Millikelvin Magnetisation studies of Low Dimensional Systems

Submitted by Tristan James Kershaw to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Physics August 2008.

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Abstract

This thesis presents a study of two-dimensional electron systems in GaAs-(Al,Ga)As heterojunctions and quasi-two-dimensional electron and hole systems in graphite within the quantum Hall effect regime of low temperature and high magnetic field. This thesis covers three main sets of experimental work as well as details of the experimental methods (chapter 2) used and the background theory behind the observed results (chapter 1).

The first experimental results presented in this thesis in chapter 3 focus on contactless measurement of the equilibrium magnetisation of sample A2268, a ten layer multiple quantum well sample. Fitting the shape of dHvA oscillations at various temperatures to different models for the density of states, various properties of the system can be estimated, such as the shape of the disorder-broadened density of states and the presence of a background density of states between the Landau levels.

Chapter 4 focuses on measurements of the decay of induced circulating currents in the quasi-dissipationless quantum Hall regime in two samples, V0049 and T73. The induced current is measured via contactless measurement of the associated magnetic moment. The magnitude of the induced current is found to be affected by the sweep rate of the magnetic field and also the distance of approach. The decay of the induced currents is observed at several temperatures and for different magnetic field sweep rates and distances of approach. Decays are observed for up to several days at time, far longer than previously possible. Information about the rate of decay can be used to build a picture of the decay mechanisms present in the quantum Hall regime. The presence of a power-law decay regime indicates many decay mechanisms contribute to the decay of a circulating current in the quasi-dissipationless quantum Hall regime.

Chapter 5 focuses on both contactless magnetometry and transport experiments carried out on a graphite sample. The experiments aim to confirm or dispute recent claims of Dirac fermions in graphite. Experiments are carried out at temperatures in the range 30 mK to ~4 K and at two different angles to the applied magnetic field. Phase analysis of both Shubnikov de Haas and de Haas van Alphen oscillations is used to distinguish
between normal and Dirac fermions. Observation of quantum Hall effect displays the presence of a half-integer quantum Hall staircase similar to that observed in graphene.
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