Coulomb Drag, Mesoscopic Physics, and Electron-Electron Interaction.

Submitted by Adam S. Price to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Physics
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A. S. Price
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Abstract

The first part of this thesis deals with the study of mesoscopic fluctuations of the Coulomb drag resistance in double-layer GaAs/AlGaAs heterostructures, both in weak magnetic fields and strong magnetic fields. In the second part, measurements are made in a monolayer graphene structure, specifically of the quantum lifetime, and the mesoscopic resistance fluctuations at quantising magnetic fields.

In weak fields, we perform the first measurements of the fluctuations of the drag as a function of changing concentration and magnetic field. The amplitude of the fluctuations was seen to exceed the size of the average drag at low temperatures, resulting in random but reproducible changes in the sign of the drag with varying concentration and magnetic field. The variance and correlation magnetic field were studied as a function of temperature. Comparison was made to existing theories for drag fluctuations in the diffusive regime of the drag (where the electron mean free path is much smaller than the separation between layers). We observe a large enhancement of the magnitude of the fluctuations of four orders of magnitude compared to the theoretical expectation, as well as a different temperature dependence. Our results prompted further theoretical studies, extending the understanding of drag fluctuations into the ballistic regime, where the local properties of the layers become important. We compare our results to this theory and find good agreement.

We extend these measurements to the regime of strong magnetic fields where transport involves composite-fermion quasiparticles. We study the temperature dependence of the amplitude and correlation magnetic field of the fluctuations, and analyse our results using existing theoretical models for drag fluctuations in the diffusive regime of drag.

Much controversy exists over the dominant scattering mechanisms in graphene. We use measurements of the concentration and temperature dependence of the quantum lifetime and transport time in graphene to identify the dominant electron scattering mechanisms. We then perform measurements of resistance fluctuations in the integer quantum Hall effect regime of magnetic fields, and show our results to be well described by existing models developed for conventional 2D systems. Using these models we make estimates for the properties of the disorder in graphene.
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4.1 Theoretical prediction for the $n$-dependence of $\tau_p$ for different sources of scattering in conventional 2D systems [83]. For the case of remote impurity scattering at high concentrations, $a$ is the quantum well width of the 2DEG. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 112

4.2 Expected ratio of $\tau_p$ to $\tau_q$ for different sources of scattering in conventional 2D systems [83]. For the case of background impurities, $q_{TF}$ is the Thomas-Fermi screening wavenumber, and $C_2$ is a coefficient that depends on the quantum well width, $q_{TF}$ and $k_F$. (The ratio $\tau_p/\tau_q$ increases linearly with increasing $n$ for the case of background impurities [83].) For the case of remote impurity scattering, $z_i$ is the distance of the impurities from the 2DEG. For the case of roughness scattering, $\Lambda$ is the correlation length of the fluctuations in the height of the SiO$_2$ interface. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 113
Symbols and Abbreviations Used

ACF : autocorrelation function
CF : composite fermion
$D$ : diffusion coefficient
DOS : density of states
e-e : electron-electron
$E_{Th}$ : Thoules energy
$g$ : dimensionless conductance, $\sigma/(e^2/h)$
$g(E)$ : The density of states
$g_s$ : The spin degeneracy, taking values of 1 or 2
$g_v$ : The valley degeneracy, taking values of 1 or 2
$l$ : elastic mean free path
$L$ : distance between voltage probes; equivalently, the sample size
LL : Landau level
$L_\varphi$ : coherence length
$L_T$ : thermal length
$\mu$ : mobility
$n$ : carrier concentration
$\nu$ : Landau-level filling factor
PSD : power spectral density
$\rho_D$ : drag resistivity
SdH : Shubnikov-de Haas
$T$ : temperature
$\tau$ : momentum relaxation time.
$\tau_q$ : quantum lifetime.
$\tau_\varphi$ : coherence time
UCF : universal conductance fluctuations
$\omega_c$ : cyclotron frequency
$W$ : sample width
List of publications

Publications


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