On the Membership of Decision-Making Committees

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Abstract: The decision of a committee is determined jointly by the voting process it adopts and the composition of its membership. The paper analyses the process through which committee members emerge from the eligible population and traces the consequences of this for the decisions of the committee. It is shown that the equilibrium committee will be composed of representatives from the extremes of the taste distribution. These extremes balance each other and the committee reaches a moderate decision. However, this mutual negation by the extremes is a socially wasteful use of time. Data from the UK House of Lords is used to illustrate these results.

Keywords: Committees, membership, voting

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1. Introduction

From the seminal work of Black (1958) onwards, the analysis of voting in committees invariably takes as its starting point a given set of committee members. Each member is characterized by their preferences and, possibly, by their voting weight in a hierarchical system. The equilibria of alternative methods of voting are then determined and the voting procedures evaluated according to criteria such as the efficiency of the chosen outcome and the potential for manipulability of the mechanism. Peleg (1984) and Miller (1995) provide comprehensive surveys of such results. What is missing from this analysis is an explanation of how the committee members come to be there in the first place. At some prior point a decision on whether or not to join the committee must have been made. In a population with heterogeneous preferences, the question of who joins the committee must be at least as important for the final outcome as the voting mechanism used by the committee.

Whether or not an individual joins a committee must depend upon what they perceive will be their influence upon the committee and the decisions that emanate from the committee. In short, committee membership and committee process are fundamentally interdependent and should not be treated in isolation. To fully understand the decisions that emerge from committees it is therefore necessary to model how the voting process interacts with the membership decision. Achieving this requires an investigation of the motives for membership. Some committees may have membership motivated by financial inducement but a great many do not. Our concern is this paper is with those committees where membership is a voluntary act. In such committees, membership actually carries a direct cost through the time that has to be expended on committee work.

To see what is meant by this, it is worth considering a number of everyday examples of voluntary committees. Anyone with experience of sporting and social clubs will know that they are usually run by the voluntary efforts of their members. Generally, a small subset of the total membership give-up some time to serve on the management committee. Although some professional services may be hired, most of the management effort is provided free of charge. Such management-by-volunteer is not confined to clubs, but is also witnessed, for example, in universities, charities, and professional associations, all of which have committee structures but no direct reward for membership. Further examples are school governors who denote their time to be the school's ultimate decision-making body and editorial boards for academic journals.

This phenomenon can also be witnessed in government. Local politicians are rewarded expenses for committee attendance but are not otherwise compensated for the time they expend. The functioning of local government relies on the willingness of individuals to forego work or leisure. The same is broadly true of peers acting in the House of Lords in the

UK. This example is developed further is Section 6. They receive a daily attendance allowance but the level of this falls well below what would be sufficient compensation. Similar examples abound in other political systems.

As well as illustrating the scenario under consideration, these examples introduce a further consideration: the act of voluntary membership seems to be in apparent contradiction to the principle of self-interest. Those joining the committee seem willing to bear a private cost in order to supply a public good; the public good being the provision of management for the club or university or government. In such situations, the theory of the voluntary provision of public goods (see Cornes and Sandler (1996) or Myles (1995)) would predict that free-riding would be the dominant strategy, so making the observed behaviour appear irrational. No doubt many of those who serve in the capacities described would view their behaviour as motivated by a selfless devotion to public welfare. In short, they might consider themselves to be just good citizens. This, however, may not be the complete story. In each of the examples given, incurring the cost of committee membership entitles the individual to a role in the decision making process which would otherwise be denied them. The management committee running the sports club and the local politicians preparing the municipality's budget plan are both making decisions that affect the entire set of club members or residents. The influence to affect these decisions cannot be ignored in determining who provides the time input.

What we wish to do in this paper is to consider the membership of committees jointly with the decision-making process in a way which accounts for the voluntary provision of a public good. By doing this we can consider the motives of those involved and trace the consequences of their individual decisions. At the heart of the analysis is a stylized model of the decision process in which individuals with heterogeneous preferences choose whether or not to join a committee on the basis of how doing so will affect the decisions made. In essence, the heterogeneity and the ability to influence the outcome may generate a private motive for what appears on the surface to be an act of public charity.

The form in which we phrase the stylized model is to consider the problem of forming a committee to make a decision concerning what quantity of a good should be provided. Although we talk in terms of choosing the quantity of the good, the model can be interpreted equally as refer to some qualitative aspect of a good of given size. The actual provision is costless for the committee at the point at which the decision has to be taken.[i] Membership of the committee though is costly for those who join. This occurs because the decision process involves the expenditure of time which is positively valued by all individuals, so participating in the decision-making process involves a private cost. The problem that we wish to address is who out of a given population of individuals will join the committee and what quantity of the good will be chosen? It can be seen that this model captures the essence of the issues that have been raised in the discussion.

As already noted, the basic assumption is that the population of potential committee members have heterogeneous preferences so they differ in the what they view as the optimal quantity of the good. If a subset of them choose to pay the private cost of joining the committee, then the choice of quantity is found by aggregating their preferences using the median voter theorem. The framework could incorporate other voting mechanisms, but we do not do so here. If no-one joins the committee, none of the good is provided. Essentially, this refers to the case in which the lack of managerial input causes a cessation in activity. The process described can be modelled as a game in which each member of the population has the choice of either joining or not joining, with a vote undertaken by those who join. The major issue is then to characterize the Nash equilibrium of this game and the resulting choice of provision.

The Nash equilibrium of the static one-shot game is characterized by using a constructive approach. In brief, an entry process is proposed at each stage of which membership of the committee is taken up by the individual with the greatest positive gain from so doing. The process is terminated when no further membership is taken up. Conditions are given under which this entry process terminates at a Nash equilibrium of the underlying game. Since equilibrium membership is positive, this shows that preference heterogeneity can overcome the free-rider problem. Besides establishing positive membership, approaching the problem *via* the entry process has the added benefit of leading directly to a very precise characterization of the structure of equilibrium membership.

The entry process reveals a surprising dynamics of membership. Imagine preferences placed in a one-dimensional line with zero at the left-hand end. The person with the furthest preferences to the right is the first to join the committee. Their extremism then provides an incentive for the person with the most extreme preferences to the left to join. The next member is from the extreme right, then the extreme left and so on. This process continues until an equilibrium membership is reached and this is drawn from the two extremes of the taste distribution. The non-members are those with moderate tastes. However, because of the action of the median voter theorem, the views of the two extremes will aggregate into a median quantity of provision. This allows those with moderate tastes to remain free-riders on the membership decision of others but those who join cannot free-ride. In this sense, some free-riding remains but is partial.

There are clear parallels between our analysis and that of Palfrey and Rosenthal (1983) on the voting paradox. Both consider the rationally for bearing a private cost in the provision of a public good and both find the solution in heterogeneity. Although the general issues in the two are also similar, the significant differences can be found in the details. Firstly, there are only two possible outcomes in the voting model corresponding to one or the other parties winning whereas we have a continuum of possibilities. Secondly, the heterogeneity in Palfrey and Rosenthal is limited since the voters support either one party or the other. Finally, these differences in assumptions are reflected in the nature of the equilibrium outcomes. Whereas pure strategy equilibria for the voting model only arise in the limiting cases of all voting or none voting, we show that the committee membership model always possesses a pure strategy equilibrium and that this is consistent with intermediate levels of membership.

Section 2 of the paper provides a description of the model and the assumptions employed. The entry process is studied in Section 3 and equilibrium characterized in Section 4. Section 5 shows how the assumptions on utility can be relaxed. Some evidence to support our results is presented in Section 6. Conclusions and a discussion of the implications of the results are given in Section 7. All proofs are placed in the Appendix.

2. Model and assumptions

Assume that a decision has to be taken on the quantity of a good to be provided. The quantity is determined by median voting of a committee of volunteers and is denoted by $G \geq 0$. The committee is formed from the set M of individuals in the population. A committee is denoted by C, $C \subseteq M$. If the equilibrium committee $C = \phi$ then the chosen level of $G = 0$. Underlying this restriction is the observation that the good requires a committee to organize its supply.

The set M consists of m individuals. Each of these individuals has preferences represented by the utility function

$$U_i = V_i(G) - \chi(c_i), \tag{1}$$

where $V_i(G)$ is differentiable and strictly concave. The component $\chi(c_i)$ represents the utility cost of committee membership. If i is member then $c_i = 1$ and $\chi(1) > 0$. If i is not a member then $c_i = 0$ and $\chi(0) = 0$. As a normalization, the representation of preferences is chosen so that $\chi(1) = 1$.

It is assumed that for each individual there is an optimal G_i^* defined by

$$V_i'\left(G_i^*\right) \leq 0, \ 0 \leq G_i^* \leq K < \infty, \tag{2}$$

with complementary slackness.[ii] The individuals are indexed $i = 1,\ldots,m$ with the index chosen so that

$$G_1^* \le G_2^* \le \ldots \le G_m^*. \tag{3}$$

To give the problem content, it is assumed that $G_m^* > 0$, so that a positive quantity of provision is demanded by at least one individual, and $G_m^* > G_1^*$ so that there is heterogeneity between individuals.

To permit comparabilities between the individual utility indicators[iii], it is assumed that $V_i(G_i)$ satisfies

$$V_i\left(G_i^* + \gamma\right) = V_j\left(G_j^* + \gamma\right) \forall \gamma, \forall\ i, j. \tag{4}$$

The interpretation of (4) is that the utility functions are horizontal translations of a basic utility. Furthermore, it follows from (4) that the utility indicator of consumer *i* can be written

$$V\left(G - \gamma_i\right) \equiv V_i(G),\ \gamma_i = G_i{}^*. \tag{5}$$

Now assume that a committee is formed. Denote the committee by *C* and a generic member of the committee by *c*. The members of the committee can be ranked by the value of G_i^*. Employing a superscript to denote the index within a given committee, the optimal values of supply for the committee members are ranked as $G^{1*} \le \ldots \le G^{c*} \le \ldots \le G^{C*}$. For committee *C*, the median voter rule then gives the level of supply as[iv]

$$\overline{G}(C) = \begin{cases} \text{median } \left\{G^{c*}, c \in C\right\} \text{if } \#C \text{ is odd} \\ \dfrac{G^{c*} + G^{c-1*}}{2}, \text{ where } c - 1 = \dfrac{\#C}{2} \text{ if } \#C \text{ is even} \end{cases} \tag{5}$$

and the consequent levels of utility are

$$U_i\left(\overline{G}, c_i\right) = V_i\left(\overline{G}(C)\right) - 1 \text{ if } i \in C\ , \tag{6}$$

and

$$U_i\left(\overline{G}, c_i\right) = V_i\left(\overline{G}(C)\right) \text{if } i \notin C. \tag{7}$$

Within this framework we are attempting to answer two question. Firstly, do any of the consumers have an incentive to participate in the committee? Expressed alternatively, will anyone be willing to supply the time required for membership? Given the structure of the model, this time is essentially a public good, so the model is also carries implications for the private supply of public goods. Secondly, if there are consumers willing to supply time, is there any equilibrium level of committee membership? Formally, these questions are answered by investigating the Nash equilibrium of the following membership game.

Membership Game
Each player i has the strategy set $\{0,1\}$, *i.e.* they choose non-membership of committee (strategy 0) or membership (strategy 1). The strategies chosen by the players then determine the quantity of G by (5), and the payoffs are given by either (6) or (7). The decision of each player is taken with the actions of others fixed.

Clearly, this game satisfies the standard conditions required for an equilibrium to exist. However, this still does not answer the question of whether anyone joins since non-membership for all may be a Nash equilibrium. Since committee membership is a public good, non-membership is a distinct possibility if free-riding is the dominant strategy.

3. Committee formation

The Membership Game defined above is a static one-shot game in which all participants make simultaneous decisions. However, rather than proceed directly to a statement of its Nash equilibrium, it is instead more fruitful to consider a dynamic process of entry onto the committee that provides a constructive means of arriving at this. The benefits of doing this are that it provides a very clear characterization of the structure of membership and an intuitive description of how such a membership might emerge. The dynamic process is, of course, intended to be no more than a fictitious motivating device that has analytical convenience. In this respect, the fact that the entry process has ordered moves does not imply that it must be solved by backward induction. We are ultimately interested in the Nash equilibrium to the static one-shot membership game and the entry process is simply a tool to lead us to this.

To derive the dynamic entry process, assume that entry is sequential and that at each point in the process the agent with the greatest, positive gain from membership joins. When no-one has any gain from joining, the process terminates.[v] It will be shown that at the termination point none of the existing members wish to leave and, by definition, no further non-members wish to become members. This is therefore a Nash equilibrium of the static game since no player has an incentive to change their choice of strategy.

The first step in the analysis of entry is to define the gain-from-membership function. To do this, let $\tilde{C}$ be the existing committee with decision $\overline{G}(\tilde{C})$. For some i, $i \notin C$, let $C = \tilde{C} \cup \{i\}$. The benefit-from-membership function for i is defined by

$$b_i(\tilde{C}) = V_i(\overline{G}(C)) - V_i(\overline{G}(\tilde{C})) - 1. \tag{8}$$

Defining $E = \{i : i \in M / C \text{ and } b_i(\tilde{C}) > 0\}$, the entry process can then be defined formally as follows.

Entry Process

For a given initial committee $\tilde{C}$:

(i) if $E = \phi$ then the process terminates;

(ii) if $E \neq \phi$, then *j* joins the committee, where *j* is defined by: $b_j(\tilde{C})$ is the maximal element of $\{b_i(\tilde{C})\}, i \in E$;

(iii) if there is no unique maximal element in (ii), then from the set of maximizers for a given $\tilde{C}$, choose the lowest *j* from the set of maximizers if this results in *j* < median $i, i \in M$, otherwise choose the highest *j*.

It is now possible to prove a series of results about this entry process, terminating in a characterization of the Nash equilibrium.

Lemma 1

Given an initial position with no committee, so $\tilde{C} = \phi$, if entry occurs it will be by player *m*.

The reasoning behind this result is that the benefit of membership to *i* depends on how far the existing choice is from the optimal choice of *i* and how much their membership of the committee moves the choice towards their optimum. When the first member joins the committee they become the median voter so the supply is equal to their optimum. Furthermore, a supply of zero is furthest from the optimal choice of consumer *m*. Hence both of these factors combine to give *m* the greatest benefit from membership.

There is an obvious and important corollary to this result.

Corollary 1

If the inequality

$$V_m(G_m^*) - 1 > 0, \tag{9}$$

is satisfied, then committee membership must be positive at the equilibrium.

This corollary follows from the observation that if the inequality is satisfied and no committee is formed, then player *m* will have an incentive to unilaterally form a committee and enforce their preferred choice, G_m^*, on the remainder of the population. Hence this contradicts the claim that having no committee members can be a Nash equilibrium. Note carefully that this corollary does not say that player m will be a member of the equilibrium committee – this has yet to be established. Condition (9) therefore acts as a sufficient condition for the formation of a committee. Note, however, that it does not imply that any public good is actually supplied since any number of the G_i* may be 0, and in equilibrium a committee may be formed which is composed solely of such players. Only if $G_i* > 0\ \forall i$ does (9) also guarantees that a positive quantity of the good is supplied.

Now assume that (9) is satisfied so that player m joins the committee. The second result determines which of the players will follow m in joining the committee.

Lemma 2
Assume player m joins the committee. If

$$b_1(\tilde{C}) \equiv V_1\left(\frac{G_1* + G_m*}{2}\right) - V_1(G_m*) - 1 > 0,$$

then agent 1 will be the next member other wise the process will terminate.

Lemma 2 shows how the committee begins to build-up with members from the opposite extremes entering in order to offset the existing views. The next results show how this process continues as further members join. In order to describe these results it is necessary to introduce the definitions of a balanced and an unbalanced committee.

Consider a committee that is composed of set C of members where C has the property that

$$C = \{1, \ldots k\} \cup \{m - \ell, \ldots, m\}.$$

If $k = \ell - 1$ then the committee is said to be *balanced*. That is, there is an equal number of members from the left extreme as there is from the right extreme. Conversely, if $k \neq \ell - 1$ then the committee is *unbalanced*. More particularly, it is *unbalanced to the left* if $k = \ell - 2$ and *unbalanced to the right* if $k = \ell$. As the results that are demonstrated below show, it is only necessary to consider positions in which one extreme has one person more than the other.

Given these preliminaries, two further results can be established.

Lemma 3 (balanced)
Assume that the committee is balanced with membership $C = \{1, \ldots k\} \cup \{m - \ell, \ldots, m\}$.

(i) If $b_{k+1}(\tilde{C}) > 0$ and $b_{k+1}(\tilde{C}) > b_{m-\ell-1}(\tilde{C})$ then $k+1$ is the next member,

(ii) If $b_{m-\ell-1}\left(\tilde{C}\right) > 0$ and $b_{m-\ell-1}\left(\tilde{C}\right) > b_{k+1}\left(\tilde{C}\right)$ then $m-\ell-1$ is the next member,

(iii) If $b_{k+1}\left(\tilde{C}\right) < 0$ and $b_{m-\ell-1}\left(\tilde{C}\right) < 0$ then the process terminates.

Again, the proof of this lemma just relies on the fact that the benefit function is increasing the further from the existing value of *G* is the optimum of *i*.

Lemma 4 (unbalanced)
Assume that the committee is unbalanced to the left with membership $\tilde{C} = \{1,\ldots k\} \cup \{m-\ell,\ldots,m\}$ where $k = \ell + 2$. Then if $b_{m-\ell-1}\left(\tilde{C}\right) > 0$, $m-\ell-1$ will be the next member. Otherwise no-one will join. The converse argument applies if the membership is unbalanced to the right.

These results allow the entire entry process to be described. First, player *m* will assess if they have a positive benefit from initially joining the committee. If the benefit is positive, then the committee will be established. Given that *m* forms the committee, the player with the greatest benefit from joining next is player 1. Again, they will join if their benefit is positive. Lemmas 3 and 4 can then be repeatedly applied. Given that 1 and *m* are on the committee, then Lemma 3 applies and the greatest gain[vi] from membership is obtained by either worker 2 (case i) or worker *m* - 1 (case ii). If either of these joins, then Lemma 4 applies. If 2 (*m* - 1) had joined, *m* - 1 (2) will have the greatest gain from membership. Given that they choose membership, Lemma 3 applies again. This process of entry will continue until there remains no non-member for whom the net benefit of membership is positive.

4. Equilibrium

To prove that the termination point of the entry process is a Nash equilibrium of the game, it is necessary to demonstrate that no existing member, who joined at an earlier point in the process, has an incentive to leave as a result of the subsequent entrants. In order to proceed with this analysis it is necessary to introduce some further notation and definitions.

The first task is to modify the concept of the *membership benefit function* which has been used in proving the previous results. Consider a committee with membership denoted by the set $\tilde{C}$ and choice of provision $\overline{G}\left(\tilde{C}\right)$. If individual *c* joins the committee the level of provision moves to $\overline{G}\left(\tilde{C} \cup c\right)$. Defining

$$s = \overline{G}\left(\tilde{C} \cup c\right) - \overline{G}\left(\tilde{C}\right),$$

the membership benefit function for c can be re-expressed as

$$b\left(\overline{G}\left(\tilde{C}\right),s;\gamma_c\right) = V_i\left(\overline{G}\left(\tilde{C} \cup c\right)\right) - V_i\left(\overline{G}\left(\tilde{C}\right)\right) - 1$$

$$= V\left(\overline{G}\left(\tilde{C} \cup c\right) - \gamma_c\right) - V_i\left(\overline{G}\left(\tilde{C}\right) - \gamma_c\right) - 1$$

$$= V\left(\overline{G}\left(\tilde{C}\right) + s - \gamma_c\right) - V_i\left(\overline{G}\left(\tilde{C}\right) - \gamma_c\right) - 1.$$

Hence $b\left(\overline{G}\left(\tilde{C}\right),s;\gamma_c\right)$ measures the benefit of membership for c when the choice of the committee before c takes up membership is $\overline{G}\left(\tilde{C}\right)$ and the membership of c shifts it by amount s. Clearly, $b\left(\overline{G}\left(\tilde{C}\right),s;\gamma_c\right)$ can be positive or negative. It is clearly negative when $\overline{G}\left(\tilde{C}\right) = \gamma_c$ since the cost of membership is paid that attains no improvement in situation. Furthermore, if $\overline{G}\left(\tilde{C}\right) < \gamma_c$ then $b\left(\overline{G}\left(\tilde{C}\right),s;\gamma_c\right)$ can only be positive when $s > 0$. It must also be increasing in s for all s such that $\overline{G}\left(\tilde{C}\right) + s \le \gamma_c$ [vii]. The converse statements can be made when $\overline{G}\left(\tilde{C}\right) > \gamma_c$.

Given this definition and discussion, Lemma 5 can be proved.

Lemma 5
The benefit function is *monotonic*. That is, $b\left(\overline{G}\left(\tilde{C}\right),s;\gamma_c\right)$ is decreasing in $\overline{G}\left(\tilde{C}\right)$ for $\overline{G}\left(\tilde{C}\right) < \gamma_c$ when $s > 0$ and increasing in $\overline{G}\left(\tilde{C}\right)$ for $\overline{G}\left(\tilde{C}\right) > \gamma_c$ when $s < 0$.

The interpretation of this result is that a given change in provision in the direction desired is of greater benefit the further the initial position is from the optimal position.

Let the mean level of optimal provision in the population be denoted by μ, so $\mu \equiv \sum_{i=1}^{m} \frac{G_i *}{m}$. The set of optimal provision levels is said to be *symmetric about the mean* whenever $G *_{m-i+1} - \mu = \mu - G *_i$ for all i with $G *_{m-i+1} - \mu > 0$. The uniform distribution and any discrete approximation to the normal are examples of distributions that satisfy this condition.

It is now possible to state the first result which proves that an equilibrium is reached if the entry process terminates at a balanced equilibrium. The restriction of the theorem to symmetric distributions make it sensible to talk in terms of a balanced equilibrium even when all players become members. (This is called *complete coverage*. If some are not members, *incomplete coverage* occurs). In this case, the committee is viewed as being partitioned into the set of those members with a preferred quantity above the mean and those with it below the mean. The two sets will contain equal numbers. In proving the results of this section, it is assumed that $V_m(G_m*)-1>0$, so that there will always be an incentive for individual m to form a committee. Corollary 1 has already noted that under this condition, if there is an equilibrium it must have positive membership.

Theorem 1
Assume that the entry process terminates in a balanced membership. If the distribution of optimal provision levels is symmetric about the mean, then the membership constitutes a Nash equilibrium.

Theorem 1 proves that when the entry process terminates in a balanced membership and optimal provision quantities are symmetrically distributed about the mean, the membership is a Nash equilibrium. Although this does not give a uniqueness result this is not especially important. What does matter is that the analysis has demonstrated that there is *at least one equilibrium* with an individually rational motive for joining the committee. It therefore shows that the free-riding issue in committee membership can be overcome when membership affects the committee's behaviour.

To treat the case of the entry process terminating in an unbalanced equilibrium, it is necessary to introduce a further definition. The utility function is said to be *symmetric* if

$$V_i(G_i*+g)=V_i(G_i*-g).$$

To see what this implies for the benefit function, consider a public good supply $\overline{G}(\tilde{C})$ and two workers with preference parameters γ_i and γ_j satisfying

$$\gamma_i=\mu+\rho,\ \gamma_j=\mu-\rho,\ \rho>0, \tag{10}$$

and

$$\overline{G}(\tilde{C})=\gamma_i-\sigma,\ \overline{G}(\tilde{C})=\gamma_j+\sigma,\ \sigma>0. \tag{11}$$

If the utility function is symmetric it then follows that the benefit function satisfies

$$b(\gamma_i-\sigma,s,\mu+\rho)=V_i(\gamma_i-\sigma+s)-V_i(\gamma_i-\sigma)-1$$

$$=V(\gamma_i-\sigma+s-\gamma_i)-V(\gamma_i-\sigma-\gamma_i)-1$$

$$=V(-\sigma+s)-V(-\sigma)-1$$

$$=V(\sigma-s)-V(\sigma)-1$$

$$=b(\gamma_j+\sigma,-s,\mu-\rho), \tag{12}$$

for all s. The interpretation of this condition is that the benefit of starting with a provision level σ below the ideal for i and shifting it s closer to γ_i is equal to starting with it σ above the ideal for j and moving it s down when γ_i and γ_j are an equal distance above and below the mean respectively.

The following lemma can now be proved.

Lemma 6
If optimal provision levels are symmetrically distributed about the mean and the utility function is symmetric, the entry process cannot terminate with an unbalanced membership.

Combining Lemma 6 and Theorem 1 gives the following result.

Theorem 2
Assume optimal provision levels are symmetrically distributed about the mean and the utility function is symmetric. If the entry process terminates with incomplete coverage, it must terminate at a balanced membership that is an equilibrium.

The final result of this section relates to the possibility of complete coverage.

Theorem 3
Assume optimal provision levels are symmetrically distributed about the mean. If the initial number of players, m, is odd the entry process cannot terminate with complete coverage.

5. Relaxing comparability

The assumption of comparability of utility used in the previous sections is a fairly strong one. Fortunately it is not essential to any of the results that have been derived. In fact, it can be seen that the only place that this was used was in the definition of (ii) of the entry process. Its role at that point was to allow selection of the individual with the highest gain as the next committee member.

Although the comparability assumption was critical in describing the entry process, it played no role in establishing that the limit of the process was an equilibrium. Reviewing the proof of theorem 1 it can be seen that the only the fact required was whether each individual would gain by changing their decision. No comparisons were made between individuals.

Consequently, an alternative logical process would be to make no comparability assumption, posit the structure of equilibrium membership directly, and then employ the same proof as for Theorem 1. The same is true of the paper’s other theorems. Comparability is therefore a helpful assumption in facilitating the construction of equilibrium but is unnecessary in the proof of equilibrium.

6 An example

The analysis has made a number of very clear predictions a bout the membership structure of committees. It is very interesting to test whether these are supported in practice. Although many committees are observed in action, the data necessary to investigate the theory is not usually available. What is needed is a committee where membership is voluntary, there are no restrictions on who, out of the eligible population, can join and in which opinions can be measured on a one-dimensional spectrum. While the first two of these are satisfied in many situations (recall the examples in the introduction), the need for a one-dimensional spectrum *and* information about where committee members lie on this spectrum makes usable data sets very limited.

One example that does meet these requirements is the attendance of peers at the UK's House of Lords. The House of Lords is the upper chamber of Parliament and has the role of scrutinizing legislation passed by the House of Commons. Its members are not elected but are made eligible through inheritance ("hereditary peers"), through reward (created peers can be either "life" or hereditary), to act as a "Law Lord" (the House of Lords is also the highest court in the UK) or through being one of the 26 Anglican bishops and archbishops ("Lords Spiritual") entitled to attend. In parliamentary year 1997 – 98 (the year covered by the data on attendance) there were approximately 1200 lords entitled to attend.[viii] The composition is described in Table 1.

Archbishops and bishops	26
Peers by succession	750
Hereditary peers of first creation	9
Life peers under the Appellate Jurisdiction Act 1876	29
Life peers under the Life Peerages Act 1958	480
Total	1294

Table 1: Composition of the House of Lords

The lords arrange themselves according to traditional party lines but with two exceptions. Firstly, the Lords Spiritual do not align with any political party. Secondly, lords can choose to be "Cross Bench" which is an explicit statement of non-alignment or they can choose simply to not declare any political attachment.[ix] It is this declaration of political affiliation that permits a comparison with the predictions of the model since political affiliation can be placed on a left-right spectrum. This permits an inspection of whether the membership of the committee – interpreted here as attendance at the Lords – fits with the predictions of the model.

The data that is used relates to attendance during parliamentary year 1997 - 98. During that year there was a total of 228 possible attendances. Attendance is recorded upon completion of a claim for attendance expenses so there is an incentive to reveal attendance. The figures given in Table 2 show the proportional attendance rate ((actual attendances)/228, averaged across group) of Lords broken down into political affiliations and type of peerage.

		CrA	CrL	CrH	H		Total
Conservative	n		141	4	297		441
	A		0.4768	0.2259	0.3533		0.3916
Lib. Dem.	n		28		23		51
	A		0.6612		0.5793		0.6243
Labour	n		96	1	18		115
	A		0.664	0.9605	0.6791		0.6689
Cross bench	n	25	83	4	195		282
	A	0.2193	0.3044	0	0.261		0.2659
No declaration	n		9	189			198
	A		0.1174	0.0115			0.0163
Spiritual	n					20	20
	A					0.1605	0.1605

Notes: CrA: created Law Lord; CrL: created Life peer; CrH: created hereditary peer: H: hereditary peer. n = number in category, A = average attendance (attendance/228).

Table 2: Average attendance rates by category

7 Conclusions

The paper has investigated the interaction between committee membership and the voting process. Its major finding has been that heterogeneity in the population will lead to a committee that has representatives from both extremes of the preference distribution. These will be evenly matched in number so that voting will result in a median outcome. The median preference will itself never see representation on the committee.

From a social perspective, it is not possible to say whether the choice made by the committee is efficient or not. Additional structure must be added before this question can be addressed. What can be said is that committee is socially inefficient in the quantity of time that it uses. Given that it reaches a median decision, it would be socially efficient to appoint the individual with median preferences as the sole decision maker. Instead, the formation of the committee leads essentially to the presence of two opposing camps whose purpose is simply to prevent the other gaining the upper hand. This is just socially wasteful. From this perspective, the committee is a poor form of decision making.

The paper can also be viewed as providing an alternative insight into the voluntary provision of a public good and how the free-rider problem can be mitigated through heterogeneity of the population. The time cost of involved in belonging to the committee provided a disincentive to supply the membership which is a public good. With a homogeneous population, free-riding would be the dominant strategy. No-one would join the committee and none of the good would ever be provided. In contrast, with heterogeneity in preferences, there would generally be a positive number of committee members in equilibrium. What underlies this is that membership of the committee allows a say on what the outcome will be. The time cost is the price of this input, so committee membership is just like buying a vote. A vote is in essence a private good since it can directly affect the welfare of the voter.

Although the model has been described in terms of a very particular example, it has much greater applicability. The issues that it addresses arise in any instance where public goods are privately provided and the free-rider problem is in evidence. The results of the paper show why the free-rider problem is not as extreme in practice as it is often predicted to be in theory. Heterogeneity gives a private incentive for supplying the public good. Furthermore, the results also provide a powerful argument for expecting that private provision of public goods will result in intermediate rather than extreme levels of supply. This provides theoretical support for the finding of a considerable body of experimental evidence.

Appendix

Proof of Lemma 1

Since $\tilde{C} = \phi$, $\overline{G}\left(\tilde{C}\right) = 0$. The first member will be the median voter so $\overline{G}(C) = G_i*$ when $C = \{i\}$. Hence the benefit for i from becoming the sole member of the committee is

$$b_1(0) \equiv V_i(G_i*) - V_i(0) - 1,$$

$$= V(0) - V(-\gamma_i) - 1,$$

which is obviously increasing in γ_i. ‖

Proof of Lemma 2

Given that m has joined the committee, $\tilde{C} = \{m\}$ and $\overline{G}\left(\tilde{C}\right) = G_m*$. If i joins, then $C = \{i, m\}$ and $\overline{G}(C) = \frac{G_i* + G_m*}{2}$. The benefit from membership is then

$$b_i\left(\tilde{C}\right) \equiv V_i\left(\frac{G_i* + G_m*}{2}\right) - V_i(G_m*) - 1,$$

$$= V\left(\frac{G_i* + G_m*}{2} - \gamma_i\right) - V(G_m* - \gamma_i) - 1,$$

$$= V\left(\frac{G_m*}{2} - \frac{\gamma_i}{2}\right) - V(G_m* - \gamma_i) - 1$$

which is decreasing in γ_i since V is strictly concave and V' is negative for $G > G_i* = \gamma_i$. ‖

Proof of Lemma 3

If the initial committee $\tilde{C}$ is balanced, G is determined by

$$\overline{G}\left(\tilde{C}\right) = \frac{G_k* + G_{m-\ell}*}{2}.$$

The next entrant must become the median voter so if i joins $\overline{G}(C) = G_i*$. Applying the arguments of Lemma 2, the benefit increases the further γ_i is from $\overline{G}\left(\tilde{C}\right)$. The next entrant must therefore be either $k+1$ or $m-\ell-1$. Computing these benefits then gives the inequality in the statement of the theorem. ‖

Proof of Lemma 4

The new entrant becomes the median voter. Applying the monotonicity argument to the benefit function then gives the result. ||

Proof of Theorem 1

Since the entry process terminates in a balanced equilibrium, the membership for the committee is given by $C = \{1,\ldots,k,m-\ell,\ldots,m\}$ with $\ell = k-1$. The proof will be undertaken for the case in which k is the last entrant. The argument for when $m-\ell$ is the last entrant is exactly the converse.

The first step is to show that since k has just entered, all members $i < k$ will wish to remain on the committee. This is clearly true since if they remain members the chosen quantity is $\frac{G^*_{m-\ell} + G^*_k}{2}$ and if they leave it becomes $G^*_{m-\ell}$. This was exactly the choice facing k when their membership decision was made. Since the benefit of k was positive with this choice, it is positive for all agents with a lower optimal quantity.

Now if it can be shown that $m-\ell$ does not wish to leave once k has joined the argument is complete since the converse of the reasoning above shows that $m-\ell+1,\ldots,m$ will not want to leave. To show that $m-\ell$ does not want to leave, consider the effect this would have. Whilst they remain a member, the choice is $\frac{G^*_{m-\ell} + G^*_k}{2}$. If they leave it becomes G^*_k. The question of whether they wish to leave is then equivalent to considering whether they would choose to join if the membership was given by $\{1,\ldots,k,m-\ell+1,\ldots,m\}$. They will join in these circumstances if $b\left(G^*_k, s^0; \gamma_{m-\ell}\right) > 0$. The proof is now completed by showing that this inequality is satisfied and hence that $m-\ell$ will not want to leave.

To do this consider the position when $m-\ell$ originally chose to join. The composition of the union was given by $\{1,\ldots,k-1,m-\ell+1,\ldots,m\}$ with reservation wage $\frac{G^*_{m-\ell+1} + G^*_{k-1}}{2}$. Since agent $m-\ell$ joined, it must be the case that $b\left(\frac{G^*_{m-\ell+1} + G^*_{k-1}}{2}, s^1; \gamma_{m-\ell}\right) > 0$.

By definition, $s^0 = \frac{G^*_{m-\ell} + G^*_k}{2} - G^*_k$ and $s^1 = G^*_{m-\ell} - \frac{G^*_{m-\ell+1} + G^*_{k-1}}{2}$. Hence $s^0 = s^1$ if $G^*_k - G^*_{k-1} = G^*_{m-\ell+1} - G^*_{m-\ell}$, an equality which is implied by the assumption that the distribution is symmetric about the mean. Furthermore, $\frac{G^*_{m-\ell+1} + G^*_{k-1}}{2} > G^*_k$. Combining these observations with the assumption that $b\left(\overline{G}(C), s; \gamma_k\right)$ is decreasing in $\overline{G}(C)$ for $\overline{G}(C) < G^*_k$ when $s > 0$, the fact that $b\left(\frac{G^*_{m-\ell+1} + G^*_{k-1}}{2}, s^1; \gamma_{m-\ell}\right) > 0$ then implies $b\left(G^*_k, s^0; \gamma_{m-\ell}\right) > 0$. Hence $m-\ell$ will not want to leave the union. This proves that the entry process terminates with an equilibrium committee. ||

Proof of Lemma 5

Assume that the entry process terminates in a membership that is unbalanced to the right so that $C = (1,\ldots,k,m-\ell,\ldots,m)$ with $\ell = k$ and $\overline{G}(C) = G*_{m-\ell}$. Since agent $m-\ell$ must have entered when the membership was $\tilde{C} = \{1,\ldots,k,m-\ell+1,\ldots,m\}$, this shows that

$$b\left(\frac{G*_k + G*_{m-\ell+1}}{2}, G*_{m-\ell} - \frac{G*_k + G*_{m-\ell+1}}{2}; \gamma_{m-\ell}\right) > 0. \tag{28}$$

(28) can be written in the form

$$b\left(G*_{m-\ell} - \sigma^0, s^0; \mu + \rho^0\right) > 0, \tag{29}$$

where $\sigma^0 = G*_{m-\ell} - \frac{G*_k + G*_{m-\ell+1}}{2} = G*_{m-\ell} - \mu$, $s^0 = G*_{m-\ell} - \mu$ and $\rho^0 = \gamma_{m-\ell} - \mu$.

Now consider the entry decision of agent k+1 when the committee has membership C. Entry will be beneficial for them if

$$b\left(G*_{m-\ell}, \frac{G*_k + G*_{m-\ell}}{2} - G*_{m-\ell}; \gamma_k\right) > 0, \tag{30}$$

or

$$b\left(G*_k + \sigma^1, -s^1; \mu - \rho^1\right) > 0, \tag{31}$$

where $\sigma^1 = G*_{m-\ell} - G*_k$, $s^1 = G*_{m-\ell} - \mu$ and $\rho^1 = \gamma_k - \mu$.

Since $\rho^1 = \rho^0$, $s^1 = s^0$ and $\sigma^1 = 2\sigma^0$, (29), symmetry of the benefit function and the fact that $b\left(\overline{G}(C), s, \gamma_k\right)$ is increasing in $\overline{G}(C)$ when $\overline{G}(J) > G*_k$ imply that

$$b\left(G*_k + 2\sigma^0, -s^0; \mu - \rho^0\right) > 0. \tag{32}$$

This establishes that if agent $m-\ell$ joins the committee, so will k+1. The entry process cannot then terminate with a membership that is unbalanced to the right.

An identical argument applies if it is assumed that the entry process terminates with a membership that is unbalanced to the left. Therefore under the assumptions of the lemma, the entry process can only terminate with a balanced membership. ||

Proof of Theorem 2

Lemma 6 shows that under these conditions the entry process must terminate at a balanced equilibrium. Theorem 1 then guarantees that this is an equilibrium. ||

Proof of Theorem 3

Since the agent with the mean optimal provision level can leave the committee without affecting its choice, complete coverage cannot be an equilibrium. ||

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[i] Although, as will be seen, the model can be interpreted as the cost being shared between club members in some pre-determined way.

[ii] As noted in footnote 2, although we treat the good as costless this statement of preferences is consistent with the cost of a chosen quantity, G, being divided between the m members of the population according to some fixed sharing rule. The actual utility given in the main text is then a product of the strength of preference for the good and the share of cost that must be borne.

[iii] It should be stressed that comparability is only needed for the construction of equilibrium using sequential entry. None of theorems on the structure of equilibrium are dependent upon it. See section 4.

[iv] Note how we use this exactly. But as will become obvious all that is really needed is for the choice to lie somewhere between the upper and lower limits when number in C is even.

[v] Notice that we do not allow those who have joined to reconsider their decision until the process terminates. There is a parallel here with the analysis of tatonnement without recontracting.

[vi]

[vii] Of course, s is determined endogenously by $\tilde{C}$ and γ_C. But it helps to think of it as exogenous for applying the argument.

[vii] The number changes continuously through deaths, creations and leaves of absence. More is said about this later.

[vii] Here "not declare" is used literally: no information is given by the Lord about political affiliation. These are the "blanks" given below.

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