Recovery and analysis of director profiles in liquid crystal cells

Submitted by

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.
Abstract

This thesis is concerned with the determination of the director profile within a nematic liquid crystal cell from optical experiments. The larger part of the work details the development of computational methods which can be used to find the director profile, and the application of these tools to the fully leaky guided mode experiment. In a second part, a simple conoscopic device is built, and then used to undertake a novel viscodynamic experiment.

In essence, the fully leaky guided mode experiments and its relatives measure the dependence of the transmission and reflection coefficients of a liquid crystal cell upon incident angle. It is simple enough to calculate these coefficients if the director profile is known, but experimentalists actually need to achieve the opposite. That is, having measured the transmission and reflection coefficients, they must determine the director profile. However, this turns out to be an ill-posed problem, and so some additional information about the director profile is required.

There is indeed an appropriate source of additional information - the continuum theory of nematic liquid crystals - and it is exploited here to develop two computational tools. In the first, it is used to adapt a mathematical technique, Tikhonov regularization, to both steady-state and time-dependent situations, so that director profiles can be recovered having made only weak assumptions about their behaviour. A second tool makes stronger assumptions and can be deployed after the first to estimate some of the unknown parameters which appear in the continuum theory.

These tools are use in the first instance to analyze data drawn from two fully leaky guided mode experiments. In the first experiment, a hybrid aligned cell was measured during AC switching, and from its data director profiles and several phenomenological parameters including four viscosities are determined. Following that, the DC switching of the same cell is studied, which turns out to be critically affected by the motion of tiny concentrations of charged impurities. Then, having noted that only limited information about the director profile can be recovered from even the most elaborate optical experiment, a conoscopy experiment is designed to recover it quickly. Following this approach, a previously unknown flow-induced transition between topologically distinct states in a homoetropically aligned cell is observed.
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It seems quite likely that a list of acknowledgments in any PhD thesis begins with the supervisor. I do think that Roy Sambles deserves that place more than most. Throughout my time as his student, other students complained to me that their own supervisors were so elusive that some kind of exclusion principle was to be suspected. But if I managed to go a whole week without a discussion with Roy, I was probably on holiday.

Tim Taphouse, who started his PhD two years before me, also deserves special credit. Without his experimental data I might well be presenting tests of the computational techniques developed in this thesis against synthetic data - a serious crime, perhaps. Thanks are also due to Fuzi Yang and Pete Cann, who built a number of the samples I used in chapter 7, including one at about 3 hours notice after a mishap.

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Lowering himself suddenly to his knees he placed his right eye at the keyhole, and controlling the oscillation of his head and the vagaries of his left eye (which was for ever trying to dash up and down the vertical surface of the door), he was able by dint of concentration to observe, within three inches of his keyholed eye, an eye which was not his, being not only of a different colour to his own iron marble but being, which is more convincing, on the other side of the door.

— Mervyn Peake, Titus Groan
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