

# Cointegration modelling of climatic time series

Submitted by

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to the University of Exeter as a thesis for the degree of Doctor of Philosophy in  
Mathematics, September 2012

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# Abstract

This thesis has used bivariate time series models to investigate the long-run causal relationships between climatic variables. The cointegration approach, widely used in econometrics, has been shown to provide more reliable estimates for detection and attribution of trends in global mean temperature.

The traditional ordinary least squares (OLS) and total least squares (TLS) estimates from a static regression model are critically compared with the maximum likelihood (ML) estimates from a cointegrating vector autoregressive (VAR) model. Using synthetic data, generated by a simple stochastic model of the climate-carbon system, the estimates are compared against a known true value and evaluated in terms of key desirable statistical properties. Results show that the OLS estimates are strongly negatively biased, TLS estimates are less biased than OLS and positively biased compared to the VAR-ML estimates. TLS estimates are much more uncertain than those from the other approaches. VAR-ML estimates are less biased and more efficient compared to estimates from the traditional approaches.

Comparison has also been made using real historic global mean temperature data and climate model simulations from Coupled Model Intercomparison Project 5 (CMIP5) archive, and similar conclusions were found. All CMIP5 model runs were

found to have cointegrating relationship with historical observed temperature.

Another issue addressed in this thesis is the Granger causality between paleoclimate temperature and CO<sub>2</sub>. Different extensions of the VAR model were used to assess Granger causality between the two variables. This research has shown that two-way causality (feedback) is occurring between temperature and CO<sub>2</sub>, particularly during the glacial epochs. Impulse-response analysis was also carried out to quantify dynamic interactions between the variables. This showed that each variable reacted positively to a shock in another. For example, a 100ppmv increase in CO<sub>2</sub> can induce an increase of up to 4°C in temperature and a 1°C increase in temperature induces up to 2.3ppmv increase in CO<sub>2</sub> during glacial periods in particular. A shock to CO<sub>2</sub> during the warmer interglacial periods was seen to induce an explosive increase in both temperature and CO<sub>2</sub>.

# Acknowledgements

I consider myself fortunate to have been supervised by a multidisciplinary team of reputable and friendly academics: Prof. David Stephenson, Prof. James Davidson and Dr. Tim Jupp. This thesis would not have been possible without the close and consistent guidance of my principal supervisor, Prof. David Stephenson. I am also very grateful to Dr. Hugo Lambert, for providing me with climate data at the initial stage of my PhD, and to Dr. Theodoros Economou, for reading and commenting on some of the chapters in my thesis. I am very proud and honoured to have been part of the Exeter Climate Systems research centre.

Special thanks go to my wife, Eskendrawit Hailegiorgis, for her support and love at all times throughout my PhD. It is hard to list names of all the people who have contributed towards who I am today. My heartfelt thanks and appreciations go to all my family members, relatives, friends and all those who have helped me develop academically - from my primary school teachers to my postgraduate level teachers.

Finally, I would like to dedicate this thesis to my late father, Abate Turasie. He shaped my childhood life into the way I am today, in a very remote rural village of Ethiopia. Dad, I will always miss you.

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