Probabilistic Controlled Airspace Infringement Tool

Yousra S. AL-Mathami

College of Engineering, Mathematic and Physical Sciences

University of Exeter

June 2012
Abstract

A current ground based safety net called Controlled Airspace infringement Tool (CAIT) is used by Air Traffic Controllers (ATCs) in the UK. It warns them if any aircraft within uncontrolled airspace penetrates the Controlled airspace (CAS) without an advance clearance from the ATC. This penetration or ‘Infringement’ is considered as a major concern to ATCs where it may cause a possible conflict or mid-air collision. A conflict is an event which one aircraft loses its minimum separation to another. A current deficiency of CAIT is that it only warns ATCs if the aircraft has already infringed CAS, this gives the ATC minimum time to react and avoid any conflict.

In this research, we investigate a model which warns ATCs of possible future infringements accurately. We implement two Kalman filters (KF) as our tracking tool, one for each flight mode: constant velocity and constant acceleration each of which has its state error covariance. Where the state error covariance measures the uncertainty in the aircraft’s estimated position, and is therefore important for accurately predicting the aircraft future position and since each aircraft has its own characteristic and journey type, a single parameterisation of the state covariance for all aircraft is unsuitable. Therefore, we learn these covariances in an online fashion at each time step to predict the future uncertainties more accurately. Given the two Kalman filters predictions and their error covariances, we use two methods to find the probability of infringement of CAS. The first method, proposed by Macdonald (2000), is called the shortest distance method. We extend this method to be able to find the probability of infringement when the prediction location is near a CAS corner by combining it with Monte Carlo sampling. A hybrid method is introduced to retain the efficiency of the shortest distance method with the accuracy of the Monte Carlo sampling.

We also used the switching Kalman filter (SKF) method proposed by Murphy (1997) to choose between the most appropriate Kalman filter at each time step. On testing on real tracks, the SKF was found to give superior predictions of the aircraft location, permitting better estimates of the probability of CAS infringement to be made.
Acknowledgements

I would like to thank NATS for providing us with the data which aided us throughout our research.

I specially would like to thank Richard Everson and Jonathan Fieldsend for their help and support and for providing me with the knowledge for which a student would wish to gain from his educator. Thank you mom and dad for your long distant support which made my study in Exeter feels like home.
# Table of Contents

1  Introduction ........................................................................................................... 7
   1.1 Overview ........................................................................................................... 7
   1.2 Research Problem ........................................................................................... 7
   1.3 Objectives ......................................................................................................... 8
   1.4 Research Structure .......................................................................................... 8
2  Background ........................................................................................................... 9
   2.1 Airspace ............................................................................................................ 9
   2.2 Radar ................................................................................................................ 10
   2.3 Current Infringement Detection ...................................................................... 12
   2.4 Hidden Markov Models ................................................................................... 13
       2.4.1 Discrete hidden states with discrete observations .................................... 15
       2.4.2 Forward-Backward Algorithm ................................................................. 17
       2.4.3 HMM learning [Baum-Welch Algorithm] ............................................... 19
2.5  Kalman Filter and Smoother ............................................................................. 20
       2.5.1 Background ............................................................................................... 20
       2.5.2 Kalman Filter Equations .......................................................................... 23
       2.5.3 Derivation of Kalman filter errors ............................................................ 25
       2.5.4 Kalman Smoother ..................................................................................... 26
       2.5.5 Kalman Smoother Equations ................................................................... 27
2.6  Literature Review on Conflict Detection .......................................................... 29
3  Probability of Infringement ................................................................................. 36
   3.1  Current Probabilistic Infringement Detection Tool ....................................... 36
   3.2  Probabilistic Infringement Detection Tool ..................................................... 40
       3.2.1 Monte Carlo Sampling ............................................................................. 41
       3.2.2 Hybrid Method ........................................................................................ 44
   3.3  Conclusion ....................................................................................................... 53
4  Prediction with Online Learning .......................................................................... 54
   4.1  Learning Using the Expectation-Maximization Algorithm .............................. 55
   4.2  Learning Model Error covariance with Different Kalman Filters .................... 67
       4.2.1 Constant Velocity Model ......................................................................... 69
       4.2.2 Constant Acceleration Model ................................................................... 70
   4.3  Further Testing on Synthetic Track ................................................................. 71
       4.3.1 Best errors for the whole track ................................................................ 72
       4.3.2 Best error at each time \( t \) ........................................................................ 75
   4.4  Testing on Real Tracks .................................................................................... 78
4.4.1 Constant velocity ................................................................. 81
4.4.2 Constant acceleration ......................................................... 87
4.5 Conclusion .............................................................................. 92
5 Switching Kalman Filter .............................................................. 94
  5.1 Why The Switching Kalman Filter ........................................... 95
  5.2 Implementation ...................................................................... 98
    5.2.1 Objective .......................................................................... 98
    5.2.2 Switching Process ............................................................. 99
  5.3 Testing .................................................................................... 104
  5.4 Conclusion .............................................................................. 113
6 Conclusion ................................................................................ 114
  6.1 Summary .............................................................................. 114
  6.2 Future Work Recommendation .............................................. 116
7 Bibliography .............................................................................. 117