



Economics Department Discussion Papers Series

ISSN 1473 – 3307

Evaluating the Role of Firm-Specific Capital in
New Keynesian Models.

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Paper number 12/04

Evaluating the role of firm-specific capital in New Keynesian models

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September 2012

Abstract

In this paper I make use of Bayesian methods to estimate a firm-specific capital DSGE model with Calvo price and wage setting. This approach allows me to firmly conclude that firm-specific capital is highly relevant in improving the fit of New Keynesian models to the data as shown by a large increase in the value of the log marginal data density relative to the more conventional rental capital model. The introduction of firm-specific capital also has important implications for business cycle dynamics leading to increased persistence of aggregate variables and helps reduce the discrepancy between macro estimates of the NKPC and the observed frequent price adjustments in the micro data.

JEL Classification: E20, E22, E27, E30, E32, E37

Keywords: New Keynesian models, sticky prices, DSGE, business cycles, firm-specific capital, Bayesian estimation.

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1 Introduction

Svein and Weinke (2004) and Woodford (2005) demonstrated how firm-specific capital can be important in helping New Keynesian models with Calvo contracts reconcile the inertia in inflation observed in the macro data with the frequent price adjustments by firms at the micro level.¹ Also, Svein and Weinke (2005, 2007) show that firm-specific capital has implications for the desirability of alternative interest rate rules whereas Blake and Kirsanova (2008) find it associated to multiple discretionary equilibria; making the empirical relevance of firm-specific capital at the macro level for dynamic stochastic general equilibrium (DSGE) models an important question which however hasn't satisfactorily been addressed previously in the literature (Nolan and Thoenissen, 2008, found that firm-specific capital generates a less volatile, as well as more persistent series for inflation but could not firmly state if assuming firm-specific capital helps New Keynesian models match the U.S. business cycle data). In this paper I address this gap in the literature by using Bayesian methods to estimate a firm-specific capital DSGE model with Calvo price and wage setting, showing that not only is it important in improving the model's ability to match the mean price duration at the micro level but also in achieving a better fit to U.S. macro time series data (this is possible because the posterior distribution obtained from Bayesian estimation offers a particularly natural method of comparing models not available with the calibration methodology employed by Nolan and Thoenissen, 2008).

Despite being a more appealing choice of modelling capital (standard business cycle models assume that capital can be instantly and costlessly transferred across firms which is empirically unrealistic and as Woodford, 2003, states "its consequences are far from trivial") there are still few examples of DSGE models with firm-specific capital and very little empirical work on the topic.² In particular, no one has previously studied jointly the implications

¹Standard New Keynesian models imply that firms reoptimize prices roughly once every six quarters (see Galí and Gertler, 1999) or even less often (see for example, the estimates in Smets and Wouter, 2003; Rabanal and Rubio-Ramírez, 2005, for the post 1982 era, and Eichenbaum and Fisher, 2007), which is inconsistent with an average of less than two quarters found in the microeconomic data (Klenow and Kryvtsov, 2005).

²One possible reason for this is that when capital is firm-specific it is no longer possible to solve the price setting problem without considering the firm's optimal investment behavior. This makes the model

of firm-specific capital at both the macro and micro level (Altig, Christiano, Eichenbaum and Linde, 2011, focus solely on the consequences for the mean time price duration at the micro level while de Walque, Smets and Wouters, 2006, and Nolan and Thoenissen, 2008, analyze only how firm-specific capital affects the aggregate behavior of economic variables). In order to do this, the use of Bayesian methods is particularly relevant since priors work as weights on the likelihood function giving more importance to areas of the parameter subspace which are more consistent with estimated values obtained from studies using micro data. As shown by Woodford (2005) the only difference between the log-linearized equations characterizing equilibrium in the firm-specific and rental capital models pertains to the mapping between the structural parameters and the slope of the New Keynesian Phillips curve (NKPC). Altig, Christiano, Eichenbaum and Linde (2011) estimate the firm-specific capital and rental capital DSGE models in terms of the reduced form NKPC. This amounts to imposing the two models to be observationally equivalent with respect to aggregate prices and quantities. In this paper I estimate the DSGE model in terms of the structural Calvo probability θ (the likelihood of the firm not being able to optimally reset its price in a given period) using as in Smets and Wouters (2007) a prior informed by the findings for prices of Bils and Klenow (2004) and Klenow and Kryvtsov (2005).³ This effectively allows for separate identification of the two models using macro data and the assessing of their relative plausibility.

With this approach, I can firmly conclude that firm-specific capital is highly relevant in improving the fit of New Keynesian models to the data as shown by a large increase in the value of the log marginal data density relative to the more conventional rental capital model. This strongly supports the hypothesis that introducing firm-specific capital in DSGE models is highly relevant for the understanding of business cycle fluctuations and highlights the advantages of employing Bayesian methods for data analysis since Nolan and Thoenissen's (2008) results using calibrated models were not at all clear on this issue.

My analysis suggests that the improved fit to the data of the New Keynesian model

considerably less tractable but it turns out to still be possible to derive an aggregate-supply relation following the method developed in Woodford (2005).

³Eichenbaum and Fisher (2007) and Matheron (2006) also obtain direct estimates of θ . However these authors estimate only the New Keynesian Phillips curve and not a fully specified DSGE model.

seems to be behind the increased persistence of aggregate variables by the firm-specific capital model relative to the rental capital specification. This is an important point since Chari, Kehoe and McGrattan (2000) found the standard New Keynesian model to have difficulty accounting for inflation and output persistence. Firm-specific capital increases persistence because it is a real rigidity in the sense of Ball and Romer (1990). That is, firm-specific capital reduces the responsiveness of firms' profit-maximizing prices to variations in aggregate output resulting from disturbances in aggregate demand therefore increasing the responsiveness of output to exogenous shocks. As Romer (2008) observes, real rigidities "appear essential to any successful explanation of short-run macroeconomic fluctuations".

I also study how the model's responses to exogenous shocks are changed by the introduction of firm-specific capital. In this aspect the paper makes another important contribution. The study of the dynamic effects of firm-specific capital for aggregate economic variables has so far been limited to productivity and monetary shocks (Svein and Weinke, 2004, study only monetary shocks whereas Nolan and Thoenissen, 2008, look at both monetary and productivity shocks). As Nolan and Thoenissen (2008) remarked 'the overall assessment of the data-congruency of New Keynesian models in general, and of firm-specific capital models in particular, awaits the incorporation of important demand shocks', for this reason I look at a wider range of shocks than those considered in previous studies of firm-specific capital models. Besides total factor productivity and monetary shocks the model presented here includes shocks to the discount rate, labor supply, government spending, investment-specific technology and price mark-up (de Walque, Smets and Wouters, 2006, include the same set of shocks in their model but assume Taylor contracts and focus their attention solely on the responses to monetary policy shocks).

I find that the introduction of firm-specific capital has important dynamic implications. The impulse response functions show that firm-specific capital, by making firms change prices by less, aids considerably in propagating the responses of output, while dampening movements in inflation, to exogenous "demand" shocks (since these tend to move output and prices in the same direction).

Moreover, I'm able to show that firm-specific capital does improve the ability of New Keynesian DSGE models to match the frequency of price adjustment in the micro data. I found that firms reoptimize prices on average every 5.6 quarters in the firm-specific capital, which is considerably lower than the estimate under the rental capital assumption (11.1 quarters), yet it still falls very far from the average of less than two quarters found in the microeconomic data (Klenow and Kryvtsov, 2005).

In summary, my analysis indicates that the introduction of firm-specific capital is important for economists applying Bayesian methods to estimate DSGE models because it allows both an improvement of the ability of the New Keynesian model to explain the data and estimation of the degree of price frequency changes more consistent with what is observed in the micro data. With a generalized method of moments (GMM), minimum distance (estimation of the DSGE model parameters by minimizing the distance between impulse response functions of the DSGE model and those of a vector autoregression model; this was the method adopted by Altig et al., 2011) or maximum likelihood estimation (MLE) approach only the latter result would be obtained. However, the result of improved fit to the data from the introduction of firm-specific capital is also of relevance to economists working in DSGE models given that, as stated by Fernández-Villaverde (2009), the profession “mainly follows a Bayesian approach”.⁴ On the other hand, adopting a calibration approach imposes the firm-specific and rental capital models to be identical in terms of the degree of price frequency changes at the micro level (and makes an evaluation of the time series fit of the models very difficult). The Bayesian approach proves to be a more general method of comparing these models as it does not impose them to either be equivalent at the macro level (as would be the case of GMM or MLE) or fixing most parameters based solely on the micro data (the case of calibration).⁵

⁴An and Schorfheide (2007) and Fernández-Villaverde (2009) make good reviews of the advantages of Bayesian inference which resulted in it achieving such prominence.

⁵Micro evidence can be sometimes misleading when choosing parameter values for a macro model. Parameter values are only defined within the context of a model, and changes in the theory, even if minor, may have a considerable impact on the parameter values. A good example of this is Rogerson's (1988) indivisible labor model where the micro and macro elasticities of labor supply differ dramatically.

The remainder of the paper is organized as follows. Section 2 outlines the DSGE model. Section 3 describes the estimation methodology and results. In section 4 I look at the implications for business cycle dynamics. Section 5 summarizes the paper's findings.

2 The Models

In this section I describe the firm-specific and homogeneous capital models. In the last subsection I compare both models with respect to inflation dynamics and show the implications of the introduction of firm-specific capital to price frequency adjustment.

2.1 A New Keynesian Model with Firm-Specific Capital

2.1.1 Households

Consider an economy with a continuum of infinitely lived agents on the interval $[0,1]$. Their utility is:

$$\sum_{s=0}^{\infty} \beta^s \left(\frac{1}{1-\sigma} C_{t+s}^{1-\sigma} + \varepsilon_{t+s}^l v \frac{1}{1-\chi} (1 - N_{t+s})^{1-\chi} \right). \quad (1)$$

The budget constraint is:

$$C_t = (D_t + W_t N_t + T_t + TR_t - E_t \left\{ \frac{D_{t+1}}{\varepsilon_t^b R_t} \right\}) / P_t, \quad (2)$$

where C_t is the consumption of the final good, N_t is hours worked, P_t is the price of the final good, W_t is the nominal hourly wage, D_t is the nominal payoff of the portfolio held at the end of period t , TR_t are government transfers, T_t denotes firms profits and R_t denotes the gross nominal interest rate.

The resulting first order conditions are:

$$\frac{1}{\varepsilon_t^b R_t} = \beta E_t \left\{ (C_{t+1}/C_t)^{-\sigma} (P_t/P_{t+1}) \right\}, \quad (3)$$

$$C_t^{-\sigma} \frac{W_t}{P_t} = \varepsilon_t^l v (1 - N_t)^{-\chi}. \quad (4)$$

These equations contain two stochastic shocks: ε_t^b represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households and ε_t^l represents a shock to the labor supply. Both shocks are assumed to follow a first-order autoregressive process with an IID- Normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + u_t^b$ and $\varepsilon_t^l = \rho_l \varepsilon_{t-1}^l + u_t^l$.

2.1.2 Firms

Final Good Firm The final consumption good, Y_t , is produced by a perfectly competitive representative firm. The firm produces the final good by combining a continuum of intermediate goods ($Y_i, i \in [0, 1]$) using a Dixit-Stiglitz technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{1/(1+\lambda_{p,t})} di \right]^{1+\lambda_{p,t}}, \quad (5)$$

where $\lambda_{p,t}$ is a stochastic parameter that determines the time-varying mark-up in the goods market.

I assume that $\lambda_{p,t} = \lambda_p + u_t^p$ and the price mark-up shock u_t^p is an IID- Normal error term of mean zero and standard deviation σ_p . Profit maximization implies the following demand for the i th good:

$$Y_{i,t} = (P_t/P_{i,t})^{(1+\lambda_{p,t})/\lambda_{p,t}} Y_t, \quad (6)$$

where P_t is an index cost of buying a unit of Y_t :

$$P_t = \left[\int_0^1 P_{i,t}^{-1/\lambda_{p,t}} di \right]^{-\lambda_{p,t}}. \quad (7)$$

Intermediate Good Firms The i^{th} intermediate good firm production function is:

$$Y_{i,t} = A_t (\bar{K}_{i,t})^\alpha (\eta^t N_{i,t})^{1-\alpha}, \quad (8)$$

where η^t represents the labor-augmenting deterministic growth rate in the economy, $\bar{K}_t = u_t K_t$ is effective capital, K_t is the stock of capital holdings, u_t is the degree of capital utilization and A_t is total factor productivity which follows the process: $\ln(A_t) = (1 - \rho_a) \ln(A) + \rho_a \ln(A_{t-1}) + u_t^a$ with u_t^a representing an independent shock with normal distribution of mean zero and standard deviation σ_a . Since the non-stationary technology process induces a trend in output, wages, consumption, capital, investment (I_t) and government expenditures (G_t), it is convenient to express the model in terms of the detrended variables $\tilde{Y}_t = (Y_t/\eta^t)$, $\tilde{W}_t = (W_t/\eta^t)$, $\tilde{C}_t = (C_t/\eta^t)$, $\tilde{I}_t = (I_t/\eta^t)$, $\tilde{K}_t = (K_t/\eta^t)$ and $\tilde{G}_t = (G_t/\eta^t)$. Intermediate good producers are subject to Calvo price staggering and face capital adjustment costs (capital becomes productive with a one period delay and $K_{i,t}$ can be used only in the production of good i , that is capital is firm-specific as in Woodford (2005) and Sveen and Weinke (2004)):

$$\tilde{I}_{i,t} = \tilde{I}(\varepsilon_t^i \frac{\tilde{K}_{i,t+1}}{\tilde{K}_{i,t}}) \tilde{K}_{i,t},$$

where $\tilde{I}_{i,t}$ represents purchases by the firm of the final good and ε_t^i is a shock to the convex capital adjustment cost function, which is assumed to follow a first-order autoregressive process with an IID- Normal error term: $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + u_t^i$. The function $\tilde{I}(\cdot)$ is an increasing and convex function, of the usual kind assumed in neoclassical investment theory, which satisfies near a zero growth rate of the capital stock, $\tilde{I}'(1) = 1$, $\tilde{I}(1) = \delta$, and $\tilde{I}''(1) = \epsilon_\psi$ where δ is the depreciation rate and ϵ_ψ measures the convex capital adjustment costs in a log-linear approximation to the equilibrium dynamics. These assumptions are the same as those made by Woodford (2005) and Sveen and Weinke (2004).

The i^{th} intermediate good firm chooses $P_{i,t+j}$, $u_{i,t+j}$, $\tilde{K}_{i,t+j+1}$, $N_{i,t+j}$ to maximize profits subject to its production function, the demand for its good, capital adjustment costs, as well as its price setting constraints and takes P_{t+j} , \tilde{Y}_{t+j} and \tilde{W}_{t+j} as given. Formally it maximizes:

$$\sum_{j=0}^{\infty} (\beta\theta)^j E_t \left\{ \frac{P_t}{P_{t+j}} \tilde{\Lambda}_{t,j} [P_{i,t+j} \tilde{Y}_{i,t+j} - \tilde{W}_{t+j} N_{i,t+j} - P_{t+j} (\tilde{I}_{i,t+j} + \Psi(u_{i,t+j}) \tilde{K}_{t+j})] \right\}, \quad (9)$$

s.t.

$$\tilde{Y}_{i,t} = A_t(u_{i,t}\tilde{K}_{i,t})^\alpha N_{i,t}^{1-\alpha}, \quad (10)$$

$$\tilde{Y}_{i,t} = (P_t/P_{i,t})^{(1+\lambda_{p,t})/\lambda_{p,t}} \tilde{Y}_t, \quad (11)$$

$$\tilde{I}_{i,t} = \tilde{I}(\varepsilon_t^i \frac{\tilde{K}_{i,t+1}}{\tilde{K}_{i,t}}) \tilde{K}_{i,t}, \quad (12)$$

where θ is the probability the firm will not be able to optimally reset its price this period and $\tilde{\Lambda}_{t,j} = \eta^{-\sigma}(\tilde{C}_{t+j}/\tilde{C}_t)^{-\sigma}$. I assume that $\Psi(u_{i,t+j})$ is increasing and convex, capturing the idea that increased capital utilization increases the maintenance cost of capital in terms of investment goods. In the steady state $u = 1$ and $\Psi(1) = 0$; Altig et al. (2011), Smets and Wouters (2003) and Nolan and Thoenissen (2008) make the same assumptions. To solve the model, one needs only the inverse of the elasticity of the capital utilization cost function: $\Psi = \Psi'(1)/\Psi''(1)$.

The resulting first order conditions are:

$$\sum_{j=0}^{\infty} (\beta\theta)^j E_t \left\{ \frac{P_t}{P_{t+j}} \tilde{\Lambda}_{t,j} Y_{i,t+j} [P_{i,t} - (1 + \lambda_{p,t+j}) MC_{i,t+j}] \right\} = 0, \quad (13)$$

$$\frac{\tilde{W}_t/P_t}{(1-\alpha)A_t(u_{i,t}\tilde{K}_{i,t})^\alpha N_{i,t}^{1-\alpha}} = MC_{i,t}, \quad (14)$$

$$\rho_{i,t} = \Psi'(u_{i,t}), \quad (15)$$

$$\tilde{I}'(\varepsilon_t^i \frac{\tilde{K}_{i,t+1}}{\tilde{K}_{i,t}}) \varepsilon_t^i = E_t \beta \tilde{\Lambda}_{t,1} [\rho_{i,t+1} u_{i,t+1} - \Psi(u_{i,t+1}) + \frac{\tilde{K}_{i,t+2}}{\tilde{K}_{i,t+1}} \varepsilon_{t+1}^i \tilde{I}'(\varepsilon_{t+1}^i \frac{\tilde{K}_{i,t+2}}{\tilde{K}_{i,t+1}}) - \tilde{I}'(\varepsilon_{t+1}^i \frac{\tilde{K}_{i,t+2}}{\tilde{K}_{i,t+1}})], \quad (16)$$

with:

$$\rho_{i,t+1} = \frac{W_{t+1}}{P_{t+1}} \frac{MP\tilde{K}_{i,t+1}}{MPL_{i,t+1}} = \frac{\tilde{W}_{t+1}}{P_{t+1}} \frac{\alpha N_{i,t+1}}{(1-\alpha)(u_{i,t+1}\tilde{K}_{i,t+1})}, \quad (17)$$

where $MC_{i,t}$ is the Lagrange multiplier with respect to the production function. The first order condition for the firm's price setting behavior (equation 14) is similar to the standard New Keynesian model (price is a function of all future expected marginal costs). However, since a firm's choice of capital is among the determinants of its marginal product of labor,

I cannot solve the price setting problem without considering the firm's optimal investment behavior. The reason for this is that capital is not purchased on a spot market. A firm's marginal cost therefore depends on its present level of capital which in turn depends on the firm's decisions in previous periods, including its price-setting decisions. The equilibrium condition for the dynamics of the capital stock, given by (16), takes a standard form. It is noteworthy, however, that a firm's marginal return to capital is measured by the marginal savings in its labor cost, $\rho_{i,t+1}$, as opposed to its marginal revenue product of capital (note that $\rho_{i,t+1}$ would correspond to the real "rental price" for capital services if a market existed for such services). As has been emphasized by Sveen and Weinke (2004) or Woodford (2005), firms are demand constrained. This implies that the return from having an additional unit of capital in place derives from the fact that this allows to produce the quantity that happens to be demanded using less labor.

2.1.3 Aggregate Resource Constraint and Monetary Policy Rule

The economy's resource constraint is:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + \Psi(u_t)\tilde{K}_t, \quad (18)$$

where \tilde{G}_t denotes detrended government expenses which are assumed to follow an exogenous AR(1) process: $\ln(\tilde{G}_t) = (1-\rho_g)\ln(\tilde{G}) + \rho_g\ln(\tilde{G}_{t-1}) + u_t^g$ with u_t^g representing an independent shock with normal distribution of mean zero and standard deviation σ_g . I assume that government adjusts lump sum taxes to ensure that its intertemporal budget constraint holds.

Finally, when prices are sticky the equilibrium path of real variables cannot be determined independently of monetary policy. In other words: monetary policy is non-neutral. From now on, I will use lower case letters to denote variables in log deviation from the steady state. The model is closed by assuming the central bank follows a simple interest rule of the form:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\gamma_\pi \pi_{t-1} + \gamma_y \tilde{y}_t) + \gamma_{\Delta\pi}(\pi_t - \pi_{t-1}) + \gamma_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1}) + u_t^r - \mu_a u_t^a - \mu_l u_t^l, \quad (19)$$

where $\pi_t = p_t - p_{t-1}$ and the monetary policy shock u_t^r is assumed to be IID- Normal of mean zero and standard deviation σ_π . The total factor productivity and labor supply shocks were introduced to capture changes in potential output. This is similar to the monetary policy rule adopted by Smets and Wouters (2003) with the only difference being the absence of an exogenous shock to the inflation objective.

2.1.4 Wage Setting Decision

As in Altig et al. (2011), Smets and Wouters (2003) and Nolan and Thoenissen (2008) I assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector. Households' labor hours are aggregated using a Dixit-Stiglitz technology:

$$N_t = \left[\int_0^1 N_{i,t}^{\frac{(\epsilon_w - 1)}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{(\epsilon_w - 1)}}. \quad (20)$$

Total labor demand for household i 's labor is:

$$N_{i,t} = \left[\frac{W_{i,t}}{W_t} \right]^{-\epsilon_w} N_t, \quad (21)$$

where W_t is the price index cost of N_t :

$$W_t = \left[\int_0^1 W_{i,t}^{(1-\epsilon_w)} di \right]^{\frac{1}{(1-\epsilon_w)}}. \quad (22)$$

The household union takes into account the labor demand curve when setting wages. Given the monopolistically competitive structure of the labor market, if household unions have the chance to set wages every period, they will set it as a mark-up over the marginal rate of substitution of leisure for consumption. The parameter ϵ_w defines the steady state wage markup as $1 + \lambda_w = \frac{1}{1-1/\epsilon_w}$. Nominal wages are set in staggered contracts that are analogous to the price contracts described above. In particular, a constant fraction $(1 - \theta_w)$ of house-

holds renegotiate their wage contracts in each period. This yields the following maximization problem:

$$E_t \sum_{j=0}^{\infty} (\beta \theta_w)^j \Lambda_{t,j} \left\{ \frac{W_{i,t} - MRS_{t+j}^n}{P_{t+j}} N_{i,t,t+j} \right\}, \quad (23)$$

s.t.

$$N_{i,t,t+j} = \left[\frac{W_{i,t}}{W_{t+j}} \right]^{-\epsilon_w} N_{t+j}. \quad (24)$$

where $MRS_t^n = P_t C_t^\sigma \varepsilon_t^l v (1 - N_t)^{-\alpha}$ denotes the household's nominal marginal rate of substitution. The household union takes as given the paths of MRS_{t+j}^n , P_{t+j} , W_{t+j} and N_{t+j} . The first order condition with respect to $W_{i,t}$ is:

$$E_t \sum_{j=0}^{\infty} (\beta \theta_w)^j \Lambda_{t,j} \left\{ \frac{N_{i,t,t+j}}{P_{t+j}} - \epsilon_w \frac{1}{W_{t+j}} \frac{W_{i,t} - MRS_{t+j}^n}{P_{t+j}} N_{i,t,t+j} \left[\frac{W_{i,t}}{W_{t+j}} \right]^{-1} \right\} = 0. \quad (25)$$

2.2 A New Keynesian Model with a Rental Market for Capital

The rental capital model differs in very little from the firm-specific capital model. Households maximize utility as given by (1) but subject to the following budget constraint:

$$P_t (C_t + I_t + \Psi(u_t) K_t) + E_t \left\{ \frac{D_{t+1}}{\varepsilon_t^b R_t} \right\} = D_t + W_t N_t + T_t + TR_t + \rho_t P_t u_t K_t + P_t (I_t - I(\varepsilon_t^i \frac{K_{t+1}}{K_t}) K_t), \quad (26)$$

where ρ_t now corresponds to the real rental cost for capital services. The resulting first order conditions are given by equations (3), (4), (15) and (16). The firm's problem only changes slightly as well. The i^{th} intermediate good firm chooses $P_{i,t}$, $Y_{i,t+j}$, $(u_{i,t+j} \tilde{K}_{i,t+j})$, $N_{i,t+j}$ to maximize the profit function below (taking P_{t+j} , ρ_{t+j} , \tilde{Y}_{t+j} , and \tilde{W}_{t+j} as given) subject to its production function given by (10) and the demand for its good given by equation (11):

$$\sum_{j=0}^{\infty} (\beta \theta)^j E_t \left\{ \frac{P_t}{P_{t+j}} \tilde{\Lambda}_{t,j} [P_{i,t+j} \tilde{Y}_{i,t+j} - \tilde{W}_{t+j} N_{i,t+j} - \rho_{t+j} P_{t+j} u_{i,t+j} \tilde{K}_{i,t+j}] \right\}. \quad (27)$$

The resulting first order conditions are given by (13), (14) and:

$$\frac{\rho_t}{\alpha A_t (u_{i,t} \tilde{K}_{i,t})^{\alpha-1} N_{i,t}^{1-\alpha}} = MC_{i,t}. \quad (28)$$

As can be seen the only difference in the optimal conditions between the two models is in the firm's marginal return to capital, the models are identical in every other respect.

2.3 Inflation Dynamics

In both model's the economy's price inflation equation takes the form:

$$\pi_t = \beta \eta^{-\sigma} E_t \pi_{t+1} + \gamma (mc_t + u_t^p), \quad (29)$$

where γ is a function of the model's structural parameters.⁶ This equation is often referred to as the NKPC.

The dynamic relationship between inflation and average real marginal cost may be identical for both models but they differ with respect to the magnitude of γ :

$$\gamma = \frac{(1 - \theta)(1 - \theta \beta \eta^{-\sigma})}{\theta \phi}.$$

In the rental capital model $\phi = 1$. As shown in Woodford (2005) in the firm-specific capital model $\phi \geq 1$ is a non-linear function of the parameters of the model. The assumption of firm-specific capital changes the predicted slope of the Phillips curve trade-off to an extent that can be quantitatively significant; in particular, for a given value of γ the firm-specific capital implies a smaller degree of price stickiness (lower θ) relative to the rental capital model (see Woodford, 2005). The intuition is as follows: when production factors are firm-specific, a firm's marginal cost is no longer independent of its own level of output. A firm that contemplates raising its price understands that this implies less demand and therefore less output. The reduced output implies a lower level of marginal costs. Other things the

⁶Please see the web appendix for details, concerning this subsection.

same, lower marginal costs induce profit maximizing firms to post a lower price. Firm-specific capital induces price adjusting firms to keep their relative price close to the non-adjusters. Hence, the sluggishness of inflation responses to changes in output (low estimates of γ at the macro level) can be reconciled with higher flexibility in changing prices at the firm level.

3 Model Estimation

3.1 Estimation Methodology

The models presented in section 2 are estimated with Bayesian techniques.⁷ I estimate the mode and standard deviation of the posterior distribution by maximizing the log posterior function, which combines the prior information on the parameters with the likelihood of the data. In a second step, the Metropolis-Hastings algorithm is used to get a complete picture of the posterior distribution. A sample of 250 000 draws was created. The value of the scale used for the jumping distribution in Metropolis-Hastings algorithm was adjusted to yield an acceptance rate of approximately 23%, the optimal rate proposed by Gelman et al. (1996). The MCMC univariate and multivariate diagnostics indicate convergence and stability in all measures of the parameter moments.⁸ The log data density is obtained by modified harmonic mean estimation.

The dataset used consists of 7 seasonally adjusted quarterly US aggregate time series: 100 times the log difference of the GDP deflator, real consumption, real investment, real wages, real government expenses and real GDP, 100 times the log of average hours worked (for the NFB sector for all persons) and the federal funds rate. These are the same time series as in Smets and Wouters (2007) but I updated the dataset to include observations for more recent years. I will therefore estimate the model for the period 1966Q1 to 2009Q4 (whereas Smets and Wouters, 2007, estimated their model with data from 1966Q1 to 2004Q4).⁹

⁷This was implemented with the use of Dynare, which is freely available at www.dynare.org.

⁸These figures and the posterior distribution are displayed in the web appendix.

⁹Following Galí et al. (2011), I also estimated the models for the period 1966Q1 to 2007Q4 due to concerns that the non-linearities induced by the zero lower bound on the federal funds rate could distort

The corresponding measurement equations are:

$$\begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \bar{\lambda} \\ \bar{\lambda} \\ \bar{\lambda} \\ \bar{\lambda} \\ \bar{n} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} \\ \tilde{c}_t - \tilde{c}_{t-1} \\ \tilde{i}_t - \tilde{i}_{t-1} \\ \tilde{w}_t - \tilde{w}_{t-1} \\ n_t \\ \pi_t \\ r_t \end{bmatrix}, \quad (30)$$

where l and dl stand for log and log difference respectively, $\bar{\lambda}$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages, \bar{n} is the average of the log of hours worked, $\bar{\pi}$ and \bar{r} are the average values of inflation and interest rate. \bar{n} is normalized to be equal to zero. The parameters $\bar{\lambda}$, $\bar{\pi}$ and \bar{r} are related to the steady states of the model economy as follows: $\eta = 1 + \frac{\bar{\lambda}}{100}$ and $\beta = \frac{1+\bar{\pi}/100}{1+\bar{r}/100}\eta^\sigma$.

The Bayesian approach has several advantages over other methods. The calibration approach does not attach any probabilistic measures of uncertainty to the quantitative statements that it generates. Unlike GMM or the minimum distance method, Bayesian estimates are based on the likelihood function generated by the DSGE model (the Bayesian approach therefore satisfies by construction the Likelihood Principle that states that all of the information existing in a sample is contained in the likelihood function). Bayesian methods also have several advantages over MLE. Fernández-Villaverde and Rubio-Ramírez (2004) find Bayesian estimates to outperform MLE results in small samples. In addition, the Bayesian approach uses priors to incorporate additional information into the parameter estimation, thus avoiding the “dilemma of absurd parameter estimates” common when maximum likelihood is applied in DSGE estimation, and helps in identifying parameters.¹⁰

the estimates. The main results proved to be robust to the choice of sample period and are reported in the web appendix. The web appendix also shows that the main results are robust to choice of priors (namely choosing a stricter prior or looser prior for the Calvo price stickiness parameter) and modelling assumptions such as removing variable capital utilization or wage stickiness from the model.

¹⁰Likelihoods of DSGE models are full of local maxima and minima and of nearly flat surfaces. As

3.2 Prior Distribution of the Parameters

I now proceed to discuss the choice of prior distribution for the model’s parameters which are for the most part similar to Smets and Wouters (2007). Some parameters are fixed in the estimation procedure. The depreciation rate δ is fixed at 0.025, the exogenous spending-GDP ratio is set at 18%, the steady-state mark-ups of the intermediate firm and labor union are both set at 1.15. Following King and Rebelo (2000) I assume the inverse elasticity of labor supply with respect to real wages χ to be 1 and v is chosen to match steady state N , which is about 20% of the available time in the US in the postwar period.

The quarterly trend growth rate $\bar{\lambda}$ is assumed to be normal distributed of mean 0.4 while the annualized average inflation and discount rates are gamma distributed with respective prior means of 2.5% and 1%.

The priors for the exogenous processes are the same as in Smets and Wouters (2007). The standard errors of the shocks are assumed to follow an inverse-gamma distribution with a mean of 0.10 and standard deviation of two. The AR(1) parameters are assumed to be beta distributed with mean 0.5 and standard deviation 0.2.

For the Taylor rule the mean prior lagged inflation and output reaction weights are assumed to have mean 1.5 and 0.125 respectively, which is consistent with observed variations in the Federal Funds rate over the Greenspan era (see Taylor, 1999). The prior distribution on the coefficient on the lagged interest rate is assumed to follow a beta distribution with mean 0.75 which is consistent with the estimates of Clarida, Galí and Gertler (2000). As in Smets and Wouters (2003) I assume a normal distribution of mean 0.3 and standard deviation 0.1 for $\gamma_{\Delta\pi}$ the interest rule weight on the current change in inflation. I assume an identical prior for the weight on the current growth rate in output $\gamma_{\Delta y}$. The parameters μ_a and μ_l which capture a feedback effect from current innovations in the technology and labor supply shock variables are assumed to be beta distributed with mean 0.5 and standard deviation 0.2 as with the AR(1) parameters of the exogenous shocks.

Fernández-Villaverde (2009) points out this is due both to the “sparsity of the data (quarterly data do not give us the luxury of many observations that micro panels provide) and to the flexibility of DSGE models in generating similar behavior with relatively different combination of parameter values.”

The remaining prior means of the structural parameters are as follows: the intertemporal elasticity of substitution is set at 1.5; the adjustment cost parameter for capital is set around 4; the inverse of the elasticity of the capital utilization cost function is set at 0.2; the capital share is 0.3 and finally the mean Calvo probabilities are assumed to be 0.5 for both prices (a value chosen to be consistent with the evidence on prices reported by Bils and Klenow, 2004, and Klenow and Kryvtsov, 2005) and wages.

The first three columns of tables 1 and 2 give an overview of the assumptions made regarding the prior distribution (shape, mean and standard deviation) of the estimated parameters.

3.3 Parameter Estimates

For summary purposes I follow Rabanal and Rubio-Ramírez (2005) and present only the mean and the standard deviation of the posterior distributions for the parameters of both models. These numbers are also reported in tables 1 (structural parameters) and 2 (exogenous shock parameters).

For both models considered, all parameter estimates are in accordance to economic theory. The parameter of greatest interest is the Calvo price stickiness probability of nonadjustment (table 1). The Calvo price staggering assumption implies an average time period for which a price is fixed of $1/(1-\theta)$ which one can then compare to the values found in the micro data. As expected one can observe a significant reduction of θ in the firm-specific capital (FSC) relative to the rental capital (RC) model. The reason for this is that when capital is firm-specific a firm's marginal cost is increasing in its own output inducing price adjusting firms to keep their relative price close to the non-adjusters (firms will re-optimize prices more frequently and by smaller amounts). For the rental capital model θ is estimated to be 0.91 (which is broadly consistent with the values found by Galí and Gertler, 1999, and others in the literature) implying a period of price stickiness of 11.1 quarters. This contrasts sharply with findings in Bils and Klenow (2004) and Klenow and Kryvtsov (2005) who argue that firms change prices more frequently than once every two quarters. For the firm-specific

capital model θ is estimated to be only 0.82 implying that firms reoptimize prices on average every 5.6 quarters, which is considerably more reasonable than the estimate under the rental capital assumption, yet it still falls very far behind the values in the micro estimations (Klenow and Kryvtsov, 2005, find prices to change on average about every 4 months).¹¹

With respect to the other structural parameters estimates turn out to be relatively similar for both models (θ is the only structural parameter that is statistically significantly different between the two models) and in line with those found in other studies such as Rabanal and Rubio-Ramírez (2005), Smets and Wouters (2003) and Smets and Wouters (2007). The intertemporal elasticity of substitution, σ , is 0.90 for the firm-specific capital model and 0.75 for the rental capital model, which is fairly close to the value normally used in business cycle models (see for example King and Rebelo, 2000). The capital share α is 0.29 for the FSC model and 0.28 for the RC model. The curvature on capital adjustment costs, ϵ_ψ , was estimated to be 5.74 for the FSC model and 5.8 for the RC model. The inverse of the elasticity of the capital utilization cost function Ψ is 0.28 for the firm-specific capital model and 0.27 for the rental capital model. This indicates that the estimates of the costs of adjusting capital and its utilization rate are slightly lower in the firm-specific capital specification. As in Rabanal and Rubio-Ramírez (2005) wages are estimated to be substantially less sticky than prices changing on average about every 2 quarters.

Estimates of the monetary policy rule are conventional and very close for both models. The mean of the long-run reaction coefficient to inflation is estimated to be relatively high (higher than 2.0 in either model). The central bank also reacts strongly to the current change in inflation (the estimate of $\gamma_{\Delta\pi}$ higher than 0.3 in both models). As expected there is a considerable degree of interest rate smoothing as the mean of the coefficient on the lagged interest rate is estimated to be 0.71 for both the firm-specific capital and rental capital models. Policy does not appear to react very strongly to the output level (the coefficient is 0.08 in both models), but does respond strongly to changes in the current growth rate

¹¹Madeira (2008) argues that this is due to the small share capital represents in firms costs and that in order to reconcile the New Keynesian model with micro estimates of price stickiness one needs to consider employment as a firm-specific factor as well since it represents a much larger share of firms costs than capital.

(0.71 in both models) in the short run. These results are consistent with those found by Smets and Wouters (2007). In table 2 one can also see that the central bank appears to respond strongly to current innovations in technology (μ_a is 0.73 for both models) but not to innovations in the labor supply shock variable (μ_l has a value close to zero)

With respect to the estimates of the exogenous shocks parameters there are no statistically significant differences between the two models (table 2). The AR(1) coefficients are estimated to be quite high with values equal or higher than 0.9 for all shocks.

4 Implications for Business Cycle Fluctuations

4.1 Data Fit

The marginal likelihood of the model gives an indication of the overall empirical performance of the model given the data and reflects its prediction ability. It therefore forms a natural benchmark for comparing the overall fit of the two DSGE models considered here. I computed the marginal likelihood by modified harmonic mean estimation for both the firm-specific capital and rental capital models, the values are displayed in the last line of table 2. The log marginal likelihood of the model with firm-specific capital is -1243.84 which is considerably higher than that of the rental capital model (-1251.56). This suggests that firm-specific capital improves the New Keynesian model's fit to the data; but how substantial is this improvement?

One can answer this by computing the Bayes factor. The Bayes factor (BF) of model 1 against model 2 is the difference of their log marginal likelihoods. Kass and Raftery (1995) suggest that values of $2 \log \text{BF}$ above 10 can be considered very strong evidence in favor of model 1. Values between 6 and 10 represent strong evidence, between 2 and 6 positive evidence, while values below 2 are "not worth more than a bare mention". I refer to this statistic as the KR criterion. When I consider the firm-specific capital model (model 1) against the rental capital model (model 2) I obtain a KR criterion of 15.44. This conclusively supports the hypothesis that introducing firm-specific capital in DSGE models

is highly relevant for the understanding of business cycle fluctuations and highlights the advantages of employing Bayesian methods for data analysis since when using calibrated models Nolan and Thoenissen (2008) were unable to conclude that firm-specific capital was useful in matching the cyclical behavior of the main US macroeconomic variables.

What is driving the improvement in the fit to the data? Table 3 presents the key business cycle statistics (volatility, contemporaneous correlation with output and degree of first order autocorrelation) for major US aggregates: consumption, output, investment, nominal interest rates, wages, hours worked and inflation. Following King and Rebelo (2000) all variables are in logarithms (except for inflation and the interest rate) and have been detrended with the HP filter in order to isolate the cyclical component of each. To maintain consistency the theoretical moments of the firm-specific (FSC) and rental capital (RC) models displayed in table 3 are also detrended with the HP filter. In terms of volatility one can see that the firm-specific capital model implies smaller cyclical volatilities in investment, output, wages and hours worked but larger volatilities for the nominal interest and inflation rates. This is consistent with Nolan and Thoenissen (2008) who found the FSC model to have more volatile interest and inflation rates to exogenous shocks. Overall the RC model seems to match better the cyclical volatilities observed in the data (the firm-specific capital only matches better the observed cyclical volatilities in wages and the nominal interest rate). With respect to comovement it is not clear that one model outperforms the other. The rental capital model matches better the observed contemporaneous correlation with output observed in the data with respect to the nominal interest rate and hours worked. However the firm-specific capital model matches better the observed contemporaneous correlation with output of inflation. It is with respect to persistence that the firm-specific capital clearly outperforms the rental capital model. With firm-specific capital several variables become more persistent. The only exception are hourly wages which are less persistent in the firm-specific (0.85) than in the rental capital model (0.86) but this is actually more in line with the data (the degree of autocorrelation of wages in US data for the period considered is only 0.76). The FSC model matches the persistence of consumption, investment, output and nominal interest rate just as

well as the RC model but does better in matching the observed persistence of wages, hours and inflation. It is therefore the higher persistence generated by the firm-specific capital model that allows it to significantly improve the fit to the data of the New Keynesian model as reflected by the substantial increase of the log marginal likelihood.

4.2 Variance Decomposition

Table 4 displays the contribution of each of the exogenous shocks to the 20 quarter (as indicated by King and Rebelo, 2000, the cyclical component consists mainly of ‘those parts of output with periodicities between 6 and 32 quarters’, since 20 quarters is approximately the midpoint of this interval, I choose it as the variance decomposition forecast horizon in order to obtain a good characterization of the relevant sources of business cycle fluctuations) forecast error variance of the endogenous variables.

Differences between the models with respect to nominal interest and inflation rates are small. With respect to these variables price mark-up shocks explain a slightly larger portion of fluctuations in the rental capital model while labor supply shocks explain slightly more in the firm-specific capital model. For the fluctuations of other macroeconomic variables labor supply shocks seem relatively more important in the FSC model while risk premium and monetary policy shocks are relatively more important in the RC model (with the exception of the interest and inflation rates).

With respect to the driving forces of output, total factor productivity and discount rate shocks prove to be the most important (in both models taken together these shocks account for about 60% of output fluctuations). As in Ireland (2004), the discount rate shock (u_t^b) explains most of the fluctuations in inflation and the interest rate of the New Keynesian model.

4.3 Impulse Response Functions

In this section I compare the dynamic responses to exogenous shocks of the FSC and RC models. In order to understand better the role of firm-specific capital both models are sim-

ulated under the estimated mean obtained for the FSC model (since it is more plausible with respect to price setting at the micro level). Figures 1-7 display the impulse response functions of key economic variables (output, consumption, investment, capital utilization, interest rate, inflation, labor and wages) of both models to exogenous shocks. The study of the dynamic effects of firm-specific capital for aggregate economic variables has so far been limited to productivity and monetary shocks (Sveen and Weinke, 2004 study only monetary shocks whereas Nolan and Thoenissen, 2008, look at both monetary and productivity shocks). Besides total factor productivity and monetary shocks the model presented here includes shocks to the discount rate, labor supply, government spending, investment-specific technology and price mark-up (de Walque, Smets and Wouters, 2006, include the same set of shocks in their model but assume Taylor contracts and focus their attention solely on the responses to monetary policy shocks).

I find the model’s impulse response functions to exogenous shocks to be significantly altered by the introduction of firm-specific capital. Firm-specific capital makes firms adjust prices by less, thus drawing out the period of above-normal output to “demand” shocks (since these tend to move output and prices in the same direction). The impulse response functions show that firm-specific capital does indeed aid considerably in propagating the responses of output (and also of consumption, investment and hours worked), while dampening movements in inflation, to exogenous “demand” shocks (such as the risk premium, fiscal policy and monetary policy shocks) and also the price mark-up shock.

Figure 1 shows that the firm-specific capital model generates smaller reactions of output and inflation in response to productivity shocks relative to the rental capital specification (this is consistent with the results of Nolan and Thoenissen, 2008). Figure (2) shows much more persistent reactions for output, consumptions, investment and hours worked of the FSC model to risk premium shocks in comparison to the RC model. As in the productivity shock the firm-specific capital model generates smaller reactions in inflation and interest rates. This is the shock for which one observes the largest difference between the two models (these differences are quite large, 40 quarters after the shock the fall in output in the RC

model is less than 0.5% away from the steady state while in the FSC model it is still more than 1% away from the steady state). Since this shock has similar effects as the entrepreneur's net-wealth shocks in Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2010) and Nolan and Thoenissen (2009), such different reactions between the two models indicate potentially important interactions between firm-specific capital and financial frictions. Figure 3 shows that the firm-specific capital specification predicts smaller movements in output and labor in the short run but larger (clearly noticeable after 30 quarters) at higher horizons in response to fiscal shocks. Again, inflation and interest rates seem to be less volatile under firm-specific capital. Figure 4 displays the impulse response functions of both models to monetary policy shocks. As with other "demand" shocks one observes higher persistence of output under firm-specific capital and more dampened movements in inflation. The responses to the investment-specific technology shock are shown in figure 5. This is the shock for which one observes smaller differences between the two models. As with other shocks movements in inflation are smaller in the FSC model. The impulse response functions of the price mark-up shock (figure 6) are also interesting. For this shock one does not observe large differences between the two models with respect to inflation and interest rate movements. Despite the fact that it is not a "demand" shock the FSC model generates significant more persistence of output, consumption, investment, labor and wages relative to the RC model. This is the reason why this shock seems to explain a larger share of business cycle fluctuations (table 4) in the estimated firm-specific capital model. Finally, figure 7 displays the impulse response functions of both models to a labor supply shock (which could also alternatively be interpreted as a wage mark-up shock). For this shock the assumption of firm-specific capital seems to lead not just to smaller movements in inflation and interest rate but with respect to most other variables (wages is the only exception) as well.

The displayed impulse response functions seem to partly contradict the results in tables 3 (which shows the estimated FSC model generates larger volatilities for the nominal interest and inflation rates) and 4 (which shows that the risk premium and monetary policy shocks are relatively more important in the estimated RC model). The reason for this is that the

impulse response functions were obtained with identical parameter values for both models while the numbers in tables 3 and 4 are obtained under the estimated parameter values for each respective model. Under an identical value of the Calvo price stickiness probability of nonadjustment θ the FSC predicts smaller price adjustments relative to the RC assumption which translates in smaller movements in inflation. However, the introduction of firm-specific capital results in lower estimates of θ which implies more frequent price adjustments at the firm level in the FSC model. All else equal this generates larger movements in inflation (and reduced effects from monetary policy shocks for example). In the estimated model the effect due to a higher degree of price frequency dominates that of the smaller degree of price changes resulting in more volatile inflation and interest rates under firm-specific capital.

5 Conclusion

In this paper estimate a firm-specific capital DSGE model with Bayesian techniques using data for key US macro-economic time series (consumption, output, investment, hours worked, wages, nominal interest rates and inflation). I find that firm-specific capital is empirically important in order for the model to match aggregate US data even in the presence of more than one source of nominal rigidity (the model includes sticky prices and wages). By comparing the cyclical moments of both the firm-specific capital and rental capital models to the data I find that the improved fit to the data of the New Keynesian model with firm-specific capital seems to be behind an increased persistence of aggregate variables.

I also extend the analysis of the effects of firms-specific capital to other exogenous shocks besides the more conventional monetary and productivity shocks. In particular, I find that the introduction of firm-specific capital altered very significantly the behavior of economic variables in response to risk-premium disturbances. This indicates that a promising extension in the future would be to explicitly model the external finance premium as in Bernanke, Gertler and Gilchrist (1999). For example, Christiano, Trabandt and Walentin (2009) ‘conjecture that for financial frictions in working capital to be interesting’ capital needs to be

firm-specific.

Another important contribution made in this paper has to do with the ability of the firm-specific capital model to match the frequency of price adjustment in the micro data. I estimate that in the firm-specific capital model firms reoptimize prices on average every 5.6 quarters, which is considerably more reasonable than the estimate under the rental capital assumption (11.1 quarters), yet it still falls very far behind the values in the micro estimations (less than 2 quarters).

References

- [1] Altig, D., Christiano L. J., Eichenbaum, M. and Linde, J., 2011. Firm-Specific Capital, Nominal Rigidities and the Business Cycle. *Review of Economic Dynamics* 14 (2), 225-247.
- [2] An, S. and Schorfheide, F., 2007. Bayesian Analysis of DSGE Models. *Econometric Reviews* 26, 113-172.
- [3] Ball, L., and Romer, D., 1990. Real rigidities and the non-neutrality of money. *Review of Economic Studies* 57, 183-203.
- [4] Bernanke, B., Gertler, M. and Gilchrist, S., 1999. The Financial Accelerator in a Quantitative Business Cycle Framework. In: Taylor, J., and Woodford, M. (Eds.). In: *Handbook of Macroeconomics*. North-Holland, 1341-1393.
- [5] Bilts, M. and Klenow, P., 2004. Some evidence on the importance of sticky prices. *Journal of Political Economy* 112 (5), 947–985.
- [6] Blake, A. and Kirsanova, T., 2008. Discretionary Policy and Multiple Equilibria in LQ RE Models. Discussion Papers 0813, Exeter University, Department of Economics.

- [7] Chari, V., Kehoe, P. and McGrattan, E., 2000. Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?. *Econometrica* 68 (5), 1151-1180.
- [8] Christiano, L., Motto, R. and Rostagno, M., 2010. Financial factors in economic fluctuations. Working Paper Series 1192, European Central Bank.
- [9] Christiano, L., Trabandt, M. and Walentin, K., 2009. Introducing Financial Frictions and Unemployment into a Small Open Economy Model. Working Paper Series 214, Sveriges Riksbank.
- [10] Clarida, R., Galí, J. and Gertler, M., 2000. Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics* 115 (1), 147-180.
- [11] de Walque, G., Smets, F. and Wouters, R., 2006. Firm-Specific Production Factors in a DSGE Model with Taylor Price Setting. *International Journal of Central Banking* 2 (3), 107-154.
- [12] Eichenbaum, M. and Fisher, J., 2007. Estimating the frequency of price re-optimization in Calvo-style models. *Journal of Monetary Economics* 54 (7), 2032-2047.
- [13] Fernández-Villaverde, J. and Rubio-Ramírez, J. F., 2004. Comparing dynamic equilibrium models to data: a Bayesian approach. *Journal of Econometrics* 123 (1), 153-187.
- [14] Fernández-Villaverde, J., 2009. The Econometrics of DSGE Models. NBER Working Papers 14677.
- [15] Galí, J. and Gertler, M., 1999. Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics* 44 (2), 195-222.
- [16] Galí, J., Smets, F. and Wouters, R., 2011. Unemployment in an Estimated New Keynesian Model. In: Acemoglu, D. and Woodford, M. (Eds.). *NBER Macroeconomics Annual* 26. University of Chicago Press.

- [17] Gelman, A., Roberts, G. and Gilks, W., 1996. Efficient Metropolis jumping rules. In: Bernardo, J., Berger, J., David, A. and Smith, A. (Eds.). *Bayesian Statistics V*, 599-608. Oxford University Press.
- [18] Ireland, P., 2004. Technology Shocks in the New Keynesian Model. *The Review of Economics and Statistics* 86 (4), 923-936.
- [19] Kass, Robert E. and Raftery, A., 1995. Bayes Factors. *Journal of the American Statistical Association* 90, 773-795.
- [20] King, R. and Rebelo, S., 2000. Resuscitating Real Business Cycles. RCER Working Papers 467, University of Rochester - Center for Economic Research (RCER).
- [21] Klenow, P. and Kryvtsov, O., 2005. State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation? NBER Working Papers 11043.
- [22] Matheron, J., 2006. Firm-Specific Labor and Firm-Specific Capital: Implications for the Euro-Data New Phillips Curve. *International Journal of Central Banking* 2 (4), 33-64.
- [23] Madeira, J., 2008. A New Keynesian Model with Overtime Labor. Discussion Papers 0815, Exeter University, Department of Economics.
- [24] Nolan, C. and Thoenissen, C., 2008. Labour markets and firm-specific capital in New Keynesian general equilibrium models. *Journal of Macroeconomics* 30 (3), 817-843.
- [25] Nolan, C. and Thoenissen, C., 2009. Financial shocks and the US business cycle. *Journal of Monetary Economics* 56 (4), 596-604.
- [26] Rabanal, P. and Rubio-Ramírez, J., 2005. Comparing New Keynesian models of the business cycle: A Bayesian approach. *Journal of Monetary Economics* 52 (6), 1151-1166.
- [27] Rogerson, R., 1988. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21, 3-16.

- [28] Romer, D., 2008. Real Rigidities. In: Durlauf, S. and Blume, L. (Eds.). *The New Palgrave Dictionary of Economics*. Palgrave Macmillan.
- [29] Smets, F. and Wouters, R., 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association* 1 (5), 1123-1175.
- [30] Smets, F. and Wouters, R., 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review* 97 (3), 586-606.
- [31] Sveen, T. and Weinke, L., 2004. Pitfalls in the Modeling of Forward-Looking Price Setting and Investment Decisions. Working Paper 2004/01, Norges Bank.
- [32] Sveen, T. and Weinke, L., 2005. New perspectives on capital, sticky prices, and the Taylor principle. *Journal of Economic Theory* 123 (1), 21-39.
- [33] Sveen, T. and Weinke, L., 2007. Firm-specific capital, nominal rigidities, and the Taylor principle," *Journal of Economic Theory* 136 (1), 729-737.
- [34] Taylor, J. B., 1999. *An Historical Analysis of Monetary Policy Rules*. University of Chicago Press.
- [35] Woodford, M., 2003. *Interest and Prices*. Princeton University Press.
- [36] Woodford, M., 2005. Firm-Specific Capital and the New-Keynesian Phillips Curve. *International Journal of Central Banking* 1 (2), 1-46.

6 Tables

Table 1: Bayesian Estimation of Structural Parameters

	Prior Distribution			Estimated Maximum Posterior			
	Type	Mean	St. Dev.	FSC		RC	
				Mean	St. Dev.	Mean	St. Dev.
$\bar{\lambda}$	Normal	0.40	0.10	0.36	0.02	0.36	0.02
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10	0.25	0.09	0.25	0.09
$\bar{\pi}$	Gamma	0.63	0.10	0.77	0.06	0.77	0.06
\bar{n}	Normal	0.00	1.00	-0.79	0.82	-0.78	0.81
σ	Normal	1.50	0.37	0.90	0.10	0.75	0.10
Ψ	Normal	0.20	0.075	0.28	0.06	0.27	0.06
ϵ_ψ	Normal	4.00	1.50	5.74	0.92	5.80	0.93
α	Normal	0.30	0.05	0.29	0.04	0.28	0.04
θ	Beta	0.50	0.10	0.82	0.01	0.91	0.01
θ_w	Beta	0.50	0.10	0.44	0.05	0.45	0.05
ρ_r	Beta	0.75	0.10	0.71	0.03	0.71	0.03
γ_π	Normal	1.50	0.25	2.02	0.15	2.04	0.15
$\gamma_{\Delta\pi}$	Normal	0.30	0.10	0.33	0.07	0.34	0.07
γ_y	Normal	0.125	0.05	0.08	0.02	0.08	0.02
$\gamma_{\Delta y}$	Normal	0.30	0.10	0.71	0.06	0.71	0.06

Table 2: Bayesian Estimation of Exogenous Shock Parameters

		Prior Distribution		Estimated Maximum Posterior			
				FSC		RC	
	Type	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
σ_a	Inv. Gamma	0.10	2.00	0.67	0.04	0.67	0.04
σ_b	Inv. Gamma	0.10	2.00	0.16	0.01	0.16	0.01
σ_g	Inv. Gamma	0.10	2.00	3.45	0.18	3.45	0.18
σ_i	Inv. Gamma	0.10	2.00	0.13	0.02	0.13	0.02
σ_r	Inv. Gamma	0.10	2.00	0.35	0.03	0.35	0.03
σ_p	Inv. Gamma	0.10	2.00	0.33	0.02	0.33	0.02
σ_l	Inv. Gamma	0.10	2.00	1.01	0.21	0.99	0.21
ρ_a	Beta	0.50	0.20	0.91	0.01	0.91	0.01
ρ_b	Beta	0.50	0.20	0.98	0.01	0.98	0.01
ρ_g	Beta	0.50	0.20	0.93	0.01	0.93	0.01
ρ_i	Beta	0.50	0.20	0.93	0.02	0.93	0.02
ρ_l	Beta	0.50	0.20	0.96	0.01	0.95	0.01
μ_a	Beta	0.50	0.20	0.73	0.07	0.73	0.07
μ_l	Beta	0.50	0.20	0.01	0.01	0.01	0.01
Log data density (modified harmonic mean)				-1243.84		-1251.56	

Table 3: Business Cycle Statistics

	US data			FSC Model			RC Model		
	St. D.	Corr(y_t)	AC (1)	St. D.	Corr(y_t)	AC (1)	St. D.	Corr(y_t)	AC (1)
c_t	1.22	0.90	0.86	1.15	0.59	0.73	1.15	0.59	0.73
i_t	5.36	0.89	0.92	4.53	0.81	0.72	4.55	0.81	0.72
y_t	1.57	1.00	0.87	1.26	1.00	0.74	1.27	1.00	0.74
r_t	0.43	0.39	0.83	0.41	0.43	0.71	0.39	0.42	0.71
w_t	0.85	0.06	0.76	1.55	0.44	0.85	1.57	0.44	0.86
n_t	1.38	0.88	0.90	1.15	0.69	0.75	1.16	0.70	0.74
π_t	0.30	0.16	0.44	0.40	0.20	0.19	0.36	0.19	0.17

Note: all variables are in logarithms (except for inflation and the interest rate) and have been detrended with the HP filter. The data covers the period between 1966Q1 to 2009Q4

Table 4: Variance Decomposition (in percentage) 20 quarter horizon

	u_t^a	u_t^b	u_t^g	u_t^i	u_t^r	u_t^p	u_t^l
c (FSC)	29.91	7.35	17.79	18.66	5.63	2.33	18.33
i (FSC)	32.11	15.53	11.14	16.06	12.41	5.06	7.69
y (FSC)	42.19	16.31	1.89	8.35	12.71	5.28	13.27
r (FSC)	0.02	92.93	0.40	1.98	0.37	0.46	3.84
w (FSC)	16.31	7.66	5.70	8.22	5.32	3.01	53.79
n (FSC)	1.08	24.59	8.72	2.32	20.79	8.40	34.10
π (FSC)	0.36	53.65	0.25	0.28	2.59	25.31	17.56
c (RC)	30.02	8.93	17.99	18.54	7.26	2.28	14.99
i (RC)	30.89	18.15	10.82	15.15	14.90	4.64	5.45
y (RC)	41.26	19.31	1.65	7.98	15.64	4.95	9.21
r (RC)	0.02	93.33	0.45	2.21	0.37	0.38	3.26
w (RC)	15.16	8.79	5.42	7.51	6.42	2.70	54.01
n (RC)	1.19	29.31	8.02	2.31	25.65	7.91	25.61
π (RC)	0.38	53.78	0.19	0.27	2.81	27.02	15.55

7 Figures

Figure 1: Productivity shock

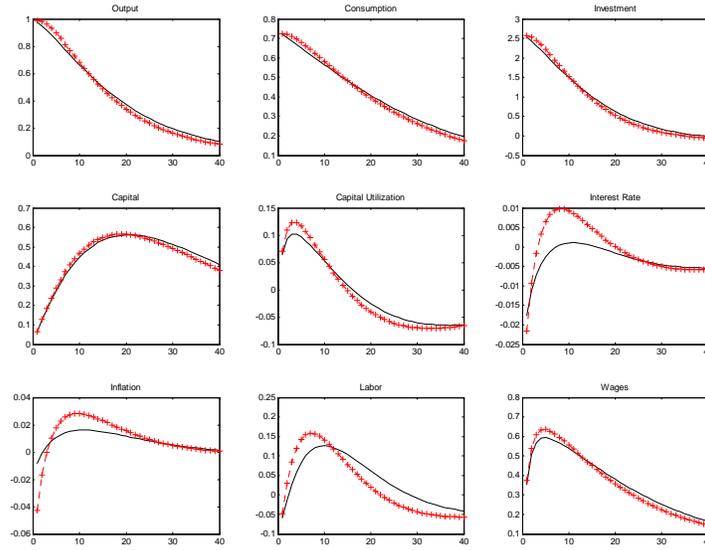
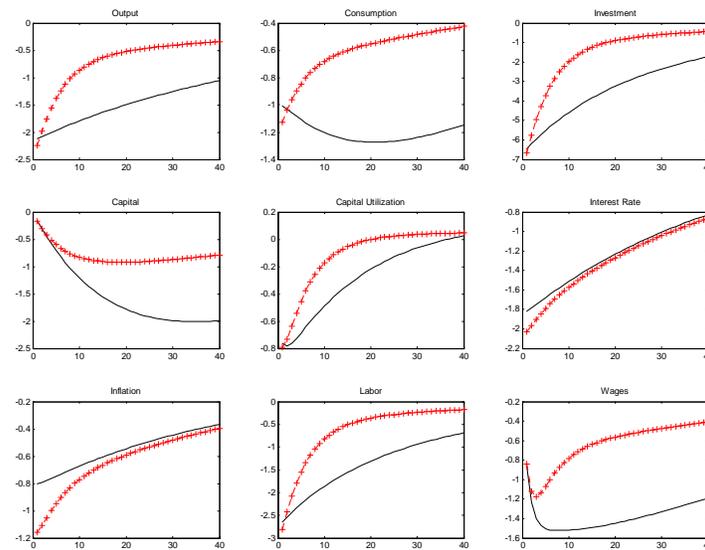


Figure 2: Risk Premium shock



Key: — FSC, + RC

Figure 3: Fiscal shock

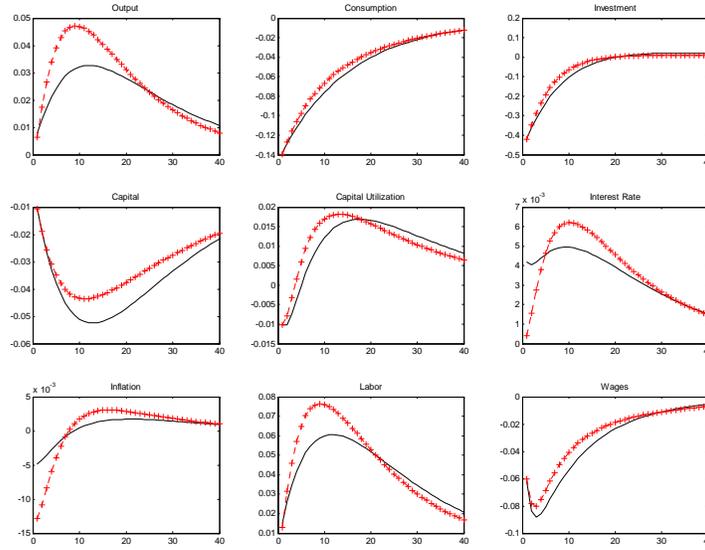
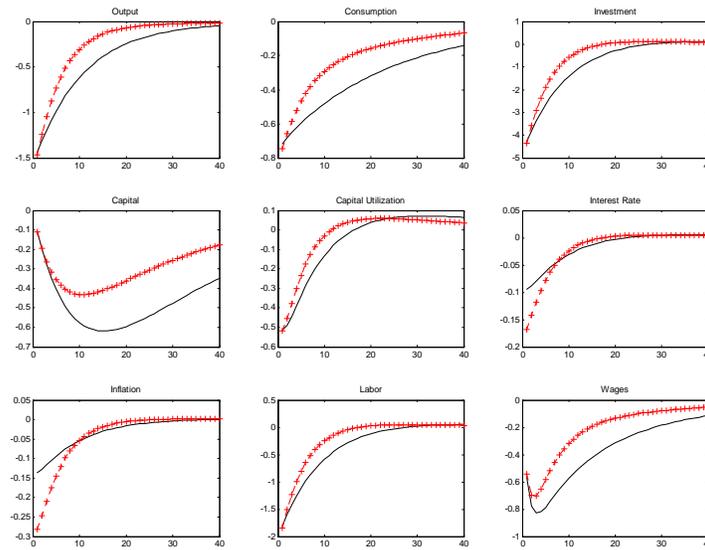


Figure 4: Monetary Policy shock



Key: — FSC, + RC

Figure 5: Investment-Specific Technology shock

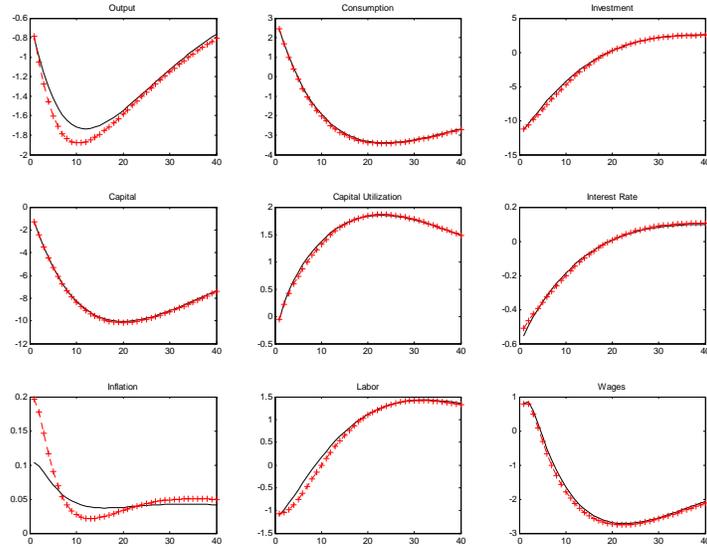
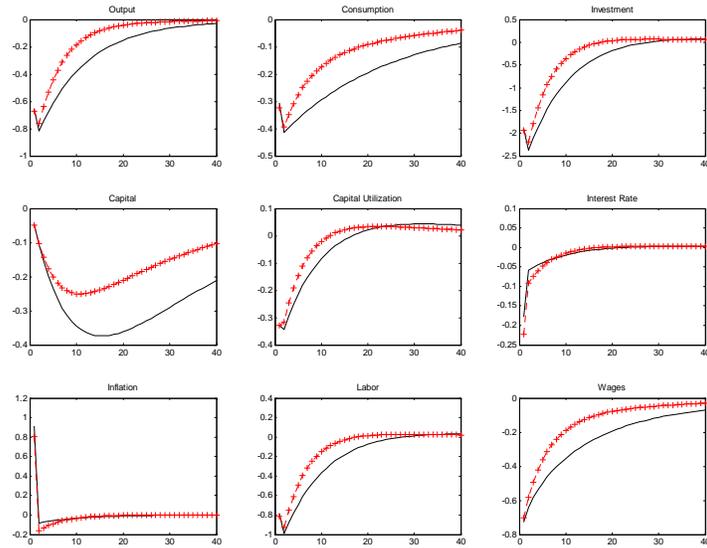
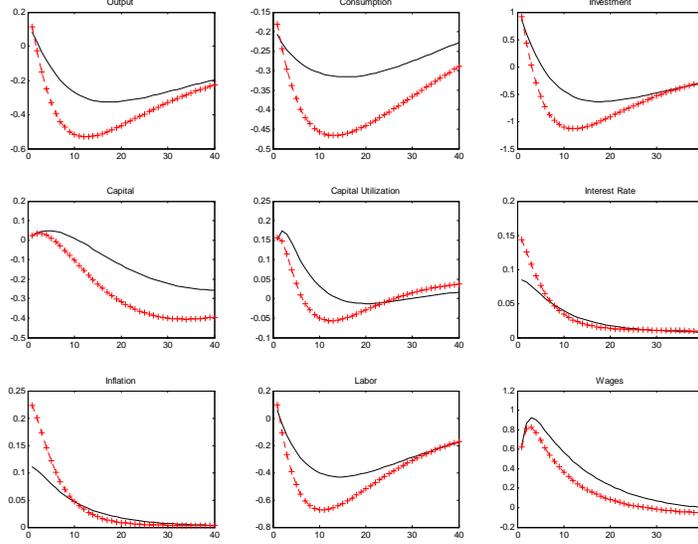


Figure 6: Price Mark-Up shock



Key: — FSC, +- RC

Figure 7: Labor Supply shock



Key: — FSC, + RC

8 Appendix

8.1 Steady State

$$MC = \frac{1}{1 + \lambda_P}$$

$$\rho = \frac{1}{\beta\eta^{-\sigma}} - (1 - \delta)$$

$$\tilde{K} = \left(\alpha \frac{MC}{\rho}\right)^{\frac{1}{1-\alpha}} N$$

$$\tilde{W} = MC(1 - \alpha)\tilde{K}^\alpha N^{-\alpha}$$

$$\tilde{C}^{-\sigma} \frac{\tilde{W}}{1 + \mu_w} = v(1 - N)^{-x}$$

$$\tilde{I} = \delta\tilde{K}$$

$$\tilde{Y} = \tilde{K}^\alpha N^{1-\alpha}$$

$$\tilde{G} = sg\tilde{Y}$$

$$\tilde{C} = \tilde{Y} - \tilde{I} - \tilde{G}$$

$$\Psi'(1) = \rho$$

In steady state $u = 1$ and it is assumed that the cost of capital utilization is zero when capital utilization is one $\Psi(1) = 0$.

8.2 Log-Linear Expansions (Aggregate Level)

8.2.1 The Firm-Specific Capital Model

Lower case letters and hats denote variables in log deviation from the steady state.

$$\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1} + \hat{\varepsilon}_t^b), \quad (\text{L1})$$

$$\omega_t = \beta\eta^{-\sigma}\omega_{t+1} + \frac{(1-\theta_w)(1-\beta\eta^{-\sigma}\theta_w)}{\theta_w}[\sigma\tilde{c}_t + \hat{\varepsilon}_t^l + \chi\frac{N}{1-N}n_t - (\tilde{w}_t - p_t)], \quad (\text{L2})$$

$$\tilde{Y}\tilde{y}_t = \tilde{C}\tilde{c}_t + \tilde{G}\tilde{g}_t + \tilde{I}\tilde{i}_t + \rho\tilde{K}\hat{u}_t, \quad (\text{L3})$$

$$\tilde{y}_t = a_t + \alpha(\hat{u}_t + \tilde{k}_t) + (1-\alpha)n_t, \quad (\text{L4})$$

$$mc_t = \tilde{w}_t - p_t - a_t + \alpha n_t - \alpha(\hat{u}_t + \tilde{k}_t), \quad (\text{L5})$$

$$\delta\tilde{i}_t = \tilde{k}_{t+1} - (1-\delta)\tilde{k}_t + \hat{\varepsilon}_t^i, \quad (\text{L6})$$

$$\begin{aligned} \tilde{k}_{t+1} = & \frac{1}{1+\beta\eta^{-\sigma}}\tilde{k}_t + \frac{\beta\eta^{-\sigma}}{1+\beta\eta^{-\sigma}}E_t\tilde{k}_{t+2} + \frac{\beta\eta^{-\sigma}\rho}{(1+\beta\eta^{-\sigma})\epsilon_\psi}E_t\hat{\rho}_{t+1} \\ & - \frac{1}{(1+\beta\eta^{-\sigma})\epsilon_\psi}(r_t - E_t\pi_{t+1} + \hat{\varepsilon}_t^b) - \frac{1+\epsilon_\psi}{(1+\beta\eta^{-\sigma})\epsilon_\psi}\hat{\varepsilon}_t^i + \frac{\beta\eta^{-\sigma}}{1+\beta\eta^{-\sigma}}E_t\hat{\varepsilon}_{t+1}^i, \end{aligned} \quad (\text{L7})$$

$$\hat{\rho}_{t+1} = \tilde{w}_{t+1} - p_{t+1} + n_{t+1} - (\hat{u}_{t+1} + \tilde{k}_{t+1}), \quad (\text{L8})$$

$$\Psi \hat{\rho}_t = \hat{u}_t, \quad (\text{L9})$$

$$\pi_t = \beta \eta^{-\sigma} E_t \pi_{t+1} + \gamma (m c_t + u_t^p), \quad (\text{L10})$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\gamma_\pi \pi_{t-1} + \gamma_y \tilde{y}_t) + \gamma_{\Delta\pi}(\pi_t - \pi_{t-1}) + \gamma_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1}) + u_t^r, \quad (\text{L11})$$

$$\hat{\varepsilon}_t^b = \rho_b \hat{\varepsilon}_{t-1}^b + u_t^b, \quad (\text{L12})$$

$$\hat{\varepsilon}_t^l = \rho_l \hat{\varepsilon}_{t-1}^l + u_t^l, \quad (\text{L13})$$

$$\hat{\varepsilon}_t^i = \rho_i \hat{\varepsilon}_{t-1}^i + u_t^i, \quad (\text{L14})$$

$$a_t = \rho_a a_{t-1} + u_t^a, \quad (\text{L15})$$

$$\tilde{g}_t = \rho_g \tilde{g}_{t-1} + u_t^g, \quad (\text{L16})$$

where $\omega_t = (\tilde{w}_t - \tilde{w}_{t-1})$, $\Psi = \Psi'(1)/\Psi''(1)$ is the inverse of the elasticity of the capital utilization cost function and the parameter γ is a function of the model's structural parameters: $\gamma = \frac{(1-\theta)(1-\theta\beta\eta^{-\sigma})}{\theta\phi}$. In the rental capital model $\phi = 1$. As shown in Woodford (2005) in the firm-specific capital model, ϕ is a non-linear function of the parameters of the model, which is computed using the undetermined coefficients method developed in Woodford (2005).

As in Smets and Wouters (2007), I normalize some of the exogenous shocks by dividing them by a constant term. The normalization consists of defining new exogenous variables, $\hat{\varepsilon}_t^{l*} = \frac{(1-\theta_w)(1-\theta_w\beta\eta^{-\sigma})}{\theta_w} \hat{\varepsilon}_t^l$ and $u_t^{p*} = \frac{(1-\theta)(1-\theta\beta\eta^{-\sigma})}{\theta\phi} u_t^p$ and estimating the standard deviation of the innovations to $\hat{\varepsilon}_t^{l*}$ and u_t^{p*} instead of $\hat{\varepsilon}_t^l$ and u_t^p .

8.2.2 The Rental Capital Model

Log-linearization of (28) results in:

$$m c_t = \hat{\rho}_t - a_t - (1 - \alpha)n_t + (1 - \alpha)(\hat{u}_t + \tilde{k}_t). \quad (\text{L17})$$

By combining the above (L17) with (L5) and updating the resulting equation one period, the outcome is an equation identical to (L8); therefore, the only difference between the two

models in the log-linear expansions at the aggregate level is in the slope of the NKPC.