THE PRICING OF SERVICES

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Abstract
This paper aims to provide a deeper conceptual understanding of demand behavior and the pricing of services. It argues why all services are sold in advance and shows how the specificities of services result in two types of risks faced by buyers that buy in advance, that of unavailability of service and a low valuation of the service at the time of consumption. Furthermore, advanced buyers run the risk of not being able to consume at the time of consumption and this relinquished capacity may be re-sold by service firms. The paper develops a theoretical model that shows that advance prices are always lower than spot prices. Also, providing a refund to advanced buyers may improve revenue. A counter intuitive result demonstrates that the firm's strategy may be pareto optimal in that a guarantee against capacity unavailability as well as a refund guarantee against valuation risk may be offered to advance buyers at a lower advance price than if a refund offer is not provided. Finally, the study also shows that the firm can earn higher revenue when the risks are asymmetric. Profits are higher when the market faces high valuation risks than when the market faces unavailability risk.

Key words: Services, Advanced Selling, Yield Management, Revenue, Refund and Pricing

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z constraints such as hotels, airlines, and restaurants. It begins with a critique of conventional pricing approaches, leading into a conceptual development of pricing in services and argues for four stylized facts. These stylized facts show how the specificities of services result in two types of risks faced by service buyers; that of unavailability of service and a low valuation of the service at the time of consumption. The risk of unavailability of service will drive buyers to buy in advance (to ensure availability) while the risk of valuation will drive buyers to buy at the time of consumption (to be sure that they want to consume it at that time (i.e. value the service)). For example, tourists buying flight tickets may buy in advance so that they are assured of a seat while business customers may prefer to buy at the time of consumption when they are more assured of the date they are required to travel.

The stylized facts also introduce the probability of advanced buyers not being able to consume the service at consumption time and how the relinquished capacity can be re-sold. Against this backdrop, a theoretical model of pricing for services is developed.

In the theoretical model, the results show that advanced prices are always lower than spot (consumption time) prices because of higher potential revenue contributed by advanced purchase as a result of the probability of non-consumption. This paper also shows that providing a refund to advanced buyers may result in higher profit. This is because the firm is able to obtain higher revenue from higher prices and increased advanced demand from the refund offer. In addition, the study uncovered that when advanced demand is highly price sensitive, the firm's strategy may be pareto optimal in that a capacity guarantee and refund offer is offered to advance buyers at a lower advance price than if a refund offer is not provided. This is because the expanded demand from the refund offer is high enough to provide higher revenue (both actual and potential) to the firm even when advanced prices are lower. Finally, the study also showed that the firm could earn higher revenue when the risks are asymmetric. In other words, if the market consists of buyers that face high valuation risks (i.e. are concerned that they would not value the service at the time of consumption), profit is higher than when the market consists of buyers that face unavailability risk (i.e. concerned about unavailability of capacity). This is because when the market faces a high valuation risk, spot prices become higher, thereby contributing to higher potential revenue earned from re-sold capacity of non-consuming advanced buyers.

The rest of the paper is organized as follows. In §1, a background of the study is presented with a critique of conventional pricing approaches. This is followed by §2, the development of a conceptual model of pricing services, putting forward four stylized facts. Following on in §3, a theoretical model formulation is presented. The model is then extended to incorporate a refund offer in §4. In §5, asymmetric demand functions are considered. Discussion of the results follows in §6, together with managerial implications. The paper then concludes with some remarks and directions for future research.

BACKGROUND OF STUDY

One of the most popular, and yet acknowledged as the most ineffective way to price a product, be it a good or a service, is the use of 'cost-plus' pricing. Essentially, this type of pricing sets a price for the product that is sufficient to recover the full costs i.e. variable and fixed costs, and adds a sufficient margin above that cost to provide the firm with some profit. Perpetuated by companies who are accounting or operations centered, this approach seems to be financially sound and logical. However, such an approach poses problems, especially in high fixed cost services. For services like transportation, airlines or 3rd Generation (3G) telecommunication services, how should one begin to price such that the fixed costs can be sufficiently covered? In transportation and airlines, fixed costs can include the cost of assets such as a cargo ship, or an airplane, whilst for 3G, the fixed cost can be the cost of acquiring the 3G license. Aside from the obviously high costs of these assets, the service may reap the benefit of the asset over the next 20, 50 years or an uncertain number of years. The cost-based price set for each unit of the service is therefore an amount that is a contribution towards the overall fixed (and sunk) costs. This contribution is not only difficult to determine, it is also inconsistent across firms. Hence, when marginal costs are zero, the 'cost' computed in the cost-based approach is an amount contributing towards fixed costs and the price is a percentage above that amount, given the volume to be sold. Such a pricing approach may lead to uncertain outcomes when firms begin to compete on price. If a service is less differentiated from its competitor and a price competition results, how low can a service firm price vis-à-vis the other, when each firm apportions its costs differently, and may change it at a whim? This can explain the downward spiraling prices experienced by the airline industry in the 1980s, after de-regulation (see Levine 1987 for an analysis of airline competition). More recently, this problem has again surfaced when aggressive pricing by European low cost airlines has resulted in losses for some, prompting the Chief Executive of Easyjet to comment that pricing by budget and full service airlines is "unprofitable and unrealistic". [1]

[1] "Low Fares Cut into Easyjet Sales", International Herald Tribune, 6 May 2004

Furthermore, firms who adopt this approach show a lack of understanding of how pricing theory functions. The simplest criticism (e.g. Nagle 2000) is that costs per unit cannot be determined without knowing the volume to be produced and the volume to produce is dependent on demand that is in turn determined by the price. By setting a 'cost-plus' price, the 'cost' is at best an approximation.

Yet, one may still be tempted to argue in its favor by pointing out that since the future is uncertain, the circularity for pricing can never be squared unless some forecast is made of the uncertain demand. Hence, it is not far fetched if the firm is to forecast the demand characteristics, based on historical data, and price its product based on the 'cost' of producing that forecasted volume.

A whole stream of research on demand forecasting and yield/revenue management in high fixed cost services such as airlines and hotels have emerged following this point. In the majority of these studies, the yield management problem is structured as one in which firms maximize payoffs/yield, given some forecasted demand profile (e.g. Badinelli and Olsen 1990; Belobaba 1989; Bodily and Weatherford 1995; Hersh and Ladany 1978; Pfeifer 1989; Toh 1979). Over time, increasingly complex demand profiles, which require increasingly sophisticated mathematical algorithms to obtain solutions, have been introduced and investigated (e.g. Alstrup et al. 1986; Hersh and Ladany 1978). Most of these studies deal with how much capacity should be allotted for a given set of prices.

The problem with this approach is three-fold. First, to use an exogenous demand profile where the profile is divorced from both the capacity allocation and pricing decision of the firm is not only unrealistic, it is also wrong. When a firm changes the capacity allocated to a particular price level, the firm should optimally revise that price. In turn, demand would be expected to adjust. If price levels and demand profile are exogenous, any optimal solution would be a false optimal.

Second, without a theoretical structure to explain why demand quantities are the way they are or why they follow a particular pattern across time, there is no assurance that the past is able to predict the future. Pricing needs to be rooted on primitive consumer behavior. Why consumers behave the way they do is just as important as to how they are behaving. Accordingly, despite tremendous computing power available today, pricing based on forecasted demand face the same old problem in conventional probability theory, where according to Bernstein (1996), "the raw material of the model is the data of the past".

Third, demand profiles are subject to a great many factors, not least the actions and strategies of the competitor. To assume that demand based on historical data can still hold for the future may be assuming too much. Consequently, since revenue management fundamentally brings in the pricing behavior of firms, concepts of consumer behavior (demand behavior) should be incorporated. Thus, revenue management is not merely an operational or optimization issue.

Given that accounting and the operations management disciplines may not provide a satisfying approach to pricing in services, do the economists have a better handle on this then? As evidenced by the Bank of England's quarterly report [2], it is clearly not the case:

> *"…some of the new service industries may have special economic properties that do not fit well with the assumptions of conventional economic models. For example, telephony and computer software production have high initial costs but very low marginal costs. As a result, pricing strategies may be more complex, and component services are sometimes embedded in customized packages that can obscure the price actually paid or the services actually bought."*

What this means is that when marginal costs are negligible, as in the case of high fixed cost services such as telecommunication, hotels, or airlines, the cost function is a straight line i.e. it does not matter how much the demand is, the cost is always negligible since all the costs to produce the service has been sunk.

Furthermore, since the service perishes immediately upon production, the optimal pricing strategy for the firm is to sell at the point on the demand curve where marginal revenue is zero, that is, if the maximum capacity of the service has not been reached. Inasmuch as conventional price theory goes, that is the advice.

Clearly, the disciplines of accounting, decision sciences and economics take a very different view of costs in services. Whilst the economists contend that sunk costs are committed and should not feature in pricing decisions, accountants and decision scientists insist that pricing decisions have to take into account the return of fixed costs. The point is, both are right. To borrow terminology from economics, ex ante, pricing decisions should not take into account sunk costs. However, ex-post, prices obtained may be used to calculate the returns to investment. The confusion arises when ex-post analyses influences ex-ante pricing decisions.

Yet, despite ex-ante decision on pricing that do not consider sunk costs, this paper argues that service pricing research have over-simplified the service firm's pricing decision. The complex pricing programs available in various service industries today clearly illustrates that more needs to be explored.

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[2] "Inflation and Growth in a Service Economy", *Bank of England Quarterly Bulletin: November 1998*

DEVELOPMENT OF A CONCEPTUAL FRAMEWORK FOR PRICING IN SERVICES

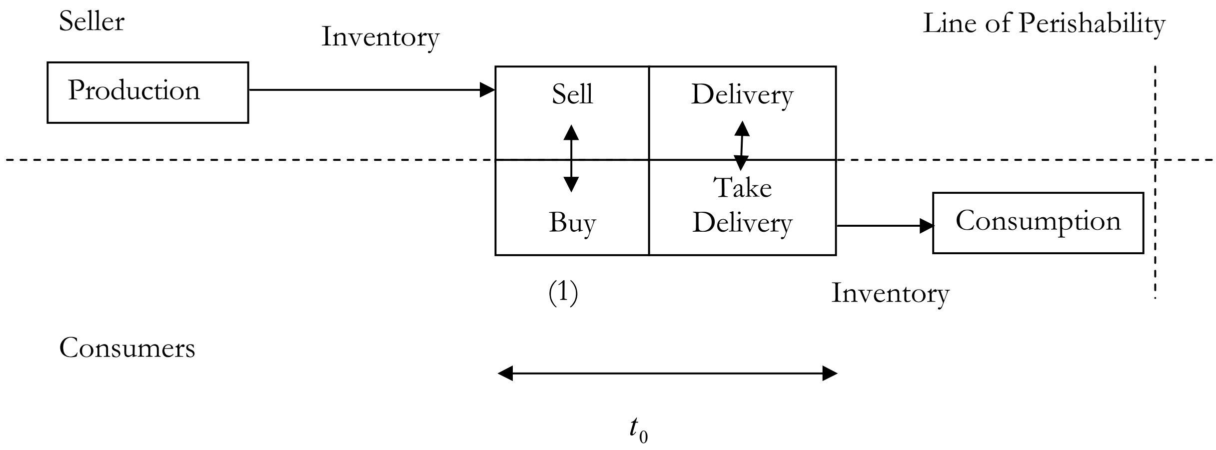
Academic service literature informs us that services are unique in that they are perishable, intangible, inseparable between production and consumption, and heterogeneous in delivery, all at once. Furthermore, other distinctively service traits (although not necessarily unique) include high fixed to variable costs ratio and largely temporal in nature. What is ambiguous about the literature is how such specificities affect pricing. Mere descriptions of service characteristics are therefore not useful, unless translated into some meaningful insights that assist firms in the pricing decision.

Let us take, as an example of a service, pricing a room in a 300-room hotel on New Year's Eve. The room could be sold 6 months or probably even a year in advance. The mere fact that it can be sold *in advance* shows that there must be something about the service that causes willingness in a customer to buy before the day of consumption, factors that will be discussed later. For now, let us think about the value the customer attaches to the room in advance, and that the firm wishes to capture that value in the asking price of the room. This value would not only differ across different customers but even for just one customer, it would differ according to when he wishes to purchase it. If it is too far in advance, he might not even be willing to buy.

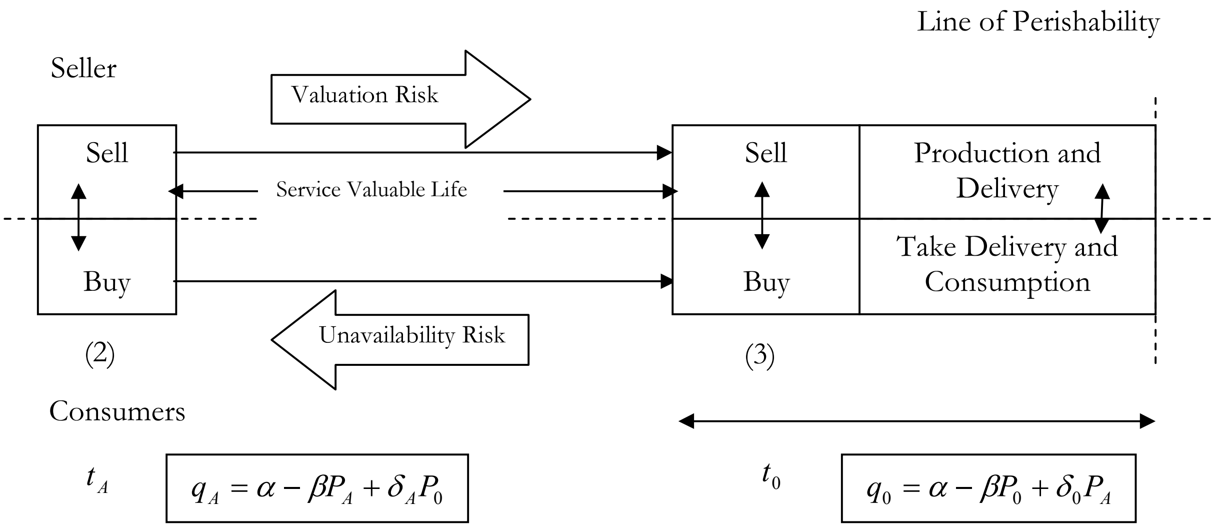
Let us label the time from when the room has some positive value all the way till its production/consumption on New Year's Eve as the service's *valuable life*, signifying the time span when the service holds some value to some customer somewhere. Figure 1 below illustrates a typical difference between a good and a service. Accordingly, the service can be sold at any time during its valuable life (i.e. selling at (1) in Figure 1). However, while it is sold at a different time, the very act of producing a service for a customer requires the source to be present, either as man or machine. This means that the production and consumption of a service is simultaneous, as it is widely established (Rathmell 1966; Regan 1963; Johnson 1970; Bateson 1977). Once the room is produced and consumed on New Year's Eve, it has perished and its valuable life has ended. Since the service can only be sold during its valuable life and its valuable life ends at the production of the service, it can then be concluded that services can only be sold *before* production (and consumption, since both are simultaneously held). This is an important point because it alludes to a key difference between services and goods. Whilst goods can also be sold before production, the manufacturing firms retain a *choice* of whether to sell before or after production. This choice is not available to services.

Figure 1: Buyer-Seller Exchange for a Typical Good and a Service

Buyer-Seller Exchange for a Typical Good



Buyer-Seller Exchange for a Service



Some may argue that certain services are actually produced in advance e.g. the production of a movie. Undeniably, many services require its materials and equipment to be produced in advance such as a hotel building, aircraft or even telecommunication towers. Yet, the value of a service is only unlocked at the point when it is performed and consumed by the customer; the same value held by the customer that can be converted into revenue to the firm through the price the customer is willing to pay. In the case of the movie, the customer values the performance of the service by the firm (through the provision of the movie, comfortable seating, quiet surroundings etc.), which is simultaneously consumed by him or her. Without the customer's consumption, which can only arise if the firm produces the service, the value of the service cannot be converted into revenue, despite the production of its equipment.

It ought to be clearer now that the issue of pricing in service is therefore the issue of *advanced pricing*, even though the time in advance may be mere minutes e.g. the purchase of a movie ticket just before the movie (cf. Edgett and Parkinson 1993) (i.e. at (3) in Figure 1).

Stylized Fact 1: The perishability and inseparability of services results in all sale of services to be advanced sales and pricing of services to be advanced pricing

As Figure 1 shows, the inventory of a good is with the seller after production and before delivery and with the buyer after taking delivery until consumption. Since services are intangible, there is no question of inventory in the exchange. Rather, if the service customer buys in advance, he faces several risks, which this study will elaborate below.

Since production and consumption is simultaneous, the consumer is unable to buy in advance to store and consume at some later date. The consumer can only buy in advance and consume later. Similarly, the firm can only sell in advance and produce later. This is an important point. Conventional economic wisdom informs us that we buy only when the utility we attach to consuming the product outweighs the price we are supposed to pay for it. However, normative economics and marketing literature often implicitly assume that buyers receive utility at the time of purchase. Since there is now a separation of time between purchase and consumption, it implies that the consumer's utility is truly obtained not at the time of purchase, but at the point of consumption (cf. Shugan & Xie 2000). Why is this significant? When there is a separation of purchase and consumption, there is a probability that a buyer who has purchased may not be able to consume, or as academics would term it – the utility becomes state dependent (cf. Karni 1983; Fishburn 1974; Cook & Graham 1977). Put simply, a buyer who buys a movie ticket an hour before the movie might find that he is unable to watch the movie when the time comes because he has fallen ill. How is this different from goods? After all, when a consumer buys a good, the consumption of his good is also dependent on his state at that time. The difference between a good and a service in this regard is that for goods, the consumer chooses the time (and the state) that is most suitable for consumption, after he has purchased the good e.g. taking a can of coke out of a fridge to drink on a hot day. This is possible because the good is not immediately perishable upon production. The service consumer may not have such a luxury since he needs to buy the service first and then consume later, when the state is uncertain. The application of state dependent utility theory into service research was first proposed by Shugan and Xie (2000), when they investigated spot and advance pricing decisions and the optimality of advanced selling.

Note that the utility of the service buyer may not drop to zero. It might be that the state of the world has rendered the consumption of the service *less* valuable e.g. an open-air concert under the rain. Since the buyer faces uncertainty in ascertaining the value of the service at the time of consumption, the buyer faces a risk, which this study terms as *valuation risk*.

Stylized Fact 2: Due to the inseparability of productions and consumption, and the separation of purchase and consumption, the (advanced) pricing of services would need to take into account the ***valuation risk*** *faced by buyers*

Of course, to mitigate valuation risk, buyers will choose the time when it is most conducive for consumption and will typically turn up to buy seconds before consumption. The study labels this time as *spot time*. However, as many service firms operate with capacity constraints, buyers may not be able to obtain the service if they all show up simultaneously. Accordingly, if a buyer waits to buy only at spot time, he faces the uncertainty that the service may not be available. In this study, this risk is termed *unavailability risk*. To alleviate this risk, he may be willing to purchase further in advance of consumption, as insurance (Png 1989), i.e. at (2) in Figure 1. Previous literature in advanced selling has shown that advanced purchasing is common in many service industries for this reason (Lee and Ng 2001; Shugan and Xie 2000, Xie and Shugan 2001). Consequently, buyers who wish to be sure of obtaining a service will buy in advance.

Stylized Fact 3: Short-term capacity constraints result in buyers facing uncertainty in the service being available, if they choose to buy at consumption time. The pricing of services would need to take into account the ***unavailability risk*** *faced by buyers*

Technically, both the purchase further in advance and the purchase at spot are deemed as advanced purchases as stylized fact 1 has explained. However, for purpose of clarity, the study terms purchases close to consumption as spot purchases and purchases further in advance as advanced purchases, consistent with the terminology used by extant literature on this phenomenon. In reality, as elaborated by Lee and Ng (2001), the point where advanced purchase ends and spot purchase begins is industry specific and is also dependent on 'rate fences' erected by the seller. Rate fences are constraints or conditions imposed by service firms to ensure minimal cannibalization of purchase. Consequently, the service industry is host to a wide range of advanced prices called "forward prices, pre-paid vouchers, super saver prices, advance ticket prices, early discounted fares, early bird specials, early booking fares, and advance purchase commitments" (Xie and Shugan 2001).

Clearly, there is a trade off between the buyer's unavailability risk and valuation risk. Hence, there exists a market for selling the service far in advance for buyers who like to ensure that the service is available, regardless of whether the seller is willing to sell to this market. Similarly, there also exists a market for selling at (close to) consumption time for buyers who like to ensure that they are able to consume.

As illustrated in Figure 1, the trade-off between unavailability risk (which drives consumers' willingness to buy further in advance) and valuation risk (which drives consumers' willingness to buy closer to consumption) means that the *distribution* of demand across the valuable life of the service becomes important in the firm's pricing decision. In this respect, the firm faces uncertainty in demand distribution across time – if they sell too much too early at too low a price, they may lose the opportunity to earn higher revenue from transient or last- minute customers but if they sell too little in advance, they may be saddled with unused capacity. To many revenue management consultants, accurate demand forecasting, coupled with dynamic optimization algorithms across time is key to better pricing decisions and higher profitability for service firms.

However, the pricing problem does not end there. The firm's decision on price also has an effect on the buyers. For simplicity, it is assumed that there exists only two times in the service's valuable life to sell – advanced time, denoting selling the service far in advance and spot time, denoting the selling of the service at a time closer to consumption. If the advanced price is low, the discount from spot price might outweigh the valuation risk faced by spot buyers. Similarly, if the spot price is low, advanced buyers might wait till spot to buy. Consequently, there is some degree of cross-time dependence between advanced and spot demand. Once this principle is extrapolated across multiple selling times in a service's valuable life, the full extent of the firm's complex pricing decision can be appreciated.

Finally, a crucial difference between pricing for services and goods is embedded in another effect of inseparability. Even if a buyer is to buy in advance, advanced selling requires the buyer to still present himself (or at least, the item that requires the service) at spot time. In other words, since services are inseparable in consumption and production, each advanced buyer has to 'show-up' to consume. Especially when the purchase is conditional upon a particular time of consumption, there will be a fraction of advanced buyers that may not be able to consume the service during that specified time. This is commonly acknowledged in various revenue management literature, where attempts have been made to structure various reservation policies to minimize the impact of the cancellation and 'no-show' concept of advanced selling. (e.g. Alstrup et al. 1986; and Belobaba 1989; Hersh and Ladany 1978; Lieberman and Yechiali 1978; Rothstein 1971 1974 1985; Thompson 1961; Toh 1985). What has not been discussed is that the existence of a non-zero probability of non-consumption by advanced buyers provides a service firm with a unique opportunity not presented to goods firm i.e. the ability to sell the capacity that was already sold in advance – again at spot. This re-selling capability may then translate into additional profit for the firm either in the additional spot sales or overselling beyond the firm's capacity in advance. If this sounds distinctively implausible, let this study illustrate this point through two examples. First, tow truck services operate with limited capacity but sell (albeit at a very low price) through the Automobile Association (AA) an enormously large number of its services in advance. Since the fraction of the market that actually requires a tow truck service may be low, the firm obviously oversells its capacity in advance as well as re-sells them at spot (at a high price for those who did not buy in advance). Second, IT support services are usually oversold to buyers in advance since the fraction of non-consumption may be high.

The implications on the pricing decision are enormous. Depending on the demand distribution across time, the level of non-consumption and capacity, firms might be prepared to manipulate advanced and spot prices to optimize profits.

Stylized Fact 4: Separation of purchase and consumption due to the inseparability of services result in a non-zero probability of advanced buyers not consuming, thus freeing up of capacity to be re-sold at spot. Pricing decisions have to take into account the non-consumption effect.

In the following section,, the paper will examine this advanced sale phenomenon, given the above propositions, through the development of a theoretical model. Few literature have investigated this phenomenon. Desiraju and Shugan (1999) evaluated strategic pricing in advanced selling and found that yield management pricing systems such as discounting, overbooking and limiting early sales work best when price insensitive customers buy later than price sensitive customers. Shugan and Xie (2000) showed that due to the state dependency of service utility, buyers are uncertain in advance and become certain at spot while sellers remain uncertain of buyer states at spot because of information asymmetry. They suggest that advance selling overcomes the informational disadvantage of sellers and is therefore a strategy to increase profit. Xie and Shugan (2001) studied when advanced selling improves profits and how advanced prices should be set. They have also investigated the optimality of advanced selling, investigating selling in a variety of situations, buyer risk aversion, second period arrivals, limited capacity, yield management and other advanced selling issues.

Png (1989) showed that costless reservations in advance is a profitable pricing strategy as it induces truth revelation on the type of valuation that consumer has for the service (which is private information). If the consumer has a high valuation i.e. ability to consume, he will exercise the reservation and pay a higher price. If not, the consumer will not exercise. In another paper, Png (1991) compared the strategies of charging a lower price for advanced sale and attaching a price premium at the date of consumption versus charging advanced buyers a premium and promising a refund to advanced buyers should consumption prices be lower than what was purchased.

However, despite various literature modeling the phenomenon, there has been no attempt to uncover the theoretical foundations that drive the primitive consumer behavior of advanced selling i.e. the notion of why advanced and spot demand exists, or how demand dynamics function within this domain. Although the above literature model sellers' pricing strategies on advanced selling, the fundamental aspect of pricing lies in demand behavior. This demand behavior, particularly in the advanced selling context, should not be exogenous and needs to be understood in at least two dimensions. First, as modeled by Xie-Shugan, Shugan-Xie and Png, it is important to understand the ways buyers react to changes in prices by their choice of buy, not-buy or switch to buy at a different time. Second, it is also important to know how many will decide on each of the choices. In the latter, the heterogeneity of buyers is a necessary factor and has yet to be studied. Lee and Ng (2001), however did study the heterogeneity of buyers through the use of demand functions. However, the model assumed that all advanced buyers were able to consume at the time of consumption, and that only such buyers were captured within the demand functions. Furthermore, the model assumed that demand at spot is independent of advanced price. This study does not make these assumptions.

Therefore in studying this phenomenon, three primary differences are highlighted between this study and those above. First, this investigation does not model the individual consumer as one (or more) set of homogeneous consumers. Instead, it models the consumers as heterogeneous, through the use of demand functions. By modeling the consumers' price sensitivity, both the decision of buyers to buy or not to buy as well as the quantities of each choice at a given price are captured. Second, the investigation also models the substitutability between advanced and spot demands, capturing the buyers switching decisions, as well as the quantities that switch for a change in price. Finally, the model explicitly captures the probability of advanced buyers who are not able to consume at the time of consumption and how this impacts pricing is analyzed. Through this model, it is hoped that there will be greater applicability in the characterization of the phenomenon.

Similar to this phenomenon is the concept of option pricing. Options are generally defined as a binding contract between two parties in which one party has the non obligated right to buy or sell some underlying asset. They are commonly a form of insurance against fluctuations in prices of commodities or some common stock. However, option prices deals only with a *price* option, and if the option is exercised, there is no uncertainty on being able to obtain the asset. In advanced selling, an option to buy is often accompanied by a capacity guarantee although there is no guarantee of high valuation at the time of consumption while in spot selling, the purchase has no guarantee that the service is available i.e. no contract.

All proofs of propositions are found in the appendix.

MODEL

The model will now be specified. The following is defined:

P_A = Price per unit of the service sold at advanced time

P_0 = Price per unit of the same service sold at spot

π = Profit to the service firm

q_A = Quantity of service demanded by the market at advanced time

q_0 = Quantity of service demanded by the market at spot

K = Capacity of the service firm and $K > 0$

t_A = Advanced time

t_0 = Spot time

A service sold in advance and at the time of consumption is not unlike two firms selling products differentiated only by the time of sale. The difference is that since there is only one service firm, the maximized profit is derived from demand at both times. Consequently, we can adapt product differentiation models derived from economics literature. Following Dixit (1979) and Singh and Vives (1984), we assume the following demand structure for selling the service at t_A and t_0:

$$q_0 = \alpha - \beta P_0 + \delta P_A$$

$$q_A = \alpha - \beta P_A + \delta P_0$$

where $\beta > 0$, $\delta > 0$ and $\beta > \delta$

Forms of this demand curve have been used in marketing modeling literature e.g. McGuire and Staelin (1983), who modeled the decision of two manufacturers and their choice to intermediate when the demand faced by both are represented by linear demand functions similar to that modeled above and Ingene and Parry (1995) who modeled two competing retailers also facing similar demand functions, and how a manufacturer would coordinate the channels.

Capacity and State effect

The parameter δ depicts the effect of increasing P_A on q_0 and increasing P_0 on q_A. The assumption $\beta > \delta$ means that the effect of increasing $P_0\,(P_A)$ on $q_0\,(q_A)$ is larger than the effect of the same increase in $P_A\,(P_0)$ i.e. own time-price effect dominates the cross time price effect. This is a reasonable assumption because the price of a service is more sensitive to a change in the quantity at its own time than to a change in the quantity across time, in other words, to borrow the terminology used in models of this nature, own-time effect dominates cross-time effect. This could be, due to several reasons, for example, the fact that the services are differentiated by time, the time difference may create other uncertainties to the buyers and thereby result in a lower cross time effect.

Note that the parameter δ, in the context of advanced selling of services, can be deemed to capture the valuation and unavailability risks faced by buyers. This means that a change in spot price would have an impact on advanced demand and the degree of impact is dependent on the magnitude of δ. It is assumed, for convenience, that the demand functions are symmetric across time. This assumption will be relaxed later. Thus, if the unavailability and valuation risks are low, δ may increase, implying that there is increased substitutability between buying in advance and at spot.

The probability that a buyer who buys in advance, but is unable to consume, is parameterized as ρ where $0 < \rho < 1$. Note that the portion of demand sold in advance who are unable to consume at t_0 can be equivalently depicted as ρq_A. This capacity could be re-sold to buyers at spot, and at the spot price of P_0, yielding a revenue of $P_0 \rho q_A$ to the firm (the assumption of full ability of the firm to re-sell relinquished capacity will be relaxed further on in the study). Finally, the firm may be constrained by its overall capacity i.e. $q_0 + q_A \leq K$.

Given the situation described above, the objective function of the service firm becomes:

$Max_{p_A, p_0}[\pi | q_0 + q_A \leq K]$ where $\pi = P_A q_A + P_0 q_0 + P_0 \rho q_A$

The following are the model assumptions.

1. While the study models the proportion of non-consuming buyers, it is assumed that this proportion, together with the consumer demand parameters, is common knowledge to the firm and the market i.e. there is perfect information.
2. The marginal cost of providing the service is negligible as service firms in general operate with high fixed costs. This is consistent with research in this area (e.g. Kimes 1989; Desiraju and Shugan 1999).
3. The capacity has no salvage value after production/consumption.
4. The service under study is a pure service (with no attributes of a good). This means that the consumer, after consumption, has no ownership of anything tangible. This is as opposed to a good/service mix where the consumer, after consumption, may also own a good (e.g. a seminar with course materials). The re-selling issue may not apply to the 'good' part of the product since the consumer may not return it after buying in advance.
5. Prices at spot and in advance are positive i.e. $P_A, P_0 > 0$.
6. The firm can credibly commit to spot prices in advance (cf. Xie and Shugan 2001)
7. Buyers who buy in advance are guaranteed the availability of capacity at time of consumption.
8. Capacity relinquished by advanced buyers can be fully re-sold at spot (this assumption is relaxed in the next section)
9. The service is not transferable.
10. The firm is a monopoly.

Furthermore, the model assumes a high congruence between what the seller sells and what the buyers believe they are buying. For example, a movie, a flight, a hotel room, tow truck service, annual auditing can all be considered (almost) homogeneous units of services because what the buyer expects to consume is similar to what the seller expects to produce. Whilst it is acknowledged that services are heterogeneous, the heterogeneity is usually at the point of consumption and it is assumed that the degree of service heterogeneity expected by prospective buyers do not sufficiently influence the value they place on the service in advance of consumption. Hence, the heterogeneity in demand lies only in buyers' valuation of the service and not because of perceived differences in the service offering.

A few noteworthy comments on the firm's objective function are necessary at this juncture. First, it is assumed the firm maximizes its profit in one stage, despite the profit being derived across two times i.e. advanced and spot. This is because of the assumption of perfect information. Since all parameters of demand at both times are common knowledge to the firm, the firm would be strategic in maximizing and manipulating its prices for both times. Certainly if information is not perfect, the model could be modeled in a variety of ways e.g. in two stages strategic pricing through backward induction, or with myopic pricing where the firm maximizes profit in advance and then at spot (cf. Jagpal 1998). The purpose here is to understand the phenomenon in its idealized form, without setting specific conditions.

The Xie-Shugan model depicts the phenomenon as a two-period process where homogeneous consumers arriving in period 1 can decide to buy or wait after the firm announces their spot and advance prices. Consumers may also arrive in period 2. In reality, buyers are not merely heterogeneous in their valuation of the service (i.e. own time price sensitivity). They are also heterogeneous in their willingness to switch between spot and advanced time (cross time sensitivity).

In Png's model, the advance buyer, should he chooses to buy, knows how much he values the service only at the time of consumption. The probability of the buyer turning out to be a low or high valuation customer is depicted as λ (in Xie-Shugan, it is q). This model incorporates this feature with ρ. If the buyer is able to consume, he is deemed to be a high valuation buyer. If he is unable to consume, he is deemed to have a low valuation. However, a key difference is that Png assumes a low valuation customer obtains a low valuation regardless of his ability to consume i.e. if he consumes, he receives a low valuation and if he does not, he will enjoy a low valuation net of any price paid for the alternative. (Png 1989, p 250) Although it may not make a difference to the customer who obtains a low valuation regardless, his willingness to consume the service has a direct effect on the firm. If he does not consume, the capacity can be relinquished and re-sold. This ability to re-sell obviously impacts on the price of the service, both in advance and at spot. In all of Png, Xie-Shugan and Shugan-Xie's models, this ability had not been considered and the study has incorporated it here.

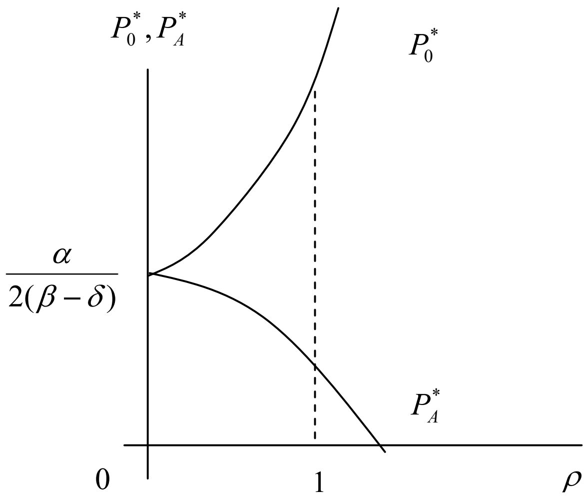
ANALYSES

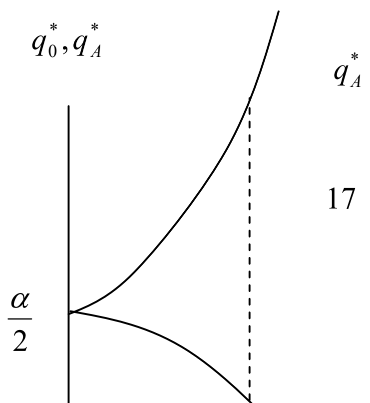
When capacity is higher than optimal demand (interior solution), the constraint is non-binding and the study provides the following lemma as a benchmark:

Lemma 1: When $\rho = 0$ *and the capacity constraint is non-binding,* $P_A^* = P_0^* = \dfrac{\alpha}{2(\beta - \delta)}$ *which we denote as* $P^*(0)$*, and* $q_A^* = q_0^* = \dfrac{\alpha}{2}$ *which we denote as* $q^*(0)$ *and* $\pi^* = \dfrac{\alpha^2}{2(\beta - \delta)}$ *which we denote as* $\pi^*(0)$.

If the fraction of non-consumption capacity is zero $\rho = 0$, i.e. all advance buyers are able to consume, prices and quantities sold at t_0 and t_A will be the same, due to the symmetric demand functions. This is illustrated in Figure 2 below.

Figure 2: Characterization of the Non-consumption effect





As the study has argued previously, ρ will always take on a positive value. Consequently, prices and quantities at t_0 and t_A start diverging, as the following proposition show:

Proposition 1: *When $\rho > 0$, the firm derives a higher profit by lowering advanced price and increasing spot price such that*

$$P_A^* = P^*(0)\left(1 - S \cdot \left[2(\beta - \delta) + \rho(\beta - 2\delta)\right]\right)$$

$$P_0^* = P^*(0)\left(1 + S \cdot \left[2(\beta - \delta) + \rho\beta\right]\right)$$

and obtains a higher advanced demand but lower spot demand such that

$$q_A^* = q^*(0)\left(1 + S \cdot \left[2(\beta + \delta) + \rho\beta\right]\right)$$

$$q_0^* = q^*(0)\left(1 - S \cdot \left[2(\beta + \delta) + \rho(\beta + 2\delta)\right]\right)$$

where

$$S = \frac{\beta\rho}{4(\beta^2 - \delta^2) - \beta^2\rho^2)]} \quad \textit{and } P_A^*, P_0^* > 0 \textit{ if and only if } \beta > \delta \cdot \frac{2}{\sqrt{4 - \rho^2}}.$$

As every unit of advanced demand provides an opportunity to the firm to re-sell, the firm chooses to lower P_A^* to obtain a higher advanced demand. Due to cross time sensitivity, a lower P_A^* decreases spot demand. However, instead of compensating by lowering P_0^* to obtain higher spot demand, the firm chooses to increase P_0^* instead since resold capacity can be sold at a premium and the marginal revenue from re-selling capacity at a premium (through higher P_0^*) is higher than marginal revenue derived from increasing spot demand through a lower P_0^*. A graphical representation of the above can be seen above in Figure 2. Notice that the solutions are conditional on $\beta > \delta \cdot \frac{2}{\sqrt{(4-\rho^2)}}$. This means that the ability of a firm to obtain positive revenue from selling in advance is dependant on the degree of own-time price sensitivity vis-à-vis the cross time price sensitivity. Notice that S exists only when $\rho > 0$. Thus, while the terms proceeding S determine the level of increase or decrease in prices and quantities, we can intuitively label S as the *non-consumption effect* attributable to the advanced purchase of services.

To highlight the impact on profit, the difference in expected profits (after some manipulation) can be written as

Lemma 2:

$\pi^* - \pi^*(0) = (1) + (2) + (3) + (4) - (5)$ *where*

(1) $\rho P^*(0) q^*(0)$

(2) $\rho P^*(0) q^*(0) S \cdot [2(\beta - \delta) + \beta\rho]$

(3) $\rho P^*(0) q^*(0) S \cdot [2(\beta + \delta) + \beta\rho]$

(4) $\rho P^*(0) q^*(0) S^2 \cdot [2(\beta + \delta) + \beta\rho][2(\beta - \delta) + \beta\rho]$

(5) $2P^*(0) q^*(0) S^2 \cdot [4(\beta^2 - \delta^2) + 4\beta^2\rho + \beta^2\rho^2 - 4\delta^2\rho]$

The first term is the added profit due to double selling the fraction of non-consuming capacity, $\rho q(0)$. The second shows that the capacity that is double-sold is sold with a price premium of $S \cdot [2(\beta - \delta) + \beta\rho]$. The third term shows that the advanced demand also increases, amplified by the combination of both own-time and cross-time sensitivities i.e. $S \cdot [2(\beta + \delta) + \beta\rho]$. This amplification is because advanced demand is made higher through both a higher spot price and a lower advanced price. The fourth term shows that the increase in advanced demand also enjoys the same price premium. Finally, the fifth term captures the loss in revenue as a result of a lower advanced price and a lower spot demand.

Lemma 1 is consistent with Xie-Shugan's model where it was shown that when marginal cost is low and capacity constraint is non-binding, advance and spot prices are the same. However, unlike Xie-Shugan, the findings show that advanced prices may be lower even when marginal costs are zero as the presence of ρ creates the divergence in advance and spot prices. Clearly, the potential revenue from one unit of advanced sale is higher than that from spot sale. Therefore, advanced price decreases to generate a higher advanced demand. The cross time effect of this is an even higher spot price.

As ρ increases, the firm has a greater incentive to price P_A lower to stimulate advanced demand. This amplifies the decrease in spot demand, pushing P_0 even higher.

Proposition 2: The greater the probability of non-consumption, the higher (lower) the quantity sold in advance (at spot) and the lower (higher) the advance (spot) price i.e. $\frac{\partial P_A^*}{\partial \rho} < 0$, $\frac{\partial P_0^*}{\partial \rho} > 0$ *and* $\frac{\partial q_A^*}{\partial \rho} >$, $\frac{\partial q_0^*}{\partial \rho} < 0$.

In Png's model, a strategy of selling firm advanced order does not maximize profit because the advanced buyer is unwilling to pay a higher price due to unavailability and valuation risk. Yet, a firm advanced order usually guarantees availability and as Xie-Shugan model showed, firm advance orders can be optimal. In addition, Png's model does not take into account the fact that a buyer who buys in advance has a non-zero probability of not consuming and that non-consumption frees up the capacity to be re-sold. By modeling in non-consumption, the study shows that the potential revenue from advanced sales increases and it may be optimal for the firm to sell in advance.

Png's model also showed that the seller's revenue from spot sales is zero because buyers would prefer the non-contingent alternative in advance rather than wait till spot where the seller would extract all the consumer's surplus (i.e. high price). Where the market is heterogeneous in the form of a demand function, the optimal price at spot assumes not all surpluses are extracted from everyone. Consequently, there is also heterogeneity in the degree to which customers may be willing to wait till spot or buy in advance. This implies that both spot and advanced demand would exist, with some degree of substitutability between buying at these two times, as modeled here. Accordingly, there is an optimal price at both times, as set out above in proposition 1.

MODEL EXTENSION: OFFERING A REFUND WHEN THE ABILITY TO RE-SELL AT SPOT IS PROBABILISTIC

Providing refunds for buyer's inability to consume is widely practiced in the airline industry. Casual enquiries by the author with airlines sales offices indicated that many airline tickets are sold with some refund value. Some tickets even provide a full refund to the customer. Generally, a full refund means that the ticket purchased can be returned to the airline for a complete reimbursement of the price at any time – even *after* the proposed date of travel. This means that if the buyer cannot make a flight for *any* reason, the airline is fully prepared to return the price of the air ticket to the customer without any penalty fee, no questions asked. Furthermore, many airlines allow a refund on *non-utilized* sectors, e.g. if the consumer has purchased a return ticket but only utilized one leg of the ticket.

There is a fundamental difference between a full refund of this nature and those given out by retail shops for goods purchased by service firms after the consumption of the service. In the latter, the refund is given (or promised) if the firm fails the consumer i.e. the compensation is provided to the buyer due to *firm's failure* to deliver the benefits, according to the buyer's perception. In the former, and also the focal point of this study, refunds are promised for *buyer failure* i.e. when the buyer fails to consume the service, through no fault of the firm. The use of the term 'buyer failure' is chosen for ease of explanation and is entirely from the perspective of the firm. From the firm's perspective, their guarantee of capacity when selling in advance is usually in return for the buyer's guarantee to consume. If the buyer doesn't, he is deemed to have 'failed'. Certainly from the buyers' perspective, they could argue that they have a right to demand for a refund since they have not yet consumed the service. What this study aims to investigate, by extending the current model, is whether there is any benefit to the firm, revenue-wise, if the firm had to provide that refund.

Png's (1989) model attempted to shed some light on this phenomenon. While he found that firms' advance orders are not optimal, his study showed the profit maximizing strategy is to insure the risk averse customer by compensating him when his valuation low and charging him high when his valuation is high. Thus the optimal result was a costless reservation at advanced time for all advanced buyers and a higher price for the high valuation customer at spot with the low valuation buyer not exercising the reservation. In this way, the advanced buyer is fully insured against capacity unavailability and partially insured against his valuation at spot time. However, this strategy requires the advanced buyer to face the risk of unavailable capacity at the time of consumption. In other words, Png's advanced buyer does not actually buy the service; he merely buys the option of purchasing the service at the time of consumption, at a stipulated price i.e. *a price option*. This is in contrast to the buyer failure refund of the type illustrated above where it is clear that the buyer buys with a firm advance order (where capacity is guaranteed) with a refund in the event of a low valuation.

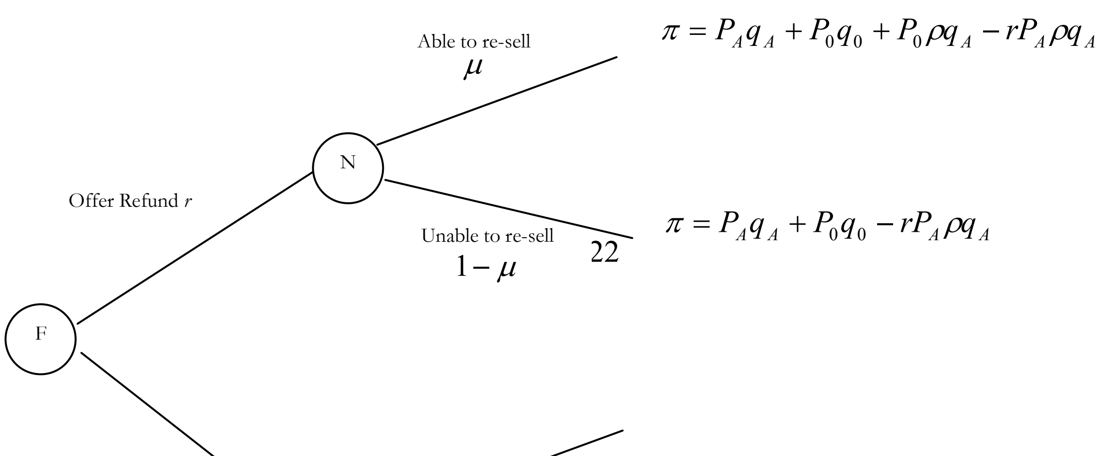
Consequently, this study extends the model to investigate the circumstances where the firm may have to provide a refund, *r*, as a fraction of the advanced price. The firm can be compelled to provide *r* due to reasons such as competition etc. The question is whether the firm can benefit from that provision. It is assumed that there is a speculative market at t_A such that it prohibits the firm from providing any refund above a full refund i.e. $0 < r \leq 1$. In addition, the condition of the firm's ability to re-sell the service at spot is relaxed and the study introduces μ, the probability of the firm being able to re-sell relinquished capacity at spot. For this extension, no assumption is made on the impact of *r* on demand. While it might be possible that a refund offer may expand advanced demand, the model makes no such assumption. Instead, this study is simply interested in when, if a refund had to be offered, would that refund be beneficial to the firm and why it could be so.

Definitions:

$P_{A,R}$	=	Price per unit of the service sold at advanced time, with a full refund offer
$P_{0,R}$	=	Price per unit of the same service sold at spot
π_R	=	Profit to the service firm when a refund is offered to advanced buyers
$q_{A,R}$	=	Quantity of service demanded by the market at advanced time
$q_{0,R}$	=	Quantity of service demanded by the market at spot
r	=	Refund offered by the firm where $0 < r \leq 1$
μ	=	Probability of capacity relinquished by advance buyers being re-sold at spot

The study models the firm's decision through an extensive form game (shown in Figure 3), with perfect information but uncertain with nature deciding if the firm is or isn't able to re-sell the relinquished capacity sold in advance.

Figure 3: Game Tree for Refund Offer when the ability to re-sell is probabilistic



ANALYSES

First, consider the impact of μ, if the firm does not have to offer a refund i.e. $r = 0$. From proposition 2, an increase in probability of non-consumption ρ decreases advanced prices and increases overall spot prices. However, for a given ρ, if μ is low, the impact of ρ is lessened even if ρ is high. The proposition below follows:

Proposition 3: *A decrease in the firm's ability to resell increases advanced prices and reduce spot prices.*

Clearly, the presence of μ reverses the effect of ρ. The above proposition is consistent with propositions 1 and 2. When the firm has a low probability of re-selling at spot market, advanced prices increase because the inability to sell reduces the firm's incentive to price lower and stimulate the advanced market. Thus, prices in advance increase and spot prices decrease to the extent that if $\mu \to 0$, $P^*_{A,R}, P^*_{0,R} \to P^*(0)$.

When is a refund offer more profitable? This should be when:

$$Max\{E[\pi|_{refund}]\} > Max\{E[\pi|_{norefund}]\}$$

The proposition below illustrates the benefit of offering a refund:

Proposition 4: *A positive refund could be more profitable to the firm if and only if the cross-time price sensitivity is low i.e. when* $\delta < \beta \cdot \phi$ *where*

$$\phi = \frac{2\mu\rho + \mu^2\rho^2 + \sqrt{(1-\rho)(2-\mu\rho)^2(4-(2-4\mu)\rho-(1-\mu)\mu\rho^2)}}{4-2\rho+\mu\rho^2}$$

and $0 < \phi < 1$ *for all permitted values of* $0 < \rho < 1$ *and* $0 < \mu < 1$

and firm prices at $P^*_{A,R} = P^*(0)\left(1 - S_R[2(\beta-\delta) + \rho(\beta\mu + \delta r) - 2\mu\delta\rho)]\right)$,

$P^*_{0,R} = P^*(0)\left(1 + S_R[2(\beta-\delta) + \rho(\beta\mu + \delta r) - 2\beta r\rho]\right)$ *and obtains*

$q^*_{A,R} = q^*(0)\left(1 + S_R[2(\beta+\delta) + \rho(\beta\mu - \delta r)]\right)$,

$q^*_{0,R} = q^*(0)\left(1 - S_R[2(\beta+\delta) + \rho(\beta\mu - \delta r) - \rho(2\beta r - 2\delta\mu)]\right)$

where $S_R = \dfrac{\rho(\beta\mu - r\delta)}{[\beta^2(4-4r\rho-\mu^2\rho^2) - \delta^2(2-r\rho)^2 + 2r\beta\delta\mu\rho^2]}$

Note that $S_R \to S$ *when* $\{r \to 0, \mu \to 1\}$.

Xie-Shugan showed that firm advanced orders with a refund offer may be optimal as the firm is able to obtain a higher price in advance to compensate for the cost of refund, as well as derive greater profits from cost savings in not having to serve these customers. The above proposition shows that the firm could derive higher revenue from either (or both) higher prices and higher advanced demand when a refund is offered.

If the cross-time sensitivity is low i.e. $\delta < \beta \cdot \phi$, the refund offer actually increases profit to the firm. This is because when the cross time impact of a price change is low (i.e. less customer switching), a change in price at (advanced) spot time would have less an impact on (spot) advanced demand. Low cross time sensitivity implies state sensitive people are more concerned about a conducive state and won't switch to advanced time even if the price is low. Similarly, capacity sensitive people are more concerned about obtaining the service, and will not switch to spot time even if the spot price is low. Consequently, a refund offer effectively reduces the price paid for the advanced buyer who may not consume without any impact on the buyer who will consume. When cross time sensitivity is low, this reduction in price does not cause as many spot buyers to switch, hence the firm obtains higher profit from higher revenue in advance without losing much spot revenue.

Proposition 5: *When* $\delta \leq \left[\frac{\mu\rho(1+\mu\rho)-2(1-\rho)}{2-\rho(1-\mu\rho)} \right] \cdot \beta$, *the firm should offer the boundary solution of a full refund (i.e.* $r^* = 1$*).*

Since the firm is constrained by speculators, it cannot increase refund beyond giving a full refund. Thus, the above proposition spells out the level of cross time sensitivity when the refund offered is a boundary solution of a full refund. As noted in proposition 3, the ability to re-sell capacity relinquished by advanced buyer contributes to the level of advanced price. Consequently, under the condition of full ability to re-sell, the study finds the following counter intuitive proposition:

Proposition 6: *When* $\mu = 1$, $\beta > \frac{\delta}{\rho}$ *and the refund offered increases profit (under the condition in proposition 4) and* $r < \frac{2\beta^3\rho(2+\rho)+4\delta^3(1+\rho)-\beta^2\delta(4+3\rho^2+\rho^3)-4\beta\delta^2\rho^2}{\delta^2\rho[2\delta+\beta(2-\rho-\rho^2)]}$, $P^*_{A,R} < P^*_A$.

In contrast to Png and Xie-Shugan models, the above proposition shows that advanced price may be lower with a refund offer. This is because the firm derives higher revenue from two sources. First, the revenue from increase demand may outweigh the revenue from increased price. Second, with an expanded advanced demand, the potential revenue from non-consumption also increases. Consequently, when price sensitivity is high relative to cross-time sensitivity i.e. $\beta > \frac{\delta}{\rho}$, the result can be pareto optimal where it is possible for the firm to provide the advanced buyer with insurance against capacity unavailability (through a positive advanced purchase price) and insurance against valuation risk (through a refund offer), at a price that may be *lower* than if a refund is not offered.

Asymmetric Cross time Sensitivity

As noted earlier, the parameter δ, in the context of advanced selling of services can be deemed to capture the trade off between valuation and unavailability risks. Clearly, they do not need to be the same i.e.

$q_0 = \alpha - \beta P_0 + \delta_0 P_A$

$q_A = \alpha - \beta P_A + \delta_A P_0$

where $\beta > 0$ and $\beta > \delta_A, \delta_0$

To investigate the implications of asymmetry, this study takes two extreme examples i.e. $\delta_A \to 0$ and $\delta_0 \to 0$. For simplicity, it is assumed no refund is provided and capacity relinquished by advanced buyers would be fully sellable i.e. $\mu = 1$, $r = 0$.

When $\delta_A \to 0$

In this case, advanced demand is not affected by spot prices. However, spot demand may be affected by prices in advance and at spot. This type of behavior can be typical of a market segment where capacity availability is extremely important. Such services may include those where the time of consumption is tied to the value of the service e.g. an important flight, a hotel room on New Year's eve, an anniversary dinner etc. In these cases, the advanced market cannot be persuaded to wait to buy at spot. If the price in advance is too high, they will seek alternatives at advanced time (and therefore drop out of the market) instead of switching to spot. This is termed as a market that faces high unavailability risk.

When $\delta_0 \to 0$

In this case, spot demand is not affected by advance prices. However, advanced demand may be affected by prices in advance and at spot. This type of behavior can be typical of a market segment where a conducive state for consumption (i.e. high valuation) is extremely important. Such services may include emergency services e.g. tow truck, or computer/equipment support services where the precise time when the service is required is often not known. In these cases, the spot market cannot be persuaded to buy in advance (high valuation risk). If the price at spot is too high, they will seek alternatives at spot time (and therefore drop out of the market for the service). This is termed as a market that faces high valuation risk.

In practice, a service can face a market with both high valuation and unavailability risk. For example, a flight from London to New York will be a service with high availability risk for a passenger that needs to get on a particular flight e.g. to attend his son's graduation. He will therefore purchase in advance. If the price is too high, he will choose an alternative (perhaps another airline) but he will not consider waiting till spot, as he needs to be in New York at that particular date. In contrast, a business executive who does not know when he may be called to go on a business trip to New York will not be swayed to buy in advance, as he is unsure if he should be traveling on that date.

Proposition 7: *A market that faces high unavailability (valuation) risk pays higher prices in advance (at spot) than a market that faces high valuation (unavailability) risk if* $\beta > \frac{\delta}{\rho}$.

Whilst the above proposition is not surprising, it is found that the firm's profit from asymmetric states is not the same. Since high unavailability risk can increase advanced price, it also serves to reduce advanced demand as a result of the increase in price. This reduces the potential revenue from non-consumption.

Proposition 8: *If the magnitude of* δ_0 *and* δ_A *are the same, profits are higher when* $\delta_0 \to 0$ *than when* $\delta_A \to 0$.

When $\delta_0 \to 0$, the market faces high valuation risk. Since spot demand is not easily persuaded to buy in advance, the firm will set spot prices higher. This, in turn, provides higher potential revenue from advanced demand (through non-consumption). However, when $\delta_A \to 0$, unavailability risk is high and this pushes up advanced prices, reducing advanced demand and thereby reducing potential revenue from non-consumption. Consequently, profits are lower than when $\delta_0 \to 0$.

DISCUSSION

This study aims to offer a more applicable model of advanced selling as a phenomenon. Stylized fact 1 justified why all services are sold in advance and that the pricing of services is in fact advanced pricing. In stylized facts 2 and 3, the study proceeded to explain how advanced demand may be distributed between a time further in advance until consumption due to the specificities of services, particularly through the heterogeneous levels of unavailability and valuation risks faced by the service consumer. In stylized fact 4, it illustrated one particular idiosyncratic factor of advanced selling in services is illustrated i.e. the fact that advanced buyers may not be able to consume and thereby releasing the capacity to be re-sold by the firm at the time of consumption. Having conceptualized the factors that influence pricing in services, the study then proceeded to model the phenomenon through a theoretical model incorporating consumer price sensitivity, degree of cross time substitutability as well as the ability of the service firm to re-sell relinquished capacity at spot time to derive the optimal pricing and quantities.

The model presented found, in proposition 1 that advanced prices are lower than spot prices even without binding capacity constraints or marginal costs because the potential revenue from one unit of advanced sale is higher than that from spot sale. Proposition 2 showed that as the probability of non-consumption increased, advanced price decreased and spot prices increased further. This result may be applied to emergency or support services where buyers can buy in advance with very low probability of consumption. As noted earlier, an example of this would be breakdown services where buyers buy in advance but their consumption date is uncertain. What may be seen as insurance will in fact be a form of advanced purchase at a very low price, Without advanced purchase, the spot price of breakdown services will be high, as many who own older vehicles will attest. Another example of advanced purchase at a low price with a low probability of consumption is the purchase of support services e.g. equipment or IT support. Such services are often sold to individuals or firms on an annual subscription basis. Again, without a support contract, the spot prices of such services are usually much higher.

The higher potential revenue from advanced demand diminishes, however, as the ability of the firm to sell relinquished capacity reduces, as shown in proposition 3. Services that required capacity scheduling or planning at the time of consumption such as removal or exhibition services may fall into this category, where spot and advanced prices may be the same. Yet, this proposition may accentuate the realization that such firms can explore creative pricing strategies or use technological advancements to improve overall profit through the ability to manipulate the parameters set out in this model.

For buyers that buy in advance but are unable to consume (i.e. buyer failure), it was found in proposition 4 that offering a refund is optimal when cross time sensitivity is low relative to own-time price sensitivity. This is because when cross time sensitivity is low, the firm benefits either from increase price and/or increased advanced demand. Proposition 5 showed that a full refund may also be optimal under conditions of very high own-time price sensitivity relative to cross time sensitivity. However, proposition 5 is a boundary solution. Consequently, if the firm is able to increase cross time sensitivity, profit may be increased with an interior solution for r. A counter intuitive result was discovered in proposition 6, where a refund offer to advanced buyers may be optimal with a *lower* advanced price than if refund was not offered. This occurred when the refund offer is able to expand demand to such a degree that for the firm could lower its advanced price.

Asymmetric conditions where unavailability and valuation risks are not equal were then investigated. Not surprisingly, the investigation found in proposition 7 that advanced prices were higher when unavailability risks were high and spot prices were higher when valuation risks were high. What was surprising at first was proposition 8 where the study showed that the two asymmetric conditions did not yield the same profit i.e. profit was higher when the market faced high valuation risk than when the market faced high unavailability risk. This was justified on the realization that high valuation risks resulted in higher spot prices which in turn increased potential revenue from the non-consumption of advanced buyers. Proposition 7 and 8 showed that the perception of high valuation and high unavailability risks may be profitable for the firm. However, if the risks are too high, buyers may not buy. In practice, such risks may be manipulated by the firm through its pricing policies, for example, by providing some degree of flexibility in the time of consumption. Where the business executive facing high valuation risk in not being certain about which date he is to fly, a flexible flight time (e.g. open tickets) might persuade him to buy in advance or lower his valuation risk, thus increasing his willingness to pay. Similarly, if the graduate's father is given a ticket that allows him to choose a range of flight times thereby reducing his unavailability risk, he might be swayed to buy at the last minute or lower his perception of risk, which will in turn increase his willingness to pay. A summary of the results of the model can be seen in Table 1 below.

Table 1: Summary of Results

	Impact of increasing non-consumption, ρ	**Impact of decreasing ability to re-sell, μ**	**Impact of increasing refund, r**	**<u>Asymmetry</u> High unavailability risk $=\delta_A \to 0$ and High valuation risk $= \delta_0 \to 0$**
	When $\beta > \delta \cdot \frac{2}{\sqrt{4-\rho^2}}$		**When** $\beta > \frac{\delta}{\rho}$ **and** $\mu = 1$	**When** $\beta > \frac{\delta}{\rho}$
Advanced Price	Decrease	Increase	Decrease	Higher when $\delta_A \to 0$ than when $\delta_0 \to 0$
Spot Price	Increase	Decrease	Increase	Lower when $\delta_A \to 0$ than when $\delta_0 \to 0$
Advanced Demand	Increase	Decrease	Increase	Lower when $\delta_A \to 0$ than when $\delta_0 \to 0$
Spot Demand	Decrease	Increase	Decrease	Higher when $\delta_A \to 0$ than when $\delta_0 \to 0$
Profit	Increase	Decrease	Increase when $\delta < \beta \cdot \phi$ (see proposition 4)	Higher when $\delta_0 \to 0$ than when $\delta_A \to 0$

This model also resolved the issue of risk aversion in buyers through the use of a demand function. Since buyers in advance are heterogeneous in their valuation of the service, their probabilities of non-consumption are also heterogeneous. As advanced buyers also discount this uncertainty in their valuation, this 'discount' alludes to the buyers' degree of risk aversion. The demand function, and its parameters, subsumes such complexities. The result was a parsimonious model that offered a theoretical framework on how service pricing could be approached.

Revenue Management

The objective of revenue management studies is to maximize yield by managing the sale of service capacity over time, through pricing, capacity allocation, and timing of sale (Badinelli and Olsen 1990; Desiraju and Shugan 1999). One of the limitations of current revenue management (RM) literature is that many RM models maximize payoffs/yield, given some forecasted demand profile that is largely exogenous, and does not explicitly capture the price/capacity relationship (e.g. Badinelli and Olsen 1990; Belobaba 1989; Bodily and Weatherford 1995; Hersh and Ladany 1978; Pfeifer 1989; Toh 1979). A widely used approach in this stream of literature is that of mathematical programming. Kimes (1989), however, commented "although the linear programming solution can be found, the assumption of deterministic demand makes the solution to the problem unrealistic". The model developed here is based on demand functions across time, hence capturing fundamental concepts of consumer behavior (cf. Chase 1999; Lieberman 1993; Relihan III 1989).

The model can be applied in a non-linear programming approach, where the firm can obtain the optimal price and quantity sold at each (continuous) point in time as:

$$P_t^* = Arg\max_{P_t}\left\{\pi \middle| \int_n^0 q(t)dt \le K\right\} \text{ for } t \in \{0,1,2...n\} \text{ and } q_t^* = \alpha(t) - \beta(t)P_t^* + \delta(t)P_0^*$$

Where

$$q_t = \alpha(t) - \beta(t)P_t + \delta(t)P_0$$

$$q_0 = \alpha(0) - \beta(0)P_0 + \int_n^0 \delta_0(t)P_t dt$$

and $\pi = \int_n^0 P_t q_t(P_t, t)dt + P_0 \int_n^0 \rho_t q_t(P_t, t)dt$

Essentially, the firm faces a demand function at any point in advance that incorporates some degree of substitutability between the price at that time and the t_0 price. Each demand parameter is time dependent and the distribution of that parameter across time may be defined exogenously. The optimal price and quantity to be sold is generated from the objective function that multiples the quantity sold at each time with the price at that time, cumulative across time. Each advanced time will also yield a fraction that may not be consumed at t_0 and that is accounted for in the objective function. Hence, the price and quantity relationship is captured explicitly as is the substitutability between times of purchase. Price discrimination, by a simple extension, can simply be incorporated into the model as the standard "affine" pricing schedule where the firm realizes a price premium at any point in time equivalent to the social surplus at the optimum at any time *t*. Similarly, multi-leg flights, 3 nights hotel stays etc. can be modeled in as a bundled product and its demand function ascertained accordingly. By using the demand function, a vast quantity of economic literature can be applied into the advanced selling context to produce richer insights into the strategy of advanced selling.

In the past, where pricing is often static, the above specification might have been difficult to implement. However, the advent of the connected economy, where data can be obtained quickly, technological innovations have made complicated algorithms possible to implement. Thus, a dynamic pricing model such as the one suggested above is not impossible. Furthermore, business to business (B2B) and business to consumer (B2C) marketplaces create new ways to exchange goods and services, and the firm has to innovate for higher revenues. What is left, is the question of how many prices should the firm introduce to the market before it becomes confused (see Desiraju and Shugan 1999). The purpose of this paper is not merely an additional model of revenue management or to capture optimal pricing strategies. What it seeks to show, by abstracting the phenomenon into this theoretical framework, is that the parameters modeled here provides the firm with various strategic levers to influence demand and plan an effective pricing strategy. As noted earlier, degree of flexibility may affect the perception of risk. Depending on the type of service industry, further research and managerial creativity may find other levers that will manipulate the demand faced by the firm and provide the firm with unique opportunities for higher revenue.

It is fairly common knowledge that many low cost airlines price according to the amount of capacity available, increasing prices as the plane fills up, and according to demand forecast. Consequently, revenue management is capacity and forecast centered rather than value centered. Previous studies (e.g. Ng, Wirtz and Lee, 1999) have shown that such practices often result in conflict between revenue managers and the sales and marketing department. This is not surprising, as this paper has shown. When revenue management takes demand as exogenous and sales and marketing strives to improve value to increase price/demand, conflicts will naturally occur. The study shows that reconciliation between the two can be obtained by way of a more satisfying approach towards revenue management, as a practice. The contention here is that the firm's focus should not merely be on revenue *management* as it should be on revenue *improvement*. Thus, only when demand behavior is incorporated into the equation, providing a more complete picture, the firm would be able to understand where and how its revenue is being obtained. As the model has demonstrated, the offer of refund can increase profit for the firm. However, firms may misunderstand that refund is merely a form of providing 'quality' service and as such, might be tempted to drop the offer (as some low cost airlines do). In doing so, they miss the opportunity to derive higher profits either through expanded advanced demand or higher advanced price.

CONCLUSION

For several decades, developed economies have been service economies. The expansion of the service sector is partially attributed to an increase in the intangible component (often known as the service component) of the production of agricultural and industrial goods. Conversely, the past decade has also shown that certain service activities are embracing some degree of industrialization. The resultant convergence has prompted some scholars to propose that the service economy is moving into "an economy based on service relationship as a mode of coordination between economic agents" (Gallouj 2002). As such an economy expands, faster than even academic analyses can hope to catch up with, fundamental questions gets left behind, to the extent that they may become less fashionable to conduct research on, although the issues may be no less important. Such fundamental questions include, though not limited to, some basic differences between goods and services. As goods and services become increasingly similar in nature (Gallouj 2002), it is more important that their differences are addressed. Only when the difference between a good and a service is properly specified, can there be a greater understanding of the broader context in which a combination of a good and a service function.

By abstracting the phenomenon of advanced pricing in services through a stylized model, the study also aims to provide a platform for a more thorough understanding of pricing in services, across various service industries. The issue of pricing for services warrants research attention as service economies mature and become more competitive. However, the cross-disciplinary nature of pricing deters many researchers from this area. Furthermore, the diversity of services results in many pricing variables and parameters being labeled differently across service industries. As a result of this contextualization within each industry, there is greater difficulty in collecting empirical data on pricing. Consequently researchers in service pricing require both experience and theoretical expertise in formulating a link between a conceptual understanding of service pricing and complex reality. However, this challenge is worthwhile to undertake as less mature service industries can benefit from the experiences of more mature ones if the pricing concepts and strategies can be understood at a more abstract level. Further research can also explore the costs of advanced selling, especially transaction costs, the impact of a binding capacity constraint (cf. Xie and Shugan 2001) and the impact of competition on advanced selling strategies.

This paper has highlighted a phenomenon that has not yet been rigorously examined in the academic context. Although the computational aspect of advanced selling (through revenue/yield management research) has been amply researched in operations research, the study aimed to provide a more applicable formulation that contributes both to operations research and pricing research in marketing.

APPENDIX (PROOFS)

Lemma 1:

(1) $$q_A = \alpha - \beta P_A + \delta P_0$$

(2) $$q_0 = \alpha - \beta P_0 + \delta P_A$$

and $\pi = P_A q_A + P_0 q_0 + P_0 \rho q_A$

$Max_{P_A, P_0}\{\pi\}$ will lead to

(3) $$P_A = \frac{\alpha + (2\delta - \beta\rho)P_0}{2\beta}$$

(4) $$P_0 = \frac{\alpha(1+\rho) + (2\delta - \beta\rho)P_A}{2(\beta - \delta\rho)}$$

Solving for (1) and (2) results in

(5) $$P_0^* = \frac{\alpha(2\delta + \beta(2+\rho))}{4(\beta^2 - \delta^2) - \beta\rho^2}$$

(6) $$P_A^* = \frac{\alpha(2\delta + \beta(2 - \rho - \rho^2))}{4(\beta^2 - \delta^2) - \beta\rho^2}$$

Substituting back to the demand functions will lead to

(7) $$q_0^* = \frac{\alpha[\beta^2(2 - \rho - \rho^2) - 2\delta^2 - \beta\delta\rho(1+\rho)}{4(\beta^2 - \delta^2) - \beta\rho^2}$$

(8) $$q_A^* = \frac{\alpha(\beta + \delta)(\beta(2+\rho) - 2\delta)}{4(\beta^2 - \delta^2) - \beta\rho^2}$$

Thus when $\rho = 0$, the above yields

$$P_A^* = P_0^* = P^*(0) = \frac{\alpha}{2(\beta - \delta)},\ q_A^* = q_0^* = q^*(0) = \frac{\alpha}{2}\ and\ \pi^*(0) = \frac{\alpha^2}{2(\beta - \delta)}$$

Proposition 1:

From the above proof of lemma 1

$$P^*(0) - P_A^* = \frac{\beta\rho[2(\beta - \delta) + \rho(\beta - 2\delta)]}{4(\beta^2 - \delta^2) - \beta\rho^2} \cdot \frac{\alpha}{2(\beta - \delta)}$$

$$\Rightarrow P^*(0) - P_A^* = S \cdot P^*(0)[2(\beta - \delta) + \rho(\beta - 2\delta)]$$

(9) $$\Rightarrow P_A^* = P^*(0)[1 - S(2(\beta-\delta) + \rho(\beta - 2\delta))]$$

$$P_0^* - P^*(0) = \frac{\beta\rho[2(\beta-\delta)+\beta\rho]}{4(\beta^2-\delta^2)-\beta\rho^2}\cdot\frac{\alpha}{2(\beta-\delta)}$$

$$\Rightarrow P_0^* - P^*(0) = S\cdot P^*(0)[2(\beta-\delta)+\beta\rho]$$

(10) $$\Rightarrow P_0^* = P^*(0)[1 + S(2(\beta-\delta)+\beta\rho)]$$

$$q^*(0) - q_0^* = \frac{\beta\rho[2(\beta+\delta)+\rho(\beta+2\delta)]}{4(\beta^2-\delta^2)-\beta\rho^2}\cdot\frac{\alpha}{2}$$

$$\Rightarrow q^*(0) - q_0^* = S\cdot q^*(0)[2(\beta+\delta)+\rho(\beta+2\delta)]$$

(11) $$\Rightarrow q_0^* = q^*(0)[1 - S(2(\beta+\delta)+\rho(\beta+2\delta))]$$

$$q_A^* - q^*(0) = \frac{\beta\rho[2(\beta+\delta)+\beta\rho]}{4(\beta^2-\delta^2)-\beta\rho^2}\cdot\frac{\alpha}{2}$$

$$\Rightarrow q_A^* - q^*(0) = S\cdot q^*(0)[2(\beta+\delta)+\beta\rho]$$

(12) $$\Rightarrow q_A^* = q^*(0)[1 + S(2(\beta+\delta)+\beta\rho)]$$

Where $S = \dfrac{\beta\rho}{4(\beta^2-\delta^2)-\beta\rho^2}$ and $S > 0$ if and only if $\beta > \delta\cdot\dfrac{2}{\sqrt{(4-\rho^2)}}$ and $\dfrac{2}{\sqrt{(4-\rho^2)}} > 1$

QED

Lemma 2:

From lemma 1 above, the optimal profit of the firm would be:

$$\pi^* = P_A^* q_A^* + P_0^* q_0^* + P_0^* \rho q_A^*$$

Replace the results of proposition 1 above (i.e. (9) – (12)) obtain the lemma after re-arranging (QED)

Proposition 2:

$\frac{\partial S}{\partial \rho} = \frac{-4\beta\delta^2 + 4\beta^3(4+\rho^2)]}{(4(\beta^2-\delta^2)-\beta\rho^2)^2}$ and $\frac{\partial S}{\partial \rho} > 0$ if $\beta > \delta \cdot \frac{2}{\sqrt{4+\rho^2}}$ which is true since $\beta > \delta$ and $\frac{2}{\sqrt{4+\rho^2}} < 1$

Hence, from (9) to (12),

$$\frac{\partial P_A^*}{\partial \rho} = -2\frac{\partial S}{\partial \rho}P^*(0)(\beta-\delta) - \frac{\partial S}{\partial \rho}P^*(0)\rho(\beta-2\delta) - SP^*(0)(\beta-2\delta) < 0,$$

$$\frac{\partial P_0^*}{\partial \rho} = 2\frac{\partial S}{\partial \rho}P^*(0)(\beta-\delta) + \frac{\partial S}{\partial \rho}P^*(0)\rho\beta + SP^*(0)\beta > 0,$$

$$\frac{\partial q_0^*}{\partial \rho} = -2\frac{\partial S}{\partial \rho}q^*(0)(\beta+\delta) - \frac{\partial S}{\partial \rho}q^*(0)\rho(\beta+2\delta) - Sq^*(0)(\beta+2\delta) < 0$$

$$\frac{\partial q_A^*}{\partial \rho} = 2\frac{\partial S}{\partial \rho}q^*(0)(\beta+\delta) + \frac{\partial S}{\partial \rho}q^*(0)\rho\beta + Sq^*(0)\beta > 0$$

Proposition 3:

When $r > 0$ and $\mu > 0$

(13) $$q_A = \alpha - \beta P_{A,R} + \delta P_{0,R}$$

(14) $$q_0 = \alpha - \beta P_{0,R} + \delta P_{A,R}$$

and

$$E[\pi] = \mu[P_{A,R}q_{A,R} + P_{0,R}q_{0,R} + P_{0,R}\rho q_{A,R} - rP_{A,R}\rho q_{A,R}] + (1-\mu)[P_{A,R}q_{A,R} + P_{0,R}q_{0,R} - rP_{A,R}\rho q_{A,R}]$$
$$\Rightarrow E[\pi] = P_{A,R}q_{A,R} + P_{0,R}q_{0,R} - rP_{A,R}\rho q_{A,R} + \mu P_{0,R}\rho q_{A,R}$$

$Max_{P_{A,R},P_{0,R}}\{E[\pi]\}$ will lead to

(15) $$P_{A,R} = \frac{\alpha(1-r\rho) + (\delta(2-r\rho) - \mu\beta\rho)P_{0,R}}{2\beta(1-r\rho)}$$

(16) $$P_{0,R} = \frac{\alpha(1+\mu\rho) + (\delta(2-r\rho) - \mu\beta\rho)P_{A,R}}{2(\beta-\mu\delta\rho)}$$

Solving for (15) and (16) results in

(17) $$P_{0,R}^{*} = \frac{\alpha(1-r\rho)[\beta(2+\mu\rho)+\delta(2-r\rho)]}{4(\beta^2-\delta^2)-4r(\beta+\delta)(\beta-\delta)\rho-(r\delta-\beta\mu)^2\rho^2}$$

(18) $$P_{A,R}^{*} = \frac{\alpha[\beta(2-\rho(2r+\mu+\mu^2\rho))+\delta(2-r\rho(1-\mu\rho))]}{4(\beta^2-\delta^2)-4r(\beta+\delta)(\beta-\delta)\rho-(r\delta-\beta\mu)^2\rho^2}$$

Substituting back to the demand functions will lead to

(19) $$q_{0,R}^{*} = \frac{\alpha(\beta+\delta)[\beta(2-\rho(2r+\mu-\mu\rho(r-\mu)))-\delta(2-r\rho(3-r\rho+\mu\rho))]}{4(\beta^2-\delta^2)-4r(\beta+\delta)(\beta-\delta)\rho-(r\delta-\beta\mu)^2\rho^2}$$

(20) $$q_{A,R}^{*} = \frac{\alpha(\beta+\delta)[\beta(2-2r\rho+\mu\rho)-\delta(2-r\rho)]}{4(\beta^2-\delta^2)-4r(\beta+\delta)(\beta-\delta)\rho-(r\delta-\beta\mu)^2\rho^2}$$

From the above, I find that

$$P^*(0)-P_{A,R}^{*} = \frac{\rho(\beta\mu-r\delta)[2(\beta-\delta)+\rho(\mu(\beta-2\delta)+r\delta)]}{\beta^2(4-4r\rho-\mu^2\rho^2)-\delta^2(2-r\rho)^2+2r\beta\delta\mu\rho^2}\cdot\frac{\alpha}{2(\beta-\delta)}$$

(21) $$\Rightarrow P_{A,R}^{*} = P^*(0)[1-S_R\cdot[2(\beta-\delta)+\rho(\mu(\beta-2\delta)+r\delta)]]$$

$$P_{0,R}^{*}-P^*(0) = \frac{\rho(\beta\mu-r\delta)[2(\beta-\delta)+\rho\mu\beta-\rho r(2\beta-\delta)]}{\beta^2(4-4r\rho-\mu^2\rho^2)-\delta^2(2-r\rho)^2+2r\beta\delta\mu\rho^2}\cdot\frac{\alpha}{2(\beta-\delta)}$$

(22) $$\Rightarrow P_{0,R}^{*} = P^*(0)[1+S_R\cdot[2(\beta-\delta)+\rho\mu\beta-\rho r(2\beta-\delta)]]$$

$$q_{A,R}^{*}-q^*(0) = \frac{\rho(\beta\mu-r\delta)[2(\beta+\delta)+\rho\mu\beta-\rho r\delta]}{\beta^2(4-4r\rho-\mu^2\rho^2)-\delta^2(2-r\rho)^2+2r\beta\delta\mu\rho^2}\cdot\frac{\alpha}{2}$$

(23) $$\Rightarrow q_{A,R}^{*} = q^*(0)[1+S_R\cdot[2(\beta+\delta)+\rho\mu\beta-\rho r\delta]]$$

$$q^*(0)-q_{0,R}^{*} = \frac{\rho(\beta\mu-r\delta)[2(\beta+\delta)+\rho\mu(\beta+2\delta)-\rho r(2\beta+\delta)]}{\beta^2(4-4r\rho-\mu^2\rho^2)-\delta^2(2-r\rho)^2+2r\beta\delta\mu\rho^2}\cdot\frac{\alpha}{2(\beta-\delta)}$$

(24) $$\Rightarrow q_{0,R}^{*} = q^*(0)[1-S_R\cdot[2(\beta+\delta)+\rho\mu(\beta+2\delta)-\rho r(2\beta+\delta)]$$

(21) above reduces to $P_{A,R}^{*} = P^*(0)\left(1-\frac{\beta\mu\rho[2(\beta-\delta)+\mu\rho(\beta-2\delta)]}{4(\beta^2-\delta^2)-\beta^2\mu^2\rho^2}\right)$ when $r=0$

This is similar in form to P_A^* in proposition 1 except that the term ρ is now $\mu\rho$. Consequently, let $k=\mu\rho$. Therefore $P_{A,R}^{*} = P^*(0)\left(1-\frac{\beta k[2(\beta-\delta)+k(\beta-2\delta)]}{4(\beta^2-\delta^2)-\beta^2k^2}\right)$. Since $\frac{\partial P_{A,R}^{*}}{\partial k}<0$ (from proposition 2) and $\frac{\partial k}{\partial\mu}=\rho>0$, $\frac{\partial P_{A,R}^{*}}{\partial\mu}<0$. Similarly, $\frac{\partial P_0^{*}}{\partial\mu}>0$, $\frac{\partial q_0^{*}}{\partial\mu}<0$, and $\frac{\partial q_A^{*}}{\partial\mu}>0$.

Proposition 4

The expected profit if a refund is offered would be

(25) $$E[\pi|_{refund}] = P_{A,R}q_{A,R} + P_{0,R}q_{0,R} - rP_{A,R}\rho q_{A,R} + \mu P_{0,R}\rho q_{A,R}$$

In contrast, if a refund is not offered, the profit would be

(26) $$E[\pi|_{norefund}] = P_A q_A + P_0 q_0 + \mu P_0 \rho q_A$$

where

(13) $$q_{A,R} = \alpha - \beta P_{A,R} + \delta P_{0,R}$$

(14) $$q_{0,R} = \alpha - \beta P_{0,R} + \delta P_{A,R}$$

(13a) $$q_A = \alpha - \beta P_A + \delta P_0$$

(14a) $$q_0 = \alpha - \beta P_0 + \delta P_A$$

Following the optimization process for (25) in proposition 3 above, we find that

(17a) $$P_0^* = \frac{\alpha[2(\beta+\delta) + \beta\mu\rho]}{4(\beta^2 - \delta^2) - \beta\mu^2\rho^2}$$

(18a) $$P_A^* = \frac{\alpha[2(\beta+\delta) - \rho(\mu + \mu^2\rho)]}{4(\beta^2 - \delta^2) - \beta\mu^2\rho^2}$$

Substituting back to the demand functions (13a) and (13b) will lead to

(19a) $$q_0^* = \frac{\alpha(\beta+\delta)[2(\beta-\delta) - \rho(\mu - \mu^2\rho))]}{4(\beta^2 - \delta^2) - \beta\mu^2\rho^2}$$

(20a) $$q_A^* = \frac{\alpha(\beta+\delta)[\beta(2 - 2r\rho + \mu\rho) - \delta(2 - r\rho)]}{4(\beta^2 - \delta^2) - 4r(\beta+\delta)(\beta-\delta)\rho - (r\delta - \beta\mu)^2\rho^2}$$

Substituting (17), (18), (19) and (20) into (25) and (17a), (18a), (19a) and (20a) into (26) yields

(27) $$\Rightarrow E[\pi^*|_{refund}] = \frac{\alpha^2(\beta+\delta)(1 - r\rho)(2 - r\rho + \mu\rho)}{\beta^2(4 - 4r\rho - \mu^2\rho^2) - \delta^2(2 - r\rho)^2 + 2r\beta\delta\mu\rho^2}$$

(28) $$\Rightarrow E[\pi^*|_{norefund}] = \frac{\alpha^2(\beta+\delta)(2 + \mu\rho)}{4(\beta^2 - \delta^2) - \beta^2\mu^2\rho^2}$$

$E[\pi^*|_{refund}] > E[\pi^*|_{norefund}]$ if and only if

$\dfrac{\alpha^2(\beta+\delta)(1-r\rho)(2-r\rho+\mu\rho)}{\beta^2(4-4r\rho-\mu^2\rho^2)-\delta^2(2-r\rho)^2+2r\beta\delta\mu\rho^2} > \dfrac{\alpha^2(\beta+\delta)(2+\mu\rho)}{4(\beta^2-\delta^2)-\beta^2\mu^2\rho^2}$ which after rearranging becomes

(29) $$r > \frac{4(\beta^2-\delta^2)-3\beta^2\mu^2\rho^2-\beta^2\mu^3\rho^3+2\beta\delta\mu\rho(2+\mu\rho)}{4\rho(\beta^2-\delta^2)-\beta^2\mu^2\rho^3+\rho\delta^2(2+\mu\rho)}$$

Check that $r > 0$ which is true if and only if $\beta > \delta \cdot \dfrac{2}{2+\mu\rho}$ (true)

However, due to speculators, $r \leq 1$ which means that the value of r (29) is constrained. So $1 \geq r > \dfrac{4(\beta^2-\delta^2)-3\beta^2\mu^2\rho^2-\beta^2\mu^3\rho^3+2\beta\delta\mu\rho(2+\mu\rho)}{4\rho(\beta^2-\delta^2)-\beta^2\mu^2\rho^3+\rho\delta^2(2+\mu\rho)}$. Therefore,

$\dfrac{4(\beta^2-\delta^2)-3\beta^2\mu^2\rho^2-\beta^2\mu^3\rho^3+2\beta\delta\mu\rho(2+\mu\rho)}{4\rho(\beta^2-\delta^2)-\beta^2\mu^2\rho^3+\rho\delta^2(2+\mu\rho)} \leq 1$ leading to

$\delta < \beta\phi$ where $\phi = \dfrac{2\mu\rho+\mu^2\rho^2+\sqrt{(1-\rho)(2-\mu\rho)^2(4-(2-4\mu)\rho-(1-\mu)\mu\rho^2)}}{4-2\rho+\mu\rho^2}$ and $\phi < 1$ for all permitted values of $0 < \rho < 1$ and $0 < \mu < 1$ QED.

Proof for optimal prices and quantities are found in proposition 3 i.e. (21) – (24)

Proposition 5

Consider the expected profit to the firm when a refund is offered i.e.

$$E[\pi^*|_{refund}] = E[\pi^*] = \frac{\alpha^2(\beta+\delta)(1-r\rho)(2-r\rho+\mu\rho)}{\beta^2(4-4r\rho-\mu^2\rho^2)-\delta^2(2-r\rho)^2+2r\beta\delta\mu\rho^2}$$

$$\frac{\partial E[\pi^*]}{\partial r} = \frac{[\alpha^2\rho(\beta+\delta)][\beta(2-2r\rho+\mu\rho)-\delta(2-r\rho)][\delta(-2+r\rho(1-\mu\rho))+\beta(-2+\rho(2r+\mu+\mu^2\rho))]}{[\beta^2(4-4r\rho-\mu^2\rho^2)-\delta^2(2-r\rho)^2+2r\beta\delta\mu\rho^2]^2}$$

The denominator is positive and so is the first term in the numerator. Therefore

$\frac{\partial E[\pi^*]}{\partial r} > 0$ if and only if the second term, $[\beta(2-2r\rho+\mu\rho)-\delta(2-r\rho)]>0$ and the third term $[\delta(-2+r\rho(1-\mu\rho))+\beta(-2+\rho(2r+\mu+\mu^2\rho))]>0$.

Second term gives

$\beta > \delta \cdot \frac{2-r\rho}{2-r\rho+\mu\rho}$ which is true since $\frac{2-r\rho}{2-r\rho+\mu\rho} < 1$ and $\beta > \delta$

Third term gives

$[\delta(-2+r\rho(1-\mu\rho))+\beta(-2+\rho(2r+\mu+\mu^2\rho))]>0$ if and only if

$r > \frac{2(\beta+\delta)-\beta\mu\rho(1+\mu\rho)}{\delta\rho(1-\mu\rho)+2\beta\rho}$. However, $r \le 1$ therefore the condition would only hold if $\frac{2(\beta+\delta)-\beta\mu\rho(1+\mu\rho)}{\delta\rho(1-\mu\rho)+2\beta\rho} < 1$ which is when $\beta < \delta \cdot \frac{[2-\rho(1-\mu\rho)]}{\mu\rho(1+\mu\rho)-2(1-\rho)}$ where $\frac{[2-\rho(1-\mu\rho)]}{\mu\rho(1+\mu\rho)-2(1-\rho)} > 1$. Hence, $\frac{\partial E[\pi^*]}{\partial r} > 0$ holds if $\delta < \beta < \delta \cdot \frac{[2-\rho(1-\mu\rho)]}{\mu\rho(1+\mu\rho)-2(1-\rho)}$. In this condition, since $\frac{\partial E[\pi^*]}{\partial r} > 0$, the firm would choose the highest possible refund i.e. $r = 1$. Hence $r = 1$ if and only if $\delta < \beta < \delta \cdot \frac{[2-\rho(1-\mu\rho)]}{\mu\rho(1+\mu\rho)-2(1-\rho)}$

Proposition 6

From (6), $P_A^* = \frac{\alpha(2\delta+\beta(2-\rho-\rho^2))}{4(\beta^2-\delta^2)-\beta\rho^2}$

From (18), $P_{A,R}^* = \frac{\alpha[\beta(2-\rho(2r+\mu+\mu^2\rho))+\delta(2-r\rho(1-\mu\rho))]}{4(\beta^2-\delta^2)-4r(\beta+\delta)(\beta-\delta)\rho-(r\delta-\beta\mu)^2\rho^2}$

Let $\mu = 1$ reducing $P_{A,R}^*$ to $P_{A,R}^* = \frac{\alpha[\beta(2-\rho(2r+1+\rho))+\delta(2-r\rho(1-\rho))]}{4(\beta^2-\delta^2)-4r(\beta+\delta)(\beta-\delta)\rho-(r\delta-\beta)^2\rho^2}$.

Therefore $P_{A,R}^* < P_A^*$ iff

$r < \frac{2\beta^3\rho(2+\rho)+4\delta^3(1+\rho)-\beta^2\delta(4+3\rho^2+\rho^3)-4\beta\delta^2\rho^2}{\delta^2\rho[2\delta+\beta(2-\rho-\rho^2)}$. It remains for us to show that condition (29) holds i.e.

$$\frac{2\beta^3\rho(2+\rho)+4\delta^3(1+\rho)-\beta^2\delta(4+3\rho^2+\rho^3)-4\beta\delta^2\rho^2}{\delta^2\rho[2\delta+\beta(2-\rho-\rho^2)} > \frac{4(\beta^2-\delta^2)-3\beta^2\mu^2\rho^2-\beta^2\mu^3\rho^3+2\beta\delta\mu\rho(2+\mu\rho)}{4\rho(\beta^2-\delta^2)-\beta^2\mu^2\rho^3+\rho\delta^2(2+\mu\rho)}$$

which leads to

$$\frac{[2\beta+\delta(1-\rho)][\beta\rho-\delta][\beta(2+\rho)-\delta\rho][\beta^2(4-\rho^2)-4\delta^2]}{\delta^2(2-\rho)\rho[\beta^2(2+\rho)-\delta^2][\beta(2-\rho-\rho^2)+2\delta]} > 0$$

which is true iff

$\beta > \delta\cdot\frac{1}{\sqrt{2+\rho}}$ which is true since $\frac{1}{\sqrt{2+\rho}} < 1$ and $\beta > \delta$

$\beta > \delta\cdot\frac{\rho}{2+\rho}$ which is true since $\frac{\rho}{2+\rho} < 1$ and $\beta > \delta$

$\beta > \delta\cdot\frac{2}{\sqrt{4-\rho^2}}$ which is true under proposition 2

$\beta > \frac{\delta}{\rho}$ (binding)

Proposition 7

(30) $$q_A = \alpha - \beta P_A + \delta_A P_0$$

(31) $$q_0 = \alpha - \beta P_0 + \delta_0 P_A$$

and $\pi = P_A q_A + P_0 q_0 + P_0 \rho q_A$

$Max_{P_A,P_0}\{\pi\}$ will lead to

(32) $$P_A = \frac{\alpha + (\delta_0 + \delta_A - \beta\mu\rho)P_0}{2\beta}$$

(33) $$P_0 = \frac{\alpha(1+\mu\rho) + (\delta_0 + \delta_A - \beta\mu\rho)P_A}{2(\beta - \mu\rho\delta_A)}$$

Solving for (32) and (33) results in

(34) $$P_0^* = \frac{\alpha(\delta_A + \delta_0 + \beta(2+\mu\rho))}{\beta^2(4-\mu^2\rho^2) - \delta_0{}^2 - \delta_A{}^2 - 2\beta\mu\rho\delta_A + \delta_0(2\beta\mu\rho - 2\delta_A)}$$

(35) $$P_A^* = \frac{\alpha(\delta_0(1+\mu\rho) + (1-\mu\rho)(\beta(2+\mu\rho+\delta_A))}{\beta^2(4-\mu^2\rho^2) - \delta_0{}^2 - \delta_A{}^2 - 2\beta\mu\rho\delta_A + \delta_0(2\beta\mu\rho - 2\delta_A)}$$

Substituting back to the demand functions will lead to

(36) $$q_A^* = \frac{\alpha[\beta(\beta(2+\mu\rho)+\delta_A)-\delta_0^2-\delta_0(\beta-\beta\mu\rho+\delta_A)]}{\beta^2(4-\mu^2\rho^2)-\delta_0{}^2-\delta_A{}^2-2\beta\mu\rho\delta_A+\delta_0(2\beta\mu\rho-2\delta_A)}$$

(37) and $$\pi^* = \frac{\alpha^2[\beta(2+\mu\rho)+(1+\mu\rho)\delta_0+\delta_A]}{\beta^2(4-\mu^2\rho^2)-\delta_0{}^2-\delta_A{}^2-2\beta\mu\rho\delta_A+\delta_0(2\beta\mu\rho-2\delta_A)}$$

When $\delta_A \to 0$

$$P_A^* = \frac{\alpha[\beta(1-\mu\rho)(2+\mu\rho)+(1+\mu\rho)\delta_0]}{\beta^2(4-\mu^2\rho^2)-\delta_0^2+2\beta\mu\rho\delta_0}$$

$$P_0^* = \frac{\alpha[\beta(2+\mu\rho)+\delta_0]}{\beta^2(4-\mu^2\rho^2)-\delta_0^2+2\beta\mu\rho\delta_0}$$

$$q_A^* = \frac{\alpha[\beta^2(2+\mu\rho)-\beta\delta_0(1-\mu\rho)-\delta_0^2]}{\beta^2(4-\mu^2\rho^2)-\delta_0^2+2\beta\mu\rho\delta_0}$$

$$q_0^* = \frac{\alpha[\beta^2(2-\mu\rho-\mu^2\rho^2)+\beta\delta_0(1+\mu\rho-\mu^2\rho^2)+\mu\rho\delta_0^2]}{\beta^2(4-\mu^2\rho^2)-\delta_0^2+2\beta\mu\rho\delta_0}$$

When $\delta_0 \to 0$

$$P_A^* = \frac{\alpha(1-\mu\rho)}{\beta(2-\mu\rho)-\delta_A}$$

$$P_0^* = \frac{\alpha}{\beta(2-\mu\rho)-\delta_A}$$

$$q_A^* = \frac{\alpha\beta}{\beta(2-\mu\rho)-\delta_A}$$

$$q_0^* = \frac{\alpha[\beta(1-\mu\rho)-\delta_A]}{\beta(2-\mu\rho)-\delta_A}$$

The proof needs to show that $P_A^*\big|_{\delta_A=0} > P_A^*\big|_{\delta_0=0}$

Which is $\dfrac{\alpha[\beta(1-\mu\rho)(2+\mu\rho)+(1+\mu\rho)\delta_0]}{\beta^2(4-\mu^2\rho^2)-\delta_0^2+2\beta\mu\rho\delta_0} > \dfrac{\alpha(1-\mu\rho)}{\beta(2-\mu\rho)-\delta_A}$

Let $\mu = 1$ and assume that $\delta_A = \delta_0 = \delta$ which would then lead to

$$P_A^*\Big|_{\delta_A=0} - P_A^*\Big|_{\delta_0=0} = \frac{2\alpha\rho(\beta\rho - \delta)\delta}{[\beta(2-\rho)-\delta][\beta(2-\rho)+\delta][\beta(2+\rho)-\delta]}$$

and $P_A^*\Big|_{\delta_A=0} - P_A^*\Big|_{\delta_0=0} > 0$ iff

$\beta > \delta \cdot \dfrac{1}{2+\rho}$ which is true since $\beta > \delta$

$\beta > \delta \cdot \dfrac{1}{2-\rho}$ which is true since $\beta > \delta$

and $\beta > \dfrac{\delta}{\rho}$

Similarly,

$$P_0^*\Big|_{\delta_0=0} - P_0^*\Big|_{\delta_A=0} = \frac{4\alpha\beta\rho\delta}{[\beta(2-\rho)-\delta][\beta(2-\rho)+\delta][\beta(2+\rho)-\delta]}$$

$P_0^*\Big|_{\delta_0=0} - P_0^*\Big|_{\delta_A=0} > 0$ which is true iff $\beta > \dfrac{\delta}{\rho}$

Proposition 8

From (38), assuming $\mu = 1$

$$\pi^*\Big|_{\delta_A=0} = \frac{\alpha^2[\beta(2+\rho)+\delta_0(1+\rho)]}{\beta^2(4-\rho^2)+2\beta\delta_0\rho-\delta_0^2}$$

$$\pi^*\Big|_{\delta_0=0} = \frac{\alpha^2}{\beta(2-\rho)-\delta_A}$$

Assume that $\delta_A = \delta_0 = \delta$

and $\pi^*\Big|_{\delta_0=0} - \pi^*\Big|_{\delta_A=0} = \dfrac{\alpha^2\rho\delta[\beta(2+\rho)+\delta]}{[\beta(2-\rho)-\delta][\beta(2-\rho)+\delta][\beta(2+\rho)-\delta]} > 0$ from proof of proposition 7 above

REFERENCES

Alstrup, Jens et al. (1986), "Booking Policy for Flights with Two Types of Passengers," *European Journal of Operations Research,* 27, 274-288.

Badinelli, Ralph D. and Michael D. Olsen (1990), "Hotel Yield Management Using Optimal Decision Rules," *Journal of International Academy of Hospitality Research*, November 26), 1, (accessed July 4, 2004), [available at http://scholar.lib.vt.edu/ejournals/JIAHR/issue1.html].

Bateson, John (1977), "Do We Need Service Marketing," in *Marketing Services: New Insights*, Pierre Eiglier et al., eds. Cambridge: Marketing Science Institute.

Belobaba, Peter P. (1989), "Application of a Probabilistic Decision Model to Airline Seat Inventory Control," *Operations Research,* 37 (2), 183-197.

Berstein, Peter L. (1996), *Against the Gods: A Remarkable Story of Risk*. New York: John Wiley and Sons.

Bodily, Samuel E. and Lawrence R. Weatherford (1995), "Perishable-asset Revenue Management: Generic and Multiple-price Yield Management with Diversion," *Omega, International Journal of Management Science,* 23 (2), 173-185.

Chase Jr., Charles W. (1999), "Revenue Management: A Review," *Journal of Business Forecasting Methods and Systems*, 18 (2), 2 and 28.

Cook, Phillip J. and Daniel A. Graham (1977), "The Demand for Insurance and Protection: The Case of Irreplaceable Commodities," *Quarterly Journal of Economics*, 91 (February), 143-154.

Desiraju, Ramarao and Steven M. Shugan (1999), "Strategic Service Pricing and Yield Management," *Journal of Marketing*, 63 (January), 44-56.

Dixit, Avinash (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers," *Bell Journal of Economics,* Vol. 10, No. 1, pp20-32

Edgett, Scott and Stephen Parkinson (1993), "Marketing for Service Industries - A Review," *The Service Industries Journal*, 13(3), 19.

Fishburn, Peter C. (1974), "On the Foundations of Decision Making under Uncertainty," in *Essays on Economic Behavior under Uncertainty*, M.Balch, D. McFadden, S. Wu, eds. North Holland Publishing Co.

Gallouj, Faiz (2002), *Innovation in the Service Economy: The New Wealth of Nations*, Edward Elgar Publishing.

Hersh, Marvin and Shaul P. Ladany (1978), "Optimal Seat Allocation for Flights with One Intermediate Stop," *Computers and Operations Research*, 5, 31-37.

Ingene, Charles A. and Mark E. Parry (1995), "Channel Coordination When Retailers Compete," *Marketing Science*, 14 (4), 360-377.

Jagpal, Sharan (1998), *Marketing Strategy and Uncertainty,* USA: Oxford University Press.

Johnson, E. (1970), "The Selling of Services" in *Handbook of Modern Marketing*, Victor P Buell. New York: McGraw-Hill.

Karni, Edi (1983), "Risk Aversion for State-Dependent Utility Functions: Measurement and Applications," *International Economic Review*, 24 (3) (October), 637-647.

Kimes, Sheryl E. (1989), "Yield Management: A Tool for Capacity-Constrained Service Firms," *Journal of Operations Management*, 8 (4) (October), 348-363.

Lee, Khai Sheang and Irene C.L. Ng (2001), "Advanced Sale of Service Capacities: A Theoretical Analysis of The Impact of Price Sensitivity on Pricing and Capacity Allocations," *Journal of Business Research*, 54 (3), 219-225.

Levine, Michael E. (1987), “Airline Competition in Deregulated Markets: Theory, Firm Strategy and Public Policy,” *Yale Journal on Regulation*, 4, 393-494.
Lieberman, Warren H. (1993), “Debunking the Myths of Yield Management,” *Cornell Hotel and Restaurant Administration Quarterly*, (February), 34-41.
Lieberman, Varda and Uri Yechiali (1978), “On the Hotel Overbooking Problem: An Inventory Problem with Stochastic Cancellations,” *Management Science*, 24, 1117-1126.
McGuire, Timothy W. and Staelin, Richard (1983), “An Industry Equilibrium Analysis of Downstream Vertical Integration,” *Marketing Science*, 2 (2) (Spring), 161-191.
Nagle, Thomas T, Reed K. Holden, Reed Holden (2002), *The Strategy and Tactics of Pricing: A Guide to Profitable Decision Making*, 3rd Edition, Prentice Hall**.**
Pfeifer, Philip E. (1989), “The Airline Discount Fare Allocation Problem,” *Decision Sciences*, 20, 149-157.
Png, Ivan P.L. (1989), “Reservations: Customer Insurance in the Marketing of Capacity,” *Marketing Science*, 8 (3) (Summer), 248-264.
---- 1991,“Most-Favored-Customer Protection versus Price Discrimination over Time,” *Journal of Political Economy*, 99(5), 1010-1028.
Rathmell, John (1966), “What is Meant by Services?”, *Journal of Marketing*, 30(4), 32-36.
Regan, W(1963), “The Service Revolution,” *Journal of Marketing*, 27(3), 57-62.
Relihan III, Walter J. (1989), “The Yield Management Approach to Hotel-Room Pricing,” *Cornell Hotel and Restaurant Administration Quarterly*, (May), 40-45.
Rothstein, Marvin (1971), “An Airline Overbooking Model,” *Transportation Science*, 5, 180-192.
---- 1974, “Hotel Overbooking as a Markovian Sequential Decision Process,” *Decision Sciences*, 5,389-394.
---- 1985, “Operations Research and the Airline Overbooking Problem,” *Operations Research*, 33 (2), 237-248.
Shugan, Steven M. and Xie, Jinhong (2000), “Advance pricing of services and other implications of separating purchase and consumption,” *Journal of Service Research*, 2 (3), 227-239.
Nirvikar Singh & Xavier Vives, (1984), "Price and Quantity Competition in a DifferentiatedDuopoly," *RAND Journal of Economics*, 15 (4), pp. 546-554.
Thompson, H.R. (1961), “Statistical Problems in Airline Reservation Control,” *Operational Research Quarterly*, 12, 167-185.
Toh, Rex (1979), “Airline Revenue Yield Protection: Joint Reservation Control Over Full and Discount Fare Sales,” *Transportation Journal*, (Winter), 74-80.
---- 1985, “An Inventory Depletion Overbooking Model For the Hotel Industry,” *Journal of Travel Research*, (Spring), 24-30.
Xie, Jinhong and Shugan, Steven M. (2001), “Electronic Tickets, Smart Cards and Online Prepayments: When and How to Advance Sell,” *Marketing Science*, 20(3) Summer, 219-243.