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Vote or Shout

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Abstract

We examine an environment with n voters each with a private value over two alternatives. We compare the social surplus of two mechanisms for deciding between them: majority voting and shouting. In majority voting, the choice with the most votes wins. With shouting, the voter who shouts the loudest (sends the costliest wasteful signal) chooses the outcome. We find that it is optimal to use voting in the case where n is large and value for each particular alternative of the voters is bounded. For other cases, the superior mechanism depends upon the order statistics of the distribution of values.

Key words: Voting, Lobbying, Order Statistics.

1 Introduction

Voting is one of the pillars of democracy.[1,2] While voting is desirable for determining a country's government, it is less clear whether policies should be decided by direct voting (referendum) or indirectly through elected politicians which then may incorporate the use of petitions or lobbying activity. While majoritarian voting procedures such as first past the post aggregates opinions of the individual voters, they have the disadvantage that they ignore how much an individual voter cares about the issues (see Chakravarty, Kaplan and Myles, 2010).

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[1] The definition of democracy is provided by the U.S. Department of State at http://usinfo.org/mirror/usinfo.state.gov/products/pubs/whatsdem/whatdm2.htm

[2] Voting procedures are also used in dictatorships. For instance, a dictator may make use of committees in order to prevent an individual from single-handedly reaching a decision. With such commitees, decisions are made through voting (see Tullock, 1998).

An alternative method of reaching a group decision is by allowing would-be voters to send a signal of how much they care: we call this a *shout*.[3] With this method, we would expect that voters with strong preferences or special interest groups to have greater influence on the outcome than with voting. This is due to the ability of voters with more extreme preferences to send a stronger signal by shouting louder. Such undue influence is not necessarily harmful; shouting may be welfare enhancing over voting since under voting the outcome can be determined by a large number of voters that do not strongly care about the outcome or vote without any information about the specific issues.[4] The objective of this paper is determine under which conditions, if any, *shouting* leads to a more efficient solution than voting.[5]

In our analysis, we make three key modelling assumptions. First, each voter has a private valuation over the outcome of a decision. Second, while in order for shouting to act as a signal for a voter it must be costly to that voter, we also assume that it is completely socially wasteful in that no one benefits from the noise of shouting.[6] Hence, we have in mind a class of problems where voters should not be able to buy votes (as with shareholders of companies), but rather one where there would be a moral repugnance if the decision could be bought such as with a safety law, jury verdict, a worker promotion or hiring decision.[7,8] Third, we assume that shouting does not add, therefore, only he who shouts the loudest is heard. From this, one can see that our notion of shouting is that of a contest.[9]

[3] Shouting can also be thought of as lobbying. See the discussion in the conclusion.

[4] Tullock gives the example that according to Pew poll in 1996 elections only 75% of the voters claimed that they were well informed enough to select the candidates (Tullock, 1998, page 148-149).

[5] We follow papers such as Börgers (2004) in making Pareto comparisions of voting and shouting by analyzing all the voters' ex-ante expected utility. Hence, we say a mechanism is more efficient if all the voters have higher ex-ante expected utility. In doing so, we also depart from the pairwise independence condition of Arrow (1951) in that we claim if two voters (very) weakly prefer A to B and one voter strongly prefers B to A, then society should choose B, while if the strength of preferences is the same for all voters, then society should choose A.

[6] See Austen-Smith and Wright (1992) as one of the earlier papers where lobbying activity provides information to the legislators.

[7] Alternatively, allowing votes to be bought or sold could create an opportunity for exploitation. Thus, even if voting buying were accepted morally, there could be an economic justification for banning such behaviour. For instance, a land owner may significantly gain from rezoning to permit construction of large luxury apartments buildings; however, these building could block the view of neighboring buildings and their grandiose size could annoy the populace. With vote buying, this landowner could profit by buying votes, even if him winning was socially inefficient due to the externalities. Dekel, Jackson and Wolinsky (2008) model vote buying when it is socially acceptable using a sequential game.

[8] Note that acceptability may change with time. For instance, between 1683 and 1871, commissions were often bought and sold in the British army.

[9] While it may be also worthwhile to investigate when shouting adds, doing so would be modelling shouting as a public good (for those who favour the same outcome). This would also introduce free-rider problems into shouting. Because of this, we feel that non-additive shouting is the cleanest case to examine.

In this paper, we compare the surplus of voting to that of shouting. We find that it is optimal to use voting in the case where the number of voters is large and value of each voter is bounded. Furthermore, if the hazard rate is increasing, then, for any n, voting is superior to shouting if the expected value of a voter (for his preferred option) is higher than two-thirds the expected maximum of two voters' values. More generally, for n voters, voting is superior to shouting if and only if the expected value times $\frac{n!!}{(n-1)!!}$ if n is odd ($\frac{(n-1)!!}{(n-2)!!}$ if n is even) is higher than the expected highest value minus the expected second-highest value.[10]

The general intuition for our results is that voting has the advantage of lower costs and aggregating everyone's rough preference. Shouting has the advantage of taking into account strength of preference but the disadvantage of being wasteful. For an illustration, take the discrete distribution of there being a p chance of having a value of 1 and a $(1-p)$ chance of having value 0. The expected value is p, while for two voters, the expected difference between the highest and second highest value is $2p(1-p)$. Hence, shouting is superior if and only if $p \leq 2p(1-p)$ or $p \leq 1/2$. This shows that when there is a lower likelihood of an extreme value, a high-value voter can then shout and win at a lower cost.

The structure of the paper is as follows. In the next section, we will describe the model and equilibrium. In section 3, we will specify the results and in the last section, we conclude.

2 Model

2.1 Description

In our model, there are $n \geq 2$ voters that as a group must decide between two alternatives. Let us call the two alternatives A and B. Each voter has a utility (value v_i) for B over A that is privately known. This difference is drawn independently on $V = [-\overline{v}, \overline{v}]$ with negative numbers implying that the voter prefers A to B. The cumulative distribution of values is identical for each voter and denoted by G. We assume that the distribution is symmetric, that is, $G'(x) = G'(-x)$. Denote F as the conditional distribution given that v is positive, that is, $F = (G(x) - G(0))/(G(\overline{v}) - G(0))$.

Given the above group decision problem, we consider two possible methods for selecting an alternative: voting by majority rule or selection by shouting. Under majority rule, each voter

[10] Note that the double factorial, $n!!$, is either all strictly positive even numbers up to n multiplied together or all strictly positive odd numbers up to n multiplied together depending upon whether n is even or odd.

costlessly casts a ballot for his preferred option. Choice A wins if the number of votes it receives, denoted by $\#_A$, is strictly greater the number of votes choice B receives, denoted by $\#_B$. Choice B wins $\#_B > \#_A$. There is a tie if $\#_B = \#_A$ and in such a case, the winner is determined randomly with equal probability. Under shouting, each voter i chooses an alternative to shout for and a level to shout. We use s_i to represent both by having the absolute value as the strength of shouting and the sign as the chosen option to shout for (positive for option B and negative for option A). By shouting, the utility of voter i is lowered by $|s_i|$. The voter with the loudest individual shout chooses.

2.2 Equilibrium and Social Surplus

Voting

If voting is used, in equilibrium, voter i will vote for his preferred candidate: i.e., for B, if $v_i > 0$ and A if $v_i < 0$. To examine social surplus under such an equilibrium, first consider three voters. There are two possible margins of victory: all three voters are in agreement or there is a two-to-one majority. In the case where all three voters are in agreement, the expected sum of values is $3E[v|v > 0]$. When there is a two-to-one majority, the expected surplus is $E[v|v > 0]$. Combining these yields the social surplus (the ex-ante expected utility):

$$\begin{aligned}(1/4)3E[v|v &> 0] + (3/4)E[v|v > 0] \\ &= \frac{3}{2}\int_0^{\overline{v}} vdF.\end{aligned}$$

For example, if F is uniform on $[0, 1]$, then the social surplus is 3/4. In a similar manner, we find the social surplus from voting in the case of n voters is:

$$S_{vote} \overset{def}{=} \sum_{i=1}^{n} i \cdot w_i E[v|v > 0] = \sum_{i=1}^{n} i \cdot w_i \cdot \int_0^{\overline{v}} vdF,$$

where w_i is the chance of winning by i votes and given by

$$w_i = \begin{cases} 0 & \text{if } n - i \text{ is odd,} \\ \binom{n}{(n-i)/2}\frac{1}{2^{n-1}} & \text{otherwise.} \end{cases}$$

Shouting

Now consider the alternative mechanism of selecting an alternative for the group: shouting. According to this the voter who shouts the loudest gets heard and his choice wins. Each voter will have a shouting strategy of a shouting function based upon value, $s_i : V \mapsto \mathbb{R}$. A Bayes-Nash equilibrium is the set of shouting functions $\{s_i\}$, such that given the other shouting functions, no voter has incentive to change his shouting function.

Let us look at the social surplus in a symmetric equilibrium where the equilibrium is symmetric in two distinct ways. First, all voters use the same shouting function, s. Second, the shouting function is symmetric w.r.t. to either option, that is, $s(v) = -s(-v)$. Since only the loudest voter counts, the gains from the outcome is the expected value of the highest voter: $\int_0^{\overline{v}} v dF^n$ (due to independence, the expected value of the winning option for the other voters is zero). The cost incurred from the shouting equilibrium is $n \int_0^{\overline{v}} s(v) dF$. Hence, the social surplus from shouting is

$$S_{shout} \stackrel{def}{=} \int_0^{\overline{v}} v dF^n - n \int_0^{\overline{v}} s(v) dF. \tag{1}$$

3 Results

In this section, we determine under which conditions the social surplus of shouting is superior to the social surplus of voting and vice-versa. We start by writing S_{shout} in terms of order statistics. In doing so, we will be able to investigate certain properties as a function of the number of voters.

Lemma 1 *The social surplus of shouting, S_{shout}, equals $X_{n,n} - X_{n-1,n}$, where $X_{i,j}$ denotes the $i-$th-order statistic of j random variables drawn iid from distribution F.*

Proof. If we have n voters and the group chooses according to the loudest member of the group then each voter will face the following problem,

$$\pi(v) = \max_s F(v(s))^{n-1} v - s.$$

This is the same problem that a bidder faces in a standard all-pay auction of incomplete information. Using the envelope theorem $\pi'(v) = F(v)^{n-1}$. Thus,

$$\pi(v) = \int_0^v F(v)^{n-1} dv = F(v)^{n-1} v - s.$$

This implies

$$s(v) = F(v)^{n-1} v - \int_0^v F(v)^{n-1} dv.$$

Substituting this into equation (1) yields

$$S_{shout} = n \int_0^{\overline{v}} \int_0^v F(v)^{n-1} dv dF. \tag{2}$$

Integration by parts yields

$$S_{shout} = n \int_0^{\overline{v}} F(v)^{n-1} dv - n \int_0^{\overline{v}} F(v)^n dv. \tag{3}$$

This also equals the expected value of the highest voter minus the expected value of the second highest voter. This difference is

$$\int_0^{\overline{v}} v dF^n - \int_0^{\overline{v}} v d(F^n + n(1-F)F^{n-1}) =$$
$$n \int_0^{\overline{v}} v dF^n - n \int_0^{\overline{v}} v dF^{n-1}.$$

Integration by parts yields (3). Note that the expected social surplus by shouting will equal the expected surplus of the highest bidder in a standard auction framework. ■

Lemma 2 $X_{n,n} - X_{n-1,n}$ *is decreasing (increasing) in* n *if the hazard rate,* $F'/(1-F)$*, is increasing (decreasing) .*

Proof. From (2), $X_{n,n} - X_{n-1,n} = n \int F^{n-1} \cdot \frac{1-F}{F'} dF$. The derivative of this w.r.t. n is $\int (n \cdot \ln F + 1) F^{n-1} \cdot \frac{1-F}{F'} dF$. Notice that $\int_0^1 (n \cdot \ln F + 1) F^{n-1} dF = F^n \ln F|_0^1 - \int_0^1 F^{n-1} dF + \int_0^1 F^{n-1} dF = 0$. Since $(n \cdot \ln F + 1) F^{n-1}$ is increasing in F, then if $\frac{1-F}{F'}$ is increasing (decreasing), the derivative is positive (negative). Thus, if $F'/(1-F)$ is increasing (decreasing), the derivative is negative (positive) and $X_{n,n} - X_{n-1,n}$ is decreasing (increasing) in n. ■

The first lemma shows that the social surplus of shouting is the expected value of the highest-valued voter, $X_{n,n}$, minus the waste of signalling which is the expected value of the second highest-valued voter, $X_{n-1,n}$. The second lemma shows the sign of the hazard rate determines whether the gain of an increased highest value outweighs the increased cost of signalling. Together, these two lemmas show that when the hazard rate is increasing (decreasing), social surplus of shouting is decreasing in the number of voters. Next we show properties of voting in relation to n.

Lemma 3 *S_{vote} is weakly increasing n and* $\lim_{n\to\infty} S_{vote} = \infty$.

Proof. Notice that $\sum_{i=1}^{n} i \cdot w_i$ is the expected absolute value of the distance of the random walk from zero. We make use of known results in this proof. (See Weisstein, 2010, for an excellent review of one-dimensional random walks.) This equals

$$S_{vote} = \int_0^{\overline{v}} v dF \cdot \begin{cases} \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even,} \\ \frac{n!!}{(n-1)!!} & \text{if } n \text{ is odd.} \end{cases} \tag{4}$$

This is weakly increasing in n: if n is odd, S_{vote} does not increase going to $n+1$. The expression $\frac{n!!}{(n-1)!!}$ increases by a factor of $\frac{n+2}{n+1}$ going to $\frac{(n+2)!!}{(n+1)!!}$. As $n \to \infty$, S_{vote} approximates $\int_0^{\overline{v}} v dF \cdot \sqrt{\frac{2n}{\pi}}$. Hence, $\lim_{n\to\infty} S_{vote} = \infty$. ■

Unlike shouting, the surplus from voting unambiguously improves with the number of voters. We can now compare the two surpluses.

Proposition 1 *For large n, if $\overline{v}$ is bounded, then voting is superior to shouting.*

Proof. Since S_{vote} is increasing and $\lim_{n\to\infty} S_{vote} = \infty$, there is an n^* where $S_{vote} > \overline{v}$ such that for all $n \geq n^*$, $S_{vote} > \overline{v} \geq S_{shout}$. ■

The intuition of the prior proposition is that as the number of voters increase, the expected number of voters favoring one option over another increases and there is no bound to this increase. Thus, the gains from choosing the most popular option goes up and eventually surpasses the gains from shouting since these gains is always limited to one voter's value. In the following proposition, we can compare voting to shouting for any particular n.

Proposition 2 *For n voters, voting is superior to shouting if and only if $X_{1,1} \cdot \frac{n!!}{(n-1)!!} \geq X_{n,n} - X_{n-1,n}$ if n is odd, otherwise if and only if $X_{1,1} \cdot \frac{(n-1)!!}{(n-2)!!} \geq X_{n,n} - X_{n-1,n}$ if n is even.*

Proof. This follows directly from Lemma 1 and equation (4). ■

Corollary 1 *For two voters, voting is superior to shouting if and only if the expected value of a voter is higher than two-thirds the expected maximum of the two voter's values.*

Proof. For two voters, $S_{vote} = \int_0^{\overline{v}} vdF$ and $S_{shout} = 2\int_0^{\overline{v}} F(v)dv - 2\int_0^{\overline{v}} F(v)^2 dv$. Integration by parts yields, $S_{shout} = 2\int_0^{\overline{v}} vdF^2 - 2\int_0^{\overline{v}} vdF$. Comparison, yields voting is superior iff $\frac{2}{3}\int_0^{\overline{v}} vdF^2 < \int_0^{\overline{v}} vdF$. (Note that this does not imply $X_{n,n} - X_{n-1,n} = 2/3 \cdot X_{n,n}$.) ■

Proposition 3 *If the hazard rate is increasing and $X_{1,1} \geq X_{2,2} - X_{1,2}$, then voting is superior for all $n \geq 2$.*

Proof. This follows directly from Proposition 2 and Lemma 2. ■

Note as with the above Corollary, $X_{1,1} \geq X_{2,2} - X_{1,2}$ is equivalent to the expected value of a voter being higher than two-thirds the expected maximum of two voter's values.

The following example shows when the social surplus under shouting would be greater than that under voting.

Example 2 $n = 2$, $F(v) = v^{\alpha}$, $0 < \alpha < 1/2$.

For shouting, we have:

$$S_{shout} = n\int_0^{\overline{v}} F(v)^{n-1}dv - n\int_0^{\overline{v}} F(v)^n dv = n\int_0^1 (v^{\alpha})^{n-1}dv - n\int_0^1 (v^{\alpha})^n dv =$$

$$\frac{n}{\alpha(n-1)+1} - \frac{n}{\alpha n+1} = \frac{\alpha n}{(\alpha(n-1)+1)(\alpha n+1)} = \frac{2\alpha}{(\alpha+1)(2\alpha+1)}.$$

For voting, the surplus is $S_{vote} = \sum_{i=1}^{n} i \cdot w_i \cdot \int_0^{\overline{v}} vdF = \int_0^1 v \cdot \alpha(v^{\alpha-1})dv = \frac{\alpha}{\alpha+1}$. Thus,

$S_{shout} > S_{vote} \iff \frac{2}{2\alpha+1} > 1 \iff 1/2 > \alpha$.

When $\alpha < 1/2$ there is a higher likelihood of a large mass of low value voters. In such a case when there are large number of poorly informed voters, who do not value the decision too highly, then a decision by shouting is more socially efficient. From data cited in the New York Times regarding minimum wage legislation, while 84% of American citizens believed that minimum wage should be increased, 22% actually knew what the current minimum wage was.[11] In such a case it is

[11] This example is taken from Tullock (1998). For the story check New York Times April 19, 1996. http://www.nytimes.com/1996/04/19/us/notebook-the-minimum-wage-a-portrait.html?scp=2&sq=april%2019%201996&st=cse

probable that decision making by shouting or lobbying with the better informed legislators would be more efficient than direct vote on the issue by uninformed voters.

Next we provide an example where voting is more efficient than shouting.

Example 3 $n \geq 2$, $F(v) = v^{\alpha}$, $\alpha \geq 1$.

From the previous example, when $n = 2$ and $\alpha \geq 1$, voting is superior (actually, voting is superior for $1 \geq \alpha \geq 1/2$, as well). The hazard rate $F'/(1 - F)$ is increasing for all $\alpha \geq 1$. Hence, voting is superior for all n.

Voting is more efficient in case it is more likely that there is a large number of voters who value the outcome highly and therefore will be better informed. Direct voting (referendum) is observed in number of places including Switzerland and California. In most cases of direct voting a significant number of signatures are required to put a particular issue for vote in the ballot. As a result, voters here are likely to have strong feelings regarding issues which are put up for direct voting in a referendum. For example, Oregon passed a highly divisive issue, assisted suicide, into law in 1994 in a referendum.

4 Conclusion

In this paper, we delineate the conditions when voting is superior to shouting and vice-versa by means of order statistics. It is also possible to interpret these conditions in terms of likelihood of voters having extreme values. Consider the example of an academic department deciding on the date for a departmental picnic. For most members, most dates will be suitable (or the member would not lose significant utility missing it). Though less likely, it is possible that a member would have a conflict that cannot be resolved and this member has a strong preference for attending. In such a case, a shouting mechanism will result in the preferred choice of this member. If instead the department used voting, then the decision would be dominated by those without conflicts or strong interest increasing the likelihood that the member with a tight schedule and a strong interest would not be able to attend.

Now instead of choosing a department picnic date if the department were considering hiring a full professor and had to choose from a variety of fields. In this case, we argue that the decision should be made by voting and not shouting. In such a case, it is more than likely that department members would have extreme values or special interests for particular candidates (for instance, fields such as experimental economics or econometric theory). This increases the chance of a high degree of shouting which should favor simple majority voting.

Our notion of shouting mirrors the process of lobbying. Normally, lobbying is thought to be socially wasteful and policies for reducing it have been analyzed, for instance putting a cap on the amount of lobbying (see Che and Gale, 1998, 2006, and Kaplan and Wettstein, 2006). This paper shows that in fact while wasteful in itself, lobbying can be a useful tool for making decisions since it signals value.

Another way to look at our results, which is left for future work is in terms of common value and private value of the choices. If the alternatives have common values for the voters then cost of collecting signals through shouting can be avoided and the decision can be reached by voting without much loss of allocative efficiency. While in case of private values, the costly signals sent through shouting can play an important role in reaching a better decision.

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