Individually-rational union membership

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April 2000.

Abstract: The analysis of the determination of union membership has typically met difficulties with the fact that union membership is not individually rational and free-riding is the dominant strategy. We assume that workers differ in their reservation wages and hence in their preferred choice of contract, so preventing free-riding on the contract choice of others. This implies that joining a union is equivalent to buying a vote on the contract and provides an individual incentive to join the union. An equilibrium trade union membership is characterized in which membership is taken up by those with relatively "extreme" tastes. The union achieves a centralist objective even though no member precisely supports such a view.

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Acknowledgements: We are grateful to Jun-ichi Itaya, David Kelsey, David de Meza, Robin Naylor, two anonymous referees and seminar participants as Birmingham, Exeter and Maastricht.
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1. Introduction

Considerable progress has now been made in analyzing the impact of a trade union on wages and employment. However the determinants of the union membership decision for the individual are less well understood. What motive have self-interested workers for joining a trade union? Why pay union dues when it is possible to free-ride on the union contract? And where unions are formed, coverage is invariably only partial leading to the question of why, if there is a motive for some to join, this does not result in all joining. These are questions of interest in their own right but the answers may also be relevant to a better understanding of union objectives since members' motives for joining a union have implications for the policies pursued by that union.

One explanation of union membership, proposed by Akerlof (1980), Booth (1985) and Naylor (1989), is that individuals may not be motivated solely by a narrow definition of utility but may join in order to conform to a social custom. Although this is an interesting suggestion, it leaves unexplained the formation of the social custom. Furthermore, in order to explain partial union coverage this approach must appeal to heterogeneous, but unobservable, pay-offs from conforming to group norms (Naylor and Cripps 1993). Consequently, it remains of interest to explain membership within the neo-classical paradigm.

The model of the union membership decision which we develop in this paper offers a solution to the two problems of identifying an individually-rational motive for membership under a conventional utility function and of explaining partial coverage, without appealing to unobservable differences in tastes. If different membership decisions are not to be explained by differences in utility functions, then we must assume workers differ in their opportunities. Specifically, we assume that workers face varied outside options so that they differ in their reservation wages. This in turn implies that they will have different optimal points in the trade-off between an increased wage and a decreased probability of employment. These differences can provide the incentive required for an individual to join the union since membership makes their opinion count towards the formulation of union objectives. Free-riding is no longer possible if the union members are working towards different objectives to the outsiders.

Once it is allowed that members have different reservation wages, we have to make a judgement about how these are aggregated into a single objective for the union. To do this, we assume that the objectives of the union are determined by majority voting of its members and apply the Median Voter Theorem.[1] The principle aim of this paper is to show how diverse reservation wages and majority voting give rise to an individually-rational incentive to join a union through the influence membership exerts over the objectives of the union. An additional aspect of the model is that the paradox of voting does not apply. Workers join the union because of the private benefit a vote brings. Once they have joined, they will always exercise their vote - it is irrational to pay the cost of membership and then decline to vote.

The assumption that workers are heterogeneous raises the question of which types of workers will join the union. In the context of this model, will it typically be high or low reservation wage workers who become union members? In the equilibrium that we characterize, membership is taken up by those with relatively “extreme” reservation wages. However, these extremes tend to be balanced so that the majority voting results in the union pursuing a centralist objective despite none of its members supporting this viewpoint. In some cases not all workers will choose to join the union. Those that don't join have centralist objectives and are able to free-ride on the activities of the union. Consequently, although an incentive to join is created for some, a degree of free-riding may remain at the equilibrium.

Although we consider only the issue of union membership in the present paper, the approach to free-riding adopted here can be applied more generally. For instance, the provision of (non-excludable) public goods by clubs is very closely related to what is described here. Indeed, the trade union is just a special case of this. Provided the cost of providing the public good is positive for club members, the model can be easily adapted. For excludable public goods, the model explains why some members of a club are willing to expend time to provide management services. More generally, if the union membership fee is interpreted as a time cost, it is possible to see parallels between the model in this paper and the willingness to serve on public decision-making bodies. An empirical application of such a model in the context of the UK House of Lords is pursued in Bulkley, Myles and Pearson (2000). When extended to these cases, our results suggest how the conflict between previous theoretical predictions of extensive free-riding can be reconciled with the limited extent of this phenomenon in empirical and experimental evidence. In these extensions, and the

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[1] Although this is not the only possible choice, it is in accord with the fact that most unions do practice majority voting. In addition, the choice only affects the details of the analysis not the main message.

union case upon which we focus, membership involves bundling of a public good and a private. This is an idea that can also be found in Olson (1965) and Stigler (1974).

The analysis considers a set of workers who, in the absence of a union, would be employed, with some fixed probability, at a wage unilaterally determined by the firm. If a union is formed the wage will be higher but the probability of employment will be lower. The decision on whether or not to join the union is modeled as a game in which each worker has to decide whether or not to join the union. At the equilibrium of this game no worker who has joined the union will wish to leave and no further workers will wish to take up membership. We derive a (weak) sufficient condition for equilibrium membership to be positive and characterize a Nash equilibrium union membership. This equilibrium is unique.

The initial dispersion of reservation wages which drives the model that follows merits some comment. The fact that these differences are not reflected in payment for the existing form of employment could arise in a number of alternative labor market structures. For instance, in a human-capital framework it could be due to all the workers possessing the same specific skill for this job but having different productivities in their next best employment. Alternatively, reservation wages could be equal to productivity within this firm but asymmetries of information prevent the employer from observing individual ability and paying a corresponding wage (Weiss (1981)).[2]

In the next section we describe the set of workers from which the union may be formed and show how workers' preferences over the wage-employment trade-off are affected by their reservation wages. In Section 3 the formation of union preferences is described and a sufficient condition for positive membership constructed. Section 4 characterizes a Nash equilibrium structure of union membership and proves that it is unique. The consequences of some modifications of our basic assumptions are considered in Section 5. Conclusions are given in Section 6. All proofs can be found in the Appendix.

2. Workers' preferences

[2] The equilibrium wage in the absence of a union would then be determined via efficiency wage arguments. Note that in this latter interpretation if the union raises the wage then it might then be argued that a higher ability pool of workers will be attracted. In order to abstract from this issue this interpretation would require the assumption, conventional in the union literature, that it could not be formed in the first place unless it is strong enough to prevent the replacement of its members with outsiders.

The purpose of this section is to introduce the structure of preferences of the individual workers for the wage the union sets, and to prove some results about the relation of a worker's optimal wage demand to their reservation wage. These results will form the basis of the arguments establishing the structure of equilibrium union membership in sections 3 and 4.

We consider a group of workers for whom, without loss of generality, the probability of employment is unity at the wage set in the absence of a union. This group can be interpreted, for example, as the set of workers who are currently employed. For descriptive purposes, we assume that before the period of employment commences there is a decision interval, the "pre-contract period", during which time each worker has the option of joining the union. At the end of the pre-contract period, after the equilibrium membership is determined, the union, reflecting the preferences of those who have joined, determines the wage. The firm then chooses employment and union members who are employed pay their membership dues. If no union is formed, the whole group expect employment in the post-contract period at the wage set unilaterally by the firm.

Denote the total number of employed workers by *m* when the wage w^c is set by the firm. This is the set of workers from which a union may be formed. At wage w^c the expected probability of employment in the post-contract period is assumed to be unity. These workers are indexed by $i = 1, \ldots, m$. For any higher level of the wage the labor demand of the firm is determined by the differentiable function $n = n(w)$, $w \geq w^c$, $n(w^c) = m$, $n'(w) < 0$. At a wage of *w* the probability of employment is therefore $n(w)/m$. The reservation wage of worker *i* is denoted by r_i and the labeling of the workers is chosen so that $r_1 < r_2 < \ldots < r_m$.

The expected utility of worker *i* receiving wage *w* if they join the union is given by

$$V(w, r_i) = \frac{n(w)}{m} U(w) + \left[1 - \frac{n(w)}{m}\right] U(r_i), \tag{1}$$

where the utility of income function satisfies $U'(\cdot) > 0$ and $U''(\cdot) \leq 0$.[3] Similarly, the expected utility of a worker joining the union and paying a membership fee, *c*, if they are subsequently employed, is given by

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[3] Notice that no employment advantage is conferred on union members. We assume this is order to remove this as an incentive for union membership. If it were included, it would further reduce the likelihood of free-riding. For a model that does include such an advantage, see Moreton (1998).

$$V(w-c;r_i)=\frac{n(w)}{m}U(w-c)+\left[1-\frac{n(w)}{m}\right]U(r_i). \tag{2}$$

In what follows, the wage rate that maximizes (2) will be of significance. Assuming expected utility is a strictly concave function of the wage rate, from (2) the optimal wage rate, w_i^*, satisfies

$$\frac{n'(w_i^*)}{m}\left[U(w_i^*-c)-U(r_i)\right]+\frac{n(w_i^*)}{m}U'(w_i^*-c)\leq 0,\ \ w_i^*\geq w^c\,, \tag{3}$$

with complementary slackness. The interpretation of the complementary slackness condition in (3) is that some workers with low values of r_i prefer the complete job security offered by wage w^c to the prospect of a higher wage but some probability of lay-off. At an interior solution, with $w_i^* > w^c$, the effect of an increase in r_i upon w_i^* can be found from (3) to be

$$\frac{dw_i^*}{dr_i}=\frac{\frac{n'}{m}U'}{S}>0\,, \tag{4}$$

where $S<0$. Hence the optimal wage increases with the reservation wage.

The maximum value function for the optimization conditional upon r_i is denoted by $V^*(r_i)\equiv\frac{n(w_i^*)}{m}U(w_i^*-c)+\left[1-\frac{n(w_i^*)}{m}\right]U(r_i)$. Employing the envelope theorem it follows that

$$\frac{dV^*(r_i)}{dr_i}=\left[1-\frac{n(w_i^*)}{m}\right]U'(r_i)>0\,. \tag{5}$$

Although the results in (4) and (5) have been proved for the continuous case, their extension to the discrete case is immediate. Making this extension, the optimal wages of the workers and the resulting utilities can be ranked according to

$$w^c\leq w_1^*\leq w_2^*\leq\ldots\leq w_m^*,V^*(r_1)<V^*(r_2)<\ldots<V^*(r_m). \tag{6}$$

We prove below that $w^c<w_m^*$, so in all cases there will be some strict inequalities in the ranking of wages in (6).

3. Union preferences and positive membership

We model the decision on whether to join the union as a game in which each worker has two strategies: joining the union or not joining. The motive for joining the union is that it entitles the worker a vote on the reservation wage that the union carries into negotiations with the firm. This is a valuable right since each worker benefits if the union's reservation wage is aligned with their own. If it is not aligned, the union will choose a point on the wage/employment trade-off away from the workers preferred position. This section begins by describing the process used for forming the union's preferences and then establishes a sufficient condition for union membership to be positive at any equilibrium.

To analyze voting we employ the standard form of the Median Voter Theorem. As described in Mueller (1989) for example, this states that when there is an odd number of voters, whose preferences can be ranked along a single dimension, the aggregate preference will be exactly that of the median voter. This follows since a majority will be found that can vote down any other alternative. When there is an even number of voters, the Median Voter Theorem is not as clear in its predictions. It is, however, certain that the outcome must be such that half of the population fall on each side of it. When the union has an even number of members, we choose to set the voting outcome as lying halfway between the preferred outcomes of the two voters either side of the median. With a symmetric distribution of preferences, a justification for this approach is that this is the expected outcome if an additional voter were added to achieve an odd number of voters in total. Adopting this procedure provides a precisely defined outcome in all possible cases. As argued in Section 5, the precise specification chosen does not affect the qualitative conclusions of the paper.

The first step in the argument is to determine the objective of the union for some given set of members. It is now standard in the union-firm bargaining literature to assume that the objectives of the union are captured by a utility function that has wages and employment as its arguments. We also follow this approach, with the extension that the utility of the union is found by aggregating the preferences of its members. The chosen aggregation process follows from noting that all the workers have the same utility of income functions, $U(\cdot)$, but differ in their reservation wages. We assume that the union has a utility function of the same form as its members. The reservation wage of the union is then determined by voting of its members. It is to this voting process that we apply the Median Voter Theorem. In this way, voting by members determines the preferences that are taken by the union into the bargaining process with the firm. The union utility function is therefore written as

$$V(w;\bar{r}) = \frac{n(w)}{m}U(w-c) + \left[1 - \frac{n(w)}{m}\right]U(\bar{r}), \tag{7}$$

where $\bar{r}$ is the union reservation wage determined by applying the Median Voter Theorem.

To describe the voting process, it is necessary to introduce some additional notation. Consider a union composed of some subset, *J,* of the set of workers. Now rank the workers within the set *J* by their reservation wages and re-label them by the index $j = 1,\dots,J$. For those in *J* the reservation wages can now be ranked according to $r^1 < r^2 < \dots < r^J$. The use of the superscript denotes that this refers to the index *within J*. The function $\bar{r}(J)$ is then defined as follows

$$\bar{r}(J) = \begin{cases} \text{median}\{r^j, j \in J\} \text{if } \# J \text{ is odd,} \\ \dfrac{r^j + r^{j-1}}{2}, \text{where } j-1 = \dfrac{\# J}{2} \text{ if } \# J \text{ is even.} \end{cases} \tag{8}$$

From its definition, it can be seen that setting $\bar{r} = \bar{r}(J)$ defines the median-voter solution to the problem of union reservation wage determination. For example, if $J = \{1\}$ then $\bar{r}(J) = r^1$ and if $J = \{1,2,3,4\}$ then $\bar{r}(J) = \dfrac{r^3 + r^2}{2}$. For a given membership defined by some set *J*, the objective of the union therefore becomes

$$V(w; \bar{r}(J)) \equiv \frac{n(w)}{m} U(w-c) + \left[1 - \frac{n(w)}{m}\right] U(\bar{r}(J)). \tag{9}$$

Under the monopoly union assumption, maximizing $V(w; \bar{r}(J))$ over w then determines the optimal union wage rate. This is wage rate denoted $w(\bar{r}(J))$.

In order to conduct the analysis of membership we start with a result concerning the optimal wage for the worker with the highest reservation wage. We assume r_m is just equal to the wage set by the firm (it cannot be higher or else the worker would never had joined the firm), and then show that the optimal wage of worker *m* is strictly greater than the firm-determined wage. This is an important result for the analysis since it has been shown in Section 2 that the optimal wages can be ranked as in (6). If there is to be any motive for the formation of the union a necessary condition is that *if m alone formed a union* $w_m^* > w^c$. Otherwise no worker would have any incentive for trying to increase the wage.

Lemma 1. *Define* $w_m^* \equiv \arg\max_{\{w\}} \dfrac{n(w)}{m} U(w-c) + \left[1 - \dfrac{n(w)}{m}\right] U(r_m)$. *Then*

$$w_m^* > w^c. \tag{10}$$

The proof of Lemma 1, and all other results, is in the Appendix.

This result allows the statement of a sufficient condition for the union to have a positive membership.

Proposition 1. *If $V^*(r_m) > V(w^c; r_m)$, the equilibrium membership of the union must be positive.*

The importance of Proposition 1 is that it establishes an individually-rational motive can exist for a worker to join a union. Through their solo membership of the union, worker *m* can set its agenda and derive a benefit from the ensuing increase in wage. Note that it does not say that *m* will be a member in equilibrium, but only that if no-one else joins then *m* will have an incentive to do so. Many of the arguments that follow can now be motivated by considering how the other workers would react to worker *m* joining the union. Worker *m* will set a union reservation wage r_m and achieve a wage of w_m^*. This wage is higher than any other worker wishes to see - all would prefer a lower wage but a higher probability of employment. The only way they can affect the outcome is to join the union and use their vote to counteract that of worker *m*. This incentive is strongest for the worker furthest from *m* in reservation wage space. Hence worker 1 now has strong motivation to also join the union. This reasoning provides the grounds of the argument that eventually shows that equilibrium union membership is made up of workers from the two extremes of the reservation wage distribution.

One point worth noting is that we have assumed that the single-worker union has bargaining power with the firm. In one sense this is irrelevant since the equilibrium membership we establish below always consists of more than one member so *m* will never be the only worker choosing the join strategy. In another, this assumption could be justified if the worker joins an extant union with pre-existing bargaining power. It will also be argued in Section 5 that the basic results of the analysis still apply when union bargaining power is dependent on the number of members. There will always be some minimum degree of power that will allow the wage to be raised sufficiently to make union formation beneficial.

4. Equilibrium

The previous section has derived a sufficient condition for union membership to be positive at any Nash equilibrium of the game played between workers. We now proceed to a characterization of an equilibrium membership. Equilibrium in this context is taken to mean that no further workers have an incentive to join the union and none of its members wish to leave.

In order to proceed some further notation and definitions need to be introduced. The first task is to define the concept of a *membership benefit function*. Consider a union with membership denoted by the set J and reservation wage $\bar{r}(J)$. If worker k joins the union the reservation wage moves to $\bar{r}(J \cup k)$. Defining

$$s = \bar{r}(J \cup k) - \bar{r}(J), \tag{11}$$

the membership benefit function for k is given by

$$\begin{aligned} b(\bar{r}(J), s; r_k) &= \frac{n(w(\bar{r}(J \cup k)))}{m}\left[U(w(\bar{r}(J \cup k)) - c; r_k) - U(r_k)\right] \\ &\quad - \frac{n(w(\bar{r}(J)))}{m}\left[U(w(\bar{r}(J)); r_k) - U(r_k)\right] \\ &= \frac{n(w(\bar{r}(J) + s))}{m}\left[U(w(\bar{r}(J) + s) - c; r_k) - U(r_k)\right] \\ &\quad - \frac{n(w(\bar{r}(J)))}{m}\left[U(w(\bar{r}(J)); r_k) - U(r_k)\right]. \end{aligned} \tag{12}$$

Hence $b(\bar{r}(J), s; r_k)$ measures the benefit of membership for k when the reservation wage of the union before k takes up membership is $\bar{r}(J)$ and the membership of k shifts it by amount s. Clearly, $b(\bar{r}(J), s; r_k)$ can be positive or negative. It is clearly negative when $\bar{r}(J) = r_k$ since a membership fee is paid and no improvement in situation is attained. Furthermore, if $\bar{r}(J) < r_k$ then $b(\bar{r}(J), s; r_k)$ can only be positive when $s > 0$. It must also be increasing in s for all s such that $\bar{r}(J) + s \leq r_k$ [4]. The converse statements can be made when $\bar{r}(J) > r_k$.

Let the mean reservation wage of the entire set of workers be μ. The set of reservation wages is said to be *symmetric about the mean* whenever $r_{m-i+1} - \mu = \mu - r_i$ for all i with $r_{m-i+1} - \mu > 0$. The uniform distribution and any discrete approximation to the normal are examples of distributions that satisfy this condition. For the benefit function, term it *monotonic* if $b(\bar{r}(J), s; r_k)$ is decreasing in $\bar{r}(J)$ for $\bar{r}(J) < r_k$ when $s > 0$ and increasing in $\bar{r}(J)$ for $\bar{r}(J) > r_k$ when $s < 0$. Finally, consider a union reservation wage $\bar{r}(J)$ and two workers with reservation wages r_i and r_j satisfying

$$r_i = \mu + \rho, r_j = \mu - \rho, \ \rho > 0, \tag{13}$$

and

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[4] Of course, s is determined endogenously by J and r_k. But it helps to think of it as exogenous for applying the argument.

$$\bar{r}(J) = r_i - \sigma, \bar{r}(J) = r_j + \sigma, \sigma > 0 \,. \tag{14}$$

The membership benefit function is said to be *symmetric* if

$$b(r_i - \sigma, s, \mu + \rho) = b(r_j + \sigma, -s, \mu - \rho), \tag{15}$$

for all *s*. The interpretation of this condition is that the benefit of starting with a union reservation wage σ below the ideal for *i* and shifting it *s* closer to r_i is equal to starting with it σ above the ideal for *j* and moving it *s* down when r_i and r_j are an equal distance above and below the mean respectively.

Now consider some membership of the union. Separate the membership into those with a reservation wage below the mean for the entire set of workers and those with a wage above the mean. A worker with a wage equal to the mean is added to the side with most members. If the group with reservation wages below the mean is in equal in number to those in the group with wage above the mean, we say the membership is *balanced*. If the groups are not equal in number, it is *unbalanced*. When the group below the mean is larger the union is termed *unbalanced to the left* and, when it is smaller, *unbalanced to the right*. *Complete coverage* is said to arise when all of the initial set of workers join the union. *Incomplete coverage* arises when only a strict subset join. The two groups formed be the separation can (i) either be connected or can have gaps, where a gap is one or more workers who are non-members located in reservation space between two workers who are members (or between a member and the end point of the distribution of reservation wages), (ii) be balanced or unbalanced.

The following theorem can now be proved.

Theorem 1. *If reservation wages are symmetrically distributed about the mean and the benefit function is monotonic and symmetric, an unbalanced membership cannot be an equilibrium.*

The next result proves that there is an equilibrium with a positive and balanced membership.

Theorem 2. *If reservation wages are symmetrically distributed about the mean, the benefit function is monotonic and symmetric and* $V^*(r_m) > V(w^c; r_m)$*, then there is an equilibrium where the number of union members is positive and the membership is balanced and connected.*

The proof of Theorem 2 is constructive. It adopts an entry process in which entry is taken up sequentially and shows that when the entry process ceases, the membership structure is a Nash equilibrium. This membership consists of two equally sized groups which contain the workers with the *k* lowest reservation wages from the population and the workers with the *k* highest reservation wages. This process follows the logic developed earlier: worker *m* enters first, then 1 enters to counteract their effect. Next either *m* - 1 or 2 will join, followed by the other to counteract them. This is repeated until no further worker finds it beneficial to join, leaving a union with two balanced but disjoint halves. The theorem shows that positive membership can exist with an individually-rational motive for joining the union under a conventional utility function. Hence the free-riding issue in union membership can be overcome when membership affects the union's behavior.

Theorem 2 characterizes an equilibrium membership but does not show that it is the only equilibrium. The analysis now proceeds towards a proof of uniqueness. It is assumed from this point onwards that $V^*(r_m) > V(w^c; r_m)$. Define by the *endpoints* the worker with the highest reservation wage in the low reservation group and the worker with the lowest reservation wage in the high reservation wage group. The endpoints of the membership identified in Theorem 2 are denoted r_{k*} (highest reservation wage in low group) and $r_{\ell*}$ (lowest reservation wage in high group). Theorem 3 shows that if the distribution of reservation wages is uniform, then these are the only possible equilibrium endpoints and the membership can have no gaps.

Theorem 3. *If reservation wages are uniformly distributed and the benefit function is monotonic and symmetric, the equilibrium is unique.*

The final result of this section relates to the possibility of complete coverage.

Theorem 4. *Assume reservation wages are symmetrically distributed about the mean. If the initial number of workers, m, is odd the union cannot have complete coverage.*

The value of this theorem is that it gives a sufficient condition for there to be incomplete coverage at equilibrium. That is, some of the workers will rationally choose not to join the union whereas some rationally choose to join.

5. Discussion and extensions

In this section we first give two examples designed to illustrate the reasoning and the role of the assumptions. We then note a number of extensions that could be made and discuss how the analysis can be modified to cope with them. These show that the results developed in the previous sections can be extended in a number of ways without affecting the main message of the paper.

The theorems have demonstrated that a union drawing its membership from the two extremes, with all non-members lying in between, is the unique equilibrium under certain restrictions. We now consider two examples that offer different perspectives on this result.

Example 1. The first example provide an illustration of the theorems by deriving the equilibrium for a simple situation with a small number of workers. Consider four workers (so m = 4) with reservation wages 2, 4, 6, and 8, and utility function

$$U = \frac{n}{m} b[w - \delta c] + \left[1 - \frac{n}{m}\right] br \, , \tag{21}$$

where δ is a dummy variable with value 1 for a member, 0 for a non-member. The level of employment at wage w is $m + w^c - w$ and, given the reservation wages, $w^c = 8$. For b = 1, calculations show that for $c \in [0.072, 1.608]$, a membership of $J = (2,8)$ is a Nash equilibrium with a wage of 9. The uniqueness of this equilibrium can be verified by directly testing all 14 alternative membership structures.

Example 2. The second example shows that if the distribution of reservation wage is not assumed to be symmetric, an equilibrium can arise where there are more than two connected sets of workers in the union. Consider a workforce of 5 and assume that 1, 3 and 5 are members. As shown in figure 1, if workers 2 and 4 are close in reservation wage to 3 then this can be an equilibrium. 3 has no incentive to leave since this would entail a significant move in the union preference towards that of 5, nor have 2 and 4 sufficient incentive to join. What is happening is that, with 3 the median voter, the addition of either 2 or 4 changes the reservation wage so little that the gain does not offset the membership cost. Therefore they do not join. The message of this example is that disconnected unions will arise if workers are in clumps in reservation wage space. As 2 and 4 are separated from 3, the gain from then joining increases. However, note that if all five workers are symmetrically spaced, 3 will have an incentive to leave: the union reservation wage remains unchanged when they do. This is the role that the uniform distribution assumption plays in Theorem 3.

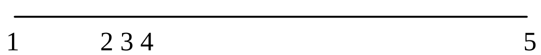

Figure 1

One possible criticism of the model is that it may be applicable only to situations with small numbers of workers. There are two points to make in this context. Firstly, although many union may be large in terms of numbers of members, it should be remembered that they will be making many wage agreements that cover smaller sub-groups of their workers. These will typically be at individual employer or even at plant level and it is the workers at this stage who will vote over any proposed offer. Secondly, the quantitative size of the effects that we discussed can be surprisingly large. To see this consider an example of a workforce of 100 who have reservation wages uniformly distributed between \$201 and \$400. After fifty members have joined the union (25 from each extreme), the union reservation wage is \$300.5. The addition of a fifty first member succeeds in moving the reservation wage to either \$253 or \$348, in either case this is a move of \$47.5. Even after this shift is worked through the bargaining process, it can potentially offset a significant membership cost.

Turning now to extending the assumptions upon which the analysis was based, the most immediate of these that warrants discussion was that the power of the union was independent of the size of its membership and the adoption of the monopoly union model. There are two defenses that can be made here. Firstly, the former assumption has already been motivated on the grounds that the dynamic process is just a form of constructive proof rather than a literal description of entry. In this respect, all that eventually matters is that the union has the specified degree of power with its equilibrium membership. Secondly, the assumption can be justified by appeal to the fact that the union formed in the firm we study is just one part of a much larger union covering many such firms. Even if it has only one member in the firm, a large union can have power in the firm through the threat of outside action. Such reasoning provides some justification for the formation of large unions encompassing numerous firms.[5]

It is also possible to argue that the assumption can be modified without affecting the results of the analysis. To see this, consider the structure of the argument that was employed. The motive for worker *m* initially joining the union was that they could then set the objective of the union and move the wage towards their favored outcome. The assumption on union power ensured that worker *m* succeeded in moving the wage exactly to their most preferred point but the central point of the argument was that any increase in the wage from the initial position benefited worker *m* more than any other worker. Consequently, if a one-member union can only secure a limited increase in the wage, it will still be worker *m* who joins first. Similarly, once

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[5] Though it is interesting to note that Vannetelbosch (1997) provides evidence to show that efficiency losses are lower if unions bargain at the firm level than at the industry level.

worker m has joined, it will be worker 1 who joins next: the same monotonicity of payoff is still present. The same reasoning can also be applied to the monopoly union assumption. It provides clarity for the analysis but it is the monotonicity that is fundamental and this will be preserved under most alternative scenarios about union-firm bargaining.

One way to formally accommodate both of these points is to adopt a generalized Nash bargaining framework. If the power of the union to influence the bargain is assumed to take the form $\mu = \mu(m)$, where m is the number of members, the maximand for the bargain can be written

$$V = U^{\mu(m)} \Pi^{1-\mu(m)}. \tag{22}$$

The wage that emerges will then be a trade-off between the objective of the union and those of the firm. Our arguments in the previous section can then be interpreted as holding when $\mu \equiv 1$. Since worker m, and the other high-reservation wage workers who subsequently join the union, will achieve less of an increase in the wage in this framework, this form of variation in union power will tend to lead to proportionately more high-reservation wage workers joining than under the fixed power assumption. This would appear to be the only distinction between the two scenarios.

These observations raise the question of why large unions, covering workers in many firms, exist at all if local unions are able to achieve monopoly benefits for their members. An answer to this can be found in the increased power that a larger union has in negotiations with the firm.

The next point that needs addressing is our interpretation of the Median Voter Theorem. The theorem is clear on what the outcome should be if there are an odd number of voters but does not offer such precise guidance when the number of voters is even. In the latter case, we have taken the outcome to fall midway between the preferences of the two voters straddling what would be the median. It should be clear that this assumption does not play any role in the reasoning above. All that matters is that the outcome lies *somewhere* between the two sets of preferences. For example, a kind of partial adjustment rule could be adopted under which it would always lie on the side closer to its old value. In any case, the nature of the argument would be unaffected and the same conclusions would emerge.

The cost of union membership has been assumed throughout to be given at some exogenously fixed value. A more satisfactory alternative might be to assume the existence of some economies of scale so that costs are a decreasing function of membership. This modification would also not affect the nature of our conclusions since it would still preserve the monotonicity property of benefits to union membership. All that will be modified is the extent of union membership, with a decreasing cost tending to lead to a larger union in equilibrium.

6. Conclusions

This paper was motivated by the observation that free-riding on a union contract is only feasible if the union bargains for a contract which exactly reflects the preferences of the outsiders. We argued that a natural assumption in many instances is rather that workers have different preferences over the wage-employment trade-off. Joining the union serves then to "buy" a vote on the union bargaining strategy.

Recognition of the possibility that workers may have different reservation wages leads to the interesting question of which types of workers will join the union. It was shown that although there may be multiple equilibrium a general property of these equilibrium is that they will typically be characterized by union members having preferences which are relatively extreme, and drawn from both ends of the distribution. The union will approximately represent the interests of the average worker, although the average worker will not actually join. This also has the important implication that if workers have different abilities observing the union membership decision conveys no signal about whether the particular worker's ability is above or below the average.

We have presented these results as an explanation for membership of a trade union. As already noted, we would also argue that they have implications for other decisions that seemingly involve a conflict between individual and group rationality. These extensions remain to be explored in future research.

Appendix

Proof of Lemma 1.

From (2)

$$\frac{\partial V(w;r_m)}{\partial w}\Big|_{w=w^c} = \frac{n'(w^c)}{m}\left[U(w^c-c)-U(r_m)\right]+\frac{n(w^c)}{m}U(w^c-c)>0, \qquad \text{(A1)}$$

where the inequality follows from the fact that $\frac{n'(w^c)}{m} < 0$ and that $w^c = r_m$ implies $U(w^c - c) < U(r_m)$. ||

Proof of Proposition 1.

Assume an equilibrium in which all workers choose the strategy of not joining the union. This gives *m* a payoff of $V(w^c; r_m)$. Holding other workers' decisions constant, a deviation by *m* to the strategy of join would lead to the wage rate $w(r_m)$ since *m* will be the only member and through majority voting $\bar{r}(m) = r_m$. The worker's payoff is $V(w(r_m), r_m) = V^*(r_m)$ since $w(r_m) = w_m^*$. If $V^*(r_m) > V(w^c; r_m)$, this deviation raises *m*'s payoff. Hence the initial strategy choices could not be an equilibrium. ||

Proof of Theorem 1.

The proof proceeds by first showing that a connected and unbalanced membership cannot be an equilibrium and then extending the argument to cover an unbalanced membership with gaps.

i. *Membership connected and unbalanced*. First consider the case in which the large group has one more member than the smaller. Assume that the larger group lies above the mean so membership is unbalanced to the right. Thus the equilibrium membership structure is of the form $J = (1, \ldots, k, m - \ell, \ldots, m)$ with $\ell = k$ and $\bar{r}(J) = r_{m-\ell}$. If worker $m - \ell$ were not in the union, its membership would become $J' = (1, \ldots, k, m - \ell + 1, \ldots, m)$. Since $m - \ell$ chose to join this shows

$$b\left(\frac{r_k + r_{m-\ell+1}}{2}, r_{m-\ell} - \frac{r_k + r_{m-\ell+1}}{2}; r_{m-\ell}\right) > 0. \tag{A2}$$

(A2) can be written in the form

$$b\left(r_{m-\ell} - \sigma^0, s^0; \mu + \rho^0\right) > 0, \tag{A3}$$

where $\sigma^0 = r_{m-\ell} - \frac{r_k + r_{m-\ell+1}}{2} = r_{m-\ell} - \mu$, $s^0 = r_{m-\ell} - \mu$ and $\rho^0 = r_{m-\ell} - \mu$.

Now consider the strategy choice of worker *k*+1 when the union has membership *J*. Joining will be beneficial for them if

$$b\left(r_{m-\ell}, \frac{r_k + r_{m-\ell}}{2} - r_{m-\ell}; r_k\right) > 0, \tag{A4}$$

or

$$b\left(r_k + \sigma^1, -s^1; \mu - \rho^1\right) > 0\,, \tag{A5}$$

where $\sigma^1 = r_{m-\ell} - r_k$, $s^1 = r_{m-\ell} - \mu$ and $\rho^1 = r_k - \mu$.

Since $\rho^1 = \rho^0$, $s^1 = s^0$ and $\sigma^1 = 2\sigma^0$, (A3), symmetry of the benefit function and the fact that $b\left(\bar{r}(J), s, r_k\right)$ is increasing in $\bar{r}(J)$ when $\bar{r}(J) > r_i$ imply that

$$b\left(r_k + 2\sigma^0, -s^0; \mu - \rho^0\right) > 0\,. \tag{A6}$$

This establishes that if it is beneficial for worker $m - \ell$ to join the union, then it will also be so for k+1. A membership that is unbalanced to the right cannot therefore be an equilibrium. An identical argument applies if it is assumed that the equilibrium membership that is unbalanced to the left.

Now assume that the larger group is at least two members larger. Consider the worker with the lowest reservation wage who is not already a member of the union. Denote their reservation wage by r'. By construction the worker with reservation wage $r'' = \mu + [\mu - r']$ is in the larger group and a member of the union (call this worker the *symmetric partner* of r'). Hence the benefit for r'' must be positive so

$$b\left(\bar{r}(J / r''), \bar{r}(J) - \bar{r}(J / r''); r''\right) \equiv b\left(r'' - \sigma, s; \mu + \rho\right) > 0\,, \tag{A7}$$

where $\sigma = r'' - \bar{r}(J / r'')$, $s = \bar{r}(J) - \bar{r}(J / r'')$ and $\rho = r'' - \mu$. Symmetry of the benefit function then implies $b\left(r' + \sigma, -s; \mu - \sigma\right) > 0$. Since $\bar{r}(J) > r' - \sigma$,

$$b\left(r' - \sigma, -s; \mu - \sigma\right) < b\left(\bar{r}(J), -s; \mu - \sigma\right), \tag{A8}$$

and using $\left|\bar{r}(J \cup r') - \bar{r}(J)\right| > |s|$

$$0 < b\left(\bar{r}(J), -s; \mu - \sigma\right) < b\left(\bar{r}(J), \bar{r}(J \cup r') - \bar{r}(J); \mu - \sigma\right), \tag{A9}$$

so that r' will join the union. The initial membership is therefore not an equilibrium.

ii. *Membership not connected and unbalanced*. The previous argument can be applied to this case too with some modification. Proceed as before by assuming that the larger group lies above the mean (the same argument can be modified if the larger group is below the mean). Consider the worker with the lowest reservation wage who is not already a member of the union. Let them have reservation wage r'. If their symmetric partner is a member of the union, then the remaining arguments of (i) can be applied directly since they did not rely on connectedness. If the symmetric partner of r' is not a member of the union, move to the non-member with the next highest reservation wage. Proceed in this way until the first non-member is reached whose symmetric partner is a member. Such a non-member must exist. The arguments of (i) are then applied to show that this worker would want to join the union. ||

Proof of Theorem 2

Proposition 1 has shown that condition (i) guarantees membership must be positive at any equilibrium. Equilibrium membership is now characterized by a constructive proof. The construction employs an entry process for the union in which new members enter sequentially. It is then proved that the termination point of this process is an equilibrium. The details of the process are as follows: At each stage, workers who have not joined the union evaluate their benefit from joining. The worker with the greatest positive benefit then joins. This is repeated until no non-members have a positive benefit from membership.

Lemma A1. *The first member of the union will be worker m.*

Proof. If worker *m* joins the union, the union wage rate will be $w(r_m)$ since $\bar{r}(m) = r_m$ and the workers payoff is $V(w(r_m), r_m) = V^*(r_m)$ since $w(r_m) = w_m^*$. Their benefit from being the single person to join is then $V^*(r_m) - V(w^c; r_m)$ which is positive under the assumed conditions. The same reasoning shows that if any other worker, *i*, were to be the single member their benefit would be $V^*(r_i) - V(w^c; r_i)$. From the definitions of utility

$$[V^*(r_m) - V(w^c; r_m)] - [V^*(r_i) - V(w^c; r_i)]$$

$$= \left[1 - \frac{n(w^c)}{m}\right][U(r_m) - U(r_i)] + [V^*(r_m) - V^*(r_i)] > 0, \qquad \text{(A10)}$$

where the inequality in (A10) follows from the fact that $r_m > r_i$ and the ranking of maximum values in (6). The inequality in (A10) must hold for all $i \neq m$. ||

Lemma A2. *Given a union consisting of worker m, the maximal benefit of being the next member will be obtained by worker 1. They will join if* $V(w(\bar{r}(1,m)) - c; r_1) > V(w_m^*; r_1)$.

Proof. Prior to the second worker joining, the wage determined by the union is given by w_m^* and $\bar{r}(m) = r_m$. If worker 1 joins, then $\bar{r}(1,m) = \frac{r_m + r_1}{2}$ and the wage is $w(\bar{r}(1,m))$. Similarly, if any other worker *i* joins then the union reservation wage is $\bar{r}(i,m) = \frac{r_m + r_i}{2}$ and the wage rate equals $w(\bar{r}(i,m))$. Employing these definitions, the gain to worker 1 will exceed that to any other worker *i* if

$$V\left(w(\bar{r}(1,m))-c;r_1\right)-V\left(w_m^*,r_1\right)>V\left(w(\bar{r}(i,m))-c;r_1\right)-V\left(w_m^*,r_i\right),\ \forall i\neq 1,m\,. \quad \text{(A11)}$$

Using (2) and simplifying, (A11) can be reduced to

$$\frac{n(w(\bar{r}(1,m)))}{m}\left[U\left(w(\bar{r}(1,m))-c\right)-U(r_1)\right]-\frac{n(w(\bar{r}(i,m)))}{m}\left[U\left(w(\bar{r}(i,m))-c\right)-U(r_i)\right]$$

$$+\ \left[U(r_i)-U(r_1)\right]\left[1-\frac{n(w(\bar{r}(m)))}{m}\right]>0\,. \qquad \text{(A12)}$$

Since $r_i > r_1$, the third term of (A12) is positive and the inequality can be established by showing that $g(\rho)=\dfrac{n\left(w\left(\frac{r_m+\rho}{2}\right)\right)}{m}\left[U\left(w\left(\frac{r_m+\rho}{2}\right)-c\right)-U(\rho)\right]$ is decreasing in ρ.

Taking the derivative of $g(\rho)$ gives

$$\frac{dg(\rho)}{d\rho}=\frac{n'w'}{2m}\left[U\left(w\left(\frac{r_m+\rho}{2}\right)-c\right)-U(\rho)\right]+\frac{n}{m}\left[U'\left(w\left(\frac{r_m+\rho}{2}\right)-c\right)\frac{w'}{2}-U'(\rho)\right]. \quad \text{(A13)}$$

To establish that this is negative, consider the effect of a variation in the wage upon the maximal utility. Since $\frac{r_m+\rho}{2}>\rho$ it follows that $w\left(\frac{r_m+\rho}{2}\right)>w_\rho^*$ and hence

$$\frac{dV(\rho)}{d\rho}\Big|_{w=w\left(\frac{r_m+\rho}{2}\right)}=\frac{n'}{m}\left[U\left(w\left(\frac{r_m+\rho}{2}\right)-c\right)-U(\rho)\right]+\frac{n}{m}U'\left(w\left(\frac{r_m+\rho}{2}\right)-c\right)<0\,. \quad \text{(A14)}$$

Noting that (4) establishes $w'\geq 0$, evaluating (A13) using (A14) shows $\frac{dg}{d\rho}<0$, which proves the first part of the lemma. The second part follows directly from the definitions. ||

Lemma A3. If the union membership is composed of the set of workers $J=\{1,\ldots,k,m-\ell,\ldots,m\}$, with $\ell=k-1$, then $k+1$ will obtain the greatest return from being the next member if

$$V\left(w(\bar{r}(J\cup k+1))-c,r_{k+1}\right)-V\left(w(\bar{r}(J))-c,r_{k+1}\right)$$

$$>V\left(w(\bar{r}(J\cup m-\ell-1))-c,r_{m-\ell-1}\right)-V\left(w(\bar{r}(J))-c,r_{m-\ell-1}\right), \qquad \text{(A15)}$$

otherwise the greatest return will be obtained by $m-\ell-1$.

Proof. Denote by *J* the existing set of union members and by *J'* the set after the addition of another member. To prove the lemma, it is necessary to show that the gain from membership is decreasing in *r* if $r < \bar{r}(J)$ and increasing in *r* if $r > \bar{r}(J)$.

Given that $\ell = k - 1$, the next member of the union becomes the median voter and their preferences become the union's preferences. Hence the gain for a worker with reservation wage *r* from joining the union is given by

$$G(r) \equiv \frac{n(w(r))}{m} U(w(r) - c) + \left[1 - \frac{n(w(r))}{m}\right] U(r)$$

$$- \frac{n(w(\bar{r}(J)))}{m} U(w(\bar{r}(J))) + \left[1 - \frac{n(w(\bar{r}(J)))}{m}\right] U(r). \quad \text{(A16)}$$

The effect of an increase in *r* is

$$\frac{\partial G}{\partial r} = \frac{n' w'}{m} [U(w(r) - c) - U(r)] + \frac{nw'}{m} U'(w(r) - c)$$

$$+ \left[1 - \frac{n(w(r))}{m}\right] U'(r) - \left[1 - \frac{n(w(\bar{r}(J)))}{m}\right] U'(r), \quad \text{(A17)}$$

but since $w(r)$ is optimal for *r*, application of (3) reduces (A17) to

$$\frac{\partial G(r)}{\partial r} = U'(r) \left[\frac{n(w(\bar{r}(J)))}{m} - \frac{n(w(r))}{m}\right]. \quad \text{(A18)}$$

Now if $r > \bar{r}(J)$ then $n(w(\bar{r}(J))) > n(w(r))$ and $G(r)$ is decreasing in *r*. The converse reasoning applies if $\bar{r}(J) > r$. This establishes the lemma. ||

Lemma A4. If the union membership is composed of the set of workers $J = \{1, \ldots, k, m - \ell, \ldots, m\}$, with $\ell = k$, then $k + 1$ will obtain the greatest return from being the next member. Conversely, if $\ell = k - 2$ then $m - \ell - 1$ will obtain the greatest return from being the next member.

Proof. Consider the case of $\ell = k$. From the structure of the membership it follows that ℓ is the median voter and so $\bar{r}(J) = r_\ell$. Hence for all remaining non-members, $r_i < \bar{r}(J) = r_\ell$. Denoting by J' membership after the addition of a further member, the gain from a worker of reservation wage *r* joining the union is given by

$$G(r) \equiv \frac{n(w(\bar{r}(J')))}{m} U(w(\bar{r}(J')) - c) + \left[1 - \frac{n(w(\bar{r}(J')))}{m}\right] U(r)$$

$$-\frac{n(w(r_\ell))}{m}U(w(\bar{r}(r_\ell)))+\left[1-\frac{n(w(r_\ell))}{m}\right]U(r), \tag{A19}$$

where $\bar{r}(J')=\frac{r_\ell+r}{2}$. Differentiation of (A19) gives

$$\frac{\partial G(r)}{\partial r}\equiv\frac{n'w'}{2m}\left[U(w(\bar{r}(J'))-c)-U(r)\right]+\frac{n'w'}{2m}U'(w(\bar{r}(J'))-c)$$

$$+\left[1-\frac{n(w(\bar{r}(J')))}{m}\right]U'(r)-\left[1-\frac{n(w(\bar{r}(J)))}{m}\right]U'(r). \tag{A20}$$

Since $r<r_\ell$, $w(r_\ell)>w(\bar{r}(J'))$. Hence

$$\left[1-\frac{n(w(\bar{r}(J')))}{m}\right]U'(r)-\left[1-\frac{n(w(\bar{r}(J)))}{m}\right]U'(r)<0. \tag{A21}$$

Finally, as $w(\bar{r}(J'))>w_r*$ it follows from (3) and (A21) that

$$\frac{\partial G(r)}{\partial r}<0. \tag{A22}$$

As the benefit is decreasing in *r*, it follows that if $\ell=k$, then the greatest benefit from being the next member will accrue to $k+1$.

Precisely the converse argument applies when $\ell=k-2$ and $G(r)$ can be shown to be increasing in *r*. ||

These results allow the entire entry process to be described. First, worker *m* will assess if they have a positive benefit from initially forming a union. If the benefit is positive, then the union will be established. Given the union is formed, the worker with the greatest benefit from joining next is worker 1. Again, they will join if their benefit is positive. Lemmas A3 and A4 can then be repeatedly applied. Given that 1 and *m* are in the union, then Lemma A3 applies and the greatest benefit from membership is obtained by either worker 2 or worker *m* - 1. If either of these joins, Lemma A4 then applies. If 2 (*m* - 1) had joined, *m* - 1 (2) will have the greatest gain from membership. Given that they choose membership, Lemma A3 applies again. This process of entry will continue until either there remains no non-member for whom the net benefit of membership is positive or until the entire workforce have joined the union.

Applying Theorem 1 shows that the process cannot terminate at a position with an unbalanced membership. Therefore it must terminate with a balanced membership and either complete or incomplete coverage. It will now be proved that if it terminates with a balanced membership an equilibrium is reached. The same argument then shows that if it terminates with complete coverage an equilibrium is again reached. Therefore there exists an equilibrium as claimed.

Denote the membership at the termination point by $J = \{1,\ldots,k,m-\ell,\ldots,m\}$, with $\ell = k-1$. Assume that *k* is the last entrant (the argument is easily modified to apply when $m-\ell$ is last). Since *k* has just entered, all workers *i* < *k* will wish to remain in the union: if they remain members the union reservation wage is $\frac{r_{m-\ell}+r_k}{2}$ and if they leave it is $r_{m-\ell}$. Since the benefit of *k* was positive with this choice, it must also be positive for all workers *i* < *k*.

It must now be shown that $m-\ell$ will not leave once *k* has joined (and be extension $m-\ell+1,\ldots,m$ will not want to leave). If $m-\ell$ remains a member, $\bar{r}(J)$ $\frac{r_{m-\ell}+r_k}{2}$ if they leave it becomes r_k. Whether they wish to leave is equivalent to whether they would join if the membership were given by $\{1,\ldots,k,m-\ell+1,\ldots,m\}$. They will join in these circumstances if $b(r_k, s^0; r_{m-\ell}) > 0$. To show that this inequality is satisfied, consider the position when $m-\ell$ chose to join: the union was given by $\{1,\ldots,k,m-\ell+1,\ldots,m\}$ with reservation wage $\frac{r_{m-\ell+1}+r_{k-1}}{2}$. Since $m-\ell$ joined, it follows that $b\left(\frac{r_{m-\ell+1}-r_{k-1}}{2}, s^1; r_{m-\ell}\right) > 0$. Given that $s^0 = \frac{r_{m-\ell+1}+r_k}{2} - r_k$ and $s^1 = r_{m-\ell} - \frac{r_{m-\ell+1}+r_{k-1}}{2}$, the assumption of symmetry implies $s^0 = s^1$. Noting $\frac{r_{m-\ell+1}+r_{k-1}}{2} > r_k$ and using the assumption that $b(\bar{r}(J), s; r_k)$ is decreasing in $\bar{r}(J)$ for $\bar{r}(J) < r_k$ when *s* > 0, the fact that $b\left(\frac{r_{m-\ell+1}+r_{k-1}}{2}, s^1; r_{m-\ell}\right) > 0$ implies $b(r_k, s^0; r_{m-\ell}) > 0$. So $m-\ell$ will not want to leave the union which proves that the entry process terminates with an equilibrium membership. ||

Proof of Theorem 3

The proof is in two steps. The first part shows that any equilibrium must have a membership with endpoints r_{k*} and $r_{\ell *}$. Given this, the second part establishes uniqueness by showing there can be no gaps in an equilibrium membership.

Assume an equilibrium membership in which the left endpoint is given by $r_k \le r_{k*}$ and the right endpoint by $r_\ell \ge r_{\ell *}$ (with at least one strict inequality). But then the argument of Lemma A4 can be applied to show that the benefit from membership is positive for at least one of r_{k*+1} and $r_{\ell *-1}$. The initial position cannot, therefore, have been an equilibrium. Alternatively, assume an equilibrium where $r_k \ge r_{k*}$ and $r_\ell \le r_{\ell *}$ (with at least one strict inequality). Then it follows from Theorem 1 that the benefit of entry must be negative for at least one of r_k and r_ℓ. Hence, this cannot be an equilibrium.

This leaves only the cases $r_k > r_{k*}$, $r_\ell > r_{\ell*}$ and $r_\ell < r_{k*}$, $r_\ell < r_{\ell*}$. Consider the first of these. Since any potential equilibrium membership must be balanced, the set of members with reservation wages below the mean must have at least one gap. Now assume that the left endpoint is given by r_{k*+1} and the first gap occurs at r_{k*}. Since the argument to be given holds even more strongly for a higher left endpoint and a lower first gap, it is sufficient to consider only this case. The assumed membership structure will not be an equilibrium if

$$b\left(\frac{r_{k*+1}+r_\ell}{2}, r_{k*+1}-\frac{r_{k*+1}+r_\ell}{2}, r_{k*}\right) > 0\,. \tag{A23}$$

It follows from Theorem 2 that

$$b(\mu, r_{k*}-\mu, r_{k*}) > 0\,. \tag{A24}$$

Since $\frac{r_{k*+1}+r_\ell}{2} > \mu$, a sufficient condition for (A24) to imply (A23) is that

$$r_{k*+1}-\frac{r_{k*+1}+r_\ell}{2} \le r_{k*}-\mu\,. \tag{A25}$$

Since $\mu = \frac{r_{k*}+r_{\ell*}}{2}$ (A25) is equivalent to

$$r_{k*+1}-r_{k*} \le r_\ell - r_{\ell*}, \tag{A26}$$

which is true when the distribution is uniform since $\ell \ge \ell*+1$.

To prove there are no gaps, assume worker $\ell*+1$ is not a member. From the fact that $\ell*$ is a member and that $r_{\ell*+1} > r_{\ell*}$

$$b(\mu, r_{\ell*}-\mu, r_{\ell*+1}) > b(\mu, r_{\ell*}-\mu, r_{\ell*}) > 0\,. \tag{A27}$$

Hence $\ell*+1$ will wish to join the union contradicting the assumption they were not a member. Symmetry can be used to extend this argument to all other workers. ||

Proof of Theorem 4.

Since the worker with the mean reservation wage can leave the union without affecting its reservation wage, complete coverage cannot be an equilibrium. ||

References

Akerlof, G. A., 1980. A theory of social customs of which unemployment may be one consequence. *Quarterly Journal of Economics* **95**, 749 - 775.

Booth, A.,1985. The free rider problem and a social custom theory of trade union membership. *Quarterly Journal of Economics* **100**, 253 261.

Booth, A., 1995 *The Economic Analysis of Trade Unions*. Cambridge University Press, Cambridge.

Bulkley, G. and Myles, G.D., 1997. Bargaining over effort. *European Journal of Political Economy* **13**, 375 - 384.

Bulkley, G., Myles, G.D., and Pearson, B.R., 2000. On the membership of decision-making committees. *Public Choice* (forthcoming).

Moreton, D.R., 1998. An open shop trade union model of wages, effort and membership. *European Journal of Political Economy* **14**, 511 - 527.

Mueller, D.C., 1989. *Public Choice II*. Cambridge University Press, Cambridge.

Naylor, R.A., 1989. Strikes, free riders and social customs. *Quarterly Journal of Economics*. **104**, 771 - 786.

Naylor, R.A., and Cripps, M.W., 1993. An economic theory of the open shop trade union. *European Economic Review* **37**, 1599 - 1620.

Olson, M., 1965. *The Logic of Collective Action*. Harvard University Press, Cambridge.

Stigler, G.J., 1974. Free riders and collective action: an appendix to theories of economic regulation. *Bell Journal of Economics* **5**, 359 - 365.

Vannetelbosch, V.J., 1997. Wage bargaining with incomplete information in a unionized Cournot oligopoly. *European Journal of Political Economy* **13**, 353 - 374.

Weiss, A., 1981. Job queues and layoffs in labor markets with flexible wages. *Journal of Political Economy* **71**, 393 - 410.