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Relative Price Distortions and Inflation Persistence*

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ABSTRACT

Many sticky-price models suggest that relative price distortion is one of the major costs of inflation. We show that this resource misallocation is costly even at quite low rates of inflation. This is because inflation strongly affects price dispersion which in turn has an impact on the economy qualitatively similar to, and of the order of magnitude of, a negative shift in productivity. Similarly, the utility cost of price dispersion is large. We incorporate price dispersion in a linearized model. This radically affects how shocks are transmitted through the economy. Notably, a contractionary nominal shock has a persistent, negative hump-shaped impact on inflation, but may have a *positive* hump-shaped impact on output. Observed persistence in the policy rate is not due to the policy rule *per se*.

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1. Overview

In this paper we investigate the macroeconomic implications of relative price distortions as this is where many, though not all, sticky-price models locate the costs of inflation[1]. The first thing we do is quantify how costly price dispersion is in a standard macroeconomic model with imperfect competition and price rigidity as in Calvo (1983). Despite being very costly in welfare terms, price dispersion is generally considered to be a term of second-order importance in linearized models. That is why many economists conclude that the direct impact of price dispersion on welfare is small (e.g., Canzoneri, Cumby and Diba, 2004). However, in economies with, say, trend inflation of $2-3\%$, no indexation and a degree of nominal price inertia, price dispersion, viewed through the lens of our simple model, will be an important (first-order) variable. Therefore, we develop a log-linear approximate model which includes price dispersion as a first-order term. We find that a negative nominal shock under an interest rate rule has an effect similar to a *positive* productivity shock, driving output *up* and inflation *down*. But, unlike in the model with no price dispersion, these responses are often persistent and hump-shaped. We trace these and other surprising results to the fact that the economy is being perturbed from a steady state that is distorted by price dispersion.

1.1. The analysis in more detail

In the basic sticky-price model that we set out, private consumption is not maximized in the presence of relative price distortion ('price dispersion', for short), for a given amount of nominal expenditure. The reflection on the supply-side of

[1] For example, in the sticky-price model of Rotemberg (1982) all firms charge the same price, even though that price differs from the price that would have been charged had price changes not been costly. So, there is no dispersion of prices *across* firms which is the focus of this paper.

the economy of that reduction in consumption is that labour is allocated away from 'high-price' firms to 'low-price' firms. Due to diminishing returns, average labour productivity is lower than it would be were all firms facing the same level of demand. In a sense, then, at the aggregate level the economy uses too much labour to produce a given level of output. Given increasing disutility of labour, there is upward pressure on the equilibrium real wage and hence the economy incurs higher total costs of production compared with an economy with no price dispersion.

In short, for a given output level, the economy with price dispersion behaves in a manner *qualitatively* similar to a low productivity economy, needing to employ more labour input to meet demand. We demonstrate this argument formally in Section 3 by forming a Ramsey problem which allows us easily to inspect the general equilibrium impact of price dispersion. In section 4 we then show that price dispersion also has an impact on outcomes *quantitatively* of the order of magnitude of a negative shift in productivity. We observe that price dispersion is itself sensitive even to relatively low rates of inflation and increases sharply in the level of inflation. In section 5 we then enquire, following Lucas (1987), what the consumption-equivalent impact is of a given level of price dispersion and confirm that it is indeed very costly. Of course, unlike productivity, price dispersion is not exogenous and so in section 6 we analyze the impact that price dispersion has on the optimal monetary policy. We recover a result like Yun's (2005), demonstrating that in the presence of price dispersion, disinflation may be the optimal policy. Typically, linearized models do not come to that conclusion as price dispersion is absent from these models. We indicate why even a full second-order approximation to our model's equations would not recover Yun's or our result and would continue to conclude that the impact of price dispersion on welfare is quantitatively very small.

In order to analyze the impact of price dispersion on dynamics, in sections 7 and 8 we develop our linearized model around a non-indexed, inflationary steady-state[2]; as a result, price dispersion is of first-order significance. We simply take as given that trend inflation is positive. We find that the impact of a persistent, negative nominal shock appears similar to a persistent, positive productivity shock, which is consistent with our analysis in sections 3 and 4. However, there is a marked difference between the models with and without price dispersion: We find that inflation follows a hump-shaped response following *both* a nominal and real shocks in the model with price dispersion; its maximal response is not in the period following the shock. Interest rates also respond more gradually following shocks in the model with price dispersion. Underlying these results is the fact that any shock which decreases price dispersion will impart upward momentum to output and downward momentum to inflation, and because price dispersion is a persistent process, this momentum will itself be persistent. Section 9 offers some conclusions.

2. The Model

In this section we present a standard sticky-price model. There are a large number of identical agents in the economy who evaluate their utility in accordance with the following criterion:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t(i)) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{\lambda_t}{1+v} \int_i N_t^{1+v}(i) di \right). \tag{2.1}$$

[2] Some recent contributions have incorporated indexation of some prices as a means to impart peristence into inflation. However, as Blanchard and Gali (2005) note, there is probably little empirical justification for this assumption in low inflation economies. Indexing in this manner also seems to make price dispersion a second-order term.

E_t denotes the expectations operator at time t, β is the discount factor, C_t is consumption and $N_t(i)$ is the quantity of labour supplied to firm i; labour is firm specific. $\upsilon \geq 0$ measures the labour supply elasticity while λ_t is a 'preference' parameter.

Consumption is defined over a basket of goods and indexed by i, in the manner of Spence-Dixit-Stiglitz

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{2.2}$$

The average price-level, P_t, is known to be

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{2.3}$$

The demand for each good is given by

$$Y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d, \tag{2.4}$$

where $p_t(i)$ is the nominal price of the final good produced by firm i and Y_t^d denotes aggregate demand. We assume that the labour market is such that all the firms pay the same real wage for the same labour. As a result, we may write $w_t(i) = w_t, \quad \forall i$. Further, as in Benigno and Woodford (2003), all households provide the same share of labour to all firms. It follows that we may write the agent's flow budget constraint as

$$\int_0^1 p_t(i)c_t(i)di + B_t = [1 + i_{t-1}] B_{t-1} + W_t N_t(1 - \tau_t^h) + \Pi_t. \tag{2.5}$$

As all agents are identical, the only financial assets traded in equilibrium will be those issued by the fiscal authority. Here B_t denotes the nominal value at the end of date t of government bond holdings, $1 + i_t$ is the nominal interest rate on this 'riskless' one-period nominal asset, W_t is the nominal wage in period t, and Π_t is profits remitted to the individual. We denote the tax rate applied to labour income by τ_t^h. We also impose the following familiar restriction on the equilibrium plan of the representative agent:

$$\lim_{J\to\infty} E_t \prod_{j=0}^{J} R_{t+j-1} B_{t+J} \geq 0, \qquad R_t \equiv (1+i_t)^{-1}. \tag{2.6}$$

Hence, the necessary conditions for an optimum include:

$$N_t = \left[w_t \left(1-\tau_t^h\right) (\lambda_t C_t)^{-1} \right]^{1/v}; \tag{2.7}$$

and

$$E_t \left\{ \frac{\beta C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} = \frac{1}{1+i_t}. \tag{2.8}$$

The complete markets assumption implies the existence of a unique stochastic discount factor,

$$Q_{t,t+k} = \beta \frac{C_t P_t}{C_{t+k} P_{t+k}} \tag{2.9}$$

where

$$E_t \{Q_{t,t+k}\} = E_t \prod_{j=0}^{k} \frac{1}{1+i_{t+j}}.$$

2.1. Representative firm: factor demand

Labour is the only factor of production. Firms are monopolistic competitors who produce their distinctive goods according to the following technology

$$Y_t(i) = A_t \left[N_t(i)\right]^{1/\phi}, \tag{2.10}$$

where $N_t(i)$ denotes the amount of labour hired by firm i in period t, A_t is a stochastic productivity shock and $\phi > 1$.

The demand for output determines the demand for labour. Hence we find that

$$N_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta\phi} \left(\frac{Y_t}{A_t} \right)^{\phi}. \tag{2.11}$$

It follows that the total amount of labour demanded will be

$$N_t = \int N_t(i)di = \left(\frac{Y_t}{A_t}\right)^{\phi} \int \left(\frac{P_t(i)}{P_t}\right)^{-\theta\phi} di = N_t^* \Delta_t \langle -\theta\phi \rangle , \tag{2.12}$$

where we define $\Delta_t \langle -\theta\phi \rangle \equiv \Delta_t$ as our measure of price dispersion:

$$\Delta_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta\phi} di. \tag{2.13}$$

From an empirical point of view this is not a natural measure of price dispersion and so in section 4.1 we shall map this into the coefficient of variation for prices. In this simple set-up, as we confirm below, were all firms given the chance to re-price at any instant in time, they would all choose the same price. In that case, if all prices are similar, then for a given level of output the labour supply would be

$$N_t^* = \left(A_t^{-1} Y_t\right)^{\phi} . \tag{2.14}$$

If we substitute (2.14) into (2.12) we receive

$$N_t = \left(A_t^{-\phi} \Delta_t\right) Y_t^{\phi}. \tag{2.15}$$

This corresponds to the amount of labour which would be employed to produce quantity Y_t should prices not be equal across industries. Finally, it follows that the equilibrium wage can be written as

$$w_t = \lambda_t \frac{1}{1-\tau_t^h} C_t \Delta_t^v \left(\frac{Y_t}{A_t}\right)^{\phi v} . \tag{2.16}$$

2.2. Representative firm: price setting

We adopt the Calvo (1983) approach to price-stickiness. This is a convenient and familiar approach to modelling sticky prices but the same basic issues that we are interested in would seem to arise in any model where price dispersion is present.

Each period a measure, $1-\alpha$, of firms is allowed to adjust prices. Those firms choose the nominal price which maximizes their expected profit given that they may have to charge the same price in $k-$periods time with probability α^k.

Importantly, we are assuming that firms are cost-takers and that they do not anticipate the change in equilibrium wages in reaction to their price setting decision, evident from (2.16). The price setting problem can then be characterized as follows:

$$\max_{p'_t} E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left(Y_{t+k} \left(\frac{p'_t}{P_{t+k}} \right)^{1-\theta} - w_{t+k} A_{t+k}^{-\phi} Y_{t+k}^{\phi} \left(\frac{p'_t}{P_{t+k}} \right)^{-\theta\phi} \right), \qquad (2.17)$$

where p'_t is the price chosen by firms which update prices. There is no need to index this nominal price on i as it is clear that this will be a function solely of variables that affect all firms symmetrically. The first order condition with respect to p'_t implies

$$\left(\frac{p'_t}{P_t} \right)^{1+\theta(\phi-1)} = \left(\frac{\theta}{\theta-1} \right) \frac{\sum_{k=0}^{\infty} (\alpha\beta)^k E_t C_{t+k}^{-1} \left[\phi w_{t+k} A_{t+k}^{-\phi} Y_{t+k}^{\phi} (P_t/P_{t+k})^{-\theta\phi} \right]}{\sum_{k=0}^{\infty} (\alpha\beta)^k E_t C_{t+k}^{-1} \left[Y_{t+k} (P_t/P_{t+k})^{1-\theta} \right]}. \qquad (2.18)$$

The price index then evolves according to the law of motion

$$P_t = \left[(1-\alpha)\, p_t'^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}. \qquad (2.19)$$

Because the relative prices of the firms that do not change their prices in period t fall by the rate of inflation, we may derive a law of motion for our measure of price dispersion,

$$\Delta_t = \alpha \Delta_{t-1} \pi_t^{\theta\phi} + (1-\alpha) \left(\frac{p'_t}{P_t} \right)^{-\theta\phi}. \qquad (2.20)$$

2.3. Fiscal Authorities

The government purchases goods in the same proportions as do private agents. These purchases yield no utility to agents nor do they boost the productive potential of the economy. Further, government expenditure is assumed exogenous and stochastic. For now, we assume that government raises revenue solely through taxes on labour income. We assume that the government can borrow by issuing a one period risk-free nominal bond. The nominal value of government debt evolves according to the law of motion

$$B_t = (1 + i_{t-1})B_{t-1} - S_t. \tag{2.21}$$

B_t and i_t were defined above, and S_t is the (primary) budget surplus,

$$S_t = \tau_t^h W_t N_t - G_t P_t.$$

We assume that the expected path of government surpluses satisfies an intertemporal solvency condition, by design, for all feasible paths of the model's endogenous variables. There is a sequence of intertemporal constraints for all t of the following sort,

$$(1 + i_{t-1})\, B_{t-1} = E_t \sum_{k=0}^{\infty} Q_{t,t+k} \left(S_{t+k}\right), \tag{2.22}$$

which we can simplify as

$$(1 + i_{t-1})\, \frac{b_{t-1}}{C_t \pi_t} = E_t \sum_{k=0}^{\infty} \beta^k \frac{1}{C_{t+k}} \left(\tau_{t+k}^h w_{t+k} N_{t+k} - G_{t+k}\right), \tag{2.23}$$

and where b_{t-1} is a measure of the real value of debt inherited from the previous period, $b_{t-1} = B_{t-1}/P_{t-1}$, while π_t is inflation, $\pi_t = P_t/P_{t-1}$.

Associated with this sequence, is a sequence of transversality conditions. This sequence is ultimately related to the incompleteness of (government debt) markets (see Hahn, 1971). Finally, there is an economy-wide resource constraint such that total output is equal to (private plus government) consumption:

$$Y_t = C_t + G_t. \tag{2.24}$$

2.4. A policy problem

We now formulate the policy problem as a search for the best macroeconomic policy for a competitive equilibrium defined as follows:

Definition 2.1. *A competitive equilibrium is defined as a set of plans,* $\left\{C_{t+k}, Y_{t+k}, N_{t+k}, w_{t+k}, \Delta_{t+k,} B_{t+k}, p'_{t+k}, P_{t+k}, \right\}_{k=0}^{\infty}$ *, given initial conditions,* $\{b_{t-1}, i_{t-1}, \Delta_{t-1}, P_{t-1}\}$ *, and expected dynamics of future policy variables,* $\{E_t P_{t+k}, E_t \tau_{t+k}, \}_{k=0}^{\infty}$ *, and exogenous shocks,* $\{E_t A_{t+k}, E_t G_{t+k}, E_t \lambda_{t+k}\}_{k=0}^{\infty}$ *, and satisfying conditions (2.15), (2.16), (2.18), (2.19), (2.20), (2.23) and (2.24).*

We are now able to set out the Ramsey problem in Proposition 2.1:

Proposition 2.1 *The Ramsey plan is a choice of state contingent paths for the endogenous variables* $\left\{P_{t+k}, C_{t+k}, \Delta_{t+k}, \tau^h_{t+k}, \right\}_{k=0}^{\infty}$ *from date t onwards given* $\{E_t A_{t+k}, E_t G_{t+k}, E_t \lambda_{t+k}, b_{t-1}, i_{t-1}, \Delta_{t-1}, P_{t-1}\}_{k=0}^{\infty}$ *so as to maximize social welfare function (2.25) subject to constraints (2.26)-(2.28):*

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left(\log\left(C_{t+k}\right) - \lambda_{t+k} \Delta_{t+k}^{v+1} \frac{\left(A_t^{-1}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi}}{v+1} \right); \qquad (2.25)$$

subject to:

- *Solvency Constraint*

$$\begin{aligned} & \left(1+i_{t-1}\right) \frac{b_{t-1}}{C_t \pi_t} \qquad (2.26) \\ = \quad & E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{\tau^h_{t+k}}{1-\tau^h_{t+k}} \lambda_{t+k} \Delta_{t+k}^{v+1} \left(A_{t+k}^{-1}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi} - \frac{G_{t+k}}{C_{t+k}} \right); \end{aligned}$$

- *Phillips Curve*

$$\begin{aligned} & \left(\frac{1-\alpha \pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{-\theta+\theta\phi+1}{1-\theta}} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{C_{t+k}+G_{t+k}}{C_{t+k}} \left(\frac{P_t}{P_{t+k}} \right)^{1-\theta} \qquad (2.27) \\ = \quad & \frac{\theta}{(1-\theta)} \phi E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{\lambda_{t+k}}{1-\tau^h_{t+k}} \Delta_{t+k}^{v} \left(A_{t+k}^{-1}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi} \left(\frac{P_t}{P_{t+k}} \right)^{-\theta\phi}; \end{aligned}$$

- *Law of Motion of Prices*

$$\Delta_t = \alpha \Delta_{t-1} \pi_t^{\theta\phi} + (1-\alpha)\left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}}. \tag{2.28}$$

Proof. See Appendix. ■

The foregoing formulation of the policy problem brings out very clearly the impact of price dispersion and the sense in which its impact is like a drag on the level of factor productivity. Indeed, the following change of variables, $A_t^R := A_t \Delta_t^{-\frac{1}{\phi}}$, demonstrates that any degree of price dispersion greater than unity impacts in the utility function and the solvency constraint exactly like a *downward* shift in the level of productivity; as proposition 3.1 below establishes, $\Delta_t \geq 1$. This change of variables does not quite work in the Phillips curve where price dispersion enters as Δ_t^v (as opposed to Δ_t^{v+1} in the utility function and solvency constraint). One may be tempted to conclude that this simply points to the fact that optimal monetary policy ought to ensure that price dispersion is minimized, or set to zero (i.e., perfect price-level stability). However, in an appendix available from the authors, we demonstrate that this analogy between price dispersion and productivity shocks goes through when one incorporates nominal wage stickiness in the manner of Erceg, Henderson and Levin (2000). This is important since in the presence of more than one source of nominal rigidity some systematic deviation from price stability will in general be optimal. Additionally, if we derive a log-linear approximation to this model economy around a non-zero inflation steady state then we shall find that in general a policy of ensuring perfect price stability will not be part of a Ramsey program. We pursue this issue further in section 6.

We also note, in passing, that price dispersion also bears a close similarity to a preference shift into leisure. If we employ the following change of variables, $\lambda_t^R := \lambda_t \Delta_t^{v+1}$, we see that the problem facing the policymaker is almost identical to that facing a policymaker in an economy with a higher preference for leisure. Again, this change of variables does not quite work in the Phillips relation; here the price dispersion term enters in a less quantitatively significant way: Δ_{t+k}^{v}, $\forall k$, as opposed to Δ_{t+k}^{v+1}, $\forall k$. We prefer to emphasize the similarity between price dispersion and productivity since in the appendix to which we have just referred, we show that in the presence of nominal wage rigidity the wage dispersion term in naturally 'paired' with the preference shifter while the price dispersion term is naturally linked, as above, with productivity.

We return to the implications for price dispersion of this policy problem in section 6. First, we investigate the quantitative impact of price dispersion in the model.

3. The Costs of Price Dispersion

We begin by establishing the following proposition:

Proposition 3.1. *Price dispersion is always greater than or equal to one,* $\Delta_t \geq 1$, *while equality can only happen when all prices are equal.*

Proof. This is a consequence of Jensen's inequality. We need to demonstrate that $\Delta_t \langle x \rangle > 1$, for $x = -\theta\phi$. For this purpose we will use Jensen's inequality that $f\left(\int u_i di\right) \leq \int f(u_i)\, di$ which holds for any convex function, f. Consider $f(u) = u^{\frac{x}{1-\theta}}$, and $u_i = \left(\frac{p_t(i)}{P_t}\right)^{1-\theta}$. We can easily show that $f(u)$ is convex for $x < 1-\theta$, since $f''(u) = \left(\frac{1}{1-\theta}\right)^2 x\,(x-1+\theta)\, u^{\frac{x}{1-\theta}-2} > 0$. Now we can apply

Jensen's inequality

$$
\begin{aligned}
f\left(\int u_i di\right) &= f\left(\int \left(\frac{p_t(i)}{P_t}\right)^{1-\theta} di\right) = f(1) = 1 \\
&\leq \int f\left[\left(\frac{p_t(i)}{P_t}\right)^{1-\theta}\right] di = \int \left(\left(\frac{p_t(i)}{P_t}\right)^{1-\theta}\right)^{\frac{x}{1-\theta}} di = \int \left(\frac{p_t(i)}{P_t}\right)^{x} di \\
&= \Delta_t \langle x \rangle
\end{aligned}
$$

In our particular case $x = -\theta\phi < -\theta < 1 - \theta$, since $\phi \geq 1$. ■

3.1. Higher Production Costs

We now demonstrate Proposition 3.2 which establishes that rising price dispersion, ceteris paribus, increases marginal production costs and distorts the demand for and supply of labour.

Proposition 3.2. *At the economy-wide level, for a given output level*

(i) *the labour input employed;*

(ii) *the aggregate production costs;*

(iii) *the disutility from labour,*

all increase in price dispersion.

Proof. The proof of (i) follows immediately from (2.15)

Not surprisingly, total production costs are increasing in labour employed. Combining (2.15) with (2.16) we can calculate total production costs

$$
TC_t := w_t N_t = \mu_t \lambda_t \frac{1}{1-\tau_t^h} C_t \left(A_t^{-\phi} Y_t^{\phi} \Delta_t\right)^{1+v}. \tag{3.1}
$$

It follows immediately that $[\partial TC_t / \partial \Delta_t] > 0$.

Finally, the higher is employment the less time households have for leisure. The aggregate disutility from labour, for a given level of output, is given in (3.2) and it is clear that this also is increasing in price dispersion.

$$\lambda_t \frac{1}{1+v} N_t^{1+v} = \lambda_t \frac{1}{1+v} \left(A_t^{-\phi} Y_t^{\phi} \Delta_t \right)^{1+v}. \tag{3.2}$$

■

The implications of this proposition will be useful in interpreting the impulse responses that we report in Section 8.

4. Price Dispersion and Productivity shocks: Some Back-of-the-Envelope Calculations

4.1. Theoretical Calculations

We can use the law of motion (2.28) to make some inference on the impact of price dispersion. We do this by mapping a given average level of inflation, via its impact on price dispersion, into an equivalent decrease in productivity using the change of variable deduced above. This is shown in the bottom line of Table 4.1[3]. That is, we use the following expression:

$$\Delta_t = \Delta_{t-1} = (1-\alpha) \left(\frac{1-\alpha \pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{\theta\phi}{\theta-1}} / \left(1 - \alpha \pi_t^{\theta\phi} \right). \tag{4.1}$$

In Table 4.1 column I corresponds to a benchmark economy, while column II shows that a higher level of competition, θ, makes price dispersion more costly, as does, respectively, the degree of concavity of the production function, ϕ, (column III), inflation, π_t, (column IV) and the degree of price stickiness, α (column V). The final row in the figure, under the maintained assumptions, maps a given degree of price dispersion into an equivalent percentage decrease in productivity. These numbers, and those in subsequent tables, are in terms of annualized percentage

[3] The values for the parameters in column I in this table and in the subsequent tables correspond to those we used in conducting the simulations reported in Section 8. These appear to be in line with much of the literature.

decreases. It is striking that a steady-state inflation rate of 2.5% maps into an almost equivalent (2.4%) decrease in factor productivity in the base case (column I).

Table 4.1	I	II	III	IV	V
θ	7	10	7	7	7
ϕ	1.38	1.38	1.6	1.38	1.38
π	2.5%	2.5%	2.5%	5%	2.5%
α	0.5	0.5	0.5	0.5	0.6
Δ	1.034	1.09	1.06	1.28	1.08
$1-\Delta^{-1/\phi}$	-2.4%	-5.8%	-3.6%	-16.6%	-5.4%

However, an obvious question follows from this simple analysis: How large is price dispersion in the data? Unfortunately, so far as we are aware, there is little direct empirical guidance on this issue, although there is some general evidence on price dispersion. For instance, Baye, Morgan and Scholten (2004) calculate the coefficient of variation ($cvar$) for online products in the USA. They find it equals 10% on average. And it may well be the case that the coefficient of variation could be significantly larger in European countries. Gatti and Kattuman (2003), for example, find that the coefficient of variation for online products in the Netherlands is 12.6%, although they also report that the coefficient of variation for online bookstores can be up to 30%. We can, in fact, map these numbers into our productivity equivalent measure, making no assumptions about trend inflation. We recall the definition of the coefficient of variation:

$$\begin{aligned} cvar &= \frac{\left(\int p^2(i)di - \left(\int p(i)di\right)^2\right)^{1/2}}{\int p(i)di} = \\ &= \frac{\left(\int \left(\frac{p(i)}{P}\right)^2 di - \left(\int \frac{p(i)}{P} di\right)^2\right)^{1/2}}{\int \frac{p(i)}{P} di}. \end{aligned}$$

In the appendix we show how one can relate this measure to our model's measure of price dispersion to arrive at the following expression:

$$\Delta \simeq 1 + \frac{1}{2}\frac{\theta\phi}{\theta+1}\left(\theta\phi - \theta + 1\right)\frac{cvar^2}{1 - \frac{1}{2}\frac{\theta}{\theta+1}\left(cvar^2 + 1\right)}. \tag{4.2}$$

Applying formula (4.2) we can estimate the effect of price dispersion in terms of productivity permitting the coefficient of variation to go from 5% to 20% (recall the studies above suggest a range of something like 10% to 30%). The results are reported in Table 4.2.

Table 4.2	*I*	*II*	*III*	*IV*	*V*
cvar	0.05	0.1	0.1	0.1	0.2
θ	7	7	10	7	7
ϕ	1.38	1.38	1.38	1.6	1.38
Δ	1.01	1.04	1.06	1.07	1.16
$1-\Delta^{-1/\phi}$	-0.7%	-2.7%	-3.8%	-3.9%	-10.3%

Interestingly, column *II* in Table 4.2 corresponds quite closely to column *I* in Table 4.1, in terms of the ultimate productivity-equivalent impact, suggesting that a coefficient of variation of 10%, or a little lower, may be a realistic number. And we emphasize, we made *no* assumption about inflation in constructing Table 4.2. Taken together, the complementary evidence in Tables 4.1 and 4.2 indicate that an empirically plausible level of price dispersion is a potentially very costly in welfare terms. We may ask the question, in the spirit of Lucas (1987), how costly in terms of utility a given degree of price dispersion might be.

5. The Consumption Equivalent Cost of price Dispersion

We will compare two economies; one corresponding to the situation when all firms charge the same price, the other with a degree of price dispersion corresponding to what we hope are reasonable levels for actual price dispersion. Due to price dispersion, households in the second economy would work more, and, ceteris paribus, therefore have lower welfare. We would like to know how damaging to welfare is a given degree of price dispersion.

So, we would like to calculate a quantity of consumption, $\Phi\%$, which represents the percentage point amount by which consumption would need to be higher every period, to achieve the same level of utility as in the case when all firms charge the same price, $\Delta_{t+k}^{v+1} = 1$[4]. To calculate this welfare equivalent we set

$$U_t\left(\Phi,\left\{\widehat{\Delta}_{t+k}\right\}\right) = E_t\sum_{t=0}^{\infty}\beta^{t+k}\left(\log(C_{t+k}) + \log\Phi - \lambda_{t+k}\Delta_{t+k}^{v+1}\frac{\left(A_{t+k}^{-1}Y_{t+k}\right)^{(v+1)\phi}}{v+1}\right)$$

such that $U_t\left(\Phi,\left\{\widehat{\Delta}_{t+k}\right\}\right) = U_t(1,0)$. Note that this calculation is quite general as it may be interpreted as assuming that the optimal degree of price dispersion is necessarily positive–that the policymaker is required to deliver a positive degree of price dispersion perhaps most plausibly as a result of numerous sources of nominal rigidity. Our calculation, therefore, reflects the remaining welfare cost of such rigidities. It follows then that[5]

$$\begin{aligned}
\log\Phi &= (1-\beta)\,E_t\sum_{t=0}^{\infty}\beta^{t+k}\lambda_{t+k}\frac{\left(A_{t+k}^{-1}Y_{t+k}\right)^{(v+1)\phi}}{v+1}; \\
&= \frac{\lambda}{1+v}Y^{(v+1)\phi}\left(\Delta^{v+1}-1\right) = \frac{1-\tau}{1-g}\frac{(\theta-1)}{\theta\phi\,(1+v)}\left(\Delta^{v+1}-1\right). \quad (5.1)
\end{aligned}$$

This is what we are seeking: It shows by how much consumption would need to be increased to compensate for a given degree of price dispersion in the economy.

[4] Of course, Lucas (1987) considered the mean-variance trade-off in consumption; our thought experiment is trading off mean consumption and mean price dispersion.

[5] In formula (5.1) Y denotes the steady-state level of output, g denotes government expenditure to output ratio and τ is the steady pay-roll tax rate. The Phillips curve (2.27) and an assumption of price stability, help us to solve for the steady state output value $\lambda Y^{(v+1)\phi} = \frac{1-\tau}{1-g}\frac{\theta-1}{\theta\phi}$.

Table 5.1 provides details of the calculations based on this expression. The required change in consumption appears far from negligible. Indeed, even on relatively moderate assumptions that number does not fall below 0.5%, and may rise substantially above it; column *II*, assuming a coefficient of variation of prices of 10%, implies a consumption equivalent of 2.2%.

Table 5.1	*I*	*II*	*III*	*IV*	*V*
cvar	0.05	0.1	0.1	0.1	0.2
θ	7	7	10	7	7
ϕ	1.38	1.3	1.38	1.6	1.38
Δ_t	1.01	1.04	1.06	1.07	1.16
g	0.15	0.15	0.15	0.15	0.15
τ	0.25	0.25	0.3	0.25	0.25
$1+\nu$	2.8	2.8	2.8	2.8	2.8
$\Delta^{\nu+1}-1$	2.8%	11.5%	16.4%	19.4%	52.3%
$\frac{1-\tau}{1-g}\frac{1-\theta}{\theta\phi(1+\nu)}$	0.2	0.2	0.19	0.17	0.2
$\Phi\%$	0.5%	2.2%	3.1%	3.3%	10.2%

6. Optimal monetary policy under price dispersion

We now return to the problem of section 2.4. Following Tack Yun (2005), we consider an economy with an initial degree of price dispersion, $\Delta_{t-1} > 1$ and access to lump-sum taxation (the full problem is set out and solved in the appendix). Yun (2005) considers an economy with linear production, $\phi = 1$,while we consider the more general case of concave production technology. The next proposition shows that optimization over price dispersion implies *negative* inflation in a transition period. Lump sum taxes are employed to meet the solvency requirement attached to the policy program. The price setting constraint in this case can be supported by payroll subsidies, τ_{t+k}. Yun (2005) shows that with competitive labour markets the optimal subsidy rate should correct for the distortion associated with imperfect competition, $\tau_{t+k}^{h} = -\frac{1}{(\theta-1)}$.[6]

Proposition 6.1. *(Tack Yun, 2005) Given initial price dispersion, the optimal policy corresponds to negative inflation.*

Proof. We can easily recover his result by writing the first-order condition for the law of motion (2.28)

$$\frac{\partial \Delta_t}{\partial \pi_t} = \theta\phi\alpha \left(\Delta_{t-1}\pi_t^{\theta\phi-1} - \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}-1} \pi_t^{\theta-2} \right) = 0. \tag{6.1}$$

We can simplify (6.1), which gives us the optimal rate of inflation

$$\pi_t = \left[(1-\alpha)\,\Delta_{t-1}^{\frac{\theta-1}{\theta\phi+1-\theta}} + \alpha \right]^{\frac{1}{1-\theta}}. \tag{6.2}$$

Clearly, this implies that $\pi_t < 1$ iff $\Delta_{t-1} > 1$. Finally, this optimal path for inflation is feasible: See the formal proof in the appendix. ∎

Substituting the expression for optimal inflation (6.2) into the law of motion (2.28) we obtain the optimal level of price dispersion to be achieved in the next period

$$\Delta_t = \Delta_{t-1} \left[(1-\alpha)\,(\Delta_{t-1})^{\frac{\theta-1}{\theta\phi+1-\theta}} + \alpha \right]^{\frac{\theta\phi+1-\theta}{1-\theta}}, \tag{6.3}$$

which implies the following dynamic relation between inflation and price dispersion:

$$\pi_t^{\theta\phi+1-\theta} = \frac{\Delta_t}{\Delta_{t-1}}. \tag{6.4}$$

We emphasize that one still cannot recover an optimal stabilization policy for price dispersion should one adopt a second-order approximation around a zero-inflation

[6] It is straightforward to demonstrate that when the labour market is imperfectly competitive, as in Erceg et al., $\tau_{t+k}^{h} = 1 - \frac{\theta\mu_{t+k}}{\theta-1}$ and that this result holds for any $\phi > 1$. Here μ refers to the wage markup. An appendix is available from the authors.

steady state. The logarithmic second-order approximation to the law of motion is given by (6.5)

$$\widehat{\Delta_t} = \alpha\widehat{\Delta_{t-1}} + \frac{1}{2}\frac{\alpha}{1-\alpha}\theta\phi\left(\theta\phi + 1 - \theta\right)\widehat{\pi}_t^2 + O\left(\left\|\xi^3\right\|\right) \tag{6.5}$$

and the policy that minimizes price dispersion implies immediate inflation stabilization: $\widehat{\pi}_t = 0$.

The usual linear-quadratic approach drops the law of motion (6.5) as a "second-order constraint", and therefore does not allow one to investigate the dynamics of price dispersion at all. As we noted in the introduction, this assumption lies at the heart of the usual conclusion in the literature that the direct impact of price dispersion on welfare is close to negligible.

7. Reincorporating price dispersion into linearized models

In economies with low inflation and no indexation, price dispersion has a large impact on the economy and welfare. Optimal monetary policy, if it could, drives price dispersion to zero. However, there may be good reasons to expect that it cannot (for example, lack of enough policy tools–lump-sum taxation, as above, fear of a liquidity trap, etc.). Hence, if optimal inflation is non-zero and pricing behaviour cannot be fully synchronized, then some price distortion (in steady-state and in the dynamics) is unavoidable.

The reason why price dispersion is generally excluded from linearized models is because the linearization takes place around a steady state in which there is no price dispersion[7]. In the previous sections we have tried to indicate that price dispersion can be significant even at relatively low rates of inflation. In the remainder of the paper, we develop a log-linear version of our model in which

[7] Of course, price dispersion may not be entirely absent in L-Q approximate models. That is because price dispersion is the source of the inflation stabilization objective in quadratic approximations to the representative agent's utility function. See Woodford (2003).

price dispersion is no longer of second-order importance. Crucially, we linearize the model around an inflationary steady state in which there remains some price dispersion.

First, consider price adjustment in the Calvo-Yun set-up. Each period firms who are unable to reprice adjust their price for steady state inflation, $\overline{\pi}$. Other firms are allowed to adjust prices in a more sophisticated way, optimally choosing their price. The aggregate price-level, (2.19) implies

$$\left(\frac{1-\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} = \left(\frac{p_t'}{P_t}\right). \tag{7.1}$$

Thus, the dynamics of price dispersion can be shown to be given by:

$$\begin{aligned}\Delta_t &= \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta\phi} di \\ &= \alpha\left(\pi_t/\overline{\pi}\right)^{\theta\phi}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}},\end{aligned} \tag{7.2}$$

where $\overline{\pi}$ is steady-state inflation. See the appendix for the full derivation. The steady state value of Δ is given by

$$\Delta = \alpha\left(\overline{\pi}/\overline{\pi}\right)^{\theta\varphi}\Delta + (1-\alpha)\left(\frac{1-\alpha\left(\overline{\pi}/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\varphi}{\theta-1}}, \tag{7.3}$$

which implies that $\Delta = 1$ in steady state. Hence, this is also consistent with the case in which steady-state inflation is zero. Linearizing this expression around this steady state results in (again, see the appendix for the details):

$$\widehat{\Delta}_t = \alpha^k \widehat{\Delta}_{t-k} + O2 \simeq O2.$$

$O2$ indicates terms of second-order, or higher. Now let us consider the approximation to the law of motion around a steady state with positive inflation and no indexation. This seems a reasonable approach given that we observe little or no indexation in low inflation economies and that most monetary authorities, to put it mildly, do not seem to wish to achieve zero inflation. We find that

$$\Delta_t = \alpha \pi_t^{\theta\phi} \Delta_{t-1} + (1-\alpha) \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{\theta\phi}{\theta-1}}. \tag{7.4}$$

The steady state price dispersion is then

$$\Delta = \frac{(1-\alpha)}{(1-\alpha\pi^{\theta\phi})} \left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha} \right)^{\frac{\theta\phi}{\theta-1}}. \tag{7.5}$$

And what we now find is that

$$\widehat{\Delta}_t = \alpha\pi^{\theta\phi} \widehat{\Delta}_{t-1} + \alpha\theta\phi \frac{\left(\pi^{\theta\phi} - \pi^{\theta-1}\right)}{\left(1-\alpha\pi^{\theta-1}\right)} \widehat{\pi}_t + O2. \tag{7.6}$$

So, price dispersion is not a second order term any longer. Hence an approximate log-linear model will include price dispersion and it follows that: The law of motion (7.6) has to be part of the linear system of the model's equations; the inflation rate which reduces price dispersion is necessarily below trend inflation, so that we may recover a version of Yun's (2005) result (although we do not pursue that issue in this paper); price dispersion and inflation will *directly* affect production costs in the same way as a negative productivity shock. We now present the equations of the full linearized model, with the details left to the appendix.

8. The log-linear model

1. The expression for the real wage (8.1) is obtained using (2.16)

$$\widehat{\lambda}_t + v\widehat{N}_t + \widehat{C}_t = \widehat{w}_t + \widehat{s}_t, \tag{8.1}$$

where we define $\widehat{s}_t = \widehat{1-\tau_t}$.

2. The log-linear form of labour demand is derived from (2.15):

$$\widehat{N}_t = \widehat{\Delta_t} + \phi\left(\widehat{Y}_t - \widehat{A}_t\right). \tag{8.2}$$

3. Market clearing is derived using (2.24):

$$\widehat{Y}_t = (1-g)\widehat{C}_t + g\widehat{G}_t. \tag{8.3}$$

4. In the appendix we show how one can construct the following log-linear form of the Phillips relation:

$$-\Lambda_1\widehat{\pi}_t - \Lambda_2 Z_t + \Lambda_3 X_t = 0; \tag{8.4}$$

$$Z_t - \widehat{Y}_t + \widehat{C}_t - \frac{\theta-1}{1-\alpha\beta\pi^{\theta-1}}\widehat{\pi}_t = \alpha\beta\pi^{\theta-1}E_t Z_{t+1}; \tag{8.5}$$

$$X_t - \widehat{w}_{t+k} + \widehat{C}_{t+k} - \phi\left(\widehat{Y}_{t+k} - \widehat{A}_{t+k}\right) - \left(\frac{\theta\phi}{1-\alpha\beta\pi^{\theta\phi}}\right)\widehat{\pi}_{t+k} = \alpha\beta\pi^{\theta\phi}E_t X_{t+1}. \tag{8.6}$$

Λ_1, Λ_2 and Λ_3 are parameters defined in the appendix, and where

$$\begin{aligned} Z_t &= \sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k\left(\widehat{Y}_{t+k} - \widehat{C}_{t+k} + \frac{\theta-1}{1-\alpha\beta\pi^{\theta-1}}\widehat{\pi}_{t+k}\right); \\ X_t &= \sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta\phi})^k\left(\widehat{w}_{t+k} - \widehat{C}_{t+k} + \phi\left(\widehat{Y}_{t+k} - \widehat{A}_{t+k}\right) + \left(\frac{\theta\phi}{1-\alpha\beta\pi^{\theta\phi}}\right)\widehat{\pi}_{t+k}\right). \end{aligned}$$

5. Approximating equation (2.8) yields

$$E_t\widehat{C}_{t+1} + E_t\widehat{\pi}_{t+1} = \widehat{C}_t + \widehat{i}_t, \tag{8.7}$$

where $\widehat{i}_t$ is the gross nominal interest rate, $\widehat{i}_t = \log\left(\frac{\beta}{\pi}(1+i_t)\right)$.

6. We log linearize (8.8) to yield (8.9)

$$E_t b_t \pi_{t+1}\frac{1}{1+i_{t-1}} = b_{t-1} - \tau_t^h w_t N_t + G_t; \tag{8.8}$$

$$\frac{b}{C}\beta\left(\widehat{b}_t + E_t\widehat{\pi}_{t+1} - \widehat{i_{t-1}}\right) = \frac{b}{C}\widehat{b}_{t-1} - \tau\frac{wN}{C}\left(\frac{\tau-1}{\tau}\widehat{s}_t + \widehat{w}_t + \widehat{N}_t\right) + \frac{g}{1-g}\widehat{G}_t. \tag{8.9}$$

7. The log-linear dynamics of price dispersion is

$$\widehat{\Delta}_{t+1}\pi^{-\theta\phi} - \alpha\theta\phi\frac{\left(\pi^{\theta\phi} - \pi^{\theta-1}\right)}{\left(1 - \alpha\pi^{\theta-1}\right)}\widehat{\pi}_{t+1} = \alpha\widehat{\Delta}_t. \tag{8.10}$$

To close the system we need to specify the actions of the fiscal and monetary authorities.

8. Monetary policy may be taken to follow a simple Taylor-type rule:

$$\begin{aligned}\widehat{i}_t &= i_t^* + (\psi_\pi + 1)\widehat{\pi}_t + \psi_y\widehat{Y}_t;\\ i_t^* &= \rho i_{t-1}^* + \widehat{m}_t.\end{aligned}$$

Here, $\widehat{m}_t$ is a white-noise, serially uncorrelated shock; i_t^* is an exogenous stochastic process as in Woodford (2001) which reflects many potential factors such as shifts in the natural rate of output, preference shocks, and such like, and we assume $\rho = 0.9$, consistent with the analysis in Rudebusch (2002)[8]. There is some debate about which output gap monetary authorities actually do react to, so in what follows we simply set $\psi_y = 0$; in effect we assume a simple Wicksell-Woodford reaction function[9]. Had we set $\psi_y = 0.5$, none of our conclusions below would be altered.

9. We assume that fiscal authorities respond to lagged debt in the following way:

$$\widehat{s}_t = -\xi\widehat{b}_{t-1}.$$

10. Productivity follows an AR(1) process with white-noise shock term:

$$\widehat{A}_{t+1} = \rho_A\widehat{A}_t + \varepsilon_{t+1}^A;$$

Full details of the calibration are in the appendix. First, we consider a shock to the interest rate target. In each graph we compare the model with price dispersion

[8] In fact, Rudebusch's results suggest that a value for ρ slightly higher than 0.9 is plausible.
[9] See Woodford (2003) chapter 4.

(the solid line) with the model in which price dispersion is absent (broken line). Figure 8.1 looks at how inflation and interest rates respond to our 'nominal' shock. Following the target interest rate shock inflation falls in both model economies, but by more in the no-price-dispersion (npd) model. More interestingly, it follows a hump-shaped path in the economy with price dispersion (pd), and appears to be more persistent. The lower panel of Figure 8.1 shows that this hump-shaped pattern shows up in the path of interest rates, which are thus somewhat more smoothed than one observes in the npd model; that is, the initial changes in interest rates are somewhat more gradual.

The impact of this shock on price dispersion is persistent and long-lasting (top panel, Figure 8.2). Although Proposition 3.1 above was established taking as given the level of output, it provides some clues as to the implications of this fall in price dispersion. Producers anticipate a persistent decline in price dispersion and as a result a period of lower than average production costs. This means that firms increase production (so that equilibrium production costs actually rise). As a result, labour input (top panel Figure 8.3) rises as does output (lower panel, Figure 8.3).

The rise in output in the pd economy is again hump-shaped and is a rather striking finding. The reduction in price dispersion, from a distorted steady-state, acts like a positive productivity shock, so long as the change in the target rate is sufficiently persistent. And it is this increase in output (and hence demand) that accounts for the smaller initial fall in inflation in the pd economy. This result is somewhat reminiscent of the disinflationary booms found by Ball (1994), Ireland (1997) and Nicolae and Nolan (2006). We stress, however, that our result is distinct in the sense that both economies (i.e., the pd and the npd economies) will display the behaviour identified by Ball for a future anticipated tightening in monetary policy; the channel we have identified is over and above that identified by Ball.

Less persistent shocks to the target rate, ceteris paribus, tend to make inflation persistence less pronounced, although the hump-shaped pattern to interest rates may still be present.

Following a productivity shock inflation and interest rates again follow the hump-shaped path back to base (Figure 8.4). The deviation of price dispersion is again persistent (top panel, Figure 8.5), whilst output responds maximally in the first period in both model economies (lower panel, Figure 8.6).

We conclude that the expected impact of a nominal shock looks to be highly dependent on both the persistence of that shock and on the steady state from which the economy is perturbed. If that steady state is distorted by what appears to be an empirically plausible amount of relative price dispersion (here we assumed an economy with a trend inflation of 2.5% and no indexation) then one may obtain some surprising results. By incorporating price dispersion, we are able to account for a persistent and gradual response in inflation to two familiar types of shocks. However, the response of output to a persistent, contractionary 'nominal' shock is striking and further work is required to understand this and reconcile it with how one typically thinks the economy responds to such a shock.

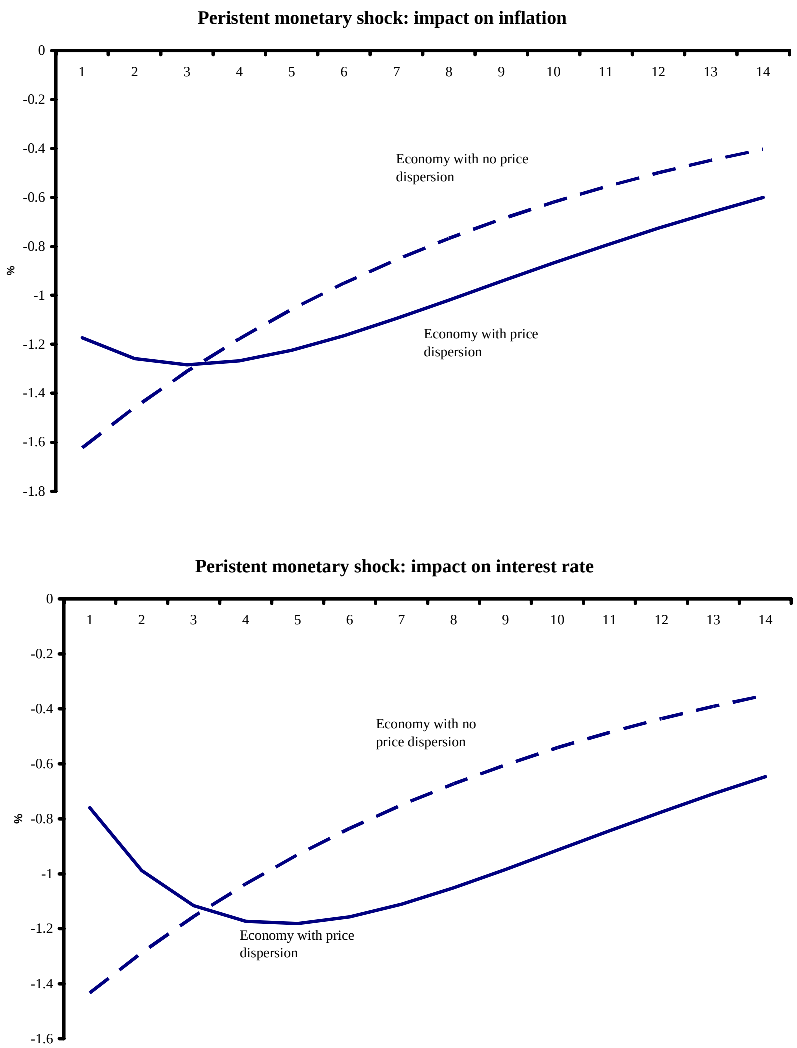

Peristent monetary shock: impact on inflation
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
%
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Economy with no price dispersion
Economy with price dispersion
Peristent monetary shock: impact on interest rate
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
%
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Economy with no price dispersion
Economy with price dispersion

Figure 8.1:

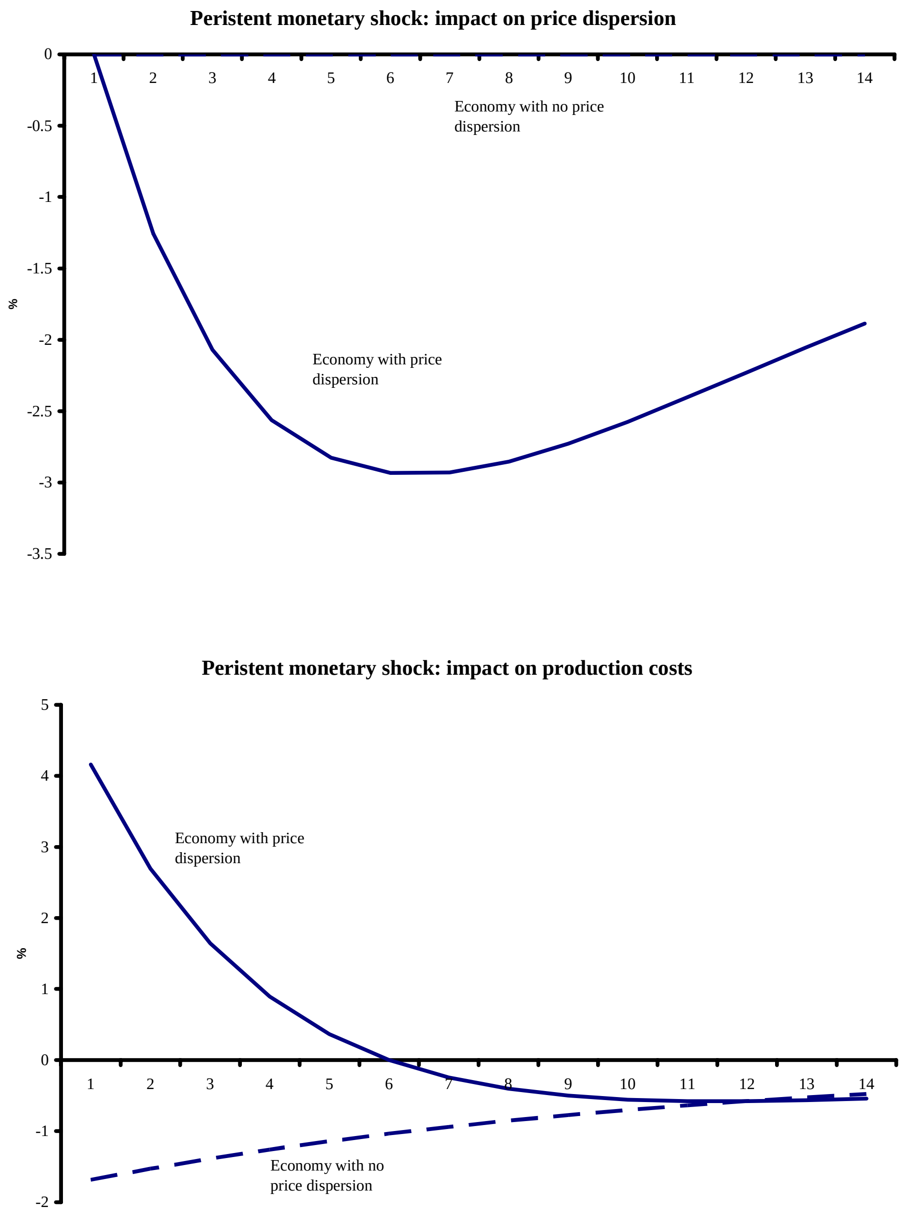
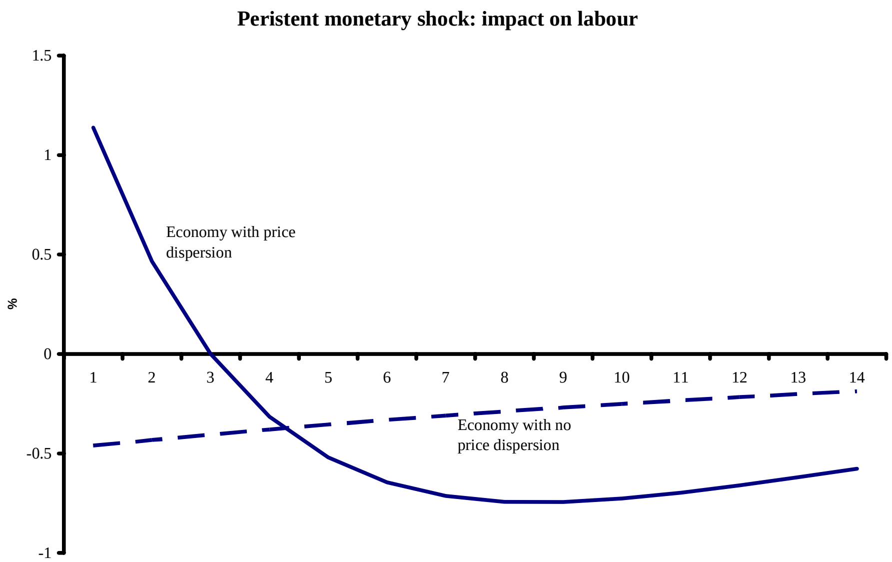

Peristent monetary shock: impact on price dispersion
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
%
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Economy with no price dispersion
Economy with price dispersion
Peristent monetary shock: impact on production costs
5
4
3
2
1
0
-1
-2
%
Economy with price dispersion
Economy with no price dispersion

Figure 8.2:


Peristent monetary shock: impact on labour
1.5
1
0.5
0
-0.5
-1
%
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Economy with price dispersion
Economy with no price dispersion

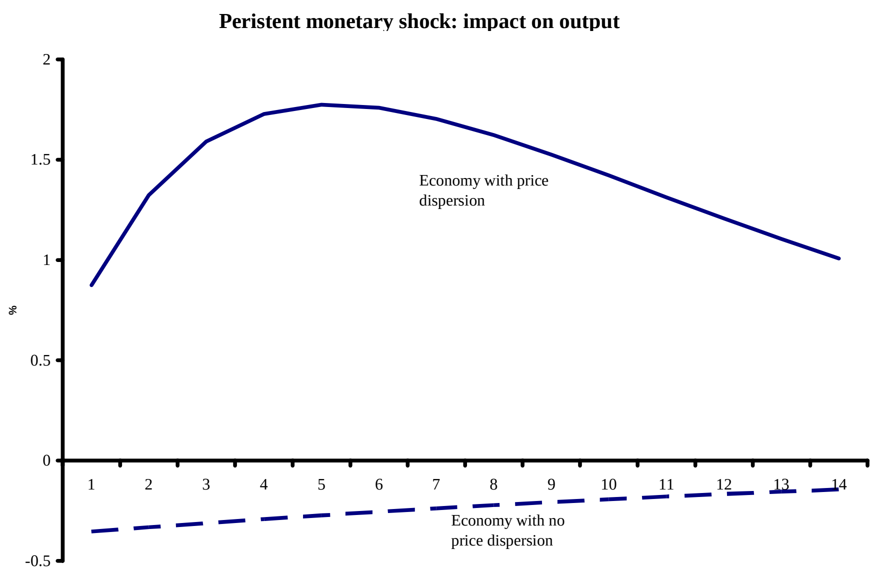
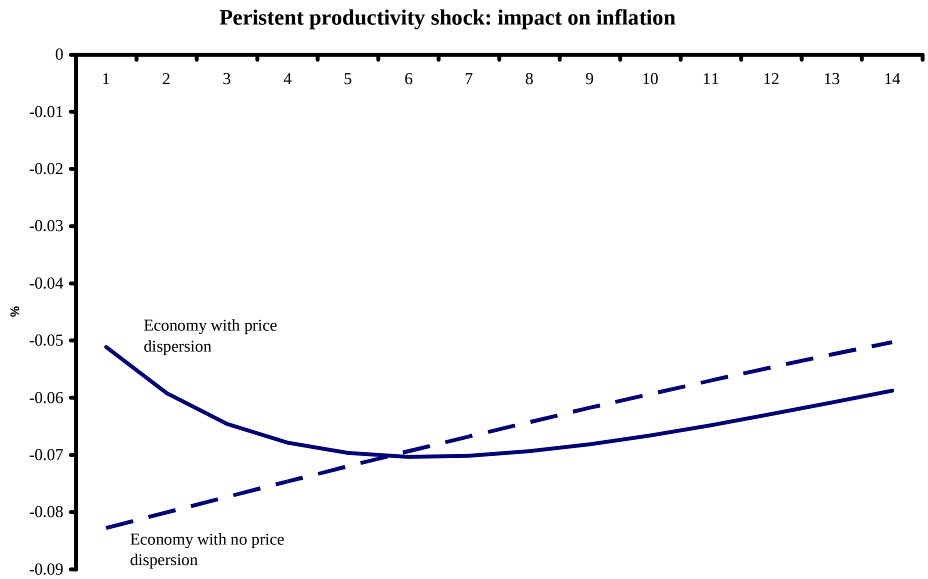

Peristent monetary shock: impact on output
2
1.5
1
0.5
0
-0.5
%
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Economy with price dispersion
Economy with no price dispersion
time

Figure 8.3:


Peristent productivity shock: impact on inflation
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
%
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Economy with price dispersion
Economy with no price dispersion

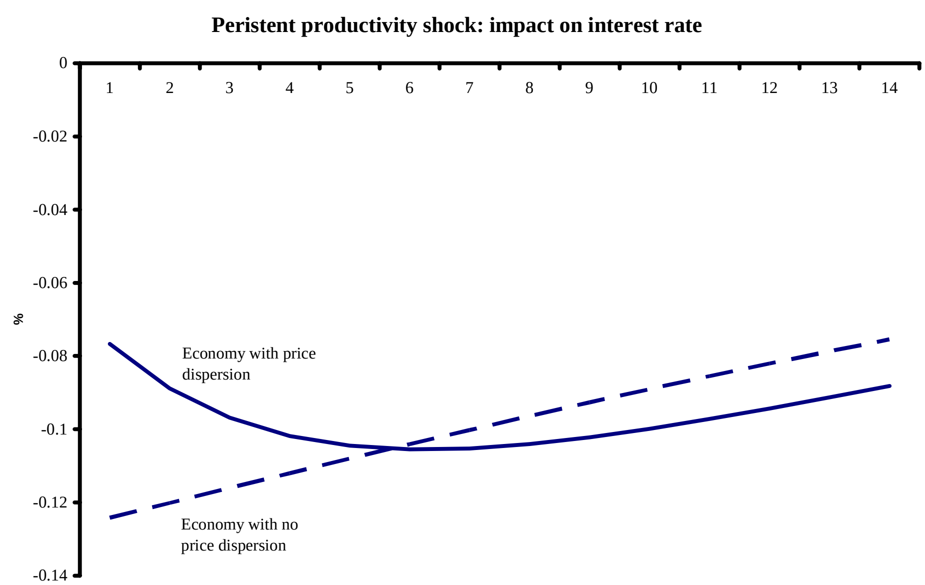

Peristent productivity shock: impact on interest rate
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
%
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Economy with price dispersion
Economy with no price dispersion

Figure 8.4:

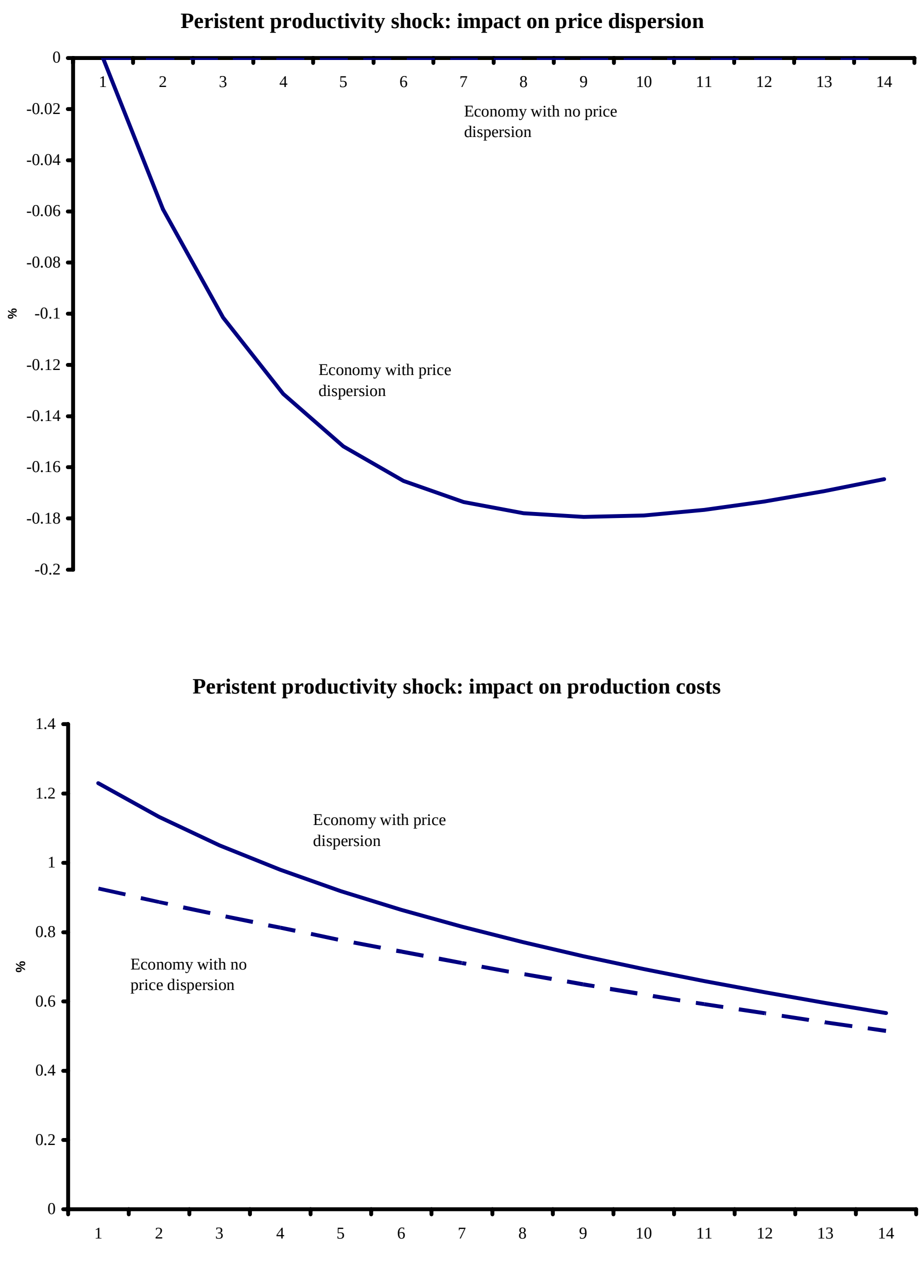
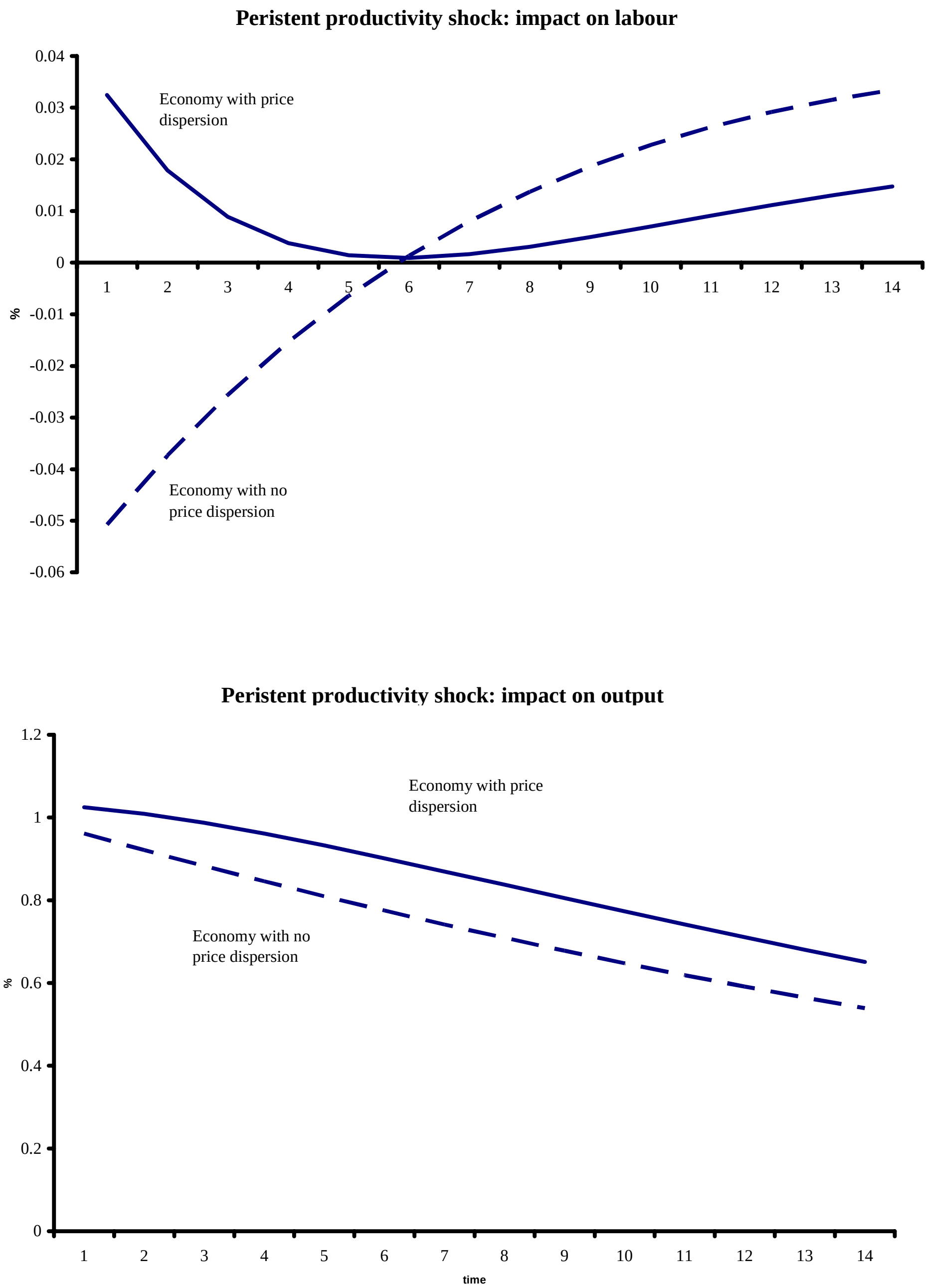

Peristent productivity shock: impact on price dispersion
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
%
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Economy with no price dispersion
Economy with price dispersion
Peristent productivity shock: impact on production costs
1.4
1.2
1
0.8
0.6
0.4
0.2
0
%
Economy with price dispersion
Economy with no price dispersion
1 2 3 4 5 6 7 8 9 10 11 12 13 14

Figure 8.5:



Peristent productivity shock: impact on labour
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
%
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Economy with price dispersion
Economy with no price dispersion
Peristent productivity shock: impact on output
1.2
1
0.8
0.6
0.4
0.2
0
%
time

Figure 8.6:

9. Conclusion

This paper has attempted to clarify the impact of price dispersion in a simple economic environment. We went to some effort to try to establish a rough order of magnitude for price dispersion. We found that the impact of price dispersion, on welfare and the dynamics of the simple model we set out, is substantial. Our model with price dispersion seemed to make the economy evolve in a more sluggish manner than the model with no price dispersion. Notably, inflation followed a hump-shaped path following either a real or a persistent nominal shock, and so any observed persistence in the policy rate was ultimately due to the persistence in the nominal shock, and not 'sluggish' policy decisions. These sorts of issues have been of concern to quantitative theorists recently; see the insightful discussion in Mash (2004). However, the expansionary impact on output of a persistent nominal contraction may be a challenge for the positive properties of the set-up. A number of research questions appear important. It would be especially interesting to have a better feel for how dispersed are actual prices through time, how that changes with inflation and the persistence of actual monetary shocks. Also, to slow the response of output one may think of incorporating sticky wages, as it seems reasonable to suppose that this will stop production costs from falling so quickly following a monetary contraction. Incorporating learning may also be useful in this regard[10].

[10] Nicolae and Nolan (2006) showed in a related, but simpler, model to the one presented here that one could 'avoid' disinflationary booms by incorporating a period of learning into the model.

References

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10. Appendix

10.1. Appendix to section 2.4

Proof of Proposition 2.4

Proof. The Ramsey plan is a policy plan $\left\{P_{t+k},\ \tau^h_{t+k},\right\}_{k=0}^{\infty}$ which is a competitive equilibrium corresponding to Definition 2.1 and which maximizes (2.1). We recall that a competitive equilibrium is a path for endogenous variables $\left\{C_{t+k},\ Y_{t+k},\ N_{t+k},\ w_{t+k},\ \Delta_{t+k},\ p'_{t+k},\ P_{t+k},\right\}_{k=0}^{\infty}$ satisfying conditions (2.16), (2.15), (2.18), (2.19), (2.20), (2.23) and (2.24). To receive a simpler system we will first substitute for $Y_{t+k},\ N_{t+k},\ w_{t+k}$ using (2.16), (2.15) and (2.24). This operation will immediately result in revised expressions for social welfare (2.25), the solvency constraint (2.26) and the Phillips Curve (10.1)

$$\begin{aligned} & \left(p'_t/P_t\right)^{-\theta+\theta\phi+1} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{C_{t+k}+G_{t+k}}{C_{t+k}} (1-\theta) \left(\frac{P_t}{P_{t+k}}\right)^{1-\theta} \\ = \quad & \theta\phi E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{\lambda_{t+k}}{1-\tau^h_{t+k}} \Delta^v_{pt+k} \left(A^{-1}_{t+k}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi} \left(\frac{P_t}{P_{t+k}}\right)^{-\theta\phi}. \end{aligned} \tag{10.1}$$

Then, using (2.19) we can calculate the optimal relative price,

$$p'_t/P_t = \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}}, \tag{10.2}$$

which can be plugged into (2.20) to receive the law of motion as in (2.28). Finally, we plug (10.2) into the transformed Phillips curve (10.1) to receive (2.27). ■

10.1.1. The relationship between the coefficient of variation and the measurement of price dispersion: derivation of (4.2)

We recall that $\Delta_t \langle -\theta\phi \rangle$ is our measure of price dispersion

$$\Delta_t \langle -\theta\phi \rangle = \int \left(\frac{p_t(i)}{P_t}\right)^{-\theta\phi} di. \tag{10.3}$$

For any x relation (10.4) is true up to second order:

$$\Delta_t \langle x \rangle = 1 + x \int \left(\widehat{p_t}(i) - \widehat{P}_t \right) di + \frac{1}{2} x^2 \int \left(\widehat{p_t}(i) - \widehat{P}_t \right)^2 di + O3. \tag{10.4}$$

Furthermore, from the definition of average price (2.3) implies that $\Delta_t \langle 1 - \theta \rangle = 1$, which together with (10.4) gives (10.5)

$$\int \left(\widehat{p_t}(i) - \widehat{P}_t \right) di = \frac{\theta - 1}{2} \int \left(\widehat{p_t}(i) - \widehat{P}_t \right)^2 di + O3. \tag{10.5}$$

In turn, expression (10.5) and (10.4) result in (10.6)

$$\Delta_t \langle x \rangle = 1 + \frac{x}{2} (\theta - 1 + x) \int \left(\widehat{p_t}(i) - \widehat{P}_t \right)^2 di + O3, \tag{10.6}$$

which can be rewritten as (10.7) for $x = -\theta\phi$

$$\Delta_t \langle -\theta\phi \rangle \simeq 1 + \frac{1}{2} \theta\phi \left(\theta \left(\phi - 1 \right) - 1 \right) \int \left(\widehat{p_t}(i) - \widehat{P}_t \right)^2 di \tag{10.7}$$

By definition, the coefficient of variation is the ratio of standard deviation to mean. Formally

$$\begin{aligned} cvar &= \frac{\left(\int p_t^2(i) di - \left(\int p_t(i) di \right)^2 \right)^{1/2}}{\int p_t(i) di} = \\ &= \frac{\left(\int \left(\frac{p(i)}{P_t} \right)^2 di - \left(\int \frac{p(i)}{P_t} di \right)^2 \right)^{1/2}}{\int \frac{p(i)}{P_t} di}, \end{aligned}$$

which we rewrite as (10.8)

$$cvar = \frac{\sqrt{\Delta_t \langle 2 \rangle - \Delta_t^2 \langle 1 \rangle}}{\Delta_t \langle 1 \rangle}, \tag{10.8}$$

where we can express $\Delta_t \langle 2 \rangle$ and $\Delta_t \langle 1 \rangle$ using relation (10.6)

$$\Delta_t \langle 2 \rangle = 1 + (\theta + 1) \int \left(\widehat{p_t}(i) - \widehat{P}_t \right)^2 di + O3; \tag{10.9}$$

$$\Delta_t \langle 1 \rangle = 1 + \frac{\theta}{2} \int \left(\widehat{p}_t(i) - \widehat{P}_t \right)^2 di + O3. \tag{10.10}$$

Combining (10.9) and (10.10) we receive

$$\Delta_t \langle 1 \rangle \simeq 1 + \frac{1}{2} \frac{\theta}{\theta + 1} \left(\Delta_t \langle 2 \rangle - 1 \right). \tag{10.11}$$

Expressions (10.8) and (10.11) help us to relate $\Delta_t \langle 2 \rangle$ and the coefficient of variation, *cvar*:

$$\left(cvar^2 + 1 \right) \left(1 + \frac{1}{2} \frac{\theta}{\theta + 1} \left(\Delta_t \langle 2 \rangle - 1 \right) \right) = \Delta_t \langle 2 \rangle,$$

$$cvar^2 + \frac{1}{2} \frac{\theta}{\theta + 1} \left(\Delta_t \langle 2 \rangle - 1 \right) \left(cvar^2 + 1 \right) = \Delta_t \langle 2 \rangle - 1$$

$$\frac{cvar^2}{1 - \frac{1}{2} \frac{\theta}{\theta+1} \left(cvar^2 + 1 \right)} = \Delta_t \langle 2 \rangle - 1 \tag{10.12}$$

Finally, we can combine (10.6) and (10.9) to receive (10.13)

$$\Delta_t \langle -\theta\phi \rangle \simeq 1 + \frac{1}{2} \frac{\theta\phi}{\theta + 1} \left(\theta\phi - \theta + 1 \right) \left(\Delta_t \langle 2 \rangle - 1 \right). \tag{10.13}$$

Now, plugging (10.12) into (10.13) we receive the final expression, (4.2), used in the main text,

$$\Delta_t \langle -\theta\phi \rangle \simeq 1 + \frac{1}{2} \frac{\theta\phi}{\theta + 1} \left(\theta\phi - \theta + 1 \right) \frac{cvar^2}{1 - \frac{1}{2} \frac{\theta}{\theta+1} \left(cvar^2 + 1 \right)}.$$

10.2. Appendix to Section 6

The Lagrangian for the Ramsey policy problem may be written as:

$$
\begin{aligned}
L \;=\; & E_t \sum_{k=0}^{\infty} \beta^k \left(\log\left(C_{t+k}\right) - \lambda_{t+k} \Delta_{t+k}^{v+1} \frac{\left(A_t^{-1}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi}}{v+1} \right) \\
& +\psi \left[\left(1+i_{t-1}\right) \frac{b_{t-1}}{C_t \pi_t} - E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{\tau_{t+k}^h \lambda_{t+k} \Delta_{pt+k}^{v+1}}{1-\tau_{t+k}^h} \left(\frac{C_{t+k}+G_{t+k}}{A_{t+k}} \right)^{(v+1)\phi} - \frac{G_{t+k}+T_{t+k}}{C_{t+k}} \right) \right] \\
& \mu \langle \left(\frac{1-\alpha \pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{-\theta+\theta\phi+1}{1-\theta}} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{C_{t+k}+G_{t+k}}{C_{t+k}} \left(\frac{P_t}{P_{t+k}} \right)^{1-\theta} - \\
& -\frac{\theta}{(1-\theta)} \phi E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \left(\frac{\lambda_{t+k}}{1-\tau_{t+k}^h} \Delta_{t+k}^{v} \left(A_{t+k}^{-1}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi} \left(\frac{P_t}{P_{t+k}} \right)^{-\theta\phi} \right) \\
& +E_t \sum_{k=0}^{\infty} \beta^k \eta_{t+k} \left(\Delta_{t+k} - \alpha \Delta_{t+k-1} \pi_{t+k}^{\theta\phi} - (1-\alpha) \left(\frac{1-\alpha \pi_{t+k}^{\theta-1}}{1-\alpha} \right)^{\frac{\theta\phi}{\theta-1}} \right)
\end{aligned}
$$

The first order condition with respect to T_t, implies that the solvency constraint is not binding, $\psi = 0$. Similarly, the first order condition with respect to τ_{t+k}^h, implies that price-setting curve is not binding either so that $\mu = 0$; we can always 'correct' it by adjusting the labour tax/subsidy rate, τ_{t+k}^h. The first order condition with respect to consumption implies (10.14)

$$
\frac{C_{t+k}+G_{t+k}}{C_{t+k}} = \phi \lambda_{t+k} \Delta_{t+k}^{v+1} \frac{\left(A_t^{-1}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi}}{v+1} \tag{10.14}
$$

which implies that consumption is bigger when price dispersion is smaller. Finally, the first order condition with respect to π_t gives us expression (6.2), which together with the law of motion (2.28) implies

$$
\left(\frac{1-\alpha \pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{-\theta+\theta\phi+1}{1-\theta}} = \Delta_t. \tag{10.15}
$$

From equation (6.4) we derive the following dynamic relationship between the level of price dispersion and average price inflation

$$\frac{P_t}{P_{t+k}} = \left(\frac{\Delta_t}{\Delta_{t+k}}\right)^{\frac{1}{\theta\phi+1-\theta}}. \tag{10.16}$$

Now we plug (10.14) ,(10.15) and (10.16) into the price setting curve (2.27) and receive condition (10.17)

$$\begin{aligned} 0 &= \Delta_t E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{C_{t+k}+G_{t+k}}{C_{t+k}} \left(\frac{\Delta_t}{\Delta_{t+k}}\right)^{\frac{1-\theta}{\theta\phi+1-\theta}} - \\ & -\frac{\theta}{(\theta-1)} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{\Delta_{t+k}^{-1}}{1-\tau_{t+k}^h} \frac{(C_{t+k}+G_{t+k})}{C_{t+k}} \left(\frac{\Delta_t}{\Delta_{t+k}}\right)^{\frac{-\theta\phi}{\theta\phi+1-\theta}} = \\ & E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{C_{t+k}+G_{t+k}}{C_{t+k}} \Delta_t \left(\frac{\Delta_t}{\Delta_{t+k}}\right)^{\frac{1-\theta}{\theta\phi+1-\theta}} \left(1 - \frac{\theta}{(1-\theta)} \frac{1}{1-\tau_{t+k}}\right) \end{aligned} \tag{10.17}$$

which is always true if $\tau_{t+k} = 1 - \frac{\theta}{1-\theta}$.

This proves that when the solvency constraint is not binding, the optimal policy minimizes price dispersion.

10.2.1. The Law of motion for the Calvo-Yun model

The dynamic of price dispersion is derived as follows:

$$\begin{aligned} \Delta_t &= \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta\phi} di \\ &= \alpha \int_0^1 \left(\frac{p_{t-1}(i)\overline{\pi}}{P_t}\right)^{-\theta\phi} + (1-\alpha) \int_0^1 \left(\frac{p_t'}{P_t}\right)^{-\theta\phi} \\ &= \alpha \left(\pi_t/\overline{\pi}\right)^{\theta\phi} \int_0^1 \left(\frac{p_{t-1}(i)}{P_{t-1}}\right)^{-\theta\phi} + (1-\alpha) \left(\frac{p_t'}{P_t}\right)^{-\theta\phi} \\ &= \alpha \left(\pi_t/\overline{\pi}\right)^{\theta\phi} \Delta_{t-1} + (1-\alpha) \left(\frac{1-\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}}, \end{aligned}$$

where $\overline{\pi}$ is steady-state inflation.

10.3. The approximate relationship around $\overline{\pi}$

$$\Delta_t = \alpha \left(\pi_t/\overline{\pi}\right)^{\theta\phi} \Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}}$$

$$\begin{aligned}\left(1+\widehat{\Delta}_t\right) &= \alpha\left(1+\theta\phi\widehat{\pi}_t\right)\left(1+\widehat{\Delta}_{t-1}\right) \\ &\quad +(1-\alpha)\left(1+\frac{\theta\phi}{\theta-1}\left(1-\widehat{\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}\right)\right)+O2\end{aligned}$$

where

$$\left(1-\widehat{\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}\right) = -\alpha\widehat{\pi}_t\frac{\theta-1}{1-\alpha}+O2$$

and therefore

$$\widehat{\Delta}_t = \alpha\left(\theta\phi\widehat{\pi}_t+\widehat{\Delta}_{t-1}\right)-(1-\alpha)\frac{\theta\phi}{\theta-1}\alpha\widehat{\pi}_t\frac{\theta-1}{1-\alpha}+O2$$

$$\widehat{\Delta}_t = \alpha\widehat{\Delta}_{t-1}+O2$$

Since our variables are bounded, backward recursive substitution leads us to conclude that price dispersion is of second-order significance.

11. Calibration

Our baseline settings are as follows: Preference parameters: $v = 1.8$, $\lambda = 1$, $\beta = 0.96$. Technology parameters: $\phi = 1.38$, $\theta = 7$, $\alpha = 0.5$. Fiscal policy in a steady state: $b/Y = 0.4$, $g = 0.15$. Monetary policy parameters: $\psi_\pi = 0.5$, $\xi = 0.1$. Persistence of stochastic shocks: $\rho_A = 0.9$.

The model is linearized around two steady states, one where steady-state price dispersion is zero, and inflation is zero, and another where inflation is 2.5% and there is price dispersion in steady state: i.e., $\pi = 1$, or $\pi = 1.025$.

12. Steady states of the model

As we explained in the text, when the model is linearized around a non-inflationary steady state, the law of motion for price dispersion is dropped as price dispersion is a second-order variable. However, if we consider the model with a small yet positive trend inflation and no indexation then price dispersion is of first-order importance. Moreover it impacts on the model economy like a negative productivity shock and in addition to being persistent it also increases with inflation.

13. Appendices to Section 8

13.1. The steady state

First, for any given level of the steady state inflation, π, we can find price dispersion using the law of motion equation (2.28)

$$\Delta = \frac{(1-\alpha)}{(1-\alpha\pi^{\theta\phi})}\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}}.$$

Without loss of generality we assume the following normalization in the steady state of the economy $A = 1$, $\lambda = 1$. Then, we may calculate the steady-state value of output from the Phillips Curve (2.18).

$$\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{1-\theta+\theta\phi}{1-\theta}}\frac{1}{1-\alpha\beta\pi^{\theta-1}}\frac{1}{1-g} = \left(\frac{\theta\phi}{\theta-1}\right)\frac{1}{1-\alpha\beta\pi^{\theta\phi}}\frac{w}{1-g}Y^{\varphi-1}. \quad (13.1)$$

Where g is the government consumption to GDP ratio, $g = G/Y$.

The steady state labour supply and real wage follow from (2.15) and (2.16):

$$\begin{aligned} N &= \Delta Y^{\phi}; \\ w &= \frac{1}{1-\tau^h}\lambda C N^v = \frac{1-g}{1-\tau^h}\lambda Y N^v = \frac{1-g}{1-\tau}\lambda\Delta^v Y^{\phi v+1}. \end{aligned}$$

These allow us to compute output as a function of the steady state tax rate, τ;

$$\frac{1}{1-g}\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{1-\theta+\theta\phi}{1-\theta}}\frac{1-\alpha\beta\pi^{\theta\phi}}{1-\alpha\beta\pi^{\theta-1}}\left(\frac{\theta-1}{\phi\theta}\right)=\frac{\lambda\Delta^{v}Y^{\phi(v+1)}}{1-\tau}. \tag{13.2}$$

From the solvency constraint (2.26) we can then relate the debt to GDP ratio to the levels of tax and output

$$\frac{b}{C}(1-\beta)=\frac{\tau}{1-\tau}\lambda\left(\Delta Y^{\phi}\right)^{v+1}-\frac{g}{1-g}. \tag{13.3}$$

Combining the Phillips curve (13.2) and the solvency constraint (13.3) we receive the equilibrium level of output

$$\begin{aligned}&\frac{b}{C}(1-\beta)+\frac{g}{1-g}+\lambda\left(\Delta Y^{\phi}\right)^{v+1}\\ =\;&\frac{1}{1-g}\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{1-\theta+\theta\phi}{1-\theta}}\Delta\frac{1-\alpha\beta\pi^{\theta\phi}}{1-\alpha\beta\pi^{\theta-1}}\left(\frac{\theta-1}{\phi\theta}\right).\end{aligned}$$

13.2. Appendix 2: Linearization of the Phillips curve

A convenient way to linearize the Phillips curve is as follows. First re-write expression (2.18) as follows, rebundling the terms in current-period inflation:

$$\left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1-\theta+\theta\phi}{1-\theta}}\pi_t^{1-\theta+\theta\phi} \tag{13.4}$$

$$\sum_{k=0}^{\infty}(\alpha\beta)^k E_t\frac{Y_{t+k}}{C_{t+k}}\left[(P_{t-1}/P_{t+k})^{1-\theta}\right] \tag{13.5}$$

$$=\left(\frac{\theta}{\theta-1}\right)\sum_{k=0}^{\infty}(\alpha\beta)^k\phi E_t\frac{w_{t+k}}{C_{t+k}}\left(\frac{Y_{t+k}}{A_{t+k}}\right)^{\phi}(P_{t-1}/P_{t+k})^{-\theta\phi}. \tag{13.6}$$

Hence, linearizing the first expression above (13.4):

$$\begin{aligned}&\left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{-\theta+\theta\phi+1}{1-\theta}}\pi_t^{1-\theta+\theta\phi}\\ =\;&\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{1-\theta}}\pi^{1-\theta+\theta\phi}\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}+\frac{-\theta+\theta\phi+1}{1-\alpha}\widehat{\pi}_t\right).\end{aligned}$$

Similarly, linearizing the second expression (13.5):

$$\begin{aligned}
&\sum_{k=0}^{\infty}(\alpha\beta)^k E_t \frac{Y_{t+k}}{C_{t+k}} \left[(P_{t-1}/P_{t+k})^{1-\theta}\right] \\
&= \sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k \frac{\pi^{\theta-1}}{1-g} E_t \left(1+\widehat{Y}_{t+k}-\widehat{C}_{t+k}+(\theta-1)\sum_{i=0}^{k}\widehat{\pi}_{t+i}\right) \\
&= \sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k \frac{\pi^{\theta-1}}{1-g} E_t \left(\widehat{Y}_{t+k}-\widehat{C}_{t+k}\right) + \frac{1}{1-\alpha\beta\pi^{\theta-1}}\frac{\pi^{\theta-1}}{1-g} \\
&\quad + \frac{\pi^{\theta-1}(\theta-1)}{1-g}\sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k E_t \sum_{i=0}^{k}\widehat{\pi}_{t+i} \qquad (13.7)
\end{aligned}$$

We may change the order of integration in the last line of (13.7)

$$\begin{aligned}
&: \sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k E_t \sum_{i=0}^{k}\widehat{\pi}_{t+i} = E_t \sum_{i=0}^{\infty}\sum_{k=i}^{\infty}\widehat{\pi}_{t+i}(\alpha\beta\pi^{\theta-1})^k \\
&= E_t \sum_{i=0}^{\infty}\widehat{\pi}_{t+i}\sum_{k=i}^{\infty}(\alpha\beta\pi^{\theta-1})^k = \frac{1}{1-\alpha\beta\pi^{\theta-1}} E_t \sum_{i=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k \widehat{\pi}_{t+k}.
\end{aligned}$$

And so we find that:

$$\begin{aligned}
&\sum_{k=0}^{\infty}(\alpha\beta)^k \frac{Y_{t+k}}{C_{t+k}} \left[(P_{t-1}/P_{t+k})^{1-\theta}\right] \\
&= \frac{1}{1-g}\left[\sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k \left(\widehat{Y}_{t+k}-\widehat{C}_{t+k}+\frac{1}{1-\alpha\beta\pi^{\theta-1}}\frac{\theta-1}{1-g}\widehat{\pi}_{t+k}\right)+\frac{1}{1-\alpha\beta\pi^{\theta-1}}\right] \qquad (13.8)
\end{aligned}$$

Hence, the product of (the approximations of) expressions (13.4) and (13.5) is

$$
\begin{aligned}
&\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{1-\theta}}\pi^{\theta\phi}\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}+\frac{-\theta+\theta\phi+1}{1-\alpha}\widehat{\pi}_t\right)\frac{1}{1-g}\times\\
&\sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k E_t\left(\widehat{Y}_{t+k}-\widehat{C}_{t+k}+\frac{1}{1-\alpha\beta\pi^{\theta-1}}\frac{\theta-1}{1-g}\widehat{\pi}_{t+k}\right)+\frac{1}{1-\alpha\beta\pi^{\theta-1}}\\
=\;&\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{1-\theta}}\frac{-\theta+\theta\phi+1}{1-\alpha}\frac{\pi^{1-\theta+\theta\phi}}{1-\alpha\beta\pi^{\theta-1}}\frac{1}{1-g}\widehat{\pi}_t\\
&+\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi+1-\theta}{1-\theta}}\frac{\pi^{1-\theta+\theta\phi}}{(1-g)^2}\sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k E_t\left(\widehat{Y}_{t+k}-\widehat{C}_{t+k}+\frac{1}{1-\alpha\beta\pi^{\theta-1}}\frac{\theta-1}{1-g}\widehat{\pi}_{t+k}\right).
\end{aligned}
$$

We turn now to the third component (13.6) of the Phillips curve:

$$
\begin{aligned}
&\left(\frac{\theta}{\theta-1}\right)\sum_{k=0}^{\infty}(\alpha\beta)^k\phi E_t\frac{w_{t+k}}{C_{t+k}}\left(\frac{Y_{t+k}}{A_{t+k}}\right)^{\phi}(P_{t-1}/P_{t+k})^{-\theta\phi}\\
=\;&\left(\frac{\theta}{\theta-1}\right)\frac{\phi wY^{\phi}\pi^{\theta\phi}}{C\,(1-g)}\sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta\phi})^k E_t\left(\widehat{w}_{t+k}-\widehat{C}_{t+k}+\phi\left(\widehat{Y}_{t+k}-\widehat{A}_{t+k}\right)+\left(\frac{\theta\phi}{1-\alpha\beta\pi^{\theta\phi}}\right)\widehat{\pi}_{t+k}\right).
\end{aligned}
$$

And so, the log linearization of the Phillips curve is completed as

$$
\begin{aligned}
&\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{-\theta+\theta\phi+1}{1-\theta}}\frac{\pi^{\theta\phi}}{1-\alpha\beta\pi^{\theta-1}}\frac{-\theta+\theta\phi+1}{1-\alpha\pi^{\theta-1}}\widehat{\pi}_t\\
&+\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{-\theta+\theta\phi+1}{1-\theta}}\pi^{\theta\phi}\sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta-1})^k E_t\left(\widehat{Y}_{t+k}-\widehat{C}_{t+k}+\frac{\theta-1}{1-\alpha\beta\pi^{\theta-1}}\widehat{\pi}_{t+k}\right)\\
=\;&\left(\frac{\theta\phi}{\theta-1}\right)wY^{\phi-1}\pi^{\theta\phi}\sum_{k=0}^{\infty}(\alpha\beta\pi^{\theta\phi})^k E_t\left(\widehat{w}_{t+k}-\widehat{C}_{t+k}+\phi\left(\widehat{Y}_{t+k}-\widehat{A}_{t+k}\right)+\frac{\theta\phi}{1-\alpha\beta\pi^{\theta\phi}}\widehat{\pi}_{t+k}\right)
\end{aligned}
$$

We use the following notational simplifications:

$$
\begin{aligned}
\Lambda_1 &: = \Lambda_2 \frac{1}{1-\alpha\beta\pi^{\theta-1}} \frac{-\theta+\theta\phi+1}{1-\alpha\pi^{\theta-1}}; \\
\Lambda_2 &: = \left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{-\theta+\theta\phi+1}{1-\theta}}; \\
\Lambda_3 &= \left(\frac{\theta\phi}{\theta-1}\right) w \frac{Y^{\phi}}{C} (1-g); \\
Z_t &= \sum_{k=0}^{\infty} (\alpha\beta\pi^{\theta-1})^k E_t \left(\widehat{Y}_{t+k} - \widehat{C}_{t+k} + \frac{\theta-1}{1-\alpha\beta\pi^{\theta-1}} \widehat{\pi}_{t+k}\right); \\
X_t &= \sum_{k=0}^{\infty} (\alpha\beta\pi^{\theta\phi})^k E_t \left(\widehat{w}_{t+k} - \widehat{C}_{t+k} + \phi\left(\widehat{Y}_{t+k} - \widehat{A}_{t+k}\right) + \left(\frac{\theta\phi}{1-\alpha\beta\pi^{\theta\varphi}}\right) \widehat{\pi}_{t+k}\right).
\end{aligned}
$$

The following bloc of equations thus comprise our Phillips relation:

$$
\begin{aligned}
-\Lambda_1 \widehat{\pi}_t - \Lambda_2 Z_t + \Lambda_3 X_t &= 0; \\
Z_t - \widehat{Y}_t + \widehat{C}_t - \frac{\theta-1}{1-\alpha\beta\pi^{\theta-1}} \widehat{\pi}_t &= \alpha\beta\pi^{\theta-1} E_t Z_{t+1}; \\
X_t - \widehat{w}_t + \widehat{C}_t - \phi\left(\widehat{Y}_t - \widehat{A}_t\right) - \frac{\theta\phi}{1-\alpha\beta\pi^{\theta\phi}} \widehat{\pi}_t &= \alpha\beta\pi^{\theta\phi} E_t X_{t+1}.
\end{aligned}
$$

When $\pi = 1$, we see that $\Lambda_3 = 1$ and $\Lambda_2 = 1$, and therefore we can recover a 'standard' Phillips relation:

$$
\widehat{\pi}_t - \kappa \widehat{Y}_t + \mu\left(\widehat{s}_t - \widehat{\lambda}_t + \frac{g}{1-g}\widehat{G}_t + (1+v)\,\phi \widehat{A}_t\right) = \beta E_t \widehat{\pi_{t+1}},
$$

where we define

$$
\begin{aligned}
\mu &= \frac{(1-\alpha\beta)(1-\alpha)}{-\theta+\theta\phi+1}; \\
\kappa &= \mu\left((v+1)\,\phi + \frac{g}{1-g}\right).
\end{aligned}
$$

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