over time, but supply chain relationships also evolve over time.
4. System variations over time are also important considerations. Even when demand is known precisely, the planning process needs to account for demand and cost parameters varying over time due to the impact of seasonal fluctuations, trends, advertising and promotions, competitors' pricing strategies, and so forth. The time-varying demand and cost parameters make it difficult to determine the most effective supply chain management strategy to minimise system wide costs and conform to customer requirements.

## 2.3 Key Issues in Supply Chain Management

Since SCM deals with how to manage and control the flows of product, information, and funds in the supply chain, there are some key issues that we need to be concerned what in achieving the objective of the supply chain. Some key issues of SCM are distribution network configuration, inventory control, production sourcing, supply contracts, distribution strategies, supply chain integration and strategic partnering, outsourcing and offshoring strategies, product design, information technology and decision-support systems, customer value, and smart pricing. These issues focus on either the development chain or the supply chain and achieving a globally optimised supply chain or managing risk and uncertainty in the supply chain, or both as shown in Table 2.1. We discuss some of the key issues which are related to this research below.

Table 2.1 Key supply chain management issues. Source: Simchi-Levi et al. (2007), page 15

| | Chain | Global optimization | Managing risk and uncertainty |
|---|---|---|---|
| Distribution network configuration | Supply | Yes | |
| Inventory control | Supply | | Yes |
| Production sourcing | Supply | Yes | |
| Supply contracts | Both | Yes | Yes |
| Distribution strategies | Supply | Yes | Yes |
| Strategic partnering | Development | Yes | |
| Outsourcing and offshoring | Development | Yes | |
| Product design | Development | | Yes |
| Information technology | Supply | Yes | Yes |
| Customer value | Both | Yes | Yes |
| Smart pricing | Supply | Yes | |

### 2.3.1 Distribution network configuration and strategies

Distribution network configuration may involve issues relating to plant, warehouse, and retailer location. Some key strategic decisions with this configuration are as follows:

- Determining the appropriate number of warehouses
- Determining the location of each warehouse
- Determining the size of each warehouse
- Allocating space for products in each warehouse
- Determining which products customers will receive from each warehouse

Distribution refers to the steps taken to move and store a product from the supplier stage to a customer stage in the supply chain (Chopra and Meindl, 2004). In designing distribution network, we should evaluate along two dimensions of customer needs that are meeting customer needs and cost of meeting customer needs. Factors relevant in designing a distribution network are as follows:

- Response time. This is the time between when a customer places an order and receives delivery. This time is also named as lead time.
- Product variety. This is the number of different products/ configurations that a customer desires from the distribution network.
- Product availability. This is the probability of having a product in stock when a customer order arrives.
- Customer experience. It includes the ease with which customer can place and receive their order and purely experiential aspects such as the possibility of getting a cup of coffee and the value that the sales staff provides.
- Order visibility. This is the ability of the customer to track their order from placement to delivery.
- Returnability. This is the ease with which a customer can return unsatisfactory merchandise and the ability of the network to handle such returns.
- Inventories costs. This cost does not add value to the product. But, we have to keep this to fulfill demand when an order comes.

- Transportation costs. This cost is any costs incurred to process and deliver products to other company.
- Facilities and handling costs. This cost is incurred to process products at warehouse or storage.

Each factor mentioned above has trade-offs with other factors. For example, if a company focuses on short response time, it must have facilities close to customers. Thus, the companies must determine which strategy they have to use to respond to customers' needs.

In today's competitive market, a modern distribution network design needs to deal with the trade-offs between a variety of factors. Romeijn et al. (2007) listed some factors consisting of:

- Location and associated (fixed) operating cost of distribution centres (DCs)
- Total transportation cost
- Storage holding and replenishment costs at DCs and retailers
- Stock outs by setting appropriate levels of safety stocks
- Capacity concerns, which may affect operating costs in the form of congestion costs

Distribution occurs between every pair of stages in the supply chain. The objective of this issue is to design or reconfigure the logistics network so as to minimise system-wide costs, including production and purchasing costs, inventory holding costs, facility costs (storage, handling, and fixed costs), and transportation costs, subject to a variety of service level requirements.

This network configuration involves a large amount of data, including information on

- Location of customers, retailers, existing warehouses and distribution centres, manufacturing facilities and suppliers.
- All products, including volumes, and special transport modes (e.g refrigerated)
- Annual demand for each product by customer location
- Transportation rates by modes ( truckload, referred to as TL and less than truckload, referred to as LTL)

- Warehousing costs, including labor, inventory carrying charges, and fixed operating costs and warehouse capacities
- Shipment sizes and frequencies for customer delivery
- Order processing costs
- Customer service requirements and goals.

To achieve the objective of distribution network configuration, we can use a certain distribution strategy to distribute products or item to customers (Simchi-Levi et al., 2007). Some distribution strategies are discussed below.

- *Direct shipment*. In this strategy, items or products are directly shipped from the supplier to the retailer stores without going through distribution centres. The advantages of this strategy are that retailers avoid the expenses of operating a distribution centre. Otherwise, the disadvantages are risk-pooling effects, the manufacturer and distributor transportation costs increase. This strategy is common if the retailer store requires fully loaded trucks, which implies that the warehouse does not help in reducing transportation cost. This is also common if the lead time is critical and the retailer has bargaining power.
- *Warehousing*. This is the classical strategy in which warehouses keep stock and provide customers with items or products as required.
- *Cross-docking*. In this system warehouses function as inventory coordination points rather than as inventory storage points. Goods arrive at warehouses from the manufacturer, are transferred to vehicles serving the retailers and are delivered to the retailers as rapidly as possible. Goods might spend less than 12 hours in the warehouses. This strategy needs significant and difficult efforts to manage. For example, a fast and responsive transportation system is necessary for a cross-docking system to work. Forecasts are critical, necessitating the sharing of information. Distribution centres, retailers and suppliers must be linked with advanced information system to ensure that all pickups and deliveries are made within the required time windows.

Moreover, in designing a distribution strategy for the supply chain, we also need to determine transportation decisions to support it. Transportation refers to the movement of product from one location to another as it makes its way from the beginning of a supply chain to the customer's hands (Chopra and Meindl, 2004). Any costs affecting transportation decisions are as follows:

- *Vehicle-related cost*. This is the cost a carrier incurs for the purchase or lease of the vehicle used to transport products.
- *Fixed operating cost*. This includes any cost associated with terminals, airport gates, and labour that are incurred whether vehicles are in operation or not.
- *Trip-related cost*. This cost includes the price of labour and fuel incurred for each trip independent of the quantity transported.
- *Quantity-related cost*. This includes loading/unloading costs and a portion of the fuel cost that varies with the quantity being transported.
- *Overhead cost*. This includes the cost of planning and scheduling a transportation network as well as any investment in information technology.

All above costs are considered in this thesis in ordering cost and transportation cost.

Furthermore, supply chain network design is the next important step relating to distribution network configuration. In supply chain network design we consider different general strategies for the operation of a centralized supply chain network versus decentralized operation, alternative ways to utilize warehouse and strategies to eliminate them completely and different approaches to meeting customer demand. In a centralized system, decisions are made at a central location for the entire supply chain network. The objective of this strategy is to minimize the total cost of the system subject to satisfying some service-level requirements. In this strategy, the saving, or profits, needs to be allocated across the supply chain network using contractual mechanism. Similarly, in a decentralized system each facility or company determines its most effective strategy without considering the impact on the other facilities or companies. This strategy leads to local optimization. The centralized system is only possible to apply if each facility or company can access all information in the supply chain. With advances in information technology, the centralized system can have access to such data. There are some considerations before choosing which strategy needs to apply.

- *Safety stock*. In general, this means that the more centralized an operation is, the lower safety stock levels there will be.
- *Overhead*. Operating a few large central warehouses leads to lower total overhead cost relative to operating many smaller warehouses.
- *Economy of scale*. It is often much more expensive to operate many small manufacturing facilities than to operate a few large facilities with the same total capacity.
- *Lead time*. Lead time to market can often be reduced if a large number of warehouses are located closer to the market areas.
- *Service*. It depends on how service is defined. Centralized warehousing enables the utilization of risk pooling, which means that more orders can be met with a lower total inventory level. On the other hand, shipping time from warehouse to the retailer will be longer.
- *Transportation costs*. As the number of warehouses increases, transportation costs between the production facilities and warehouses also increases because the total distance travelled is greater and quantity discounts are less likely to apply.

The supply chain designs or network configurations are often categorized as push or pull systems (Simchi-Levi et al., 2007).

- Push system

  In push system, production decisions are based on long-term forecasts. The manufacturer uses orders received from the retailer's warehouses to forecast customer demand. It takes a long time for a push system to react to changing marketplace therefore it is unable to meet changing demand patterns. Supply chain inventory can become obsolete as demand for certain products disappears. In addition, the variability of orders received from the retailers and the warehouses lead to excessive inventories due to the need for large safety stock, larger and more variable production batches, unacceptable service levels and product obsolescence. In this system we often find increased transportation costs, high inventory levels, and/ or high manufacturing costs, due to the needs for emergency production changeovers. Figure 2.3 below shows how this system works.

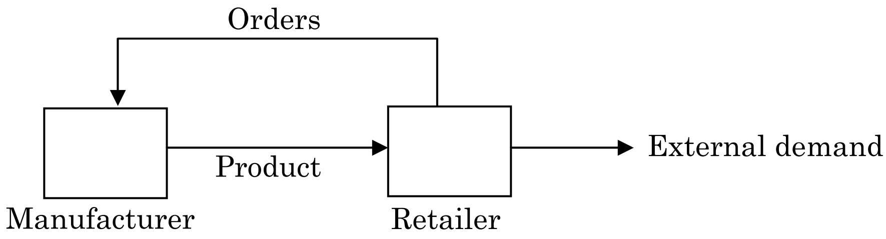


Figure 2.3 A push system. Source: Simchi-Levi et al. (2007)

- Pull system

  Unlike the push system, in a pull system production decision is demand driven so that it is coordinated with the actual customer demand rather than a forecast. The supply chain uses fast information flow mechanism to transfer information about customer demand to the manufacturing facilities through information technology such as decision support system (DSS) and agent-based system. For example, Akanle and Zhang (2008) proposed agent-based model for optimising supply chain configurations. This system leads to decrease in lead times achieved through the ability to better anticipate incoming orders from the retailers, decrease in inventory at retailers, decrease in variability in the system, in particular, variability faced by manufacturers and decrease inventory at the manufacturer due to the reduction in variability. Therefore, this system can significantly reduce system inventory level, system costs and enhance ability to manage resources. However, this system is often difficult to implement when lead times are so long that it is impractical to react to demand information. Figure 2.4 below shows how this system works.

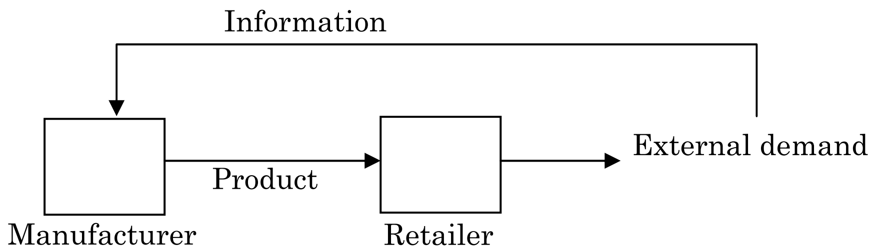


Figure 2.4 A pull system. Source: Simchi-Levi et al. (2007)

### 2.3.2 Supply chain integration

As mentioned in the previous section, integrating the supply chain is quite difficult because of its dynamics and the conflicting objectives employed by different facilities and companies in the supply chain. However, in today's competitive markets, most companies have no choice. They are forced to integrate their supply chain and engage in strategic partnering. Information sharing and operational planning are the keys to a successfully integrated supply chain (Simchi-Levi et al., 2007). By carefully using the available information, we can reduce the cost of the system while accounting for the conflicting goals and objectives of each level of the supply chain. It can be easier to do in a centralized system, but even in a decentralized system it may be necessary to find incentives to bring about the integration of supply chain facilities.

For designing the supply chain, some of the objectives have to be sacrificed. The supply chain is viewed as a set of trade-offs that have to be made (Simchi-Levi et al., 2007). We discuss some trade-offs and how through the use of advanced information technology and creative network design as follows:

1. *The lot size-inventory trade-off.* As previously described, manufacturers would like to have large lot sizes so that per unit setup costs are reduced, manufacturing expertise for a particular product increases, and processes are easy to control. Unfortunately, typical demand doesn't come in large lot sizes, so large lot sizes lead to high inventory. Retailers and distributors would like short delivery lead times and wide product variety to respond to the needs of their customers. By applying setup time reduction, Kanban and CONWIP (constant work in progress) system and others it is possible for manufacturers to meet these needs by enabling them to respond more rapidly to customer needs.

2. *The inventory-transportation cost trade-off.* There is a similar trade-off between inventory and transportation costs. If a company operates its own fleet of trucks which have some fixed cost of operation (e.g., depreciation, driver time) and some variable cost (e.g., gas) with the full capacity, it will minimize transportation costs but it leads to higher inventory costs. Unfortunately, this trade-off can't be eliminated completely. However, we can use advanced information technology to reduce this effect.

3. *The lead time-transportation cost.* As mentioned above, transportation costs are lowest when large quantities of items are transported between stages of the supply chain. However, lead times can often be reduced if items are transported immediately after they are manufactured or arrive from suppliers. Thus, there is a trade-off between holding items until enough accumulate to reduce transportation costs, and shipping them immediately to reduce lead time.

Again, this trade-off cannot be completely eliminated, but information can be used to reduce its effect.

4. *The product variety-inventory trade-off*. Product variety greatly increases the complexity of supply chain management. Manufacturers that make a multitude of different products with small lot sizes find their manufacturing costs increase and their manufacturing efficiency decreases. A company can maintain the same lead times with smaller amounts which will probably be shipped so warehouses will need to hold a larger variety of products. Thus, increasing product variety increases both transportation and warehousing costs. Because it is usually difficult to accurately forecast the demand for each product, higher inventory levels must be maintained to ensure the same service level.

5. *The cost-customer service trade-off*. Reducing inventories, manufacturing costs and transportation costs typically comes at the expense of customer service. The level of customer service can be maintained while decreasing these costs by using information and appropriate supply chain designs.

Indeed, as described before, transportation and inventory costs are often critical supply chain cost drivers, particularly when inventory levels must be kept fairly high to ensure high service levels.

### 2.3.3 Inventory Management

Inventories control and management is the common problem for every company in many sectors of organizations including agribusiness, industries, military etc. There are some reasons why each company needs to manage them. The basic reason is that it is impossible physically and economically to receive a product or service while the product is ordered (Hadley and Whitin, 1963). Two common questions in inventories control and management are when the product or service is ordered and how many products or services are ordered?

Tersine (1994) mentioned that there are different types of inventory. They are supplies, raw materials, work-in-process and finished products. Supplies are types of inventory which are consumed in the organization which are not part of finished product such as pens, paper, disk, etc. Raw materials are items which are supplied by suppliers to be used as input in production such as woods, paints, steel, etc. Work-in-process includes items which are part of finished product which are still to be processed. Finished products are items which are ready to be sold, distributed or put in the inventory.

Tersine (1994) addressed four factors of functionality of inventory as follows:

1. Time, including production and distribution. It is calculated from time taken for designing production schedule, ordering and delivering raw materials, raw materials inspection, production process, shipping products to customers.
2. Discontinuity, allowing a treatment of many different operations (retailing, distributing, warehousing, manufacturing and buying).
3. Uncertainty, focusing on unpredictable events which can change the schedule of organization. These include demand, variables of production, resources breakdown, delaying to deliver, and changing natural condition.
4. Economic, allow the company to gain the profit from many alternatives for reducing costs.

Inventories can be also classified according to their function (Silver et al, 1998):

1. *Cycle stock,* number of inventories which are ordered in a lot size
2. *Congestion stock*, inventories of products which are produced caused by limitation of production capabilities
3. *Safety stock*, inventories of products to meet the uncertainty demand and supply in short term period
4. *Anticipation inventory*, inventories which are used to anticipate the high demand.
5. *Pipeline inventory*, including inventories in delivery time between two players in a supply chain
6. *Decoupling stock*, used in multi-echelon inventory system to allow each level to make its own decision regarding inventory level.

According to those statements mentioned above, inventory decisions are important and must be managed by each company or firm. There are some relevant costs incurred caused by the needs to handle the inventory (Hadley and Whitin, 1963).

1. Price. This is the cost to buy product per unit if the product is received from another company. For a manufacturing company, these costs include direct labor cost, material cost and overhead.
2. Ordering / setup cost. This cost includes any costs for ordering product such as vendor analysis, writing an order, receiving material, inspecting material, and any costs in order transaction.
3. Holding / carrying cost. This cost is any costs related to investment in inventory and maintenance of product in warehouse. This cost includes cost of capital, tax, insurance, product handling, warehouse, deteriorating products.

4. Shortage cost. This cost is the economic consequence of stock-out product internally or externally.

*2.3.3.1 Managing inventory in the supply chain*

As inventory management is one of the key issues in designing and managing the supply chain and the models described above assume a single facility or company managing its inventory in order to minimize its own cost as much as possible, the need for the coordination of inventory and production decisions and transportation policies among entities in the supply chain has been evident for many years, referred to multi-echelon or multi-level supply chain inventory system. Multi-echelon or multi-level supply chain consists of some players in different levels managing to minimise the total cost of the supply chain. Managing inventory in a complex supply chain is typically quite difficult and may have a significant impact on the customer service level and supply chain system-wide cost. In the supply chain, the main objective is to reduce system-wide cost, but it is important to consider the interaction of various facilities or companies and the impact this interaction has on the inventory policy.

A supply chain can consist of suppliers and manufacturers who convert raw materials into finished products, and distribution centres and warehouses, from which finished products are distributed to customers, we define total supply chain inventories as the sum of raw materials, work-in process and finished products held by parts suppliers, plus raw materials and work-in process held by assemblers and finished products held by distributors and retailers. Each of these forms of inventories mentioned above needs its own inventory control mechanism. The difficulty in determining these mechanisms is that efficient production, distribution and inventory control strategies that reduce system-wide costs and improve service levels must take into account the interactions of various levels in the supply chain.

Managing inventory effectively in this environment is often difficult. It is caused by two important issues in inventory management which are demand forecasting and order quantity calculation. While customer demand for products does not vary much in retailers, inventory and back-order levels fluctuate considerably across the supply chain. However, the distributors' orders placed to the manufacturer fluctuate much more than retailers so that the manufacturer's orders to its suppliers fluctuate even more. This increase in variability as we travel up in the supply chain is referred to as the *bullwhip effect*. Lee et al. (1997a, 1997b) identify four major causes of the bullwhip effect as: (1) demand forecast updating,

(2) order batching, (3) price fluctuation, and (4) rationing and shortage gaming. Therefore, we need to make two important inventory decisions with the objective of minimising system-wide cost. To reduce the bullwhip effect in the supply chain, coordination in the supply chain is needed. The decision maker needs to have access to inventory information at each level of the supply chain.

To manage and control inventory level, there are five strategies to reduce inventory level (Simchi-Levi et al, 2007). These strategies are:

1. Periodic inventory review policy. In this policy, inventory is reviewed at a fixed time interval and every time it is reviewed, a decision is made on the order size. This policy makes it possible to identify slow-moving and obsolete products and allows management to continuously reduce inventory levels.
2. Tight management of usage rates, lead time, and safety stock. This allows the company to make sure inventory is kept at the appropriate level.
3. ABC approach. In this strategy, items are classified into three categories. Class A items include all high-value products which typically account for about 80 percent of annual sales and represent about 20 percent of inventory. Periodic review policy is appropriate for this class. Class B items include products which account for about 15 percent of annual sales while Class C items represent low-value items, whose value is no more than 5 percent of sales. Periodic review policy is also appropriate for Class B but would not be applied to Class C.
4. Reduce safety stock levels. This strategy can be accomplished by focusing on lead time reduction.
5. Quantitative approaches. These approaches are similar to the models described above which focus on the right balance between inventory holding and ordering costs. Many past and current researches had been done regarding quantitative approaches to manage and control inventory in the supply chain. We will discuss these in more details in the next chapter.

## 2.4 Summary

This chapter presents a review of supply chain management. It includes the definition of the supply chain and supply chain management and key issues in supply chain management. The supply chain is defined as a network consisting of all companies involved, directly and indirectly, in fulfilling a customer request. Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements. To achieve this aim, there are three levels or phases to manage that are supply chain strategy or design, supply chain planning and supply chain operations.

However, the integration of the supply chain is difficult. Some reasons for that are: supply chain strategies cannot be determined in isolation, uncertainty and risk are inherent, system variations over time are also important consideration, different facilities in the supply chain frequently have different, conflicting objectives, supply chain is a dynamic system that evolves over time and supply chain is a complex network of facilities and organizations.

Some key issues in achieving the objective of the supply chain to manage and control the flows of product, information, and funds in the supply chain must be considered. These issues are distribution network configuration, inventory control, production sourcing, supply contracts, distribution strategies, supply chain integration and strategic partnering, outsourcing and offshoring strategies, product design, information technology and decision-support systems and customer value. We discuss some issues related to this research.

Inventories management is one issue which is a common problem for every company in many sectors of organizations including agribusiness, industries, military, etc. There are some reasons why each company as well as the supply chain needs to manage them. The basic reason is that it is impossible physically and economically to receive a product or service as fast as possible while the product is ordered. If inventory is not managed properly, the product can probably be out of stock or inventory cost can be higher. Managing inventory involve two common questions which are when the product or service is ordered and how many products or services are ordered

# Chapter 3

# Managing Inventory in Supply Chains: a Literature Review

## 3.1 Introduction

In this chapter, we review and discuss in more detail about inventory management in supply chains involving reverse logistics focusing on quantitative models which have been presented in relevant literature. State-of-the-art of models which have been developed in coordinating inventory decisions in supply chains is presented. Section 3.2 introduces basic Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models. In section 3.3, we discuss the needs to integrate inventory decisions among all players in the supply chain and classify the levels of production and inventory models in the supply chain. Next, the buyer-vendor production and inventory models (two-level supply chain) are reviewed and discussed in section 3.4 while the three-level production and inventory models are discussed in section 3.5. In section 3.6, a review of models involving reverse logistics in coordinating production and inventory system in supply chains is presented. Section 3.7 summarises the chapter.

## 3.2 Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) Models

### *3.2.1 Economic Order Quantity/ Economic Lot Size (EOQ) Model*

Since each company needs to have the inventory in anticipating demand from customer, the next step that we need to do is determining the economic lot size or economic order quantity to minimize the inventory cost incurred. Firstly, the model to determine the economic lot size or order quantity was introduced by Harris (1915). He developed a simple model that illustrates trade-offs between ordering and storage costs. Then, Wilson (1934) derived independently the model known as economic order quantity (EOQ) model which is the classic model for solving such as this problem. There are some assumptions of the model.

1. Deterministic and constant rate demand (*D*)
2. Lot sizes or order quantities *(I)* are fixed per order.
3. Ordering/ setup cost (*A*) is fixed.
4. Lead time is zero.
5. Initial inventory is zero.
6. Horizon period is infinite.

Based on these assumptions, inventory level for the model can be shown in Fig. 3.1 below (Tersine, 1994).

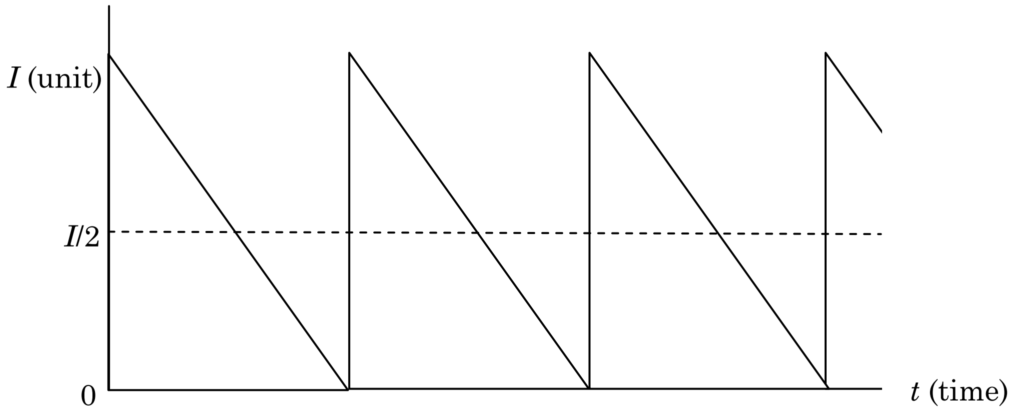

Figure 3.1 Inventory level at EOQ model.

Total inventory costs function (*TC*) per period is

$$TC\ (I) = C \cdot D + \frac{A \cdot D}{I} + \frac{h \cdot I}{2} \tag{3.1}$$

First term is total price per period. Total price per period is price per unit (*C*) times demand (*D*). Second term is ordering cost per period. Ordering cost per period is ordering cost per order (*A*) times number of the order (*D/I*). The last term is holding cost per period. Holding cost per period is holding cost per unit per period (*h*) times average inventory in unit (*I/2*). The level of these inventory costs can be shown in Fig. 3.2 below.

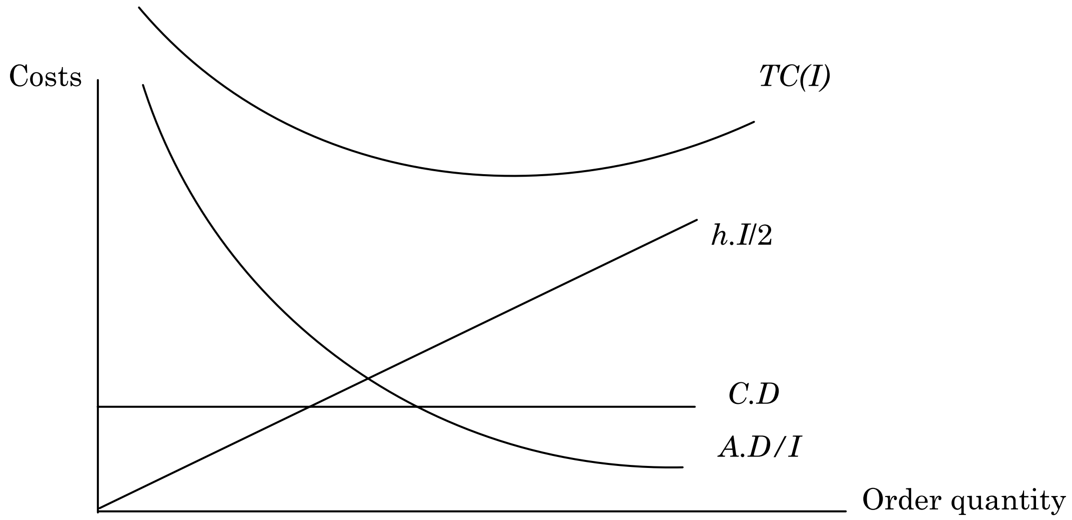


Figure 3.2 Inventory costs levels

Then, optimal lot size or order quantity *(I*)* is

$$I^* = \sqrt{\frac{2 \cdot A \cdot D}{h}} = \sqrt{\frac{2 \cdot A \cdot D}{C \cdot r}} \tag{3.2}$$

where,

$r$ = percentage of price of product ($C$)

Optimal total inventory costs per period is

$$TC\,(I^*) = \frac{A \cdot D}{I^*} + \frac{h \cdot I^*}{2} \tag{3.3}$$

### *3.2.2 Economic Production/Manufacturing Quantity (EPQ/EMQ) Model*

EOQ model which has been described above assumes lot size coming into warehouse instantaneously as amount of order quantity (*I*). For the manufacturing company, this assumption is not realistic so that EOQ model is modified to take into consideration the production rate. The model is known as Economic Production/Manufacturing Quantity (EMQ/EPQ) model (Tersine, 1994).

EMQ model assumes production rate is limited and constant. In the model, production rate is presented by $P$ along production time ($t_p$). Decision variable for this model is production lot size ($I$). Inventory level of this model can be shown in Fig. 3.3 below.

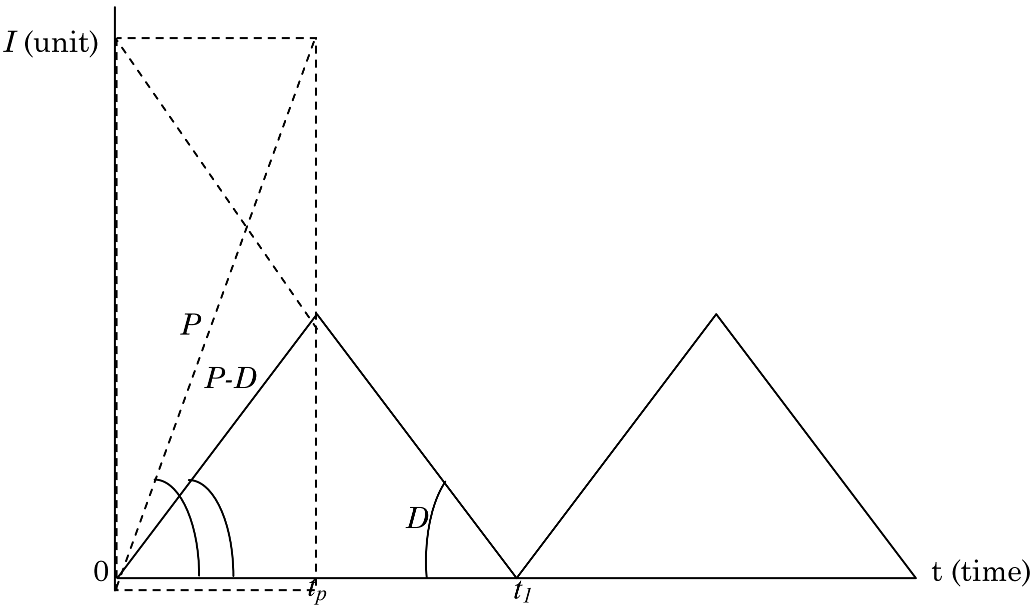


Figure 3.3 Inventory level of EPQ/EMQ model

Total inventory costs can be stated as the following equation.

$$TC\ (I) = C \cdot D + \frac{A \cdot D}{I} + \frac{h \cdot I \cdot (P - D)}{2 \cdot P} \tag{3.4}$$

Then, optimal production lot size is as the following equation.

$$I^* = \sqrt{\frac{2 \cdot A \cdot D \cdot P}{h \cdot (P - D)}} \tag{3.5}$$

Substituting $I$ value with $I^*$, optimal total inventory costs per period is

$$TC\ (I^*) = C \cdot D + \frac{A \cdot D}{I^*} + \frac{h \cdot I^* \cdot (P - D)}{2 \cdot P} \tag{3.6}$$

## 3.3 Integrated Production Inventory Models in Supply Chains

As discussed in the previous chapter, inventory models developed so far assume a single facility or single company (e.g., a warehouse or a retail outlet) managing its inventory in order to minimize its own cost as much as possible. In a typical supply chain, the main objective is to reduce system-wide cost; thus it is important to consider the interaction of various facilities and/or companies and the impact of this interaction on the inventory policy that should be employed by each facility. Each company in the supply chain must decide its inventory decision. However, they can use appropriate EOQ or EPQ models to solve the inventory problem. One problem in using this approach is that it may result in inventory cycles or orders which are not coordinated, as a result the supply chain have to hold more inventory than required to anticipate demand at different cycles. There are three traditional stages in a supply chain: procurement, production and distribution. Based on these stages, there are three categories of operational coordination that is buyer-vendor coordination, production-distribution and inventory-distribution coordination (Thomas and Griffin, 1996).

Since the main objective of the supply chain is to reduce system-wide cost, the companies in the supply chain need to coordinate their own objectives with other companies. Particularly in inventory decisions, they need to coordinate their inventory cycles among all companies in the system. Therefore, inventory models which can coordinate inventory decisions in the supply chain are needed.

To address the issue, many research studies have been carried out. Over thirty years since Goyal (1977) first developed an integrated inventory model for single supplier-single customer problem the research in coordinating production and inventory system in the supply chain have interested many researchers throughout the world. Goyal (1977) developed the model for a simple supply chain consisting of one supplier and one customer (buyer). Based on the research which has been carried out, we classify them into three categories. These categories are coordinating inventory decisions in a two-level supply chain (buyer-vendor coordination), inventory decisions in a three-level supply chain and inventory decisions in a multi-level supply chain. We review and discuss these categories in the next sections.

## 3.4 The Buyer-Vendor Coordination

As mentioned in the previous section, the integration of inventory models in supply chains was first developed by Goyal (1977). He suggested a joint economic lot size model where the objective is to minimize the total relevant costs for both the vendor and the buyer. The model is suitable when a collaborative arrangement between the buyer and the vendor is enforced by some contractual agreement. Goyal assumed an infinite replenishment rate for the vendor. It meant that the vendor does not manufacture the items himself but in turn buys it from his vendor and ignored the effect of a finite production rate in computing his inventory carrying costs. Moreover, he assumed that the inventory holding costs are independent of the price of the item (the price of item was assumed fixed). Lee and Rosenblatt (1986) then developed a generalized quantity discount pricing model in Goyal's model. The inventory holding costs are now no longer constant.

Banerjee (1986) generalized Goyal's model by incorporating a finite production rate. To illustrate how the model works, he considered a simple purchasing scenario. A purchaser (buyer) periodically orders some quantity, $Q$, of an inventory item from a vendor (supplier). The vendor follows a lot-for-lot policy from the purchaser, and on completion of a batch, ships the entire lot to the buyer. Fig. 3.4 shows the inventory behaviour between a purchaser and a vendor.

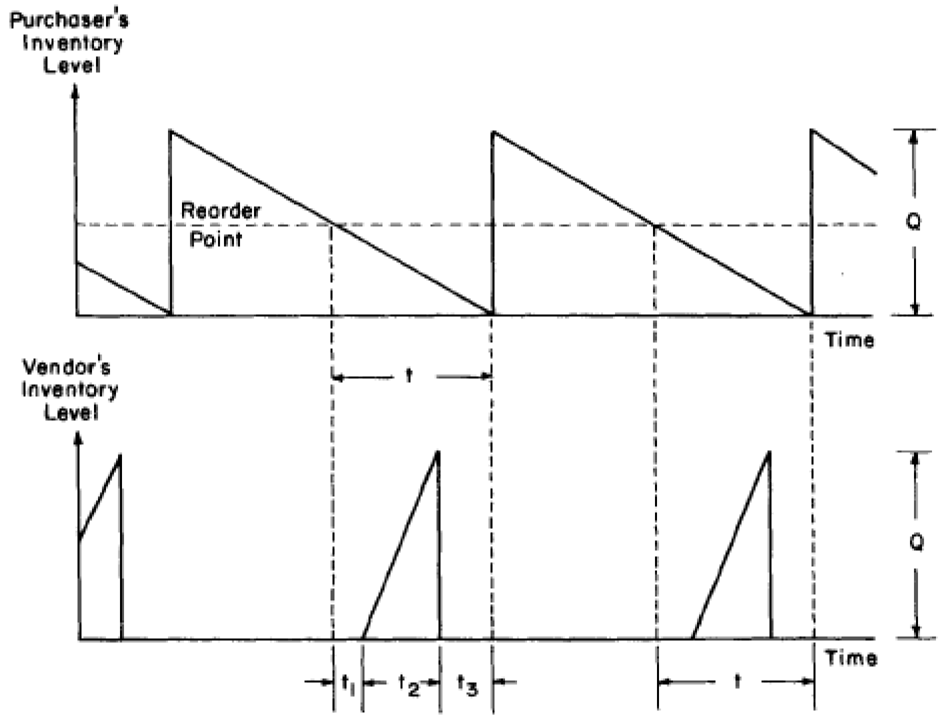


Figure 3.4 Purchaser's and vendor's inventory behaviour. Source: Banerjee (1986)

The supply lead time $t$ as shown in the diagram consists of three components: $t_1$ represents the time it takes to transmit a purchase order and set up a production lot, $t_2$ is the actual production time, and $t_3$ is the time it takes to deliver the completed lot to the buyer. Here, the purchaser and the vendor have to determine the coordination of when the purchaser places an order and when the vendor sets up a production lot. Joint total relevant cost function for the purchaser and the vendor is expressed as follows:

$$JTRC = \frac{D}{Q}A + \frac{Q}{2}rC_p + \frac{D}{Q}S + \frac{DQ}{2P}rC_v \tag{3.7}$$

where,

$JTRC$ = joint total relevant cost

$D$ = annual demand for the item

$P$ = vendor's annual rate of production for the item

$C_v$ = unit production cost for the item

$C_p$ = unit purchase cost paid by the purchaser

$A$ = purchaser's ordering cost per order

$S$ = vendor's setup cost per setup

$r$ = annual inventory carrying cost per unit cost invested in stocks

$Q$ = production lot size for the vendor (or order quantity for the purchaser)

When the vendor undertakes a production setup every production cycle time an order is placed by the purchaser, then economic order quantity for the purchaser or the production lot size for the vendor is given by

$$Q_j^* = \left[ \frac{2D(S+A)}{r\left(C_v \frac{D}{P} + C_p\right)} \right]^{1/2} \tag{3.8}$$

The minimum *JTRC* is given by

$$JTRC = \frac{D}{Q_j^*} A + \frac{Q_j^*}{2} rC_p + \frac{D}{Q_j^*} S + \frac{DQ_j^*}{2P} rC_v \tag{3.7}$$

Banerjee also modelled joint economic consequences of individual optimization as well as individual economic consequences of the joint optimization. These models can examine the cost trade-offs associated with joint optimization from both perspectives of individual optimization and joint optimization so that each player in the supply chain can determine which policy can be applied to the players.

Since the assumption of a lot-for-lot policy in Banerjee's model is restrictive in nature and it is possible for the vendor to produce in a lot to supply an integer number of orders of the purchaser, Goyal (1988) generalized Banerjee's model. If the order quantity for the purchaser is $Q$, then the production lot for the vendor can be $Qn$ where $n$ is an integer as mentioned in Goyal (1977) for the case of infinite production rate. Joint total relevant cost function for the purchaser and the vendor will be

$$JTRC\left(Q,n\right)=\frac{D}{Q}A+\frac{Q}{2}rC_p+\frac{D}{Qn}S+\frac{Q}{2}rC_v\left(n\left(1+\frac{D}{P}\right)-1\right) \tag{3.10}$$

and at a particular value of $n$, the economic order quantity for the purchaser or the production lot size for the vendor is given by

$$Q(n)=\left[\frac{2D\left(\frac{S}{n}+A\right)}{r\left(C_p-C_v+nC_v\left(1+\frac{D}{P}\right)\right)}\right]^{1/2} \tag{3.11}$$

*JTRC(n)* is given by

$$JTRC\left(n\right)=\left[2Dr\left(A+\frac{S}{n}\right)\left(C_p-C_v+nC_v\left(1+\frac{(D)}{P}\right)\right)\right]^{1/2} \tag{3.12}$$

$n^*$ is obtained by meeting the following condition.

$$n^*\left(n^*+1\right)\geq\frac{S\left(C_p-C_v\right)}{AC_v\left(1+\frac{(D)}{P}\right)}\geq n^*\left(n^*+1\right) \tag{3.13}$$

Following works which had been carried out by Goyal (1977), Banerjee (1986), Lee and Rosenblatt (1986) and Goyal (1986), many research studies in the buyer-vendor coordination have been carried out. Goyal (1989) classified the models which deal with integrated buyer-vendor coordination in four categories, that is models which deal with joint economic lot sizing policies, models which deal with coordination of inventory by simultaneously determining the order quantity of the buyer and the vendor, models which deal with integrated problem but do not determine simultaneously the order quantity of the buyer and the vendor, and models which deal with buyer-vendor coordination due to marketing. Furthermore, Rau and Ouyang (2008) considered one vendor and one buyer inventory system where the vendor makes a single product and supplies to the buyer with non-periodic and just-in-time replenishment policy under finite horizon period and a linear trend in demand.

The models described above have some common assumptions such as: demand rate is independent of the price changes and is continuous, buyer and vendor's inventory policies can be described by a simple EOQ model, demand is deterministic, shortages are not allowed, backlogs are not allowed, lead times are either deterministic or replenishment is continuous, and the vendor has knowledge of the holding and ordering costs governing the buyer's ordering policy. The following researches were carried out to relax or eliminate some of these assumptions.

Sarmah et al. (2006) investigated supply chain models for buyer-vendor coordination that use quantity discount as a coordination tool under deterministic environment. These also included some integrated buyer-vendor models that have similar type of objective functions to achieve production distribution coordination and that improve the performance of the supply chain.

Due to the quantity discount in the buyer-vendor coordination, Chakrabarty and Martin (1988) developed a joint buyer seller discount pricing model in the buyer-seller coordination. They modelled discounted pricing for joint buyer seller. Joglekar (1988) modelled a quantity discount pricing problem to increase vendor profits. He showed that an optimal production lot size policy is superior to the policy of optimal price discounts particularly when the setup cost of the manufacturer is substantially larger than the ordering cost of the buyer. Kim and Hwang (1989) suggested the improvement solution simultaneously of supplier's profit and buyer's cost by utilizing quantity discounts. They examined the effects of price and order size on the inventory related cost of a customer and the profit of a supplier. Lam and Wong (1999) applied fuzzy mathematical programming to solve the joint economic lot size problem with multiple price breaks. They determined the number of price breaks, as well as quantity discount and order quantity at each price break, to achieve the optimal joint costs. Fuzzy mathematical programming provides a very efficient algorithm to solve problems simultaneously from the perspectives of the seller and the buyer. Duan et al (2010) applied buyer-vendor coordination model with quantity discount incentive for products with fixed lifetime. They formulated the centralized decision-making model to examine the effectiveness of the proposed quantity discount model for fixed lifetime product. Also, Tsao (2010) considered a two-level supply chain between one supplier and one retailer subject to supplier's credit period and retailer's promotional effort. He analysed two trade allowances, the promotion cost sharing and the cost discount, which are designed for managing players' behaviour in the supply chain.

In addition, Lee and Wu (2006) analyzed bullwhip effect, order batching, in a one supplier one retailer supply chain. This bullwhip effect causes excessive inventory due to information distortion. They used two types of inventory replenishment methods, the traditional methods and the statistical process control based replenishment method.

Later, Hill (1997) considered a more general policy for the single-vendor single-buyer production-inventory model with multiple shipments within a single production lot or batch. The production lot or batch increases in the next cycles by a fixed factor. This fixed factor equals to the production rate divided by the demand rate. Hill (1999) then extended the model by deriving the structure of the globally-optimal solution and then setting out an algorithm for obtaining the solution. Goyal (2000) extended Hill (1977) by suggesting a generalised policy to improve the single-vendor single-buyer integrated production inventory model. He applied the procedure given in Hill (1997). Differently, Hoque (2000) considered the capacity of the transport equipment in the single vendor single buyer integrated production inventory system. This constraint on transport capacity affects decisions of optimal order size and production lot size.

Since models mentioned above assume that the payment for an order is settled when the order is placed, Jaber and Osman (2006) proposed a two-level supply chain model with delay in payments to coordinate orders to minimize local costs and that of the chain with centralized decision. They also included a profit sharing scenario for the distribution of generated net savings amongst players in the supply chain. Huang et al. (2010) also considered permissible delay in payments in the single vendor single buyer coordination model. In addition, they considered order-processing cost reduction at an extra crashing cost which varies with the reduction in the order-processing time length. Chen and Kang (2007) then extended the models with delay in payments. They considered various permissible delays in payments in the model.

Chen and Chen (2005, 2008) extended the two-level production inventory model by formulating several models based on several policies (non-cooperative and cooperative policies) with multiple products. They also considered raw material ordering and holding cost for each product type. In addition, they proposed a saving-sharing mechanism, through a quantity discount scheme so that one party is better off and the other is no worse off. To illustrate inventory levels of finished products of the retailer and the manufacturer and raw materials of the manufacturer, see Fig. 3.5 below.

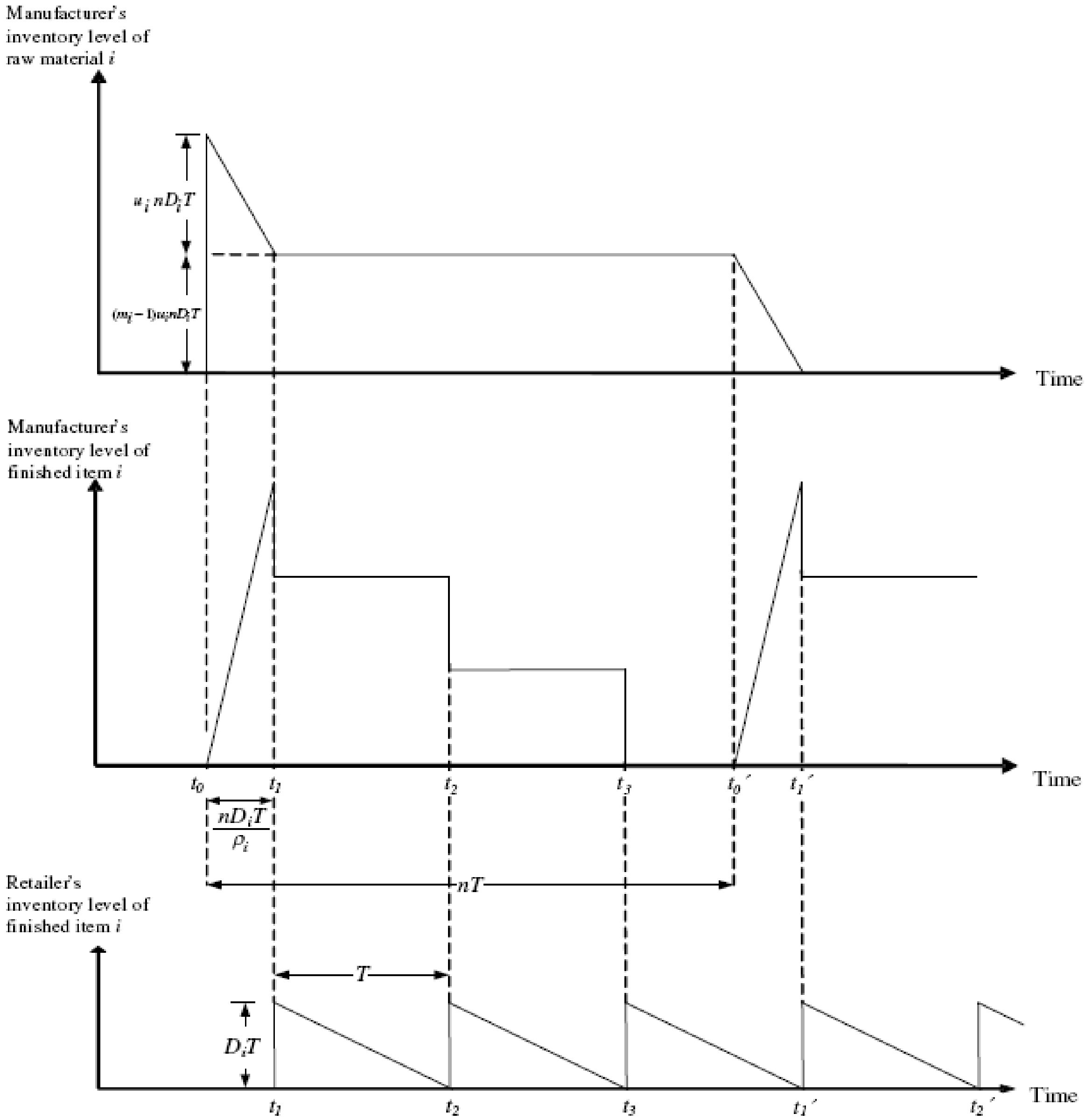


Figure 3.5 Inventory levels of finished products and raw materials in a two-level supply chain with $n$= 3 and $m_i$ = 2 where production stops when the first order of delivery at time $t_1$

Source: Chen and Chen (2008)

As seen in Fig. 3.5, the retailer orders finished product $i$ from the manufacturer every cycle time $T$ with order quantity $D_iT$ units. The manufacturer produces finished product every production cycle time $nT$ with the production lot size $nD_iT$ units and production rate $\rho_i$ units/period during the production time $\frac{nD_iT}{\rho_i}$. The first order in each production cycle time is delivered to the retailer after the manufacturer finishes the production. The $(n-1)D_iT$ units of finished products is stored in inventory for fulfilling the next orders. To produce the finished products the manufacturer needs raw materials. The manufacturer orders raw materials from the suppliers every order cycle time $m_i n D_i T$ for each finished product $i$. The $(m_i\text{-}1)u_i n D_i T$ units of raw materials is stored in the inventory for the next production cycle time.

In the model, they considered four policies that are individual item non-cooperative replenishment, joint items non-cooperative replenishment, individual item cooperative replenishment and joint items cooperative replenishment. The problem for this model is to determine the common or individual replenishment cycles for finished products at retailer's end, depending on which policy is being employed, and the production and procurement cycles at the manufacturer's end, with the objective of minimizing total relevant costs in the supply chain. The total relevant cost function of the supply chain for the individual item non-cooperative replenishment policy of the two-level supply chain between the manufacturer and the retailer can be shown as follows:

$$TC_{chain} = TC_R + TC_M \tag{3.14}$$

where,

$TC_{chain}$ = annual total relevant costs in the supply chain

$TC_R$ = annual total relevant costs of retailer

$TC_M$ = annual total relevant costs of manufacturer.

For the retailer, the annual relevant costs function is:

$$\begin{aligned} TC_R &= \sum_{i=1}^{k} TC_i(T_i) \\ &= \sum_{i=1}^{k} \left( \frac{A + a_i}{T_i} + \frac{h_i D_i T_i}{2} \right) \end{aligned} \tag{3.15}$$

where,

$A$ = the major ordering cost per order

$a_i$ = the minor ordering cost of product $i$

$T_i$ = the replenishment cycle time of product $i$

$h_i$ = the holding cost of product $i$

$D_i$ = the demand of product $i$

For the manufacturer, annual total relevant costs are:

$$TC_M = \sum_{i=1}^{k} \left[ TC_{fi}(n, T_i^*) + TC_{ri}(n, m_i, T_i^*) \right]$$

$$=\sum_{i=1}^{k}\left[\frac{B+b_i}{nT_i^*}+\frac{h_{fi}D_iT_i^*}{2}\left(n(1+\frac{D_i}{\rho_i})-1\right)+\frac{s_{ri}}{m_inT_i^*}+\frac{u_ih_{ri}nT_i^*}{2}\left(\frac{D_i^2}{\rho_i}+(m_i-1)D_i\right)\right] \tag{3.16}$$

where,

$T_i^*$ = the optimal replenishment cycle time of product $i$

$b_i$ = the minor setup cost of product $i$

$\rho_i$ = the production rate of product $i$

$s_{ri}$ = the ordering cost of raw material for product $i$

$u_i$ = the usage rate of raw material for product $i$

$m_i$ = the integer multiplier of production quantity for product $i$

$h_i$ = the holding cost of product $i$ for the retailer

$h_{fi}$ = the holding cost of product $i$ for the manufacturer

$h_{ri}$ = the holding cost of raw material for product $i$ for the manufacturer

$n$ = the integer multiplier of ordering quantity for all products produced by the manufacturer

Since an order from the retailer can be delivered before the manufacturer finishes producing one production lot, Chen and Chen (2008) improved their previous model by considering that the first and the next orders from the retailer can be delivered before the manufacturer finishes one production lot. The illustration for this condition can be seen in Fig. 3.6. The change of the delivery time for the first order causes the change of the inventory holding cost for the products of the manufacturer. The total inventory cost of the manufacturer is expressed as follows:

$$TC_M = \sum_{i=1}^{k}\left[TC_{fi}(n,T_i^*)+TC_{ri}(n,m_i,T_i^*)\right]$$

$$=\sum_{i=1}^{k}\left[\frac{B+b_i}{nT_i^*}+\frac{h_{fi}D_iT_i^*}{2}\left(n-1-\frac{(n-2)D_i}{\rho_i}\right)+\frac{s_{ri}}{m_inT_i^*}+\frac{u_ih_{ri}nT_i^*}{2}\left(\frac{D_i^2}{\rho_i}+(m_i-1)D_i\right)\right] \tag{3.17}$$

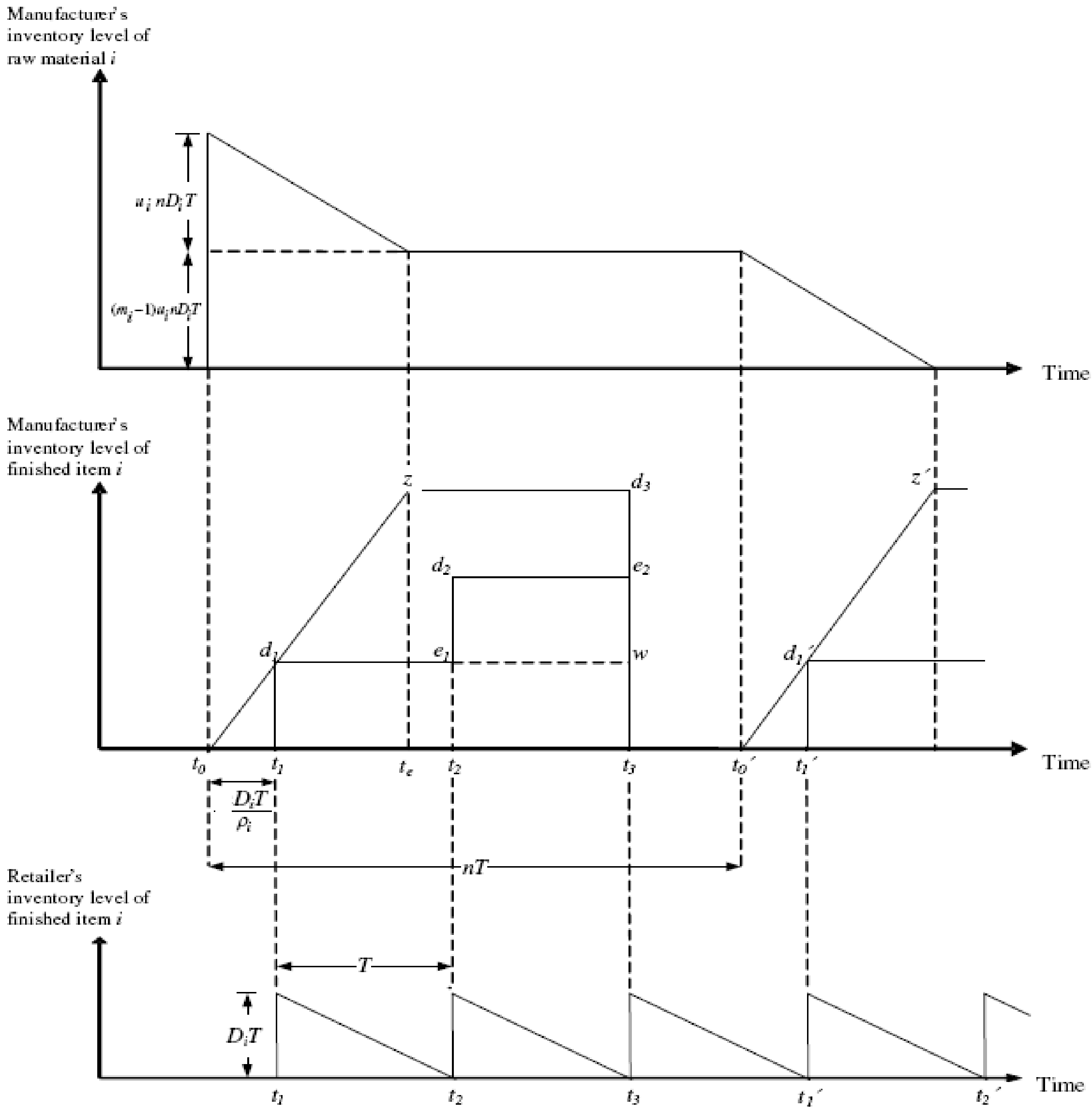


Figure 3.6 Inventory levels of finished products and raw materials in a two-level supply chain with $n$= 3 and $m_i$ = 2 where production stops after the first order of delivery at time $t_e$
Source: Chen and Chen (2008)

Works that have been carried out in Chen and Chen (2005, 2008) are some of the references which will be referred and developed in this research.

The next extension of the buyer-vendor coordination model is coordinating production inventory model between a single vendor and multiple buyers and between multiple vendors and a single buyer. Banerjee and Burton (1994) first addressed this issue. They developed a production inventory model between a single vendor and multiple buyers (industrial customers buying in discrete lots or orders). Within this context, two alternative sets of production/ inventory policies are examined. Firstly, each buyer independently determines and adopts its individual optimal ordering policy. To respond to this, the vendor also determines its own individual production policy. A simulation method was then used to evaluate this policy. Secondly, the vendor and multiple buyers cooperate and jointly derive an integrated or coordinated production/ inventory decision system, with the objective of minimizing the total cost incurred by all parties. Since some of parties are at a cost disadvantage, they are compensated adequately through a price discount or side payment scheme to ensure their participation in the system. In the result, the total system cost values under coordinated policy are substantially lower than those yielded by individual optimization, in every case examined. The supply chain's total cost for coordinated policy is expressed as follows:

$$JTRC\,(T, K) = \frac{\left[ S/K + \sum_{i=1}^{n} C_i \right]}{T} + \left(T/2\right)\left[ Dh\left\{\left(D/P\right)(2 - K) + K - 1\right\} + \sum_{i=1}^{n} h_i D_i \right] \tag{3.18}$$

where,

| | |
|---|---|
| $JTRC$ | = joint total relevant cost |
| $T$ | = the common order cycle time for buyers |
| $D_i$ | = annual demand for buyer $i$ |
| $P$ | = vendor's annual rate of production |
| $h$ | = holding cost per unit per year of the vendor |
| $h_i$ | = holding cost per unit per year of the buyer $i$ |
| $C_i$ | = ordering cost per order for buyer $i$ |
| $S$ | = vendor's setup cost per setup |
| $K$ | = an integer multiplier of the vendor's production cycle time |

Lu (1995) considered one vendor and multiple buyers with different types of items or product. The model developed is subject to the maximum costs which buyers are prepared to incur. The vendor only needs to know buyer's annual demand and previous order quantity which can be found from the buyer's past purchasing information.

Similarly, Abdul-Jalbar et al. (2007) considered an integrated production inventory model between a single vendor and two buyers which is the simplest case within the single vendor multi-buyer system. In their model, replenishment interval at any buyer is allowed to be greater than the replenishment interval at the vendor. They assumed that both buyers order the same item from the vendor and that the vendor can supply to the buyers before the whole lot is produced. Replenishment interval of each buyer can be different depending on the optimal solution of the system. The time interval between two consecutive setups of the vendor can be either constant or non-constant as illustrated in Fig. 3.7 and 3.8. Under these assumptions, they formulated the problem in terms of integer-ratio policies. In Fig. 3.7, the vendor anticipates demand from buyer 1 ($B_1$) by producing two orders before delivering them so that the time interval between two consecutive setups is constant. In Fig. 3.8, the vendor delivers the order to buyer 1 immediately after the production reaches one order so that the time interval between two consecutive setups is non-constant. We can see that the replenishment interval of buyer 2 ($B_2$) is four times the replenishment interval of buyer 1 and the production interval of the vendor is twice of the replenishment interval of buyer 1.

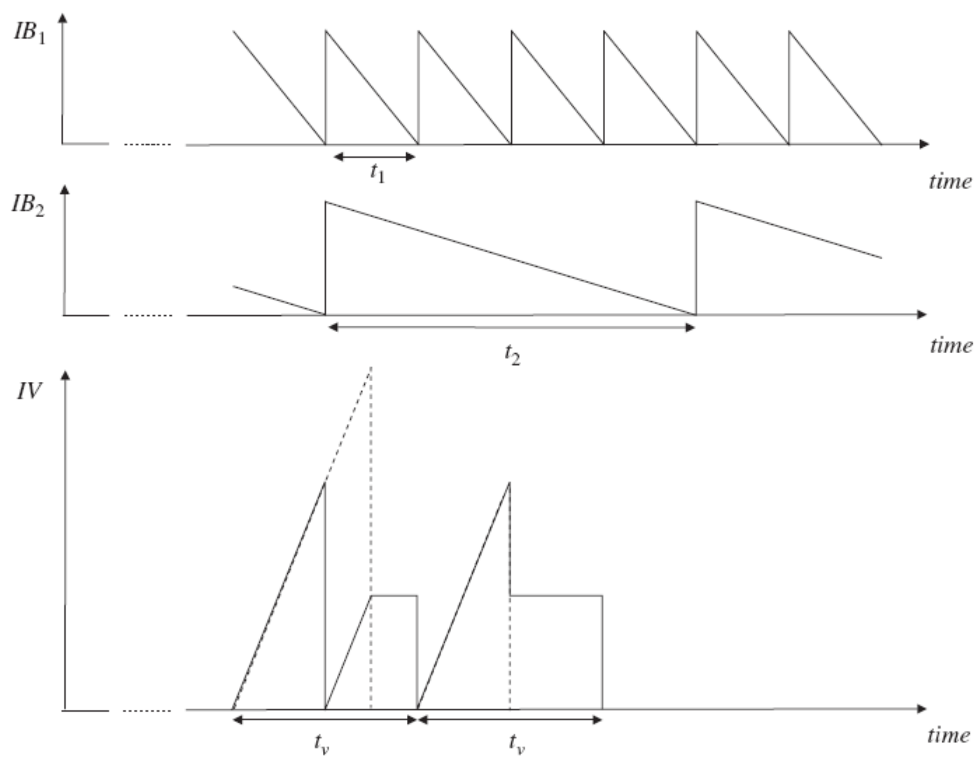


Figure 3.7 Inventory levels at the vendor and at two buyers considering that the replenishment interval $t_v$ is constant. Source: Abdul-Jalbar et al (2007)

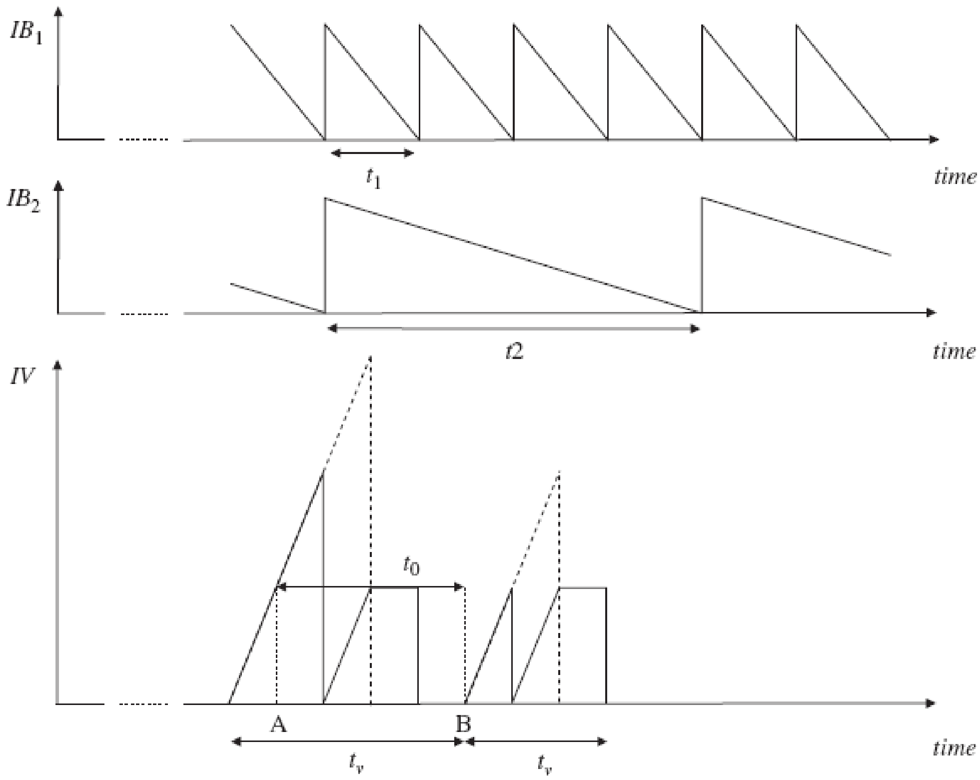


Figure 3.8 Inventory levels at the vendor and at two buyers considering that the replenishment interval $t_v$ is non-constant. Source: Abdul-Jalbar et al (2007)

Furthermore, Siajadi et al. (2006) proposed a model of one vendor multiple buyers with multiple size shipments from vendor to all buyers. Only one specific item is considered. The production is organized in such a way that the first shipment for each buyer is carried out in a sequence. Following the sequence, the first delivery starts from the first buyer followed by the second buyer, the third and so on. The duration from one delivery to the next is fixed for each buyer. It is also assumed that the order cycle time for each buyer and the production cycle time for the vendor is equal.

Differently, Sarmah et al (2008) developed a model for the coordination of a single manufacturer and multi-buyer supply chain considering credit option as the mechanism to develop coordination between parties of the supply chain. Unlike existing inventory models with credit option, they developed two new models that integrate the transportation cost explicitly in the single vendor multiple-buyer's situation. In the first model, the transportation cost is borne by the manufacturer whilst the transportation cost is borne by buyers in the second one. Chan and Kingsman (2007) also developed a single vendor multi-buyer coordination which allowed buyers to have different order cycle times but there is still a relationship between them using integer multipliers. The buyers in the supply chain determine their lot sizes independently but they synchronize their delivery and production times. Each buyer in determining the size for its deliveries also fixes the intervals between its deliveries. The buyers then allow the vendor to schedule exactly when their delivery days will occur, subject to the delivery interval fixed by the buyer. The cycle time of each buyer must be an integer multiple of some basic time period and an integer factor of the vendor's cycle time.

Since some players in a coordinated supply chain can be better off and others can be worse off than in an uncoordinated chain, Chan and Lee (2012) proposed an order-frequency-based price discount scheme which is incorporated into the synchronized cycles model developed in Chan and Kingsman (2007), to motivate buyers to change their policies so as to allow the saving from co-ordination to be achieved. The discount offered to a buyer depends on the deviation of the buyer's new ordering cycle from the one under independent policy. If the buyer's new ordering cycle deviates from its original one to a large extent, the vendor would offer a larger discount. This discount is to compensate for the increased cost incurred by the buyers due to the change of the ordering cycle. Moreover, Chan et al. (2010) proposed to incorporate a delayed payment period and a cost-sharing scheme into the synchronized cycle model developed in Chan and Kingsman (2007), to guarantee that every buyer will not be worse off when compared with independent optimization. This is also an incentive to motivate the buyers to participate in the co-ordination. In this model the manufacturer does not require any cost information from the buyer. In addition, the delayed payment period for each buyer is different such that the savings achieved from the co-ordination can be shared in an equitable sense.

Unlike a single-vendor multiple-buyer coordination Glock (2011, 2012a) developed models to coordinate production inventory system between the single-buyer multiple-vendor. In the first paper, he proposed one buyer sourcing a product from heterogeneous suppliers and tackled both the supplier selection and lot size decision with the objective to minimize total system cost. A two stage solution procedure is suggested. The second paper considered a single buyer and a network of homogeneous suppliers. He assumed a close and cooperative relationship and suggested two coordination mechanism that is overlapping production cycles with immediate delivery (OPCI) and overlapping production cycles with delayed delivery

(OPCD) which differently affect where inventory is held in the system. He also derived analytical and heuristic solutions for both alternatives.

More extensions of models for coordinating production inventory system in a supply chain consisting of a single buyer single vendor, a single vendor multiple buyers and a single buyer multiple vendors can be found in the following works. Woo et al. (2001) proposed an integrated inventory model for a single vendor and multiple buyers with ordering cost reduction. Similarly, Zhang et al. (2007) proposed an integrated vendor-managed inventory (VMI) model for a single vendor and multiple buyers where vendor purchases and processes raw materials and then delivers finished products to multiple buyers. Buyers' ordering cycles may be different and each buyer can replenish more than once in one production cycle. Investment decision is also considered with ordering cost reduction of the buyers on operating the new ordering system. Hoque (2008) proposed synchronization of production and delivery time in the single-manufacturer and multiple buyer integrated inventory system. An improved synchronization for generalized single vendor multi-buyer problem was proposed in Hoque (2011).

Work that has considered deteriorating items in buyer-vendor coordination can be found in Yang and Wee (2000, 2002, 2003), Wee et al. (2009), Zhou and Wang (2007), Rau et al. (2003, 2004), Lo et al (2007), and Zanoni and Zavanella (2007). Work that considers stochastic demand and/or stochastic lead time in the buyer-vendor coordination model is found in Sharafali and Co (2000) who considered a Poisson-demand distribution, Ouyang et al. (2004) presented integrated single-vendor single-buyer integrated production inventory models with stochastic demand following the normal distribution in controllable lead time and Ben-Daya and Hariga (2004) who proposed that the lead time is varying linearly with the lot size and that demand during lead time is stochastic and follows a normal distribution. This model was extended by Glock (2009) to account for unequal-sized batches. Another extension of this model is Taleizadeh et al. (2010) who studied the case of multiple products and included budget and service level constraints as well as the option of reducing lead time in the model. The effect of fuzzy annual demand and/or a fuzzy production rate was analysed by Pan and Yang (2008) with demand during lead time following a normal distribution. The average demand per year and the production rate are treated as fuzzy numbers. Glock (2012c) studied alternative methods for reducing lead time with lot size-dependent lead times and stochastic demand.

Research that considers quality of products in coordinating production inventory models was carried out in Huang (2002, 2004) who analysed buyer vendor system under assumptions that equal-sized shipments are transferred between vendor and buyer, and that a constant fraction of defective items is delivered with every shipment. Quality is divided into conformance and non-conformance. Alternative defective rates were studied by Ouyang et al. (2006) who assumed that defective rate is either known or fuzzy in nature or that a confidence interval should be used for it which combines the case of certain and fuzzy defective rates. Other extensions that take into consideration product quality were studied in Affisco et al. (2002), Goyal et al (2003), Ouyang et al. (2007), Liu and Cetinkaya (2007), Ben-Daya and Noman (2008), Wu et al. (2007) and El Saadany and Jaber (2008).

## 3.5 Three-Level Supply Chain Coordination

In this section, we review and discuss integrated production inventory models in three-level supply chains. Banerjee and Kim (1995) was the first joint economics lot size (JELS) model that consider more than a two level supply chain. They included raw material ordering in the single buyer single vendor system. This supply chain still consists of buyer and supplier or vendor. Banerjee et al. (2007) extended this model by including multiple buyers in their analysis. They assumed that a common delivery cycle is implemented and that all buyers are replenished with a single shipment at regular interval. Also, Lee (2005) analyzed raw material ordering. In contrast to Banerjee and Kim, Lee assumed that the manufacturer can order an integer multiple of his production lot size at the raw material supplier.

For three levels of players in a supply chain consisting of materials supplier/s, finished product vendor and buyer/s, research studies on coordinating production inventory system model are few. Munson and Rosenblatt (2001) considered a single-product centralized three-level supply chain consisting of a single supplier, a single manufacturer, and a single retailer. They assumed that the manufacturer is the most influential channel player who would be able to obtain a quantity discount from the supplier without worsening the supplier's financial performance. They also suggested the compensation to be paid to retailer. Jaber et al. (2006) extended the work of Munson and Rosenblatt (2001) by adopting a profit rather than a cost function, discount-dependent demand, and profit sharing. Prices and order quantities are decision variables in this model. They assumed one player in each level as the same in Lee and Moon (2006) who also assumed one player at each level of the supply chain.

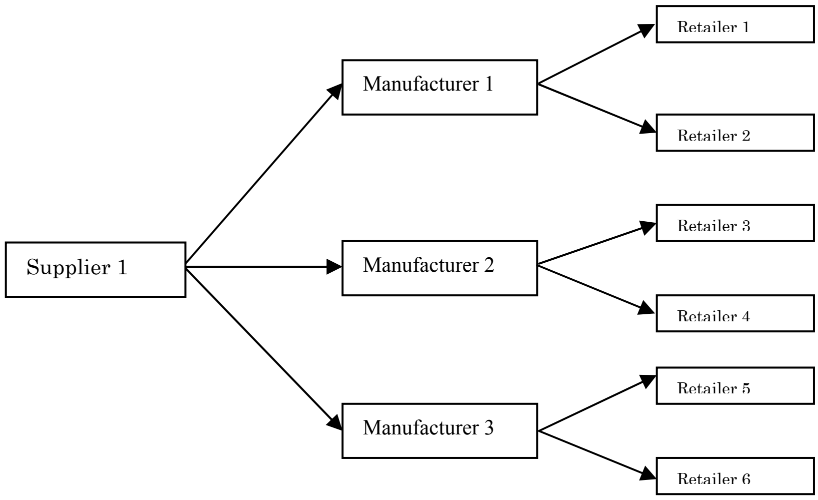


Figure 3.9 An example of the supply chain configuration. Source: Khouja (2003)

Khouja (2003) then studied a supply chain which has multiple firms and a firm can supply two or more customers. An example of such a supply chain configuration consisting of one supplier, three manufacturers, and six retailers can be seen in Figure 3.9. A supplier supplies raw material to three manufacturers and then each manufacturer delivers a single product to two retailers.

Furthermore, Jaber and Goyal (2008) extended those works in three level supply chains by assuming multiple suppliers at the first level, a single manufacturer or vendor at second level and multiple buyers at the third level. A supplier may supply one or more items to the vendor who will manufacture/assemble these items into a single product that is shipped to buyer as seen in Fig. 3.10. Each supplier supplies unique items which are never identical among suppliers. Total supply chain cost for the coordination of multiple suppliers, a manufacturer, and multiple buyers is expressed as follows:

$$C_{chain} = \begin{array}{l} \sum_{j=1}^{n}\left(A_{b,j} + h_{b,j}\frac{D_j T}{2}\right) + \frac{A_v + \sum_{i=1}^{k} a_{v,i}}{\lambda_v T} + \frac{h_v}{2}(\lambda_v - 1)T\sum_{j=1}^{n} D_j \\ + (\lambda_v - 1)T\sum_{j=1}^{n}\frac{\sum_{i=1}^{k} h_{v,i}u_i}{2} D_j + \frac{1}{\lambda_v T}\sum_{s=1}^{m}\frac{A_s}{\lambda_s} + \frac{\lambda_s}{2}T\sum_{s=1}^{m}\sum_{j=1}^{n}(\lambda_s - 1)D_j\left(\sum_{i=1}^{k} h_{s,i}u_i\right) \end{array} \tag{3.19}$$

where

$T$ = the common order cycle time across buyers

$D_j$ = annual demand rate for buyer $j$

$A_{b,j}$ = order cost per cycle

$h_{b,j}$ = holding cost per unit per year

$A_v$ = fixed order/setup cost per cycle for the vendor

$h_v$ = holding cost per unit per year

$\lambda_v$ = an integer multiplier to adjust the order quantity of the buyers

$k$ = number of items required by the manufacturer to assemble into product

$a_{v,i}$ = the cost of placing a purchase order for item $i$

$u_i$ = number of units required in one unit of the product

$h_{v,i}$ = holding cost per unit per year for item $i$

$A_s$ = order cost for supplier $s$ for items

$h_{s,i}$ = holding cost per unit per year of item $i$ supplied by supplier $s$

$\lambda_s$ = an integer multiplier to adjust the order quantity of the vendor

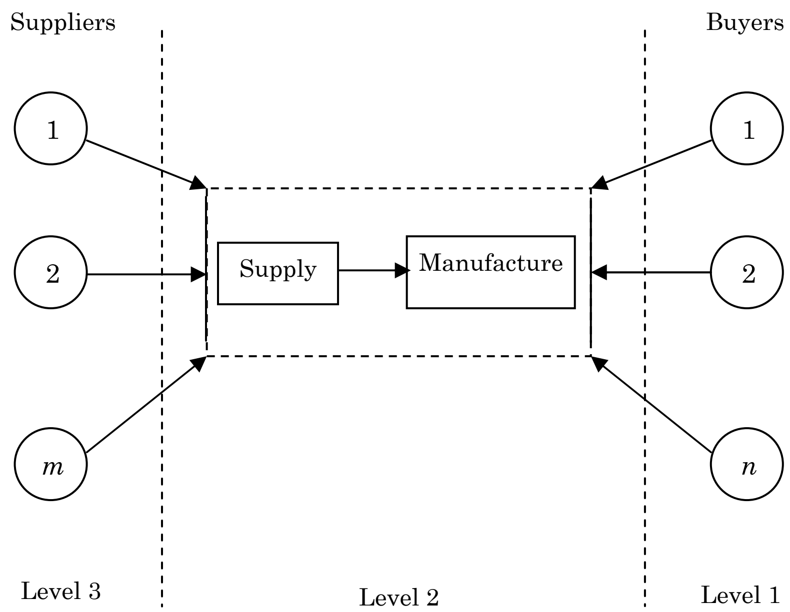


Figure 3.10 System description of the three-level supply chain.
Source: Jaber and Goyal (2008)

The optimal solution is obtained using two solution procedures that are with and without coordination. They also computed compensations and savings among all players to make the coordination fair among all players. Following this work, Jaber et al. (2010) considered a learning-based continuous improvement process for the manufacturing operations. Improvements are characterized by enhanced capacity utilization, reductions in set-ups time and improved product quality through the elimination of rework.

Similar to works in Khouja (2003), Jaber and Goyal (2008) and Jaber et al. (2010), Ben-Daya et al. (2010) incorporated Lee (2005) and Ben-Daya and Al-Nassar (2008) for a three-level supply chain consisting of one supplier, one manufacturer and multiple retailers. The supplier receives raw materials from his supplier and transforms it to semi-finished products at certain production rate. The manufacturer receives those semi-finished products in equal size batches and transforms them to finished products at a rate. The finished products are shipped to the retailers at common replenishment time and they are used by the retailers to fulfill end customers' demand. The order of products received by the retailer is shipped in a number of shipments of equal size. Similarly, the semi-finished product received by the manufacturer is also shipped in a number of shipments of equal size. Kim et al. (2006) proposed an analytical model to integrate and synchronize the procurement, production and deliveries activities in the supply chain also consisting of a single supplier, a single manufacturer and multiple retailers. This model is a variant of the classical economic lot scheduling problem. Later, Chung and Wee (2007) proposed an optimized inventory system in a three-stage supply chain allowing backordering. They derived backordering without derivatives. Then, Ganeshan (1999) proposed ($s$, $Q$) inventory policy to manage the supply chain consisting of multiple suppliers, one warehouse and multiple retailers. The model analyzes inventory at retailers and suppliers and demand at warehouse and integrate them to analyze simple supply chains. The decisions in the model include the inventory, transportation and transit components of the supply chain.

However, the works presented and discussed above considered a single finished product in their models. In fact, coordination among players in the supply chain particularly in three-level supply chain manages multiple items or finished products such as automotive and electrical industry. Therefore, we need to consider this in coordinating production and inventory decisions.

## 3.6 Involving Reverse Logistics in the Supply Chain Coordination

Green Supply Chain Management (GrSCM) is gaining increasing interest among researchers and practitioners of operations and supply chain management. Three drivers, economic, regulatory, and customer pressure, drive GrSCM worldwide (Srivastava, 2008). The growing importance of GrSCM is driven mainly by the escalating deterioration of environment, e.g. diminishing raw material resources, overflowing waste sites and increasing levels of pollution (Srivastava, 2007). Srivastava (2007) defined GrSCM as "Integrating environmental thinking into supply chain management including product design, material sourcing and selection, manufacturing processes, delivery of the final product to the customers as well as end-of-life management of the product after its useful life". An interesting and significant trend in GrSCM has been the recognition of the strategic importance of reverse logistics (RL) as evident from Fig. 3.11.

Reverse logistics is the collective noun for logistic environments with recovery of products and materials (Teunter, 2001). There are different types of recovery: repair, refurbishing, remanufacturing, cannibalization, recycling and reuse. A major issue in reverse logistics in distribution systems is the question if and how forward and reverse channels should be integrated (Fleischmann et al., 1997). Forward channel refers to new items or products channel. See Fig. 3.12 to illustrate it. To set up an efficient reverse distribution channel, decisions have to be made with respect to:

- Who are the actors in the reverse distribution channel?

  Actors may be members of the forward channel (e.g. manufacturers, retailers, logistics service providers) or specialized parties or the third parties.

- Which functions have to be carried out in the reverse distribution channel and where?

  Possible functions in the reverse distribution channel are: collection, testing, sorting, transportation, and processing.

- What is the relation between the forward and the reverse distribution channels?

Recycling can often be described as an open-loop system. Remanufacturing and reuse often lead to closed-loop systems.

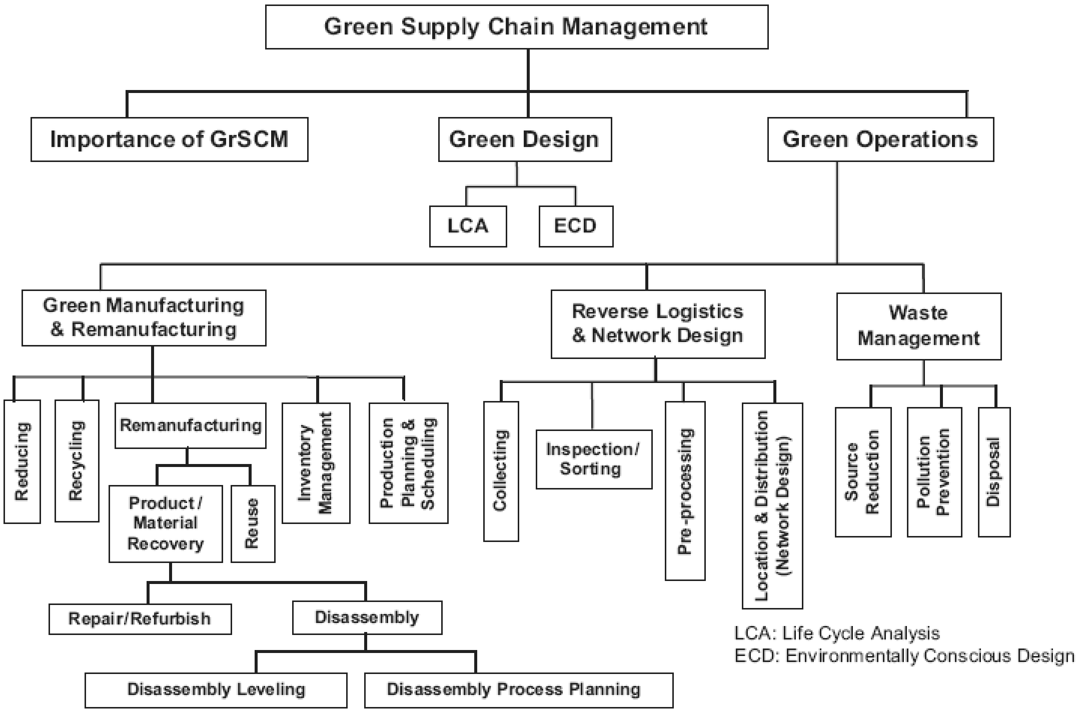


Figure 3.11 Classification and categorization of existing GrSCM literature. Source: Srivastava (2007)

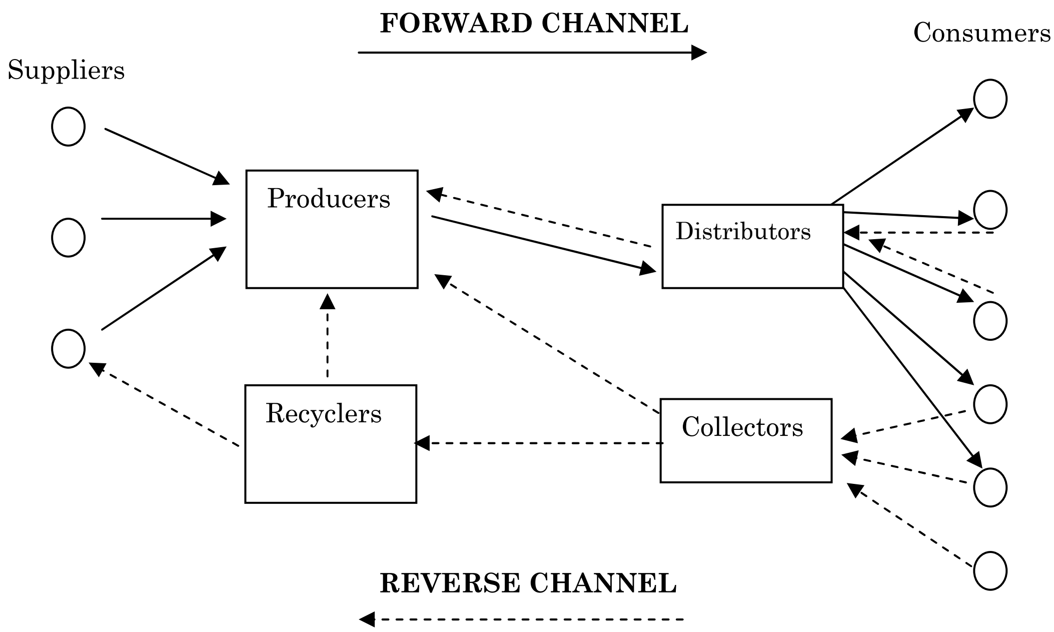


Figure 3.12 Framework reverse logistics. Source: Fleischmann et al. (1997)

How to integrate forward and reverse production inventory system in the supply chain is the next issue. To address the issue, there are only a few research studies that have been carried out. Many research studies which had been carried out are only considering a single player inventory system such as Teunter (2001) who proposed EOQ model of inventory system with items that can be recovered (repaired/refurbishment/remanufactured). He used different holding cost rates for manufactured and recovered items, and included disposal. The optimal solution is obtained by joining the inventory cost for new items and recoverable items. Fleischmann and Kuik (2003) then considered independent stochastic item returns from customers in inventory control. Kleber et al. (2002) proposed a continuous time inventory model for a product recovery system with multiple options. Later, Koh et al. (2002) developed optimal ordering and recovery policies for reusable items. The paper deals with a join EOQ and EPQ model. The model assumes stationary demand which can be satisfied by recycled products and newly purchased products with a fixed proportion of used products collected from customers and later recovered for reuse. It has adopted both new products and recycled products. This is similarly with Wang and Hsu (2010). Choi et al. (2007), proposed a generalized policy in ordering and recovery for reusable items while Demirel and Gökçen (2003) proposed a mixed integer linear programming model to solve remanufacturing problem in reverse logistics environment. Ching et al. (2003) considered lateral transshipments in returning used product in an inventory model with returns. This model only considered returned used products with a single item and one player. Roy et al. (2009) also proposed a production-inventory model with remanufacturing for defective and usable items in a fuzzy-environment where rate of defectiveness can be approximated by a constant or fuzzy parameter and El Saadany and Jaber (2011) considered a production/ remanufacturing model for subassemblies of returns which is managed differently. Finally, Teunter and Van der Laan (2002) analyzed non-optimality of the average approach for inventory models with remanufacturing.

For coordinating a production inventory system in the supply chain involving reverse logistics, Savaskan et al. (2004) considered a manufacturer and a retailer system. The manufacturer has three options for collecting such product: (1) they can collect by themselves directly from the customers, (2) they can provide suitable incentives to an existing retailer (who already has a distribution channel) to introduce the collection, or (3) they can subcontract the collection activity to a third party. Savaskan et al. (2004) modelled three options above as decentralized decision-making systems with the manufacturer being the leader. They found option (2) is the most effective undertaker of product collection activity for the manufacturer. In addition, they showed that a simple coordination mechanism can be designed such that the collection effort of the retailer and the supply chain profits are attained at the same level as in a centrally coordinated system. This model considers only single product and single retailer. There are no suppliers and components, whereas in modern and complex supply chain configuration suppliers and components hold important functions. Chung and Wee (2011) developed an integrated production inventory model for deteriorating item with short life-cycles between a supplier and a buyer considering green product design and remanufacturing with re-use concept whilst Wee et al. (2011) developed vendor managed inventory strategy between one supplier and one buyer for deteriorating product and conducted life cycle cost and benefits analysis.

Later, Chung et al. (2008) developed an inventory system with traditional forward-oriented material flow as well as a reverse material flow supply chain. In the reverse material flow, the used products are returned, remanufactured and shipped to the retailer for resale. The supply chain consists of the supplier, the manufacturer, the third party recycle dealer, and the retailer under contractual design. They considered only one single product without components. Fig. 3.13 shows an integrated closed-loop supply chain inventory system.

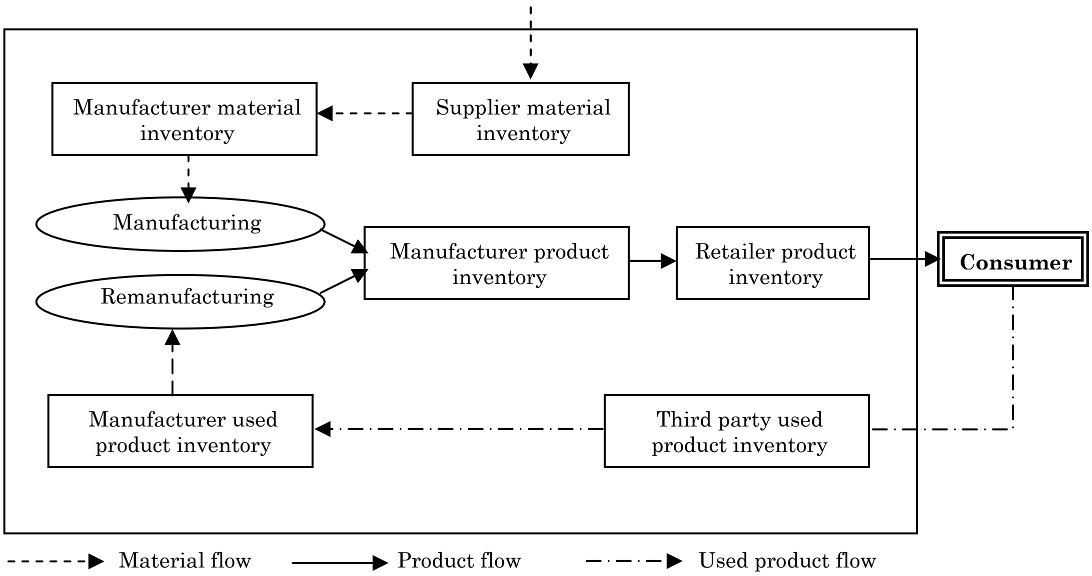


Figure 3.13 The integrated closed-loop supply chain inventory system.

Source: Chung et al. (2008)

The third-party collects used-products from customers and delivers them to the manufacturer every reproduction period $T_{R1}$ with number of deliveries $k$ times. $T_{R2}$ is non-reproduction period of the manufacturer every reproduction cycle time $nT_r$.

After finishing reproduction period, the production stops during $T_{R2}$ and then starts production period to produce new products during $T_{M1}$ period. The supplier supplies raw material to the manufacturer as many as $l$ deliveries during $T_{M1}$. After finishing production period, the production stops during $T_{M2}$ period every production cycle time $mT_r$. Then, the manufacturer delivers finished product to retailer every order cycle time of retailer $T_r$ with $I$ deliveries. $I = m + n$. The inventory levels of the third-party, the supplier and the manufacturer can be seen in Fig. 3.14 below.

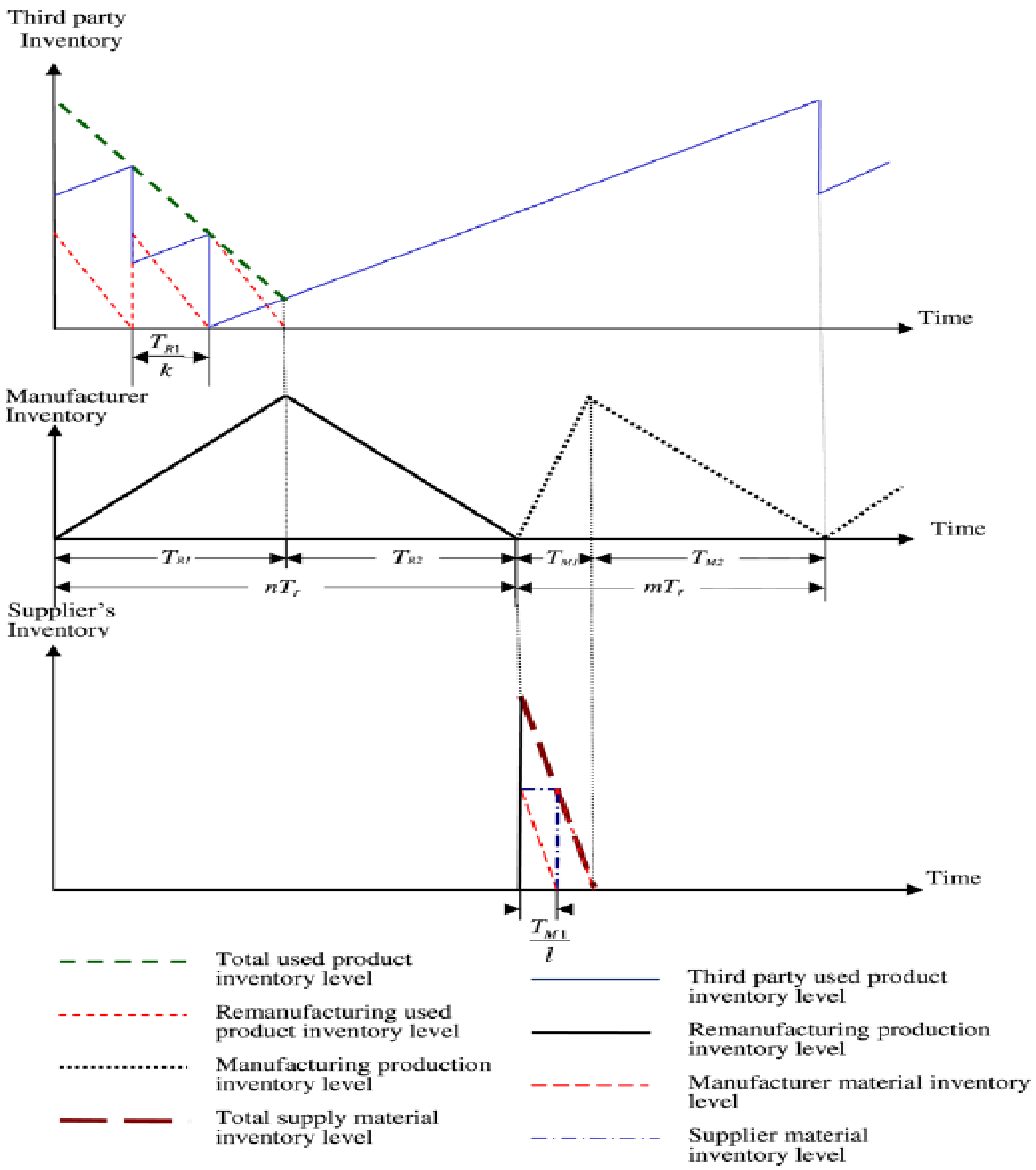


Figure 3.14 The inventory levels of the third-party, the manufacturer, the supplier with manufacturing and remanufacturing. Source: Chung et al. (2008)

Based on the literature review above, research studies have been carried out to coordinate and integrate production and inventory system in the multi-level supply chain considering some aspects such as reverse logistics, transportation cost, limited horizon period, multiple products and multiple sources. However, no research has been carried out to coordinate and integrate production and inventory system in a complex supply chain which is more than three-level supply chain considering all aspects mentioned above. Many research studies described considered only a part of the system studied. The summary table for the literature review can be seen in Appendix A.

This research therefore proposes and develops coordinated and integrated production inventory models in a complex manufacturing supply chain involving reverse logistics considering limited horizon period and transportation costs. In this research, we also consider multiple items (raw materials, parts and finished products as well as used products) and multiple sources (tier-2 and tier-1 suppliers). Works that have been carried out in Chen and Chen (2005,2008), Jaber and Goyal (2008), Chan and Kingsman (2007), Chan et al. (2010, Teunter (2001), Chung et al. (2008), Rieksts and Ventura (2008), and Ertogral (2011) are the models on which to build modelling framework in this thesis.

## 3.7 . Summary

In this chapter, research studies that had been carried out to coordinate production and inventory in the supply chain are reviewed. As described, there are two types of coordination in the supply chain. There are buyer-vendor coordination and multi-level supply chain coordination. The simple integrated production inventory model between vendor and buyer is how to determine common order cycle between the buyer and the vendor to minimise the total cost for both of them. The model assumes an infinite replenishment rate, constant demand rate, no shortages cost, and fixed price. Many extensions of the model have been carried out such as considering a finite production rate, offering a quantity discount scheme, multiple shipments/ deliveries, finite horizon period, non-constant demand rate, permissible delays in payment and multiple products.

The next extension of the buyer-vendor coordination model is coordinating production inventory models between a single vendor and multiple buyers and multiple vendors and a single buyer. For single vendor and multiple buyers, the models consider common cycle time for all players, different order cycles for each buyer, multiple shipments from vendor to all buyers, credit option and quantity discount scheme to compensate disadvantageous players and transportation cost explicitly. Unlike the single vendor multi-buyer coordination, a single buyer and multiple vendors coordination model is to determine both the vendor selection and lot size decision with the objective to minimise total system cost.

For multi-level supply chain, many research studies that have been carried out are to coordinate production inventory system in three-level supply chains. The supply chain consists of supplier/s, manufacturer/s and buyers. Similarly with buyer-vendor coordination, three-level supply chain model also considers multiple deliveries of a production lot, a quantity discount, learning-based continuous improvement, backordering, and transportation costs.

More extensions in the supply chain coordination are considering reverse logistics. Three drivers, economic, regulatory and customer pressure are forcing all companies to consider reverse logistics. Considering this issue, there are only few researches which had been carried out in the supply chain coordination such as between a manufacturer and a retailer, and between the supplier, the manufacturer and the third party recycle dealer.

It is clear that many research studies which had been carried out considered only a part of the system studied. Therefore, this research proposes and develops coordinated and integrated production inventory models in a complex manufacturing supply chain involving reverse logistics considering limited horizon period and transportation costs for multiple items (raw materials, parts and finished products) and multiple sources (tier-2 and tier-1 suppliers).

# Chapter 4

# Mathematical Modelling of Inventory System in a Complex Manufacturing Supply Chain

## 4.1 Introduction

In this chapter a mathematical modelling for coordinating production and inventory cycles in a complex manufacturing supply chain without involving reverse logistics is derived. In section 4.2 a description of the system studied is provided. The mathematical model of the system is developed and described in section 4.3. Section 4.4 summarises the chapter.

## 4.2 A Complex Manufacturing Supply Chain System

A complex manufacturing supply chain without involving reverse logistics consists of tier-2 suppliers, tier-1 suppliers, a manufacturer, distributors and retailers as shown in Fig. 4.1. Tier-2 suppliers produce and supply multiple-raw materials to tier-1 suppliers producing multiple-parts. Parts from tier-1 suppliers are then supplied to a manufacturer which manufactures and assembles parts into multiple finished products. The finished products are then delivered to distributors distributing them to retailers.

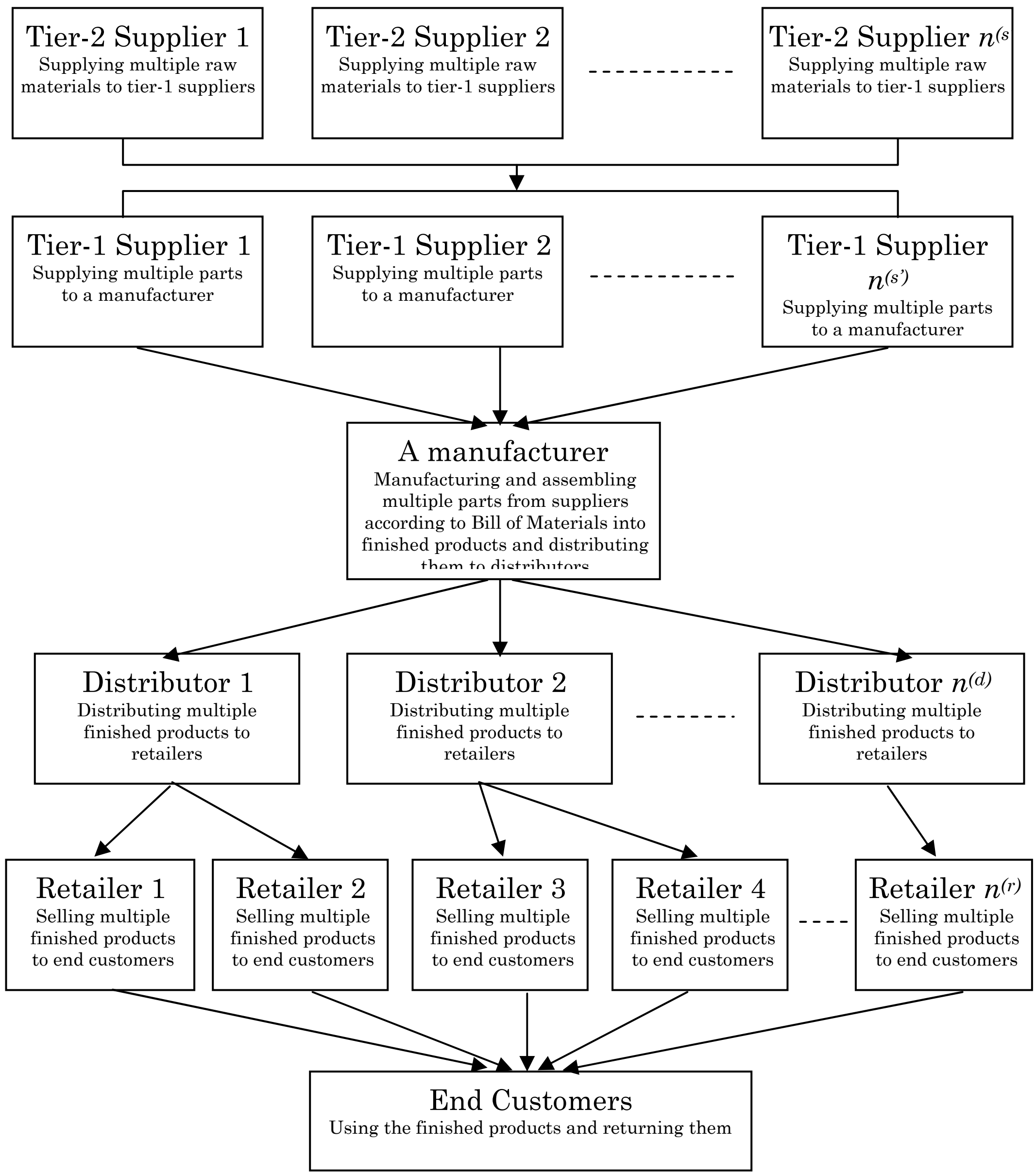


Figure 4.1 System description of a complex manufacturing supply chain without involving reverse logistics

## 4.3 Mathematical Modeling of the System

### 4.3.1 Assumptions and limitations

Before building the mathematical model of the system we explain and summarize all assumptions and limitations used. We consider a complex manufacturing supply chain consisting of tier-2 suppliers, tier-1 suppliers, the manufacturer, distributors and retailers. Tier-2 suppliers are specified to produce raw materials. Each tier-2 supplier may produce one or more types of raw materials and a type of raw materials may be produced by one or more tier-2 suppliers. Raw materials then are supplied to tier-1 suppliers according to the quantity needed by each tier-1 supplier. Like tier-2 suppliers, tier-1 suppliers are specified to produce parts. Each tier-1 supplier may produce one or more types of parts. A type of parts may also be produced by one or more tier-1 suppliers depending on the production capacity of each tier-1 supplier. Limited production capacity issue in a production inventory model had been addressed earlier in Ishii and Imori (1996). Then, parts produced by tier-1 suppliers are supplied to the manufacturer which manufactures and assembles them into finished products. A type of parts may be used in some types of finished products depending on the types of finished products. To determine how long lead times to produce raw materials, parts and finished we represent production rate term. Production rates for the manufacturer and all suppliers are limited. Since production rate term is used in modeling the system, we just need the data of the number of raw materials in units needed to produce one unit of parts and the number of parts in units needed to produce one unit of finished products. The data can be taken from bill of materials (BOM) of the products. Therefore, we ignore process sequences to produce each part or each finished product. In this chapter we first assume a constant demand and no shortages allowed and in the next chapter we eliminate their limitations.

In this section, we build the model of the cost function per unit time for retailers, distributors, the manufacturer, tier-1 suppliers, tier-2 suppliers, the third party and the whole supply chain which is the sum of all players cost function. Especially, we extend and develop works that have been carried out in Chen and Chen (2005), Jaber and (2008), Chang and Kingsman (2007) and Chang et al. (2010). We derive the cost function under an independent policy first and then under coordinated policies. The model under independent policy is provided so as to compare the performance of coordinated policies with that of the independent policy to identify whether coordination will lead to better performance. The mathematical model will be developed using notations listed comprehensively below.

### 4.3.2 Notations

The input parameters and decision variables for retailers, distributors, the manufacturer, tier-1 suppliers and tier-2 suppliers are as shown below, respectively.

Parameters:

$r$ index for retailers, $r$ = 1,2,..., $n^{(r)}_d$, where $n^{(r)}_d$ is the total number of retailers supplied by distributor d and $n^{(r)}$ is the total number of retailers supplied by $n^{(d)}$ distributors, $n^{(r)} = \sum_{d=1}^{n^{(d)}} n_d^{(r)}$

$d$ index for distributors, $d$ = 1,2,..., $n^{(d)}$, where $n^{(d)}$ is the total number of distributors.

$s'$ index for tier-1 suppliers, $s'$ = 1,2,..., $n^{(s')}$, where $n^{(s')}$ is the total number of tier-1 suppliers.

$s''$ index for tier-2 suppliers, $s''$ = 1,2,..., $n^{(s'')}$, where $n^{(s'')}$ is the total number of tier-2 suppliers.

$i$ index for product types, $i$ = 1,2,..., $k^{(i)}$, where $k^{(i)}$ is the number of product types.

$p$ index for part types, $p$ = 1,2,..., $k^{(p)}$, where $k^{(p)}$ is the number of part types.

$w$ index for raw material types, $w$ = 1,2,..., $k^{(w)}$, where $k^{(w)}$ is the number of types.

$D^{(r)}_{r,i}$ demand rate of retailer $r$ for product $i$

$D^{(d)}_{d,i}$ demand rate of distributor $d$ for product $i$, where $D_{d,i}^{(d)} = \sum_{r=1}^{n_d^{(r)}} D_{r,i}^{(r)}$

$D^{(m)}_i$ demand rate on the manufacturer for product $i$, where $D_i^{(m)} = \sum D_{d,i}^{(d)}$

$P^{(m)}_i$ production rate of the manufacturer for product $i$

$P^{(s')}_{s',p}$ production rate of the tier-1 supplier $s'$ for part $p$

$P^{(s'')}_{s'',w}$ production rate of the tier-2 supplier $s''$ for raw material $w$

$A^{(r)}_r$ ordering cost per cycle time of retailer $r$

$a_{r,i}^{(r)}$ the cost of placing an order for product $i$ from retailer $r$

$A^{(d)}_d$ ordering cost per cycle time of distributor $d$

$a_{d,i}^{(d)}$ the cost of placing an order for product $i$ from distributor $d$

$A_M$ ordering cost for all parts per cycle time of the manufacturer

$a_p^{(m)}$ the cost of placing an order for part $p$ from the manufacturer

$A^{(s')}_{s'}$ ordering cost for all raw materials of tier-1 supplier $s'$

$a_{s',w}^{(s')}$ the cost of placing an order for raw material $w$ from tier-1 supplier $s'$

$S^{(m)}$ setup cost per cycle time of the manufacturer for all finished products

$s^{(m)}_i$ setup cost for producing product $i$ per cycle time of the manufacturer

$S^{(s')}_{s'}$ setup cost per cycle time of tier-1 supplier $s'$ for all parts

$s^{(s')}_{s',p}$ setup cost for producing part $p$ of tier-1 supplier $s'$

$S^{(s")}_{s"}$ setup cost per cycle time of tier-2 supplier *s"*

$s^{(s")}_{s",w}$ setup cost for producing raw material *w* of tier-2 supplier *s"* for all raw materials

$h^{(r)}_{r,i}$ holding cost of retailer *r* for product *i*

$h^{(d)}_{d,i}$ holding cost of distributor *d* for product *i*

$h_i$ holding cost of the manufacturer for product *i*

$h^{(m)}_p$ holding cost of the manufacturer for part *p*

$h_{s',p}$ holding cost of tier-1 supplier *s'* for part *p*

$h^{(s')}_{s',w}$ holding cost of tier-1 supplier *s'* for raw material *w*

$h^{(s")}_{s",w}$ holding cost of tier-2 supplier *s"* for raw material *w*

$h^{(3)}_i$ holding cost for product *i* of the third party

$\beta^{(I)}_{p,i}$ the usage rate of part *p* per unit product *i*, where $\beta_{p,i} = \beta_{p',i}$

$\beta^{(II)}_{p,w}$ the usage rate of raw material *w* per unit part *p*

$e^{(s')}_{s',p}$ the proportion of part *p* supplied by tier-1 supplier *s'*

$e^{(s")}_{s",w}$ the proportion of raw material *w* supplied by tier-2 supplier *s"*

Decision variables:

$T^{(r)}_r$ cycle time of retailer *r*

$Q^{(r)}_{r,i}$ order quantity for product *i* of retailer *r*

$T$ common cycle time for all retailers

$T^{(d)}_d$ cycle time for distributor *d*

$Q^{(d)}_{d,i}$ order quantity for product *i* of distributor *d*

$T_D$ common cycle time for all distributors

$T_M$ cycle time of the manufacturer

$Q_i$ order quantity for product *i* of the manufacturer

$Q^{(m)}_p$ order quantity for part *p* of the manufacturer

$T^{(s')}_{s'}$ cycle time for tier-1 supplier *s'*

$Q_{s',p}$ order quantity for part *p* of tier-1 supplier *s'*

$Q^{(s')}_{s',w}$ order quantity for raw material *w* of tier-1 supplier *s'*

$T_{S'}$ common cycle time for all tier-1 suppliers

$T^{(s")}_{s"}$ cycle time for tier-2 supplier *s"*

$Q^{(s")}_{s",w}$ order quantity for raw material *w* of tier-2 supplier *s"*

$T_{S"}$ common cycle time for all tier-2 suppliers

$a_D$ integer multiplier of the cycle time of all distributors

$a_M$ integer multiplier of the cycle time of the manufacturer

$a_P$ integer multiplier of the manufacturer's cycle time for all parts

$a_{S'P}$ integer multiplier of the cycle time of all tier-1 suppliers

$a_{S'W}$ integer multiplier of the cycle time of all tier-1 suppliers for all raw materials

$a_{S''W}$ integer multiplier of the cycle time of all tier-2 suppliers

Objective functions*:*

$TCR_r$, $TCR$, $TCD_d$, $TCD$, $TCM$, $TCS'_{s'}$ , $TCS'$, $TCS''_{s''}$, $TCS''$, *TCChain* are total associated cost of retailer *r*, all retailers, distributor *d*, all distributors, the manufacturer, tier-1 supplier *s'*, all tier-1 suppliers, tier-2 supplier *s''*, all tier-2 suppliers, and the whole supply chain respectively.

### 4.3.3 Independent policy

Under an independent policy, each player of the supply chain minimises its own inventory cost by its own model without considering the interests of other players. Each player determines each optimal order and/or production cycle and quantity without considering optimal order and/or production cycle and quantity of other players. We derive formulations based on economic order quantity (EOQ) model, Economic Production Quantity (EPQ) and works that have been carried out in Chan and Kingsman (2007) and Chan et al. (2010).

#### *4.3.3.1 The cost of retailers*

Retailers clearly incur only two types of costs; ordering cost and finished products holding cost. We derive the cost function for each retailer for both single item and joint items policy based on traditional Economic Order Quantity (EOQ) model. Under single item policy each retailer orders each finished product independently. Retailer *r* orders $Q^{(r)}_{r,i}$ units of each finished product *i* from distributor *d* every cycle time $T^{(r)}_{r,i}$. Total cost function for retailer *r* for finished product *i*, $TCR_{r,i}$, for multiple retailers and multiple products is given by

$$TCR_{r,i} = \frac{A^{(r)}_{r} + a^{(r)}_{r,i}}{T^{(r)}_{r,i}} + \frac{T^{(r)}_{r,i} h^{(r)}_{r,i} D^{(r)}_{r,i}}{2} \tag{4.1}$$

The first term is ordering cost per unit time and the second term is finished products holding cost per unit time.

Based on standard method for calculating economic order interval and quantity, the economic order interval and quantity for each retailer for each finished product are derived. Differentiating $TCR_{r,i}$ with respect to $T^{(r)}_{r,i}$

$$\frac{\partial TCR_{r,i}}{\partial T^{(r)}_{r,i}} = -\frac{A^{(r)}_{r} + a^{(r)}_{r,i}}{T^{(r)2}_{r,i}} + \frac{h^{(r)}_{r,i} D^{(r)}_{r,i}}{2}$$

By setting

$$\frac{\partial TCR_{r,i}}{\partial T_{r,i}^{(r)}} = 0 \text{, then}$$

$$T_{r,i}^{(r)*} = \sqrt{\frac{2\left(A_r^{(r)} + a_{r,i}^{(r)}\right)}{h_{r,i}^{(r)} D_{r,i}^{(r)}}} \tag{4.2}$$

Based on traditional economic order quantity (EOQ) model

$$Q_{r,i}^{(r)} = T_r^{(r)} D_{r,i}^{(r)} \tag{4.3}$$

By substituting Eq. (4.2) to Eq. (4.3),

$$Q_{r,i}^{(r)*} = \sqrt{\frac{2D_{r,i}^{(r)2}\left(A_r^{(r)} + a_{r,i}^{(r)}\right)}{h_{r,i}^{(r)} D_{r,i}^{(r)}}} \tag{4.4}$$

where $T^{(r)}{}_{r,i}*$ and $Q^{(r)}{}_{r,i}*$ are optimal cycle time and order quantity of finished product $i$ for retailer $r$.

Eq. (4.1) is a convex function when the second derivation of it with respect to $T^{(r)}{}_{r,i}$ is more than zero.

$$\frac{\partial^2 TCR_{r,i}}{\partial T_{r,i}^{(r)2}} = 2\frac{A_r^{(r)} + a_{r,i}^{(r)}}{T_{r,i}^{(r)3}} > 0$$

Therefore, the optimal solution for Eq. (4.1) is a global optimum.

For all finished products and retailers, the total cost function can be expressed as follows, respectively

$$TCR_r = \sum_{i=1}^{k^{(i)}} \left( \frac{A_r^{(r)} + a_{r,i}^{(r)}}{T_{r,i}^{(r)}} + \frac{T_{r,i}^{(r)} h_{r,i}^{(r)} D_{r,i}^{(r)}}{2} \right) \tag{4.5}$$

$$TCR = \sum_{r=1}^{n^{(r)}} \sum_{i=1}^{k^{(i)}} \left( \frac{A_r^{(r)} + a_{r,i}^{(r)}}{T_{r,i}^{(r)}} + \frac{T_{r,i}^{(r)} h_{r,i}^{(r)} D_{r,i}^{(r)}}{2} \right) \tag{4.6}$$

Furthermore, under a joint item policy each retailer orders each finished product at joint order cycle time for all finished products. Retailer $r$ orders $Q^{(r)}{}_{r,i}$ units of each finished product $i$ from distributor $d$ every joint cycle time $T^{(r)}{}_r$. Similarly the total cost function for retailer $r$, $TCR_r$, is given by

$$TCR_r = \frac{A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)}}{T_r^{(r)}} + \frac{T_r^{(r)} \sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)}}{2} \tag{4.7}$$

Then, economic order interval and quantity will be

$$T_r^{(r)^*} = \sqrt{\frac{2\left(A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)}\right)}{\sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)}}} \tag{4.8}$$

$$Q_{r,i}^{(r)^*} = \sqrt{\frac{2D_{r,i}^{(r)^2}\left(A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)}\right)}{\sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)}}} \tag{4.9}$$

For all retailers, the total cost function is given by

$$TCR = \sum_{r=1}^{n^{(r)}} \left( \frac{A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)}}{T_r^{(r)}} + \frac{T_r^{(r)} \sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)}}{2} \right) \tag{4.10}$$

*4.3.3.2 The cost of distributors*

Distributor $d$ is faced with orders from each of the retailers supplied by it based on their demand rates and so

$$D_{d,i}^{(d)} = \sum_{r=1}^{n_d^{(r)}} D_{r,i}^{(r)} \tag{4.11}$$

Similarly with retailers, distributors incur also two types of costs; ordering cost and finished products holding costs. When retailers and their distributors are operating independently inventory cycles of distributors may be different with retailers' ones. In order to anticipate orders from retailers at the same time which may be different with inventory cycles of distributors they need a stock to fulfill these orders. The largest possible aggregate stock quantity needed to satisfy all orders from retailers at the same time is $\sum_{r=1}^{n_d^{(r)}} Q_{r,i}^{(r)^*}$ units for finished product $i$. Since orders from retailers are processed and delivered depending on the optimal order cycle time of retailers $T_{r,i}^{(r)*}$ we need order processing and fixed shipment cost which is separated from the ordering cost. Chan and Kingsman (2007) and Chan et al. (2010) had addressed these issues. Therefore, the cost function per unit time incurred by distributor $d$ for finished product $i$ under single item policy is

$$TCD_{d,i} = \frac{A_d^{(d)} + a_{d,i}^{(d)}}{T_{d,i}^{(d)}} + \frac{T_{d,i}^{(d)} h_{d,i}^{(d)} D_{d,i}^{(d)}}{2} + h_{d,i}^{(d)} \sum_{r=1}^{n_d^{(r)}} Q_{r,i}^{(r)^*} + \sum_{r=1}^{n_d^{(r)}} \frac{B_d^{(d)} + b_{d,i}^{(d)}}{T_{r,i}^{(r)^*}} \tag{4.12}$$

The third term is holding cost of the stock for anticipating the orders from retailers at the same time per unit time for distributor *d* if the distributor is to have zero stock outs. The fourth term is order processing and fixed shipment cost to supply orders to retailers. We develop works that have been carried out in Chan and Kingsman (2007) and Chan et al. (2010) for the third and fourth terms for multiple products and multiple players. Again based on standard method for calculating economic order interval and quantity, the economic order interval and quantity are given.

$$\frac{\partial TCD_{d,i}}{\partial T_{d,i}^{(d)}} = -\frac{A_d^{(d)} + a_{d,i}^{(d)}}{T_{d,i}^{(d)2}} + \frac{h_{d,i}^{(d)} D_{d,i}^{(d)}}{2}$$

By setting

$\frac{\partial TCD_{d,i}}{\partial T_{d,i}^{(d)}} = 0$, then

$$T_{d,i}^{(d)*} = \sqrt{\frac{2\left(A_d^{(d)} + a_{d,i}^{(d)}\right)}{h_{d,i}^{(d)} D_{d,i}^{(d)}}} \tag{4.13}$$

$$Q_{d,i}^{(d)*} = \sqrt{\frac{2 D_{d,i}^{(d)2}\left(A_d^{(d)} + a_{d,i}^{(d)}\right)}{h_{d,i}^{(d)} D_{d,i}^{(d)}}} \tag{4.14}$$

where $T^{(d)}_{d,i}*$ and $Q^{(d)}_{d,i}*$ are optimal cycle time and order quantity of finished product *i* for distributor *d*. Here, holding cost for the stock to anticipate orders from retailers at the same time and the order processing and fixed shipment cost do not affect the optimal cycle time and order quantity.

For all finished products and distributors, the total cost function will be

$$TCD = \sum_{d=1}^{n^{(d)}} \sum_{i=i}^{k^{(i)}} \left( \frac{A_d^{(d)} + a_{d,i}^{(d)}}{T_{d,i}^{(d)}} + \frac{T_{d,i}^{(d)} h_{d,i}^{(d)} D_{d,i}^{(d)}}{2} + h_{d,i}^{(d)} \sum_{r=1}^{n_d^{(r)}} Q_{r,i}^{(r)*} + \sum_{r=1}^{n_d^{(r)}} \frac{B_d^{(d)} + b_{d,i}^{(d)}}{T_{r,i}^{(r)*}} \right) \tag{4.15}$$

The maximum inventory positioning at each distributor will be economic order quantity plus the stock for anticipating orders at the same time from retailers.

For joint items policy, the total cost function will be

$$TCD_d = \frac{A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)}}{T_d^{(d)}} + \frac{T_d^{(d)} \sum_{i=1}^{k^{(i)}} \left( h_{d,i}^{(d)} D_{d,i}^{(d)} \right)}{2} + \sum_{i=1}^{k^{(i)}} h_{d,i}^{(d)} \sum_{r=1}^{n_d^{(r)}} Q_{r,i}^{(r)^*} + \sum_{r=1}^{n_d^{(r)}} \frac{B_d^{(d)} + b_{d,i}^{(d)}}{T_r^{(r)^*}} \tag{4.16}$$

and economic order interval and quantity are

$$T_d^{(d)^*} = \sqrt{\frac{2\left( A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)} \right)}{\sum_{i=1}^{k^{(i)}} \left( h_{d,i}^{(d)} D_{d,i}^{(d)} \right)}} \tag{4.17}$$

$$Q_{d,i}^{(d)^*} = \sqrt{\frac{2 D_{d,i}^{(d)^2} \left( A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)} \right)}{\sum_{i=1}^{k^{(i)}} h_{d,i}^{(d)} D_{d,i}^{(d)}}} \tag{4.18}$$

where $T^{(d)}{}_d{}^*$ is optimal order cycle time for distributor $d$

For all distributors, the total cost function is

$$TCD = \sum_{d=1}^{n^{(d)}} \left( \frac{A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)}}{T_d^{(d)}} + \frac{T_d^{(d)} \sum_{i=1}^{k^{(i)}} \left( h_{d,i}^{(d)} D_{d,i}^{(d)} \right)}{2} + \sum_{i=1}^{k^{(i)}} h_{d,i}^{(d)} \sum_{r=1}^{n_d^{(r)}} Q_{r,i}^{(r)^*} + \sum_{r=1}^{n_d^{(r)}} \frac{B_d^{(d)} + b_{d,i}^{(d)}}{T_r^{(r)^*}} \right) \tag{4.19}$$

*4.3.3.3 The cost of manufacturer*

The manufacturer is faced with orders from distributors based on demand rate $D^{(m)}{}_i$ where

$$D_i^{(m)} = \sum_{d=1}^{n^{(d)}} D_{d,i}^{(d)} \tag{4.20}$$

The manufacturer manufactures and assembles parts from tier-1 suppliers into finished products at a rate of $P^{(m)}{}_i$ per unit time for each finished product $i$ with $P^{(m)}{}_i$ > $D^{(m)}{}_i$. We assume that each finished product $i$ is manufactured and assembled separately in a different production line. When the manufacturer and distributors are operating independently, even the manufacturer also needs to carry a stock of finished products to satisfy orders from distributors at the same time. Similar to distributors, the largest possible aggregate stock quantity is $\sum_{d=1}^{n^{(d)}} Q_{d,i}^{(d)^*}$ units of stock for anticipating those orders. Since the optimal production cycle time of the manufacturer may not be the same with the optimal order cycle times of distributors the manufacturer needs to hold this stock to anticipate orders from distributors when the manufacturer either has not started or just starts to produce the products as described in Chan and Kingsman (2007).

Unlike retailers and distributors, the manufacturer incurs production setup cost, ordering cost for parts, holding cost for finished products, holding cost for parts, holding cost of the stock for anticipating orders from distributors and order processing and fixed shipment cost. The detailed derivation for each cost is as follows;

*Production setup cost*: the manufacturer produces finished product $i$ every production cycle time $T_i$. The manufacturer incurs major setup cost $S^{(m)}$ for the production line and minor setup cost $s^{(m)}_i$ for each finished product every production cycle time. Under single item policy, production setup cost incurred by the manufacturer for finished product $i$ per unit time is $\left(S^{(m)} + s_i^{(m)}\right)/T_i$.

*Finished products holding cost*: The manufacturer produces each finished product with the production rate per unit time $P^{(m)}_i$ to fulfill the demand $D^{(m)}_i$ per unit time. Based on traditional economic production/ manufacturing quantity (EPQ/EMQ) model average inventory for finished product $i$ is $\dfrac{D_i^{(m)}T_i}{2}\left(1-\dfrac{D_i^{(m)}}{P_i^{(m)}}\right)$. Hence, holding cost for finished product $i$ per unit time is given by $\dfrac{h_i D_i^{(m)}T_i}{2}\left(1-\dfrac{D_i^{(m)}}{P_i^{(m)}}\right)$.

Then, the inventory cost for finished product $i$ is $\dfrac{\left(S^{(m)} + s_i^{(m)}\right)}{T_i} + \dfrac{h_i D_i^{(m)}T_i}{2}\left(1-\dfrac{D_i^{(m)}}{P_i^{(m)}}\right)$

*Parts ordering cost*: The manufacturer orders parts from tier-1 suppliers to manufacture and assemble into finished products. Parts are ordered every order cycle time $\alpha_i T_i$ which is multiple integer of $T_i$, production cycle time for finished product $i$. The manufacturer incurs major ordering cost $A_M$ per cycle time and minor ordering cost $a^{(m)}_p$ for part $p$ per cycle time. Ordering cost for part $p$ incurred by the manufacturer per unit time is $\left(A_M + \sum_{p\in i} a_p^{(m)}\right)/\alpha_i T_i$

*Parts holding cost*: Given the usage rate of part $p$ per unit finished product $i$ $\beta^{(I)}_{p,i}$ the demand for part $p$ per unit time is $\sum_{i=1}^{k^{(i)}} \beta^{(I)}_{p,i} D^{(m)}_{i}$. Under single finished product policy order quantity per order cycle time for part $p$ is $\beta^{(I)}_{p,i} D^{(m)}_{i} \alpha_i T_i$. Parts are consumed during $\frac{D^{(m)}_{i}}{P^{(m)}_{i}} T_i$ period and stored in stock of $(\alpha_i - 1)\beta^{(I)}_{p,i} D^{(m)}_{i} T_i$ units to be consumed during the next $\alpha_i$ production cycle time. Average inventory for part $p$ for finished product $i$ is $\frac{\beta^{(I)}_{p,i} D^{(m)}_{i} T_i}{2}\left(\frac{D^{(m)}_{i}}{P^{(m)}_{i}} + (\alpha_i - 1)\right)$.

So, the holding cost for part $p$ for finished product $i$ per unit time is $\frac{h^{(m)}_{p} \beta^{(I)}_{p,i} D^{(m)}_{i} T_i}{2}\left(\frac{D^{(m)}_{i}}{P^{(m)}_{i}} + (\alpha_i - 1)\right)$.

Similarly, since orders from distributors are processed and delivered depending on the optimal order cycle time of retailers $T^{(d)}_{d,i}{}^*$ we need order processing and fixed shipment cost which is separated from the production setup cost. The order processing and fixed shipment cost will be $\frac{B^{(m)} + b^{(m)}_{i}}{T^{(d)^*}_{d,i}}$. The holding cost of stock for anticipating orders from distributors is $h_i \sum_{d=1}^{n^{(d)}} Q^{(d)^*}_{d,i}$.

Then, the total cost function for the manufacturer for finished product $i$

$$TCM_i = \frac{S^{(m)} + s^{(m)}_{i}}{T_i} + \frac{h_i D^{(m)}_{i} T_i}{2}\left(1 - \frac{D^{(m)}_{i}}{P^{(m)}_{i}}\right) + h_i \sum_{d=1}^{n^{(d)}} Q^{(d)^*}_{d,i} + \sum_{d=1}^{n^{(d)}} \frac{B^{(m)} + b^{(m)}_{i}}{T^{(d)^*}_{d,i}} + \frac{A_M + \sum_{p \in i} a^{(m)}_{p}}{\alpha_i T_i}$$
$$+ \sum_{p=1}^{k^{(p)}} \left( \frac{\beta^{(I)}_{p,i} h^{(m)}_{p} D^{(m)}_{i} T_i}{2} \left( \frac{D^{(m)}_{i}}{P^{(m)}_{i}} + (\alpha_i - 1) \right) \right) \tag{4.21}$$

The first term is the manufacturer's production setup cost for finished product $i$ per unit time. The second term is the manufacturer's holding cost for finished product $i$. The third term is the manufacturer's holding cost for the stock of finished product $i$ for anticipating orders that might come from all distributors simultaneously. The fourth term is order processing and fixed shipment cost to supply orders to distributors. The fifth term is the manufacturer's ordering cost for parts and the last term is the manufacturer's holding cost for parts. Again, we develop works that have been carried out in Chan and Kingsman (2007) and Chan et al. (2010) for the the third and fourth terms for multiple products.

Likewise, based on standard method for calculating economic production interval and quantity the economic production interval and quantity for finished product $i$ ($T_i^*$) are given.

In Eq. (4.22), Let

$$A1 = \left(S^{(m)} + s_i^{(m)}\right) \; A2 = \left(A_M + \sum_{p \in i} a_p^{(m)}\right) \; H1 = \frac{h_i D_i^{(m)}}{2}\left(1 - \frac{D_i^{(m)}}{P_i^{(m)}}\right)$$

$$H2 = \sum_{p=1}^{k^{(p)}} \left( \frac{\beta_{p,i}^{(I)} h_p^{(m)} D_i^{(m)}}{2} \left( \frac{D_i^{(m)}}{P_i^{(m)}} + (\alpha_i - 1) \right) \right) \text{then}$$

$$TCM_i = \frac{A1}{T_i} + H1 T_i + h_i \sum_{d=1}^{n^{(d)}} Q_{d,i}^{(d)^*} + \sum_{d=1}^{n^{(d)}} \frac{B^{(m)} + b_i^{(m)}}{T_{d,i}^{(d)^*}} + \frac{A2}{\alpha_i T_i} + H2 T_i$$

Differentiating $TCM$ with respect to $T_i$,

$$\frac{\partial TCM_i}{\partial T_i} = -\frac{A1}{T_i^2} + H1 - \frac{A2}{\alpha_i T_i^2} + H2$$

By setting

$\frac{\partial TCM_i}{\partial T_i} = 0$, then

$$T_i^* = \sqrt{\frac{\alpha_i A1 + A2}{\alpha_i (H1 + H2)}} \tag{4.22}$$

and the economic production and order quantity for finished products and parts, respectively, are

$$Q_i^* = T_i^* D_i^{(m)} \tag{4.23}$$

$$Q_{p,i}^{(m)^*} = T_i^* \beta_{p,i}^{(I)} D_i^{(m)} \tag{4.24}$$

For all finished products the total cost function will be

$$TCM_i = \sum_{i=1}^{k^{(i)}} \left( \begin{array}{l} \frac{S^{(m)} + s_i^{(m)}}{T_i} + \frac{h_i D_i^{(m)} T_i}{2}\left(1 - \frac{D_i^{(m)}}{P_i^{(m)}}\right) + h_i \sum_{d=1}^{n^{(d)}} Q_{d,i}^{(d)^*} + \sum_{d=1}^{n^{(d)}} \frac{B^{(m)} + b_i^{(m)}}{T_{d,i}^{(d)^*}} + \frac{A_M + \sum_{p \in i} a_p^{(m)}}{\alpha_i T_i} \\ + \sum_{p=1}^{k^{(p)}} \left( \frac{\beta_{p,i}^{(I)} h_p^{(m)} D_i^{(m)} T_i}{2} \left( \frac{D_i^{(m)}}{P_i^{(m)}} + (\alpha_i - 1) \right) \right) \end{array} \right) \tag{4.25}$$

Similarly, under joint items policy the total cost function for the manufacturer is

$$TCM=\sum_{i=1}^{k^{(i)}}\left(\begin{array}{l}\sum_{i=1}^{k^{(i)}}\frac{S^{(m)}+\sum_{i=1}^{k^{(i)}}s_i^{(m)}}{T_M}+\sum_{i=1}^{k^{(i)}}\frac{h_iD_i^{(m)}T_M}{2}\left(1-\frac{D_i^{(m)}}{P_i^{(m)}}\right)+\sum_{i=1}^{k^{(i)}}h_i\sum_{d=1}^{n^{(d)}}Q_{d,i}^{(d)^*}+\sum_{i}^{k^{(i)}}\sum_{d=1}^{n^{(d)}}\frac{B^{(m)}+b_i^{(m)}}{T_d^{(d)^*}}+\frac{A_M+\sum_{p=1}^{k^{(p)}}a_p^{(m)}}{\alpha_PT_M}\\+\sum_{i=1}^{k^{(i)}}\sum_{p=1}^{k^{(p)}}\left(\frac{\beta_{p,i}^{(I)}h_p^{(m)}D_i^{(m)}T_i}{2}\left(\frac{D_i^{(m)}}{P_i^{(m)}}+(\alpha_P-1)\right)\right)\end{array}\right) \quad (4.26)$$

and economic production interval and quantity for finished products is

$$T_M{}^*=\sqrt{\frac{\alpha_P\left(S^{(m)}+\sum_{i=1}^{k^{(i)}}s_i^{(m)}\right)+\left(A_M+\sum_{p=1}^{k^{(p)}}a_p^{(m)}\right)}{\alpha_P\left(\sum_{i=1}^{k^{(i)}}\frac{h_iD_i^{(m)}}{2}\left(1-\frac{D_i^{(m)}}{P_i^{(m)}}\right)+\sum_{i=1}^{k^{(i)}}\sum_{p=1}^{k^{(p)}}\left(\frac{\beta_{p,i}^{(I)}h_p^{(m)}D_i^{(m)}T_i}{2}\left(\frac{D_i^{(m)}}{P_i^{(m)}}+(\alpha_P-1)\right)\right)\right)}} \quad (4.27)$$

$$Q_i^*=T_M{}^*D_i^{(m)} \quad (4.28)$$

and the economic order quantity for parts is

$$Q_p^{(m)^*}=\alpha_P{}^*T_M{}^*\sum_{i=1}^{k^{(i)}}\beta_{p,i}^{(I)}D_i^{(m)} \quad (4.29)$$

*4.3.3.4 The cost of tier-1 suppliers*

Each tier-1 supplier produces one or more types of the parts needed by the manufacturer to manufacture and assemble finished products. Under the condition that each tier-1 supplier has limited production capacity, it is possible that one or more types of parts can be supplied by more than one tier-1 supplier. Thus, the number of part *p* supplied by tier-1 supplier *s* per unit time is $e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)$ units where $e_{s',p}^{(s')}$ is a proportion of parts supplied by tier-1 supplier *s'* and $\beta_{p,i}^{(I)}D_i^{(m)}$ is the number of parts needed by the manufacturer to produce finished product *i*. Once again, because tier-1 suppliers and the manufacturer are operating the inventory independently, each of the tier-1 suppliers needs to carry $e_{s',p}^{(s')}Q_p^{(m)^*}$ units of stock for anticipating order from the manufacturer for each part. Tier-1 suppliers incur parts production setup cost, raw materials ordering cost, parts holding cost and raw materials holding cost.

Similar to the manufacturer, the total cost function per unit time incurred by tier-1 suppliers is given.

$$TCS=\sum_{s'=1}^{n^{(s')}}\frac{S_{s'}^{(s')}+\sum_{p=1}^{k^{(p)}}s_{s',p}^{(s')}}{T_{s'}^{(s')}}+\sum_{s'=1}^{n^{(s')}}\sum_{p=1}^{k^{(p)}}\left(\frac{h_{s',p}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)T_{s'}^{(s')}}{2}\left(1-\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}}\right)\right)+\sum_{s'=1}^{n^{(s')}}\sum_{p=1}^{k^{(p)}}h_{s',p}e_{s',p}^{(s')}Q_p^{(m)*}$$
$$+\sum_{s'=1}^{n^{(s')}}\frac{A_{s'}^{(s')}+\sum_{w=1}^{k^{(w)}}a_{s',w}^{(s')}}{\alpha_{S'W}T_{s'}^{(s')}}+\sum_{s'=1}^{n^{(s')}}\frac{B_{(s')}^{(s')}+\sum_{p=1}^{k^{(p)}}b_{s',p}^{(s')}}{\alpha_p{}^{*}T^{*}}\sum_{s'=1}^{n^{(s')}}\sum_{w=1}^{k^{(w)}}\sum_{p=1}^{k^{(p)}}\left(\frac{\beta_{p,w}^{(II)}h_{s',w}^{(s')}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)T_{s'}^{(s')}}{2}\left(\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}}+\left(\alpha_{S'W}-1\right)\right)\right) \tag{4.30}$$

The economic production interval and quantity for parts and raw materials are given

$$T_{s'}^{(s')^*}=\sqrt{\frac{\alpha_{S'W}B1+B2}{\alpha_{S'W}\left(G1+G2\right)}}$$

where $B1=\sum_{s'=1}^{n^{(s')}}S_{s'}^{(s')}+\sum_{p=1}^{k^{(p)}}s_{s',p}^{(s')}$ $B2=\sum_{s'=1}^{n^{(s')}}A_{s'}^{(s')}+\sum_{w=1}^{k^{(w)}}a_{s',w}^{(s')}$

$$H1=\sum_{p=1}^{k^{(p)}}\left(\frac{h_{s',p}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)}{2}\left(1-\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}}\right)\right) \quad \text{and}$$

$$H2=\sum_{w=1}^{k^{(w)}}\sum_{p=1}^{k^{(p)}}\left(\frac{\beta_{p,w}^{(II)}h_{s',w}^{(s')}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)}{2}\left(\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}}+\left(\alpha_{S'W}-1\right)\right)\right)$$

$$Q_{s',p}^{(s')^*}=T_{s'}^{(s')^*}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right) \tag{4.31}$$

$$Q_{s',w}^{(s')^*}=\alpha_{S'W}{}^{*}T_{s'}^{(s')^*}\beta_{p,w}^{(II)}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right) \tag{4.32}$$

*4.3.3.5 The cost of tier-2 suppliers*

Raw materials supplied to tier-1 suppliers are produced by tier-2 suppliers. Every tier-2 supplier *s"* can supply raw materials to one or more tier-1 suppliers. Thus it can apply that a number of raw material *w* supplied by tier-2 supplier *s"* per unit time is $e_{s'',w}^{(s'')}\sum_{p=1}^{k^{(p)}}\beta_{p,w}^{(II)}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)$ units where $e_{s'',w}^{(s'')}$ is the proportion of raw materials supplied by tier-2 supplier *s''* and $\beta_{p,w}^{(II)}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)$ is the number of raw materials needed by the tier-1 suppliers to produce part $p$. To satisfy all demand of tier-1 suppliers on time, tier-2 supplier $s''$ needs to carry a large stock of $e^{(s'')}_{s'',w}\sum_{s'=1}^{n^{(s')}} Q^{(s')^*}_{s',w}$ units for each raw material $w$. The total cost function per unit time incurred by tier-2 suppliers is as follows:

$$TCS' = \sum_{s''=1}^{n^{(s'')}} \frac{S^{(s'')}_{s''} + \sum_{w=1}^{k^{(w)}} s^{(s'')}_{s'',w}}{T^{(s'')}_{s''}} + \sum_{s''=1}^{n^{(s'')}}\sum_{w=1}^{k^{(w)}} \left( \frac{h^{(s'')}_{s'',w} e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \left(\beta^{(I)}_{p,i} D^{(m)}_{i}\right) T^{(s'')}_{s''}}{2} \left( 1 - \frac{e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \left(\beta^{(I)}_{p,i} D^{(m)}_{i}\right)}{P^{(s'')}_{s'',w}} \right)\right) \tag{4.33}$$

$$+ \sum_{s''=1}^{n^{(s'')}}\sum_{w=1}^{k^{(w)}} h^{(s'')}_{s'',w} e^{(s'')}_{s'',w} \sum_{s'=1}^{n^{(s')}} Q^{(s')^*}_{s',w} + \sum_{s''=1}^{n^{(s'')}}\sum_{s'=1}^{n^{(s')}} \frac{B^{(s'')}_{(s'')} + \sum_{w=1}^{k^{(w)}} b^{(s'')}_{s'',w}}{\alpha_{S'W}{}^{*} T^{(s')^*}_{s'}}$$

### 4.3.4 Coordinated policy

Under coordinated policy there is cooperation between all players in the supply chain in determining production and inventory cycles. Inventory and/or production cycles of downstream players have a correlation with production and/or inventory cycles of upstream players using integer multipliers of inventory and/or production cycles of the lower level players in the supply chain. In each level of the supply chain we assume that all players use common transportation units to deliver raw materials, parts and finished products from upstream level to immediate downstream level of the supply chain. Therefore they can reduce the number of transportation units used.

*4.3.4.1 Retailers' cost components*

Under coordinated policy all retailers apply a common order cycle time $T$. By applying $T$ to Eq. (4.10) the cost function will be

$$TCR = \frac{\sum_{r=1}^{n^{(r)}} \left( A^{(r)}_{r} + \sum_{i=1}^{k^{(i)}} a^{(r)}_{r,i} \right)}{T} + \frac{\sum_{r=1}^{n^{(r)}} \left( T \sum_{i=1}^{k^{(i)}} h^{(r)}_{r,i} D^{(r)}_{r,i} \right)}{2} \tag{4.34}$$

Similarly with independent policy the economic order interval and quantity are respectively

$$T^* = \sqrt{\frac{2\sum_{r=1}^{n^{(r)}} \left( A^{(r)}_{r} + \sum_{i=1}^{k^{(i)}} a^{(r)}_{r,i} \right)}{\sum_{r=1}^{n^{(r)}} \sum_{i=1}^{k^{(i)}} h^{(r)}_{r,i} D^{(r)}_{r,i}}} \tag{4.35}$$

and

$$Q_{r,i}^{(r)^*} = \sqrt{\frac{2D_{r,i}^{(r)^2}\sum_{r=1}^{n^{(r)}}\left(A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)}\right)}{\sum_{r=1}^{n^{(r)}}\sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)}}} \tag{4.36}$$

where $T^*$ is optimal common order cycle time for all retailers.

*4.3.4.2 Distributors' cost components*

Since there is the relationship of production and/or inventory cycles between all players in the supply chain distributors apply also a common order cycle time which is an integer multiplier of $T$, common order cycle time of retailers, $\alpha_D T$ for all distributors. Distributors incur three types of costs; ordering cost, finished products holding cost and order processing and fixed shipment cost. The last term of these costs is the cost to process and deliver finished products to retailers. Usually, this cost can be included in ordering cost. Since the value of $\alpha_D$ can be more than one, it means that the number of orders processed and delivered to retailers can be more than the number of orders from distributors to the manufacturer. So we need to separate this cost from the ordering cost. As decisions for distributors and retailers are coordinated, orders from retailers are anticipated so there will be no need to keep the stock. The detailed derivation for each cost is as follows:

*Ordering cost*: Distributor $d$ orders all finished products every order cycle time $\alpha_D T$ with ordering cost for all finished products per cycle time $A_d^{(d)}$ and the cost for placing an order for finished product $i$ per cycle time $a_{d,i}^{(d)}$. Ordering cost incurred by distributor d per unit time is $\left(A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)}\right) \Big/ \alpha_D T$

*Finished products holding cost*: A distributor orders $D^{(d)}_{d,i}\alpha_D T$ units of finished product $i$ every cycle time $\alpha_D T$ from the manufacturer and $D^{(d)}_{d,i}T$ units of finished product $i$ will be immediately delivered to satisfy the first order from retailers. The maximum stock stored for next common order cycle of retailers will be $D^{(d)}_{d,i}T(\alpha_D-1)$ units as shown in Fig. 4.2. Fig. 4.2 shows inventory behaviour of finished product $i$ for retailer $r$ and distributor $d$, with $\alpha_D = 3$. Following the basic EOQ model, average inventory for distributor $d$ for finished product $i$ based on Fig. 4.2 is calculated as follows:

Average inventory for distributor $d$

$$= \frac{D_{d,i}^{(d)}T(\alpha_D - 1)T + D_{d,i}^{(d)}T(\alpha_D - 2)T + D_{d,i}^{(d)}T(\alpha_D - 3)T + \ldots + D_{d,i}^{(d)}T^2}{\alpha_D T}$$

$$= \frac{D_{d,i}^{(d)}T((\alpha_D - 1) + (\alpha_D - 2) + (\alpha_D - 3) + \ldots + 1)}{\alpha_D}$$

$$= \frac{D_{d,i}^{(d)}T(\alpha_D - 1)}{2} \tag{4.37}$$

Hence, finished product $i$ inventory holding cost for distributor $d$ is given by $h_{d,i}^{(d)} \frac{D_{d,i}^{(d)}T(\alpha_D - 1)}{2}$.

*Order processing and fixed shipment cost*: A distributor supplies and delivers orders $D^{(d)}_{d,i}T$ units of finished product $i$ every cycle time $T$, common order cycle time of retailers, to retailers with order processing and fixed shipment cost per unit time $B_d^{(d)}$ and per finished product $i$ $b_{d,i}^{(d)}$ for distributor $d$. Order processing and fixed shipment cost incurred by distributor $d$ per unit time is $\left(B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)}\right) \Big/ T$. We develop work that has been carried out in Chan and Kingsman (2007) for this cost.

Then, the total cost incurred by all distributors for all finished products is

$$TCD = \frac{\sum_{d=1}^{n_d}\left(A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)}\right)}{\alpha_D T} + \frac{T\sum_{d=1}^{n^{(d)}}\sum_{i=1}^{k^{(i)}}\left(h_{d,i}^{(d)} D_{d,i}^{(d)}\right)}{2}(\alpha_D - 1) + \frac{\sum_{d=1}^{n^{(d)}}\left(B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)}\right)}{T} \tag{4.38}$$

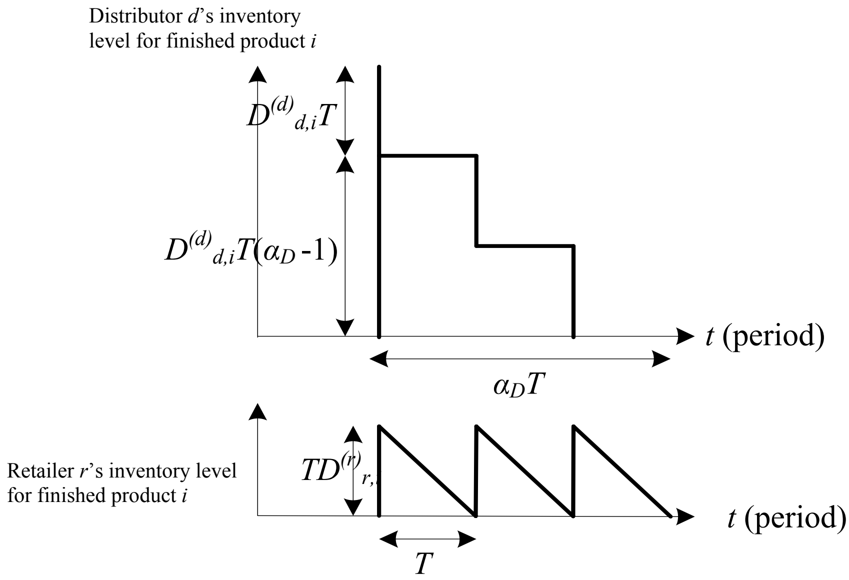


Figure 4.2 Inventories behaviour of finished product $i$ for retailer $r$ and distributor $d$ with $n^{(r)} = 1$, $n^{(d)} = 1$ and $\alpha_D = 3$

*4.3.4.3 The manufacturer's cost components*

The manufacturer applies joint production cycle time for all finished products which is an integer multiplier of $a_D T$, common order cycle time of distributors, $a_M a_D T$ for the finished products and joint order cycle time, $a_P a_M a_D T$, for parts. Here, $a_M$ is the multiplier of the common order cycle time of distributors to obtain the cycle time of the production for finished products, and $a_P$ is the multiplier of the production cycle time for finished products to obtain order cycle time for ordering parts. These cycle times $a_M a_D T$ and $a_P a_M a_D T$ are applied to replace cycle times in Eq. (4.26). Fig. 4.3 shows inventories behaviour between the manufacturer and distributor $d$.

Since there is coordination between the manufacturer and distributors, the manufacturer does not need to keep large stock of finished goods for anticipating order from distributors at the same time. The manufacturer just need to keep stock based on economic production quantity. Therefore, the manufacturer incurs production setup cost, ordering cost for parts, finished products holding cost, parts holding cost, and order processing and fixed shipment cost. The detailed derivation for each cost is as follows:

*Production setup cost*: Similarly with distributors the manufacturer applies production cycle time $a_M a_D T$ which is a multiple integer of $a_D T$, common order cycle time of distributors, to produce finished products as shown in Fig. 4.3 (b). The manufacturer incurs setup cost for production process once at every production cycle time. Production setup cost incurred by the manufacturer per unit time is

$$\left(S^{(m)} + \sum_{i=1}^{k^{(i)}} s_i^{(m)}\right) \Big/ \alpha_M \alpha_D T$$

*Finished products holding cost*: The manufacturer produces finished products with the production rate per unit time $P^{(m)}_i$. Production quantity per cycle time for finished product $i$ is $D^{(m)}_i a_M a_D T$. Since $P^{(m)}_i > D^{(m)}_i$ the length of production time for every production cycle time is $(D^{(m)}_i / P^{(m)}_i) a_M a_D T$. The manufacturer stops production once the production quantity reaches $D^{(m)}_i a_M a_D T$ unit. At this point, finished product $i$ inventory immediately drops one order of the distributors since the manufacturer supplies and delivers it to distributors. Similarly with distributors, remaining finished product $i$ inventory will be $D^{(m)}_i a_D T(a_M - 1)$ unit as shown in Figure 4.3 (b). Average inventory for finished product $i$ is calculated as follows:

Average inventory for finished product $i$

$$= \left( \frac{\dfrac{\left(D_i^{(m)}\alpha_M\alpha_D T\right)\left( D_i^{(m)} \big/ P_i^{(m)}\, \alpha_M\alpha_D T \right)}{2}}{\alpha_M\alpha_D T} \right) + \left( \frac{D_i^{(m)}\alpha_D T(\alpha_M - 1)\alpha_D T + D_i^{(m)}\alpha_D T(\alpha_M - 2)\alpha_D T + \ldots + D_i^{(m)}(\alpha_D T)^2}{\alpha_M\alpha_D T} \right)$$

$$= \left( \frac{\left(D_i^{(m)}\alpha_M\alpha_D T\right)}{2}\left( \frac{D_i^{(m)}}{P_i^{(m)}} \right) \right) + \left( \frac{D_i^{(m)}\alpha_D T\left((\alpha_M - 1) + (\alpha_M - 2) + \ldots + 1\right)}{\alpha_M} \right)$$

$$= \left( \frac{\left(D_i^{(m)}\alpha_D T\right)}{2}\left( \alpha_M \frac{D_i^{(m)}}{P_i^{(m)}} \right) \right) + \left( \frac{D_i^{(m)}\alpha_D T\left(\alpha_M - 1\right)}{2} \right)$$

$$= \left( \frac{\left(D_i^{(m)}\alpha_D T\right)}{2}\left( \alpha_M \frac{D_i^{(m)}}{P_i^{(m)}} + \left(\alpha_M - 1\right) \right) \right) \tag{4.39}$$

This formulation is similar as shown in Chen and Chen (2005).

Hence, the finished product $i$ inventory holding cost per unit time is given by $h_i\left( \frac{\left(D_i^{(m)}\alpha_D T\right)}{2}\left( \alpha_M \frac{D_i^{(m)}}{P_i^{(m)}} + \left(\alpha_M - 1\right) \right) \right)$

*Parts ordering cost*: The manufacturer orders parts from tier-1 suppliers to manufacture and assemble finished products. Parts are ordered every order cycle time $\alpha_P\alpha_M\alpha_D T$ which is multiple integer of $\alpha_M\alpha_D T$, common production cycle time. Ordering cost for parts incurred by the manufacturer per unit time is $\left( A_M + \sum_{p=1}^{k^{(p)}} a_p^{(m)} \right) \Big/ \alpha_P\alpha_M\alpha_D T$.

*Parts holding cost*: The manufacturer orders parts from tier-1 suppliers to manufacture and assemble finished products. Given the usage rate of part $p$ per unit finished product $I$, $\beta^{(I)}_{p,i}$, the demand for part $p$ per unit time is $\sum_{i=1}^{k^{(i)}} \beta_{p,i}^{(I)} D_i^{(m)}$ units. Since the manufacturer orders parts every order cycle time $\alpha_P\alpha_M\alpha_D T$ order quantity per cycle time for part $p$ is $\sum_{i=1}^{k^{(i)}} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_P\alpha_M\alpha_D T$. As the ratio between demand and production rate for each finished product, $\frac{D_i^{(m)}}{P_i^{(m)}}$, might be

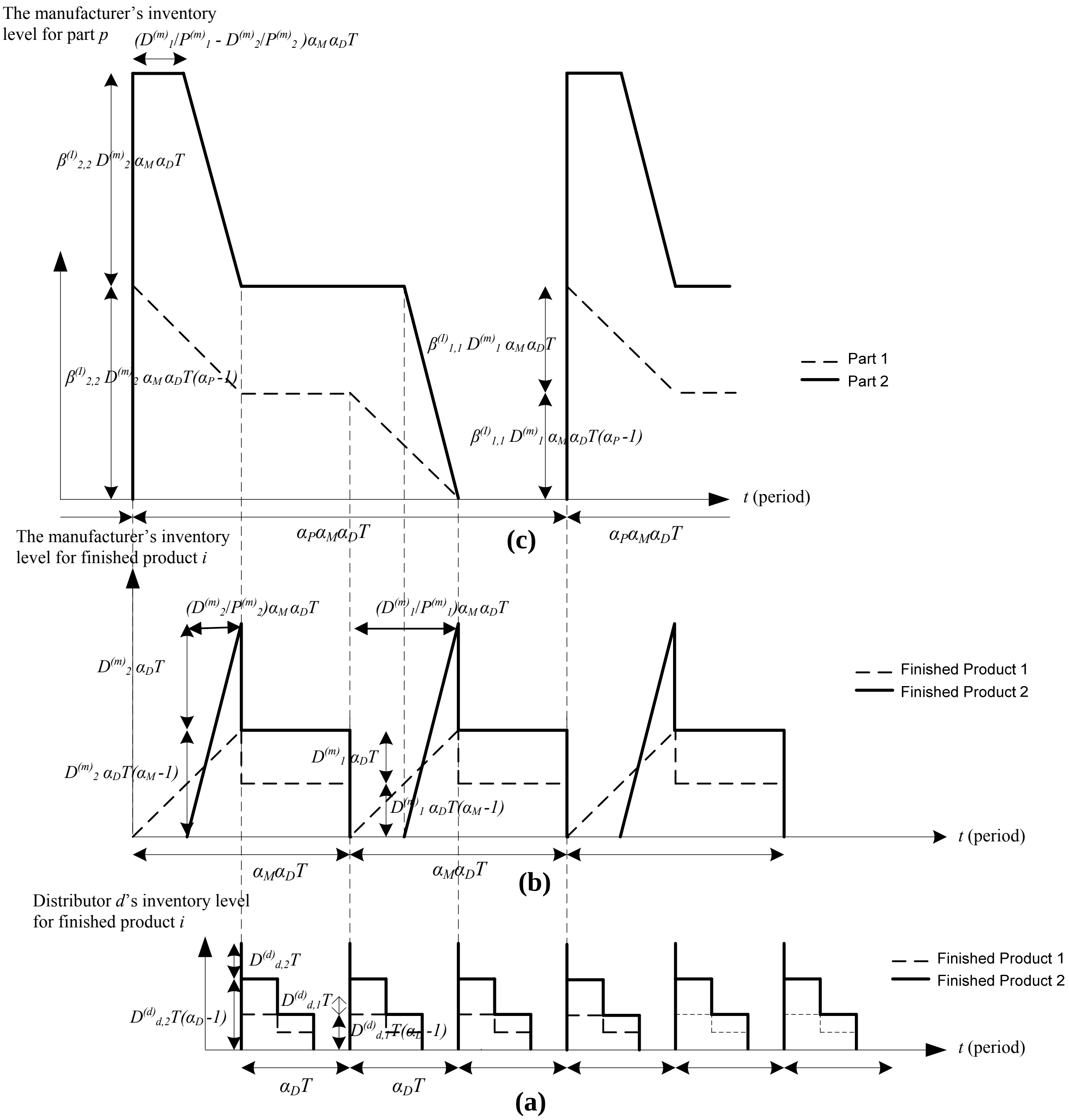


Figure 4.3 Inventory behaviour of finished product $i$ and part $p$ for distributor $d$ and the manufacturer with $n^{(d)} = 1$, $\alpha_D = 3$, $\alpha_M = 2$ and $\alpha_P = 2$

either different or the same, the production of each finished product may start either at different or the same time to set the production of all finished products will finish at the same time per cycle time. Furthermore, since the order for all parts to tier-1 suppliers is at the same time there will be one or more types of parts will keep storing in inventory until they are manufactured and assembled in the production. Therefore, we need to calculate inventory level for each part type from the time they arrive until they start to be manufactured and assembled in the production. We derive a formulation to calculate this inventory level. Then, we calculate inventory level for all parts during the production time. Lastly, similar to distributors, since the order cycle time for parts is $\alpha_P\alpha_M\alpha_D T$ which is multiple integer of $\alpha_M\alpha_D T$ remaining inventory level for part $p$ for finished product $i$ after the first production cycle time will be $\beta^{(I)}_{p,i} D^{(m)}_{i} \alpha_M \alpha_D T(\alpha_P - 1)$ as shown in Figure 4.3 (c). For last two formulations to calculate the inventory level, we develop work that has been carried out in Chen and Chen (2005). Therefore, average inventory for part $p$ for finished product $i$ is calculated as follows:

Average inventory for part $p$ for finished product $i$

$$= \left( \frac{\left( \left( \frac{D^{(m)}_i}{P^{(m)}_i} \right)^{\max} - \left( \frac{D^{(m)}_i}{P^{(m)}_i} \right) \right) (\alpha_M \alpha_D T) \beta^{(I)}_{p,i} D^{(m)}_i \alpha_P \alpha_M \alpha_D T}{\alpha_P \alpha_M \alpha_D T} \right) + \left( \frac{\dfrac{\beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T \left( D^{(m)}_i \big/ P^{(m)}_i \, \alpha_M \alpha_D T \right)}{2}}{\alpha_M \alpha_D T} \right) +$$

$$\left( \frac{\beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T (\alpha_P - 1) \alpha_M \alpha_D T + \beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T (\alpha_P - 2) \alpha_M \alpha_D T + \ldots + \beta^{(I)}_{p,i} D^{(m)}_i (\alpha_M \alpha_D T)^2}{\alpha_P \alpha_M \alpha_D T} \right)$$

$$= \left( \beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T \left( \left( \frac{D^{(m)}_i}{P^{(m)}_i} \right)^{\max} - \left( \frac{D^{(m)}_i}{P^{(m)}_i} \right) \right) \right) + \left( \frac{\beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T}{2} \left( \frac{D^{(m)}_i}{P^{(m)}_i} \right) \right) +$$

$$\left( \frac{\beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T \left( (\alpha_P - 1) + (\alpha_P - 2) + \ldots + 1 \right)}{\alpha_P} \right)$$

$$= \left( \frac{\beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T}{2} \left( 2 \left( \left( \frac{D^{(m)}_i}{P^{(m)}_i} \right)^{\max} - \frac{D^{(m)}_i}{P^{(m)}_i} \right) + \frac{D^{(m)}_i}{P^{(m)}_i} + (\alpha_P - 1) \right) \right) \tag{4.40}$$

Part $p$ for finished product i inventory holding cost per unit time is therefore given by $h^{(m)}_p \left( \frac{\beta^{(I)}_{p,i} D^{(m)}_i \alpha_M \alpha_D T}{2} \left( 2 \left( \left( \frac{D^{(m)}_i}{P^{(m)}_i} \right)^{\max} - \frac{D^{(m)}_i}{P^{(m)}_i} \right) + \frac{D^{(m)}_i}{P^{(m)}_i} + (\alpha_P - 1) \right) \right)$.

*Order processing and fixed shipment cost*: The manufacturer supplies and delivers orders of $D^{(m)}_i \alpha_D T$ units of finished product $i$ every cycle time $\alpha_D T$, common order cycle time of distributors, to distributors with order processing and fixed shipment cost per unit time $B_M$ across all finished products and $b_i$ for each finished product. Order processing and fixed shipment cost incurred by the manufacturer per unit time is $\left( B_M + \sum_{i=1}^{k^{(i)}} b_i \right) \Big/ \alpha_D T$.

Then, the total cost incurred by the manufacturer for all finished products and parts is

$$TCM=\left(\begin{array}{l}\dfrac{S^{(m)}+\sum\limits_{i=1}^{k^{(i)}}s_i^{(m)}}{\alpha_M\alpha_D T}+\sum\limits_{i=1}^{k^{(i)}}\dfrac{h_i D_i^{(m)}\alpha_D T}{2}\left(\alpha_M\dfrac{D_i^{(m)}}{P_i^{(m)}}+\left(\alpha_M-1\right)\right)+\dfrac{B_M+\sum\limits_{i=1}^{k^{(i)}}b_i}{\alpha_D T}+\dfrac{A_M+\sum\limits_{p=1}^{k^{(p)}}a_p^{(m)}}{\alpha_P\alpha_M\alpha_D T}+\\ \sum\limits_{p=1}^{k^{(p)}}\sum\limits_{i=1}^{k^{(i)}}\left(\dfrac{\beta_{p,i}^{(I)}h_p^{(m)}D_i^{(m)}\alpha_M\alpha_D T}{2}\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}+\left(\alpha_p-1\right)+2\left(\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}\right)^{\max}-\dfrac{D_i^{(m)}}{p_i^{(m)}}\right)\right)\right)\end{array}\right) \tag{4.41}$$

The fifth term is order processing and fixed shipment cost to supply orders to distributors as described in Chan and Kingsman (2007).

*4.3.4.4 Tier-1 suppliers' cost components*

For coordination between all suppliers and the manufacturer, tier-1 suppliers apply common production cycle time $\alpha_{PS}\alpha_P\alpha_M\alpha_D T$ for parts which is an integer multiplier of $\alpha_P\alpha_M\alpha_D T$, common order cycle time for parts of the manufacturer, and common order cycle time $\alpha_{WS}\alpha_{PS}\alpha_P\alpha_M\alpha_D T$ for sourcing raw materials which is an integer multiplier of $\alpha_{PS}\alpha_P\alpha_M\alpha_D T$ as shown in Fig. 4.4. In the figure, we set $\alpha_{S'P} = 1$ and $\alpha_{S'W} = 2$. Again, by replacing cycle times in Eq. (4.30) with the common production cycle time for parts $\alpha_{PS}\alpha_P\alpha_M\alpha_D T$ and common order cycle time for raw materials $\alpha_{WS}\alpha_{PS}\alpha_P\alpha_M\alpha_D T$ similarly we can derive the cost function for tier-1 suppliers. Under coordinated policy, tier-1 suppliers incur parts production setup cost, raw materials ordering cost, parts holding cost, raw materials holding cost and order processing and fixed shipment cost.

The detailed derivation for each cost is as follows:

*Production setup cost*: Similar to the manufacturer, tier-1 suppliers apply production cycle time $\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ to produce finished products as shown in Fig. 4.4 (b). Production setup cost for parts incurred by tier-1 supplier *s'* per unit time is $\dfrac{S_{s'}^{(s')}+\sum\limits_{p=1}^{k^{(p)}}s_{s',p}^{(s')}}{\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}$

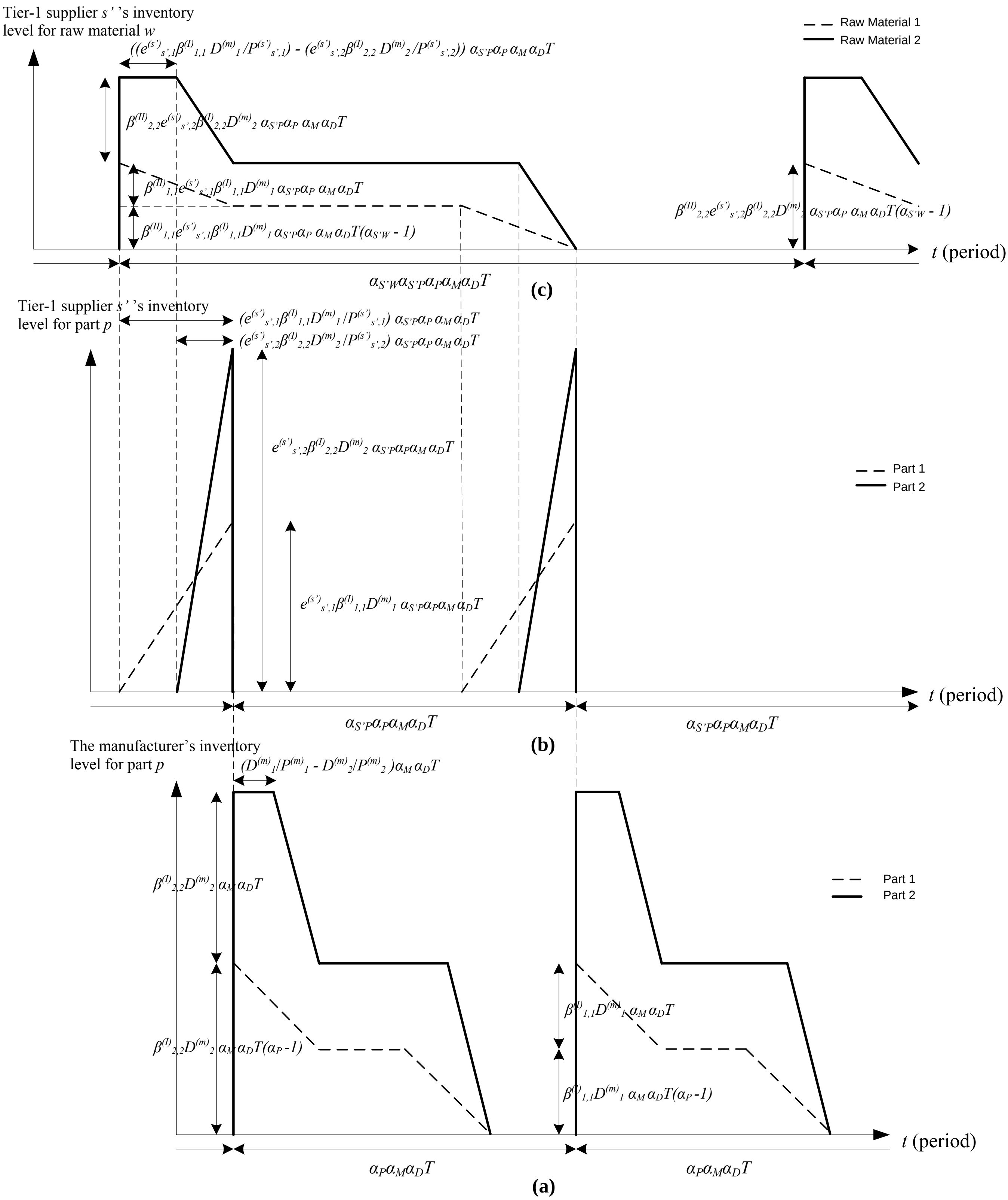


Figure 4.4 Inventory behaviour of part $p$ and raw material $w$ of the manufacturer and the tier-1 supplier $s'$ with $\alpha_{S'P} = 1$ and $\alpha_{S'W} = 2$

*Parts holding cost*: Tier-1 suppliers produce finished products every cycle time with the production rate per unit time $P^{(s')}_{s',p}$. Given a proportion of part $p$ supplied by tier-1 supplier $s'$ $e^{(s')}_{s',p}$ the quantity of part $p$ produced per unit time is $e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)$.

Production quantity per cycle time for part $p$ is $e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ as shown in Figure 4.4 (b). Since $P^{(s')}_{s',p} > e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)$ the production time for every cycle time is $\left(\dfrac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$. Similarly with the manufacturer, average inventory for part $p$ for tier-1 supplier $s'$ is calculated as follows:

Average inventory for part $p$ for tier-1 supplier $s'$

$$
= \left(\frac{\dfrac{\left(\left(\dfrac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right)e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2}}{\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}\right)+
$$

$$
\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_P\alpha_M\alpha_D T(\alpha_{S'P}-1)\alpha_P\alpha_M\alpha_D T+e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_P\alpha_M\alpha_D T(\alpha_{S'P}-2)\alpha_P\alpha_M\alpha_D T+\ldots+e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\left(\alpha_P\alpha_M\alpha_D T\right)^2}{\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}\right)
$$

$$
= \left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2}\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\right)+
$$

$$
\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_P\alpha_M\alpha_D T\left((\alpha_{S'P}-1)+(\alpha_{S'P}-2)+\ldots+1\right)}{\alpha_{S'P}}\right)
$$

$$
= \left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_P\alpha_M\alpha_D T}{2}\left(\alpha_{S'P}\frac{D^{(m)}_{i}}{P^{(m)}_{i}}+\left(\alpha_{S'P}-1\right)\right)\right) \tag{4.42}
$$

Hence, part $p$ inventory holding cost per unit time for tier-1 suppliers $s'$ is given by

$$
h_{s',p}\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_P\alpha_M\alpha_D T}{2}\left(\alpha_{S'P}\frac{D^{(m)}_{i}}{P^{(m)}_{i}}+\left(\alpha_{S'P}-1\right)\right)\right)
$$

*Raw materials ordering cost*: Tier-1 suppliers order raw materials from tier-2 suppliers to manufacture them into parts. Raw materials are ordered every order cycle time $\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ which is multiple integer of $\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ as shown in

Figure 4.4 (c). Ordering cost incurred by tier-1 supplier *s'* per unit time is $\dfrac{A^{(s')}_{s'} + \sum_{w=1}^{k^{(w)}} a^{(s')}_{s',w}}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}$

*Raw materials holding cost*: Similarly with parts ordering cost of the manufacturer, average inventory for raw material $w$ for tier-1 supplier *s'* as shown in Figure 4.4 (c) is calculated as follows:

Average inventory for raw material $w$ for tier-1 supplier *s'*

$$= \left( \frac{\left(\left(\left( e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right) \middle/ P^{(s')}_{s',p} \right)^{\max} - \left( e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right) \middle/ P^{(s')}_{s',p} \right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right) \beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right)}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T} + \frac{\dfrac{\left(\left(\left( e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right) \middle/ P^{(s')}_{s',p} \right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right) \beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right)}{2}}{\alpha_{S'P}\alpha_P\alpha_M\alpha_D T} + \frac{\left( \begin{array}{l} \beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T(\alpha_{S'W}-1)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T + \\ \beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T(\alpha_{S'W}-2)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T + \ldots + \beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\left(\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right)^2 \end{array} \right)}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T} \right)$$

$$= \left( \left(\left(\left( e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right) \middle/ P^{(s')}_{s',p} \right)^{\max} - \left( e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right) \middle/ P^{(s')}_{s',p} \right)\right) \beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T + \frac{\left( e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right) \middle/ P^{(s')}_{s',p} \right) \beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2} + \left( \frac{\beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\left((\alpha_{S'W}-1)+(\alpha_{S'W}-1)+\ldots+1\right)}{\alpha_{S'W}} \right) \right)$$

$$= \left( \frac{\beta^{(II)}_{p,w} e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2} \left( 2\left(\left( \frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)}{P^{(s')}_{s',p}} \right)^{\max} - \left( \frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)}{P^{(s')}_{s',p}} \right)\right) + \left( \frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)}{P^{(s')}_{s',p}} \right) + (\alpha_{S'W}-1) \right) \right)$$

Again, raw material $w$ for part $p$ inventory holding cost per unit time is given by

$$h^{(s')}_{s',w}\left(\frac{\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(2\left(\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)^{\max}-\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\right)+\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)+\left(\alpha_{S'W}-1\right)\right)\right)$$

*Order processing and fixed shipment cost*: Tier-1 supplier *s'* supplies and delivers orders $e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{P}\alpha_{M}\alpha_{D}T$ units of part $p$ every cycle time $\alpha_P\alpha_M\alpha_D T$, joint order cycle time of the manufacturer, to the manufacturer with order processing and fixed shipment cost per unit time $B^{(s')}_{s'}$ for all parts and $b^{(s')}_{s',p}$ for each part. Order processing and fixed shipment cost incurred by tier-1 supplier *s'* per unit time is

$$\left(B^{(s')}_{s'}+\sum_{p=1}^{k^{(p)}}b^{(s')}_{s',p}\right)\Big/\alpha_{P}\alpha_{M}\alpha_{D}T$$

Then, the total cost incurred by tier-1 suppliers for all parts and raw materials is

$$TCS=\sum_{s'=1}^{n^{(s')}}\frac{S^{(s')}_{s'}+\sum_{p=1}^{k^{(p)}}s^{(s')}_{s',p}}{\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}+\sum_{s'=1}^{n^{(s')}}\sum_{p=1}^{k^{(p)}}\left(\frac{h_{s',p}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(\alpha_{S'P}\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}+\left(\alpha_{S'P}-1\right)\right)\right)+\sum_{s'=1}^{n^{(s')}}\frac{A^{(s')}_{s'}+\sum_{w=1}^{k^{(w)}}a^{(s')}_{s',w}}{\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}+$$

$$\sum_{s'=1}^{n^{(s')}}\frac{\left(B^{(s')}_{s'}+\sum_{p=1}^{k^{(p)}}b^{(s')}_{s',p}\right)}{\alpha_{P}\alpha_{M}\alpha_{D}T}+\sum_{s'=1}^{n^{(s')}}\sum_{w=1}^{k^{(w)}}\sum_{p=1}^{k^{(p)}}\left(\frac{\beta^{(II)}_{p,w}h^{(s')}_{s',w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\beta^{(I)}_{p,i}D^{(m)}_{i}}{P^{(s')}_{s',p}}+\left(\alpha_{S'W}-1\right)+\left(2\left(\gamma\max_{s'}-\gamma_{s',p}\right)\right)\right)\right)$$

(4.43)

where

$$\gamma_{s',p}=\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}} \tag{4.44}$$

$$\gamma\max_{s'}=\max\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}},\ p=1,2,..,k^{(p)}\right) \tag{4.45}$$

*4.3.4.5 Tier-2 suppliers' cost components*

Since there is coordination between each tier-2 supplier and tier-1 suppliers, each tier-2 supplier applies the production cycle time $\alpha_{S''W}\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ for raw materials which is an integer multiplier of $\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$, common order cycle time for raw materials of tier-1 suppliers as shown in Fig. 4.5. In this figure, we set $\alpha_{S''W}$ = 1. Once again, by replacing production cycle times in Eq. (4.33) with the common production cycle time for all tier-2 suppliers $\alpha_{S''W}\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ similarly with the distributors, the manufacturer, and tier-1 suppliers, the total cost function for the tier-2 suppliers is derived.

Tier-2 suppliers incur raw materials production setup cost, raw materials holding cost and order processing and fixed shipment cost. The detailed derivation for each cost is as follows:

*Production setup cost*: Similar to tier-1 suppliers, tier-2 suppliers apply production cycle time $\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ to produce raw materials as shown in Figure 4.5 (b). Production setup cost for raw materials incurred by tier-2 supplier *s"* per unit time is $\dfrac{S_{s"}^{(s")} + \sum_{w=1}^{k^{(w)}} s_{s",w}^{(s")}}{\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}$

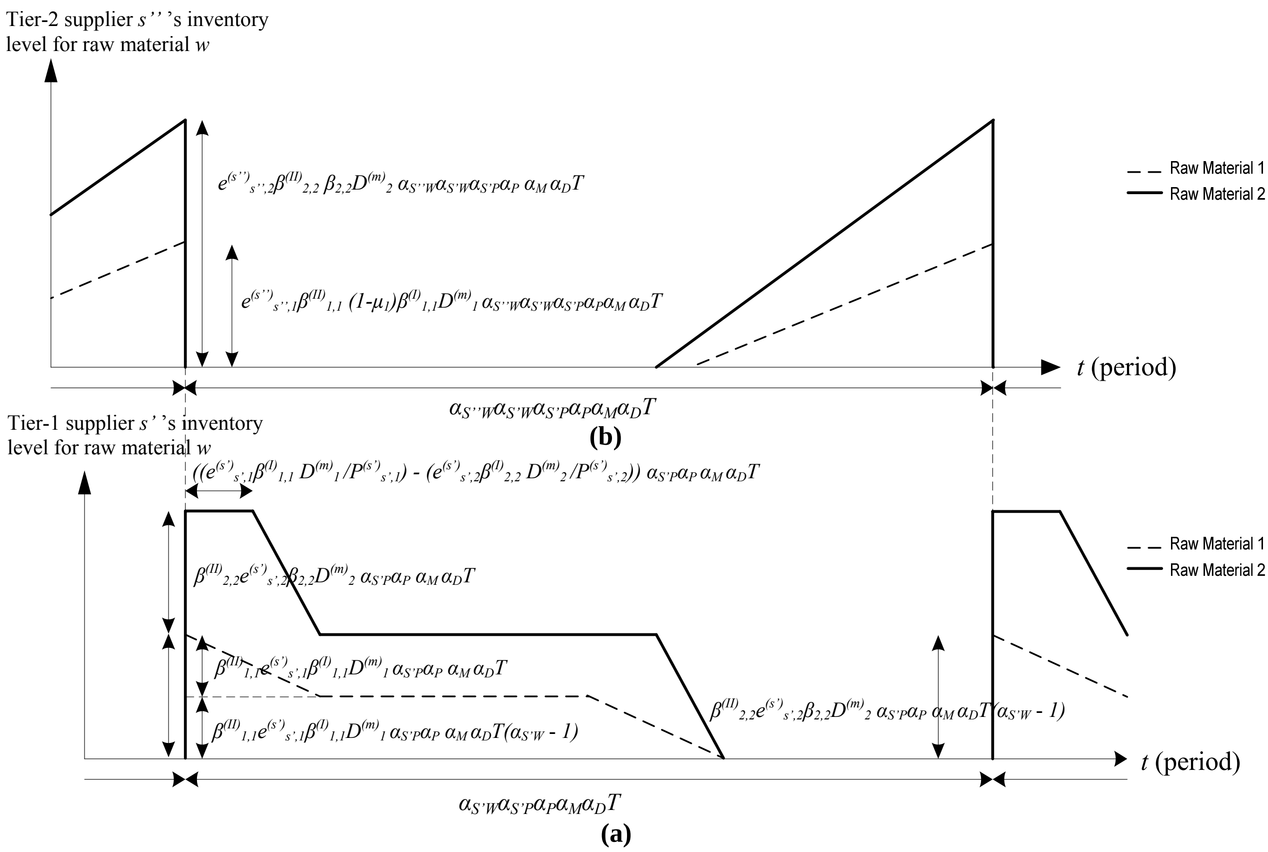


Figure 4.5 Inventory behavior of raw materials for the tier-1 supplier *s'* and the tier-2 supplier *s"* with $\alpha_{S'W} = 2$ and $\alpha_{S"W} = 1$

*Raw materials holding cost*: Tier-2 suppliers produce raw materials every cycle time with the production rate per unit time $P^{(s")}_{s",w}$. Given the proportion of raw material *w* supplied by tier-2 supplier *s"* $e^{(s")}_{s",w}$ the quantity of raw material *w* produced per unit time is $e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)$. Given production cycle time $\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ the production quantity per cycle time for raw material *w* will be $e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_i\right)\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T$ as shown in Figure 4.5 (b). Similarly, since

$P^{(s")}_{s",w} > e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)$ the production time for every cycle time is $\left(e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right) \Big/ P^{(s")}_{s",w}\right)\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$. Therefore, average inventory for raw material $w$ for tier-2 supplier $s"$ is calculated as follows:

Average inventory for raw material $w$ for tier-2 supplier $s"$

$$= \left(\frac{e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(\frac{e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s")}_{s",w}}\right)\right) +$$

$$\left(\frac{e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T\left((\alpha_{S"W}-1)+(\alpha_{S"W}-2)+\ldots+1\right)}{\alpha_{S"W}}\right)$$

$$= \left(\frac{e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(\alpha_{S"W}\frac{e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s")}_{s",w}}+\left(\alpha_{S"W}-1\right)\right)\right) \quad (4.46)$$

Hence, raw material $w$ inventory holding cost per unit time for tier-2 supplier $s"$ is given by $h^{s(")}_{s",w}\left(\frac{e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(\alpha_{S"W}\frac{e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s")}_{s",w}}+\left(\alpha_{S"W}-1\right)\right)\right)$

*Order processing and fixed shipment cost*: Tier-2 supplier *s'* supplies and delivers orders $e^{(s")}_{s",w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$ units of part $p$ every cycle time $\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$, common order cycle time of tier-1 suppliers, to tier-1 suppliers with order processing and fixed shipment cost per unit time $B^{(s")}_{s"}$ for all raw materials and $b^{(s")}_{s",w}$ for each raw material. Order processing and fixed shipment cost incurred by tier-2 supplier $s"$ per unit time is $\left(B^{(s")}_{s"}+\sum_{w=1}^{k^{(w)}}b^{(s")}_{s",w}\right)\Big/\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$

Then, the total cost incurred by tier-2 suppliers for all parts and raw materials is

$$TCS'' = \left( \begin{array}{l} \sum_{s''=1}^{n^{(s'')}} \frac{S_{s''}^{(s'')} + \sum_{w=1}^{k^{(w)}} s_{s'',w}^{(s'')}}{\alpha_{S''W}\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T} + \sum_{s''=1}^{n^{(s'')}} \frac{\left( B_{s''}^{(s'')} + \sum_{w=1}^{k^{(w)}} b_{s'',w}^{(s'')} \right)}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T} + \\ \sum_{s''=1}^{n^{(s'')}} \sum_{w=1}^{k^{(w)}} \left( \frac{h_{s'',w}^{(s'')} e_{s'',w}^{(s'')} \sum_{p=1}^{k^{(p)}} \beta_{p,w}^{(II)} \sum_{i=1}^{k^{(i)}} \left( \beta_{p,i}^{(I)} D_i^{(m)} \right) \alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2} \left( \alpha_{S''W} \frac{e_{s'',w}^{(s'')} \sum_{p=1}^{k^{(p)}} \beta_{p,w}^{(II)} \sum_{i=1}^{k^{(i)}} \beta_{p,i}^{(I)} D_i^{(m)}}{P_{s'',w}^{(s'')}} + \left( \alpha_{S''W} - 1 \right) \right) \right) \end{array} \right) \quad (4.47)$$

*4.3.4.6 The whole supply chain's total cost function*

The total cost function per unit time for the whole manufacturing supply chain under coordinated policy is determined by summing the total cost of all players. The whole supply chain's total cost function is the sum of equations (4.34), (4.38), (4.41), (4.43) and (4.47) as follows:

$$TCChain = TCR + TCD + TCM + TCS' + TCS'' \quad (4.48)$$

**Lemma 1**: Eq. (4.48) is convex function over $T > 0$ for any values of $a_D$, $a_M$, $a_P$, $a3$, $a_{S'P}$, $a_{S'W}$, $a_{S''W} \geq 1$.

**Proof**. See Appendix B.

## 4.4 Summary

In this chapter, the description of the system studied and the mathematical modelling of production inventory model in a complex manufacturing supply chain for multiple items and multiple sources are provided. The models are derived under independent and coordinated policies. The model under independent policy is derived to compare the system's total cost with coordinated one. Under independent policy, each player in the supply chain determines their own objectives without considering other players. Otherwise, under coordinated policy each player in the supply chain determines their own objectives with considering other players.

Under independent policy, upstream level players need only demand information from downstream level players. Under coordinated policy, information needed depends on the solution method selected. For centralized decision making process, all information about costs and demand have to be known by a decision maker in the supply chain. For decentralized one, upstream level players just need information about optimal cycle time from immediate downstream level players. The derivation of total cost function for each player is started from retailers until tier-2 suppliers. The total cost functions for each level and the supply chain are derived based on standard economic order quantity (EOQ) and economic production quantity (EPQ) models and works that have been carried out previously. Some references are referred and developed in this research. The model uses common order cycle time for each level in the supply chain. Upstream level players use a multiple integer of common order cycle time from immediate downstream level players to be their common cycle time. The total cost function of the supply chain is the sum of all total cost function of all players.

# Chapter 5

# Considering Reverse Logistics, Transportation Cost, Finite Horizon Period and Stochastic Demand in the Complex Manufacturing Supply Chain

## 5.1 Introduction

In this chapter we consider reverse logistics, transportation cost, finite horizon period and stochastic demand in the system being studied. We derive the mathematical modelling of parts of the system which are affected by these issues. In section 5.2 we describe and formulate the model considering reverse logistics and their effect on the system. In section 5.3 we describe and formulate the model considering transportation cost which is separated from ordering and processing cost. In section 5.4 we describe and formulate the model considering finite horizon period in the system. Considering stochastic demand in the model is described in section 5.5. Section 5.6 summarises the chapter.

## 5.2 Considering Reverse Logistics in the System

### 5.2.1 Description of the system

In this section we describe the system studied involving reverse logistics. Fig. 5.1 shows the description of the whole manufacturing supply chain involving reverse logistics. The manufacturer uses a proportion of reusable parts from used finished products which are collected by a third party which reduce the need of new parts for tier-1 suppliers. The third party will now be the part of the supply chain. The third party collects used finished products from end customers after certain period of the use. Used finished products are disassembled into parts. Some parts can be used again in new finished products. This scenario will affect the total costs incurred by the manufacturer, tier-1 suppliers and tier-2 suppliers as some parts used in finished products are from the used finished products. Therefore, there are the changes to the cost functions of the manufacturer, tier-1 suppliers and tier-2 suppliers as well as the whole supply chain. The cost function of the third party will be included in the total cost function of the whole system. These changes and the total cost function of the third party then will affect the optimal solution as well as the objective function of the supply chain.

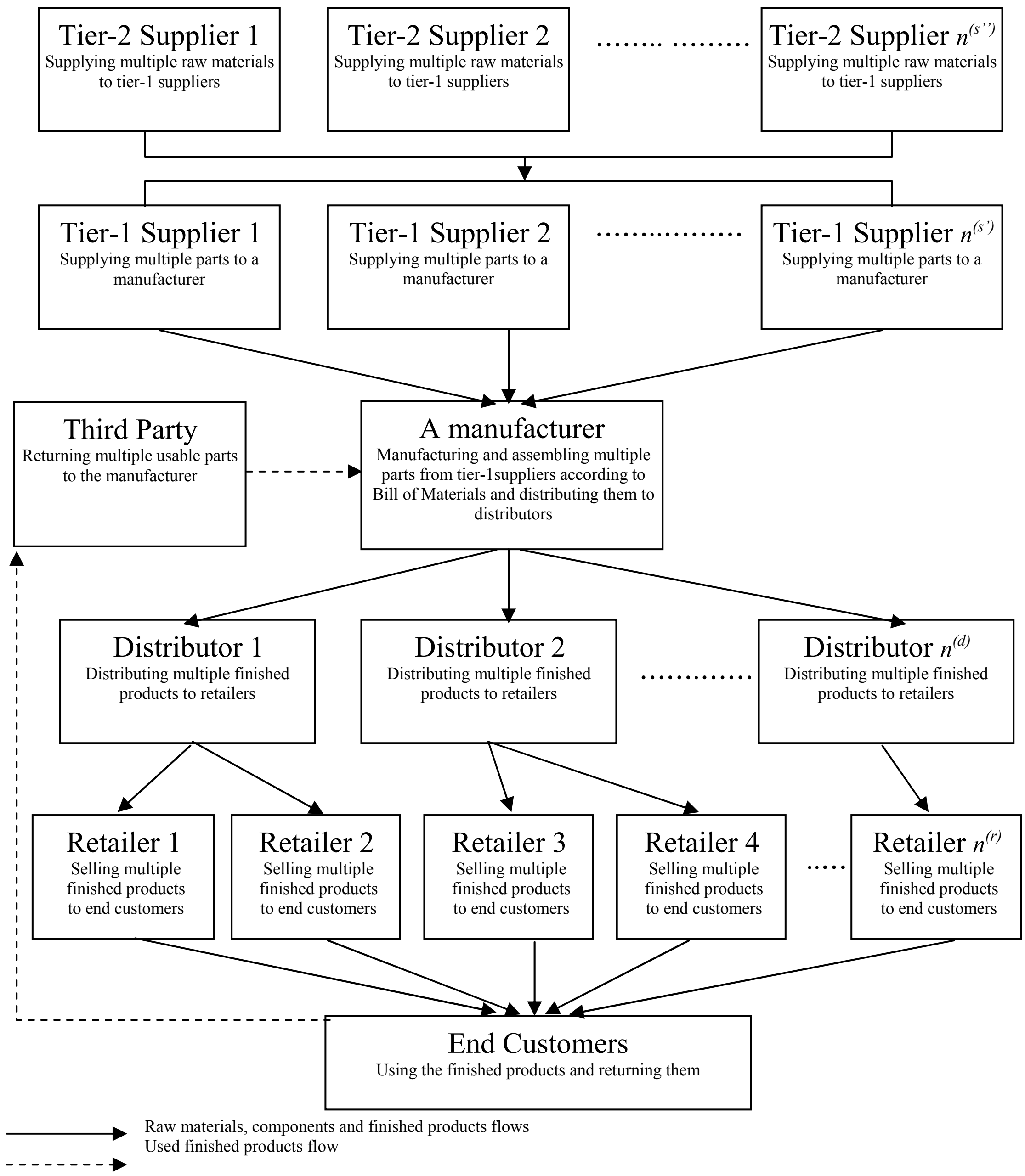


Figure 5.1 System description of a complex manufacturing supply chain involving reverse logistics

### 5.2.2 Notations

Parameters:

$p'$ index for returned part types, $p' = 1, 2, \ldots, k^{(p')}$, where $k^{(p')}$ is the number of types, $p = p'$

$R_i$ returning rate of the returned product $i$ from customers per period

$C_i$ collecting rate for the returned product $i$ of the third party

$S^{(3)}$ setup cost for collecting all returned products by the third party

$s^{(3)}_i$ the cost of processing returned product $i$ of the third party

$h^{(3)}_i$ holding cost for product $i$ of the third party

$u_{p',i}$ a portion of returned part $p'$ of finished product $i$ which are reusable into the products

Decision variable:

$a3$ integer multiplier of the cycle time of the third party

Objective functions:

$TC3$ total associated cost of the third party

### 5.2.3 Mathematical modelling of the system

As before, when considering the supply chain involving reverse logistics, there are also a number of assumptions that are applied. We assume that collected used finished products can be perfectly disassembled into parts which can be used in new finished products. Since disassembly and collection costs per unit used product do not affect the optimal solutions we ignore the costs in the model. The quality of usable returned parts is considered to be as good as new parts. As the reverse logistics affect the supply chain from the manufacturer to upstream levels we derive the cost functions of the whole manufacturing supply chain involving reverse logistics for the manufacturer, tier-1 suppliers, tier-2 suppliers and the third party. We derive the formulations for the costs incurred based on coordinated policy.

#### *5.2.3.1 The manufacturer's cost components*

In the system with reverse logistics, the manufacturer incurs production setup cost, ordering cost for parts, ordering cost for returned parts, parts holding cost, returned parts holding cost, finished products holding cost and order processing and fixed shipment cost. There are no changes to production setup cost function, ordering cost function for parts and order processing and fixed shipment cost function so that we keep these costs as derived in chapter 4. The detailed derivations for other costs are as follows:

*Finished products holding cost*: A proportion $u_{p',i}$ of total parts, returned parts, are used in new finished products so that we need to calculate how much saving of holding cost we can obtain. For every production cycle time we use first the proportion $u_{p',i}$ of total parts from returned parts and then new parts supplied by tier-1 suppliers to manufacture and assemble all these parts into new finished products as shown in Fig. 5.2 (a). To calculate finished products holding cost involving reusable returned parts (adjusted) we first calculate finished products without considering returned parts. Second, we calculate the saving of the use of returned parts in new finished products. The derivation of formulations is developed from Chen and Chen (2005), Teunter (2001) and Chung et al. (2008). Chen and Chen (2005) derived the formulation to calculate holding cost for raw materials which are consumed in the finished products. Chen and chen (2005) did not consider returned parts in the models. Teunter (2001) and Chung et al. (2008) consider remanufacturing in their model but they separated remanufacturing and manufacturing process in each cycle time. In this model, we use returned and new parts at the same production cycle time. Therefore, finished products holding cost is the finished holding cost without considering returned parts minus the saving of the use of returned parts in new finished products. For the first term we have done in chapter 4. The saving of the use of returned parts is calculated by the saving of holding cost per unit time for returned part *p'*, $\left(h_p^{(m)} - h_{p'}^{(m')}\right)$, times average inventory for returned part *p'*. We propose the formulation for first term and we developed what has been carried out in Chen and Chen (2005) for multiple items involving reverse logistics for the second and third terms. Average inventory for returned part *p'* is as follows:

Average inventory for returned part *p'*

$$= \left( \frac{\frac{\left(u_{p',i}\beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T\right)\left(u_{p',i} \frac{D_i^{(m)}}{P_i^{(m)}} \alpha_M \alpha_D T\right)}{2}}{\alpha_M \alpha_D T} \right) + \left( \frac{\left(u_{p',i}\beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T\right)\left(\left(1-u_{p',i}\right)\frac{D_i^{(m)}}{P_i^{(m)}} \alpha_M \alpha_D T\right)}{\alpha_M \alpha_D T} \right)$$

$$\left( \frac{u_{p',i}\beta_{p,i}^{(I)} D_i^{(m)} \alpha_D T(\alpha_M - 1)\alpha_D T + u_{p',i}\beta_{p,i}^{(I)} D_i^{(m)} \alpha_D T(\alpha_M - 2)\alpha_D T + \ldots + u_{p',i}\beta_{p,i}^{(I)} D_i^{(m)} (\alpha_D T)^2}{\alpha_M \alpha_D T} \right)$$

$$= \left( \frac{\left(u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_M\alpha_D T\right)}{2}\left(\frac{D^{(m)}_{i}}{P^{(m)}_{i}}u_{p',i}\right)\right) + \left(\left(u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_M\alpha_D T\right)\left(\frac{D^{(m)}_{i}}{P^{(m)}_{i}}\left(1-u_{p',i}\right)\right)\right) +$$

$$\left( \frac{u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_D T\left((\alpha_M - 1) + (\alpha_M - 2) + \ldots + 1\right)}{\alpha_M} \right)$$

$$= \left( \frac{\left(u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_D T\right)}{2}\left(\alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}}u_{p',i}\right)\right) + \left( \frac{\left(u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_M\alpha_D T\right)}{2}\left(\alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}}2\left(1-u_{p',i}\right)\right)\right) +$$

$$\left( \frac{u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_D T\left(\alpha_M - 1\right)}{2} \right)$$

$$= \left( \frac{\left(u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_D T\right)}{2}\left(\alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}}u_{p',i} + \alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}}2\left(1-u_{p',i}\right) + \left(\alpha_M - 1\right)\right)\right)$$

$$= \frac{u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_D T}{2}\left(\alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}}\left(2-u_{p',i}\right) + \left(\alpha_M - 1\right)\right) \tag{5.1}$$

Hence, the saving of finished product $i$ inventory holding cost for returned part $p'$ per unit time is given by $\frac{\left(h^{(m)}_{p} - h^{(m')}_{p'}\right)u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_D T}{2}\left(\alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}}\left(2-u_{p',i}\right) + \left(\alpha_M - 1\right)\right)$.

Then, finished product $i$ inventory holding cost adjusted will be

$$\frac{h_i D^{(m)}_{i}\alpha_D T}{2}\left(\alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}} + \left(\alpha_M - 1\right)\right) - \sum_{p=1}^{k(p)} \frac{\left(h^{(m)}_{p} - h^{(m')}_{p'}\right)u_{p',i}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_D T}{2}\left(\alpha_M \frac{D^{(m)}_{i}}{P^{(m)}_{i}}\left(2-u_{p',i}\right) + \left(\alpha_M - 1\right)\right)$$

*Parts holding cost*: Given the usage rate of part $p$ per unit finished product $i$ $\beta^{(I)}_{p,i}$ and the proportion $u_{p',i}$ of total parts, returned parts, used in new finished products the demand for part $p$ per unit time is $\left(1-u_{p',i}\right)\sum_{i=1}^{k(i)}\beta^{(I)}_{p,i}D^{(m)}_{i}$. The manufacturer orders parts every order cycle time $\alpha_P\alpha_M\alpha_D T$. Order quantity per cycle time for part $p$ is $\left(1-u_{p',i}\right)\sum_{i=1}^{k(i)}\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_P\alpha_M\alpha_D T$ Parts are consumed as many as the proportion $(1-u_{p',i})$ of total parts (new parts and returned parts). As the ratio of demand rate and production rate for each finished product could be either different or the same we need to calculate inventory level for each part for each finished product which consumes the parts. Also, since the order for all parts are placed at the same time we also need to calculate inventory for them from the time they come until they are consumed in the production. Since order cycle time for parts is $\alpha_P\alpha_M\alpha_D T$ remaining inventory for part $p$ for finished product $i$ after the first production cycle time will be $\left(1-u_{p',i}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\alpha_M\alpha_D T\left(\alpha_P - 1\right)$ as shown in Figure 5.2 (c). Similar to returned parts, we propose the formulation for first term and we developed what has been carried out in Chen and Chen (2005) for multiple items involving reverse logistics for the second and third terms. Average inventory for part $p$ for finished product $i$ is calculated as follows:

Average inventory for part $p$ for finished product $i$

=

$$\left(\frac{\left(\left(\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)^{\max}-\left(\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)\right)(\alpha_M\alpha_D T)(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_P\alpha_M\alpha_D T}{\alpha_P\alpha_M\alpha_D T}\right)+\left(\frac{\frac{(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T\left(D_i^{(m)}\Big/P_i^{(m)}\,\alpha_M\alpha_D T\right)}{2}}{\alpha_M\alpha_D T}\right)$$

$$+\left(\frac{(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T(\alpha_P-1)\alpha_M\alpha_D T+(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T(\alpha_P-2)\alpha_M\alpha_D T+\ldots+(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}(\alpha_M\alpha_D T)^2}{\alpha_P\alpha_M\alpha_D T}\right)$$

$$=\left((1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T\left(\left(\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)^{\max}-\left(\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)\right)\right)+\left(\frac{(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T}{2}\left(\frac{D_i^{(m)}}{P_i^{(m)}}\right)\right)+$$
$$\left(\frac{(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T((\alpha_P-1)+(\alpha_P-2)+\ldots+1)}{\alpha_P}\right)$$

$$=\left(\frac{(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T}{2}\left(2\left(\left(\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)^{\max}-\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)+\frac{D_i^{(m)}}{P_i^{(m)}}+(\alpha_P-1)\right)\right)\qquad(5.2)$$

Again, part $p$ for finished product i inventory holding cost per unit time is given by $h_p^{(m)}\left(\frac{(1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\alpha_M\alpha_D T}{2}\left(2\left(\left(\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)^{\max}-\frac{D_i^{(m)}}{P_i^{(m)}}(1-u_{p',i})\right)+\frac{D_i^{(m)}}{P_i^{(m)}}+(\alpha_P-1)\right)\right)$

*Returned parts ordering cost*: Similarly, the manufacturer orders returned parts from the third party. Returned parts are ordered every order cycle time $\alpha_P\alpha_M\alpha_D T$ which is multiple integer of $\alpha_M\alpha_D T$, common production cycle time. Ordering cost for returned parts incurred by the manufacturer per unit time is $\left(A_{M'}+\sum_{p'=1}^{k^{(p)}}a_{p'}^{(m')}\right)\Big/\alpha_P\alpha_M\alpha_D T$

*Returned parts holding cost*: Given the usage rate of part $p$ per unit finished product $i$ $\beta^{(I)}_{p,i}$ the demand for returned part $p'$ per unit time is $u_{p',i}\sum_{i=1}^{k^{(i)}}\beta_{p,i}^{(I)}D_i^{(m)}$ Order quantity per cycle time for returned part $p'$ is $u_{p',i}\sum_{i=1}^{k^{(i)}}\beta_{p,i}^{(I)}D_i^{(m)}\alpha_P\alpha_M\alpha_D T$ Returned parts are consumed as many as the proportion $u_{p',i}$ of total parts (new parts and returned parts) as shown in Figure 5.2 (b). Similar to parts holding cost, average inventory for returned part $p'$ for finished product $i$ is calculated as follows:

Average inventory for returned part $p'$ for finished product $i$

$$= \left( \frac{\left( \left( \frac{D_i^{(m)}}{P_i^{(m)}} \right)^{\max} - \left( \frac{D_i^{(m)}}{P_i^{(m)}} \right) \right) (\alpha_M \alpha_D T) u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_P \alpha_M \alpha_D T}{\alpha_P \alpha_M \alpha_D T} \right) + \left( \frac{\frac{u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T \left( D_i^{(m)} / P_i^{(m)} \alpha_M \alpha_D T \right)}{2}}{\alpha_M \alpha_D T} \right)$$

$$+ \left( \frac{u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T (\alpha_P - 1) \alpha_M \alpha_D T + u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T (\alpha_P - 2) \alpha_M \alpha_D T + \ldots + u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} (\alpha_M \alpha_D T)^2}{\alpha_P \alpha_M \alpha_D T} \right)$$

$$= \left( u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T \left( \left( \frac{D_i^{(m)}}{P_i^{(m)}} \right)^{\max} - \left( \frac{D_i^{(m)}}{P_i^{(m)}} \right) \right) \right) + \left( \frac{u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T}{2} \left( \frac{D_i^{(m)}}{P_i^{(m)}} \right) \right) +$$

$$\left( \frac{u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T \left( (\alpha_P - 1) + (\alpha_P - 2) + \ldots + 1 \right)}{\alpha_P} \right)$$

$$= \left( \frac{u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T}{2} \left( 2 \left( \left( \frac{D_i^{(m)}}{P_i^{(m)}} \right)^{\max} - \frac{D_i^{(m)}}{P_i^{(m)}} \right) + \frac{D_i^{(m)}}{P_i^{(m)}} + (\alpha_P - 1) \right) \right) \tag{5.3}$$

Again, returned part *p'* for finished product *i* inventory holding cost per unit time is given by $h_{p'}^{(m')} \left( \frac{u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_M \alpha_D T}{2} \left( 2 \left( \left( \frac{D_i^{(m)}}{P_i^{(m)}} \right)^{\max} - \frac{D_i^{(m)}}{P_i^{(m)}} \right) + \frac{D_i^{(m)}}{P_i^{(m)}} + (\alpha_P - 1) \right) \right)$

Then, the total cost incurred by the manufacturer for all finished products, parts and returned parts is

$$TCM = \left( \begin{array}{l} \frac{S^{(m)} + \sum_{i=1}^{k^{(i)}} s_i^{(m)}}{\alpha_M \alpha_D T} + \sum_{i=1}^{k^{(i)}} \frac{h_i D_i^{(m)} \alpha_D T}{2} \left( \alpha_M \frac{D_i^{(m)}}{P_i^{(m)}} + (\alpha_M - 1) \right) - \sum_{i=1}^{k^{(i)}} \sum_{p=1}^{k^{(p)}} \frac{\left( h_p^{(m)} - h_{p'}^{(m')} \right) u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_D T}{2} \left( \alpha_M \frac{D_i^{(m)}}{P_i^{(m)}} \left( 2 - u_{p',i} \right) + (\alpha_M - 1) \right) \\ + \frac{B_M + \sum_{i=1}^{k^{(i)}} b_i}{\alpha_D T} + \frac{A_M + \sum_{p=1}^{k^{(p)}} a_p^{(m)}}{\alpha_P \alpha_M \alpha_D T} + \sum_{p=1}^{k^{(p)}} \sum_{i=1}^{k^{(i)}} \left( \frac{\left( 1 - u_{p',i} \right) \beta_{p,i}^{(I)} h_p^{(m)} D_i^{(m)} \alpha_M \alpha_D T}{2} \left( \frac{D_i^{(m)}}{p_i^{(m)}} + (\alpha_p - 1) + 2 \left( \left( \frac{D_i^{(m)}}{p_i^{(m)}} \left( 1 - u_{p',i} \right) \right)^{\max} - \left( \frac{D_i^{(m)}}{p_i^{(m)}} \left( 1 - u_{p',i} \right) \right) \right) \right) \right) \\ + \frac{A_{M'} + \sum_{p'=1}^{k^{(p')}} a_{p'}^{(m')}}{\alpha_P \alpha_M \alpha_D T} + \sum_{p'=1}^{k^{(p')}} \sum_{i=1}^{k^{(i)}} \left( \frac{u_{p',i} \beta_{p,i}^{(I)} h_{p'}^{(m')} D_i^{(m)} \alpha_M \alpha_D T}{2} \left( \frac{D_i^{(m)}}{p_i^{(m)}} + (\alpha_P - 1) + 2 \left( \left( \frac{D_i^{(m)}}{p_i^{(m)}} \right)^{\max} - \frac{D_i^{(m)}}{p_i^{(m)}} \right) \right) \right) \end{array} \right) \tag{5.4}$$

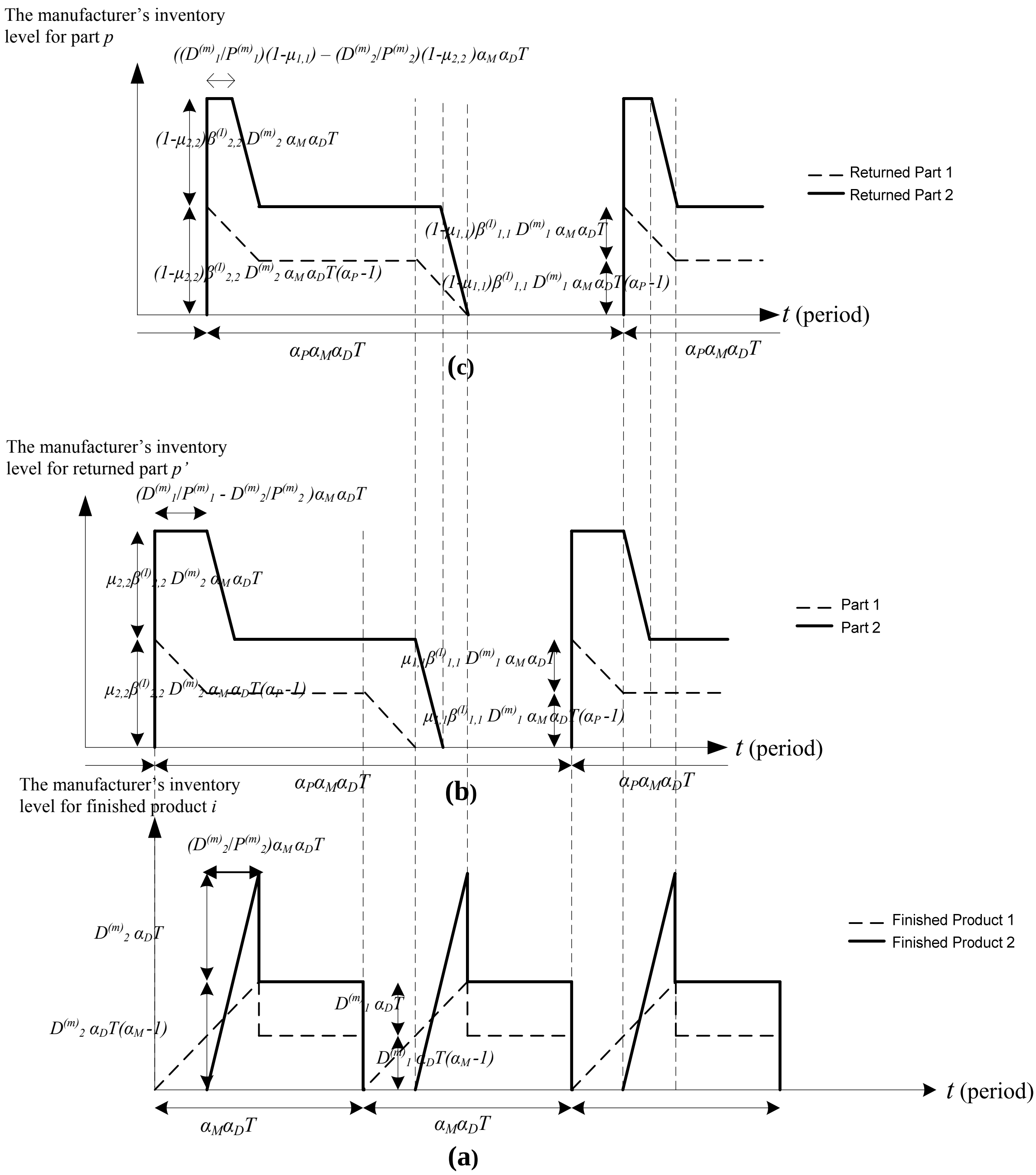


Figure 5.2 Inventories behavior of finished product $i$, returned part $p'$ and part $p$ for the manufacturer with $\alpha_M = 2$ and $\alpha_P = 2$

#### *5.2.2.2 Tier-1 suppliers' cost components*

Tier-1 suppliers incur parts production setup cost, raw materials ordering cost, parts holding cost, raw materials holding cost and order processing and fixed shipment cost. There are no changes to parts production setup cost function, raw materials ordering cost function and order processing and fixed shipment cost function so we keep these costs as derived in chapter 4. The detailed derivations for other costs are as follows:

*Parts holding cost*: Tier-1 suppliers produce finished products every cycle time with the production rate per unit time $P^{(s')}_{s',p}$. Given the proportion of part $p$ supplied by tier-1 supplier $s'$ $e^{(s')}_{s',p}$ the quantity of part $p$ supplied is $e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)$. Production quantity per cycle time for part $p$ is $e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$ as shown in Fig. 5.3 (b). Since $P^{(s')}_{s',p} > e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)$ production time for every production cycle time is $\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$. Similar to parts holding cost in the model without considering reverse logistics in this cost function we use a parameter, $(1-u_{p',i})$, the proportion of part $p$ for finished product $i$. Average inventory for part $p$ for tier-1 supplier $s'$ is calculated as follows:

Average inventory for part $p$ for tier-1 supplier $s'$

$$
= \left(\frac{\frac{\left(\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T\right)e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2}}{\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}\right)
$$

$$
+\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{P}\alpha_{M}\alpha_{D}T(\alpha_{S'P}-1)\alpha_{P}\alpha_{M}\alpha_{D}T+\ldots+e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)(\alpha_{P}\alpha_{M}\alpha_{D}T)^{2}}{\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}\right)
$$

$$
= \left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\right)+
$$

$$
\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{P}\alpha_{M}\alpha_{D}T\left((\alpha_{S'P}-1)+(\alpha_{S'P}-2)+\ldots+1\right)}{\alpha_{S'P}}\right)
$$

$$
= \left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{P}\alpha_{M}\alpha_{D}T}{2}\left(\alpha_{S'P}\frac{D^{(m)}_{i}}{P^{(m)}_{i}}+(\alpha_{S'P}-1)\right)\right) \tag{5.5}
$$

Hence, part $p$ inventory holding cost per unit time for tier-1 suppliers $s'$ is given by $h_{s',p}\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_P\alpha_M\alpha_D T}{2}\left(\alpha_{S'P}\frac{D^{(m)}_{i}}{P^{(m)}_{i}}+(\alpha_{S'P}-1)\right)\right)$

*Raw materials holding cost*: Similar to parts holding cost of the manufacturer, average inventory for raw material $w$ for tier-1 supplier $s'$ as shown in Figure 5.3 (c) is calculated as follows:

Average inventory for raw material $w$ for tier-1 supplier $s'$

$$\left(\frac{\left(\left(\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)^{\max}-\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right)\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}\right)+$$

$$\left(\frac{\frac{\left(\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right)\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2}}{\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}\right)+$$

$$=\left(\frac{\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T(\alpha_{S'W}-1)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T+\ldots+\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)(\alpha_{S'P}\alpha_P\alpha_M\alpha_D T)^2}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}\right)$$

$$\left(\left(\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)^{\max}-\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\right)\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\right)+$$

$$=\frac{\left(\frac{e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s')}_{s',p}}\right)\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2}+$$

$$\left(\frac{\beta^{(II)}_{p,w}e^{(s')}_{s',p}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T\left((\alpha_{S'W}-1)+(\alpha_{S'W}-1)+\ldots+1\right)}{\alpha_{S'W}}\right)$$

$$
= \left( \frac{\beta^{(II)}_{p,w} e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \alpha_{S'P} \alpha_P \alpha_M \alpha_D T}{2} \left( \begin{array}{l} 2\left( \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} \right)^{\max} - \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} \right) \right) + \\ \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} \right) + \left( \alpha_{S'W} - 1 \right) \end{array} \right) \right) \tag{5.6}
$$

Again, raw material $w$ for part $p$ inventory holding cost per unit time is given by

$$
h^{(s')}_{s',w} \left( \frac{\beta^{(II)}_{p,w} e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \alpha_{S'P} \alpha_P \alpha_M \alpha_D T}{2} \left( \begin{array}{l} 2\left( \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} \right)^{\max} - \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} \right) \right) + \\ \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} \right) + \left( \alpha_{S'W} - 1 \right) \end{array} \right) \right)
$$

Then, the total cost incurred by tier-1 suppliers for all parts and raw materials is

$$
\begin{aligned}
TCS' = & \sum_{s'=1}^{n^{(s')}} \frac{S^{(s')}_{s'} + \sum_{p=1}^{k^{(p)}} s^{(s')}_{s',p}}{\alpha_{S'P} \alpha_P \alpha_M \alpha_D T} + \sum_{s'=1}^{n^{(s')}} \sum_{p=1}^{k^{(p)}} \left( \frac{h_{s',p} e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \alpha_P \alpha_M \alpha_D T}{2} \left( \alpha_{S'P} \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} + \left( \alpha_{S'P} - 1 \right) \right) \right) + \\
& \sum_{s'=1}^{n^{(s')}} \frac{\left( B^{(s')}_{s'} + \sum_{p=1}^{k^{(p)}} b^{(p)}_{s',p} \right)}{\alpha_P \alpha_M \alpha_D T} \sum_{s'=1}^{n^{(s')}} \frac{A^{(s')}_{s'} + \sum_{w=1}^{k^{(w)}} a^{(s')}_{s',w}}{\alpha_{S'W} \alpha_{S'P} \alpha_P \alpha_M \alpha_D T} + \\
& \sum_{s'=1}^{n^{(s')}} \sum_{w=1}^{k^{(w)}} \sum_{p=1}^{k^{(p)}} \left( \frac{\beta^{(II)}_{p,w} h^{(s')}_{s',w} e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \alpha_{S'P} \alpha_P \alpha_M \alpha_D T}{2} \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} + \left( \alpha_{S'W} - 1 \right) + \left( 2\left( \gamma \max_{s'} - \gamma_{s',p} \right) \right) \right) \right)
\end{aligned} \tag{5.7}
$$

where

$$
\gamma_{s',p} = \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}} \tag{5.8}
$$

$$
\gamma \max_{s'} = \max \left( \frac{e^{(s')}_{s',p} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right)}{P^{(s')}_{s',p}}, \; p = 1,2,..,k^{(p)} \right) \tag{5.9}
$$

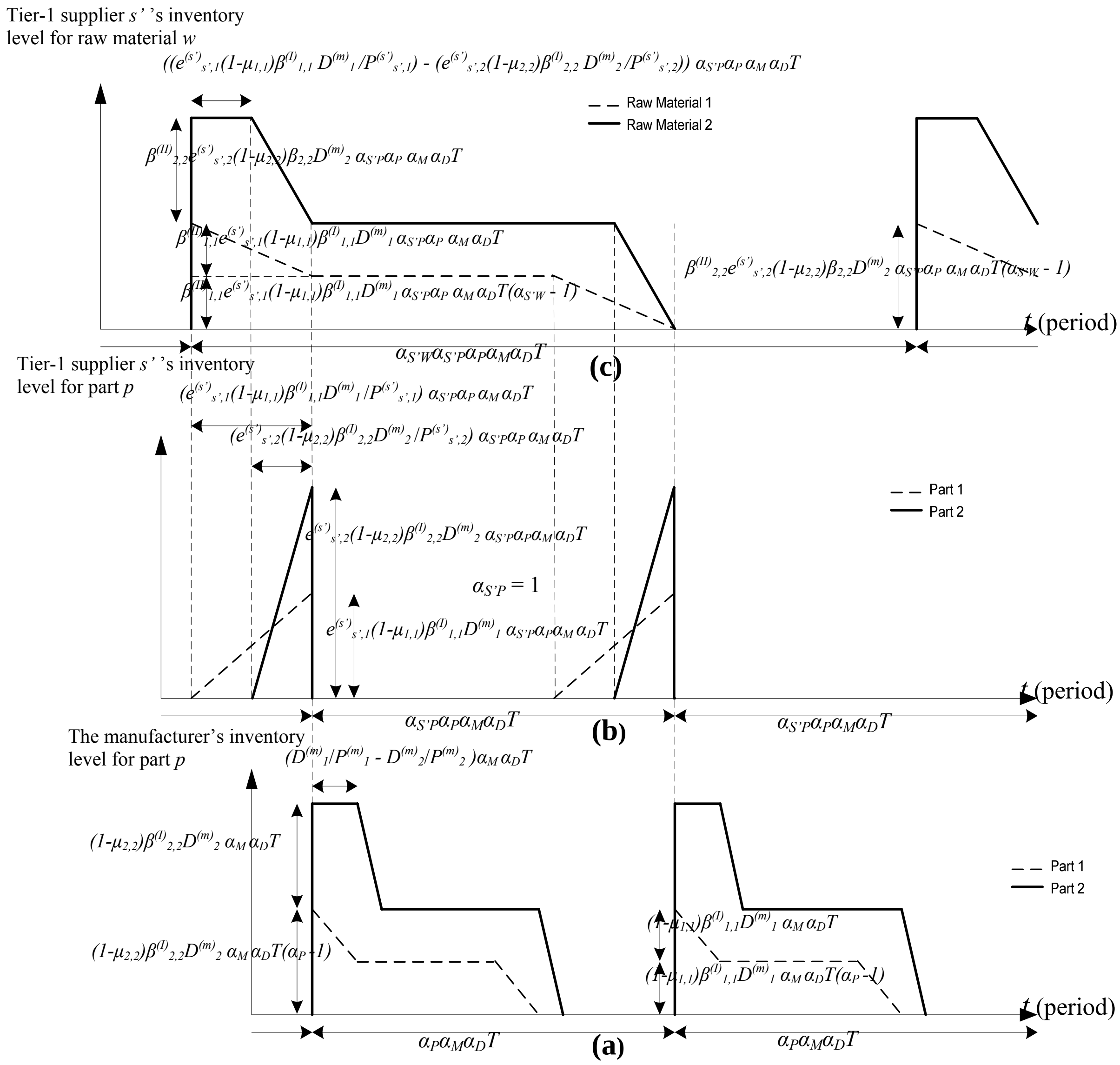


Figure 5.3 Inventories behavior of part $p$ and raw material $w$ of the manufacturer and the tier-1 supplier $s'$ with $\alpha_{S'P} = 1$ and $\alpha_{S'W} = 2$

### *5.2.2.3 Tier-2 suppliers' cost components*

Similar to tier-1 suppliers, tier-2 suppliers incur raw materials production setup cost, raw materials holding cost and order processing and fixed shipment cost. There are no changes to raw materials production setup cost function and order processing and fixed shipment cost function so we keep these costs as derived in chapter 4. The detailed derivation for raw materials holding cost is as follows:

*Raw materials holding cost*: Tier-2 suppliers produce raw materials every cycle time with the production rate per unit time $P^{(s'')}_{s'',w}$. Given the proportion of raw material $w$ supplied by tier-2 supplier $s''$, $e^{(s'')}_{s'',w}$ the quantity of raw material $w$ supplied is $e^{(s'')}_{s'',w}\sum_{p=1}^{k^{(p)}}\beta^{(II)}_{p,w}\sum_{i=1}^{k^{(i)}}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)$. Production quantity per cycle time for raw material $w$ is $e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$ as shown in Figure 5.4 (b).

Since $P^{(s")}_{s",w} > e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)$ the production time for every cycle time is $\left( e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right) \Big/ P^{(s")}_{s",w} \right)\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T$. Similarly, average inventory for raw material $w$ for tier-2 supplier $s"$ is calculated as follows:

Average inventory for raw material $w$ for tier-2 supplier $s"$

$$= \left( \frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S"W}\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2} \left( \frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s")}_{s",w}} \right) \right) +$$

$$\left( \frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T\left((\alpha_{S"W}-1)+(\alpha_{S"W}-2)+\ldots+1\right)}{\alpha_{S"W}} \right)$$

$$= \left( \frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2} \left( \alpha_{S"W}\frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s")}_{s",w}} + \left(\alpha_{S"W}-1\right) \right) \right) \tag{5.10}$$

Hence, raw material $w$ inventory holding cost per unit time for tier-2 supplier $s"$ is given by $h^{s(")}_{s",w}\left( \frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2} \left( \alpha_{S"W}\frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)}{P^{(s")}_{s",w}} + \left(\alpha_{S"W}-1\right) \right) \right)$

Then, the total cost incurred by tier-2 suppliers for all parts and raw materials is

$$TCS" = \left( \begin{array}{l} \sum_{s"=1}^{n(s")}\frac{S^{(s")}_{s"}+\sum_{w=1}^{k(w)}s^{(s")}_{s",w}}{\alpha_{S"W}a_{S'W}a_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T} + \sum_{s"=1}^{n(s")}\frac{\left(B^{(s")}_{s"}+\sum_{w=1}^{k(w)}b^{(s")}_{s",w}\right)}{\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T} + \\ \sum_{s"=1}^{n(s")}\sum_{w=1}^{k(w)}\left( \frac{h^{(s")}_{s",w}e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}\right)\alpha_{S'W}\alpha_{S'P}\alpha_{P}\alpha_{M}\alpha_{D}T}{2} \left( \alpha_{S"W}\frac{e^{(s")}_{s",w}\sum_{p=1}^{k(p)}\beta^{(II)}_{p,w}\sum_{i=1}^{k(i)}\left(1-u_{p'}\right)\beta^{(I)}_{p,i}D^{(m)}_{i}}{P^{(s")}_{s",w}} + \left(\alpha_{S"W}-1\right) \right) \right) \end{array} \right) \tag{5.11}$$

### 5.2.2.4 The third party's cost components

Unlike other players, the third party incurs the setup cost for collecting used products from end customers, used products holding cost and order processing and fixed shipment cost. Since disassembly and collection costs per unit used product do not affect the optimal solutions we ignore the costs in the total cost function. The detailed derivation for each cost is as follows:

*Collecting setup cost*: The third party applies the cycle time $\alpha 3\alpha_P\alpha_M\alpha_D T$ which is a multiple integer of $\alpha_P\alpha_M\alpha_D T$, common order cycle time of the manufacturer for parts, to collect used products from end customers with collecting setup cost per cycle time $S^{(3)}$ and processing cost per unit used product $i$ per unit time $s_i^{(3)}$. Collecting setup cost incurred by the third party per unit time is $\dfrac{S^{(3)} + \sum_{i=1}^{k^{(i)}} s_i^{(3)}}{\alpha 3\alpha_P\alpha_M\alpha_D T}$

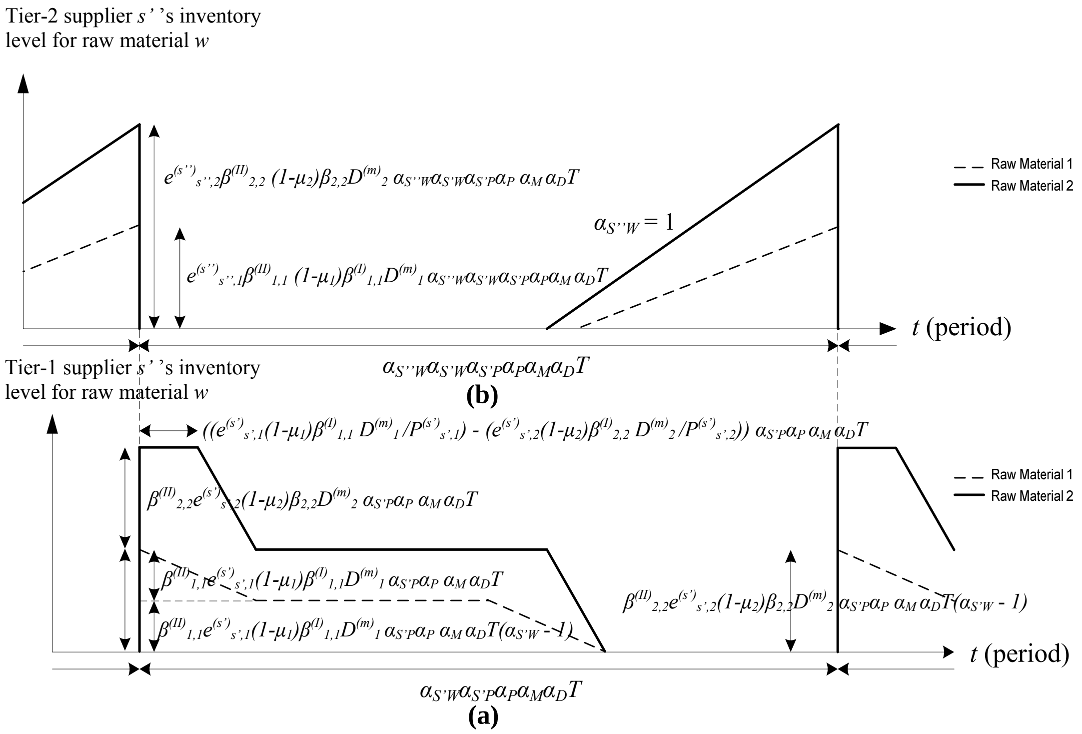


Figure 5.4 Inventory behavior of raw materials for the tier-1 supplier *s'* and the tier-2 supplier *s"* with $\alpha_{S'W} = 2$ and $\alpha_{S''W} = 1$

*Used products holding cost*: The third party collects used products every cycle time with the collecting rate per unit time $C_i$. Given the returning rate of used products from end customers $R_i$ per unit time, the collection quantity per cycle time for used product $i$ is $R_i\alpha 3\alpha_P\alpha_M\alpha_D T$. Since $C_i > R_i$ collecting time for every cycle time is $(R_i/C_i)\alpha 3\alpha_P\alpha_M\alpha_D T$. The third party stops the collection once the quantity reaches $R_i\alpha 3\alpha_P\alpha_M\alpha_D T$ units every cycle time. At this point used products inventory immediately drops as many as one order for parts of the manufacturer to satisfy the order from the manufacturer. As the collection quantity may be more than the order from the manufacturer every cycle time ($R_i\beta_{p',i}^{(I)} >= u_{p',i}\beta_{p',i}^{(I)}D_i^{(m)}$) the remaining quantities are ordered and processed by other players excluding in the supply chain. We do not consider this in the model. Average inventory for used product $i$ is calculated as follows:

Average inventory for used product $i$

$$= \left( \frac{\frac{(R_i \alpha 3 \alpha_P \alpha_M \alpha_D T)\left( R_i \big/ C_i \, \alpha 3 \alpha_P \alpha_M \alpha_D T \right)}{2}}{\alpha 3 \alpha_P \alpha_M \alpha_D T} \right) +$$

$$\left( \frac{(R_i \alpha_P \alpha_M \alpha_D T)(\alpha 3 - 1)\alpha_P \alpha_M \alpha_D T + (R_i \alpha_P \alpha_M \alpha_D T)(\alpha 3 - 2)\alpha_P \alpha_M \alpha_D T + \ldots + \left( R_i (\alpha_P \alpha_M \alpha_D T)^2 \right)}{\alpha 3 \alpha_P \alpha_M \alpha_D T} \right)$$

$$= \left( \frac{(R_i \alpha 3 \alpha_P \alpha_M \alpha_D T)}{2} \left( \frac{R_i}{C_i} \right) \right) + \left( \frac{R_i \alpha_P \alpha_M \alpha_D T((\alpha 3 - 1) + (\alpha 3 - 2) + \ldots + 1)}{\alpha 3} \right)$$

$$= \left( \frac{(R_i \alpha_P \alpha_M \alpha_D T)}{2} \left( \alpha 3 \frac{R_i}{C_i} \right) \right) + \left( \frac{R_i \alpha_P \alpha_M \alpha_D T(\alpha 3 - 1)}{2} \right)$$

$$= \left( \frac{(R_i \alpha_P \alpha_M \alpha_D T)}{2} \left( \alpha 3 \frac{R_i}{C_i} + (\alpha 3 - 1) \right) \right) \tag{5.12}$$

Hence, used product $i$ inventory holding cost per unit time is given by

$$h_i^{(3)} \left( \frac{(R_i \alpha_P \alpha_M \alpha_D T)}{2} \left( \alpha 3 \frac{R_i}{C_i} + (\alpha 3 - 1) \right) \right)$$

Then, the total cost incurred by the third party for all used products is

$$TC3 = \frac{S^{(3)} + \sum_{i=1}^{k(i)} s_i^{(3)}}{\alpha 3 \alpha_P \alpha_M \alpha_D T} + \frac{\left( B^{(3)} + \sum_{p'=1}^{k(p')} b_{p'}^{(3)} \right)}{\alpha_P \alpha_M \alpha_D T} + \sum_{i=1}^{k(i)} \left( \frac{h_i^{(3)} R_i \alpha_P \alpha_M \alpha_D T}{2} \right) \left( \alpha 3 \frac{R_i}{C_i} + (\alpha 3 - 1) \right) \tag{5.13}$$

where $$R_i \beta_{p',i}^{(I)} >= u_{p',i} \beta_{p',i}^{(I)} D_i^{(m)} \tag{5.14}$$

The first term is the collecting setup cost per unit time. The second term is order processing and fixed shipment cost to deliver returned parts to the manufacturer and the last term is the holding cost of used products.

*5.2.2.5 The cost function of the system involving reverse logistics*

The total cost function for the whole supply chain involving reverse logistics is determined by summing equations (4.34), (4.38), (5.4), (5.7), (5.11) and (5.13).

$$TCChain = TCR + TCD + TCM + TCS' + TCS'' + TC3 \tag{5.15}$$

**Lemma 2**: Eq. (5.15) is also convex function over $T > 0$ for any values of $\alpha_D$, $\alpha_M$, $\alpha_P$, $\alpha 3$, $\alpha_{S'P}$, $\alpha_{S'W}$, $\alpha_{S''W} \geq 1$.

**Proof**. See Appendix C.

## 5.3 Considering Transportation Costs

### 5.3.1 Notations

The input parameters and decision variables for retailers, distributors, the manufacturer, tier-1 suppliers and tier-2 suppliers are as shown below, respectively.

Parameters:

$F$ fixed transportation cost per unit delivery

$V$ fixed transportation cost per cycle time

$L^{(i)}_i$ the length of pack size of product $i$

$L^{(p)}_p$ the length of pack size of part $p$

$L^{(w)}_w$ the length of pack size of raw material $w$

$L_F$ the length of the container of the delivery unit

$W^{(i)}_i$ the width of pack size of product $i$

$W^{(p)}_p$ the width of pack size of part $p$

$W^{(w)}_w$ the width of pack size of raw material $w$

$W_F$ the width of the container of the delivery unit

$H^{(i)}_i$ the height of pack size of product $i$

$H^{(p)}_p$ the height of pack size of part $p$

$H^{(w)}_w$ the height of pack size of raw material $w$

$H_F$ the height of the container of the delivery unit

$g^{(i)}_i$ the number of product $i$ per pack

$g^{(p)}_p$ the number of part $p$ per pack

$g^{(w)}_w$ the number of raw material $w$ per pack

$a_d$min, $a_P$min, $a_{P'}$min, $a_W$min are the minimum capacity allowance per unit delivery for each distributor, all parts for the manufacturer, all returned parts for the manufacturer, all raw materials for all tier-1 suppliers respectively.

Decision variables:

$a_d$, $a_M$, $a_{S'}$, $a_{S''}$, $a3$ are capacity allowances per unit delivery for all retailers, all distributors, all parts for the manufacturer, all returned parts for the manufacturer, all raw materials for all tier-1 suppliers respectively.

$N_d$, $N_M$, $N_{S'}$, $N_{S''}$, $N3$ are the numbers of delivery units per cycle time for all retailers, all distributors, all parts for the manufacturer, all returned parts for the manufacturer, all raw materials for all tier-1 suppliers respectively.

### 5.3.2 Mathematical Modelling of Transportation Costs

Transportation is one of the major issues in a supply chain as described in chapter 2. In many research studies about inventory control and models developed, transportation cost is commonly either included in the products price or ordering cost which is fixed for any order quantity and assumed to be independent of the size of the shipment/ delivery (Ertogral et al., 2007). However, for some cases where an order quantity is more than the capacity of a transportation unit we can not include the transportation cost in ordering cost. We need to separate the calculation of this cost in the model. There are two different modes of shipping freight typically categorized as either truckload (TL) transportation or less than truckload (LTL) transportation (Rieksts and Ventura, 2008). In this work we use truckload (TL) transportation category. When we use truckload (TL) transportation category, the costs incurred for each transportation unit which is excluded from ordering cost and product price are fuel cost, driver cost, fixed operation cost, and road taxes. Then, the transportation costs which is still included in ordering cost and product price are loading and unloading cost, overhead cost related to transportation such as transportation planning cost. We developed the approach which has been carried out in Rieksts and Ventura (2008). Rieksts and Ventura (2008) developed transportation cost for a single stage and single product only so that capacity of transportation unit can be directly determined in the quantity of products. In this research, we develop formulations to determine the number of transportation units needed by calculating the volume of a container of a transportation unit and the volume of each item (raw material, part, finished product) which will be delivered by the transportation unit.

First, we derive the formulation to calculate the number of transportation units needed by distributors to deliver finished products to retailers. We calculate the number of transportation units needed based on total volumes of finished products and the volume of the container of transportation such as a truck. The detailed derivation of this formulation is as follow:

Total volumes of packages of finished products is either less than or equal to the volume of the container of transportation unit minus the capacity allowance minimum for handling space and equipments.

Total volumes of packages of finished product $i = \frac{\left(TD^{(d)}_{d,i}(L^{(i)}_{i}W^{(i)}_{i}H^{(i)}_{i}\right)}{g^{(i)}_{i}}$ (5.16)

The number of transportation units for distributor $d$, $N_d$, needed to deliver products every common order cycle time of retailers is determined by satisfying the formulation as follow.

$$\sum_{i=1}^{k^{(i)}} \frac{\left(TD_{d,i}^{(d)}(L_i^{(i)}W_i^{(i)}H_i^{(i)}\right)}{g_i^{(i)}} \le N_d(L_F W_F H_F) - a_d \min\left(N_d(L_F W_F H_F)\right) \tag{5.17}$$

Then, transportation cost per unit time incurred by distributor $d$ to deliver finished products to a subgroup of retailers is given by $\frac{(V + N_d F)}{T}$. Order processing and fixed shipment cost in Eq. (4.38) is included in this transportation cost so that we eliminate the equation from the total cost function where $V$ is fixed transportation cost per order cycle time and $F$ is fixed transportation per transportation unit. The similar situation is applied to other players in the supply chain.

The cost function of distributors with transportation cost considered will be

$$TCD = \frac{\sum_{d=1}^{n_d}\left(A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)}\right)}{\alpha_D T} + \frac{T\sum_{d=1}^{n^{(d)}}\sum_{i=1}^{k^{(i)}}\left(h_{d,i}^{(d)} D_{d,i}^{(d)}\right)}{2}(\alpha_D - 1) + \frac{\sum_{d=1}^{n^{(d)}}(V + N_d F)}{T} \tag{5.18}$$

Similarly, we derive the formulation to calculate the number of transportation units needed by the manufacturer to deliver finished products to distributors. The detailed derivation is as follow:

Total volumes of packages of finished products is either less than or equal to the volume of the container of transportation unit minus the capacity allowance minimum for handling space and equipments.

Total volumes of packages of finished product $i$ = $\frac{\alpha_D TD_i^{(m)}(L_i^{(i)}W_i^{(i)}H_i^{(i)})}{g_i^{(i)}}$ (5.19)

The number of transportation units needed to deliver finished products every common order cycle time of distributors is determined by satisfying the formulation as follow.

$$\sum_{i=1}^{k^{(i)}} \frac{\left(\alpha_D TD_i^{(m)}(L_i^{(i)}W_i^{(i)}H_i^{(i)})\right)}{g_i^{(i)}} \le N_M(L_F W_F H_F) - a_M \min\left(N_M(L_F W_F H_F)\right) \tag{5.20}$$

Then, transportation cost per unit time incurred by the manufacturer to deliver finished products to distributors is given by $\frac{(V + N_M F)}{\alpha_D T}$.

The cost function of the manufacturer without reverse logistics with transportation cost considered will therefore be

$$TCM=\left(\begin{array}{l}\dfrac{S^{(m)}+\sum_{i=1}^{k^{(i)}}s_i^{(m)}}{\alpha_M\alpha_D T}+\sum_{i=1}^{k^{(i)}}\dfrac{h_i D_i^{(m)}\alpha_D T}{2}\left(\alpha_M\dfrac{D_i^{(m)}}{P_i^{(m)}}+(\alpha_M-1)\right)+\dfrac{A_M+\sum_{p=1}^{k^{(p)}}a_p^{(m)}}{\alpha_P\alpha_M\alpha_D T}+\dfrac{(V+N_M F)}{\alpha_D T}+\\ \sum_{p=1}^{k^{(p)}}\sum_{i=1}^{k^{(i)}}\left(\dfrac{\beta_{p,i}^{(I)}h_p^{(m)}D_i^{(m)}\alpha_M\alpha_D T}{2}\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}+(\alpha_p-1)+2\left(\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}\right)^{\max}-\dfrac{D_i^{(m)}}{p_i^{(m)}}\right)\right)\right)\end{array}\right) \quad (5.21)$$

and

the cost function of the manufacturer involving reverse logistics, when transportation cost is considered will therefore be

$$TCM=\left(\begin{array}{l}\dfrac{S^{(m)}+\sum_{i=1}^{k^{(i)}}s_i^{(m)}}{\alpha_M\alpha_D T}+\sum_{i=1}^{k^{(i)}}\dfrac{h_i D_i^{(m)}\alpha_D T}{2}\left(\alpha_M\dfrac{D_i^{(m)}}{P_i^{(m)}}+(\alpha_M-1)\right)-\sum_{i=1}^{k^{(i)}}\sum_{p=1}^{k^{(p)}}\dfrac{\left(h_p^{(m)}-h_{p'}^{(m')}\right)u_{p',i}\beta_{p,i}^{(I)}D_i^{(m)}\alpha_D T}{2}\left(\alpha_M\dfrac{D_i^{(m)}}{P_i^{(m)}}(2-u_{p',i})+(\alpha_M-1)\right)+\dfrac{(V+N_M F)}{\alpha_D T}+\\ \dfrac{A_M+\sum_{p=1}^{k^{(p)}}a_p^{(m)}}{\alpha_P\alpha_M\alpha_D T}+\sum_{p=1}^{k^{(p)}}\sum_{i=1}^{k^{(i)}}\left(\dfrac{(1-u_{p',i})\beta_{p,i}^{(I)}h_p^{(m)}D_i^{(m)}\alpha_M\alpha_D T}{2}\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}+(\alpha_p-1)+2\left(\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}(1-u_{p',i})\right)^{\max}-\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}(1-u_{p',i})\right)\right)\right)\right)+\\ \dfrac{A_{M'}+\sum_{p'=1}^{k^{(p')}}a_{p'}^{(m')}}{\alpha_P\alpha_M\alpha_D T}+\sum_{p'=1}^{k^{(p')}}\sum_{i=1}^{k^{(i)}}\left(\dfrac{u_{p',i}\beta_{p,i}^{(I)}h_{p'}^{(m')}D_i^{(m)}\alpha_M\alpha_D T}{2}\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}+(\alpha_P-1)+2\left(\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}\right)^{\max}-\dfrac{D_i^{(m)}}{p_i^{(m)}}\right)\right)\right)\end{array}\right)$$

(5.22)

Furthermore, we derive the formulation to calculate the number of transportation units needed by tier-1 suppliers to deliver parts to the manufacturer. The detailed derivations for both situations (involving and without involving reverse logistics) are as follow:

Total volume of packages of parts is either less than or equal to the volume of the container of transportation unit minus the capacity allowance minimum for handling space and equipments.

For the system without involving reverse logistics,

$$\text{Total volume of packages of part } p = \frac{\alpha_P\alpha_M\alpha_D T\sum_{s'=1}^{n^{(s')}}\left(e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\beta_{p,i}^{(I)}D_i^{(m)}\right)\left(L_p^{(p)}W_p^{(p)}H_p^{(p)}\right)}{g_p^{(p)}} \quad (5.23)$$

The number of transportation units needed to deliver parts every common order cycle time of the manufacturer is determined by satisfying the formulation as follow.

$$\sum_{p=1}^{m^{(p)}}\left(\frac{\alpha_P\alpha_M\alpha_D T\sum_{s'=1}^{n^{(s')}}\left(e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\beta_{p,i}^{(I)}D_i^{(m)}\right)\left(L_p^{(p)}W_p^{(p)}H_p^{(p)}\right)}{g_p^{(p)}}\right)\le N_{S'}(L_F W_F H_F)-a_{S'}\min\left(N_{S'}(L_F W_F H_F)\right) \quad (5.24)$$

For the system involving reverse logistics, Total volumes of packages of part $p$ = $\dfrac{\alpha_P \alpha_M \alpha_D T \sum_{s'=1}^{n^{(s')}} \left( e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} (1-u_{p',i}) \beta_{p,i}^{(I)} D_i^{(m)} \right) \left( L_p^{(p)} W_p^{(p)} H_p^{(p)} \right)}{g_p^{(p)}}$ (5.25)

The number of transportation units needed to deliver parts every common order cycle time of the manufacturer is determined by satisfying the formulation as follow.

$$\sum_{p=1}^{m^{(p)}} \left( \frac{\alpha_P \alpha_M \alpha_D T \sum_{s'=1}^{n^{(s')}} \left( e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} (1-u_{p',i}) \beta_{p,i}^{(I)} D_i^{(m)} \right) \left( L_p^{(p)} W_p^{(p)} H_p^{(p)} \right)}{g_p^{(p)}} \right) \le N_{S'} (L_F W_F H_F) - a_{S'} \min \left( N_{S'} (L_F W_F H_F) \right) \tag{5.26}$$

Then, transportation cost per unit time incurred by tier-1 suppliers to deliver parts to the manufacturer is given by $\dfrac{(V + N_{S'} F)}{\alpha_P \alpha_M \alpha_D T}$.

The cost function of tier-1 suppliers in the system that does not involve reverse logistics, with transportation cost taken into consideration will therefore be

$$\begin{aligned} TCS = & \sum_{s'=1}^{n^{(s')}} \frac{S_{s'}^{(s')} + \sum_{p=1}^{k^{(p)}} s_{s',p}^{(s')}}{\alpha_{S'P} \alpha_P \alpha_M \alpha_D T} + \sum_{s'=1}^{n^{(s')}} \sum_{p=1}^{k^{(p)}} \left( \frac{h_{s',p} e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \left( \beta_{p,i}^{(I)} D_i^{(m)} \right) \alpha_P \alpha_M \alpha_D T}{2} \left( \alpha_{S'P} \frac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \left( \beta_{p,i}^{(I)} D_i^{(m)} \right)}{P_{s',p}^{(s')}} + (\alpha_{S'P} - 1) \right) \right) + \sum_{s'=1}^{n^{(s')}} \frac{A_{s'}^{(s')} + \sum_{w=1}^{k^{(w)}} a_{s',w}^{(s')}}{\alpha_{S'W} \alpha_{S'P} \alpha_P \alpha_M \alpha_D T} + \\ & \frac{(V + N_{S'} F)}{\alpha_P \alpha_M \alpha_D T} + \sum_{s'=1}^{n^{(s')}} \sum_{w=1}^{k^{(w)}} \sum_{p=1}^{k^{(p)}} \left( \frac{\beta_{p,w}^{(II)} h_{s',w}^{(s')} e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \left( \beta_{p,i}^{(I)} D_i^{(m)} \right) \alpha_{S'P} \alpha_P \alpha_M \alpha_D T}{2} \left( \frac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \beta_{p,i}^{(I)} D_i^{(m)}}{P_{s',p}^{(s')}} + (\alpha_{S'W} - 1) + \left( 2 \left( \gamma \max_{s'} - \gamma_{s',p} \right) \right) \right) \right) \end{aligned} \tag{5.27}$$

and

the cost function of tier-1 suppliers in a system involving reverse logistics, with transportation cost taken into consideration will therefore be

$$\begin{aligned} TCS' = & \sum_{s'=1}^{n^{(s')}} \frac{S_{s'}^{(s')} + \sum_{p=1}^{k^{(p)}} s_{s',p}^{(s')}}{\alpha_{S'P} \alpha_P \alpha_M \alpha_D T} + \sum_{s'=1}^{n^{(s')}} \sum_{p=1}^{k^{(p)}} \left( \frac{h_{s',p} e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta_{p,i}^{(I)} D_i^{(m)} \right) \alpha_P \alpha_M \alpha_D T}{2} \left( \alpha_{S'P} \frac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta_{p,i}^{(I)} D_i^{(m)} \right)}{P_{s',p}^{(s')}} + (\alpha_{S'P} - 1) \right) \right) + \\ & \frac{(V + N_{S'} F)}{\alpha_P \alpha_M \alpha_D T} + \sum_{s'=1}^{n^{(s')}} \frac{A_{s'}^{(s')} + \sum_{w=1}^{k^{(w)}} a_{s',w}^{(s')}}{\alpha_{S'W} \alpha_{S'P} \alpha_P \alpha_M \alpha_D T} + \\ & \sum_{s'=1}^{n^{(s')}} \sum_{w=1}^{k^{(w)}} \sum_{p=1}^{k^{(p)}} \left( \frac{\beta_{p,w}^{(II)} h_{s',w}^{(s')} e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta_{p,i}^{(I)} D_i^{(m)} \right) \alpha_{S'P} \alpha_P \alpha_M \alpha_D T}{2} \left( \frac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \left( (1-u_{p',i}) \beta_{p,i}^{(I)} D_i^{(m)} \right)}{P_{s',p}^{(s')}} + (\alpha_{S'W} - 1) + \left( 2 \left( \gamma \max_{s'} - \gamma_{s',p} \right) \right) \right) \right) \end{aligned} \tag{5.28}$$

Again, we derive the formulation to calculate the number of transportation units needed by tier-2 suppliers to deliver raw materials to tier-1 suppliers. The detailed derivations for both the system involving and without involving reverse logistics are as follows:

Total volume of packages of raw materials is either less than or equal to the volume of the container of transportation unit minus the capacity allowance minimum for handling space and equipments.

For the system without involving reverse logistics,

Total volumes of packages of raw material $w$

$$= \frac{\alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T \sum_{s''=1}^{n^{(s'')}} \left( e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \left( \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \right) \left( L^{(w)}_{w} W^{(w)}_{w} H^{(w)}_{w} \right)}{g^{(w)}_{w}} \tag{5.29}$$

The number of transportation units needed to deliver raw materials every common order cycle time of tier-1 suppliers is determined by satisfying the formulation as follows.

$$\sum_{w=1}^{k^{(w)}} \left( \frac{\alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T \sum_{s''=1}^{n^{(s'')}} \left( e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \left( \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \right) \left( L^{(w)}_{w} W^{(w)}_{w} H^{(w)}_{w} \right)}{g^{(w)}_{w}} \right) \leq N_{S''}(L_F W_F H_F) - a_{S''} \min\left( N_{S''}(L_F W_F H_F) \right) \tag{5.30}$$

For the system involving reverse logistics,

Total volumes of packages of raw material $w$

$$= \frac{\alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T \sum_{s''=1}^{n^{(s'')}} \left( e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \left( (1 - u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \right) \left( L^{(w)}_{w} W^{(w)}_{w} H^{(w)}_{w} \right)}{g^{(w)}_{w}} \tag{5.31}$$

The number of transportation units needed to deliver raw materials every common order cycle time of tier-1 suppliers is determined by satisfying the formulation as follow.

$$\sum_{w=1}^{k^{(w)}} \left( \frac{\alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T \sum_{s''=1}^{n^{(s'')}} \left( e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \left( (1 - u_{p',i}) \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \right) \left( L^{(w)}_{w} W^{(w)}_{w} H^{(w)}_{w} \right)}{g^{(w)}_{w}} \right) \leq N_{S''}(L_F W_F H_F) - a_{S''} \min\left( N_{S''}(L_F W_F H_F) \right) \tag{5.32}$$

Then, transportation cost per unit time incurred by tier-2 suppliers to deliver raw materials to tier-1 suppliers is given by $\dfrac{(V + N_{S''}F)}{\alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T}$.

The cost function of tier-2 suppliers without involving reverse logistics, with transportation cost taken into consideration will therefore be

$$TCS'' = \left( \begin{array}{l} \displaystyle \sum_{s''=1}^{n^{(s'')}} \frac{S^{(s'')}_{s''} + \sum_{w=1}^{k^{(w)}} s^{(s'')}_{s'',w}}{\alpha_{S''W} \alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T} + \frac{(V + N_{S''}F)}{\alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T} \\ \displaystyle \sum_{s''=1}^{n^{(s'')}} \sum_{w=1}^{k^{(w)}} \left( \frac{h^{(s'')}_{s'',w} e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \left( \beta^{(I)}_{p,i} D^{(m)}_{i} \right) \alpha_{S'W} \alpha_{S'P} \alpha_P \alpha_M \alpha_D T}{2} \left( \alpha_{S''W} \frac{e^{(s'')}_{s'',w} \sum_{p=1}^{k^{(p)}} \beta^{(II)}_{p,w} \sum_{i=1}^{k^{(i)}} \beta^{(I)}_{p,i} D^{(m)}_{i}}{P^{(s'')}_{s'',w}} + (\alpha_{S''W} - 1) \right) \right) \end{array} \right) \tag{5.33}$$

and

the cost function of tier-2 suppliers for the system involving reverse logistics, with transportation cost taken into consideration will therefore be

$$TCS' = \left( \begin{array}{l} \sum_{s''=1}^{n^{(s'')}} \dfrac{S_{s''}^{(s'')} + \sum_{w=1}^{k^{(w)}} s_{s'',w}^{(s'')}}{\alpha_{S''W}\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T} + \dfrac{\left(V + N_{S''}F\right)}{\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T} \\ \sum_{s''=1}^{n^{(s'')}}\sum_{w=1}^{k^{(w)}} \left( \dfrac{h_{s'',w}^{(s'')} e_{s'',w}^{(s'')} \sum_{p=1}^{k^{(p)}} \beta_{p,w}^{(II)} \sum_{i=1}^{k^{(i)}} \left( \left(1-u_{p',i}\right) \beta_{p,i}^{(I)} D_i^{(m)} \right) \alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2} \left( \alpha_{S''W} \dfrac{e_{s'',w}^{(s'')} \sum_{p=1}^{k^{(p)}} \beta_{p,w}^{(II)} \sum_{i=1}^{k^{(i)}} \left(1-u_{p',i}\right) \beta_{p,i}^{(I)} D_i^{(m)}}{P_{s'',w}^{(s'')}} + \left(\alpha_{S''W} - 1\right) \right) \right) \end{array} \right) \tag{5.34}$$

Finally, we derive the formulation to calculate the number of transportation units needed by the third party to deliver reusable returned parts to the manufacturer. The detailed derivation for the system involving reverse logistics is as follow:

Total volume of packages of reusable returned parts is either less than or equal to the volume of the container of transportation unit minus the capacity allowance minimum for handling space and equipments.

Total volume of packages of reusable returned part *p'*

$$= \frac{\alpha_P\alpha_M\alpha_D T\left(\sum_{i=1}^{k^{(i)}} u_{p',i}\beta_{p,i}^{(I)} D_i^{(m)}\right)\left(L_{p'}^{(p')} W_{p'}^{(p')} H_{p'}^{(p')}\right)}{g_{p'}^{(p')}} \tag{5.35}$$

The number of transportation units needed to deliver reusable returned parts every common order cycle time of the manufacturer is determined by satisfying the formulation as follow.

$$\sum_{p'=1}^{k^{(p')}} \left( \frac{\alpha_P\alpha_M\alpha_D T\left(\sum_{i=1}^{k^{(i)}} u_{p',i}\beta_{p,i}^{(I)} D_i^{(m)}\right)\left(L_{p'}^{(p')} W_{p'}^{(p')} H_{p'}^{(p')}\right)}{g_{p'}^{(p')}} \right) = N3(L_F W_F H_F) - a3\min\left(N3(L_F W_F H_F)\right) \tag{5.36}$$

Then, transportation cost per unit time incurred by the third party to deliver reusable returned parts to the manufacturer is given by $\dfrac{\left(V + N3F\right)}{\alpha_P\alpha_M\alpha_D T}$.

The cost function of the third party with transportation cost will therefore be

$$TC3 = \frac{S^{(3)} + \sum_{i=1}^{k^{(i)}} s_i^{(3)}}{\alpha 3\alpha_P\alpha_M\alpha_D T} + \frac{\left(V + N3F\right)}{\alpha_P\alpha_M\alpha_D T} + \sum_{i=1}^{k^{(i)}} \left( \frac{h_i^{(3)} R_i \alpha_P\alpha_M\alpha_D T}{2} \right) \left( \alpha 3 \frac{R_i}{C_i} + \left(\alpha 3 - 1\right) \right) \tag{5.37}$$

## 5.4 Considering Finite Horizon Period

In the previous inventory models we assumed that the horizon period of the model is infinite. This assumption is not applicable in certain situations. In a supply chain often there will be a limited period during that cooperation takes place between all players in the system so that how many items (raw materials, parts, finished products) supplied and/or delivered to other players during the period have to be exactly the same with the demand. To take this situation into consideration, we formulate some equations as constraints for the developed model which is similar with work has been carried out in Rieksts and Ventura (2008) for single stage and single product model (a retailer). Here, we derive the formulas for each level of the supply chain.

For retailers, since the length of horizon/ contract period is limited the number of the common order cycle for all finished products for all retailers during this horizon period has to be an integer number. It means that quantity of finished products ordered during the period is only to satisfy the demand during the period. Therefore, the common order cycle time $T$ times the number of the common order cycles $V^1$ must be equal to the length of horizon period $N$.

$$N = TV^1 \tag{5.38}$$

Similarly, the number of the distributors' common order cycles $V^2$ for all finished products within the horizon period times the distributors' common order cycle time $a_D T$ must be equal to the length of horizon period $N$

$$N = \alpha_D TV^2 \tag{5.39}$$

Then, the number of the manufacturer's common production cycles $V^3$ for all finished products times the manufacturer's common production cycle time $a_M a_D T$ and the number of the manufacturer's common order cycles $V^4$ for all parts times the manufacturer's common order cycle time $a_P a_F a_D T$ must be equal to the length of horizon period $N$.

$$N = \alpha_M \alpha_D TV^3 \tag{5.40}$$

$$N = \alpha_P \alpha_M \alpha_D TV^4 \tag{5.41}$$

Furthermore, the number of tier-1 suppliers' common production cycles $V^5$ for all parts times tier-1 suppliers' common production cycle time $a_{S'P} a_P a_M a_D T$ and the number of tier-1 suppliers' common order cycles $V^6$ for all raw materials times the tier-1 suppliers' common order cycle time $a_{S'W} a_{S'P} a_P a_M a_D T$ must be equal to the length of horizon period $N$.

$$N = a_{S'P} \alpha_P \alpha_M \alpha_D TV^5 \tag{5.42}$$

$$N = \alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D TV^6 \tag{5.43}$$

Again, the number of tier-2 suppliers' common production cycles $V^7$ for all raw materials times tier-2 suppliers' common production cycle time $a_{S''W}a_{S'W}a_{S'P}a_{P}a_{M}a_{D}T$ must be equal to the length of horizon period $N$.

$$N = \alpha_{S''W}\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D TV^7 \tag{5.44}$$

Finally, the number of the third party's common collecting cycles $V^8$ for all used finished products times the third party's common collecting cycle time $a3a_{P}a_{M}a_{D}T$ must be equal to the length of horizon period $N$.

$$N = a3\alpha_P\alpha_M\alpha_D TV^8 \tag{5.45}$$

where $V^j$ (integer numbers) ($j$ = 1, 2, …, 8) ≥ 1.

## 5.5 Considering Stochastic Demand

In this section, we develop the model considering stochastic demand. We assume that the stochastic demand follows the normal distribution. Since the stochastic demand only affects the total cost of retailers we only derive the total function of retailers. Other total cost functions of other players are kept the same. To anticipate stochastic demand from end customers we add safety stock to the retailers' inventory. This is similar to the work that has been carried out in Ertogral (2011). The safety stock is assumed based on a service level policy notated by $k$ factor. For an example, $k$ = 3 indicates 99.86 % of service level. This policy is that probability of running out of inventory during the retailers' common cycle time should be less than a specific value (e.g. 99.86 %). It means that (1-99.86) % of the demand is not satisfied during the cycle time (stock outs). Stock outs can be either treated as back order or lost sales policies. In this model, stock outs are treated as back order policy.

Given $E(D^{(r)}_{r,i})$ average demand for finished product $i$ of retailer $r$ per unit time, $\sigma_{r,i}$ standard deviation of the demand for finished $i$ of retailer $r$, $C_{r,i}$ back order cost for finished product $i$ of retailer $r$ per unit product, the safety stock $ss_{r,i}$ and quantity of back order per cycle time $\pi_{r,i}$ for finished product $i$ of retailer $r$ as adopted from Chopra and Meindl (2004) are as follows:

$$ss_{r,i} = k\sigma_{r,i}\sqrt{T} \tag{5.46}$$

$$\pi_{r,i} = -ss_{r,i}\left(1 - F_s(k)\right) + \sigma_{r,i}\sqrt{T} f_s(k) \tag{5.47}$$

where $F_s$ is the standard normal cumulative distribution function and $f_s$ is the standard normal density function.

Then, the expected total cost, *ETCR*, incurred by retailers per unit time is

$$ETCR = \frac{\sum_{r=1}^{n^{(r)}}\left(A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)}\right)}{T} + \frac{\sum_{r=1}^{n^{(r)}}\left(T\sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)}\right)}{2} + \sum_{r=1}^{n^{(r)}}\sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} k\sigma_{r,i}\sqrt{T} + \sum_{r=1}^{n^{(r)}}\sum_{i=1}^{k^{(i)}} \frac{C_{r,i}\left(-\left(k\sigma_{r,i}\sqrt{T}\right)\left(1 - F_s(k)\right) + \sigma_{r,i}\sqrt{T} f_s(k)\right)}{T} \tag{5.48}$$

The third term is safety stock cost per unit time. The last term is backorder cost per unit time. The whole supply chain's total cost function involving reverse logistics with stochastic demand is the sum of Eq. (4.38), Eq. (5.4), Eq. (5.7), Eq. (5.11), Eq. (5.13) and Eq. (5.48)

## 5.6 Summary

Following previous chapter, in this chapter the description of the system studied and the mathematical modelling of production inventory model in a complex manufacturing supply chain for multiple items and multiple sources considering reverse logistics, transportation costs, finite horizon period and stochastic demand are provided. The derivation of total cost function involving reverse logistics is given from the manufacturer until the third party since there are no changes to total cost function of retailers and distributors derived in the previous chapter.

Then, the derivation of the model considering transportation costs excluding ordering and order processing cost is presented. Formulating of the transportation costs is started from distributors until the third party. Formulations obtained are as constraints of the model. Finally, the derivation of formulations for limited horizon period is provided. Again, formulations obtained for limited horizon period are also as constraints of the model.

# Chapter 6

# Solution Methods

## 6.1 Introduction

In this chapter, solution methods of the models developed are described. In section 6.2, we present a centralised decision making process based on a mixed integer non-linear programming method. Solution method based on decentralised decision making process is developed in section 6.3. In section 6.4 we propose semi-centralised decision making process which is a mixture of centralised and decentralised decision making processes. Section 6.5 summarises the chapter.

## 6.2 Centralised Decision Making Process

In centralized decision-making process there are no dominant players in determining the optimal production and inventory cycles so that the entire system determines all decision variables simultaneously. In this work we use mixed integer non-linear programming (MINLP) method to solve the models developed.

*6.2.1 The complex manufacturing supply chain without involving reverse logistics (Model-1)*

Objective function for this supply chain is the minimum of *TCChain* in Eq. (4.48) subject to $T > 0$ and $a_D$, $a_M$, $a_P$, $a3$, $a_{S'P}$, $a_{S'W}$, $a_{S''W}$ $\geq 1$. The MINLP formulation is listed as follows;

Min $TCChain = TCR + TCD + TCM + TCS' + TCS''$

s/t.

$T > 0$

$a_D, a_M, a_P, a3, a_{S'P}, a_{S'W}, a_{S''W} \geq 1$

where $a_D$, $a_M$, $a_P$, $a3$, $a_{S'P}$, $a_{S'W}$, $a_{S''W}$ are integer numbers

*6.2.2 The complex manufacturing supply chain involving reverse logistics (Model-2)*

Objective function for this supply chain is the minimum of *TCChain* in Eq. (5.15). The MINLP formulation is listed as follows;

Min *TCChain = TCR + TCD + TCM +TCS' + TCS" + TC3*

s/t.

$T > 0$

$a_D, a_M, a_P, a3, a_{S'P}, a_{S'W}, a_{S"W} \geq 1$

where $a_D$, $a_M$, $a_P$, $a3$, $a_{S'P}$, $a_{S'W}$, $a_{S"W}$ are integer numbers

### 6.2.3 The complex manufacturing supply chain involving reverse logistics with transportation cost considered (Model-3)

Objective function for this supply chain is the minimum of *TCChain* which considers transportation costs. The MINLP formulation is listed as follows;

Min *TCChain = TCR + TCD + TCM +TCS' + TCS" + TC3* (Eq. (4.34) + Eq. (5.18) + Eq. (5.22) + Eq. (5.28) + Eq. (5.34) + Eq. (5.37))

s/t.

$$\sum_{i=1}^{k^{(i)}} \frac{\left(TD_{d,i}^{(d)}(L_i^{(i)}W_i^{(i)}H_i^{(i)})\right)}{g_i^{(i)}} \leq N_d(L_F W_F H_F) - a_d \min\left(N_d(L_F W_F H_F)\right) \quad 6.1$$

$$\sum_{i=1}^{k^{(i)}} \frac{\left(\alpha_D TD_i^{(m)}(L_i^{(i)}W_i^{(i)}H_i^{(i)})\right)}{g_i^{(i)}} \leq N_M(L_F W_F H_F) - a_M \min\left(N_M(L_F W_F H_F)\right) \quad 6.2$$

$$\sum_{p=1}^{m^{(p)}} \left( \frac{\alpha_P \alpha_M \alpha_D T \sum_{s'=1}^{n^{(s')}} \left( e_{s',p}^{(s')} (1-u_{p'}) \sum_{i=1}^{k^{(i)}} \beta_{p,i}^{(I)} D_i^{(m)} \right) \left(L_p^{(p)} W_p^{(p)} H_p^{(p)}\right)}{g_p^{(p)}} \right) \leq N_{S'}(L_F W_F H_F) - a_{S'} \min\left(N_{S'}(L_F W_F H_F)\right) \quad 6.3$$

$$\sum_{w=1}^{k^{(w)}} \left( \frac{\alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D T \sum_{s"=1}^{n^{(s")}} \left( e_{s",w}^{(s")} \sum_{p=1}^{k^{(p)}} \beta_{p,w}^{(II)} (1-u_{p'}) \sum_{i=1}^{k^{(i)}} \left(\beta_{p,i}^{(I)} D_i^{(m)}\right) \right) \left(L_w^{(w)} W_w^{(w)} H_w^{(w)}\right)}{g_w^{(w)}} \right) \leq N_{S"}(L_F W_F H_F) - a_{S"} \min\left(N_{S"}(L_F W_F H_F)\right) \quad 6.4$$

$$\sum_{p'=1}^{k^{(p')}} \left( \frac{\alpha_P \alpha_M \alpha_D T \left( u_{p'} \sum_{i=1}^{k^{(i)}} \beta_{p,i}^{(I)} D_i^{(m)} \right) \left(L_{p'}^{(p')} W_{p'}^{(p')} H_{p'}^{(p')}\right)}{g_{p'}^{(p')}} \right) = N3(L_F W_F H_F) - a3 \min\left(N3(L_F W_F H_F)\right) \quad 6.5$$

$T > 0$

$a_D, a_M, a_P, a3, a_{S'P}, a_{S'W}, a_{S"W} \geq 1$

$N_d, N_M, N_{S'}, N_{S"}, N_3 \geq 1$

where $a_D$, $a_M$, $a_P$, $a3$, $a_{S'P}$, $a_{S'W}$, $a_{S"W}, N_d, N_M, N_{S'}, N_{S"}, N_3$ are integer numbers

### 6.2.4 The complex manufacturing supply chain involving reverse logistics with finite horizon period (Model-4)

Objective function for this supply chain is the minimum of *TCChain* in Eq. (5.15) with finite horizon period. The MINLP formulation is listed as follows;

Min *TCChain = TCR + TCD + TCM +TCS' + TCS" + TC3* s/t.

$$N = TV^1 \tag{6.6}$$
$$N = \alpha_D TV^2 \tag{6.7}$$
$$N = \alpha_M \alpha_D TV^3 \tag{6.8}$$
$$N = \alpha_P \alpha_M \alpha_D TV^4 \tag{6.9}$$
$$N = a_{S'P} \alpha_P \alpha_M \alpha_D TV^5 \tag{6.10}$$
$$N = \alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D TV^6 \tag{6.11}$$
$$N = \alpha_{S''W} \alpha_{S'W} a_{S'P} \alpha_P \alpha_M \alpha_D TV^7 \tag{6.12}$$
$$N = a3 \alpha_P \alpha_M \alpha_D TV^8 \tag{6.13}$$
$$T \le N \tag{6.14}$$
$$T > 0$$

$a_D, a_M, a_P, a3, a_{S'P}, a_{S'W}, a_{S''W} \ge 1$

$N_d, N_M, N_{S'}, N_{S''}, N_3 \ge 1$

$V^1, V^2, V^3, V^4, V^5, V^6, V^7, V^8 \ge 1$

where $a_D$, $a_M$, $a_P$, $a3$, $a_{S'P}$, $a_{S'W}$, $a_{S''W}$, $N_d$, $N_M$, $N_{S'}$, $N_{S''}$, $N_3$, $V^1$, $V^2$, $V^3$, $V^4$, $V^5$, $V^6$, $V^7$, $V^8$ are integer numbers.

### *6.2.5 The complex manufacturing supply chain involving reverse logistics with stochastic demand (Model-5)*

Objective function for this supply chain is the minimum of *TCChain* which considers stochastic demand. The MINLP formulation is listed as follows;

Min *ETCChain* = *ETCR* + *TCD* + *TCM* +*TCS'* + *TCS"* + *TC3* (Eq. (5.48) + Eq. (4.38) + Eq. (5.4) + Eq. (5.7) + Eq. (5.11) + Eq. (5.13))

s/t.

$$T > 0$$

$a_D, a_M, a_P, a3, a_{S'P}, a_{S'W}, a_{S''W} \ge 1$

where $a_D$, $a_M$, $a_P$, $a3$, $a_{S'P}$, $a_{S'W}$, $a_{S''W}$ are integer numbers

The formulations are listed above is to solve the problem of any number of players in each level of the supply chain and any number of raw material types, part types and finished product types. We use LINGO version 12 software package to solve mixed integer non-linear programming formulations above. In the next chapter we list how long it takes to run the models uses LINGO version 12 software package for scenarios studied.

## 6.3 Decentralised Decision Making Process

Unlike centralised decision making process, in decentralized one downstream level players dominate their immediate upstream level players in determining the optimal production and inventory cycles. Therefore, the optimal solution is determined by sequencing process from retailers to tier-2 suppliers. Detailed solution procedures for all scenarios above are as follows:

### *6.3.1 The complex manufacturing supply chain without involving reverse logistics (Model-1)*

1. All retailers determine their optimal common order cycle time, $T^*$ from Eq. (4.34) and subsequently their economic order quantity. Then, retailers' total cost, *TCR* is computed in Eq. (4.34).
2. $T^*$ which is obtained from Eq. (4.34) is subtituted to Eq. (4.38). Since the value of $a_D$ is relatively small Eq. (4.38) can be minimized by searching the optimal value of $a_D{}^*$ where $TCD(a_D{}^*\text{-}1) > TCD(a_D{}^*) < TCD(a_D{}^*+1)$. Distributors' total cost, *TCD* is computed in Eq. (4.38).
3. $T^*$ and $a_D{}^*$ which are obtained from Eq. (4.34) and Eq. (4.38) are subtituted to Eq. (4.41). Similarly with step 2 Eq. (4.41) is minimized by searching optimal values of $a_M{}^*$ and $a_P{}^*$ where $TCM((a_M{}^*+a_P{}^*)\text{-}1) > TCD(a_M{}^*+a_P{}^*) < TCD((a_M{}^*+a_P{}^*)+1)$. The manufacturer's total cost, *TCM* is computed in Eq. (4.41).
4. $T^*$, $a_D{}^*$, $a_M{}^*$ and $a_P{}^*$ which are obtained from Eq. (4.34), (4.38) and (4.41) are subtituted to Eq. (4.43). Again, Eq. (4.43) is minimized by searching optimal values of $a_{S'P}{}^*$ and $a_{S'W}{}^*$ where $TCS'\ ((a_{S'P}{}^*+a_{S'W}{}^*)\text{-}1) > TCS'\ (a_{S'P}{}^*+a_{S'W}{}^*) < TCS'\ ((a_{S'P}{}^*+a_{S'W}{}^*)+1)$. Tier-1 suppliers' total cost, *TCS'* is computed in Eq. (4.43).
5. $T^*$, $a_D{}^*$, $a_M{}^*$, $a_P{}^*$, $a_{S'P}{}^*$ $a_{S'W}{}^*$ which are obtained from Eq. (4.34), (4.38), (4.41) and (4.43) are subtituted to Eq. (4.47). Eq. (4.47) is minimised by searching the optimal value of $a_{S''W}{}^*$ where $TCS''(a_{S''W}{}^*\text{-}1) > TCS''(a_{S''W}{}^*) < TCD(a_{S''W}{}^*+1)$. Tier-2 suppliers' total cost, *TCS"* is computed in Eq. (4.47).

### *6.3.2 The complex manufacturing supply chain involving reverse logistics (Model-2)*

1. All retailers determine their optimal common order cycle time, $T^*$ from Eq. (4.34) and subsequently their economic order quantity. Then, retailers' total cost, *TCR* is computed in Eq. (4.34).
2. $T^*$ which is obtained from Eq. (4.34) is subtituted to Eq. (4.38). Since the value of $a_D$ is relatively small Eq. (4.38) can be minimized by searching the optimal value of $a_D{}^*$ where $TCD(a_D{}^*\text{-}1) > TCD(a_D{}^*) < TCD(a_D{}^*+1)$. Distributors' total cost, *TCD* is computed in Eq. (4.38).
3. $T^*$ and $a_D{}^*$ which are obtained from Eq. (4.34) and Eq. (4.38) are subtituted to Eq. (5.4). Similarly, Eq. (5.4) is minimized by searching optimal values of

$a_M$* and $a_P$* where $TCM((a_M*+a_P*)\text{-}1) > TCD(a_M*+a_P*) < TCD((a_M*+a_P*)+1)$. The manufacturer's total cost, *TCM* is computed in Eq. (5.4).

4. $T$*, $a_D$*, $a_M$* and $a_P$* which are obtained from Eq. (4.34), (4.38) and (5.4) are subtituted to Eq. (5.7). Again, Eq. (5.7) is minimized by searching optimal values of $a_{S'P}$* and $a_{S'W}$* where $TCS'((a_{S'P}*+a_{S'W}*)\text{-}1) > TCS'(a_{S'P}*+a_{S'W}*) < TCS'((a_{S'P}*+a_{S'W}*)+1)$ . Tier-1 suppliers' total cost, *TCS'* is computed in Eq. (5.7).
5. $T$*, $a_D$*, $a_M$*, $a_P$*, $a_{S'P}$* $a_{S'W}$* which are obtained from Eq. (4.34), (4.38), (5.4) and (5.7) are subtituted to Eq. (5.11). Eq. (5.11) is minimised by searching the optimal value of $a_{S''W}$* where $TCS''(a_{S''W}*\text{-}1) > TCS''(a_{S''W}*) < TCD(a_{S''W}*+1)$. Tier-2 suppliers' total cost, *TCS"* is computed in Eq. (5.11).
6. $T$*, $a_D$*, $a_M$* and $a_P$* are substituted to Eq. (5.13). Eq. (5.13) is minimised by searching the optimal value of *a3** where $TC3(a3*\text{-}1) > TC3(a3*) < TC3(a3*+1)$. The third party's total cost, *TC3* is computed in Eq. (5.13).

*6.3.3 The complex manufacturing supply chain involving reverse logistics with transportation cost considered (Model-3)*

1. All retailers determine their optimal common order cycle time, $T$* from Eq. (4.34) and subsequently their economic order quantity. Then, retailers' total cost, *TCR* is computed in Eq. (4.34).
2. $T$* which is obtained from Eq. (4.34) is subtituted to Eq. (5.18). Since the value of $a_D$ is relatively small Eq. (5.18) can be minimized by searching the optimal value of $a_D$* and $N_d$ where $TCD(a_D*\text{-}1) > TCD(a_D*) < TCD(a_D*+1)$ and they satisfy Eq. (5.17) as a constraint of Eq (5.18). Distributors' total cost, *TCD* is computed in Eq. (5.18).
3. $T$* and $a_D$* which are obtained from Eq. (4.34) and Eq. (5.18) are subtituted to Eq. (5.22). Similarly, Eq. (5.22) is minimized by searching optimal values of $a_M$*, $a_P$* and $N_M$ where $TCM((a_M*+a_P*)\text{-}1) > TCD(a_M*+a_P*) < TCD((a_M*+a_P*)+1)$ and they satisfy Eq. (5.20) as a constraint of Eq. (5.22). The manufacturer's total cost, *TCM* is computed in Eq. (5.22).
4. $T$*, $a_D$*, $a_M$* and $a_P$* which are obtained from Eq. (4.34), (5.18) and (5.22) are subtituted to Eq. (5.28). Again, Eq. (5.28) is minimized by searching optimal values of $a_{S'P}$*, $a_{S'W}$* and $N_{S'}$ where $TCS'((a_{S'P}*+a_{S'W}*)\text{-}1) > TCS'(a_{S'P}*+a_{S'W}*) < TCS'((a_{S'P}*+a_{S'W}*)+1)$ and they satisfy Eq. (5.26) as a constraint of Eq. (5.28). Tier-1 suppliers' total cost, *TCS'* is computed in Eq. (5.28).
5. $T$*, $a_D$*, $a_M$*, $a_P$*, $a_{S'P}$* $a_{S'W}$* which are obtained from Eq. (4.34), (5.18), (5.22) and (5.28) are subtituted to Eq. (5.34). Eq. (5.34) is minimised by searching

the optimal value of $a_{S''W}$* and $N_{S''}$ where $TCS''(a_{S''W}*-1) > TCS''(a_{S''W}*) < TCD(a_{S''W}*+1)$ and they satisfy Eq. (5.32) as a constraint of Eq. (5.34). Tier-2 suppliers' total cost, $TCS''$ is computed in Eq. (5.34).

6. $T$*, $a_D$*, $a_M$* and $a_P$* are substituted to Eq. (5.37). Eq. (5.37) is minimised by searching the optimal value of $a3$* and $N3$ where $TC3(a3*-1) > TC3(a3*) < TC3(a3*+1)$ and they satisfy Eq. (5.36) as a constraint of Eq. (5.37). The third party's total cost, $TC3$ is computed in Eq. (5.37).

*6.3.4 The complex manufacturing supply chain involving reverse logistics with finite horizon period (Model-4)*

1. All retailers determine their initial optimal common order cycle time $T^0$ in (4.40) where $0 < T^0 \leq N$ (the length of horizon period). If $T^0$ found in (4.34) > $N$, then $T^0 = N$.
2. Calculate noninteger number, $V1^0$, in (5.38) by substituting $T^0$.
3. Then, the integers $V1$ and $V1 - 1$ are calculated, where $V1$ is the nearest integer $\geq V1^0$.
4. Substitute $V1$ and $V1 - 1$ to (5.38) to result two candidates of decision variables of $T$, $T_A$ and $T_B$.
5. Compute $TCR(T_A)$ and $TCR(T_B)$ in (4.34).
6. If $TCR(T_A) < TCR(T_B)$ then $T_A = T^*$, otherwise if $TCR(T_B) < TCR(T_A)$ then $T_B = T^*$.
7. $T$* which is obtained in step 6 is subtituted to Eq. (4.38). Eq. (4.38) can be minimized by searching the optimal value of $a_D$* where $TCD(a_D*-1) > TCD(a_D*) < TCD(a_D*+1)$ and it satisfies Eq. (5.39) as a constraint of Eq. (4.43). Distributors' total cost, $TCD$ is computed in Eq. (4.38).
8. $T$* and $a_D$* are subtituted to Eq. (5.4). Similarly, Eq. (5.4) is minimized by searching optimal values of $a_M$* and $a_P$* where $TCM((a_M*+a_P*)-1) > TCD(a_M*+a_P*) < TCD((a_M*+a_P*)+1)$ and they satisfy Eq. (5.40) and (5.41) as constraints of Eq. (5.4). The manufacturer's total cost, $TCM$ is computed in Eq. (5.4).
9. $T$*, $a_D$*, $a_M$* and $a_P$* are subtituted to Eq. (5.7). Again, Eq. (5.7) is minimized by searching optimal values of $a_{S'P}$* and $a_{S'W}$* where $TCS'((a_{S'P}*+a_{S'W}*)-1) > TCS'(a_{S'P}*+a_{S'W}*) < TCS'((a_{S'P}*+a_{S'W}*)+1)$ and they satisfy Eq. (5.42) and (5.43) as constraints of Eq. (5.7). Tier-1 suppliers' total cost, $TCS'$ is computed in Eq. (5.7).
10. $T$*, $a_D$*, $a_M$*, $a_P$*, $a_{S'P}$* $a_{S'W}$* are subtituted to Eq. (5.11). Eq. (5.11) is minimised by searching the optimal value of $a_{S''W}$* where $TCS''(a_{S''W}*-1) >$

$TCS''(a_{S''W}*) < TCD(a_{S''W}*+1)$ and it satisfies Eq. (5.44) as a constraint of Eq. (5.11). Tier-2 suppliers' total cost, $TCS''$ is computed in Eq. (5.11).

11. $T*$, $a_D*$, $a_M*$ and $a_P*$ are substituted to Eq. (5.13). Eq. (5.13) is minimised by searching the optimal value of $a3*$ where $TC3(a3*-1) > TC3(a3*) < TC3(a3*+1)$ and it satisfies Eq. (5.45) as a constraint of Eq. (5.13). The third party's total cost, $TC3$ is computed in Eq. (5.13).

*6.3.5 The complex manufacturing supply chain involving reverse logistics with stochastic demand (Model-5)*

The solution method for this model is similar with model-2. Exemption is only for step 1. In this model, we use Eq. (5.48) for step 1.

## 6.4 Semi-centralised Decision Making Process

The semi-centralized decision-making process is a combination of decentralized and centralized decision-making process. This solution method is applied if there are some players in the supply chain that dominate some others in determining production and inventory cycles. In real cases this solution method is more practicable than other ones. General solution method for semi-centralised decision making process can be described as follow:

1. The first sub-chain determines the optimal solution under centralised decision making process.
2. Other players in the supply chain determine their optimal solutions under decentralised decision making process.

The specific solution procedure for this method depends on which players are included in a sub-chain and which players are dominant to others. In this work, we set a scenario where retailers and distributors to be first sub-chain and the manufacturer, tier-1 suppliers, tier-2 suppliers and the third party to be second sub-chain. The solution method for this scenario is as follow:

1. Distributors and retailers (the first sub-chain) determine their optimal production and inventory cycles simultaneously using mixed integer non-linear programming method.
2. The manufacturer, tier-1 suppliers, tier-2 suppliers and the third party (the second sub-chain) will use the optimal decision variables of the first sub-chain to find their optimal solution together.
3. The supply chain's total cost is computed by summing the first sub-chain's total cost and the second sub-chain's total cost.

## 6.5 Summary

This chapter provides the solution methods proposed to solve the models developed in chapter 4 and 5. The solution methods are proposed based on centralised and decentralised decision making process. Under centralised decision making process, all players determine all decision variables simultaneously with using mixed integer nonlinear programming (MINLP) method. Under decentralised decision making process downstream-level players dominate their immediate upstream-level players in determining their decision variables. Therefore, the optimal solution is determined by sequencing process from retailers to tier-2 suppliers.

Finally, semi-centralised decision making process is proposed as an alternative solution method of the model. This method is a combination of decentralized and centralized decision-making process. This solution method is applied if there are only some players in the supply chain that dominate some others in determining their optimal solutions. In real cases this solution method may be more practicable than other ones.

# Chapter 7

# Analysis and Discussion

## 7.1 Introduction

In this chapter, we use numerical examples to test, analyse and discuss models developed in chapter 4 and 5 with solution methods presented in chapter 6. In section 7.2, numerical examples are used to illustrate how the models work. Analysis of results of numerical examples and discussions about the results and models are provided in section 7.3. Section 7.4 summarises the chapter.

## 7.2 Numerical Examples

In order to test models developed in this thesis, two numerical examples are solved. Example 1 consists of four retailers ($r$ = 1, 2, 3, 4), two distributors ($d$ = 1, 2) , a manufacturer, two tier-1 suppliers ($s'$ = 1, 2) , two tier-2 suppliers ($s''$ = 1, 2) and a third party. Example 2 consists of eight retailers ($r$ = 1, 2, 3,... , 8), two distributors ($d$ = 1, 2) , a manufacturer, four tier-1 suppliers ($s'$ = 1, 2, 3, 4) , two tier-2 suppliers ($s''$ = 1, 2) and a third party. Both examples have two types of finished products ($i$ = 1, 2). The values of the input parameters for example 1 are partly adopted from the literature (Jaber and Goyal, 2008). These two numerical examples are used to test models described in chapter 6: the complex manufacturing supply chain system, the system involving reverse logistics, the system involving reverse logistics with transportation cost considered and the system involving reverse logistics with finite horizon period. The detailed data for these numerical examples are as follows:

Tabel 7.1 Input parameters of retailers and distributors

**Example 1**

| $r$ | $A^{(r)}_r$ | $h^{(r)}_{r,i}$ | | $D^{(r)}_{r,i}$ | |
|---|---|---|---|---|---|
| | | $i=1$ | $i=2$ | $i=1$ | $i=2$ |
| 1 | 3 | 16.00 | 15.00 | 100,000 | 75,000 |
| 2 | 4 | 14.00 | 15.00 | 75,000 | 100,000 |
| 3 | 5 | 12.00 | 13.00 | 50,000 | 75,000 |
| 4 | 4 | 13.00 | 14.00 | 75,000 | 100,000 |

| $d$ | $A^{(d)}_d$ | $B^{(d)}_d$ | $h^{(d)}_{d,i}$ | | $D^{(d)}_{d,i}$ | |
|---|---|---|---|---|---|---|
| | | | $i=1$ | $i=2$ | $i=1$ | $i=2$ |
| 1 | 2.5 | 1 | 14.00 | 13.00 | 175,000 | 175,000 |
| 2 | 3 | 2 | 13.00 | 13.00 | 125,000 | 175,000 |

**Example 2**

| $r$ | $A^{(r)}_r$ | $h^{(r)}_{r,i}$ | | $D^{(r)}_{r,i}$ | |
|---|---|---|---|---|---|
| | | $i=1$ | $i=2$ | $i=1$ | $i=2$ |
| 1 | 30 | 160.00 | 150.00 | 10,000 | 7,500 |
| 2 | 40 | 140.00 | 150.00 | 7,500 | 10,000 |
| 3 | 50 | 120.00 | 130.00 | 5,000 | 7,500 |
| 4 | 40 | 130.00 | 140.00 | 7,500 | 10,000 |
| 5 | 40 | 150.00 | 160.00 | 8,000 | 8,000 |
| 6 | 35 | 140.00 | 170.00 | 10,000 | 6,000 |
| 7 | 45 | 145.00 | 160.00 | 8,000 | 6,500 |
| 8 | 55 | 155.00 | 150.00 | 7,000 | 9,000 |

| $d$ | $A^{(d)}_d$ | $B^{(d)}_d$ | $h^{(d)}_{d,i}$ | | $D^{(d)}_{d,i}$ | |
|---|---|---|---|---|---|---|
| | | | $i=1$ | $i=2$ | $i=1$ | $i=2$ |
| 1 | 15 | 20 | 130.00 | 120.00 | 30,000 | 35,000 |
| 2 | 20 | 30 | 135.00 | 125.00 | 33,000 | 31,500 |

In example 1, for the manufacturer, the setup costs for products are $s^{(m)}_1$ = 0.75, $s^{(m)}_2$ = 1.0, order processing cost to deliver products to distributors $b^{(m)}_1$ = 0.25, $b^{(m)}_2$ = 0.5, the holding costs for products $h_1$ = 8, $h_2$ = 7, and the production rates for products $P^{(m)}_1$ = 500,000 and $P^{(m)}_2$ = 600,000. In example 2, the setup costs for products are $s^{(m)}_1$ = 5, $s^{(m)}_2$ = 7.5, order processing cost to deliver products to distributors $b^{(m)}_1$ = 5, $b^{(m)}_2$ = 7.5, the holding costs for products $h_1$ = 18, $h_2$ = 17, and production rates for products $P^{(m)}_1$ = 100,000 and $P^{(m)}_2$ = 200,000. Each product 1 or 2 requires 3 types of parts ($p$ = 1, 2, 3). Each part requires two types of raw materials ($w$ = 1, 2). The details of input parameters of parts are as follows:

Tabel 7.2 Input parameters for parts

**Example 1**

| $p$ | $B^{(I)}_{p,i}$ | | $B^{(II)}_{p,w}$ | | $e^{(s')}_{s',p}$ | | $u_{p',i}$ | | $h^{(m')}_{p'}$ | $h^{(m)}_p$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | $i=1$ | $i=2$ | $w=1$ | $w=2$ | $s'=1$ | $s'=2$ | $i=1$ | $i=2$ | | |
| 1 | 3 | 2 | 2 | 1 | 0.5 | 0.5 | 0.4 | 0.4 | 0,06 | 0.208 |
| 2 | 2 | 1 | 2 | 2 | 0.5 | 0.5 | 0.4 | 0.4 | 0.08 | 0.416 |
| 3 | 2 | 1 | 1 | 1 | 0.4 | 0.6 | 0.4 | 0.4 | 0.06 | 0.250 |

**Example 2**

| $p$ | $B^{(I)}_{p,i}$ | | $B^{(II)}_{p,w}$ | | $e^{(s')}_{s',p}$ | | | | $u_{p',i}$ | | $h^{(m')}_{p'}$ | $h^{(m)}_p$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $i=1$ | $i=2$ | $w=1$ | $w=2$ | $s'=1$ | $s'=2$ | $s'=3$ | $s'=4$ | $i=1$ | $i=2$ | | |
| 1 | 4 | 2 | 2 | 4 | 0.2 | 0.3 | 0.3 | 0.2 | 0.3 | 0.3 | 0.6 | 2,08 |
| 2 | 3 | 4 | 2 | 4 | 0.3 | 0.2 | 0.2 | 0.3 | 0.3 | 0.3 | 0.8 | 4,16 |
| 3 | 5 | 2 | 2 | 2 | 0.2 | 0.3 | 0.3 | 0.2 | 0.3 | 0.3 | 0.8 | 2,50 |

In example 1, the cost of ordering and placing an order for each part $p$ and returned part p' are $A_M$ = 3, $a^{(m)}_p$ = 1 ($p$ = 1, 2, 3) and $a^{(m')}_{p'}$ = 0.3. In example 2 these are $A_M = 8$, $a^{(m)}{}_p = 2.5$ ($p$ = 1, 2, 3) and $a^{(m')}{}_{p'} = 1$, respectively. The three parts are supplied by two tier-1 suppliers who have the following parameters.

Tabel 7.3 Input parameters of tier-1 suppliers

| $p$ | Example 1 | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $s^{(s')}{}_{s',p}$ | | $b^{(s')}{}_{s',p}$ | | $P_{s',p}$ | | $h_{s',p}$ | |
| | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ |
| 1 | 1.5 | 1.5 | 0.5 | 0.5 | 1,300,000 | 1,400,000 | 0.10 | 0.15 |
| 2 | 1.5 | 0.75 | 0.5 | 0.25 | 800,000 | 700,000 | 0.20 | 0.18 |
| 3 | 0.75 | 0.75 | 0.25 | 0.25 | 1,000,000 | 1,200,000 | 0.12 | 0.15 |

| $p$ | Example 2 | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $s^{(s')}{}_{s',p}$ | | | | $b^{(s')}{}_{s',p}$ | | | | $P^{(s')}{}_{s',p}$ | | | | $h_{s',p}$ | | | |
| | $s'=1$ | $s'=2$ | $s'=3$ | $s'=4$ | $s'=1$ | $s'=2$ | $s'=3$ | $s'=4$ | $s'=1$ | $s'=2$ | $s'=3$ | $s'=4$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ |
| 1 | 6 | 7.5 | 7 | 8 | 6 | 7.5 | 7 | 8 | 500,000 | 700,000 | 500,000 | 400,000 | 1.0 | 1.5 | 1.5 | 2.0 |
| 2 | 5 | 7.5 | 8 | 9 | 5 | 7.5 | 8 | 9 | 400,000 | 800,000 | 600,000 | 600,000 | 2.5 | 1.7 | 2.0 | 2.5 |
| 3 | 7.5 | 10 | 8 | 10 | 7.5 | 10 | 8 | 10 | 600,000 | 600,000 | 400,000 | 400,000 | 1.2 | 1.5 | 1.0 | 1.5 |

Input parameters for raw materials are listed as follows:

Tabel 7.4 Input parameters of raw materials

| $w$ | Example 1 | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $e^{(s'')}{}_{s'',w}$ | | $A^{(s')}{}_{s'}$ | | $a^{(s')}{}_{s',w}$ | | $h^{(s')}{}_{s',w}$ | |
| | $s''=1$ | $s''=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ |
| 1 | 0.4 | 0.6 | | | 1 | 0.5 | 0.05 | 0.05 |
| 2 | 0.5 | 0.5 | 2 | 1.5 | 1 | 0.5 | 0.05 | 0.06 |

| $w$ | Example 2 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $e^{(s'')}{}_{s'',w}$ | | $A^{(s')}{}_{s'}$ | | | | $a^{(s')}{}_{s',w}$ | | | | $h^{(s')}{}_{s',w}$ | | | |
| | $s''=1$ | $s''=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ | $s'=1$ | $s'=2$ |
| 1 | 0.5 | 0.5 | 6 | 5 | 5 | 8 | 5 | 4 | 4 | 5 | 0.5 | 0.6 | 0.8 | 1 |
| 2 | 0.5 | 0.5 | | | | | 4 | 6 | 3 | 4 | 0.5 | 0.5 | 0.7 | 1.2 |

The raw materials are supplied by two tier-2 suppliers who have the following input parameters:

Tabel 7.5 Input parameters for tier-2 suppliers

| $s''$ | Example 1 | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $s^{(s'')}{}_{s'',w}$ | | $b^{(s'')}{}_{s'',w}$ | | $h^{(s'')}{}_{s'',w}$ | | $P^{(s'')}{}_{s'',w}$ | |
| | $w=1$ | $w=2$ | $w=1$ | $w=2$ | $w=1$ | $w=2$ | $w=1$ | $w=2$ |
| 1 | 1.5 | 1.5 | 0.5 | 0.5 | 0.03 | 0.035 | 5,000,000 | 7,000,000 |
| 2 | 1.25 | 1.5 | 0.25 | 0.5 | 0.03 | 0.035 | 6,000,000 | 8,000,000 |
| $s''$ | Example 2 | | | | | | | |
| | $s^{(s'')}{}_{s'',w}$ | | $b^{(s'')}{}_{s'',w}$ | | $h^{(s'')}{}_{s'',w}$ | | $P^{(s'')}{}_{s'',w}$ | |
| | $w=1$ | $w=2$ | $w=1$ | $w=2$ | $w=1$ | $w=2$ | $w=1$ | $w=2$ |
| 1 | 8 | 10 | 4 | 5 | 0.5 | 0.6 | 400,000 | 500,000 |
| 2 | 10 | 10 | 5 | 5 | 0.5 | 0.6 | 500,000 | 400,000 |

For the third party, in example 1, the setup cost for collecting all used products per cycle time, $S^{(3)} = 1$, the cost for processing each returned used product, $s^{(3)}{}_1 = 0.25$, $s^{(3)}{}_2 = 0.25$, order processing cost to deliver returned parts to the manufacturer $B^{(3)} = 0.5$, $b^{(3)}{}_1 = 0.25$, $b^{(3)}{}_2 = 0.25$, return rate of each used product, $R_1 = 150{,}000$, $R_2 = 200{,}000$, collecting rate for each used product, $C_1 = 500{,}000$ $C_2 = 600{,}000$, and holding cost for each returned used product, $h^{(3)}{}_1 = 0.02$ and $h^{(3)}{}_2 =$

0.02. In example 2, these are: $S^{(3)} = 7.5$, $s^{(3)}_1 = 2$, $s^{(3)}_2 = 2.5$, $B^{(3)} = 7.5$, $b^{(3)}_1 = 1$, $b^{(3)}_2 = 1.5$, $R_1 = 30{,}000$, $R_2 = 35{,}000$, $C_1 = 100{,}000$ $C_2 = 120{,}000$, $h^{(3)}_1 = 0.4$ and $h^{(3)}_2 = 0.5$

The capacity of delivery unit and the sizes of items are as follows:

Tabel 7.6 Input parameters of delivery unit and items

| **Example 1** | $L$ | $W$ | $H$ | $g$ | **Example 2** | $L$ | $W$ | $H$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|
| Transportation unit ($F$) | 250 | 100 | 100 | | Transportation unit($F$) | 250 | 100 | 100 | |
| Products ($i$): | | | | | Products ($i$): | | | | |
| 1 | 15 | 10 | 5 | 1 | 1 | 20 | 10 | 10 | 2 |
| 2 | 10 | 7.5 | 5 | 1 | 2 | 15 | 7.5 | 10 | 2 |
| Parts ($p$): | | | | | Parts ($p$): | | | | |
| 1 | 5 | 4 | 1 | 1 | 1 | 8 | 4 | 5 | 1 |
| 2 | 3 | 3 | 1.5 | 1 | 2 | 5 | 6 | 5 | 1 |
| 3 | 5 | 3 | 1 | 1 | 3 | 5 | 8 | 5 | 1 |
| Raw materials ($w$): | | | | | Raw materials ($w$): | | | | |
| 1 | 10 | 5 | 1 | 1 | 1 | 15 | 5 | 2 | 1 |
| 2 | 10 | 10 | 1 | 1 | 2 | 20 | 10 | 2 | 1 |

Fixed transportation cost ($F$) per unit delivery and fixed transportation cost (V) per cycle time and the capacity allowances minimum ($a_d$min, $a_M$min, $a_{S'}$min, $a_{S''}$min, $a_3$min) are 20, 5 and 0.04 in example 1 and 50, 10 and 0.5 in example 2 respectively.

For stochastic demand case, the input parameters are as follows:

Tabel 7.7 Input parameters of standard deviations and backorder costs

| **Example 1** | | | |
|---|---|---|---|
| $\sigma_{r,i}$ | | $C_{r,i}$ | |
| $i = 1$ | $i = 2$ | $i = 1$ | $i = 2$ |
| 100 | 75 | 100 | 120 |
| 75 | 100 | 110 | 125 |
| 50 | 75 | 105 | 130 |
| 75 | 100 | 110 | 125 |
| **Example 2** | | | |
| $\sigma_{r,i}$ | | $C_{r,i}$ | |
| $i = 1$ | $i = 2$ | $i = 1$ | $i = 2$ |
| 100 | 75 | 800 | 750 |
| 75 | 100 | 700 | 750 |
| 50 | 75 | 600 | 650 |
| 75 | 100 | 650 | 700 |
| 80 | 80 | 750 | 800 |
| 100 | 60 | 700 | 850 |
| 80 | 65 | 705 | 800 |
| 70 | 90 | 755 | 750 |

In two numerical examples, we use service level = 95.05 % so that $k = 1.65$, $F_s(k) = 0.9505$, and $f_s(k) = 0.102265$.

Results of numerical examples for all models are listed in Table 7.8, 7.9, 7.10 and 7.11 below.

Tabel 7.8 Results of numerical examples for the complex manufacturing supply chain (Model-1)

| Decision variables | Example 1 | | | Example 2 | | |
|---|---|---|---|---|---|---|
| | Decentralized | Semi-centralized | Centralized | Decentralized | Semi-centralized | Centralized |
| $T^*$ | 0.001862481 | 0.002304702 | 0.002225485 | 0.005976143 | 0.006691496 | 0.006763413 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 2 | 2 | 2 | 1 | 2 | 2 |
| $a_{S'P}^*$ | 1 | 1 | 1 | 2 | 1 | 1 |
| $a_{S'W}^*$ | 2 | 2 | 2 | 1 | 1 | 1 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| Objective functions | | | | | | |
| *TCR* | 17181.39 | 17572.77 | 17454.49 | 112112.4 | 112829.8 | 112972.0 |
| *TCD* | 4563.805 | 3688.112 | 3819.391 | 14223.22 | 12702.69 | 12567.62 |
| *TCM* | 7069.392 | 7480.138 | 7390.003 | 16231.26 | 25650.02 | 25821.05 |
| *TCS'* | 4740.596 | 4454.333 | 4487.175 | 36504.72 | 21634.94 | 21474.45 |
| *TCS"* | 1137.507 | 975.3927 | 998.7886 | 6211.864 | 5874.757 | 5846.832 |
| *TCChain* | 34692.69 | 34170.74 | 34149.85 | 185283.5 | 178692.21 | 178682.0 |

Tabel 7.9 Results of numerical examples for the complex manufacturing supply chain involving reverse logistics (Model-2)

| Decision variables | Example 1 | | | Example 2 | | |
|---|---|---|---|---|---|---|
| | Decentralized | Semi-centralized | Centralized | Decentralized | Semi-centralized | Centralized |
| $T^*$ | 0.001862481 | 0.002304702 | 0.002183319 | 0.005976143 | 0.006691496 | 0.006923409 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 2 | 1 |
| $a_P^*$ | 2 | 3 | 3 | 1 | 1 | 2 |
| $a_{S'P}^*$ | 2 | 1 | 1 | 2 | 1 | 1 |
| $a_{S'W}^*$ | 1 | 2 | 2 | 1 | 2 | 2 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a3^*$ | 1 | 3 | 3 | 4 | 2 | 2 |
| Objective functions | | | | | | |
| *TCR* | 17181.39 | 17572.77 | 17398.85 | 112112.40 | 112829.8 | 113328.1 |
| *TCD* | 4563.805 | 3688.112 | 3893.155 | 14223.22 | 12702.69 | 12277.19 |
| *TCM* | 6606.937 | 7078.283 | 6930.966 | 11934.36 | 17498.81 | 17778.12 |
| *TCS'* | 3626.136 | 2973.102 | 3008.330 | 33464.52 | 20743.17 | 20359.53 |
| *TCS"* | 1052.804 | 629.7623 | 655.3129 | 5474.984 | 3712.876 | 3696.416 |
| *TC3* | 421.9440 | 288.5088 | 296.8012 | 2543.795 | 1509.434 | 1480.266 |
| *TCChain* | 33453.02 | 32230.54 | 32183.42 | 179736.28 | 168996.79 | 168919.7 |

Tabel 7.10 Results of numerical examples for the complex manufacturing supply chain involving reverse logistics with transportation cost considered (Model-3)

| Decision variables | Example 1 | | | Example 2 | | |
|---|---|---|---|---|---|---|
| | Decentralized | Semi-centralized | Centralized | Decentralized | Semi-centralized | Centralized |
| $T^*$ | 0.001862481 | 0.003937176 | 0.004143223 | 0.005976143 | 0.007227642 | 0.008036518 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 2 | 4 | 4 | 1 | 2 | 2 |
| $a_{S'P}^*$ | 2 | 1 | 1 | 2 | 1 | 1 |
| $a_{S'W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a3^*$ | 5 | 1 | 1 | 4 | 2 | 2 |
| $N_d$ | 1,1 | 1,1 | 1,1 | 1,1 | 1,1 | 1,1 |
| $N_M$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $N_{S'}$ | 1 | 1 | 1 | 1 | 2 | 2 |
| $N_{S''}$ | 1 | 2 | 2 | 2 | 2 | 2 |
| $N3$ | 1 | 1 | 1 | 1 | 1 | 1 |
| Objective functions | | | | | | |
| *TCR* | 17181.39 | 22224.05 | 22972.34 | 112112.40 | 114145.1 | 117067.3 |
| *TCD* | 29798.96 | 14096.40 | 13395.37 | 25936.46 | 21445.45 | 19286.96 |
| *TCM* | 19627.20 | 17087.14 | 17243.20 | 19882.63 | 24705.72 | 25066.38 |
| *TCS'* | 9733.581 | 3849.032 | 3802.574 | 27858.89 | 19881.34 | 18246.30 |
| *TCS"* | 4174.642 | 3322.017 | 3166.969 | 13089.59 | 11093.55 | 10158.75 |
| *TC3* | 6864.963 | 1700.264 | 1617.503 | 10910.40 | 4904.841 | 4483.236 |
| *TCChain* | 87380.74 | 62278.9 | 62197.96 | 209790.37 | 196176.06 | 194308.9 |

Tabel 7.11 Results of numerical examples for the complex manufacturing supply chain involving reverse logistics with finite horizon period (N=1) (Model-4)

| Decision variables | **Example 1** | | | **Example 2** | | |
|---|---|---|---|---|---|---|
| | Decentralized | Semi-centralized | Centralized | Decentralized | Semi-centralized | Centralized |
| $T^*$ | 0.001862197 | 0.002304147 | 0.002222222 | 0.005988024 | 0.006711409 | 0.006944444 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 1 | 2 | 3 | 1 | 1 | 2 |
| $a_{SP}^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_{S'W}^*$ | 1 | 7 | 2 | 1 | 1 | 2 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a3^*$ | 1 | 7 | 3 | 1 | 1 | 2 |
| Objective functions | | | | | | |
| *TCR* | 17181.39 | 17571.88 | 17450.00 | 112112.70 | 112868.0 | 113378.9 |
| *TCD* | 4564.501 | 3689.000 | 3825.000 | 14195.00 | 12665.00 | 12240.00 |
| *TCM* | 7834.880 | 6803.757 | 6976.804 | 11929.18 | 11709.69 | 17802.12 |
| *TCS'* | 8491.440 | 4880.849 | 2995.868 | 41795.29 | 37453.46 | 20326.52 |
| *TCS"* | 4039.270 | 436.3674 | 646.7652 | 9873.214 | 8890.005 | 3695.158 |
| *TC3* | 1344.580 | 396.2957 | 294.0000 | 3700.060 | 3307.209 | 1477.752 |
| *TCChain* | 43456.06 | 33778.15 | 32188.44 | 193605.44 | 186893.36 | 168920.4 |

Tabel 7.12 Results of numerical examples for the complex manufacturing supply chain involving reverse logistics with stochastic demand (Model-5)

| Decision variables | **Example 1** | | | **Example 2** | | |
|---|---|---|---|---|---|---|
| | Decentralized | Semi-centralized | Centralized | Decentralized | Semi-centralized | Centralized |
| $T^*$ | 0.003643935 | 0.003933649 | 0.003496220 | 0.01116625 | 0.01160605 | 0.01218014 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 1 | 2 | 2 | 1 | 1 | 1 |
| $a_{SP}^*$ | 2 | 1 | 1 | 1 | 1 | 1 |
| $a_{S'W}^*$ | 1 | 2 | 2 | 2 | 2 | 2 |
| $a_{S''W}^*$ | 3 | 1 | 1 | 1 | 1 | 1 |
| $a3^*$ | 5 | 2 | 3 | 2 | 2 | 2 |
| Objective functions | | | | | | |
| *TCR* | 47869.76 | 47962.03 | 47893.51 | 349635.5 | 349776.7 | 350353.8 |
| *TCD* | 2332.643 | 2160.843 | 2431.197 | 7612.224 | 7323.769 | 6978.574 |
| *TCM* | 8032.367 | 8719.475 | 8107.303 | 12594.19 | 12789.17 | 13059.73 |
| *TCS'* | 3649.065 | 2924.062 | 2966.865 | 23198.41 | 22613.74 | 21930.38 |
| *TCS"* | 1452.108 | 576.0976 | 624.6764 | 3873.384 | 3828.697 | 3780.875 |
| *TC3* | 428.176 | 267.5456 | 286.8892 | 1694.784 | 1650.802 | 1599.289 |
| *TCChain* | 63335.94 | 62600.05 | 62310.44 | 398608.49 | 397982.81 | 397702.6 |

For centralised decision making process, we use LINGO version 12 software package to solve the models. For model-1, it takes around 18 seconds to run the model for example 1 and around 29 seconds for example 2. For model-2, it takes around 38 seconds to run the model for example 1 and around 71 seconds for example 2. For model-3, it takes around 30 seconds to run the model for example 1 and around 65 seconds for example 2. For model-4, it takes around 70 seconds to run the model for example 1 and around 131 seconds for example 2. For model-5, it takes around 55 seconds to run the model for example 1 and around 90 seconds for example 2. Based on computational times above we can conclude that the computational time increases when the model size increases as shown in example 1 and 2 and the computational time increase when the complexity of the model increase as shown in model-1, model-2, and so on.

## 7.3 Analysis and Discussion

Firstly, we analyse all models mentioned in chapter 6. From Table 7.8 to 7.12 above we can see that each player can be better off with one decision process while doing less well with another decision process. Since there are no dominant players in determining production and inventory cycles under the centralised decision making process, which players benefits more depends on how much costs are incurred by each players. For these two numerical examples, the centralised decision making process is more favourable for distributors, tier-1 suppliers, tier-2 suppliers and the third party because these players can reduce inventory holding cost caused by large stock for anticipating demand from retailers when independent and other coordinated policies. For retailers and the manufacturer, decentralized decision-making process is the best. Under decentralized decision-making process, retailers are dominant players to distributors. Retailers determine their own optimal common order cycle time first and then distributors will follow the cycle time so that total cost for retailers will be the minimum.

Under semi-centralised decision masking process, results are varied. Since semi-centralised decision making process is a combination of decentralised and centralised decision making process, the results of the system depend on which players dominate other players so that optimal solutions for each scenario vary. In this work, we set the scenario where distributors and retailers, named as first sub-chain, dominate another sub-chain in the supply chain. Distributors and retailers determine their optimal decision variables simultaneously and their solutions are used by another sub-chain to obtain its optimal solutions. Another sub-chain, named as second sub-chain, consists of the manufacturer, tier-1 suppliers, tier-2 suppliers and third party. There are no dominant players in the second sub-chain so that they determine their optimal solutions simultaneously too. Therefore, distributors and retailers apply centralised decision making process to obtain their optimal solutions. Then, the manufacturer, tier-1 suppliers, tier-2 suppliers and third party also apply centralised one to obtain their decisions. But, the second sub-chain applies decentralised decision making process since it uses the optimal solution of the first sub-chain to obtain its optimal solutions. Hence, for the scenario above the whole supply chain's total cost for the semi-centralised decision making process is lower than the whole supply chain's total cost for the decentralised one.

The costs with semi-centralised decision process appear to be higher than that under decentralised decision process for retailers and the manufacturer. For retailers, since they determine their own optimal solutions by their own model under decentralised decision making process decentralised policy is the best policy for them. Furthermore, since the manufacturer uses the optimal solutions of the first sub-chain and solves its optimal solutions simultaneously with other players in the second sub-chain the manufacturer's total cost under semi-centralised decision making process may be higher than decentralised one.

Overall, the centralized decision-making process is the best amongst all strategies and the coordinated policy is better than independent policy (Sarmah et al., 2006; Chan and Lee, 2010) as shown in Table 7.13. We use the model for the complex manufacturing supply chain system (Model-1) to compare coordinated policy with independent one. As shown in Table 7.13, not all players gain benefits by changing from independent decision strategy to a coordinated decision strategy. Retailers will bear higher costs in a coordinated situation than in an independent situation. In independent policy each retailer determines its optimal solution by its own model so that the total cost of each retailer is minimum whereas in coordinated policy each retailer determines its optimal solution simultaneously with other retailers and other players in the supply chain so that their optimal solutions may be the same or higher than independent policy.

Tabel 7.13 Comparison between independent and coordinated policy

| Objective functions | Independent Policy | Coordinated Policy | | |
|---|---|---|---|---|
| | | Decentralized | Semi-centralized | Centralized |
| *TCR* | 16887.68 | 17181.39 | 18398.90 | 17454.49 |
| *TCD* | 27929.08 | 4563.805 | 6635.488 | 3819.391 |
| *TCM* | 13002.87 | 7069.392 | 7190.548 | 7390.003 |
| *TCS'* | 8243.97 | 4740.596 | 4388.023 | 4487.175 |
| *TCS"* | 1520.60 | 1137.507 | 911.7602 | 998.7886 |
| *TCChain* | 67584.2 | 34692.69 | 37524.72 | 34149.85 |

Furthermore, we analyse each model developed and its relationship to other ones. The model of the complex manufacturing supply chain system (model-1) is a generalised model for multi-level supply chain. It means this model can be also applied to a smaller system with a smaller number of players such as a buyer-a vendor coordination, a three-level supply chain with multiple players and so on. To prove it, we use this model to solve the problem addressed in Jaber and Goyal (2008). Since they didn't consider the production rate in the model we set the production rate to close to infinite (big number). Also, we set other cost parameters as zero and integer variables $a_D = a_{S'W} = a_{S''W} = 1$ for players which are not involved in the supply chain. Results of the problem are shown in Table 7.14 below.

Table 7.14 Result of the problem addressed in Jaber and Goyal (2008) with coordination

| Decision variables | | | Objective functions | | | | | |
|---|---|---|---|---|---|---|---|---|
| $T^*$ | $a_M^*$ ($\lambda_v^*$) | $a_{SP}^*$ ($\lambda_s^*$) | $TCR$ ($\hat{C}_b$) | | | $TCM$ ($\hat{C}_v$) | $TCS'$ ($\hat{C}_s$) | |
| | | | Buyer 1 ($\hat{C}_{b=1}$) | Buyer 2($\hat{C}_{b=2}$) | Buyer 3($\hat{C}_{b=3}$) | | Supplier 1 ($\hat{C}_{s=1}$) | Supplier 2 ($\hat{C}_{s=2}$) |
| 0.018846 | 1 | 2 | 16669 | 12547 | 9368 | 13265 | 16337 | 11404 |
| | | | $TCR$ ($\hat{C}_b$) = 38584 | | | | $TCS'$ ($\hat{C}_s$) = 27741 | |

The results shown above are exactly the same with what has been reported in Jaber and Goyal (2008) as shown in Fig. 7.1. In table 7.14, we only show variables and objective functions for players included in the supply chain, as studied by Jaber and Goyal (2008). For other players in this work objective functions are zero as stated before. Notations in brackets are notations which are used in Jaber and Goyal (2008).

Optimal order policies for the buyers the vendor and the suppliers with coordination

| $\lambda^*_{s=1}$ | $\lambda^*_{s=1}$ | $\lambda^*_v$ | $T^*$ | $\hat{C}_b$ | $\hat{C}_v$ | $\hat{C}_{s=1}$ | $\hat{C}_{s=2}$ |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 1 | 0.01885 | 38,584 | 13,265 | 16,337 | 11,404 |

| $j$ | $\hat{Q}^*_j$ | $\hat{C}_{b,j}$ | $\delta_{b,j}$ | $\delta_s$ |
|---|---|---|---|---|
| 1 | 1885 | 16,669 | 0.0687 | 0 |
| 2 | 1413 | 12,547 | 0.0307 | |
| 3 | 942 | 9368 | 0.0041 | $\hat{C}_{\text{Chain}} = \hat{C}_b + \hat{C}_v + \hat{C}_{s=1} + \hat{C}_{s=2} = 79,590$ |

Figure 7.1 Results table of the problem had been addressed in Jaber and Goyal (2008) page. 100

Next, in order to check the relationship between model-1 and model-2 we test model-2 by setting the proportion of returned part $p'$ $u_{p'} = 0$ and setting cost parameters, which are related to returned parts, to zero. Result for model-2 in this special case is as follow:

Tabel 7.15 Results of numerical examples for the complex manufacturing supply chain involving reverse logistics (Model-2) with $u_{p',i} = 0$

| Decision variables | **Example 1** | | | **Example 2** | | |
|---|---|---|---|---|---|---|
| | Decentralized | Semi-centralized | Centralized | Decentralized | Semi-centralized | Centralized |
| $T^*$ | 0.001862481 | 0.002304702 | 0.002225485 | 0.005976143 | 0.006691496 | 0.006763413 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 2 | 2 | 2 | 1 | 2 | 2 |
| $a_{S'P}^*$ | 1 | 1 | 1 | 2 | 1 | 1 |
| $a_{S'W}^*$ | 2 | 2 | 2 | 1 | 1 | 1 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 |
| Objective functions | | | | | | |
| *TCR* | 17181.39 | 17572.77 | 17454.49 | 112112.4 | 112829.8 | 112972.0 |
| *TCD* | 4563.805 | 3688.112 | 3819.391 | 14223.22 | 12702.69 | 12567.62 |
| *TCM* | 7069.392 | 7480.138 | 7390.003 | 16231.26 | 25650.02 | 25821.05 |
| *TCS'* | 4740.596 | 4454.333 | 4487.175 | 36504.72 | 21634.94 | 21474.45 |
| *TCS"* | 1137.507 | 975.3927 | 998.7886 | 6211.864 | 5874.757 | 5846.832 |
| *TCChain* | 34692.69 | 34170.74 | 34149.85 | 185283.5 | 178692.21 | 178682.0 |

As from table 7.15 we can see the solution of model-2 is the same with the solution of model-1 (table 7.8). Thus, model-1 is a specific case of model-2 for $u_{p',i} = 0$. Furthermore, we compare the results of model-2 and model-5. From tables 7.9 and 7.12, we find that the supply chain's total cost of model-5 is higher than the supply chain's total cost of model-2. Since there is the safety stock of retailers in model-5 the total cost of retailers increase so that the supply chain's total cost is also increase.

Again, we test model-4 to see the effect of the length of horizon period to the optimal solution. We have tested the problem with indefinite horizon period as shown in Table 7.9 (model-2), and with finite horizon period varied from 1 to 10 as listed in Table 7.16.

Tabel 7.16. The computational results for centralized, semi-centralized and decentralized decision making process with any different length of horizon period (*N*)

| Decision variables | Centralized decision making process | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | N=1 | N=2 | N=3 | N=4 | N=5 | N=6 | N=7 | N=8 | N=9 | N=10 |
| $T^*$ | 0.002222222 | 0.002178649 | 0.002192982 | 0.002178649 | 0.002187227 | 0.002178649 | 0.002184769 | 0.002178649 | 0.002183406 | 0.002187227 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $a_{S'P}^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_{S'W}^*$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a3^*$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| *TCR* | 17450.00 | 17393.02 | 17411.13 | 17393.02 | 17403.78 | 17393.02 | 17400.68 | 17393.02 | 17398.96 | 17403.78 |
| *TCD* | 3825.000 | 3901.500 | 3876.000 | 3901.500 | 3886.200 | 3901.500 | 3890.571 | 3901.500 | 3893.000 | 3886.200 |
| *TCM* | 6976.804 | 6925.557 | 6942.225 | 6925.557 | 6935.509 | 6925.557 | 6932.650 | 6925.557 | 6931.068 | 6935.509 |
| *TCS'* | 2995.868 | 3009.905 | 3005.126 | 3009.905 | 3007.026 | 3009.905 | 3007.845 | 3009.905 | 3008.301 | 3007.026 |
| *TCS"* | 646.7652 | 656.3629 | 653.1565 | 656.3629 | 654.4382 | 656.3629 | 654.9879 | 656.3629 | 655.2933 | 654.4382 |
| *TC3* | 294.0000 | 297.1471 | 296.0921 | 297.1471 | 296.5134 | 297.1471 | 296.6942 | 297.1471 | 296.7948 | 296.5134 |
| ***TCChain*** | 32188.44 | 32183.49 | 32183.73 | 32183.49 | 32183.47 | 32183.49 | 32183.42 | 32183.49 | 32183.42 | 32183.47 |
| Decision variables | Semi-centralized decision making process | | | | | | | | | |
| | N=1 | N=2 | N=3 | N=4 | N=5 | N=6 | N=7 | N=8 | N=9 | N=10 |
| $T^*$ | 0.002304147 | 0.002306805 | 0.002305919 | 0.002305476 | 0.002305210 | 0.002305033 | 0.002304906 | 0.002304811 | 0.002304738 | 0.002305210 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 2 | 3 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $a_{S'P}^*$ | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 2 | 2 |
| $a_{S'W}^*$ | 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 1 | 3 | 3 |
| $a3^*$ | 7 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *TCR* | 17571.88 | 17576.14 | 17574.72 | 17574.01 | 17573.58 | 17573.30 | 17573.09 | 17572.94 | 17572.82 | 17573.58 |
| *TCD* | 3689.000 | 3684.750 | 3686.167 | 3686.875 | 3687.300 | 3687.583 | 3687.786 | 3687.938 | 3688.056 | 3687.300 |
| *TCM* | 6803.757 | 7080.943 | 7527.647 | 8226.955 | 7078.925 | 7527.887 | 7527.922 | 7527.948 | 8225.399 | 7078.925 |
| *TCS'* | 4880.849 | 2863.852 | 6929.786 | 2384.195 | 3394.283 | 6932.290 | 6932.649 | 6932.917 | 2384.293 | 2972.976 |
| *TCS"* | 436.3674 | 1127.486 | 3267.072 | 723.4766 | 492.6180 | 3268.317 | 3268.496 | 3268.629 | 723.6617 | 629.6620 |
| *TC3* | 396.2957 | 368.9778 | 1086.741 | 229.7472 | 288.4767 | 1087.157 | 1087.217 | 1087.261 | 229.8125 | 292.6996 |
| ***TCChain*** | 33778.15 | 32702.15 | 40072.13 | 32825.25 | 32515.30 | 40076.53 | 40077.16 | 40077.64 | 32824.05 | 32235.14 |
| Decision variables | Decentralized decision making process | | | | | | | | | |
| | N=1 | N=2 | N=3 | N=4 | N=5 | N=6 | N=7 | N=8 | N=9 | N=10 |
| $T^*$ | 0.001862197 | 0.001862197 | 0.001862197 | 0.001862197 | 0.001862197 | 0.001862197 | 0.001862693 | 0.001862631 | 0.001862583 | 0.001862544 |
| $a_D^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_M^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a_P^*$ | 1 | 2 | 3 | 2 | 3 | 2 | 2 | 5 | 2 | 1 |
| $a_{S'P}^*$ | 1 | 3 | 3 | 2 | 1 | 1 | 1 | 1 | 2 | 1 |
| $a_{S'W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $a_{S''W}^*$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a3^*$ | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 4 | 1 |
| *TCR* | 17181.39 | 17181.39 | 17181.39 | 17181.39 | 17181.39 | 17181.39 | 17181.39 | 17181.39 | 17181.39 | 17181.39 |
| *TCD* | 4564.501 | 4564.501 | 4564.502 | 4564.502 | 4564.502 | 4564.502 | 4563.286 | 4563.437 | 4563.555 | 4563.651 |
| *TCM* | 7834.880 | 6606.905 | 6614.029 | 6606.905 | 6614.029 | 6606.905 | 6606.962 | 7370.082 | 6606.949 | 7834.459 |
| *TCS'* | 8491.440 | 3940.868 | 4419.636 | 3626.310 | 3278.314 | 4497.627 | 4496.608 | 2504.198 | 3812.494 | 8489.92 |
| *TCS"* | 4039.270 | 741.8635 | 553.4202 | 1053.951 | 1377.807 | 2037.288 | 2036.758 | 864.1706 | 597.5038 | 4038.522 |
| *TC3* | 1344.580 | 675.4090 | 453.7385 | 441.2976 | 453.7385 | 675.4090 | 675.2314 | 278.8372 | 424.8644 | 1344.330 |
| ***TCChain*** | 43456.06 | 33710.94 | 33786.72 | 33474.36 | 33469.78 | 35563.12 | 35560.24 | 32762.15 | 33186.76 | 43452.27 |

We also conduct a comparison of finite and infinite horizon period. The comparison for three coordinated policies with infinite and finite horizon period can be seen in Figure 7.2, 7.3 and 7.4.

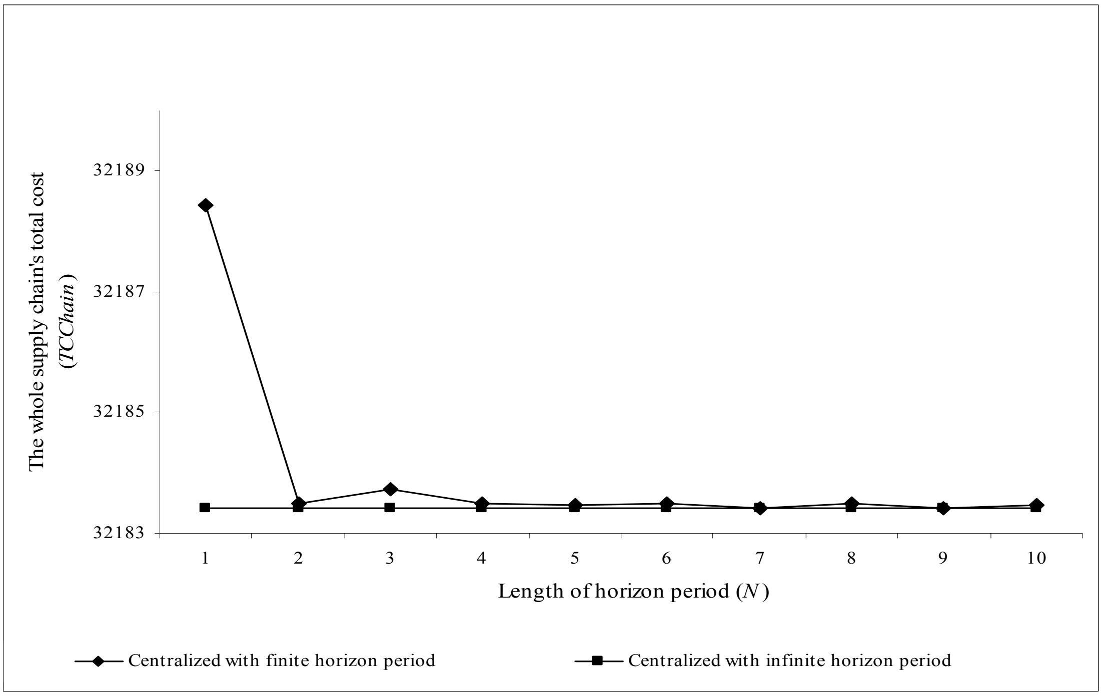


Figure 7.2 The supply chain's total cost curves for centralized decision making process with finite and infinite horizon period

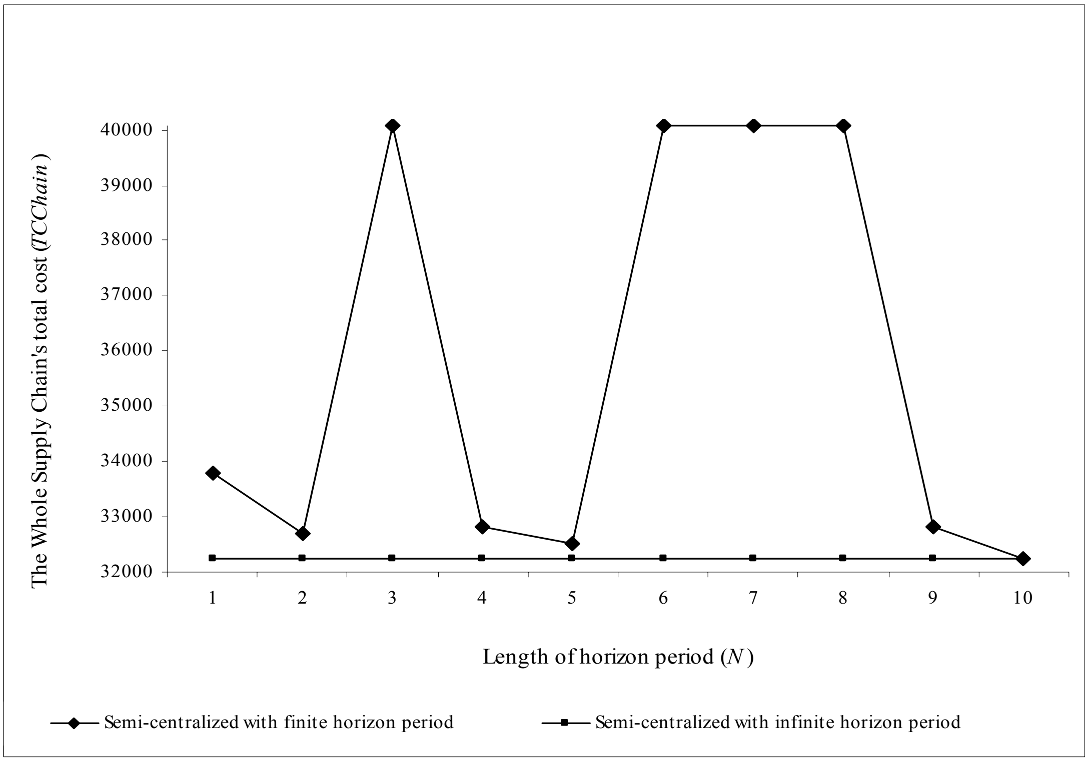


Figure 7.3 The supply chain's total cost curves for semi-centralized decision making process with finite and infinite horizon period

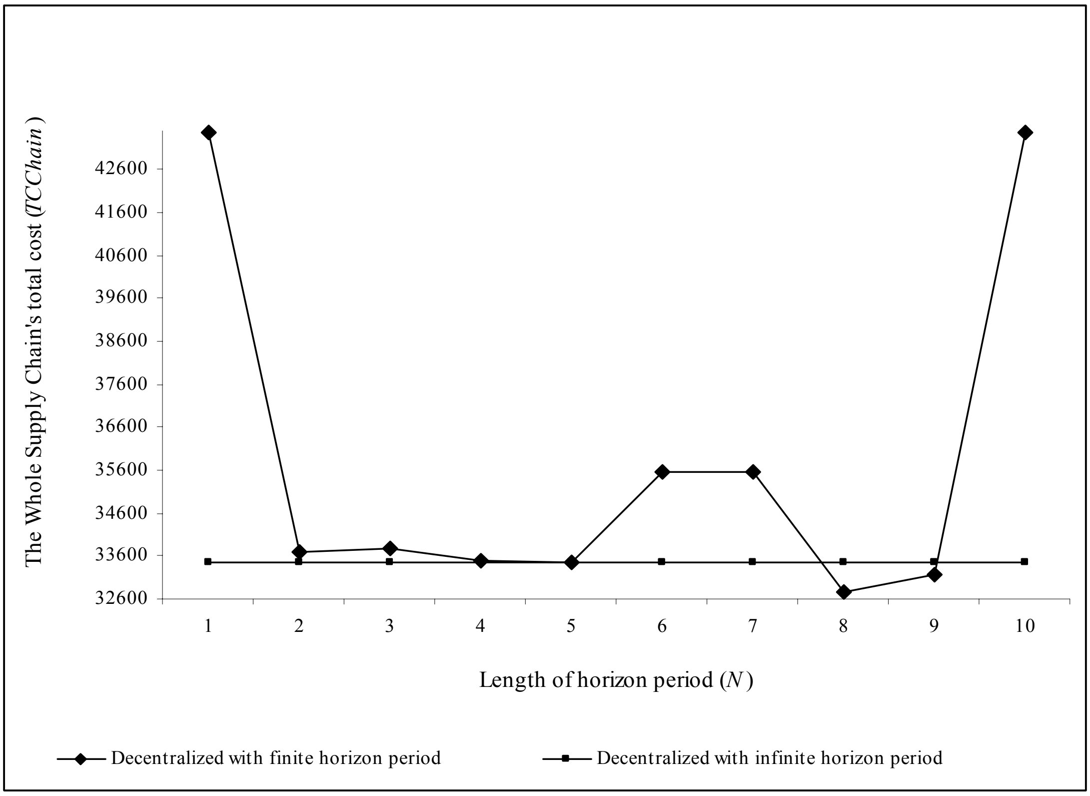


Figure 7.4 The supply chain's total cost curves for de-centralized decision making process with finite and infinite horizon period

Based on the results given in above tables, they clearly show that any different length of horizon/contract period makes only small variations to the value of the total cost of whole supply chain when the centralized policy is applied to the system. Since the optimal decision variables are solved simultaneously in the model it is clear why there are only small variations in the results as shown in Fig. 7.2. Therefore, the length of horizon/contract period does not affect significantly to the results when centralised policy is applied. Otherwise, if decentralised policy is applied, different lengths of horizon/ contract period make big variations to the values of the total cost of whole supply chain. Since the optimal decision variables for each level of the supply chain are solved consecutively the total cost of each level of the supply chain can varies significantly so that the total cost of the supply chain can varies significantly too as shown in Fig. 7.4. It therefore means the length of horizon/contract could be one of the main factors which will determine which type of the coordination between all players in the system is applied. Furthermore, semi-centralized policy can be an alternative policy to all players if both centralised and decentralised policies mentioned above are difficult to be applied to the supply chain. Therefore, the management of each player in the system should consider this horizon/ contract period in determining which policy is appropriate for adoption as well as their total inventory cost.

Finally, in order to see the sensitivity of input assumptions to the results, we use similar approach which has been used in Chung and Wee (2011). First, we change the values of certain input parameters in the numerical example by certain percentages (-7.5%, -5%, -2.5%, 2.5%, 5%, and 7.5%) for each level of the supply chain such as ordering cost and holding cost and we compare the results of the total cost of each player and the supply chain with the original results in the numerical example. Then, we change certain input parameters for all levels of the supply chain together and we compare the results the total cost of each player and the supply chain. In these scenarios, we use the results of model-2 for example 1 to see the sensitivity of the model. The results can be seen in Table 7.17, 7.18 and 7.19.

Tabel 7.17 The sensitivity of the results under varying ordering and order processing and fixed shipment costs

| Changes of $A^{(r)}_r$ (%) | Changes of total cost (%) | | | | | | |
|---|---|---|---|---|---|---|---|
| | *TCR* | *TCD* | *TCM* | *TCS'* | *TCS"* | *TC3* | *TCChain* |
| -7.50 | -3.47 | 1.75 | -0.62 | 0.44 | 1.31 | 0.96 | -1.72 |
| -5.00 | -2.31 | 1.16 | -0.41 | 0.29 | 0.87 | 0.63 | -1.15 |
| -2.50 | -1.15 | 0.57 | -0.21 | 0.14 | 0.43 | 0.31 | -0.57 |
| 2.50 | 1.14 | -0.56 | 0.21 | -0.14 | -0.42 | -0.31 | 0.57 |
| 5.00 | 2.27 | -1.12 | 0.42 | -0.27 | -0.83 | -0.61 | 1.13 |
| 7.50 | 3.39 | -1.67 | 0.63 | -0.39 | -1.24 | -0.90 | 1.69 |
| Changes of $A^{(d)}_d$ and $B^{(d)}_d$ (%) | *TCR* | *TCD* | *TCM* | *TCS'* | *TCS"* | *TC3* | *TCChain* |
| -7.5 | -0.14 | -6.54 | -0.33 | 0.68 | 0.49 | 0.49 | -0.90 |
| -5 | -0.09 | -4.42 | -0.22 | 0.46 | 0.33 | 0.33 | -0.61 |
| -2.5 | -0.05 | -2.10 | -0.11 | 0.22 | 0.16 | 0.16 | -0.29 |
| 2.5 | 0.05 | 2.29 | 0.12 | -0.24 | -0.17 | -0.17 | 0.32 |
| 5 | 0.10 | 4.37 | 0.22 | -0.45 | -0.32 | -0.32 | 0.60 |
| 7.5 | 0.15 | 6.64 | 0.34 | -0.68 | -0.49 | -0.49 | 0.92 |
| Changes of $A_M$ and $b^{(m)}_i$ (%) | *TCR* | *TCD* | *TCM* | *TCS'* | *TCS"* | *TC3* | *TCChain* |
| -7.5 | -0.03 | 0.19 | -0.95 | 0.05 | 0.14 | 0.10 | -0.19 |
| -5 | -0.02 | 0.12 | -0.59 | 0.03 | 0.09 | 0.06 | -0.12 |
| -2.5 | -0.01 | 0.06 | -0.29 | 0.01 | 0.04 | 0.03 | -0.06 |
| 2.5 | 0.01 | -0.06 | 0.29 | -0.01 | -0.04 | -0.03 | 0.06 |
| 5 | 0.02 | -0.13 | 0.66 | -0.03 | -0.10 | -0.07 | 0.13 |
| 7.5 | 0.03 | -0.19 | 0.95 | -0.05 | -0.14 | -0.10 | 0.19 |
| Changes of $A^{(s')}_{s'}$ (%) | *TCR* | *TCD* | *TCM* | *TCS'* | *TCS"* | *TC3* | *TCChain* |
| -7.5 | -0.01 | 0.06 | -0.02 | -0.65 | 0.05 | 0.03 | -0.06 |
| -5 | -0.01 | 0.04 | -0.01 | -0.42 | 0.03 | 0.02 | -0.04 |
| -2.5 | 0.00 | 0.02 | -0.01 | -0.22 | 0.02 | 0.01 | -0.02 |
| 2.5 | 0.00 | -0.02 | 0.01 | 0.22 | -0.02 | -0.01 | 0.02 |
| 5 | 0.01 | -0.04 | 0.02 | 0.45 | -0.03 | -0.02 | 0.04 |
| 7.5 | 0.01 | -0.06 | 0.02 | 0.64 | -0.05 | -0.03 | 0.06 |

Tabel 7.18 The sensitivity of the results under varying setup costs

| Changes of $S^{(m)}$, $S^{(s')}_{s'}$, $S^{(s")}_{s"}$, $S^{(3)}$ (%) | Changes of total cost (%) | | | | | | |
|---|---|---|---|---|---|---|---|
| | *TCR* | *TCD* | *TCM* | *TCS'* | *TCS"* | *TC3* | *TCChain* |
| -7.5 | -0.08 | 0.55 | -1.07 | -2.45 | -4.64 | -1.00 | -0.54 |
| -5 | -0.06 | 0.36 | -0.71 | -1.63 | -3.09 | -0.66 | -0.36 |
| -2.5 | -0.03 | 0.18 | -0.36 | -0.81 | -1.54 | -0.33 | -0.18 |
| 2.5 | 0.03 | -0.18 | 0.35 | 0.81 | 1.54 | 1.33 | 0.18 |
| 5 | 0.06 | -0.36 | 0.71 | 1.62 | 3.07 | 0.66 | 0.36 |
| 7.5 | 0.09 | -0.54 | 1.06 | 2.43 | 4.60 | 0.99 | 0.54 |

Tabel 7.19. The sensitivity of the results under varying holding costs

| Changes of $h^{(r)}_{r,i}$, $h^{(d')}_{d,i}$, $h_i$, $h_{s',p}$, $h^{(s")}_{s",w}$, $h^{(3')}_i$ (%) | Changes of total cost (%) | | | | | | |
|---|---|---|---|---|---|---|---|
| | *TCR* | *TCD* | *TCM* | *TCS'* | *TCS"* | *TC3* | *TCChain* |
| -7.5 | -3.91 | -3.24 | -2.24 | -1.63 | -3.39 | -3.51 | -3.24 |
| -5 | -2.59 | -2.15 | -1.49 | -1.09 | -2.25 | -2.32 | -2.15 |
| -2.5 | -1.29 | -1.07 | -0.74 | -0.54 | -1.12 | -1.16 | -1.07 |
| 2.5 | 1.27 | 1.06 | 0.74 | 0.54 | 1.10 | 1.14 | 1.06 |
| 5 | 2.53 | 2.11 | 1.47 | 1.09 | 2.20 | 2.27 | 2.11 |
| 7.5 | 3.77 | 3.14 | 2.20 | 1.63 | 3.28 | 3.39 | 3.14 |

First, we change the values of input parameters for each level of supply chain. In Table 7.17, we find that percentages of changes of the total cost are less than percentages of changes of input parameters. It means that the model results are not sensitive to the changes of each input parameter. In addition, changes of input parameters of each player will result in the biggest changes to the total cost of that player. For example in Table 7.17, if we change the ordering cost of retailers, $A^{(r)}_r$, the total cost of retailers, *TCR*, will have the biggest changes amongst all players.

To evaluate the effect of changing simultaneously the input parameters of all players to the total cost of the supply chain, we change setup cost of all players as can be seen in Table 7.18. Again, we find that percentages of changes of the total cost are less than percentages of changes of input parameters. As no changes to the input parameters of distributors are made in Table 7.18, the cases of changes of the total cost correspond to situations where the common order cycle time of retailers have been changed, so that common order cycle time of distributors, $a_D T$ is also changed. The same situation also happens to retailers. Therefore, we can conclude that the models developed are insensitive to changes of input parameters since percentages of changes of the supply chain's total cost are less than percentages of changes of input parameters for the scenarios studied as shown in Figure 7.5 to 7.10.

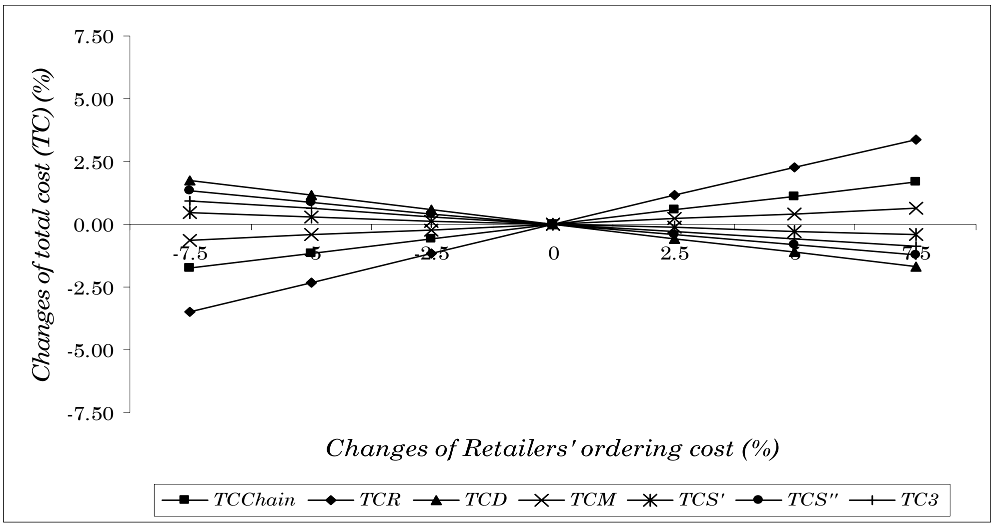


Figure 7.5 The sensitivity of the total cost under varying retailers' ordering cost

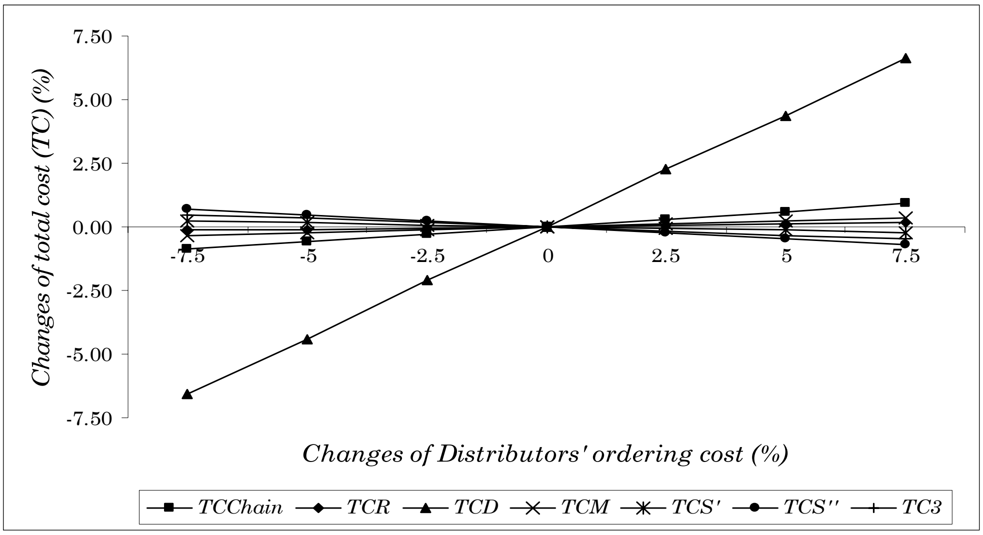


Figure 7.6 The sensitivity of the total cost under varying distributors' ordering cost

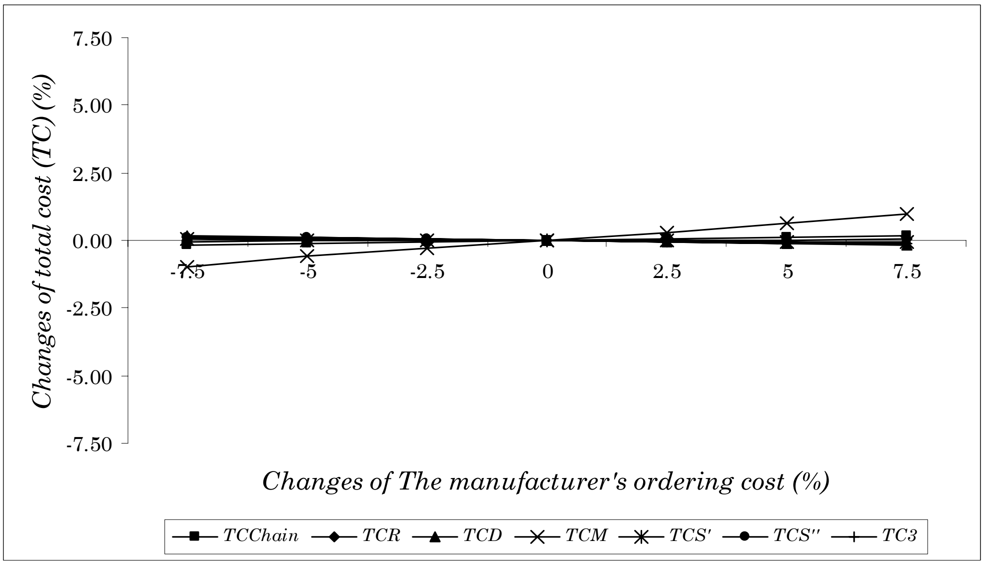


Figure 7.7 The sensitivity of the total cost under varying the manufacturer's ordering cost

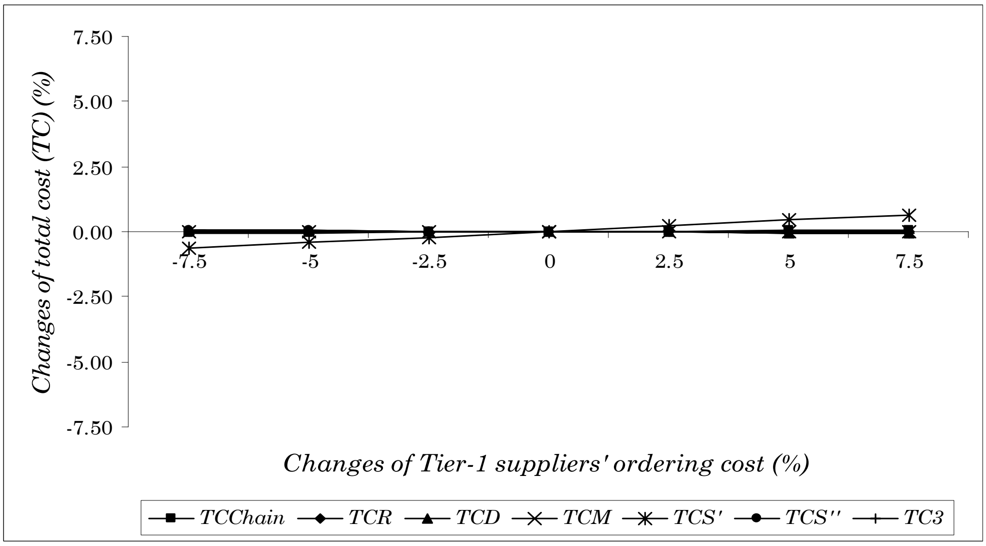


Figure 7.8 The sensitivity of the total cost under varying tier-1 suppliers' ordering cost

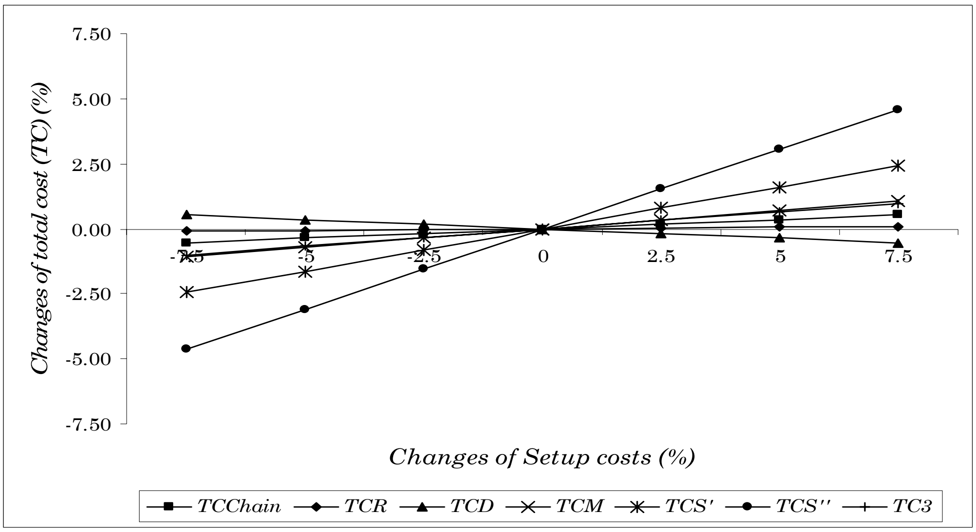


Figure 7.9 The sensitivity of the total cost under varying setup costs

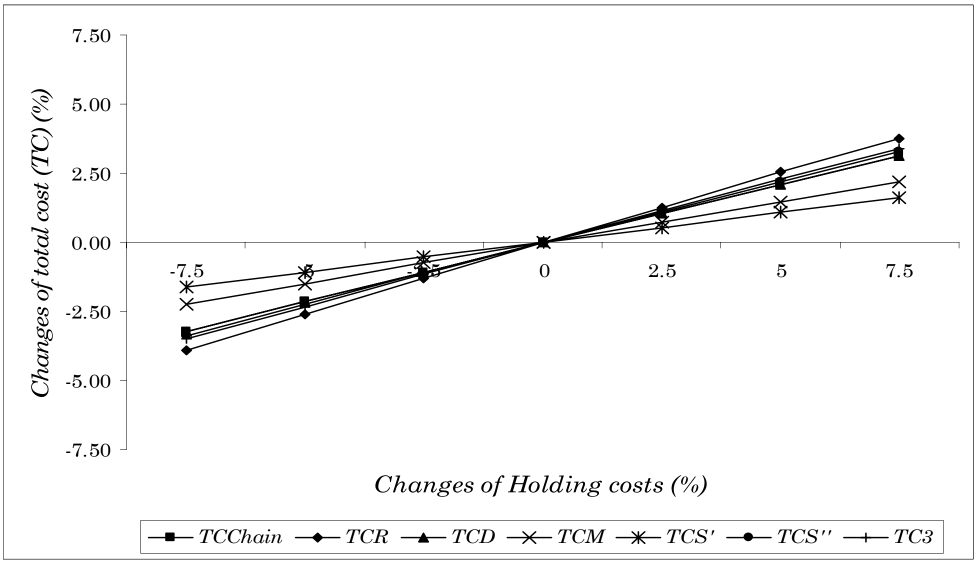


Figure 7.10 The sensitivity of the total cost under varying holding costs

## 7.4 Summary

In this chapter, numerical examples are used to demonstrate, test, and discuss the models. First, numerical examples are used in integrated production inventory model in a complex manufacturing supply chain, the system involving reverse logistics, the system considering transportation costs and the system considering limited horizon period based on three solution methods. The results are analysed and discussed. Then, the model is tested by using the data from literature to see that the model can be generalised model for multi-level supply chain. The model is also tested with different finite horizon period to see the effects to the optimal solution. Lastly, the sensitivity analysis of the model is carried out to see the sensitivity of the model to changes of input parameters for the scenarios studied.

# Chapter 8

# Conclusions and Future Work

## 8.1 Introduction

This chapter summarises and concludes this research work. The chapter also provides some directions for possible future work.

## 8.2 Conclusions

This research has presented the modeling of coordinating production inventory systems in a complex manufacturing supply chain involving reverse logistics. The necessity of this problem has been described in chapter 1. The chapter described the necessity for all companies to manage their products at competitive prices, coordinate with other players in the supply chain and include environmental and ecological consciousness and responsibility in their business through remanufacturing and reuse.

To support this research, definitions and major issues in supply chain management relate to this research have been described in chapter 2. A literature review has been carried out in chapter 3. Chapter 3 described and summarized past and current research in managing inventory in the supply chain. It included managing inventory in two-level supply chains and three-level supply chains considering aspects such as deteriorating items, credit option and delays in payment schemes, a quantity discount scheme, multiple shipments and finite horizon period. Involving reverse logistics in coordination production inventory system is also included in the literature review.

As stated in section 1.5 in chapter 1 the aim of this research is to establish models to determine coordinated and integrated production and inventory decisions in a whole manufacturing supply chain involving reverse logistics subjected to the limited contract period among all players and capacity constraint of the transportation units as described in section 1.3. To achieve this aim the following work has been carried out: (1) the building of mathematical models for coordinating and integrating the production and inventory decisions among all players in the whole manufacturing supply chain involving reverse logistics in order to minimize the supply chain's total cost subjected to a limited contract period and capacity constraints of transportation units, and (2) the development of solution methods of such models based on centralized, semi-centralized and decentralized decision making processes.

For this purpose, mathematical models with the constraints of a limited contract period and with capacity constraints of transportation units have been developed in chapter 4 and 5. In the chapter 4, the mathematical model for coordinating production inventory cycles in the complex manufacturing supply chain without involving reverse logistics was developed. In chapter 5, the mathematical models for the system considering reverse logistics, transportation units, finite horizon period and stochastic demand were developed. The solution methods to solve the models based on centralized, semi-centralized and decentralized decision making processes have been developed in section 6. Semi-centralized decision making process is a combination of centralized and decentralized decision making process which is proposed in this research.

The models have been tested in chapter 7 to analyse and discuss the results. Numerical examples are provided to demonstrate the models. The analysis of the computational results of the examples has been reported regarding the comparison of three types of coordination, the relationship between the models, the comparison between infinite and finite horizon period and between different lengths of horizon period and sensitivity analysis of the models. The analysis found that the centralized decision making process is the best solution from all types of the coordination for the scenarios studied. However, since this type of coordination has limitations, sometimes in practice, the research also suggests decentralized and semi-centralized methods for solving real problems. It is also found that any different length of horizon period makes only small variations to the value of the total cost of whole supply chain under centralized decision making process and makes big variations under decentralized one. Furthermore, in a stochastic environment adding safety stock to retailers increases the total cost of retailers and the whole system if compared with no safety stock (deterministic case). In sensitivity analysis, the models developed are insensitive to changes of input parameters since percentages of changes of the total costs are less than percentages of changes of input parameters for the scenarios studied.

Based on the work done, this research makes the contribution to the area of coordinating production and inventory models in the complex supply chain especially in a complex manufacturing supply chain involving reverse logistics for multiple raw materials, parts and finished products with considering the finite horizon period, limited capacity of transportation units and stochastic demand. This research also makes the contribution in developing the solution methods or procedures by proposing semi-centralized decision making process as well as the use of mixed integer nonlinear programming method.

Therefore, it is appropriate to conclude that in the course of this PhD research, models and solution methods for coordinating production and inventory system in a complex manufacturing supply chain involving reverse logistics, considering a limited contract period among all players and capacity constraints of transportation units have been developed. These models are applied for situation where multiple raw materials, parts, and finished products and multiple players in the complex supply chain are involved.

This research has built the models and proposed the solution methods to solve these problem: (1) when raw materials, parts and finished products should be produced and/or ordered by a company from other companies, (2) how many raw materials, parts and finished products companies should be ordered and/or produced every order and production cycle time, and (3) how many transportation units companies should use to deliver raw materials, parts, and finished products from a company to other companies subject to a finite horizon period and capacity constraint of transportation units in a complex manufacturing supply chain. A supply chain can select which solution method that is appropriate to be applied. The solution method which is used depends on the type of coordination in the supply chain. Our research has shown that the models that had been developed can be used for solving production and inventory problems in a supply chain for both deterministic and stochastic demand.

## 8.3 Discussion and Limitation

The models that have been developed can be viewed as generalised models for two up to five-level supply chains and for single or multiple products. For example, in buyer-vendor coordination case the models can be applied by setting values of input parameters of other players excluding in the system with zero and multiplier integers of common cycle time with one. If the case assumes an infinite production rate for the vendor the models can still be applied by setting the production rate with a big number which is closer to infinity. Furthermore, in production planning of manufacturing companies in the supply chain the results of the models such as the production quantity per cycle time can be used as one of inputs to determine how many labours/ workers which are needed to produce the products. For longer term period, the results of the models can be also used as basic data to design resource and capacity planning of the manufacturing companies.

As during the development of the models and solution methods certain assumptions were made such as the use of different production facilities in producing different types of products and a common cycle time of each level of the supply chain, the models have some limitations in practice. The models are applied in a system where each company produces different types of products in the different production facilities/ lines. Otherwise, if a company only produces many variations of a product such as colours, sizes and shapes (the same product family) using the same production line the models can not be applied. Furthermore, the models are also applied in a system where players in the same level of the supply chain use common order and/or production cycle time. This limitation is reasonable if demand quantities for each player in the same level have small differences between each other and location of the players is relatively contiguous.

## 8.4 Future Work

Since there are certain limitations made in this research, there are some suggestions for future work. To facilitate where a company produces many variation of products using the same production line, a batch scheduling method to design a production schedule of the products can be considered as constraints of the models. Then, inventory holding cost term in the total cost function of the supply chain is modified following such constraints. Furthermore, in the case where there is a big difference in the demand of players in the same level of the supply chain, use of different multiple integers of a certain order cycle time for each player can be applied to the models to reduce total cost of the players.

Moreover, as described in section 7.3 each company in the supply chain can be better off for one decision making process and can be worse off for another decision making process. Some incentive and/or compensation schemes models such as quantity discount, credit option, profit sharing and/or delay in payments to compensate the disadvantages can be built accompanying the models.

## 8.5 Summary

In this chapter, conclusions of the research are drawn. Discussion about the research and the models developed is also described. Some limitations of the use of the models are presented. Lastly, some suggestions for future work are summarised.

# Appendix A

**Table A. The summary of literature review on production inventory models in the supply chain**

| References | Single product | Multiple products | Single -level | Two-level supply chain | | Three-level supply chain | | Factors considered in this thesis | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Single player | Multiple players | Single player | Multiple players | Reverse logistics | Transportation costs | Finite horizon period | Deterministic demand | Stochastic demand |
| Affisco et al. (2002) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Abdul-Jalbar et al. (2007) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Banerjee (1986) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Banerjee and Burton (1994) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Banerjee et al. (2007) | √ | - | - | - | - | - | √ | - | - | - | √ | - |
| Ben-Daya and Hariga (2004) | √ | - | - | √ | - | - | - | - | - | - | - | - |
| Ben-Daya and Noman (2008) | √ | - | √ | - | - | - | - | - | - | - | - | √ |
| Ben-Daya and Al-Nassar (2008) | √ | - | - | - | - | - | √ | - | - | - | √ | √ |
| Chakrabarty and Martin (1988) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Chan et al. (2010) | √ | - | - | - | - | - | √ | - | - | - | √ | - |
| Chan and Lee (2012) | √ | - | - | - | - | - | √ | - | - | - | √ | - |
| Chan and Kingsman (2007) | √ | - | - | - | - | - | √ | - | - | - | √ | - |
| Chen and Kang (2007) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Chen and Chen (2005) | - | √ | - | √ | - | - | - | - | - | - | √ | - |
| Ching et al. (2003) | √ | - | √ | - | - | - | - | √ | - | - | √ | - |
| Choi et al. (2007) | √ | - | √ | - | - | - | - | √ | - | - | √ | - |
| Chung et al. (2008) | √ | - | - | - | - | √ | - | √ | - | - | √ | - |
| Chung and Wee (2007) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Chung and Wee (2008) | √ | - | - | √ | - | - | - | √ | - | - | √ | - |
| Chung and Wee (2011) | √ | - | - | √ | - | - | - | √ | - | - | √ | - |
| Duan et al. (2010) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| El Saadany and Jaber (2008) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Ertogral et al. (2007) | √ | - | - | √ | - | - | - | - | √ | - | √ | - |
| Ertogral (2011) | √ | - | - | √ | - | - | - | - | - | - | - | √ |
| Ganeshan (1999) | √ | - | - | - | - | - | √ | - | - | - | √ | - |
| Glock (2012a) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Glock (2011) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Goyal et al. (2003) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Goyal (1977) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Goyal (2000) | √ | - | - | √ | - | - | - | - | - | - | √ | - |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Goyal and Gupta (1989) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Hill (1997) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Hill (1999) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Hoque (2000) | √ | - | - | √ | - | - | - | - | √ | - | √ | - |
| Hoque (2008) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Huang et al. (2010) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Huang (2002) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Huang (2004) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Jaber and Goyal (2008) | √ | - | - | - | - | - | √ | - | - | - | √ | - |
| Jaber et al. (2006) | √ | - | - | - | - | √ | - | - | - | - | √ | - |
| Jaber and Osman (2006) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Jaber et al. (2010) | √ | - | - | - | - | √ | - | - | - | - | √ | - |
| Kim and Hwang (1989) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Kim et al. (2006) | - | √ | - | - | - | - | √ | - | - | - | √ | - |
| Lee and Wu (2006) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Lee and Rosenblatt (1986) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Lee (2005) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Lo et al. (2007) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Lu (1995) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Munson and Rosenblatt (2001) | √ | - | - | - | - | √ | - | - | - | - | √ | - |
| Ouyang et al. (2006) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Ouyang et al. (2007) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Pan and Yang (2008) | - | √ | - | √ | - | - | - | - | - | - | √ | - |
| Rau et al. (2003) | √ | - | - | - | - | √ | - | - | - | - | √ | - |
| Rau et al. (2004) | √ | - | - | - | - | √ | - | - | - | - | √ | - |
| Rau and OuYang (2008) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Rieksts and Ventura (2008) | √ | - | √ | - | - | - | - | - | √ | √ | - | - |
| Savaskan et al. (2004) | √ | - | - | √ | - | - | - | √ | - | - | √ | - |
| Sarmah et al. (2008) | √ | - | - | - | √ | - | - | √ | - | - | √ | - |
| Siajadi et al. (2006) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Taleizadeh et al. (2010) | √ | - | - | √ | - | - | - | - | - | - | - | √ |
| Teunter (2001) | √ | - | √ | - | - | - | - | √ | - | - | √ | - |
| Tsao (2010) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Yang and Wee (2000) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Yang and Wee (2002) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Woo et al. (2001) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Wu et al. (2007) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| Zanoni and Zavarella (2007) | √ | - | - | - | √ | - | - | - | - | - | √ | - |
| Zhang et al. (2008) | √ | - | - | - | - | - | - | - | - | - | √ | - |
| Zhou and Wang (2007) | √ | - | - | √ | - | - | - | - | - | - | √ | - |
| This Research | - | √ | - | - | - | - | - | √ | √ | √ | - | √ |

# Appendix B

Proof of Eq. (4.53) which is a convex function

Let

$TCChain = TCR + TCD + TCM + TCS' + TCS''$

$TCChain$ is the convex function over $T > 0$ only if

$$\frac{\partial^2 TCChain}{\partial T^2} > 0$$

Following the property *c(x) = f(x) + g(x)* and $\partial^2 c(x)/\partial x^2 = \partial^2 f(x)/\partial x^2 + \partial^2 g(x)/\partial x^2$ then

$$\frac{\partial^2 TCChain}{\partial T^2} = \frac{\partial^2 TCR}{\partial T^2} + \frac{\partial^2 TCD}{\partial T^2} + \frac{\partial^2 TCM}{\partial T^2} + \frac{\partial^2 TCS'}{\partial T^2} + \frac{\partial^2 TCS''}{\partial T^2}$$

The second derivation for each total cost function is given as follows:

**Retailers,**

$$TCR = \frac{\sum_{r=1}^{n^{(r)}} \left( A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)} \right)}{T} + \frac{\sum_{r=1}^{n^{(r)}} \left( T \sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)} \right)}{2}$$

$$\frac{\partial^2 TCR}{\partial T^2} = \frac{\partial^2 \left[ \frac{\sum_{r=1}^{n^{(r)}} \left( A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)} \right)}{T} \right]}{\partial T^2} + \frac{\partial^2 \left[ \frac{\sum_{r=1}^{n^{(r)}} \left( T \sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)} \right)}{2} \right]}{\partial T^2} = \frac{\partial \left[ - \frac{\sum_{r=1}^{n^{(r)}} \left( A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)} \right)}{T^2} \right]}{\partial T} + \frac{\partial \left[ \frac{\sum_{r=1}^{n^{(r)}} \left( \sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)} \right)}{2} \right]}{\partial T}$$

$$= 2 \frac{\sum_{r=1}^{n^{(r)}} \left( A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)} \right)}{T^3} > 0$$

**Distributors,**

$$TCD = \frac{\sum_{d=1}^{n_d} \left( A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)} \right)}{\alpha_D T} + \frac{T \sum_{d=1}^{n^{(d)}} \sum_{i=1}^{k^{(i)}} \left( h_{d,i}^{(d)} D_{d,i}^{(d)} \right)}{2} (\alpha_D - 1) + \frac{\left( B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)} \right)}{T}$$

$$\frac{\partial^2 TCD}{\partial T^2} = \frac{\partial^2 \left[ \frac{\sum_{d=1}^{n^{(d)}} \left( A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)} \right)}{\alpha_D T} \right]}{\partial T^2} + \frac{\partial^2 \left[ \frac{T \sum_{d=1}^{n^{(d)}} \sum_{i=1}^{k^{(i)}} \left( h_{d,i}^{(d)} D_{d,i}^{(d)} \right)}{2} (\alpha_D - 1) \right]}{\partial T^2} + \frac{\partial^2 \left[ \sum_{d=1}^{n^{(d)}} \frac{\left( B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)} \right)}{T} \right]}{\partial T^2}$$

$$
= \frac{\partial\left[ -\dfrac{\sum_{d=1}^{n_d}\left(A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)}\right)}{\alpha_D T^2} \right]}{\partial T} + \frac{\partial\left[ \dfrac{\sum_{d=1}^{n^{(d)}}\sum_{i=1}^{k^{(i)}}\left(h_{d,i}^{(d)} D_{d,i}^{(d)}\right)}{2}(\alpha_D - 1) \right]}{\partial T} + \frac{\partial\left[ -\sum_{d=1}^{n^{(d)}} \dfrac{\left(B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)}\right)}{T^2} \right]}{\partial T}
$$

$$
= 2\frac{\sum_{d=1}^{n_d}\left(A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)}\right)}{\alpha_D T^3} + 2\frac{\left(\sum_{d=1}^{n^{(d)}} B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)}\right)}{T^3} > 0
$$

Similarly for other total cost functions, then

**The manufacturer,**

$$
TCM = \left( \begin{array}{l} \dfrac{S^{(m)} + \sum_{i=1}^{k^{(i)}} s_i^{(m)}}{\alpha_M \alpha_D T} + \sum_{i=1}^{k^{(i)}} \dfrac{h_i D_i^{(m)} \alpha_D T}{2}\left(\alpha_M \dfrac{D_i^{(m)}}{P_i^{(m)}} + (\alpha_M - 1)\right) + \dfrac{B_M + \sum_{i=1}^{k^{(i)}} b_i}{\alpha_D T} + \dfrac{A_M + \sum_{p=1}^{k^{(p)}} a_p^{(m)}}{\alpha_P \alpha_M \alpha_D T} + \\ \sum_{p=1}^{k^{(p)}}\sum_{i=1}^{k^{(i)}}\left( \dfrac{\beta_{p,i}^{(I)} h_p^{(m)} D_i^{(m)} \alpha_M \alpha_D T}{2}\left( \dfrac{D_i^{(m)}}{p_i^{(m)}} + (\alpha_p - 1) + 2\left(\left(\dfrac{D_i^{(m)}}{p_i^{(m)}}\right)^{\max} - \dfrac{D_i^{(m)}}{p_i^{(m)}}\right)\right)\right) \end{array} \right)
$$

$$
\frac{\partial^2 TCM}{\partial T^2} = 2\frac{S^{(m)} + \sum_{i=1}^{k^{(i)}} s_i^{(m)}}{\alpha_M \alpha_D T^3} + 2\frac{B_M + \sum_{i=1}^{k^{(i)}} b_i^{(m)}}{\alpha_D T^3} + 2\frac{A_M + \sum_{p=1}^{k^{(p)}} a_p^{(m)}}{\alpha_P \alpha_M \alpha_D T^3} > 0
$$

**Tier-1 suppliers,**

$$
\begin{array}{l} TCS = \sum_{s'=1}^{n^{(s')}} \dfrac{S_{s'}^{(s')} + \sum_{p=1}^{k^{(p)}} s_{s',p}^{(s')}}{\alpha_{S'P}\alpha_P \alpha_M \alpha_D T} + \sum_{s'=1}^{n^{(s')}}\sum_{p=1}^{k^{(p)}}\left( \dfrac{h_{s',p} e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)} D_i^{(m)}\right)\alpha_P \alpha_M \alpha_D T}{2}\left( \alpha_{S'P} \dfrac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)} D_i^{(m)}\right)}{P_{s',p}^{(s')}} + (\alpha_{S'P} - 1)\right)\right) + \sum_{s'=1}^{n^{(s')}} \dfrac{A_{s'}^{(s')} + \sum_{w=1}^{k^{(w)}} a_{s',w}^{(s')}}{\alpha_{S'W}\alpha_{S'P}\alpha_P \alpha_M \alpha_D T} + \\ \sum_{s'=1}^{n^{(s')}} \dfrac{\left(B_{s'}^{(s')} + \sum_{p=1}^{k^{(p)}} b_{s',p}^{(s')}\right)}{\alpha_P \alpha_M \alpha_D T} + \sum_{s'=1}^{n^{(s')}}\sum_{w=1}^{k^{(w)}}\sum_{p=1}^{k^{(p)}}\left( \dfrac{\beta_{p,w}^{(II)} h_{s',w}^{(s')} e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)} D_i^{(m)}\right)\alpha_{S'P}\alpha_P \alpha_M \alpha_D T}{2}\left( \dfrac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}} \beta_{p,i}^{(I)} D_i^{(m)}}{P_{s',p}^{(s')}} + (\alpha_{S'W} - 1) + \left(2\left(\gamma \max_{s'} - \gamma_{s',p}\right)\right)\right)\right) \end{array}
$$

where
$$
\gamma_{s',p} = \frac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)} D_i^{(m)}\right)}{P_{s',p}^{(s')}}
$$

$$
\gamma \max_{s'} = \max\left( \frac{e_{s',p}^{(s')} \sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)} D_i^{(m)}\right)}{P_{s',p}^{(s')}},\ p = 1,2,..,k^{(p)} \right)
$$

$$
\frac{\partial^2 TCS'}{\partial T^2} = 2\sum_{s'=1}^{n^{(s')}} \frac{\left(S_{s'}^{(s')} + \sum_{p=1}^{k^{(p)}} s_{s',p}^{(s')}\right)}{\alpha_{S'P}\alpha_P \alpha_M \alpha_D T^3} + 2\sum_{s'=1}^{n^{(s')}} \frac{\left(A_{s'}^{(s')} + \sum_{w=1}^{k^{(w)}} a_{s',w}^{(w)}\right)}{\alpha_{S'W}\alpha_{S'P}\alpha_P \alpha_M \alpha_D T^3} + 2\sum_{s'=1}^{n^{(s')}} \frac{\left(B_{s'}^{(s')} + \sum_{p=1}^{k^{(p)}} b_{s',p}^{(s')}\right)}{\alpha_P \alpha_M \alpha_D T^3} > 0
$$

**Tier-2 suppliers,**

$$TCS''=\left(\begin{array}{l}\displaystyle\sum_{s''=1}^{n^{(s'')}}\frac{S_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}s_{s'',w}^{(s'')}}{\alpha_{S''W}\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T}+\sum_{s''=1}^{n^{(s'')}}\frac{\left(B_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}b_{s'',w}^{(s'')}\right)}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}+\\ \displaystyle\sum_{s''=1}^{n^{(s'')}}\sum_{w=1}^{k^{(w)}}\left(\frac{h_{s'',w}^{(s'')}e_{s'',w}^{(s'')}\sum_{p=1}^{k^{(p)}}\beta_{p,w}^{(II)}\sum_{i=1}^{k^{(i)}}\left(\beta_{p,i}^{(I)}D_i^{(m)}\right)\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2}\left(\alpha_{S''W}\frac{e_{s'',w}^{(s'')}\sum_{p=1}^{k^{(p)}}\beta_{p,w}^{(II)}\sum_{i=1}^{k^{(i)}}\beta_{p,i}^{(I)}D_i^{(m)}}{P_{s'',w}^{(s'')}}+\left(\alpha_{S''W}-1\right)\right)\right)\end{array}\right)$$

$$\frac{\partial^2 TCS''}{\partial T^2}=2\sum_{s''=1}^{n^{(s'')}}\frac{\left(S_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}s_{s'',w}^{(s'')}\right)}{\alpha_{S''W}\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T^3}+2\sum_{s''=1}^{n^{(s'')}}\frac{\left(B_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}b_{s'',w}^{(s'')}\right)}{\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T^3}>0$$

Since the second derivation for each total cost function is more than zero, then the second derivation of the total cost function of the whole supply chain is also more than zero as follows:

$$\frac{\partial^2 TCChain}{\partial T^2}=\frac{\partial^2 TCR}{\partial T^2}+\frac{\partial^2 TCD}{\partial T^2}+\frac{\partial^2 TCM}{\partial T^2}+\frac{\partial^2 TCS'}{\partial T^2}+\frac{\partial^2 TCS''}{\partial T^2}>0$$

so Eq. (4.53) is the convex function over $T > 0$ for any values of $\alpha_D$, $\alpha_M$, $\alpha_P$, $\alpha 3$, $\alpha_{S'P}$, $\alpha_{S'W}$, $\alpha_{S''W} \geq 1$

# Appendix C

Proof of Eq. (5.15) which is a convex function

Let

$TCChain = TCR + TCD + TCM + TCS' + TCS'' + TC3$

$TCChain$ is the convex function over $T > 0$ only if

$$\frac{\partial^2 TCChain}{\partial T^2} > 0$$

Following the property *c(x) = f(x) + g(x)* and $\partial^2 c(x)/\partial x^2 = \partial^2 f(x)/\partial x^2 + \partial^2 g(x)/\partial x^2$ then

$$\frac{\partial^2 TCChain}{\partial T^2} = \frac{\partial^2 TCR}{\partial T^2} + \frac{\partial^2 TCD}{\partial T^2} + \frac{\partial^2 TCM}{\partial T^2} + \frac{\partial^2 TCS'}{\partial T^2} + \frac{\partial^2 TCS''}{\partial T^2} + \frac{\partial^2 TC3}{\partial T^2}$$

The second derivation for each total cost function is given as follows:

**Retailers,**

$$TCR = \frac{\sum_{r=1}^{n^{(r)}} \left( A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)} \right)}{T} + \frac{\sum_{r=1}^{n^{(r)}} \left( T \sum_{i=1}^{k^{(i)}} h_{r,i}^{(r)} D_{r,i}^{(r)} \right)}{2}$$

$$\frac{\partial^2 TCR}{\partial T^2} = 2 \frac{\sum_{r=1}^{n^{(r)}} \left( A_r^{(r)} + \sum_{i=1}^{k^{(i)}} a_{r,i}^{(r)} \right)}{T^3} > 0$$

**Distributors,**

$$TCD = \frac{\sum_{d=1}^{n_d} \left( A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)} \right)}{\alpha_D T} + \frac{T \sum_{d=1}^{n^{(d)}} \sum_{i=1}^{k^{(i)}} \left( h_{d,i}^{(d)} D_{d,i}^{(d)} \right)}{2} (\alpha_D - 1) + \frac{\left( B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)} \right)}{T}$$

$$\frac{\partial^2 TCD}{\partial T^2} = 2 \frac{\sum_{d=1}^{n_d} \left( A_d^{(d)} + \sum_{i=1}^{k^{(i)}} a_{d,i}^{(d)} \right)}{\alpha_D T^3} + 2 \frac{\left( B_d^{(d)} + \sum_{i=1}^{k^{(i)}} b_{d,i}^{(d)} \right)}{T^3} > 0$$

**The manufacturer,**

$$TCM = \left( \begin{array}{l} \frac{S^{(m)} + \sum_{i=1}^{k^{(i)}} s_i^{(m)}}{\alpha_M \alpha_D T} + \sum_{i=1}^{k^{(i)}} \frac{h_i D_i^{(m)} \alpha_D T}{2} \left( \alpha_M \frac{D_i^{(m)}}{P_i^{(m)}} + (\alpha_M - 1) \right) - \sum_{i=1}^{k^{(i)}} \sum_{p=1}^{k^{(p)}} \frac{\left( h_p^{(m)} - h_{p'}^{(m')} \right) u_{p',i} \beta_{p,i}^{(I)} D_i^{(m)} \alpha_D T}{2} \left( \alpha_M \frac{D_i^{(m)}}{P_i^{(m)}} \left( 2 - u_{p',i} \right) + (\alpha_M - 1) \right) \\ + \frac{B_M + \sum_{i=1}^{k^{(i)}} b_i}{\alpha_D T} + \frac{A_M + \sum_{p=1}^{k^{(p)}} a_p^{(m)}}{\alpha_P \alpha_M \alpha_D T} + \sum_{p=1}^{k^{(p)}} \sum_{i=1}^{k^{(i)}} \left( \frac{\left( 1 - u_{p',i} \right) \beta_{p,i}^{(I)} h_p^{(m)} D_i^{(m)} \alpha_M \alpha_D T}{2} \left( \frac{D_i^{(m)}}{p_i^{(m)}} + (\alpha_p - 1) + 2 \left( \left( \frac{D_i^{(m)}}{p_i^{(m)}} \left( 1 - u_{p',i} \right) \right)^{\max} - \left( \frac{D_i^{(m)}}{p_i^{(m)}} \left( 1 - u_{p',i} \right) \right) \right) \right) \right) \\ + \frac{A_{M'} + \sum_{p'=1}^{k^{(p')}} a_{p'}^{(m')}}{\alpha_P \alpha_M \alpha_D T} + \sum_{p'=1}^{k^{(p')}} \sum_{i=1}^{k^{(i)}} \left( \frac{u_{p',i} \beta_{p,i}^{(I)} h_{p'}^{(m')} D_i^{(m)} \alpha_M \alpha_D T}{2} \left( \frac{D_i^{(m)}}{p_i^{(m)}} + (\alpha_P - 1) + 2 \left( \left( \frac{D_i^{(m)}}{p_i^{(m)}} \right)^{\max} - \frac{D_i^{(m)}}{p_i^{(m)}} \right) \right) \right) \end{array} \right)$$

$$\frac{\partial^2 TCM}{\partial T^2}=2\frac{S^{(m)}+\sum_{i=1}^{k^{(i)}}s_i^{(m)}}{\alpha_M\alpha_D T^3}+2\frac{B_M+\sum_{i=1}^{k^{(i)}}b_i^{(m)}}{\alpha_D T^3}+2\frac{A_M+\sum_{p=1}^{k^{(p)}}a_p^{(m)}}{\alpha_P\alpha_M\alpha_D T^3}+2\frac{A_{M'}+\sum_{p'=1}^{k^{(p')}}a_{p'}^{(m')}}{\alpha_P\alpha_M\alpha_D T^3}>0$$

**Tier-1 suppliers,**

$$TCS'=\sum_{s'=1}^{n^{(s')}}\frac{S_{s'}^{(s')}+\sum_{p=1}^{k^{(p)}}s_{s',p}^{(s')}}{\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}+\sum_{s'=1}^{n^{(s')}}\sum_{p=1}^{k^{(p)}}\left(\frac{h_{s',p}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\right)\alpha_P\alpha_M\alpha_D T}{2}\left(\alpha_{S'P}\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}}+(\alpha_{S'P}-1)\right)\right)+$$

$$\sum_{s'=1}^{n^{(s')}}\frac{\left(B_{s'}^{(s')}+\sum_{p=1}^{k^{(p)}}b_{s',p}^{(p)}\right)}{\alpha_P\alpha_M\alpha_D T}\sum_{s'=1}^{n^{(s')}}\frac{A_{s'}^{(s')}+\sum_{w=1}^{k^{(w)}}a_{s',w}^{(s')}}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}+$$

$$\sum_{s'=1}^{n^{(s')}}\sum_{w=1}^{k^{(w)}}\sum_{p=1}^{k^{(p)}}\left(\frac{\beta_{p,w}^{(II)}h_{s',w}^{(s')}e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\right)\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2}\left(\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}}+(\alpha_{S'W}-1)+\left(2\left(\gamma\max_{s'}-\gamma_{s',p}\right)\right)\right)\right)$$

where
$$\gamma_{s',p}=\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}}$$

$$\gamma\max_{s'}=\max\left(\frac{e_{s',p}^{(s')}\sum_{i=1}^{k^{(i)}}\left((1-u_{p',i})\beta_{p,i}^{(I)}D_i^{(m)}\right)}{P_{s',p}^{(s')}},\ p=1,2,..,k^{(p)}\right)$$

$$\frac{\partial^2 TCS'}{\partial T^2}=2\sum_{s'=1}^{n^{(s')}}\frac{\left(S_{s'}^{(s')}+\sum_{p=1}^{k^{(p)}}s_{s',p}^{(s')}\right)}{\alpha_{S'P}\alpha_P\alpha_M\alpha_D T^3}+2\sum_{s'=1}^{n^{(s')}}\frac{\left(B_{s'}^{(s')}+\sum_{p=1}^{k^{(p)}}b_{s',p}^{(s')}\right)}{\alpha_P\alpha_M\alpha_D T^3}+2\sum_{s'=1}^{n^{(s')}}\frac{\left(A_{s'}^{(s')}+\sum_{w=1}^{k^{(w)}}a_{s',w}^{(w)}\right)}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T^3}>0$$

**Tier-2 suppliers,**

$$TCS''=\left(\begin{array}{l}\sum_{s''=1}^{n^{(s'')}}\frac{S_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}s_{s'',w}^{(s'')}}{\alpha_{S''W}\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T}+\sum_{s''=1}^{n^{(s'')}}\frac{\left(B_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}b_{s'',w}^{(s'')}\right)}{\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}+\\ \sum_{s''=1}^{n^{(s'')}}\sum_{w=1}^{k^{(w)}}\left(\frac{h_{s'',w}^{(s'')}e_{s'',w}^{(s'')}\sum_{p=1}^{k^{(p)}}\beta_{p,w}^{(II)}\sum_{i=1}^{k^{(i)}}\left((1-u_{p'})\beta_{p,i}^{(I)}D_i^{(m)}\right)\alpha_{S'W}\alpha_{S'P}\alpha_P\alpha_M\alpha_D T}{2}\left(\alpha_{S''W}\frac{e_{s'',w}^{(s'')}\sum_{p=1}^{k^{(p)}}\beta_{p,w}^{(II)}\sum_{i=1}^{k^{(i)}}(1-u_{p'})\beta_{p,i}^{(I)}D_i^{(m)}}{P_{s'',w}^{(s'')}}+(\alpha_{S''W}-1)\right)\right)\end{array}\right)$$

$$\frac{\partial^2 TCS''}{\partial T^2}=2\sum_{s''=1}^{n^{(s'')}}\frac{\left(S_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}s_{s'',w}^{(s'')}\right)}{\alpha_{S''W}\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T^3}+2\sum_{s''=1}^{n^{(s'')}}\frac{\left(B_{s''}^{(s'')}+\sum_{w=1}^{k^{(w)}}b_{s'',w}^{(s'')}\right)}{\alpha_{S'W}a_{S'P}\alpha_P\alpha_M\alpha_D T^3}>0$$

**The third party,**

$$TC3=\frac{S^{(3)}+\sum_{i=1}^{k^{(i)}}s_i^{(3)}}{\alpha 3\alpha_P\alpha_M\alpha_D T}+\frac{\left(B^{(3)}+\sum_{p'=1}^{k^{(p')}}b_{p'}^{(3)}\right)}{\alpha_P\alpha_M\alpha_D T}+\sum_{i=1}^{k^{(i)}}\left(\frac{h_i^{(3)}R_i\alpha_P\alpha_M\alpha_D T}{2}\right)\left(\alpha 3\frac{R_i}{C_i}+(\alpha 3-1)\right)$$

$$\frac{\partial^2 TC3}{\partial T^2}=2\frac{S^{(3)}+\sum_{i=1}^{k^{(i)}}s_i^{(3)}}{\alpha 3\alpha_P\alpha_M\alpha_D T^3}+2\frac{\left(B^{(3)}+\sum_{p'=1}^{k^{(p')}}b_{p'}^{(3)}\right)}{\alpha_P\alpha_M\alpha_D T^3}>0$$

Similarly, since the second derivation for each total cost function is more than zero, then the second derivation of the total cost function of the whole supply chain is also more than zero as follows:

$$\frac{\partial^2 TCChain}{\partial T^2} = \frac{\partial^2 TCR}{\partial T^2} + \frac{\partial^2 TCD}{\partial T^2} + \frac{\partial^2 TCM}{\partial T^2} + \frac{\partial^2 TCS'}{\partial T^2} + \frac{\partial^2 TCS''}{\partial T^2} + \frac{\partial^2 TC3}{\partial T^2} > 0$$

so Eq. (5.15) is the convex function over $T > 0$ for any values of $\alpha_D$, $\alpha_M$, $\alpha_P$, $\alpha 3$, $\alpha_{S'P}$, $\alpha_{S'W}$, $\alpha_{S''W} \geq 1$

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