Monetary Shocks, Exchange Rates, and the Extensive Margin of Exports∗

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Abstract: This paper develops a two-country Dynamic General Equilibrium model to assess the relationship between the real exchange rate and the extensive margin of exports. Exchange rate pass-through to consumer prices governs the relative strength of a demand channel onto the exporting decision of a firm. With incomplete pass-through, a favorable movement in the real exchange rate generates increased export participation and an expansion in the extensive margin of exports. This result is consistent with firm-level studies, and contributes to an ongoing empirical debate as to the importance of changes in export participation over the business cycle.

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1. Introduction

Recent empirical studies based on firm-level data have shown that exchange rate movements induce firm entry and exit in export markets. For example, Berman et al. (2012) and Fitzgerald and Haller (2012) both find that favorable movements in the real exchange rate generate entry into foreign markets (for French and Irish firms, respectively). However, using product-level data for the US and OECD, Alessandria and Choi (2008) and Kehoe and Rhul (2009), argue that the number of exporters has little correlation with the real exchange rate and the extensive margin of exports is largely invariant to general fluctuations in the business cycle.\(^1\)

This paper attempts to discriminate between these findings using a two-country Dynamic General Equilibrium model. I start with a standard explanation for real exchange rate movements and business cycle fluctuations: the interaction of monetary shocks with sticky goods prices.\(^2\) Within this setting, I allow for endogenous export participation based on per-period export costs. I show that exchange rate pass-through to consumer prices determines the relative strength of a demand channel onto the exporting decision of a firm. When exchange rate pass-through is incomplete, a favorable movement in the exchange rate expands the set of firms that find it profitable to export because increased demand dominates rising production costs.

Given the well-documented lack of exchange rate pass-through to consumer prices this result is consistent with firm-level studies. Favorable movements in the real exchange rate are associated with expansions in the extensive margin of exports. However, the magnitude of this change also depends on the extent of pass-through.\(^3\) In this sense,

\(^1\)Alessandria et al. (2012) provide evidence on real exchange rates and extensive margins across emerging markets.

\(^2\)For example, see Chari et al. (2002) and the multi-sector model of Carvalho and Nechio (2011). Iversen and Soderstrom (2012) discuss the role of non-monetary shocks in sticky-price models.

\(^3\)If pass-through has fallen over time, as suggested by Marazzi and Sheets (2007), increased export participation following a favorable movement in the real exchange rate is more likely.
even if pass-through were relatively low it could still be possible that the extensive margin is only weakly correlated with movements in the business cycle, as suggested by product-level studies. To determine whether this is indeed the case I calibrate the model and show that the relative strength of the demand channel identified in this paper is quantitatively important. A 1 percent rise in the money stock (after one year) results in a 0.4 percent expansion in the extensive margin, on impact.

There are two features of the model that generate the pass-through based trade-off between demand and costs for firms, and thus strong results with regard to the extensive margin. First, consumed goods undergo two stages of production, with international trade in intermediate inputs.\(^4\) Intermediate firms (stage one) use labor as an input, are heterogeneous in productivity, and export subject to a per-period cost. Final firms (stage two) use domestic and available foreign intermediate goods as inputs and also sell their output in domestic and export markets. The price of final goods is sticky in either producer of local currency terms, which limits exchange rate pass-through to consumer prices.\(^5\)

The second feature is that the creation of new intermediate firms is subject to a sunk entry cost.\(^6\) Because firm creation requires resources, the export participation decision of all pre-existing intermediate firms is conditional on the mass of new entrants, and greater firm creation, all else equal, leads to fewer firms in the export market. The decision of an intermediate firm to export then depends on both the demand for it’s

\(^4\)Huang and Liu (2007) develop a model with multiple stages of production and nominal rigidities to explain business cycle co-movement across countries.

\(^5\)To obtain analytical results I begin by considering polar cases with prices preset in terms of either producer or local currencies. A reduced-form way to index pass-through is then to follow Betts and Devereux (2000) and parameterize the proportion of firms that set prices in local currency. This point is less important when I introduce staggered pricing and perform a quantitative analysis.

\(^6\)The relationship between monetary policy and firm creation is discussed in Bilbiie \textit{et al.} (2007), Bergin and Corsetti (2008), Lewis (2009), and Lewis and Poilly (2012).
output (by home and foreign final firms) and firm creation in the domestic market. For the same reason, changes in the extensive margin of exports depend on movements in international relative consumer prices, which affect demand, and the terms-of-labor, which affect costs. The demand channel dominates at low levels of exchange rate pass-through.

Ghironi and Melitz (2005) also consider an environment in which firms face sunk entry and fixed exporting costs but focus on technology shocks as the source of business cycle fluctuations. They consider both a permanent and transitory (with 0.9 persistence) 1 percent increase in home productivity. This results in a 0.55 and 0.21 percent expansion in the extensive margin, on impact, respectively, and positive co-movement between the home and foreign extensive margins. With monetary shocks and incomplete pass-through the results are similar, yet the channel is markedly different. Positive international co-movement and pro-cyclical extensive margins are generated by changes in demand.

This paper adds to a literature focused on the role of monetary uncertainty and endogenous export participation. For example, Russ (2007) analyzes the differential effect of domestic and foreign monetary uncertainty on multinational production (albeit without an exporting decision), Bergin and Lin (2009) consider the role of the exchange rate regime for adjustment along the extensive margin, and Lewis (2011) analyzes the choice to serve a foreign market by exporting or to produce as a multinational. Rodríguez-López (2011) presents a model where changes in the extensive margin of exports explain exchange rate disconnect. In this paper, the extent of exchange rate pass-through to consumer prices is given, and I assess movements in the extensive margin of exports over the business cycle.

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7See Alessandra and Choi (2007) for an analysis of sunk export costs.
8Monetary models with nominal rigidities that analyze the role of technology shocks are presented in Naknoi (2008), Cavallari (2010), and Auray et al. (2012).
The remainder of the paper is organized as follows. Section 2 develops a two-country model with heterogeneous firms, vertical specialization in production, and sticky-prices. Section 3 provides analytical results and section 4 computes impulse responses for a quantitative version of the model. Section 5 concludes.

2. The World Economy

The world consists of a home and foreign economy each populated by a unit mass of identical, infinitely lived households. Each household supplies labor and holds three domestic assets; shares in a mutual fund of firms, a risk-free bond, and money. Households consume domestic and foreign final goods. Intermediate firms use labor, are heterogenous in productivity, and export subject to a fixed cost. Final firms use home and (available) foreign intermediate goods as inputs and set prices. All final firms export.

Below, goods produced in the home economy are subscripted with an $h$, while those produced in the foreign economy are subscripted with an $f$. An asterisk denotes a foreign economy variable.

2.1. Households

A representative household consumes $C_t$ units of final goods, supplies $L_t$ units of labor, holds real money balances, $m_t \equiv M_t / P_t$, shares in a mutual fund of domestic intermediate firms, $S_t$, and a domestic risk-free bond, $B_t$. The households intertemporal utility function is,

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t + \nu \ln m_t - \kappa L_t^{1+\eta} / (1 + \eta) \right] \quad (1)$$

where $\beta \in (0, 1)$ is a subjective discount factor, $1/\eta$ is the Frisch elasticity of labor supply to wages. Utility is maximized subject to the sequence of constraints,

$$B_{t+1} + n_t S_{t+1} v_{t+1} + P_t C_t + M_t = B_t I_t + W_t L_t + n_{h,t} S_t (d_t + u_t) + d_t + M_{t-1} - T_t \quad (2)$$
where \( n_t \equiv n_{h,t} + n_{e,t} \) is the mass of intermediate firms prior to an exit shock, and is comprised of \( n_{h,t} \) pre-existing firms and \( n_{e,t} \) entrants, \( v_t \) is the share price, \( I_t \) is the gross nominal interest rate, \( W_t \) is the nominal wage, \( d_t \) and \( d_t \) are firm profits, and \( T_t \) is a lump-sum tax.

The following conditions are associated with the household’s optimization problem over shares, labor supply, money, and bonds, respectively,

\[
v_t = \beta E_t \left\{ \left( \frac{C_t P_t}{C_{t+1} P_{t+1}} \right) [(1 - \delta) v_{t+1} + d_{t+1}] \right\} \quad \text{and} \quad w_t = \kappa C_t L_t^n
\]

\[
m_t = \nu C_t I_t / (I_t - 1) \quad \text{and} \quad 1/I_t = \beta E_t \left( \frac{C_t P_t}{C_{t+1} P_{t+1}} \right)
\]

where \( w_t \equiv W_t / P_t \) is the real wage.

An aggregate of home and foreign final goods is consumed by households,

\[
C_t = \Gamma \left( \int_0^1 y_{h,t} (z)^\theta \, dz \right)^{\gamma_h/\theta} \left( \int_0^1 y_{f,t} (z)^\theta \, dz \right)^{\gamma_f/\theta}
\]

where \( y_{h,t} (z) \) (\( y_{f,t} (z) \)) is the consumption of a home (foreign) variety ‘\( z \)’ and \( \Gamma \equiv \gamma_h^n \gamma_f^n \) and \( \gamma_h + \gamma_f = 1 \). The elasticity of substitution between varieties of (home or foreign) final goods is \( \sigma \equiv 1/(1 - \theta) > 1 \) and the elasticity of substitution across the bundles of varieties is unity. Utility maximization leads to a downward-sloped demand curve for a variety \( z \) of the final good,

\[
y^i_{f,t} (z) = \gamma_i (p_{i,t} (z) / P_{i,t})^{-\theta} (P_{i,t} / P_t)^{-1} C_t, \quad \text{for } i = \{h, f\},
\]

where \( P_t \equiv \left( \int_0^1 p_{h,t} (z)^{1-\sigma} \, dz \right)^{\gamma_h/(1-\sigma)} \left( \int_0^1 p_{f,t} (z)^{1-\sigma} \, dz \right)^{\gamma_f/(1-\sigma)} \) is the consumer price index and \( p_{i,t} (z) \) is the price of an individual variety.

2.2. Final Goods Producers

Firms producing final goods use home and foreign (of which \( n_{f,t} \) are available) intermediate goods as inputs. The technology available to final firms is,

\[
y_{h,t} (z) + y_{h,t} (z) = \Gamma \left( \int_{n_{h,t}} y_{h,t} (z, a)^\theta \, da \right)^{\gamma_h/\theta} \left( \int_{n_{f,t}} y_{f,t} (z, a)^\theta \, da \right)^{\gamma_f/\theta}
\]
where $n_{i,t} = n_{t,i}^{\nu - 1/\theta}$ controls a variety effect in production. Final firms compete in a monopolistically competitive market and each firm faces a constant probability, $1 - \alpha$, of a price-adjustment opportunity. The present discounted value of expected profits for a final firm producing good $z$ is then,

$$d_0(z) = \mathbb{E}_0 \sum_{t=0}^{\infty} \alpha^t \mathcal{M}_{0,t} \left[ (p_{h,t}(z) - P_t) y_{h,t}^d(z) + (e_t p_{h,t}^*(z) - P_t) y_{h,t}^{*d}(z) \right]$$

where $P_t \equiv \left( n_{h,t}^{\sigma - 1} \int_{n_{h,t}} p_{h,t}(a)^{1-\sigma} da \right)^{\gamma_h/(1-\sigma)} \left( n_{f,t}^{*\sigma - 1} \int_{n_{f,t}} p_{f,t}(a)^{1-\sigma} da \right)^{\gamma_f/(1-\sigma)}$ is the unit cost function and $\mathcal{M}_{0,t}$ is a stochastic discount factor. Firms maximize profits subject to their technology, given by (6), and the demand for their product from households. I allow for two possibilities in terms of price setting. Either firms can set the price of output in their own currency only, and so choose $p_{h,t}(z)$. In this case, the foreign currency price of their output is, $p_{h,t}^*(z) = p_{h,t}(z) / e_t$, where $e_t$ is the nominal exchange rate. Alternatively, firms can set prices in local currency terms, and choose both $p_{h,t}(z)$ and $p_{h,t}^*(z)$. In this case, the law of one price fails and exchange rate pass-through is incomplete.

As with final goods, the demand function for any intermediate good ‘$a$’ is downward-sloped,

$$y_{i,t}^d(a) = \gamma_i n_{i,t}^{\theta - 1} \left( p_{i,t}(a) / P_t \right)^{-\theta} \left( P_{i,t} / P_t \right)^{-1} \int_0^1 [y_{h,t}(z) + y_{h,t}^*(z)] dz$$

for $i \in \{h, f\}$.

### 2.3. Intermediate Goods Producers

Firms producing intermediate goods require labor (resources) and have a linear technology, $y_{h,t}(a) + y_{h,t}^*(a) = a_{t}(a)$, where $a$ is firm-specific. Intermediate firm profits from domestic and (potential) export sales are,

$$d_t(a) = (p_{h,t}(a) - W_t) y_{h,t}^d(a) + (e_t p_{h,t}^*(a) - W_t) y_{h,t}^{*d}(a) - W_t f_{h}^*$$
where \( f_h^* > 0 \) is a fixed cost associated with exporting and \( W_t/a \) is the unit cost of production.

With fixed costs and a distribution over productivity a subset of firms do not export because they do not earn profit from doing so. I define the zero export profit (threshold) level of productivity as, \( a^*_{h,t} \equiv \inf \{ a : d^*_{h,t} (a) > 0 \} \), where \( d^*_{h,t} (a) \) are the potential export profits of a firm with productivity level \( a \). Using the first-order conditions from profit maximization - \( p_{h,t} (a) = W_t/\theta a \) and \( p^*_{h,t} (a) = p_{h,t} (a)/e_t \) - the threshold level of productivity for exporting can be written as,

\[
a^*_{h,t} = \frac{p^*_{h,t}}{\theta n^*_{h,t}} \left( \int_{\gamma}^{1} \left( \frac{1}{\sigma f_h^*} \int_{0}^{1} [y_{h,t} (z) + y^*_{h,t} (z)] \, dz \right)^{1/(1-\sigma)} \right)
\]

(9)

The threshold level of productivity depends on the demand for final goods, international relative prices, the unit costs of production, and the mass of home intermediate exporting firms, denoted \( n_{h,t}^* \), where \( n_{h,t}^* = n_{h,t}^{v-1}/\theta \).

Following Ghironi and Melitz (2005), I write firm-level variables in averages. In any period there are \( n_{h,t} \) intermediate firms producing goods, with an average level of productivity, \( \bar{a} \equiv \left( \int_{a}^{1} a^{\theta-1} g (a) \, da \right)^{1/(\theta-1)} \), where \( g (a) = \kappa a^{-(\kappa+1)} \). Because it is possible to earn zero profit from exporting, only a subset of intermediate firms export. Average productivity across firms that export is, \( \bar{a}_{h,t} \equiv \left( \frac{1}{1-G (a_{h,t}^*)} \int_{a_{h,t}^*}^{a_{h,t}} a^{\theta-1} g (a) \, da \right)^{1/(\theta-1)} \). The term \( 1 - G (a_{h,t}^*) \) is the ex-post probability of successfully exporting and is related to the ratio of intermediate exporters to all intermediate firms, \( n_{h,t}^*/n_{h,t} = 1 - G (a_{h,t}^*) \).

2.4. Intermediate Entrants

There is a competitive fringe of potential intermediate firms. The creation of intermediate firms is subject to a sunk entry cost, \( f_e > 0 \). New intermediate firms at

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9The assumption that productivity is Pareto distributed is a common assumption and matches well with micro data on firm size distribution.
time \( t \) start producing at time \( t + 1 \). Prospective intermediate firms are forward looking, and correctly anticipate their future expected profits as well as the probability of incurring an exit shock, \( \delta \), at the end of each period after they produce. Intermediate firms post-entry value is given by the present discounted value of expected profits,

\[
\nu_0 = \mathbb{E}_0 M_{0,1} d_t + \mathbb{E}_0 \sum_{t=1}^{\infty} (1 - \delta)^t M_{0,t+1} d_{t+1}.
\]

New intermediate firms enter as long as they can cover sunk costs, which implies \( \nu_t \geq f_e W_t \). Finally, the timing of entry and production implies the number of intermediate firms during period \( t \) is,

\[
n_{h,t} = (1 - \delta)n_{h,t-1} + n_{e,t-1},
\]

where \( n_{e,t-1} \) is the mass of intermediate entrants.

### 2.5. Foreign Economy and Equilibrium

In the foreign economy, there are \( n_{f,t}^\star \) intermediate firms, with average productivity \( \bar{\sigma} \). Of these firms, \( n_{f,t} \) also serve the export market, with average productivity \( \bar{\sigma}_{f,t} \). Potential intermediate firms enter the domestic market only if they can cover a sunk cost, and produce with a one period lag, subject to being hit by an exit shock. This implies,

\[
\nu_{t}^\star = \mathbb{E}_t M_{0,1}^\star d_{t+1} + \mathbb{E}_t \sum_{t=1}^{\infty} (1 - \delta)^t M_{0,t+1}^\star d_{t+1}^\star \geq f_e^\star W_t^\star \quad \text{and} \quad n_{f,t}^\star = (1 - \delta)n_{f,t-1}^\star + n_{e,t-1}^\star.
\]

Foreign intermediate firms choose prices - \( p_{f,t}^\star (a) \) and \( p_{f,t} (a) \) - to maximize profits with the input price taken as given. Foreign final firms choose output prices - \( p_{f,t}^\star (z) \), or \( p_{f,t}^\star (z) \) and \( p_{f,t} (z) \), depending on the possibility of international price discrimination - to maximize profits, taking input prices - \( p_{h,t}^\star (a) \) and \( p_{f,t}^\star (a) \) - as given.

Aggregating across home households, imposing \( S_t = 1 \) and \( B_t = 0 \), and using the government budget constraint, \( T_t = M_{t-1} - M_t \), equilibrium requires that a resource constraint is satisfied,

\[
W_t L_t + n_{h,t} d_t + d_t = P_t C_t + \nu_t n_{e,t}
\]

with an analogous condition for the foreign economy. The right-hand side of (10) represents expenditure - on consumption and investment in new intermediate firms - and the left-hand side income - from labor and firm profits.
In each economy the labor market clears, with labor used for domestic production, export production, exporting costs, and domestic entry (firm creation). In the home economy, for example, \( L_t = (n_{h,t} + n_{h,t})\hat{\lambda}_t + n_{h,t}f_h + n_{e,t}f_h \). The free entry condition holds with equality and there are two sector specific net export equations,

\[
x_t = n_{h,t}^*e_t p_{h,t} y_{h,t}^* - n_{f,t} p_{f,t} y_{f,t}^* \quad \text{and} \quad x_t = e_t p_{h,t} y_{h,t}^* - y_{f,t} p_{f,t}
\]

These conditions capture the world trade in goods.

3. Analytical Results for the Extensive Margin of Exports

In this section I present analytical results. I show how the extensive margin of exports depends on international relative consumer prices and the terms-of-labor and how the response of the extensive margin to shocks depends on exchange rate pass-through. To this end, I make a number of simplifying assumptions. I assume final firms reset the price of their output each period and new intermediate firms only produce for one period. The variety effect in the production of final goods is eliminated by assuming \( v = 1 \). Finally, I focus on an unanticipated and permanent change in the level of the home money supply (equivalent to a one period change in rate of money growth). Denoting a variable linearized around it’s steady-state value with a caret, this implies, \( \Delta \hat{M}_0 = \hat{M} \) and \( \Delta \hat{M}_t = 0 \) for \( t \geq 1 \).

3.1. A Reduced-Form for the Extensive Margin

I start by deriving a reduced-form expression for the extensive margin of exports in the home economy. The decision to export for an individual firm is driven by it’s productivity. Average productivity (across exporting intermediate firms) changes with the proportion of firms that decide to export. Since there is a time-to-build lag in

\[\text{Given the structure of the model, these assumptions imply that, in periods } t \geq 1, \text{ all real variables reach their long-run levels, monetary shocks are neutral, and there are no nominal exchange rate dynamics. As a result, I focus on the impact effect of the shock.}\]
production, the extensive margin and average productivity are linked by, \( \hat{n}_{h,0} = -\kappa \hat{w}^*_h \).

Accounting for export demand in the productivity threshold,

\[
\hat{n}_{h,0} = \hat{C}^w_0 + \left[ (\hat{c}_0 - \hat{P}_0) - \frac{1}{2} (\hat{P}_{f,0} - \hat{P}_{h,0}) \right] - (\hat{w}_0 - \hat{P}_0^*)
\]

(12)

where I have imposed that home and foreign goods have an equal weighting when used in production and for consumption. Using (12), movements in the extensive margin of exports can be decomposed into changes in, (i), global aggregate demand, (ii), international relative prices, and (iii), the costs of production, for both intermediate and final firms. All else equal, a rise in global demand, \( \hat{C}^w_0 \equiv (1/2) (\hat{C}_0 + \hat{C}^*_0) \), expands the home extensive margin of exports. Favorable movements in the real exchange rate, \( \hat{Q}_0 \equiv \hat{c}_0 + \hat{P}_0^* - \hat{P}_0 \), also expand the extensive margin (where \( \hat{P}_0^* = \frac{1}{2} (\hat{P}_{h,0} + \hat{P}_{f,0}) \) and the same for \( \hat{P}_0^* \)), but only when prices are preset in local currency terms. When consumer prices are preset in the currency of the producer, the term in square brackets in (12) is zero.

Input costs - captured by \( (\hat{w}_0 - \hat{P}_0^*) \) - introduce a role for the terms-of-labor. When the home real wage rises, all else equal, the extensive margin contracts, consistent with higher fixed costs. However, eliminating \( \hat{P}_0^* \), which is the input cost for foreign final firms, the extensive margin can be written as a function of the terms-of-labor, \( \hat{tol}_0 \equiv \hat{w}_0^R - \hat{Q}_0 \), where \( \hat{w}_0^R \) is the relative real wage.\(^{11}\) Thus, although higher home intermediate firm input costs reduce the home extensive margin of exports, higher foreign intermediate firm input costs expand the home extensive margin. In this setting, therefore, both the real exchange rate (international relative prices) and terms-of-labor (international relative input costs) play a key role in determining the response of the extensive margin to shocks.\(^{12}\)

\(^{11}\)Using the pricing equations for foreign intermediate firms, the unit cost is determined by the terms-of-labor and the extensive margin of exports. If we temporarily ignore the feedback onto the extensive margin from a change in unit costs, we can write the combined cost in terms of the relative real wage.

\(^{12}\)The terms-of-labor also plays an important role in Ghironi and Melitz (2005).
Finally, the terms-of-labor depends on the extent of firm creation. Potential intermediate firms enter their domestic market until sunk entry costs rise above the presented discounted value of total expected profits. When intermediate firms produce for one period, the average value of the firm is the future (i.e. \( t = 1 \)) level of profits, discounted by the current real interest rate, and \( \hat{v}_0 = \hat{C}_0 - \hat{C}_1 + \hat{d}_1 \). Future profits are consistent with flexible prices, so total profits can be written as, \( \hat{d}_1 = \hat{C}_1^W - \hat{n}_{e,0} \). Imposing free entry, the terms-of-labor, relative number of new entrants, and real interest rates are linked to the real exchange rate in the following way.

\[
\hat{tol}_0 = - \left( \hat{n}_{e,0}^R + \hat{r}_{e,0}^R + \hat{Q}_0 \right) \tag{13}
\]

Eliminating \( \hat{P}_0^* \) in equation (12), a higher relative real wage leads to a contraction in the home extensive margin, which is appealing because it reflects higher relative costs for home intermediate firms. However, using (13), the same negative relationship holds for new entrants, which implies a positive relationship between relative home firm creation and the home extensive margin of exports. Since (13) holds for both economies, relative firm creation and the foreign extensive margin of exports, \( \hat{n}_{f,0} \), are negatively related. In this case, we conclude that relatively more new entrants at home (abroad) generates an expansion in the home (foreign) extensive margin of exports.

### 3.2. Analysis of the Extensive Margin

I now provide an explicit solution for the home and foreign extensive margin of exports. When firms engage in producer currency pricing, own-country consumer prices do not react to shocks, there is full exchange rate pass-through, and the foreign local-currency price of traded goods is given by, \( \hat{P}_{f,0} = -\hat{P}_{h,0} = \hat{e}_0 \). When firms engage in local currency pricing, consumer price indices do not react to shocks, and there is zero pass-through. Using short-run money demand, in both cases, the reaction of global consumption is, \( \hat{C}_0^W = (1/2) \hat{M} \).\(^{13}\) I then have the following expressions for extensive

\(^{13}\)A direct result of local currency pricing is that adjustment to monetary shocks works through
margins,

Table 1: Extensive Margin of Exports

where $\theta \equiv \kappa/(2\kappa - 1) > 0$ and $\kappa$ controls the curvature of the distribution over productivity. Changes in economy-wide resources and the trade account (see equations (10) and (11)) jointly determine the pattern of firm creation across countries and the nominal exchange rate in table 1.

As I show in the Appendix, when there is full exchange pass-through, the nominal exchange rate rises proportionately less than the change in the money supply and this leads to a drop in relative entry across the two economies. Using the conditions in table 1, $\hat{n}_{h,0}^* = (\theta/2) \hat{n}_{e,0}^R$, so the home extensive margin of exports must contract as a result of a positive home monetary shock. With incomplete exchange rate pass-through, the nominal exchange rate changes proportionally with the shock, and firm creation is identical across the two economies. The result is that the home extensive margin of exports expands, and $\hat{n}_{h,0}^* = \theta \hat{M}$. The general point is that a currency depreciation is only consistent with firm entry into the export market when exchange rate pass-through to consumer prices is incomplete.

The intuition for the contraction in the home extensive margin is that when exchange rate pass-through is high a home depreciation is consistent with increased (decreased) competitiveness of home (foreign) final goods. A relative increase in the production and export volume of home final goods requires more home and foreign intermediate inputs. Home final firms have access to all home goods but only a subset of foreign consumption and consumption is uncorrelated across countries ($\hat{C}_0 = \hat{M}$ and $\hat{C}_0^* = 0$). If exchange rate pass-through were to rise, the effect of a nominal depreciation of the domestic currency would be to lower $\hat{P}_0^*$, and generate a rise in $\hat{C}_0^*$. This leads to positive correlation in consumption across countries (full pass-through implies, $\hat{C}_0 = \hat{M} - \frac{1}{2}\hat{e}_0$ and $\hat{C}_0^* = \frac{1}{2}\hat{e}_0$).
goods. The result is rise in the number of foreign exporters. For a similar reason the number of home exporters falls - there is less demand from foreign final firms. However, this does not account for the deterioration in the terms-of-labor, which has a further negative effect on home intermediate firms export decisions because they also need to cover the fixed export cost.

Because monetary shocks are expansionary for home economy, the results also imply that, depending on the extent of exchange rate pass-through, the number of exporters can be either pro or counter-cyclical and extensive margins can be positively or negatively correlated across countries. Betts and Devereux (2000) model exchange rate pass-through in a simple and appealing way by parameterizing the proportion of firms that set prices in local currencies. They show that this can have a profound effect on the international output and consumption correlations. Lower pass-through results in weaker consumption correlation across countries and positive output correlations. The same mechanism underlies the relationship between extensive margins because intermediate firms form part of an international production chain.

4. A Quantitative Example

In this section I analyze a quantitative version of the model using impulse response functions. Although the model allows for firm creation and endogenous export participation, it is consistent with a large class of open economy models that feature a fixed number of traded varieties and physical capital, such as Kollmann (2001) and Chari et al. (2002).\textsuperscript{14} Although I abstract from physical capital, firm creation represents the extensive (as opposed to intensive) margin of investment, with \( \hat{n}_{h,t} = (1-\delta)\hat{n}_{h,t-1} + \delta \hat{n}_{e,t-1} \) replacing the standard law of motion for capital.

In addition to firm creation, model dynamics are generated through price setting behavior. When final firms engage in producer currency pricing, two dynamic price

\textsuperscript{14}As a result I can undertake a standard calibration of the model.
equations determine the extent of global price dynamics at the consumer level,

\[ \Delta \hat{P}_{h,t} = \beta E_t \Delta \hat{P}_{h,t-1} + \varrho \left( \hat{P}_t - \hat{P}_{h,t} \right) \quad \text{and} \quad \Delta \hat{P}_{f,t} = \beta E_t \Delta \hat{P}_{f,t-1} + \varrho \left( \hat{P}_t - \hat{P}_{f,t} \right) \]  

(14)

where \( \varrho \equiv (1 - \alpha) (1 - \alpha \beta) / \alpha \beta \) and \( \Delta \) denotes the first difference of a variable. The consumer price indices in the home and foreign economy are given by,

\[ \hat{P}_t = \gamma_h \hat{P}_{h,t} + \gamma_f \left( \hat{P}_{f,t} + \hat{e}_t \right) \quad \text{and} \quad \hat{P}^*_t = \gamma_h \left( \hat{P}_{h,t} - \hat{e}_t \right) + \gamma_f \hat{P}_{f,t}, \]

where the nominal exchange rate is pinned down by resources, trade, and money demand. When firms engage in local currency pricing, the law of one price fails to hold, and I map price dynamics into four composite variables, \( \Delta \hat{P}_{t}^R \), \( \Delta \hat{P}_{t}^W \), \( \Delta \left( \hat{P}_{f,t} - \hat{P}_{h,t} \right) \), and \( \Delta \left( \hat{P}^*_{f,t} - \hat{P}^*_{h,t} \right) \), whose driving variables are as in (14). An important point in my analysis is that because price dynamics are driven by marginal costs, there is a role for the price of home and foreign intermediate firms outputs, \( \hat{P}_t \) and \( \hat{P}^*_t \), respectively. These prices are directly affected by changes in the number of firms producing and exporting in each economy.

Consistent with the analysis in section 3 I focus on a home monetary expansion. However, I now assume that the change in money growth follows a first-order autoregressive process, \( \Delta \hat{M}_t = \rho \Delta \hat{M}_{t-1} + \varepsilon_t, \) where \( 0 < \rho < 1 \) and \( \varepsilon_t = 0 \) for \( t \geq 1 \). I set \( \varepsilon_0 \) so that the home money stock rises by 1% one year after the shock.

Table 2 presents the parameters used to calibrate the model.

**Table 2: Calibrated Parameters for Quantitative Analysis**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.99</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.47</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.025</td>
</tr>
<tr>
<td>( \theta )</td>
<td>3.8</td>
</tr>
</tbody>
</table>

I interpret a time period in the model as a quarter and set \( \beta = 0.99 \). This implies a steady-state annualized interest rate of approximately 4%. Following Rotemberg and Woodford (1997), I assume the (inverse) Frisch elasticity of labor supply to wages is \( \eta = 0.47 \). I follow Ghironi and Melitz (2005) and match the intermediate firm exit shock with a 10% rate of job destruction per year, which implies \( \delta = 0.025 \), and assume the markup of final and intermediate goods firm is 35%, which implies \( \theta = 3.8 \). The
standard deviation of log US plant sales is 1.67, which implies setting the parameter that determines the curvature of the Pareto distribution for firm-level productivity at $\kappa = 3.4$. I also use a standard calibration for the production of final goods. I assume market power and lover-of-variety coincide, which implies $\upsilon = 1.4$, that the average length of a price contract of 3 quarters, which implies $\alpha = 0.67$, and I set $\gamma_f = 0.2$, which determines import share for intermediate and final goods. Finally, I follow Chari et al. (2002) and set $\rho = 0.68$. This further implies the monetary shock is $\varepsilon_0 = (1 - \rho) / (1 - \rho^4) = 0.41$.

Figure 1 plots the response of home and foreign variables to a monetary shock under the assumption of producer currency pricing (full exchange rate pass-through).

**Figure 1: Impulse Responses with Producer Currency Pricing**

The solid lines in figure 1 track the change in the money supply. Lines with ♦/♣ represent the home/foreign economy response of endogenous variables. The impulse responses confirm the analytical results. A positive home monetary shock causes a contraction in the home extensive margin of exports. The number of home imports expands (so the home and foreign extensive margin of exports are negatively correlated). In particular, there is a 0.3% fall (1.1% rise) in the home (foreign) extensive margin on impact, given a 1% rise in the home money supply after 4 quarters.\(^{15}\) The impulse responses also show that whilst traded variety rises at a global level, the indirect effect of a home monetary stimulus on foreign exported varieties is larger than the direct negative impact on home firms. This is a result of the change in the terms-of-labor.

\(^{15}\)The response of the nominal exchange rate to the shock is muted because of financial autarky. This assumption is made for the sake of simplicity. Adding an internationally traded bond, for example, produces a greater initial rise in the exchange rate, and strengthens the results.
Although I focus on the extensive margin of exports, new domestic entrants in each economy also embody investment by households and the total stock of firms represents accumulated capital. In IRBC and sticky-price models, it is more common to consider international co-movement rather than country-level responses. In this model, there are strong positive spillovers onto the foreign economy, with positive international co-movement at the extensive margin of investment - measured as the real value of household investment in new firms. Investment is also pro-cyclical. This is broadly similar to results produced by sticky-price models of the business cycle for investment at the intensive margin. Consumption and the real interest rate also respond as they would do in a sticky-price model. Consumption rises in both the home and foreign economy and the real interest rate falls.\textsuperscript{16}

With fixed traded variety, once local currency pricing is assumed, the international relative price and real interest rate channels of macroeconomic interdependence are weakened (Kollmann, 2001). This leads to a muted response of foreign macro-variables to home monetary shocks (i.e., there are weak international spillovers). In the analytical version of the model the producer/local currency pricing distinction is also important for the extensive margin of exports. Figure 2 plots the response of home and foreign variables to a home monetary shock under local currency pricing (incomplete exchange rate pass-through).

\textbf{Figure 2: Impulse Responses with Local Currency Pricing}

It is immediate from the impulse responses that international spillovers from home monetary shocks onto foreign consumption and investment (at the intensive margin) are relatively weak. However, this is not the case for the variety of intermediate traded

\textsuperscript{16}I have not plotted the response of inflation, but the reaction of home and foreign consumer price inflation is standard. Consumer prices rise by less in the foreign economy than in the home economy.
goods. Both home and foreign extensive margins expand as a result of the shock, and
the expansion is very similar in magnitude in both economies (the impact effect is
around 0.4%). The intuition for this result is that, under local currency pricing,
changes in the demand for final goods are driven by global patterns of demand, as
opposed to a shift in demand away from foreign final goods. In this case, there is no
large change in the terms-of-labor to drive differences in the home and foreign extensive
margin of exports as there would be under producer currency pricing.

To see this point, note that irrespective of pass-through, the home real wage rises
in response to the home monetary shock - although with local currency pricing the
foreign real wage is less sensitive to the shock. Recall also that the relative real wage
can be expressed as the sum of the terms-of labor and consumer-based real exchange
rate, \( \hat{w}_0^R = \hat{t}q_0 + \hat{Q}_0 \), that the real exchange rate is more sensitive to monetary shocks
under local currency pricing, and that the reaction of the nominal exchange rate is
broadly similar. Therefore, under producer (local) currency pricing, the terms-of-labor
exerts a strong (weak) effect on intermediate firms. This generates negative (under
producer currency pricing) or positive (under local currency pricing) co-movement in
the extensive margin of exports. Finally, the global impact on traded varieties is
similar across the two price-setting regimes. Thus, when the home extensive margin
of exports expands with a home depreciation, the rise in foreign imported varieties
into the home economy falls in magnitude, consistent with a traditional expenditure
switching effect.

4. Conclusion

This paper develops a two-country Dynamic General Equilibrium model to understand
changes in the extensive margin of exports over the business cycle - which is subject to
an ongoing empirical debate. Exchange rate pass-through to consumer prices is shown
to play a key role in determining a trade-off (between the demand for a product and the
costs of production) for firms when deciding to export. With incomplete pass-through, a favorable movement in the exchange rate generates firm entry and an expansion in the extensive margin of exports, as firm-level studies suggest.
References


Appendix on Analytical Results under Producer and Local Currency Pricing

In period $t = 0$, resources, free entry, and profits (for intermediate and final goods firms) under PCP are,

$$\hat{n}_{e,0} = \left( \frac{wL}{vn_e} - 1 \right) \hat{w}_0 + \left( \frac{n_h d_1}{vn_e} \right) \hat{d}_0 + \left( \frac{d_2}{vn_e} \right) \hat{d}_0 - \left( \frac{Y_2}{vn_e} \right) \hat{C}_0$$

$$\hat{d}_0 = \hat{C}_0^W + \omega \hat{P}_0 + (1 - \omega) \hat{P}^*_0 ; \quad \hat{d}_0 = \hat{C}_0^W - (\sigma - 1) \hat{P}_0 ; \quad \hat{w}_0 = \hat{C}_0 - \hat{n}_{e,0}$$

where $\omega \equiv \kappa / (\kappa + \theta - 1)$ and a caret denotes a variable linearized around it’s steady-state value. Steady-state variables (variables without a sub-script) are a function of the underlying parameters of the model. An equivalent set of conditions hold for $\hat{n}^*_e,0$.

Eliminating profits and wages from home and foreign conditions,

$$\frac{wL}{vn_e} \hat{n}^R_{e,0} = \left( \frac{n_h d_1}{vn_e} + \frac{d_2}{vn_e} \right) \left( \hat{e}_0 - \hat{M}^R \right) + \left[ (1 - \sigma) \left( \frac{d_2}{vn_e} \right) - (1 - 2\omega) \left( \frac{n_h d_1}{vn_e} \right) \right] \hat{P}^R_0$$

where $\hat{n}^R_{e,0} \equiv \hat{n}_{e,0} - \hat{n}^*_e,0$ is relative firm entry into respective domestic markets. The equation for the exchange rate is derived from the trade balance and relative money demand, and can be written as, $0 = \theta \hat{P}^R_0 - (\hat{M}^R - \hat{e}_0)$. Using the equations for the extensive margin of exports and relative prices for intermediate goods firms,

$$\hat{P}^R_0 = \left( \frac{\kappa + 1/2}{\kappa - 1/2} \right) \hat{w}_0^R + \left( \frac{\kappa}{\kappa - 1/2} \right) \hat{e}_0 \quad \text{and} \quad \hat{P}^W_0 = \hat{w}_0^W + \left( \frac{1/2}{\kappa - 1/2} \right) \hat{M}^W$$

where I have also imposed $\hat{C}_0^W = \hat{M}^W$. Using all of these conditions, the solution for $\hat{n}^R_{e,0}$ is,

$$\hat{n}^R_{e,0} = 2\kappa \left( \frac{\Omega}{\Omega - 1} \right) \hat{M} \quad ; \quad \Omega \equiv \left[ \frac{2\omega \sigma + 1 + \theta}{2\omega \sigma + (\beta - 1)} \right] \left[ \left( \frac{1}{1 + \theta} \right) \left( \frac{1}{\kappa - 1} \right) \right]$$

where $1 / (1 + \theta)$ and $1 / (2\kappa - 1)$ are both less than 1. The term $2\omega \sigma$ is greater than 2, and thus the numerator and denominator are both positive in the first term of $\Omega$. Moreover, the parameter $\Omega$ is always between zero and one. In this case, $\hat{n}^R_{e,0} < 0$. Once we know this, it is possible to show $\hat{M} > \hat{e}_0 > 0$. Using these condition in the
expressions for the extensive margin of exports, we can also conclude, $\hat{n}_{h,0} < 0$ and $\hat{n}_{f,0} > 0$.

For local currency pricing, I repeat the analysis. In this case, there is a unit coefficient for $\hat{M}^R$ and $(\hat{e}_0 + \hat{P}_0^R)$ in the relative resource constraint. I can use this to understand relative price fluctuations, $\hat{P}_0^R$. If we temporarily suppose $\hat{n}_{c,0} = 0$, it is immediate that $\hat{P}_0^R$ and $\hat{M}^R$ are also related by a unit coefficient. Since $\hat{n}_{c,0}^R$ is a function of $(\hat{e}_0 + \hat{P}_0^R)$ and $\hat{P}_0^R$, it follows $\hat{n}_{c,0}^R = 0$. Since $\hat{e}_0 = \hat{M}^R$, this requires $\hat{n}_{h,0}^* = \hat{n}_{f,0} > 0$, as in the main text.
Table 1: Extensive Margin of Exports

<table>
<thead>
<tr>
<th></th>
<th>Home Export Margin ($\hat{n}_{h,0}$)</th>
<th>Foreign Export Margin ($\hat{n}_{f,0}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCP</td>
<td>$\left(\frac{\theta}{2}\right)\hat{n}_{e,0}$</td>
<td>$\theta \left(\hat{M} - \frac{1}{2}\hat{n}_{e,0}\right)$</td>
</tr>
<tr>
<td>LCP</td>
<td>$\frac{\theta}{2}(\hat{n}_{e,0} + \hat{e}_0)$</td>
<td>$\theta \left[\hat{M} - \frac{1}{2}(\hat{n}_{e,0} + \hat{e}_0)\right]$</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Value</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>0.47</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Firm exit shock</td>
<td>0.025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between goods</td>
<td>3.8</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Variety effect</td>
<td>1.4</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Productivity dispersion</td>
<td>3.4</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Duration of price contracts</td>
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</tr>
<tr>
<td>$\gamma_f$</td>
<td>Import penetration</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autoregressive parameter for money growth</td>
<td>0.68</td>
</tr>
</tbody>
</table>
Figure 1: Impulse Responses with Producer Currency Pricing
Figure 2: Impulse Responses with Local Currency Pricing